Allowable Hull Loading due to Fender Contact

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by

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Abstract

Marine structures are frequently equipped with rubber fender systems, which absorb the berthing energy in order to protect both the marine structure and the berthing vessel. These fender systems absorb the kinetic berthing energy by elastic deflection and the associated reaction fender force introduces a berthing impact load acting on the vessel's side hull.

In guidelines and rules on ship design, recommendations regarding the structural resistance due to external fenders are not present. On the other hand, special requirements state minimal strengthening for tug resistance, which results in marked areas on a vessel's side hull at which tug contact is allowed. Also for ships equipped with integrated steel fenders in their side hull, also known as beltings, minimal strengthening is required. Since the use of fender systems in ports is common, and all ships require to berth in a port the maximum hull loading due to fender contact is an important factor to take into account from the vessel's perspective.

PIANC WG33 published design recommendations for the maximum allowable hull pressure in kN/m^2 for different types of vessels. The size of the fender contact area is in practice determined by dividing the design reaction force by the maximum allowable hull pressure. However, the hull pressures in this recommendation are based on the pressure on the keel of a fully laden vessel. Based on this background information, not all values are reliable, since some pressures correspond to a draft of 70 meters. Besides that, the pressure formulation does not contain information on the specific geometry of the contact area, i.e. height and width.

This thesis systematically analyses the strength of the vessels' parallel side hull for different failure modes, e.g. yielding in the stiffeners due to excessive bending- or shear stresses. The structural geometry of various vessel types is represented by various grillages. Two different pressure distributions were considered: a soft contact area, and a rigid contact area, to cover the most extreme behaviour of a fender panel.

The results of this study show that the allowable load is largely influenced by the geometry of the contact area. The fender panels designed using the PIANC WG33 fender design are not always optimal. Especially for fender panels having large widths, the current guidelines seem to be too optimistic. Consequently, it is necessary to define a maximum width to what extent the current guidelines are allowable. This study shows that a specific allowable force in kN for a specific geometry is preferred over the current guidelines.

In this study, a general formulation for the maximum allowable hull loading is proposed. This formulation requires the specific structural lay-out of the vessel that encounters the berthing facility. If the specific ship's particulars are not known, recommendations are provided for different vessel types. These recommendations consist of a new acceptance criterion for each vessel type, which can be used to optimise the geometry of fender panels.

The findings of this study can be used in the design of fender systems and the new design criterion has been submitted to the members of PIANC WG211.

Preface

The scope of this master thesis is initiated by the Port of Rotterdam Authority, as part of a working group of PIANC, the Permanent International Association of Navigation Congresses. The guidelines on design of fender systems are revised, and one specific section of these guidelines discusses the topic of maximum loading on a ship's side hull. In order to revise this section of the guidelines, the topic was presented to the department Ship and Offshore Structures of the Faculty 3mE, Delft University of Technology.

During my thesis, the COVID-19 pandemic developed to a stage that required a total lock-down in the Netherlands. While the first meetings with Erik and Carey were possible in real life, after one month the meetings changed to a digital format. In the early stage of this graduation project, Erik organised physical excursions in port that have been extremely helpful to understand the basic principles of the fenders systems. Furthermore, these informative visits to the port have been extremely helpful to become more enthusiastic for this project. Therefore, I want to thank Erik Broos, who gave me an introduction to civil engineering and port development at the Port of Rotterdam. Furthermore, Erik has been very helpful in defining the problem statements in the intermediate phases due to his practical knowledge on this topic.

Secondly, I want to thank my TU Delft supervisor, Carey Walters, for the help with the theoretical problems. During the meetings, the guidance towards simplifications and more basic mechanical principles have been helpful to understand the root of the problems that I encountered. Furthermore, the organised meetings with other students in similar stages of their graduation have been helpful to enhance the progress of my own project. Also, a broader sight of what other students worked on was interesting to broaden my horizon during the lockdown.

In the final stage of the thesis, the help of Alfred Roubos resulted in more clear guidance on how to present the results. A combination of the practical knowledge on fenders, as well as his academic background, have been helpful in describing the results more clearly.

Furthermore, family and friends should be included in this preface. First of all, I want to thank my girlfriend for her emotional support over the past year. I want to thank my family for the distractions during small vacations in our own country in these COVID times. The friends of Marine Technology were extremely helpful during my whole study with many coffees the past years, and the digital alternative during my thesis was as good as it could be. Also, the boys from HYSBAK have been helpful with discussions on all kinds of topics outside of my expertise, and of course non-academic distractions. With special thanks to my roommates, who have been very encouraging as having encountered similar challenges (or even bigger challenges) during their own thesis. My fellow Froude board members were extremely helpful with a hard deadline for my actual graduation.

Hopefully, this thesis contributes to both the academic, as well as the industrial parties showing interest in this topic.

T.C.R. IJzerman
Rotterdam, September 2021

Contents

List of Tables

List of Figures

Nomenclature

Abbreviations

DWT	Dead Weight Tonnage
FEA	Finite Elements Analysis
FEM	Finite Elements Method
PIANC	Permanent International Association of Navigation Congresses
SPC	Super Cone (Fender)

Greek Symbols

β	Plate slenderness coefficient	[-]
η	Location of patch centre in z-direction	[m]
ν	Poison's ratio	[-]
σ_Y	Yield stress	[MPa]
σ_{xx},σ_{yy}, and σ_{zz}	Principle stresses	[MPa]
σ_{xy},σ_{yz}, and σ_{xz}	Shear stresses	[MPa]
τ	Shear stress	[MPa]
ξ	Location of patch centre in x direction	[m]
ξ	Stiffener capacity coefficient	$\left[\frac{1}{N}\right]$

Roman Symbols

$\bar{z}'$	Distance from the neutral axis to the centroid of A'	[mm]
ℓ_{bdg}	Bending span of longitudinals	[m]
A	Cross sectional area	[m^2]
a	Plate width (= $s_{webframe}$)	[m]
A'	Cross sectional area below the plane where τ is determined	[mm^2]
A_p	Cross sectional area of the plate	[m^2]
A_{stiff}	Cross sectional area of the stiffener	[m^2]
b	Plate height (= $s_{stiffener}$)	[m]
B_{vessel}	Cross sectional area of the plate	[m^2]
d	Distance of cross sectional centre to neutral axis	[m]
f_{pl}	Bending moment factor	[-]
h	Height of cross section	[m]
I	Moment of inertia	[m^4]

P_{fl}	Local design force for tug contact	[kN]
q	Intensity of lateral load	[$\frac{N}{m^2}$]
S	Section modulus	[m^3]
s_{deck}	Spacing between the decks	[m]
S_{pl}	Plastic section modulus	[cm^3]
$s_{stiffener}$	Stiffener spacing	[m]
$s_{webframe}$	Transverse frame spacing	[m]
t	Thickness of the cross sectional area	[mm]
t_p	Plate thickness	[mm]
t_{s-gr}	Minimum gross offered thickness of the web plating	[mm]
T_{sc}	Scantling draught	[m]
u	Patch width (= w_{fender})	[m]
V	Internal shear force	[N]
v	Patch height (= h_{fender})	[m]
y_{stiff}	Neutral height of stiffener	[m]
z'	Distance from the neutral axis	[mm]
E	Young's modulus	[GPa]

1 Introduction

1.1. Research Background

An important aspect in the operational lifetime of a vessel is the large number of berthing operations that are required to fulfil the aim of transporting goods. In recent decades, the sizes of vessels entering ports have been growing, and the associated berthing energy has grown accordingly. A rubber fender system is a commonly used system in ports to absorb this berthing energy in order to prevent damage to both the quay wall and the vessel. In figure 1.1, a quay wall with several fenders is given in order to indicate the size and the number of fenders on a typical quay wall.

Figure 1.1: Example of fenders on a quay wall

Widely used guidelines for the design of fender systems were published by PIANC WG33 in 'Guidelines for the Design of Fender Systems' [28]. The general principles of fender systems are based on berthing energy that is defined as the kinetic energy of the vessel before impact. Rubber fenders, which are relatively soft, absorb a part of this energy by elastic deflection. The maximum deflection of the rubber fender system multiplied by the maximum reaction force and an efficiency factor equals the kinetic energy of the vessel that can be absorbed. One of the assumptions in the guidelines is that the berthing energy of a vessel is fully absorbed by the fender system. Furthermore, the vessel is assumed as a rigid body in the guidelines.

Besides the energy absorption capacity of a fender, a well-designed fender should never exert forces that damage a ship. Therefore, the strength of a ship side structure should also be taken into account. This thesis primarily focuses on this requirement. The resulting berthing loads may not exceed limits based on the internal mechanics of the vessel. Since the aim is to keep the ship intact and undeformed after a visit to a port, only the elastic region is considered, and the yield stress is used as the maximum limit. The mechanics of a vessel are a subject that is not yet properly included in the fender design. More specifically, this research focuses on the maximum allowable loading on the vessels' side hull during the berthing operation, where the influence of the geometry of the contact area between fenders and ship is of special interest.

1.2. Problem Statement

The allowable hull loading is an important design requirement for fender systems, as this may not be exceeded in order to prevent hull failure or large permanent deformations. The current design guidelines [28] state that it is assumed that the maximum allowable hull pressure equals the pressure at the keel of a fully laden ship, while the exact methodology is not given. Furthermore, the bottom structure of a vessel is usually sturdier than the deck or side due to hydrostatic and hydrodynamic loads [19], which may result in overestimations of the allowable loading. Due to the increase in vessel sizes and berthing energy, an improvement of this assumption is desirable in the form of an analytical expression that is applicable for a broad range of ship types and dimensions.

Besides the knowledge gap in the design guidelines for fenders systems, regulations regarding shipbuilding do not mention loading on a vessel due to berthing operations. External forces on a hull due to ice loads are mentioned; however, only vessels that sail in these specific conditions need to comply with these rules while every ship interacts with fenders systems. Furthermore, regulations regarding minimum strengthening for tug contact result in specified areas on the hull of a vessel that does not necessarily correspond with locations where fenders are supposed to make contact.

1.3. Objective and Research Question

As stated in the previous section, there is a desire for a simplified analytical criterion for fender impact that can be used in renewed guidelines of the Permanent International Association of Navigation Congresses, PIANC [28]. This expression needs to be widely applicable in ports all over the world and with all kinds of configurations, with respect to fender types and vessel types. Eventually, a parametric study will be the result in which the geometry of the contact area of the fender, and the vessel type are leading parameters. The main question of this thesis yields:

Research question:
What are the maximum allowable fender loads acting on a ship's side structure?
The research required to answer this question must take into account the ability of both the ship's structure and the fender structure to deform elastically. Moreover, the effect of contact location and contact area should be part of this research.

In this research, different loading locations are expected to result in different maximum hull loadings. The location where the maximum is lowest is crucial. The hypothesis is that the maximum allowable loading will be larger for larger contact areas. Eventually, it could be possible to find an optimum contact area for which larger areas will not be significantly more effective. In order to find an answer to this research question, different parts of the system are analysed separately, and the following sub-questions are set up:

Sub-questions:

1. *Which size-dependent parameters of the structural layout of a ship's parallel side hull influence the structural integrity when loading is applied perpendicular to it, and how can the differences in geometry between different types of vessels be taken into consideration?*
2. *How can the pressure distribution acting on a ship's side structure be described during fender contact?*
3. *What types of failure modes within the ship's side structure are likely to occur during fender contact?*
4. *What is the influence of the fender contact area geometry on the maximum allowable loads acting on the ship's side structure?*

1.4. Scope and Methodology

This research focuses on a one-directional loading, i.e. lateral loading, due to fender contact. Different grillages, combinations of plates and stiffeners, will be used to represent the parallel side hull for a range of vessels. The grillage is reduced to a maximum length of four web frames and a height that is bounded by strength decks or primary longitudinal stiffeners. These boundaries are assumed to be rigid and restrained in both rotation and deflection since they are found to be much more robust than the other components.

By using a set of different geometries for the fender panel, the hypothesis is that different failure mechanisms can occur, and these will be analysed separately in the analytical part of the study. In order to verify the results, different grillages will be simulated using a Finite Element Analysis (FEA).

In order to resolve the main question and the different sub-question, the methodology is schematically represented in figure 1.2. In this research, firstly one grillage, representing a specific vessel category, will be analysed based on a specific loading location. Two different pressure distributions will be used to represent the behaviour of different fender contact areas. A rigid and a soft fender contact area are represented by local loads on the edges, and a uniform pressure, respectively. For both distributions, an overview of the maximum loading for the specific vessel type on a specific loading location is found. The area, i.e. length and width of the contact area are the main variable parameters in this study.

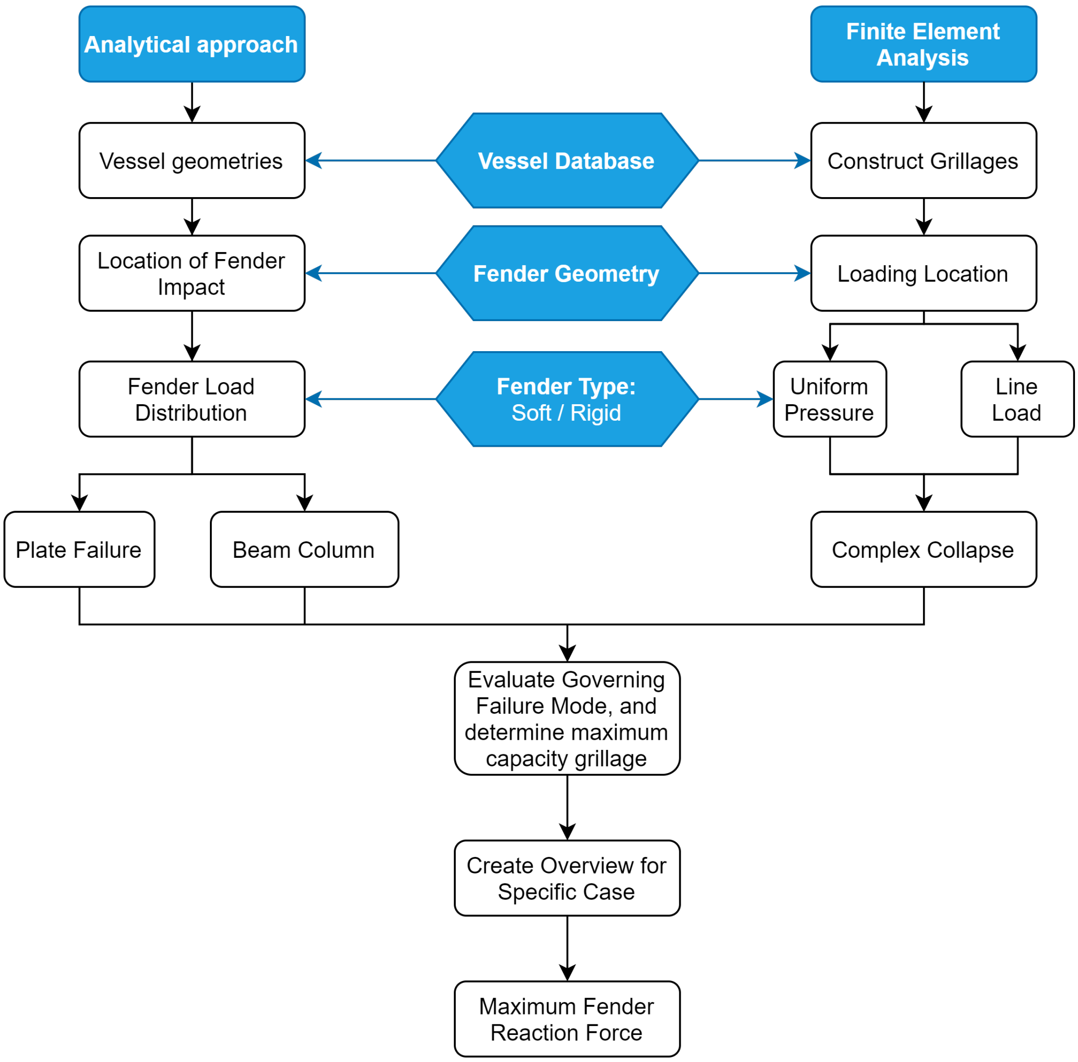


Figure 1.2: Schematic representation of the methodology

1.5. Outline of the Research

This report consists of seven chapters, including this introduction as the first chapter. Chapter 2 consists of the technical background. This chapter gives a broader explanation regarding the fenders systems and the vessel's layout. The different parameters that are taken into account will be introduced in combination with the according basic mechanical principles. The third chapter, the methodology, discusses the possible locations, and the pressure distributions that are investigated. Furthermore, the different failure modes are introduced that are expected for specific values for the parameters. Thereafter the failure modes are discussed per chapter in chapter 4 and 5. Eventually, the failure modes are combined in chapter 6, followed by the conclusion and recommendations in chapter 7.

2

Technical Background

This chapter presents the technical background of the thesis. Firstly, a short introduction to fender systems and the working principle is discussed. Section 2.2 discusses previous research that was done based on fenders. In the third section, attention is paid to the guidelines on fenders. The vessels' structural mechanics and the most important differences between vessel types are described in section 2.4. The contact between and the loading of the fender on the hull of the vessel is discussed in the fifth section. In section 2.6, a categorisation of different vessel types is introduced. The regulations and guidelines regarding this kind of loading for vessels will be discussed in section 2.7.

2.1. Fender Systems

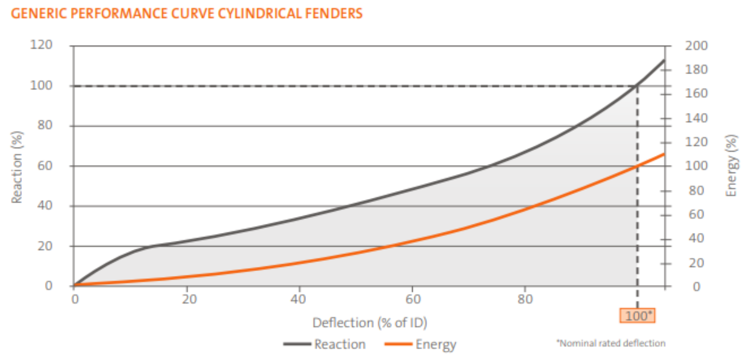
Different types of fenders can be distinguished with respect to size, material and working principle. In this thesis, the research focuses on rubber fenders, which can roughly be divided into two main categories; Fenders with a linear performance curve, for example a cylindrical fender shown in figure 2.1b, or fenders with a nonlinear performance curve, for example super cone fenders or V fenders shown in figure 2.2b, and 2.3b, respectively. Within these categories, there are different applications and geometries of the fenders themselves; however, the performance curves have a shape comparable to respectively figure 2.1a, 2.2a, and 2.3a. These performance curves are published by the manufacturers and show the relation of the reaction force as a function of the deflection of the fender. Secondly, the absorbed energy of the fender system can be found as the integral of this reaction force with respect to displacement. The deflection is defined as a percentage of the inner diameter for the cylindrical fenders and as a percentage of the length of the cone for the super cone (SPC), and V-fenders.

The maximum reaction forces of the specific fender types are also given by manufacturers. Table 2.1 gives a range of sizes and maximum reaction forces for the corresponding sizes, based on the product information of the manufacturer 'ShibataFenderTeam' [35]. These values are valid for grade 1 rubber.

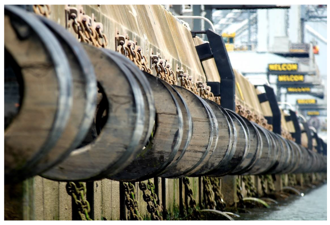
Table 2.1: Fender properties [35]

	Cylindrical fenders *	SPC fenders	V fenders *
Outer x Inner Diameter [mm]	100x50 - 2700x1300	-	-
Offset (quaywall - ship) [mm]	-	300 - 2000	250 - 1000
Height of contact area [mm] **	-	262 - 1750	200 - 800
Reaction force [kN]	43 - 1486	57 - 2544	135 - 542

* Performance of a single unit with a length of 1000 mm.
** The contact area of an SPC fender is defined by the panel in front of the rubber, in this table the diameter of the rubber directly behind the panel is given.

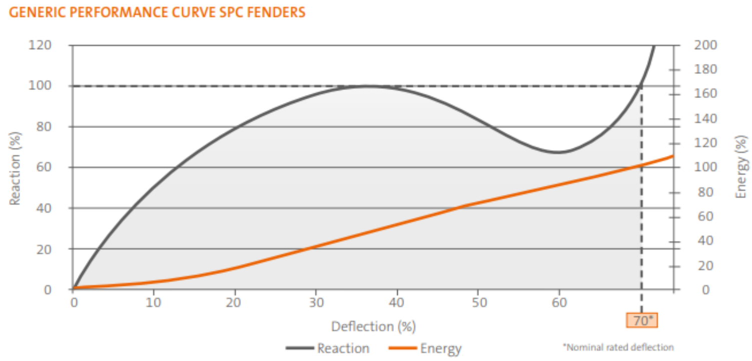


(a) Typical performance curve of cylindrical fenders [34]

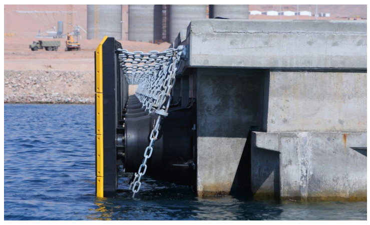


(b) Overview of cylindrical fenders on a quay wall [44]

Figure 2.1: Cylindrical fenders

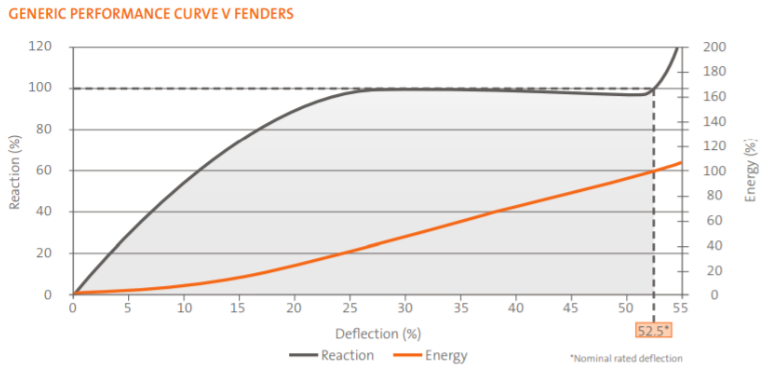


(a) Typical performance curve of Super Cone (SPC) fenders [36]



(b) Overview of Super Cone (SPC) fenders on a quay wall [35]

Figure 2.2: Super Cone (SPC) fenders



(a) Typical performance curve of V fenders [37]

(b) Overview of V fenders on a quay wall [38]

Figure 2.3: V fenders

2.2. Previous Fender Based Research

In the design procedure of fender systems, the berthing loads are determined via the berthing energy. A simplified formulation of the berthing energy is given in equation (2.1), [28].

$$E_{kin} = \frac{1}{2} \cdot M \cdot v^2 \cdot C_e \cdot C_m \cdot C_s \cdot C_c \quad (2.1)$$

In this equation, M represents the mass of the vessel, and v is equal to the velocity perpendicular to the berth. The factors C_e, C_m, C_s, and C_c represent the eccentricity factor which takes the berthing angle into account, the virtual mass factor, the softness factor and the configuration or cushion factor, respectively.

Prior studies have been performed regarding the design of fender systems [21], [27], [32]. However, these studies focused on the determination of the berthing energy by investigating the reliability of the loading factors [21], [27], or the berthing velocity [32]. Also, physical tests have been done to determine the berthing loads by measuring the displacement in the berthing facility or fender system [24], [26]. Other studies that investigated the fender's behaviour [3], [39] resulted in the energy absorption capacity of a parallel fender combination and the effect of the cross-sectional shape of V-type fenders, respectively. All just mentioned studies were primarily focused on energy absorption, where the vessel is represented as a rigid body with a certain mass. Since the strength of the vessel's hull structure is not yet implemented in the fender design, this thesis will be a valuable addition to the fender design process. By evaluating the maximum tolerable reaction force of the vessel a comparison can be made with the fenders with respect to the maximum attainable berthing energy.

The Port of Rotterdam Authority has also commissioned research into fenders and the concerning hull pressures several times, carried out by Lloyd's Register of Shipping in the 1990s [11], [12] and the Dutch research organisation, TNO, from 2015 to 2018 [43], [30] and [44]. Lloyd's investigated the ability of Panamax container vessels to resist quayside loads from two different types of fenders. The Port of Rotterdam had commissioned TNO to study the maximum berthing velocity on different kinds of vessels and quay walls. The latter resulted in a more systematic guideline covering multiple vessel characteristics and different diameters of the cylindrical fenders. Within this research, the maximum allowable hull pressure has also been a subject of discussion. The maximum allowable stress was assumed to be the minimum specified yield stress of the material. In all cases used in this study, the flange of the longitudinals was the location at which the yield stress occurs first. An interesting issue that is mentioned in this report is a future investigation into vertically oriented fenders instead of the horizontal orientation since this would increase the number of effective stiffeners. Furthermore, the number of ships considered in the investigation must be increased in order to use the specifics for statistical analyses, following the report.

Based on previous research regarding the design of cylindrical fenders [4], it was concluded that the resulting hull pressure exceeded over twice the stated value in the table of the PIANC WG33 guidelines 2.2for gas carriers, without damaging the ship. The mismatch was also found for the large container vessels, and therefore, broader research on hull pressures is desired.

The first recommendation of TNO will be covered in this thesis, and the orientation can be generalised even more in the case of SPC fenders to a rectangular geometry. This rectangular geometry represents the panel in front of the rubber. For the SPC fenders, the material properties of the rubber cone at which the panel is attached are governing for the fender performance curve. However, for the vessel's maximum allowable loading, the contact area is assumed to be governing, and only the panel is further looked into.

2.3. Guidelines on Fender Design

As mentioned in the introduction, PIANC has been working on a review of the guidelines for the design of fenders systems. One specific subject that is under revision relates to the associated hull pressures during the berthing impact. There is reason to believe that the currently used table providing the maximum allowable hull pressures, see table 2.2, is not determined correctly since the assumption is made that this pressure is equal to the water pressure at the bottom plates of the vessel in fully laden condition. First of all, the comparison between the structural layout of the bottom and the side shell of a vessel is not necessarily admissible since they can differ significantly due to different design methods and guidelines. Especially since the bottom structure of the vessel is sturdier than the deck or side hull due to hydrostatic and hydrodynamic loads [19]. Secondly, the local pressure on neither the plating in the bottom or in the side shell determines the structural design of the vessel. The structural design of a ships' hull is mainly dependent on the bending moment of the vessel, consisting of the still water bending moment and the wave bending moment. This global hull girder bending results in a larger distance to the keel than the side, and therefore higher moments. Which explains a sturdier bottom structure due to higher occurring stresses. Thirdly, the motivation for the values in the table is questionable since a pressure of 700 kN/m^2 for small general cargo vessels corresponds to a draught of seventy metres, which is not reasonable.

Table 2.2: Current hull pressure guide of PIANC WG33, 2002 [28]

Type of vessel	**Hull Pressure [kN/m^2]**
Container vessel 1st and 2nd generation	<400
3rd generation (Panamax)	<300
4th generation	<250
5th and 6th generation (Superpost Panamax)	<200
General cargo vessels	
≤ 20.000 DWT	400-700
> 20.000 DWT	<400
Oil tankers	
≤ 60.000 DWT	<300
> 60.000 DWT	<350
VLCC	150-200
Gas carriers (LNG/LPG)	<200
Bulk carriers	<200
SWATH	}
RO-RO vessels	} These vessels are usually belted
Passenger vessels	}

Another guideline on fender design and testing is published by the Japanese Coastal Development Institute of Technology [7]. In this guideline, more attention is paid to the fender panels, and cylindrical fenders, such as figure 2.1b, are not taken into account. However, different rubber geometries behind the panel are compared. This guideline is more explicit in the testing procedure of the rubber fenders and the quality of the rubber materials. This guideline dedicated more text, with respect to the allowable hull pressure. However, the table used is the same as table 2.2 from the PIANC guideline, with three additional types of vessels in the form of 'Recent examples in Japan'. An interesting addition in this section of the guideline is the distinction between five different load cases for a fender panel, which are also shown in figure 2.4.

1. Point load: Corner contact at the initial stage of berthing, etc.
2. Line load: Angular berthing, contact of hull flare, etc.
3. Two line loads: Contact of vessel with belted hull
4. Central line load: Contact with the centre of two or more vertical fenders
5. Distributed load: Full contact or partial contact with fender panel

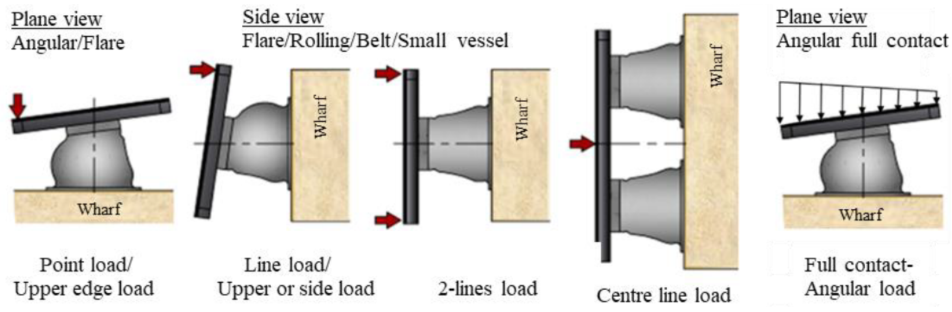


Figure 2.4: Load cases for fender panels [7]

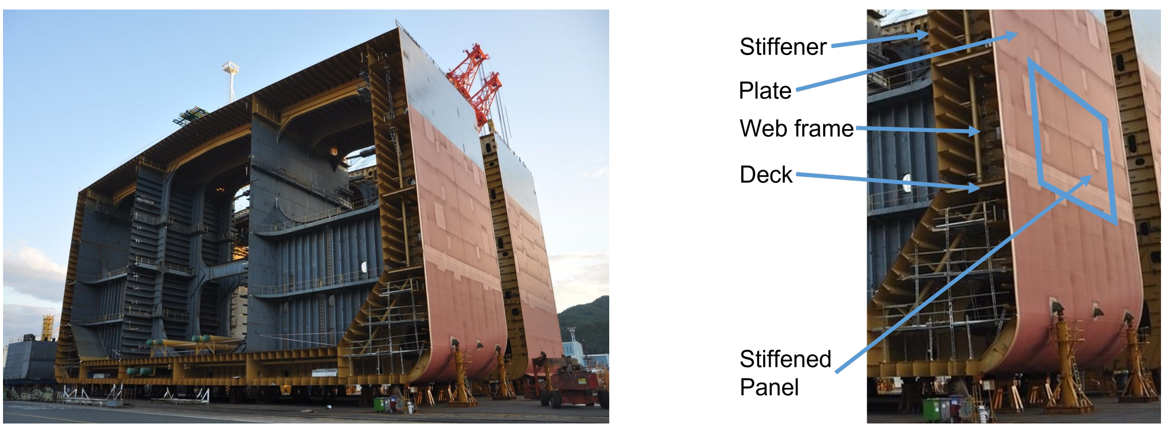
The vessel is also assumed as a rigid body in this guideline, and the rotation or displacement on the vessel is assumed as a reason for a specific load case. In this thesis, different load cases will be investigated as well. However, the material behaviour of both the vessel's structure and the fender panel will be investigated with respect to a resulting load case. This study only takes parallel berthing operations into account, and the load cases of this guideline are not relevant in that aspect. The load cases will be discussed in subsection 3.2.

2.4. Basic Mechanical Principles of a Vessel

Since this research focuses on the operation of berthing, only the parallel side panel of the vessel is taken into account, which is relatively simple and represented by figure 2.6. A grillage is used to represent the various vessel categories. The layout and basic mechanical principles of a grillage are discussed in this section. In section 2.6, the differences in grillages for different vessel types will be discussed.

An oceangoing vessel consists of a large amount of steel which is used in plating and stiffeners giving strength to the hull in the form of stiffened panels. An example of a very large crude carrier (VLCC) under construction is given in figure 2.5 in order to give an insight into the structural layout. Every vessel has its own specific layout, and even within a vessel, the layout of stiffened panels can differ.

The loading and response of a stiffened panel can be described in multiple steps. Firstly the relative deflection of the plate in between two stiffeners results in a transfer of the loading to the stiffeners. Secondly, the stiffeners function as a beam clamped between the transverse frames. The transverse frames are clamped between decks in the side hull or primary stiffeners which are longitudinals of a larger size.



(a) Total overview of the construction site

(b) Anatomy of ship's side structure

Figure 2.5: VLCC under construction showing structural lay-out [31]

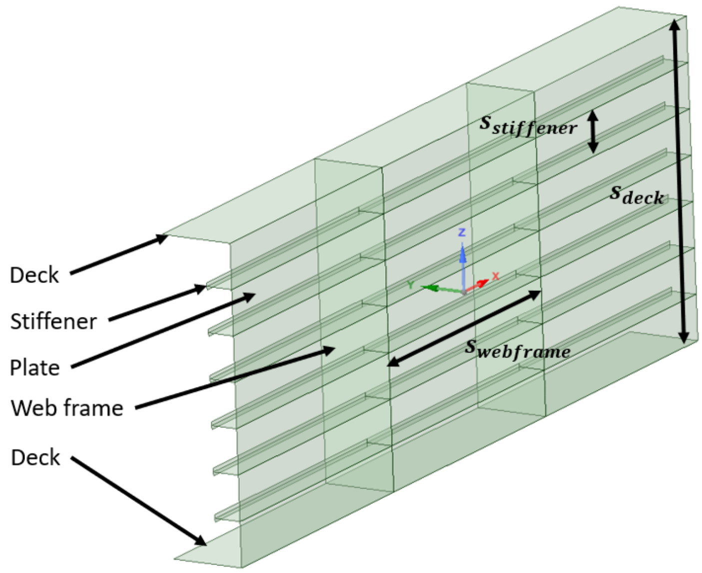


Figure 2.6: A stiffened panel/grillage consisting of longitudinal stiffeners, and transverse web frames with relevant particulars

Figure 2.6 shows different sizes, which are often referred to as scantlings. Stiffeners (longitudinals) have a spacing $s_{stiffener}$, the transverse frame spacing is denoted by $s_{webframe}$ and the distance between the decks is equal to s_{deck}. The influence of these scantlings have to be investigated in order to find an answer to the first sub-question of the study. For different vessel categories, which will be discussed in section 2.2, the most critical combination of dimensions are chosen in order to find an applicable formulation for the maximum allowable hull pressure.

1. *Which size-dependent parameters of the structural layout of a ship's parallel side hull influence the structural integrity? And how can the differences in geometry between different types of vessels be taken into consideration?*

In the design stage of a vessel, these parameters are also of interest. Putra et al. [29] derived an optimisation technique for decreasing the structural weight based on the number of stiffeners, cross-sectional area of the stiffener, plate thickness and stiffener spacing. The most important parameters that are considered in this study are:

- Stiffener spacing
- Amount of stiffeners in between the decks
- Transverse web frame spacing
- Plate thickness
- Stiffener geometry

Important mechanical properties of the plate or a beam can be found in the moment of inertia (I) in m^4, section modulus (S) in m^3 and the bending moment at the onset of yielding (M_Y) in kNm calculated by equations (2.2) to (2.4).

$$I = \frac{b \cdot h^3}{12} + d^2 \cdot A \tag{2.2}$$

$$S = \frac{I}{y_{max}} \tag{2.3}$$

$$M_Y = S \cdot \sigma_Y \tag{2.4}$$

Where, b and h equal the width and height of cross-section in metres, respectively. The perpendicular distance of the object's centre of gravity to the neutral axis is denoted with d. A is the cross-sectional area. The distance from the neutral axis to the outer fibre is denoted with y_{max}, and σ_Y equals the yield stress of the material.

2.4.1. Hooke's Law

For the elastic region, a linear relationship between the stress and strain of the material can be used following Hooke's law, equation (2.5).

$$\epsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}\right) \tag{2.5}$$

Where ϵ_{xx} is the strain in x-direction, σ_{xx}, σ_{yy}, and σ_{zz} are the normal stresses in respectively x, y, and z direction. E is the Young's modulus, and ν is the Poison's ratio.

In analytical calculations, this equation can often be simplified further if all stress components of a particular direction, the out of plane direction, are zero. This is called plane stress and is often used in the case of thin plates. Another useful method for analytical calculations is found in-plane strain, where the strain components of a particular direction are zero. This assumption can be made if a boundary condition does not allow any deformation in a particular direction. This assumption is also often used if one dimension of the geometry is much larger than the others. For instance, a length that is much larger than the width and thickness.

2.4.2. Von Mises-Yield Criterion

As mentioned in the introduction, this study focuses on the elastic region only. Therefore, it can be stated that the maximum allowable stress that is permitted in the material is equal to the yield stress of the material itself. This maximum allowable stress has to be specified with the Von Mises-yield criterion. A uni-axial stress, for instance the normal x-directed stress, on its own is not responsible for yielding. However, the equivalent stress calculated with this criterion is responsible. This equivalent stress is derived from the Cauchy stress tensor and takes all principal stresses, as well as the shear stresses into account. The formulation of the Von Mises-stress criterion is equal to equation (2.6).

$$\sigma_{eq} = \sqrt{\frac{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2 + 6\left(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2\right)}{2}} \tag{2.6}$$

Where σ_{xy}, σ_{yz}, and σ_{xz} represent the shear stresses in the material, which are also often denoted with τ_{xy}, τ_{yz}, and τ_{xz}, respectively. The principle stresses that are required for this equivalent stress, are often calculated using Hooke's law.

2.4.3. Effective Width

The grillage that is discussed can be analysed for different components, resulting in different failure modes. One of these components is the longitudinal stiffener. Once the loading on the grillage is transferred to the stiffener, the plate itself will still contribute to the rigidity. However, it is usual to model only a part of the width of this plate, the 'effective width'. A graphical example of such a plate-stiffener combination is given in figure 2.7. The total plate width is equal to the stiffener spacing, $s_{stiffener}$, graphically shown with a dashed line. The effective width is equal to the continuous line denoted with b_{eff}. The idea of the effective width concept is that this portion of the width, if the same stress would we developed as in the stiffener, carries the same load as is actually carried by the total width.

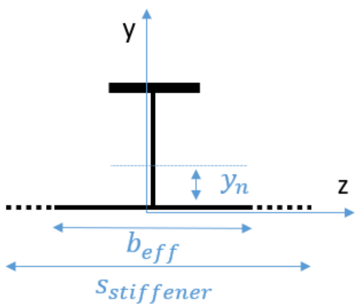


Figure 2.7: Cross-section of a plate-stiffener combination

A commonly used formulation of this effective plate width is formulated by Von Karman [42], following equation (2.7).

$$b_{eff} = \frac{\pi \cdot t_p}{2\sqrt{3\,(1-\nu^2)}}\sqrt{\frac{E}{\sigma_Y}} \tag{2.7}$$

A more recent formulation is given by Faulkner [10] as a review of b_{eff} in equation (2.8). This reviewed effective width will be used in this thesis.

$$\frac{b_{eff}}{s_{stiffener}} = \begin{cases} 1.0 \text{ for } \beta < C_3 \\ \frac{C_1}{\beta} - \frac{C_2}{\beta^2} \text{ for } \beta \geq C_3 \end{cases} \tag{2.8}$$

$$\beta = \frac{s_{stiffener}}{t_p}\sqrt{\frac{\sigma_Y}{E}} \tag{2.9}$$

Where β is the plate slenderness coefficient and the coefficients are given by: $C_1 = 2$, $C_2 = 1$, and $C_1 = 1$ for simply supported plates.

The effective width is a relevant factor in determining the height of the neutral axis, y_n. The combination with the cross-sectional area, A_{stiff}, and plate thickness, t_p, determines the height of the neutral axis following equation (2.10).

$$y_n = \frac{0.5 \cdot t_p.^2 \cdot b_{eff} + A_{stiff} \cdot y_{stiff}}{A_{eff}} \tag{2.10}$$

2.5. Fender Contact with the Vessel

This study fully focuses on the lateral loading on the parallel side hull of the vessel, due to the external fenders. Axial loading in the vessel due to global bending, hogging and sagging, of the vessel itself is neglected. The fender is simplified to a rectangular striking body, referred to as an indenter. An indenter with a bounded uniform load is also known as a patch loading, and examples that are used for patch loadings are found in wheel loadings and ice floes that are encountered by polar vessels. These patch loadings are commonly used in elastoplastic and fully plastic analyses [6, 8, 14, 16, 23, 46, 47], since the serviceability criteria for decks and hull plates are often expressed in an allowable deflection, which will be discussed in section 2.7. Furthermore, this lateral loading is assumed to be an extreme condition.

In the study of ice loadings [46] also the 'softness' of the ice floe is taken into account. Very soft actions, such as ice floes or hydrodynamic loads, are described by an uniform distributed load. This uniform load is simplified to a concentrated force on the location where in plastic analyses the plastic hinge would occur. Yu and Amdahl [46] introduce a factor γ, which should be taken into account for a patch loading that is not soft, and an adjusted length of the loading results in a reduced force on the vessel. This pressure distribution is definitely of interest for fender contact, since the softness of the contact patch are directly influenced by the choice of the fender panels material. This topic will be discussed more extensively in the methodology in section 3.2.

2.6. Vessel Categorisation

The research of TNO regarding the cylindrical fenders [44] categorises ship types following table 2.3. The data of the main parameters was provided by the Port of Rotterdam Authority with the intention to cover all vessels that may dock in the Port of Rotterdam. In table 2.3, $s_{stiffener}$ represents the stiffener spacing, the distance between the transverse web frames is denoted by $s_{webframe}$, the plate thickness by t_p, and the used stiffener type is given in the right-hand side column. The stiffeners are of the type 'HP' (Holland Profile), 'L'- or 'T'-profiles.

The so-called scantlings, the dimensions of the structural components in the vessel, are obtained from several cross-sections. These cross-sections of the vessels were designed by TNO applying the rules and specifications in accordance with Bureau Veritas [5] and Lloyds Register of Shipping [25]. Only the area of the cross-sectional drawings directly above the waterline is of interest in this thesis since this is the area where fenders are placed on the quay wall. The stiffeners do not have a constant spacing in most of the cross-sectional drawings in this region, the box enclosed by two decks directly above the waterline. The largest spacing that is found in this region is chosen in order to result in a conservative approach since this is most sensitive for high bending moments. Furthermore, a uniform plate thickness is assumed to be equal to the smallest thickness, and the increased thickness for ice-class vessels is neglected. Also steel grade A, with a minimum strength of 235 MPa, will be used in this thesis for all structural members. In reality, some of the vessels do have a few stiffeners with a higher strength due to the AH, or AH36 steel grade, which corresponds to a minimum strength of 315 and 365 MPa respectively. All these simplifications are chosen in a way that results in a conservative approach.

The cross-sections show many similarities with respect to lay-out, except for bulk carriers 'B10' and 'B10A'. Bulk carriers differ with respect to lay-out for missing longitudinals and having only transverse frames, which seem to be oddly placed together with respect to the other scantlings. However, since the longitudinal stiffeners are not present in this vessel type, the web frames are placed closer together to achieve enough strength.

Furthermore, in table 2.3 can be found that larger ships are designed with a larger stiffener-, and web frame spacing, a larger plate thickness, and larger stiffeners. Except for the second tanker from table 2.3, T2 Handysize, which does not seem to follow the general trend. However, there are fewer stiffeners between the decks, and the decks are placed closer to each other which could result in a stronger design. Furthermore, as a check of these outstanding particulars of the smallest tanker vessels, particulars were provided by the Dutch maritime design company Conoship International BV, Groningen. These are used as an addition to the series of vessel types used in the most recent research by TNO [44] in order to divide the vessels into different categories.

The different vessel types do differ with respect to the particulars themselves. However, the final strength of the side hull is defined by a combination of the structural parameters. The hypothesis is that the different vessel types do have a strength of the total grillage that should be in the same order of magnitude since they do all comply with the same regulations. A comparison needs to be made in order to find if there is a widely applicable rule with respect to the maximum allowable hull loading due to fender impact.

Table 2.3: Vessel categories considered in a research by TNO regarding cylindrical fenders [44]

		Main Parameters						Scantlings				
		Description	**Displ. [tonne]**	L_{PP} **[m]**	B_{vessel} **[m]**	**T [m]**	C_b **[-]**	$s_{stiffener}$ **[mm]**	**Stiffeners between decks**	$s_{webframe}$ **[mm]**	t_p **[mm]**	**Stiffener Type**
Tankers	**T1**	Coaster	9056	99.92	17	6.3	0.8462	650	4	2660	10	HP 180x10
	T2	Handysize	20963	136.99	22.9	8.25	0.81	575	3	1995	10.5	HP 120x8
	T3	Handymax Panamax (32 m)	45628	176	27.4	11.32	0.8361	700	7	2700	13	L 250x10 + 90x14
	T4	Aframax Suezmax	120658	239	43.8	13.6	0.8475	855	7	4300	17	L 300x13 + 90x17
	T5	VLCC	364013	320	60	22.52	0.8418	920	6	5680	20	T 550x12 + 150x20
Container Vessels	**C6**	Coaster Feeder	10893	112.29	15.9	7.21	0.8462	550	2	2840	15	HP 220x10
	C7	Panamax (32 m) Post Panamax Neo Panamax	139065	287	48.2	14.5	0.6933	860	8	3040	18	L 280x12 + 120x15
	C8	ULCV	225624	383	58.6	14.5	0.6933	850	11	3160	18	L 275x12 + 125x12
Bulk Cariers	**B9**	Coaster Handysize	47634	176.85	30	10.6	0.847	800	6	2400	13	HP 200x9
	B10	Handymax Panamax (32 m)	68047	189	32.26	13	0,8585	-	3	800	15	FB 150x15
	B10A	Panamax	200000	280	45	18.3	0.8674					
	B11	Capesize VLBC Berge Stahl/Vale max	277441	321	57	18	0.8424	844	19	4950	1850	T 450x12 + 150x20

2.7. Current Rules and Guidelines for Ships

DNV is the only classification society that has included specifications with respect to rules on berthing impact for ships [9]. In the so-called ‘special requirements’, rules for steel fenders integrated to side shell structures of a vessel are prescribed. The extent to which the side shell strengthening rules apply is in the range from ballast draught to 0.25 times the scantling draught, T_{sc}, with a maximum of 2 metres above T_{sc} where the breadth exceeds 0.9 times the moulded breadth. Based on the displacement of a ship, the included force of the fender is determined, and the minimum thickness of the side shell plating, t_p in mm, the bending moment, M_f in kNm, the minimum plastic section modulus of side shell longitudinals, S_{pl} in cm^3, and the minimum gross offered thickness of the web plating, t_{s-gr} in mm, are prescribed, see equation (2.11) to (2.14) respectively.

$$t_p = 26\left(\frac{b}{1000} + 0.7\right)\left(\frac{B_{vessel} \cdot T_{sc}}{\sigma_Y^2}\right)^{0.25} \tag{2.11}$$

$$M_f = \frac{P_f}{f_{pl}}\left(\ell_{bdg} - 0.5\right) \tag{2.12}$$

$$S_{pl} = \frac{M_f}{\sigma_Y} \cdot 10^3 \tag{2.13}$$

$$t_{s-gr} = 0.85\left(\frac{1}{\sigma_Y}\right)^{0.25}\sqrt{\frac{P_f}{c + 0.17}} + 2 \tag{2.14}$$

In these equations b is the breadth of the elementary plate panel in mm, B_{vessel} is the moulded breadth in metres, T_{sc} equals the scantling draught, σ_Y the specified minimum yield stress, P_f is the force induced by fender contact according to given formulas depending on the displacement, f_{pl} a bending moment factor, and ℓ_{bdg} the bending span of longitudinals in meters.

Whilst the rules are prescribed for fenders integrated into the side shell of the vessel, and not for external fenders mounted on the quay wall, it could be interesting to compare the prescriptions with the scantlings used in the further research into external fenders. Furthermore, an interesting comparison can be made with minimum strengthening for tug contact from DNV-GL [9], equation (2.15) to (2.17).

$$t = 0.65\sqrt{P_{fl} \cdot k} \tag{2.15}$$

$$S_{pl} = 0.3 \cdot P_{fl} \cdot \ell_{bdg} \cdot k \tag{2.16}$$

$$P_{fl} = \frac{\Delta}{100}, \ (200 \leq P_{fl} \leq 700) \tag{2.17}$$

Where k is a material factor which is 1.0 for S235 structural steel and P_{fl} the local design force in kN that should be taken in between the given boundaries.

Bulk vessels are not covered by the discussed rules, since the structural layout lacks longitudinal stiffeners. However, IACS (International Association of Classification Societies) has a very straightforward description regarding minimal scantlings of single skin bulker vessels [20]. Most of the classification societies are in line with IACS. Therefore, the IACS rules are used as a guideline in order to check the scantlings regarding bulker vessels, following equation (2.18) and (2.19),

$$t_{w,min} = C(7.0 + 0.03 \cdot L) \tag{2.18}$$

$$t_{p,min} = \sqrt{L} \tag{2.19}$$

In which $t_{w,min}$ and $t_{p,min}$ represent the web frames’ thickness and the plate thickness, respectively. C is a non-dimensional factor, which is 1.0 for web frames midships.

In table 2.4, the scantlings of the given vessel categories are compared with the discussed rules. First of all, it is found that the web frames do comply with the rules of DNV-GL with an extensive margin. Therefore, it is likely that the web frames do have a relatively high stiffness with respect to the other structural members, and it seems reasonable to neglect their compliance in this thesis. It can be seen that the plate thicknesses do comply in all cases with the rules for fender strength. However, regulations regarding strength for tug contact require larger plate thicknesses in a few cases. The plastic section moduli of the vessel categories do not comply with both regulations in most cases.

The results in this table indicate that the standard structural lay-out of vessels, following the designs from TNO, do not comply with these special requirements. Since the operations that belong to these special requirements are considered to be in line with the berthing operation that should be executed by all vessels, it is interesting to investigate the plate and stiffener behaviour due to forces induced by fenders. Also, a large difference between the two special requirements is observed. Indicating, that the rules on this kind of lateral loading on a ship's hull are not completely clear.

Table 2.4: Comparison of scantlings used by TNO [44], and current rules. Scantlings described by rules are the plate thickness (t_p), plastic section modulus of longitudinal stiffeners (S_{pl}), and the gross offered thickness of web plating of web frames ($t_{s,gr}$)

	TNO [44]			**DNV GL − Fenders [9]**			**DNV GL − Tug [9]**	
	t_p **[mm]**	S_{pl} **[cm^3]**	t_{sgr} **[mm]**	t_p **[mm]**	S_{pl} **[cm^3]**	t_{sgr} **[mm]**	t_p **[mm]**	S_{pl} **[cm^3]**
T1	10	153.5	10	7.37	520.04	2.31	17.20	558.60
T2	10.5	55.4	9	8.02	83.35	2.15	9.41	125.46
T3	13	1227.4	11	9.96	266.97	2.22	13.88	369.59
T4	17	1874.6	13	13.03	1219.42	2.36	17.20	903.00
T5	20	2589.8	19	16.66	5014.86	2.62	17.20	1192.80
C6	15	259.4	10	6.94	105.80	2.13	9.19	170.40
C7	18	745.9	12	13.60	939.43	2.39	17.20	638.40
C8	18	647.4	12	14.19	1596.17	2.49	17.20	663.60
B9	13	258.4	10	10.74	240.70	2.23	14.19	342.96
B10 *	15		15	13.75 *		12.67 *		
B10A								
B11	18	1635.3	15	14.82	3283.54	2.55	17.20	1039.50

* The B10 bulk vessel is compared with IACS rules, taking plate and frame thickness into account.

3 Methodology

In order to determine the maximum allowable fender loads, the analysis is divided into different components. As the effect of the fender's geometry is of special interest in this analysis, the contact length and width will function as the main input parameters. Other important factors that will eventually determine the maximum capacity of the hull structure are discussed per section in this methodology.

First of all, different contact locations of a fender panel on a ship's structure are discussed in section 3.1. The second step in the analysis consists of the description of the load distribution of the fender, this is discussed in section 3.2. In the third section, different failure modes are introduced that are analysed separately in this thesis. These failure modes take into account the different locations and distributions that are discussed.

3.1. Locations

The first factor that is analysed in the objective to determine the maximum loading on the ships hull is the contact location. Figure 3.1 shows a schematic view of the stiffened panel as introduced in section 2.4. The relevant contact locations are shown in red. For every contact location, a specific component of the stiffened panel is expected to function as the most vulnerable link. Such as the plate between the stiffeners is expected to be the weakest link for loading location A, and a stiffener itself is expected to be the weakest link for loading location B. These components will be discussed in section 3.3.

The locations A - D in figure 3.1 are described by:

A. Center of the loading in between both the longitudinal stiffeners and transverse frames.
B. Center of the loading on a longitudinal stiffener in between the transverse frames.
C. Center of the loading on a transverse frame in between the longitudinals.
D. Center of the loading on both a transverse frame, and the longitudinal.

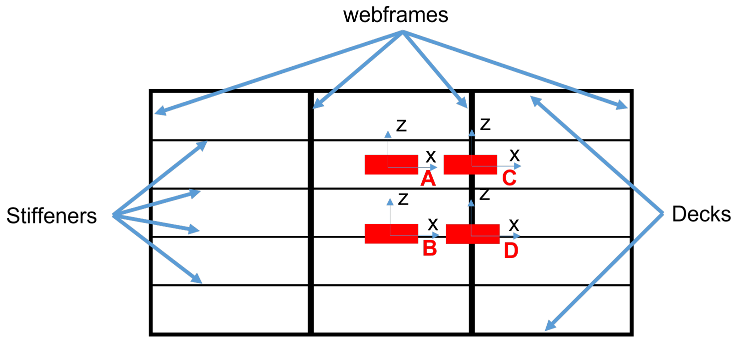


Figure 3.1: Fender contact locations, with a local coordinate system for the specific load case

In figure 3.1, only areas that do not exceed the stiffener spacing are indicated. The contact area functions as the main parameter in this study and the influence of the length and width are analysed from small to large lengths. Eventually, a fender's contact area that exceeds multiple longitudinals will also be of interest. In this thesis, the assumption is made that the force is equally divided over the least amount of stiffeners that are activated due to a specific fender height. The maximum area that is analysed has a width equal to the transverse web frame spacing, and a height equal to the distance between the strength decks.

At loading location A, the first step of the loading and response of the stiffened panel, a plate strip will be modelled in a way that the longitudinals act as simple supports with a zero deflection stiffness. The second step in the loading and response mechanism for a loading on location A is based on the reaction force at the longitudinal. The reaction force can be seen as a loading on the longitudinal which is clamped between the frames, which results in a method that equals the method for describing a load at location B.

For a loading at location B, the longitudinal stiffener is assumed with an effective width of the plate. The longitudinals are clamped in between the transverse frames and both rotation and deflection are restrained at the ends. In this thesis, the transverse web frames are assumed to be very robust, as discussed in section 2.7. Therefore clamped boundary conditions are assumed.

A loading at locations C, and D results in a major contribution of the web frame. Since section 2.7 showed that these web frames can be assumed as rigid, these locations are not considered for the determination of the maximum allowable loading.

3.2. Pressure Distribution

Besides the effect of the contact location, the pressure distribution of the fender on the hull is an important topic in this research. As introduced in section 2.5, the 'softness' of the material that makes contact with the vessel's side hull is important in order to determine the maximum allowable loading. For the wheel loadings, and ice floes that are mentioned in the technical background this property of 'softness' of the contact surface are not dependant on the design, but the vessel design is dependant on this property. On the other hand, the fender design specifically focuses on a ship's berthing, and the behaviour of the contact area is therefore of interest in this thesis. The material properties of the contact area do directly influence the pressure distribution, which leads to the second sub-question that has to be answered in order to fully understand the effect of the geometry of the contact area.

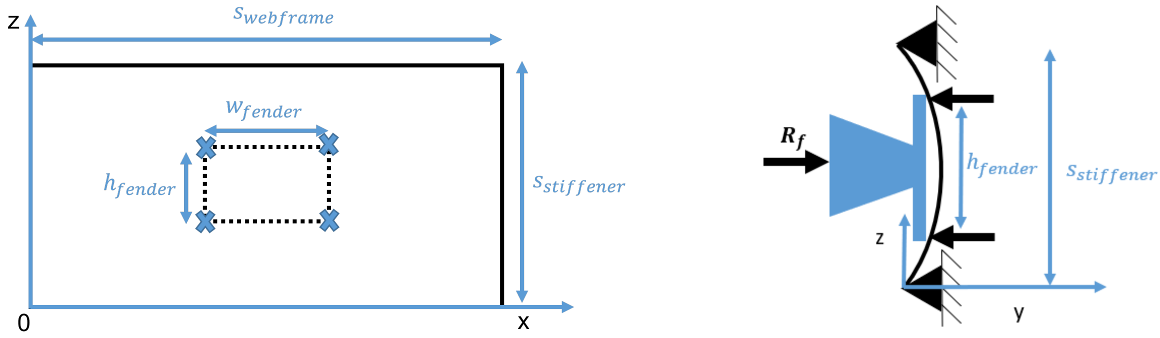
2. *How can the pressure distribution acting on a ship's side structure be described during fender contact?*

In the technical background, a distinction was made between very soft indenters that resulted in a uniform pressure, and indenters that have a higher stiffness. In this study, different kinds of loading distributions are used to represent different sorts of fenders. In order to cover a broad range of possible pressure distributions, the focus is laid on the two most extreme cases. A fender contact area that has a much higher stiffness than the vessel's side hull is modelled as a rigid fender contact area, this could be observed in practice as a very rigid structure in front of an SPC fender. The other extreme case is a fender that is much softer than the vessel's side hull. This could be represented by extreme soft rubber fenders without a panel. The different distributions are discussed per paragraph.

3.2.1. Rigid Fender Contact Area

A very rigid fender contact area is presented in figure 3.2, with a load that is applied on a plate. It is assumed that in this case, the hull plate will deflect, and the fender panel will not. After the initial bending of the hull plate the contact between the hull and the fender will only be presented at the corners of the contact area, and therefore result in concentrated loads at the corners of the contact patch. In the analytical method, this will be simplified by line loads at the location of the dashed lines in the figure. This pressure distribution will be referred to as the ”rigid fender contact area”, and will be discussed more extensively per failure mode in the following chapters.

Eventually, the reaction force of the fender is obtained by assuming a statically determinate structure, and the reaction force of the fender will be distributed over the edges of the contact area.



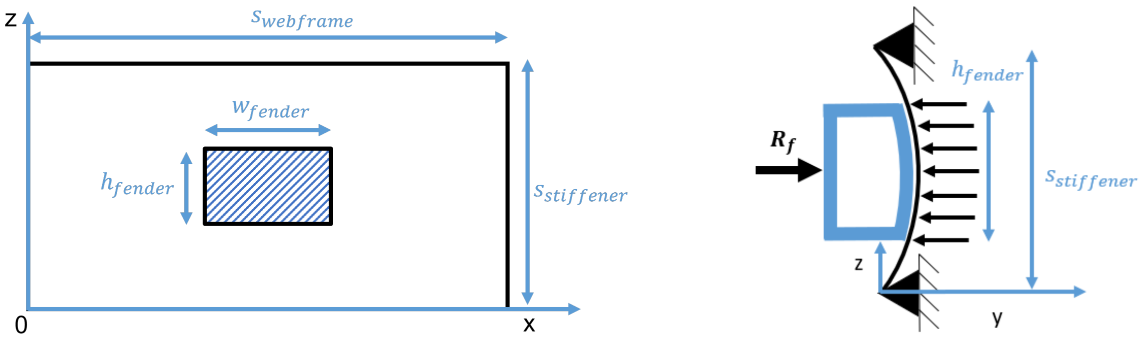
(a) Plan view (b) Side view

Figure 3.2: Rigid fender contact area

3.2.2. Soft Fender Contact Area

In the technical background, it was found that for plastic analyses, fully uniform distributed load simplifications were made, by using a concentrated load at the location where the plastic hinge would occur. This study focuses on the elastic region only, and therefore, this assumption can not be used. A formulation for a constant uniform pressure imposed on the full contact patch is used as a representation for a ”soft fender contact area”.

The total applied loading and area of a specific case are of interest and used to determine the reaction force of the fender; R_f. The reaction force can be compared with the rigid fender contact area.



(a) Plan view (b) Side view

Figure 3.3: Soft fender contact area

In first instance, the hypothesis is that a soft fender contact will result in higher allowable loading since the loading is distributed over the full contact area. However, due to the distance to the centre of the loading which is important for the occurring moment in this case, higher values can be expected than for a rigid fender where the load is closer to the boundary condition. Therefore, the distributions are both checked. Actual fenders will behave in a way that is not fully rigid or soft, but somewhere in between. With these extreme representations, it is assumed that most of the possible pressure distributions of fender contact are covered.

3.3. Failure Modes

The given loading locations (figure 3.1), vessel types (table 2.3), and pressure distributions (figure 3.2, and 3.3), are used as input for this research. The layout, width and height, of the fender panel will function as the variables. The ship's structure behaves differently per loading location and loading geometry. By analysing the results of several combinations of input, the third sub-question can be answered. The behaviour of the ship's structure is analysed to answer sub-question four.

3. *What types of failure modes within the ship's side structure are likely to occur within the ship's structure during fender contact?*
4. *What is the influence of the fender contact area geometry on the maximum allowable loads acting on the ship's side structure?*

Different failure mechanisms will be discussed, and a maximum load will be calculated for all mechanisms separately. Eventually, the maximum loads of the failure mechanisms are compared with each other, and the mechanism with the lowest resistance can be assumed critical. These failure modes are shortly introduced per subsection and discussed more extensively in the following chapters.

3.3.1. Plate Failure

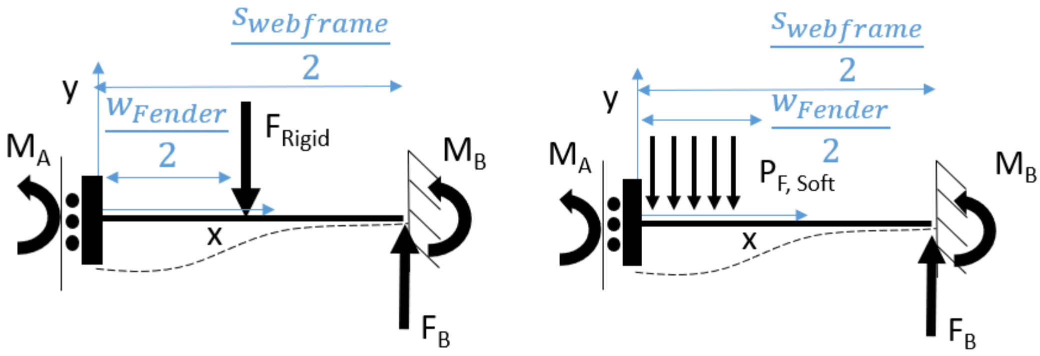
Hughes concentrated on the plating of a stiffened panel under lateral loading [16], [18], although the largest stresses are expected in the stiffeners due to the larger distance from the neutral axis of the plate stiffener combination. For the servicebility criterion, the maximum permanent deflection of the plate would be reached before the stiffeners have undergone any appreciable yield for their maximum allowable deflection. This assumption is made due to the robust scantlings the stiffeners require in order to satisfy the ultimate strength requirements. The research resulted in an expression for the capacity of welded plates under uniform pressure. The collapse mechanism that was used is described by yield line theory. This yield line theory is often used for describing a plate under lateral loading, [33], [22], [14]. These yield lines however, are formed when the full thickness of a plate has reached the yield stress, which is beyond the region of interest in this research. A more general plate bending theory of Hughes and Caldwell [17] describes the small deflection theory in the elastic region.

In this study, the plate failure is especially of interest for smaller patches at location A (figure 3.1). For each pressure distribution, figure 3.2 and 3.3, a separate formulation is used for the maximum allowable pressure on a plate.

3.3.2. Stiffener Failure

Beam theory is a topic that is well established and a lot of theory is covered in books like Hibbeler [13], Timoshenko's theory of elastic stability [41] and Roark's formulas for stress and strain [45]. This type of failure is of interest in the second load and response mechanism, for loading location B or for larger contact areas. The maximum distance from the neutral axis of the stiffener-plate combination is at the flange of a longitudinal stiffener, and failure is assumed when the yield stress is reached locally.

Following the same method as used for the plate in the vertical direction, the stiffener is analysed horizontally. The stiffener is assumed to be clamped between the transverse frames and different formulations have to be used in order to describe the deformation correctly. Figure 3.4 shows the corresponding free body diagrams.



(a) Rigid fender contact area (b) Soft fender contact area

Figure 3.4: Free body diagrams representing half the stiffener's length clamped at the transverse frame

3.3.3. Failure Modes out of the Scope

Tripping or lateral-torsional buckling of the stiffener is a phenomenon where the stiffeners tend to bend over and rotate about the location where the stiffener is connected with the plate. Tripping results in a serious decrease in the load-carrying capacity of the stiffened panel. A thin-walled open cross-section, such as a stiffener, has low flexural rigidity and therefore tripping should be checked. Furthermore, an applied load that does not intervene with the rotational centre of the stiffeners is highly expected during fender contact resulting in a larger possibility that tripping occurs.

A widely cited article on tripping goes back to the 1970s, in which Adamchack [2] found formulations for describing peak stresses for tee- and flat stiffeners in three different forms of external loading; axial force, constant moment and uniform lateral load. Hughes and Ma [15] expanded this research to an energy method for analysing the tripping behaviour of asymmetric stiffeners. The same forms of external loading are taken into account. A simplified model that ignores the rotational restraint between the stiffener and the plate is used in first instance. A rigid web assumption is said to be good, given that the plate is quite thin and buckles before tripping. Both a tee stiffener and an asymmetric stiffener in a pure bending load, are said to be keen on tripping, even for the case of loading on a stiffener, loading situation B in figure 3.1 in this thesis. Due to a lack of time in this topic could not be covered in this thesis. In examples from practice, it is observed that transverse web frames of bulk carriers are keen for this failure mode. Therefore, it would be interesting for follow-up research to include.

3.4. Finite Element Analysis

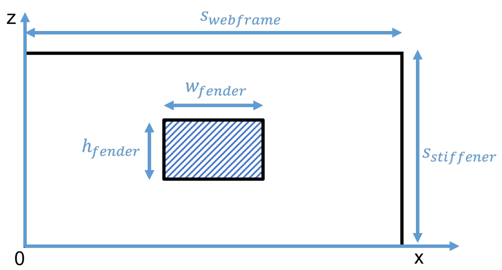
The theoretical approach will result in an analytical formulation of a maximum allowable loading for different geometries of an indenter. This will be verified for a fixed amount of cases with a Finite Element Analysis (FEA), using the software of ANSYS Workbench, version 19.2 [1].

In the ANSYS Workbench, the material is chosen to be 'structural steel NL', with an adjusted yield stress of 235 MPa, equal to the minimum strength of grade A steel. The material model has an elastic perfectly plastic stress-strain curve. Furthermore, the 'Static Structural' solver is used with the default SHELL181 type elements.

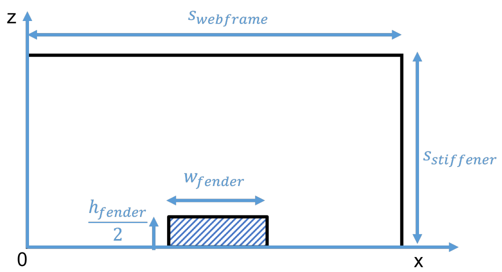
4
Plate Failure

As discussed in the methodology, this thesis distinguishes different deformation modes. This chapter elaborates on the maximum allowable loading on plate level. The different vessel types that are discussed in section 2.6 have different layouts, and the geometry of the plates of interest in this chapter is enclosed by two stiffeners and two web frames. For fender contact areas that exceed these boundaries, the maximum height is used for the soft fenders contact area, and the reaction forces due to rigid fenders are assumed to be fully absorbed by the stiffeners, chapter 5. The stiffeners and web frames are simplified to hinged boundary conditions in this chapter.

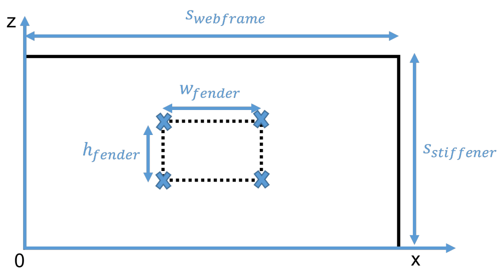
This chapter consists of three sections, which discuss the two different loading locations and two different loading distributions. Figure 4.1 gives an overview of the covered content per section. A load case in which the centre of the fender is located at the centre of a plate is described in subsection 4.1 for a uniform load, and in subsection 4.2 for a concentrated load. Subsection 4.3 describes the scenario in which the centre of the fender collides with the stiffener, that supports the plate. A fourth scenario could be thought of following figure 4.1d. However, due to the location and the representation of a rigid fender, the load is fully absorbed by the stiffener along the x-axis in this fourth scenario. This scenario will therefore be covered in chapter 5.



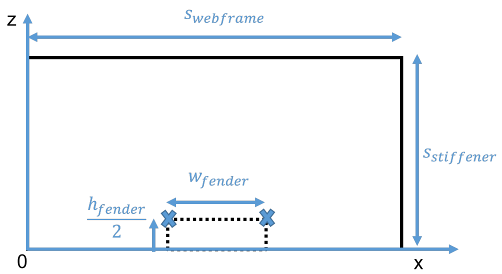
(a) Central loading with a uniform pressure



(b) Loading from the edge with a uniform pressure



(c) Central loading with a concentrated load



(d) Loading from the edge with a concentrated load

Figure 4.1: Loading locations and distribution of interest on a plate

The central loading location on the plate is the most vulnerable location with respect to the highest corresponding internal moments in the plate. However, in the scenario in which a uniform pressure is applied to the location of a stiffener, a loading at the edge of the plate is of interest in order to describe the behaviour of the total grillage more accurately.

In this chapter, one specific plate geometry is verified during the process. This plate is based on the third tanker category. The corresponding plate dimensions that are used are given by {$s_{stiffener}$ = 0.6 m, $s_{webframe}$ = 2.7 m, t_p = 9 mm, σ_Y = 235 MPa}.

4.1. Central Loading with Soft Fender

The goal of this section is to find a formulation for the maximum loading that can be applied in the centre of the plate. This formulation has to take into account the size of the contact area. The maximum is defined as the moment at which the yield stress is reached at any location.

4.1.1. Analytical 2D

A simplified analytical approach is found by a beam representation. This beam represents a strip of the plate at the centre of the width, graphically represented in red in figure 4.2. This method will be referred to as the 2D approach. A representative free body diagram is given in the right of figure 4.2. Since the plate is symmetrically loaded over the height, the centre has an angle equal to zero which is presented in figure 4.2 as a guided boundary condition with a resulting moment M_r. The hinged boundary condition represents the connection of the plate to the stiffener. The maximum moment in the plate strip is defined per unit length in the x-direction. For uniaxial state of stress in z-direction, σ_{zz} only, equation (4.1) is used to define the maximum moment.

$$(M_z)_Y = \frac{\sigma_Y \cdot t_p^2}{6} \tag{4.1}$$

However, the Von Mises criterion states that yielding will occur when the equivalent stress is equal to the yield stress. The orthogonal stress due to bending has to be taken into account as well [17]. Therefore, the maximum moment is given by equation (4.2), following plane strain conditions.

$$M_Y = \frac{\sigma_Y \cdot t_p^2}{6} \cdot \frac{1}{\sqrt{1 - \nu - \nu^2}} \tag{4.2}$$

Where ν is equal to the Poison's ratio, which is typically 0.3 for steel.

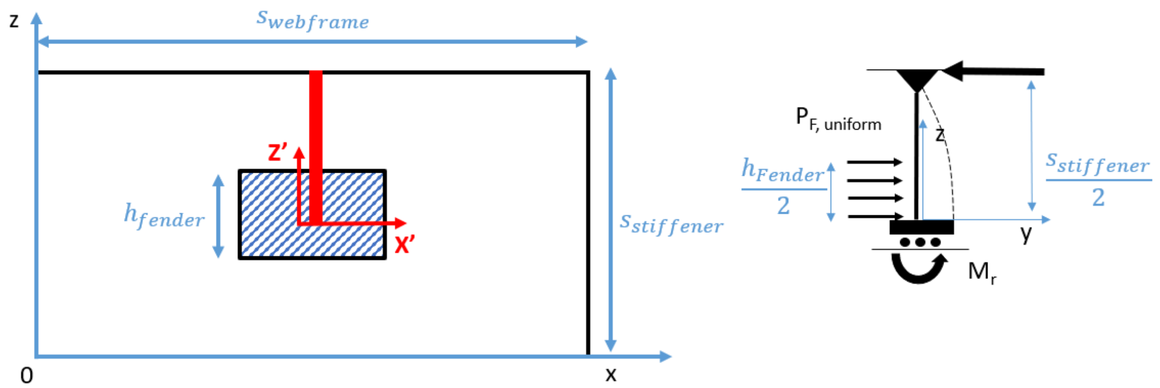


Figure 4.2: Overview of 2D strip representation, and corresponding free body diagram

Following the beam equations, the formulation for the moment at the guided end is given. The derivation and way of rewriting this formulation to the way it is used in this thesis can be found in appendix A.1. Equation (4.3) yields the formulation of the maximum allowable load in Newton per meter corresponding to a specific fender size. This is rewritten in equation (4.4) to a maximum loading in Newtons by multiplying the maximum pressure with its area, or in the case of this 2D approach with its length.

$$P_{F,uniform} = \frac{2 \cdot M_Y}{\left(\left(\frac{s_{stiffener}}{2}\right)^2 - \left(\frac{s_{stiffener}-h_{fender}}{2}\right)^2\right)} \tag{4.3}$$

$$F_{Max,uniform} = P_{F,uniform} \cdot \frac{h_{fender}}{2} \tag{4.4}$$

4.1.2. Analytical 3D

A theoretical approach of partially loaded simply supported rectangular plates is described by Timoshenko and Woinowsky-Krieger [40]. An overview of the plate and notations following Timoshenko and the ones used in this report are given in figure 4.3. This section describes the central loading of the plate, which yields $\xi = \frac{s_{webframe}}{2}$, and $\eta = \frac{s_{stiffener}}{2}$.

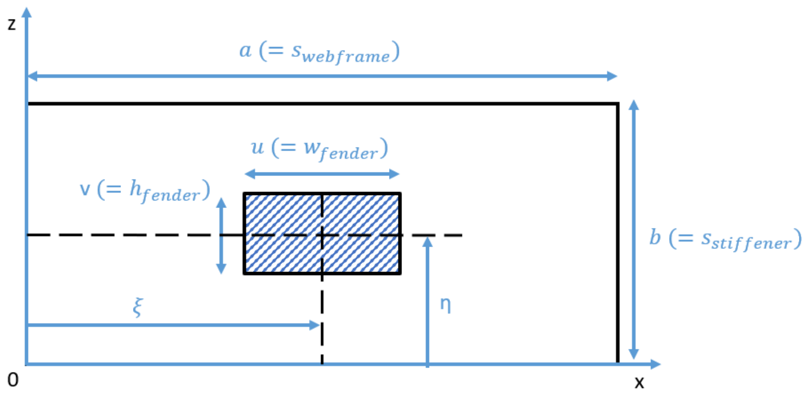


Figure 4.3: Overview of particulars of the plate

The bending moment with the highest value occurs in the centre of the plate, and the z-direction is investigated since $s_{webframe} > s_{stiffener}$. Hence the maximum bending stress is given in equation (4.5).

$$(\sigma_{zz})_{max} = \frac{6 \cdot (M_z)_{max}}{t_p^2} \tag{4.5}$$

The maximum bending moment yields equation (4.6).

$$M_z = -D \cdot \left(\frac{\partial^2 w}{\partial z^2} + \nu \cdot \frac{\partial^2 w}{\partial x^2}\right) \tag{4.6}$$

Where D is given by:

$$D = \frac{E \cdot t_p^3}{12 \cdot (1 - \nu^2)} \tag{4.7}$$

The formulation of the deflection at any point of the loaded portion is given by Timoshenko and Woinowsky-Krieger in the form of equation (4.8).

$$\begin{aligned} w = &\frac{4qa^4}{D\pi^5} \sum_{m=1,3,5,\ldots}^{\infty} \frac{(-1)^{(m-1)/2}}{m^5} \sin\frac{m\pi u}{2a} \left\{1 - \frac{\cosh\frac{m\pi y}{a}}{\cosh\alpha_m}\right. \\ &\left[\cosh(\alpha_m - 2\gamma_m) + \gamma_m \sinh(\alpha_m - 2\gamma_m) + \alpha_m \frac{\sinh 2\gamma_m}{2\cosh\alpha_m}\right] \\ &\left. + \frac{\cosh(\alpha_m - 2\gamma_m)}{2\cosh\alpha_m} \frac{m\pi y}{a} \sinh\frac{m\pi y}{a}\right\} \sin\frac{m\pi x}{a} \end{aligned} \tag{4.8}$$

Where α_m and γ_m are given by:

$$\alpha_m = \frac{m\pi b}{2a} \tag{4.9}$$

$$\gamma_m = \frac{m\pi v}{4a} \tag{4.10}$$

By assigning a numerical factor β for the moments that occur in the centre of the plate, the moment formulation can be represented by equation (4.11).

$$(M_z)_{max} = \beta\left(a, b, \frac{u}{a}, \frac{v}{a}\right) \cdot q \cdot u \cdot v \tag{4.11}$$

Where q represents the intensity of the lateral load on the plate in $\frac{N}{m^2}$.

By equating the differential equation (4.6) with (4.11), β is found. This numerical factor is dependant on both the geometry of the plate as well as the geometry of the patch divided by the plate length. The maximum stress equals the yield stress following equation (4.12).

$$(\sigma_{zz})_{max} = \sigma_Y = \beta\left(a, b, \frac{u}{a}, \frac{v}{a}\right) \cdot \frac{q \cdot u \cdot v}{t_p^2} \tag{4.12}$$

Which leads to the maximum allowable loading for a specific plate and fender size in equation (4.13).

$$F = q \cdot u \cdot v = \frac{\sigma_Y \cdot t_p^2}{\beta\left(a, b, \frac{u}{a}, \frac{v}{a}\right)} \tag{4.13}$$

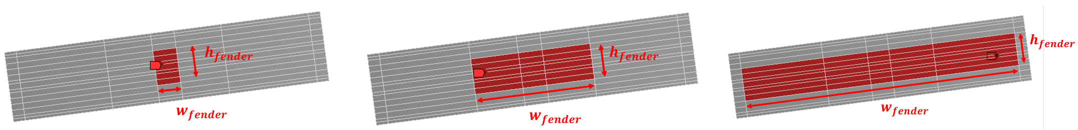
Specific values of β are given by Timoshenko and Woinowsky-Krieger [40] for three specific plate aspect ratios: $a = b$, $a = 1.4b$, and $a = 2b$. A verification of the derived function that describes β is executed with success for these specific geometries. Therefore, this function can be used for all kinds of plate geometries, such as the geometries of the vessel types of interest.

4.1.3. Finite Element Method

In order to verify the analytical approaches, an FEA is performed in ANSYS Workbench [1]. A parametric approach in which three different fender widths, and seven different fender heights is used to check the maximum allowable uniform pressure on the plate which is described in the introduction of this section. Figure 4.4 gives a representation of the verified fender widths. The area in red represents the area on which the uniform pressure is applied. The height of the red area is denoted with a variable h_{fender}, which is continuous in the analytical approach and is checked at seven different heights in FEM.

By applying a pressure that exceeds the yielding limit with multiple substeps, the substep of interest can be used. The substep of interest is at the moment at which the equivalent stress is equal to σ_Y = 235 MPa.

The mesh size in the shell model for this specific plate is set at five times the plate thickness, since this simulation runs within a couple of minutes, and is reduced until convergence is achieved. Convergence is assumed when results are not changing more than 5%, with respect to the previous result.



(a) 0.2 meter fender width (b) 1.0 meter fender width (c) 2.5 meter fender width

Figure 4.4: Overview of verified uniform loading on a plate

4.1.4. Verification

The height of the contact area is normalised by dividing the fender height with the stiffener spacing. The results are plotted in figure 4.5. The maximum allowable force for the widths is verified in ANSYS. This figure illustrates that a 2D approach is a good approach for very wide fenders, as the solid yellow line follows the marked yellow 'x'es nicely. However, for small contact widths, the 2D approach shows large differences compared to the FEM results. The 3D approach is closer to the results obtained via ANSYS. The analytical approach that results in the highest maximum allowable loadings show the most similarities with ANSYS and is therefore chosen to be most relevant.

Another important observation in this verification is the fact that the analytical approaches result in maximum loadings that are of a conservative kind. The ANSYS data results in higher loadings in all cases. This concludes that the analytical approach is a safe method for the analysis of a uniform load on a plate.

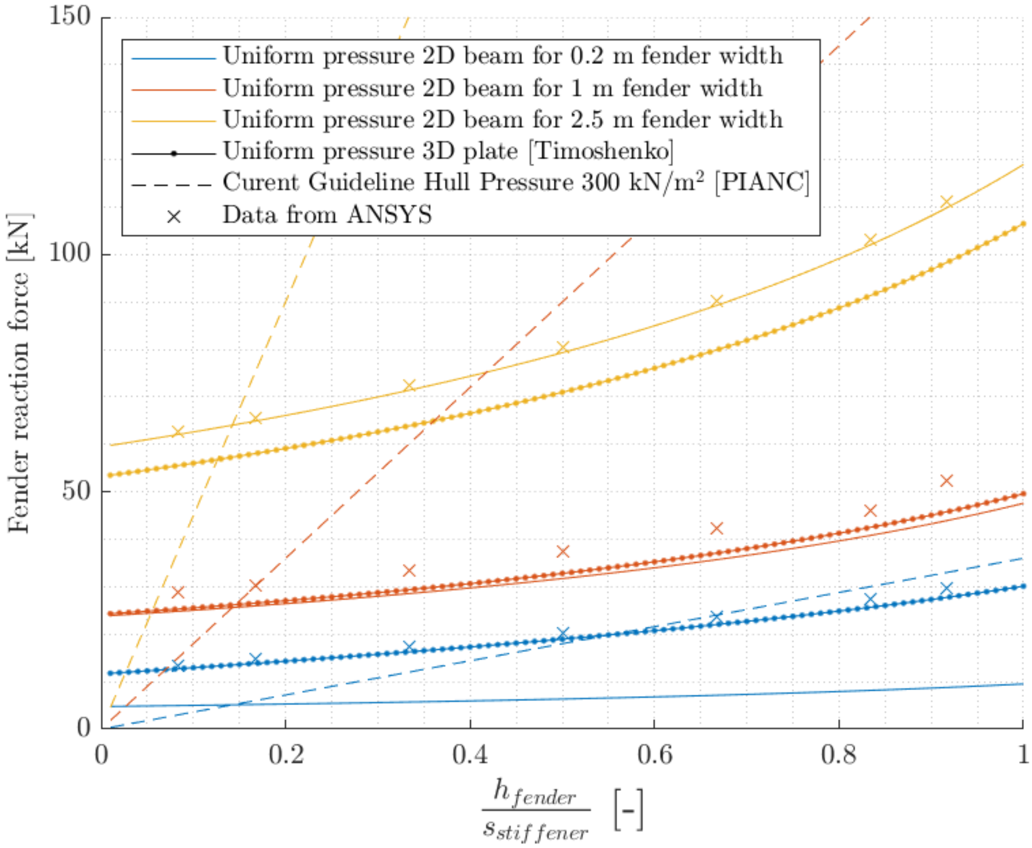


Figure 4.5: Maximum capacity of a plate on hinged boundary conditions, due to uniform pressure, $P_{F,uniform} \cdot A_{contact}$, for centred loaded case ($s_{stiffener}$ = 0.6 m, $s_{webframe}$ = 2.7 m, t_p = 9 mm, σ_Y = 235 MPa)

In figure 4.6, the maximum capacity of the reference plate is calculated for a broader range of fender widths following the analytical approach. An important difference between figure 4.5 and figure 4.6, what should be noticed is that the x-axis is now used for the normalised fender width instead of the height. The different fender heights are given in the form of separate lines. The same figure is made for all different vessel types discussed in section 2.6, and will be used to compare with the other failure modes in chapter 7.

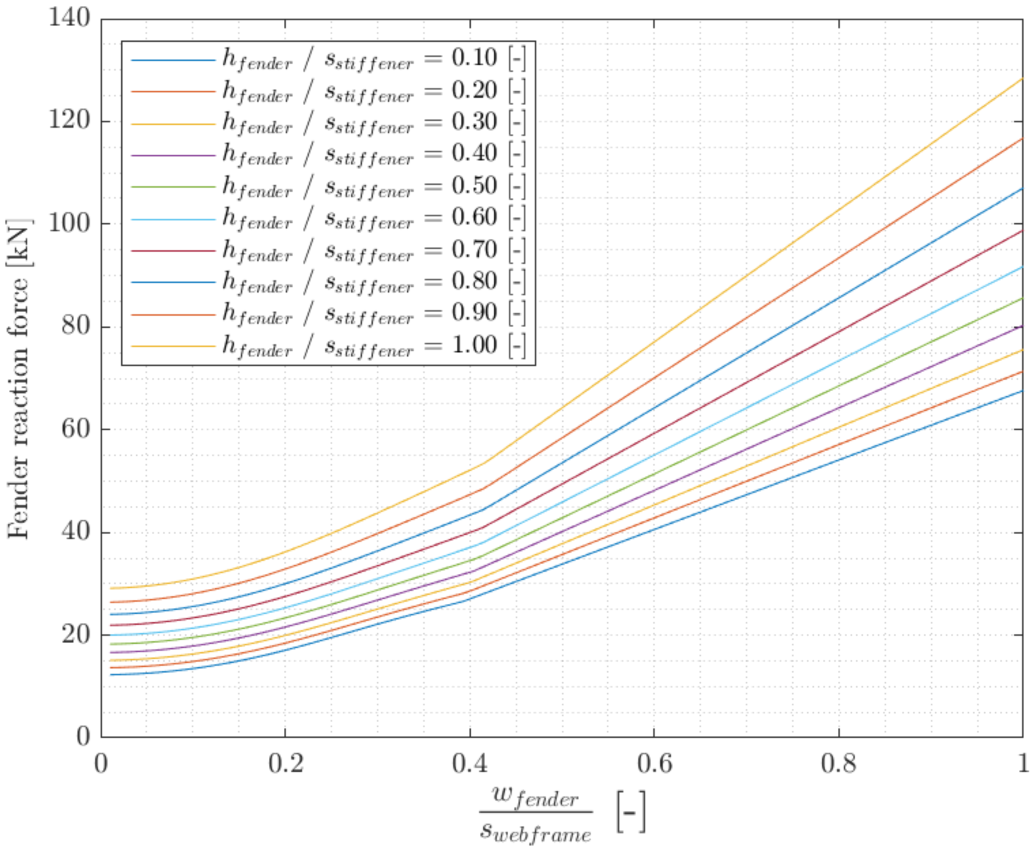


Figure 4.6: Maximum capacity of a plate on hinged boundary conditions, due to uniform pressure, $P_{F,uniform} \cdot A_{contact}$, for centred loaded case ($s_{stiffener}$ = 0.6 m, $s_{webframe}$ = 2.7 m, t_p = 9 mm, σ_Y = 235 MPa)

4.2. Central Loading with Rigid Fender

This section covers the second loading distribution as discussed in section 3.2. The same approach and subsections are used as was used for the uniform pressure.

4.2.1. Analytical 2D

Figure 4.7 gives an overview of the plate and the loading locations on the plate. The 2D strip is marked in red in the overview and is analysed on the right of figure 4.7 as a beam representation. Since the loading is symmetrical, only the upper half of the plate is considered in the 2D analysis. The centre of the plate is represented as a guided boundary condition, and the connection with the stiffener is represented by a hinge.

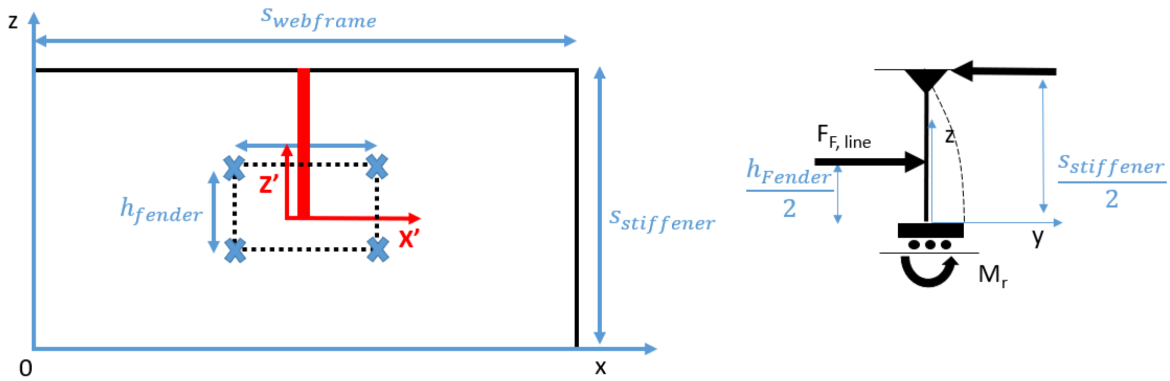


Figure 4.7: Overview of 2D strip representation, and corresponding free body diagram with a concentrated loading

First of all, the assumption is made that the forces in y-direction are in equilibrium. Followed by the equation of the reaction moment, M_r, at the centre of the plate in equation (4.14) the maximum allowable concentrated load can be calculated via equation (4.15), which follows from substituting M_Y (4.2) for M_r.

$$M_r = F_{F,line} \cdot \frac{s_{stiffener}}{2} - F_{F,line} \cdot \frac{h_{Fender}}{2} \tag{4.14}$$

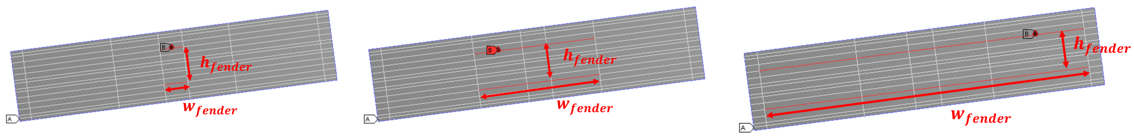
$$F_{max} = \frac{t_p^2}{3 \cdot (s_{stiffener} - h_{Fender})} \cdot \frac{\sigma_Y}{\sqrt{1 - \nu - \nu^2}} \tag{4.15}$$

Where $F_{F,line}$ is equal to a line load in $\frac{N}{m}$ with an x-directed width equal to the unit width of the plate strip.

4.2.2. Finite Element Method

As discussed in the FEM paragraph of the uniform load, ANSYS Workbench [1] is used in order to verify the analytical formulation. The loading is applied as two line loads representing the upper and lower part of a stiff fender panel. Figure 4.8 shows three different lengths of line loads that are used to represent different fender widths, and seven different heights are modelled by changing the distance between the line loads. The line loads are denoted with a thin red line, and a spacing h_{fender} which represents the fender height. In this figure, the same geometries are used as discussed in the uniform load paragraph.

The yielding limit of 235 MPa is found for a specific line load, and this load is saved. The mesh size is chosen as five times the plate thickness, and while decreasing the size; a load within a 5% range from the previous is assumed as converged.



(a) 0.2meter fender width (b) 1.0 meter fender width (c) 2.5 meter fender width

Figure 4.8: Overview of verified concentrated loading on a plate

4.2.3. Verification

The maximum allowable line loads are verified with FEM, and figure 4.9 gives the result. In this figure, the 2D approach is plotted for the three fender widths, and the height functions as the input variable. The PIANC guideline is also implemented in this figure. The loadings based on the PIANC guideline are more in line with the values calculated for the concentrated loads than it was for the uniform pressure formulation. However, the actual results of this study are lower and therefore this specific case is not safely described with the current guideline. In practice, these configurations, stiff fenders with heights smaller than 0.6 meters, are not representative for fenders used in the port.

Nevertheless, the introduced 2D approach shows a better description of the verified values and is in all cases but one a conservative estimate of the maximum loading. Only the very wide and largely spaced line loads result in an overestimation of the maximum allowable loading by 2D beam theory.

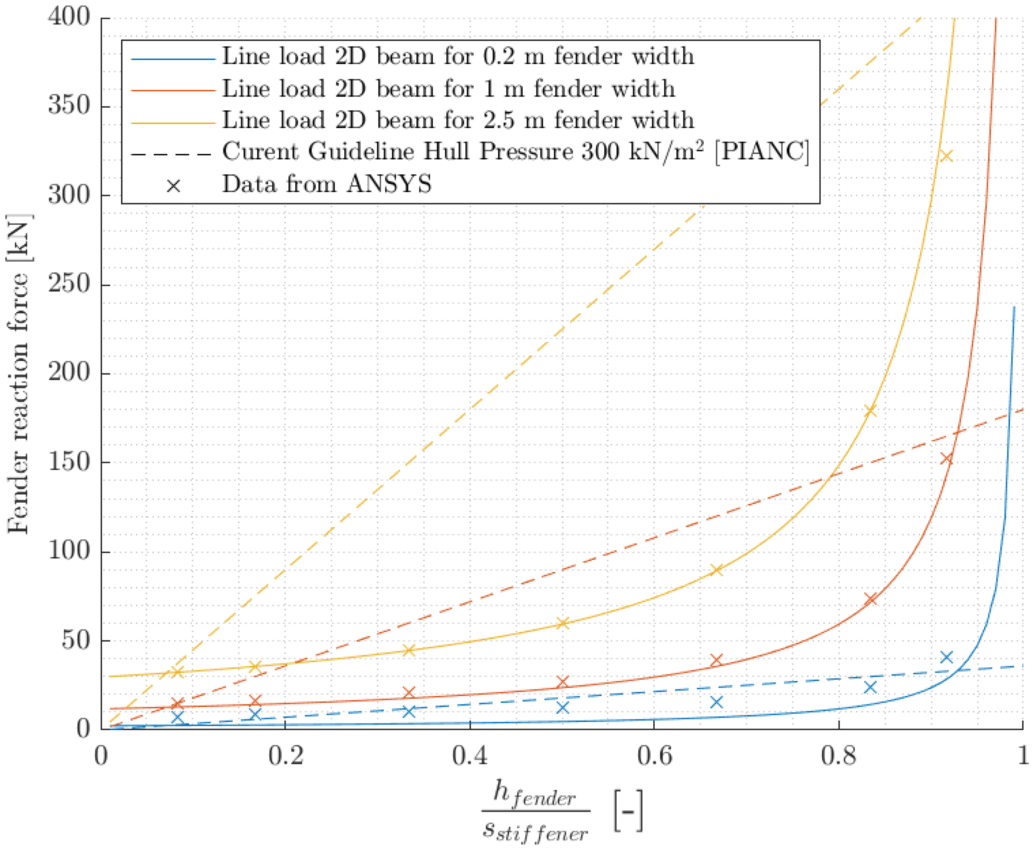


Figure 4.9: Maximum capacity of a plate on hinged boundary conditions, due to concentrated load, for centred loaded case ($s_{stiffener}$ = 0.6 [m], $s_{webframe}$ = 2.7 [m], t_p = 9 [mm], σ_Y = 235 [MPa])

Figure 4.10 gives the maximum capacity of the reference plate for a broader range of fender widths following the analytical approach. An important difference with figure 4.9 which should be noticed is that the x-axis is now used for the normalised fender width instead of the height. The different fender heights are given in the form of separate lines. The same figure is made for all different vessel types discussed in section 2.6, and will be used to compare with the other failure modes in chapter 7.

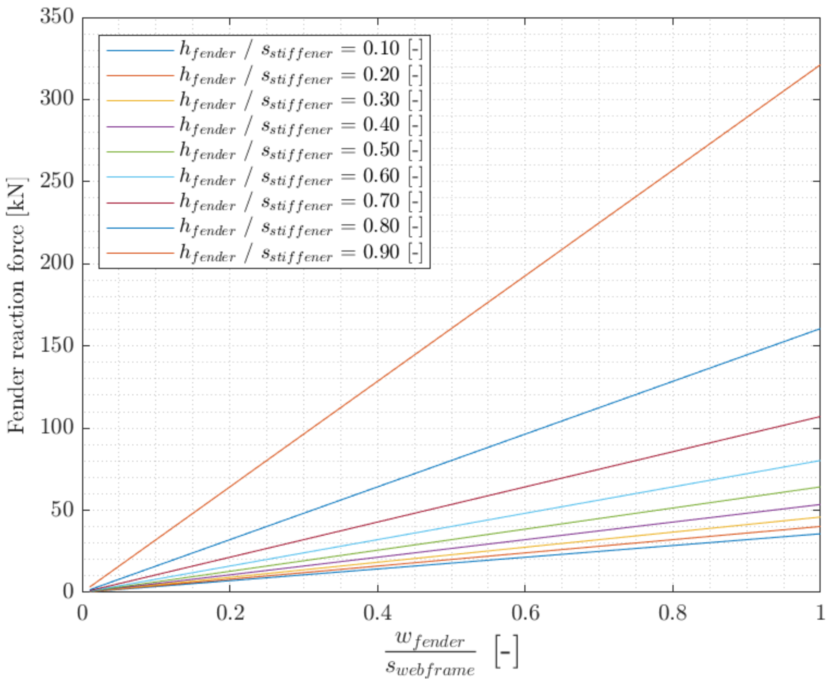


Figure 4.10: Maximum capacity of a plate with hinged boundary conditions, due to concentrated load, for centred loaded case ($s_{stiffener}$ = 0.6 [m], $s_{webframe}$ = 2.7 [m], t_p = 9 [mm], σ_Y = 235 [MPa])

4.3. Loading near the Edge with Soft Fender

This section described the behaviour of the plate when it is loaded near the edge. As mentioned in the introduction of this chapter, this will not result in the largest internal stresses of the plate. However, it is necessary to describe the behaviour of the grillage when a soft fender makes contact with a stiffener.

4.3.1. Analytical 2D

For a uniform load case in which the centre of the fender makes contact with the stiffener, the fender height will result in pressures around the stiffener. The plating will deform around this stiffener. Figure 4.11 gives a simplified beam representation for an analytical approach of the plate. The red line represents a strip of the plate at the centre of the width, resulting in a free body diagram shown on the right side of the figure. The hinged boundary represents the connections to the stiffeners.

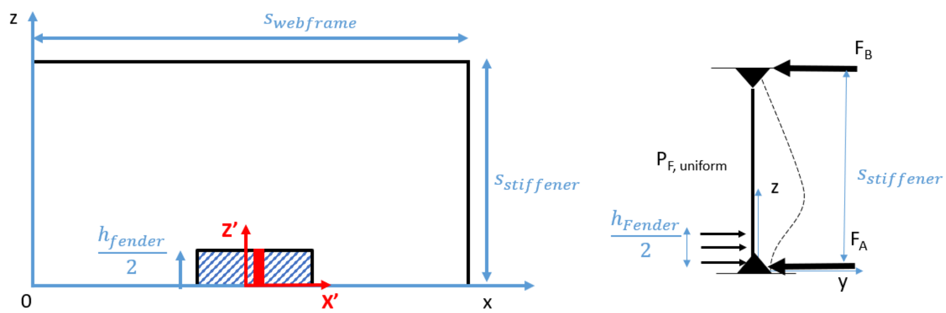


Figure 4.11: Overview of 2D strip representation, and corresponding free body diagram with a uniform loading from the edge

In order to find the maximum allowable pressure, the yielding moment is substituted as the maximum moment that occurs in the plate. This maximum moment occurs at:

$$z_{M,max} = \frac{h_{fender}}{2} - \frac{h_{fender}^2}{8 \cdot s_{stiffener}} \tag{4.16}$$

This is found by a static approach in order to find the reaction forces F_B, and F_A. A more extensive overview of the formulations used can be found in appendix A.2.

The obtained formulation of the height is used to describe the internal moment development along the z-direction in equation (4.17).

$$M(z) = -F_A \cdot z + \left(P_{F,uniform} \cdot z\right) \cdot \frac{z}{2} \tag{4.17}$$

Substituting the $z_{M,max}$-coordinate, equation (4.16), and reaction force F_A, appendix A.2, results in a formulation for the maximum moment in equation (4.18).

$$M(z_{M,max}) = -\frac{P_{F,uniform} \cdot \left(\frac{h_{fender}}{2} - \frac{h_{fender}^2}{8 \cdot s_{stiffener}}\right)^2}{2} \tag{4.18}$$

By equating this maximum moment with M_Y, the maximum allowable pressure is derived.

$$P_{F,uniform} = -\frac{2 \cdot M_Y}{\left(\frac{h_{fender}}{2} - \frac{h_{fender}^2}{8 \cdot s_{stiffener}}\right)^2} \tag{4.19}$$

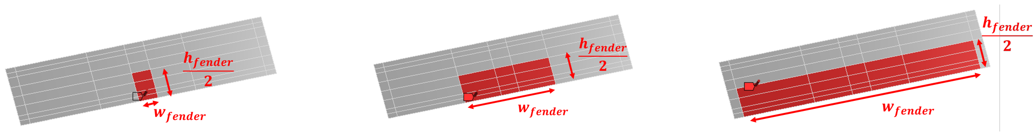
4.3.2. Analytical 3D

The theoretical approach as discussed in subsection 4.1.2 is also used for the loading from the edge. As shown in figure 4.3 the parameter η should be equal to $\frac{h_{fender}}{2}$ to describe the loading from the edge. The same formulations can be used by changing this parameter only.

4.3.3. Finite Element Method

This loading scenario is also verified with an FEA performed in ANSYS Workbench [1]. Three different fender widths and seven different fender heights are used to check the maximum allowable uniform pressure on the plate for loading from the edge, shown in figure 4.12.

The same yielding limit as the previous sections is used, 235 MPa. The mesh is once more equal to x times the plate thickness and assumed as converged when a 5% range is obtained in the actual results.



(a) 0.2meter fender width (b) 1.0 meter fender width (c) 2.5 meter fender width

Figure 4.12: Overview of verified uniform loading on a plate from the edge

4.3.4. Verification

Figure 4.13 gives the FEM data from ANSYS, in combination with the analytical formulations. The x-axis is the normalised fender height and has a maximum value of two times the stiffener spacing. This is reasonable since this situation represents the loading situation in which a fender makes contact with the stiffener at the edge of the plate. Therefore, only half the fender height will make contact at this plate, and a fender height of two times the stiffener spacing will cover the full plate height.

The figure shows that the 3D analytical approach gives an overestimation for fenders with a very small height. In general, the 2D approach is of a conservative kind, which is preferable. Only the very small fender widths show differences with FEM that can be seen as too conservative. Another detail that should be noted is the increase in maximum allowable loading for larger fender heights following the ANSYS data, and this is observed for the 2D approach as well. However, the 3D approach is decreasing for increasing fender heights.

The uniform pressure for a centrally loaded case is per definition smaller and can therefore be assumed as the only relevant failure mode for uniform pressure. Despite this, these plots are still useful in order to confirm the 3D plate theory that is used.

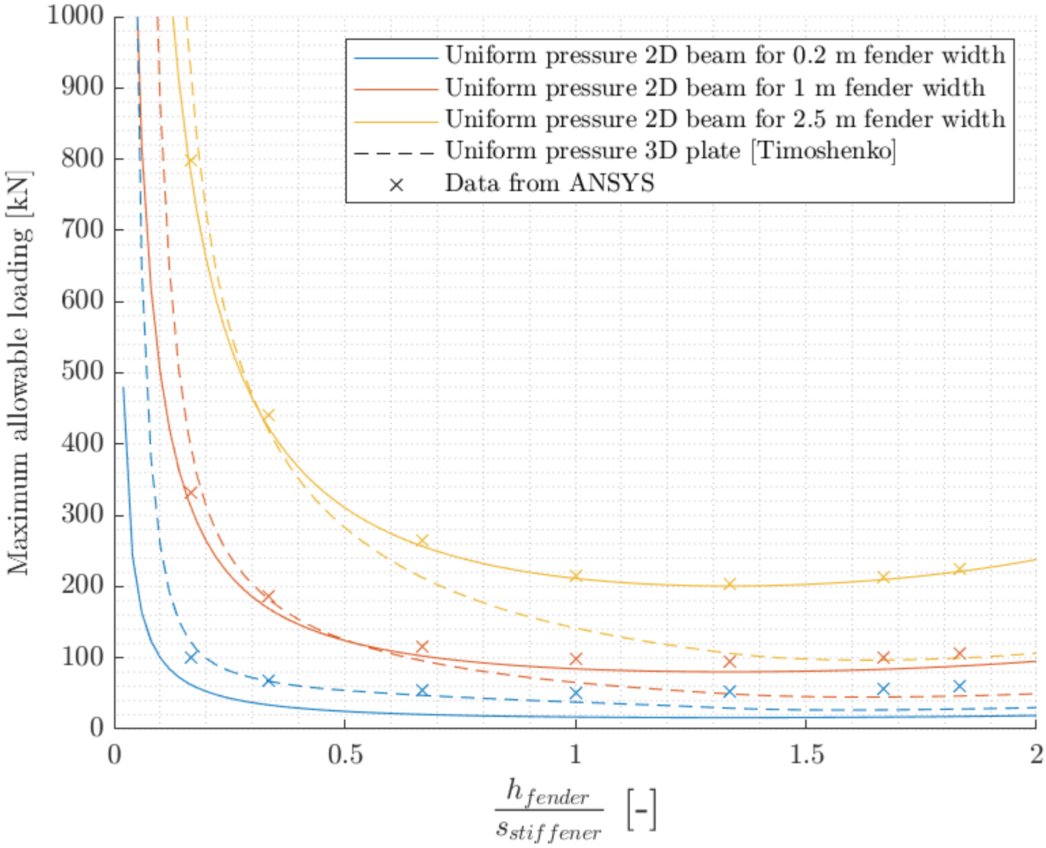


Figure 4.13: Maximum capacity of a plate on hinged boundary conditions, due to uniform pressure, $P_{F,uniform} \cdot A_{contact}$, for loaded from edge case ($s_{stiffener}$ = 0.6 m, $s_{webframe}$ = 2.7 m, t_p = 9 mm, σ_Y = 235 MPa)

Since the 2D approach is conservative in all scenarios, this approach is extended to a full analysis. Figure 4.14 gives the maximum capacity of the reference plate for a broader range of fender widths following this 2D analytical approach. An important difference with figure 4.6 which should be noticed is that the x-axis is now used for the normalised fender width instead of the height. The different fender heights are given in the form of separate lines. The same figure is made for all different vessel types discussed in section 2.6, and will be used to compare with the other failure modes in chapter 7.

As noted in figure 4.13, it can be seen in this figure that the largest fender heights do not correspond to the minimal plate capacity. In this figure, the least capacity corresponds to a fender height of 1.4 times the stiffener spacing.

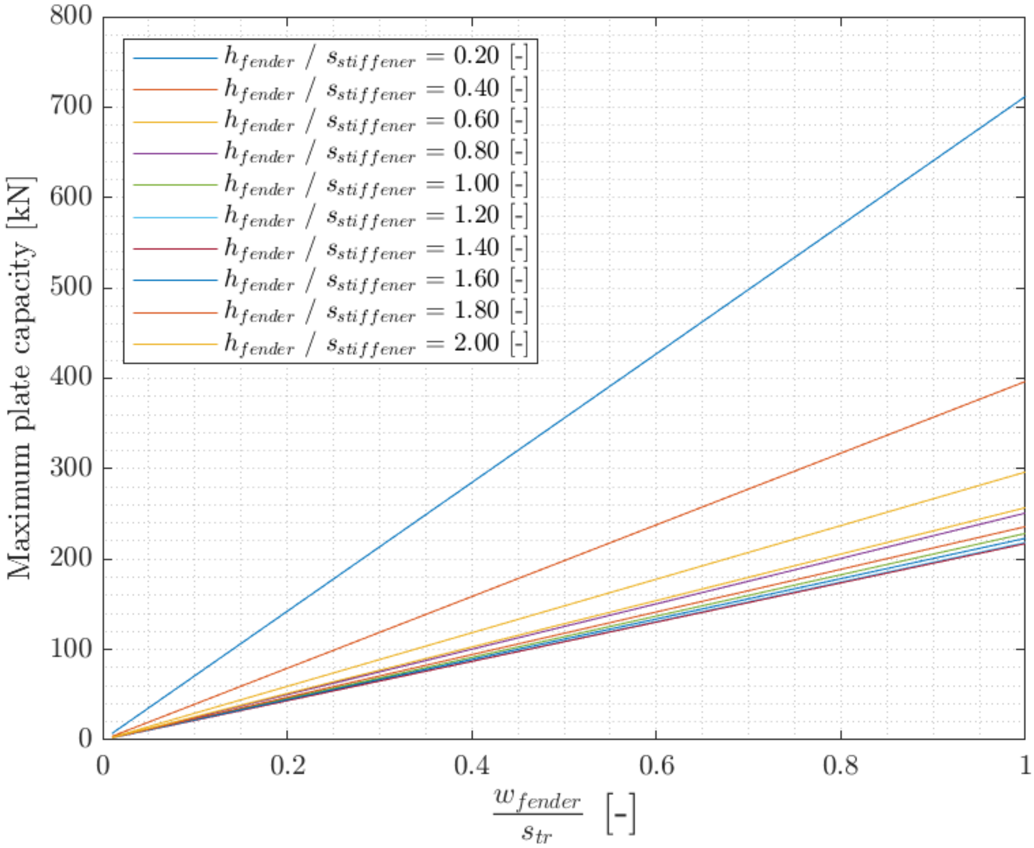


Figure 4.14: Maximum capacity of a plate with hinged boundary conditions, due to uniform pressure $P_{F,uniform} \cdot A_{contact}$, for loaded from edge case ($s_{stiffener}$ = 0.6 [m], $s_{webframe}$ = 2.7 [m], t_p = 9 [mm], σ_Y = 235 [MPa])

5
Stiffener Failure

This chapter elaborates on the rigidity of the longitudinal stiffeners in the grillage. The chapter is divided into two different sections describing the soft fender contact area and the behaviour due to a rigid fender contact area. The boundary conditions for the stiffeners are fixed since the ends of the stiffeners are assumed to be connected to vertical watertight web frames.

The yielding moment of the stiffener is highly dependant on the cross-section of the stiffener. In the different ship types that are discussed in section 2.6, nine different stiffener types are distinguished. The ships are numbered from 1 to 11 with the first letter of the ship type in front of the number, i.e. Tanker T, Container C or Bulk B. Four main categories in these stiffeners can be distinguished, and their cross-sections are presented in figure 5.1.

Holland Profiles (HP) are denoted as: HP web height x web thickness in mm.
L profiles are denoted as: L web height x web thickness + flange width x flange thickness in mm.
T profiles are denoted as: T web height x web thickness + flange width x flange thickness in mm.
Flat Bar profiles (FB) are denoted as: FB web height x web thickness in mm.

Tankers:

- **T1:** HP 180x10
- **T2:** HP 120x8
- **T3:** L 250x10 + 90x14
- **T4:** L 300x13 + 90x17
- **T5:** T 450x12 + 150x15

Container vessels:

- **C6:** HP 220x10
- **C7:** L 280x12 + 15x120
- **C8:** L 275x12 + 125x12

Bulk cariers:

- **B9:** HP 200x9
- **B10:** FB 150x15
- **B11:** T 550x12 + 150x20

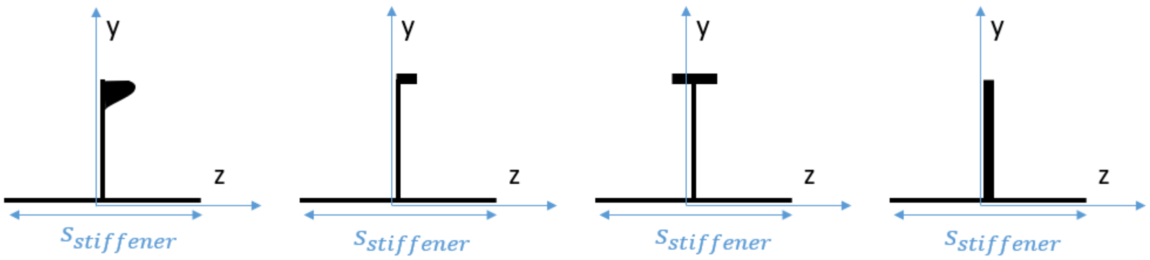


Figure 5.1: Overview of the stiffener categories. Left to right: Holland Profile (HP), L- profile (L), T- profile (T), Flat Bar (FB).

In this chapter, the T 450x12 + 150x15 stiffener is used in order to verify the analytical calculations numerically in FEA. In the analytical approach, two main criteria will be checked. First of all, the bending moments may not result in stresses exceeding the yield stress. This will be checked in both the clamped ends and in the centre of the loading. Secondly, the shear stress within the web may not cause yielding.

Bending moment

The maximum bending moment is calculated via equation (5.1).

$$M_Y = \sigma_{xx,Y} \cdot \frac{I}{y} \tag{5.1}$$

In this calculation, the effective width of the plate at which the stiffener is attached is taken into account, as discussed in the Technical Background in subsection 2.4.3. An important step in this approach is the determination of the height of the neutral axis. This is important since the maximum stresses are present in the outer fibres where y is equal to y_n (figure 5.2) in order to calculate the bending stresses at the bottom of the plate. The bending stresses in the top of the flange are calculated with an y equal to the stiffener height minus y_n. The stresses due to the bending moments will be discussed per loading distribution type in section 5.1 and section 5.2. In these sections, beam equations are used to formulate the maximum allowable loading, based on this maximum bending moment.

In equation 5.1 the yield stress, $\sigma_{xx,Y}$ is equal to the maximum normal stress in the x-direction. This stress is expressed as a function of the equivalent Von Mises stress and the Poisson's ratio by assuming plane stress perpendicular to the surface of the plate and flange because they are relatively thin. Which means:

$$\sigma_{yy} = 0 \tag{5.2}$$

Hooke's law with a zero ϵ_{zz} strain is applied since this plate stiffener combination is a repetitive part in a larger grillage and no strain in the z-direction is allowed due to other stiffeners. This results in a z-directed stress equal to:

$$\sigma_{zz} = \nu \cdot \sigma_{xx} \tag{5.3}$$

Substituting equation (5.3) in the Von Mises yield criterion:

$$\sigma_Y = \sqrt{\left(\sigma_{xx}^2 - \sigma_{xx} \cdot \sigma_{zz} + \sigma_{zz}^2\right)} \tag{5.4}$$

Which is rewritten into a formulation for the bending stress at the onset of yielding in the following equation.

$$\sigma_{xx,Y} = \frac{\sigma_Y}{\sqrt{\nu^2 - \nu + 1}} \tag{5.5}$$

Transverse shear

Transverse shear is a factor that is taken into account with the shear formula from Hibbeler [13], equation (5.6).

$$\tau_{xy} = \frac{V \cdot Q}{I \cdot t_{web}} \tag{5.6}$$

In this equation τ_{xy} is the shear stress in the web at a specific location with a distance z' from the neutral axis. V is the internal shear force, t_{web} the thickness of the cross-sectional area where τ_{xy} is to be determined. The moment of inertia is denoted by I, and the moment of the area A' about the neutral axis, which is denoted with Q, is given by:

$$Q = \bar{z}' \cdot A' = \left(z_n - \frac{t_p}{2}\right) \cdot b_{eff} \cdot t_p \tag{5.7}$$

Where A' is the area below the section plane where τ_{xy} is determined, and $\bar{z}'$ is equal to the distance from the neutral axis to the centroid of A'. For the specified location just above the plate $\bar{z}'$, and A' are specified in more generic terms.

The location at which the stress is distributed from the plate to the attached web of the stiffener is of special interest since the width of this cross-sectional area is small at this specific location. Figure 5.2 shows the location of interest.

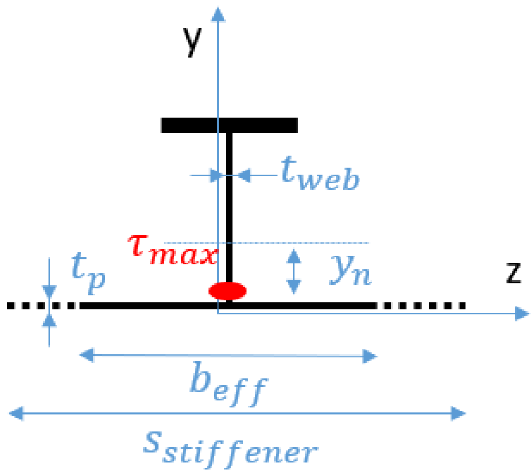


Figure 5.2: Cross-section of T stiffener

The shear stress from equation (5.6) is combined with the normal stresses in x-, and z-direction in a simplified Von Mises equation, (5.8).

$$\sigma_Y = \sqrt{\frac{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + 6 \cdot \tau_{xy}^2}{2}} \tag{5.8}$$

At the bottom of the web, the x-directed stress is assumed to be equal to the stresses in the plate discussed in the paragraph on bending. The moment is expressed as the lateral force, V, times the distance from the clamped end of the stiffener. The distance from the neutral axis to the location of interest is equal to the neutral height minus the plate thickness which leads to an x-directed stress equal to:

$$\sigma_{xx} = \frac{M \cdot y}{I} = \frac{V \cdot \left(s_{webframe} - w_{fender}\right) \cdot \left(y_n - t_p\right)}{2 \cdot I} \tag{5.9}$$

and the y-directed stress is assumed to be equal to the lateral force divided by the cross-sectional area of the plate:

$$\sigma_{yy} = \frac{V}{b_{eff} \cdot t_p} \tag{5.10}$$

Substitution of the different stress components, equation (5.6), (5.9), and (5.10) in equation (5.8), results in a maximum shear force formulation since V is the only unknown variable.

5.1. Central Loading with Soft Fender

This section describes the uniform loading on a stiffener. In this section, only the rigidity of the stiffener is of interest, and the deformation of the plate at which the stiffener is attached is not investigated. The rigidity of the plate, due to this type of loading, is discussed in section 4.3. Figure 5.3 gives an overview of the location that is discussed in this section.

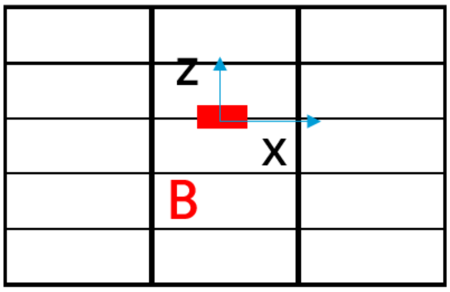


Figure 5.3: Central loading on a stiffener

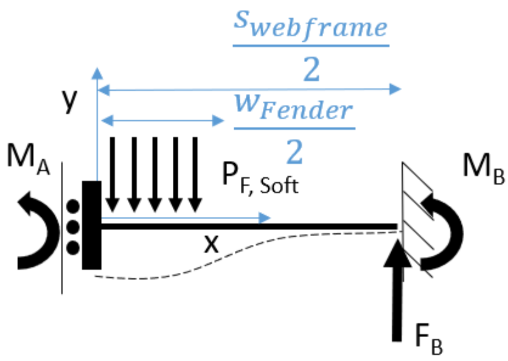


Figure 5.4: Uniform distributed pressure over the contact area

5.1.1. Analytical Beam Equations

A simplified analytical approach is found by a beam representation. Figure 5.4 represents a clamped plate stiffener combination cut in half. Since the stiffener is symmetrically loaded over the length, the centre will have an angle equal to zero. At this location, the beam is cut and simplified to a guided boundary condition with a moment equal to M_A. The clamped boundary represents the connection to the transverse web frame, with a reaction moment M_B and force F_B. The formulation for the moments at the guided end and the clamped end are used following beam equations. The derivations of these moment equations can be found in appendix A.3.

The maximum loading in Newtons is found by multiplying the maximum pressure with its area, or in the case of this 2D approach with its length. Equation (5.11) gives a formulation of the maximum allowable load for the case in which the moment A would be equal to the yield moment.

$$F_{A,max} = \left(P_{F,uniform}\right)_{A,max} \cdot \frac{w_{fender}}{2} \tag{5.11}$$

Where the maximum allowable pressure, $\left(P_{F,uniform}\right)_{A,max}$, is found by a substitution of the yield moment, equation (5.1), for both end moments. Equation (5.12), yields the formulation of the maximum allowable load in pressure form, $\frac{N}{m}$, corresponding to a specific fender size.

$$\left(P_{F,uniform}\right)_{A,max} = \frac{3 \cdot M_Y \cdot s_{webframe}}{\left(\left(\frac{s_{webframe}}{2}\right)^3 - \left(\frac{s_{webframe}}{2} - \frac{w_{fender}}{2}\right)^3\right)} \tag{5.12}$$

The derivation of the maximum allowable load for the case in which moment B would reach the yield moment is given by:

$$F_{B,max} = \left(P_{F,uniform}\right)_{B,max} \cdot \frac{w_{fender}}{2} \tag{5.13}$$

Where $\left(P_{F,uniform}\right)_{B,max}$ is given by:

$$\left(P_{F,uniform}\right)_{B,max} = \frac{24 \cdot M_y \cdot s_{webframe}}{3 \cdot s_{webframe}^2 \cdot w_{fender} - w_{fender}^3} \tag{5.14}$$

5.1.2. Verification

In order to verify the analytical approach, a FEM analysis is performed in ANSYS Workbench [1]. A plate-stiffener combination is modelled consisting of a T 450x12 + 150x15 stiffener, and a plate with an effective width of 410 mm. For this plate-stiffener combination, a parametric approach is used in which nine evenly spaced fender widths are checked along the transverse frame spacing.

In FEA, a line load is applied exactly beneath the stiffener, in order to have the main deformation of the stiffener and less deformation of the plate. Furthermore, the edges of the plate are modelled with a zero displacement in the z-direction, since another plate stiffener combination at this location is assumed to prevent the plate from any deflection in the z-direction. Lastly, the ends of the stiffener are fully clamped which represents the connection to the transverse web frames. Figure 5.5 gives an overview of the described model and corresponding boundary conditions.

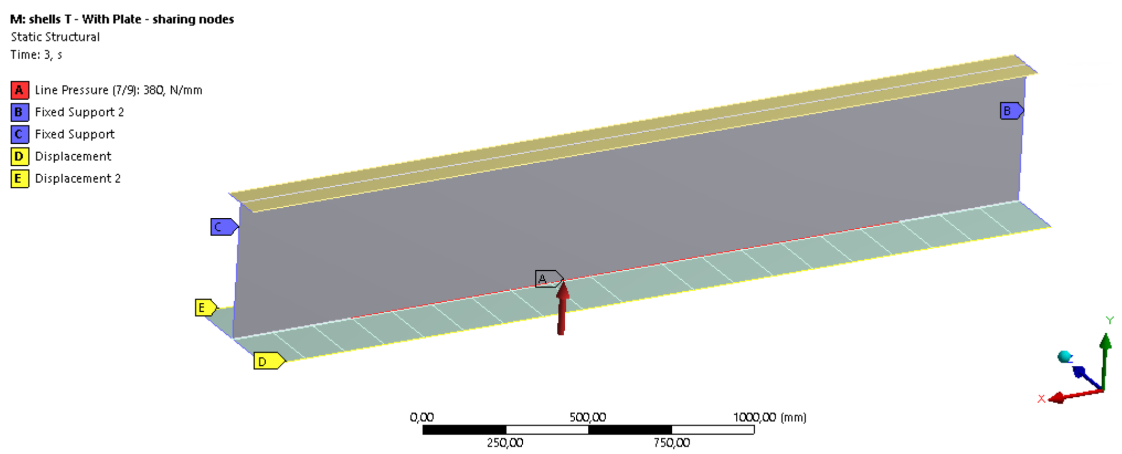


Figure 5.5: Overview of uniform loading on a stiffener in FEM

A line pressure is applied that results in stresses exceeding the yielding limit. The specific substep at which the yield stress is equal to σ_Y = 235 MPa is of interest. At this substep, the exact loading is found and a visual check of the location of this maximum stress is executed. The mesh size is chosen as five times the thickness of the plate and is reduced till the solution is within 5% in comparison with the previous result.

Figure 5.6 gives the maximum allowable pressures for all the different bending moments and the maximum shear forces. Figure 5.7 shows the resulting rigidity for this specific stiffener. This result is based on the least values from figure 5.6 since these determine where the yield stress occurs first.

It can be seen that the analytical approach shows a good comparison with respect to the global trend of the FEM results. It was seen in FEA that for small fender widths, the shear stress should be the deterministic factor. However, the analytical approach overestimated the maximum allowable shear stress in this specific case. Other differences between the FEM and analytical approach are obtained in the absolute value. A difference of 19% at the location where the differences are largest is observed. The analytical calculation results in a very conservative approach, and therefore is assumed as safe. However, if reaction forces are encountered which are not safe following the analytical approach a FEM analysis could still result in a safe outcome.

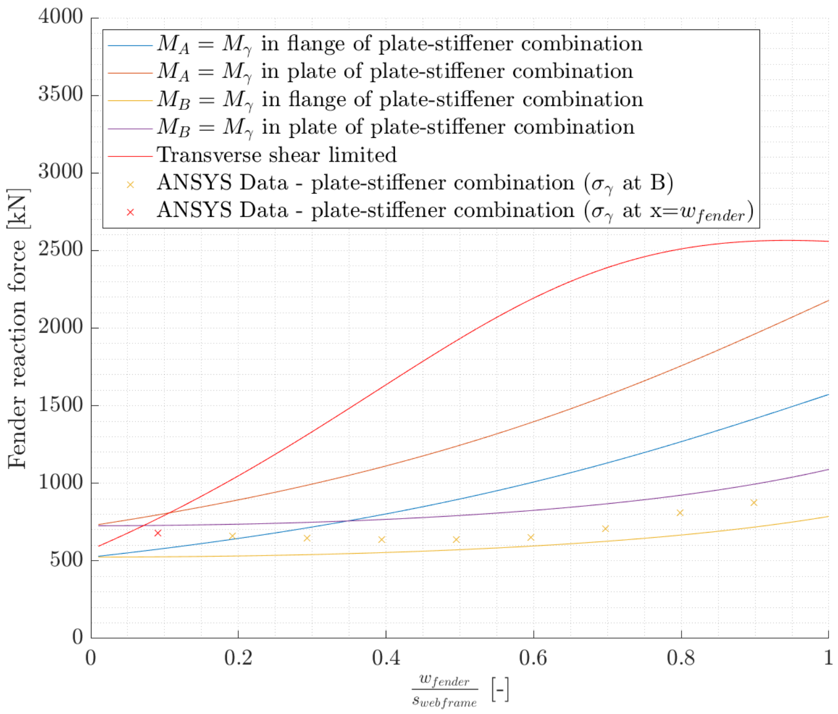


Figure 5.6: Verification of failure mode for uniform pressure on stiffener via ANSYS

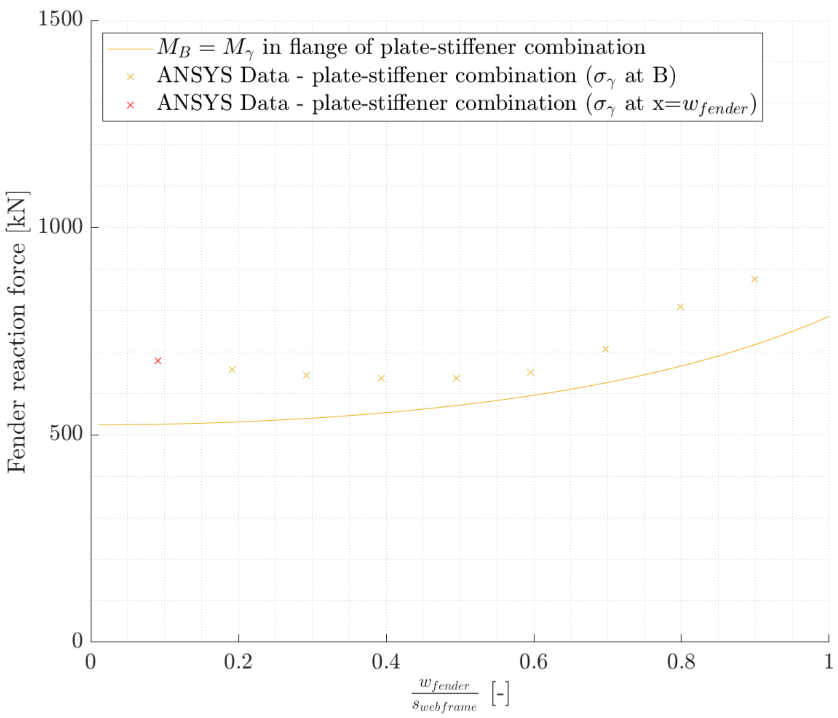


Figure 5.7: Rigidity of stiffener due to a uniform pressure

5.2. Central Loading with Rigid Fender

In this section, the rigidity of a stiffener due to concentrated load is described. With respect to the contact location on the grillage, the same figure 5.3 as used in the previous section should be kept in mind.

5.2.1. Analytical Beam Equations

A simplified analytical approach is found by a beam representation. This beam represents the plate stiffener combination. Since the stiffener is symmetrically loaded over the length, the centre will have an angle equal to zero. This is simplified to a guided boundary condition with a moment equal to M_A. The clamped boundary represents the connection to the transverse web frame, with a reaction moment M_B and force F_B. The representative free body diagram is shown in figure 5.8.

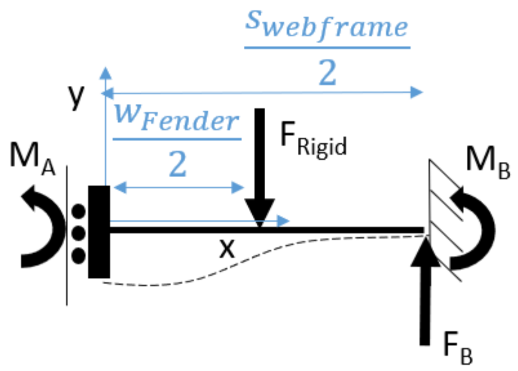


Figure 5.8: Concentrated loading at edges of the contact area

Following the beam equations, the formulation for the moment at the guided end is given by equation (5.15). Equation (5.17) yields the formulation of the maximum allowable load in Newton per meter corresponding to a specific fender size. This is rewritten to a maximum loading in Newtons by multiplying the maximum pressure with its area, or in the case of this 2D approach with its length.

Once more, the moment is calculated for locations A and B. Equation (5.16) gives a formulation of the maximum allowable load for the case in which the moment A would reach the yield moment.

$$M_A = \frac{F_{F,concentrated} \cdot \left(\frac{s_{webframe}}{2} - \frac{wfender}{2}\right)^2}{s_{webframe}} \tag{5.15}$$

$$F_{A,max} = \frac{M_Y \cdot s_{webframe}}{\left(\frac{s_{webframe}}{2} - \frac{wfender}{2}\right)^2} \tag{5.16}$$

The derivation of the maximum allowable load for the case in which moment B would reach the yield moment is given by:

$$M_B = \frac{-F_{F,concentrated} \cdot \left(\left(\frac{s_{webframe}}{2}\right)^2 - \left(\frac{wfender}{2}\right)^2\right)}{s_{webframe}} \tag{5.17}$$

$$F_{B,max} = \frac{M_Y \cdot s_{webframe}}{\left(\left(\frac{s_{webframe}}{2}\right)^2 - \left(\frac{wfender}{2}\right)^2\right)} \tag{5.18}$$

$$\tag{5.19}$$

5.2.2. Verification

The same model as discussed for the uniform load is used as the basis for the stiffener in the concentrated load case. An additional body is used in this model to exert the concentrated loads, since the application of a force on a node results in singularities. This indenter is modelled as a rigid body that can not deform. A remote displacement of this rigid body is applied in multiple substeps in order to find the exact moment at which an equivalent stress of 235 MPa is reached within the stiffener. Figure 5.9 gives an overview of the model and its boundary conditions in FEM.

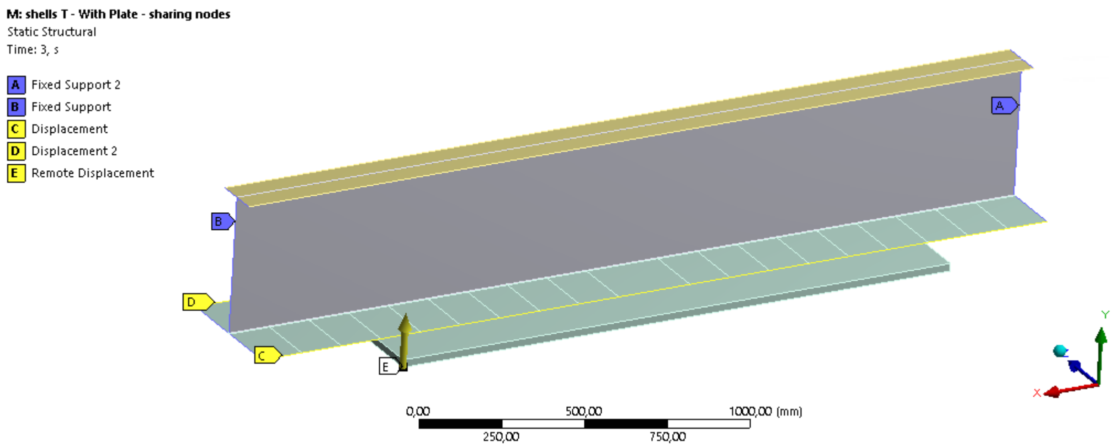


Figure 5.9: Overview of rigid indenter hitting a stiffener in FEM

The different criteria are plotted for different fender widths in figure 5.10. Furthermore, the data from ANSYS is implemented in order to verify both the magnitude of the maximum loading as well as failure location within a plate stiffener combination. The locations at which the yield stress occurs first, are denoted with different colours. These colours are in accordance with the colours of the analytical approach.

Figure 5.11 gives the maximum allowable force for the relevant location within the stiffener, by showing the least values of figure 5.10 only.

The resulting data from ANSYS shows that the shear limitation is the deterministic factor for the failure of this stiffener larger fender widths. For smaller fender widths, the bending moment is deterministic for failure. In general, the analytical approach is safe, since the values are smaller than the ones obtained with FEM. The exact failure mode does not correspond at all widths for the analytical and FEM approach. However, the FEM results in situations in which the stresses in the web due to shear, and in the flange or plate due to bending, are almost identical for these widths.

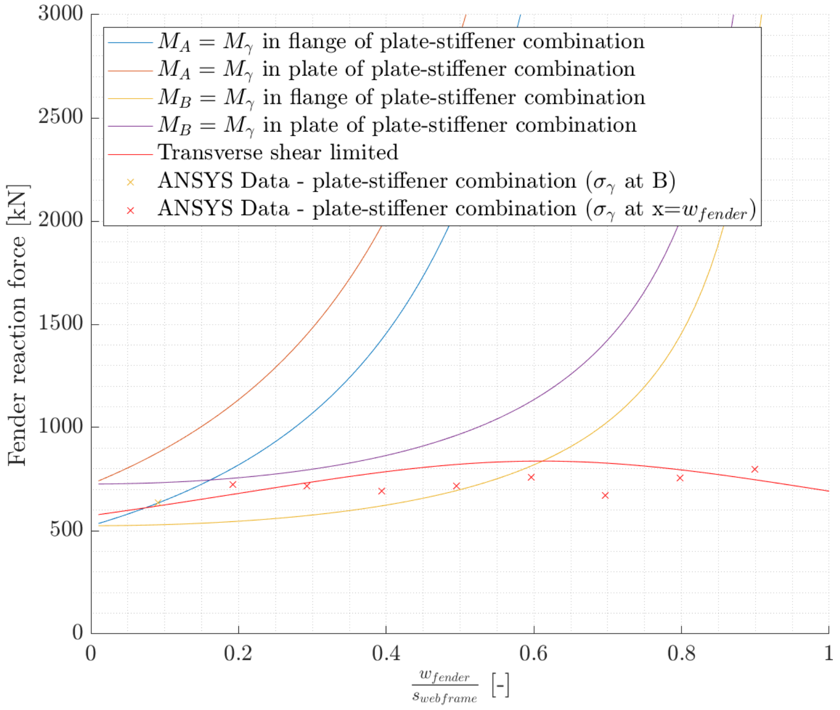


Figure 5.10: Verification of failure mode for concentrated load in stiffener via ANSYS

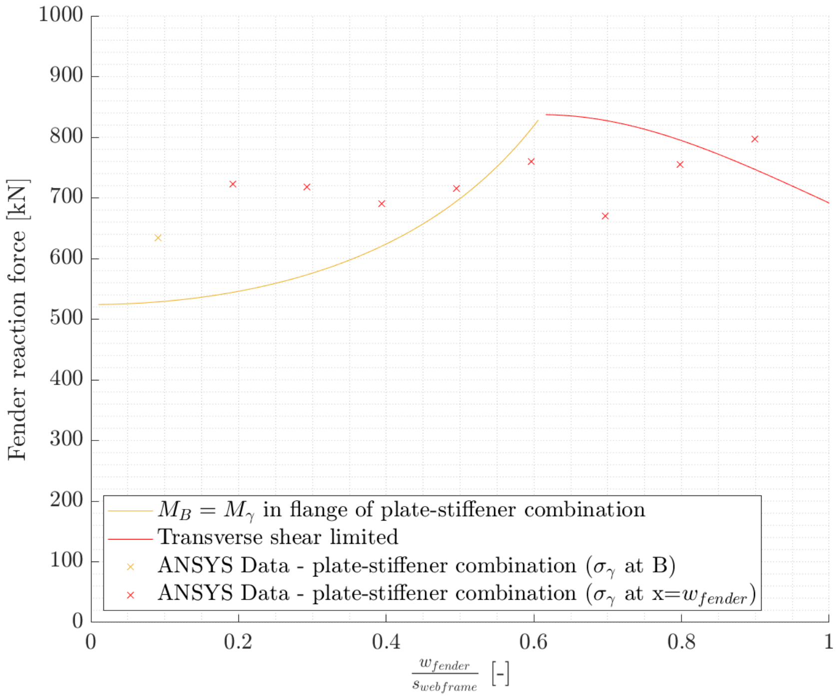


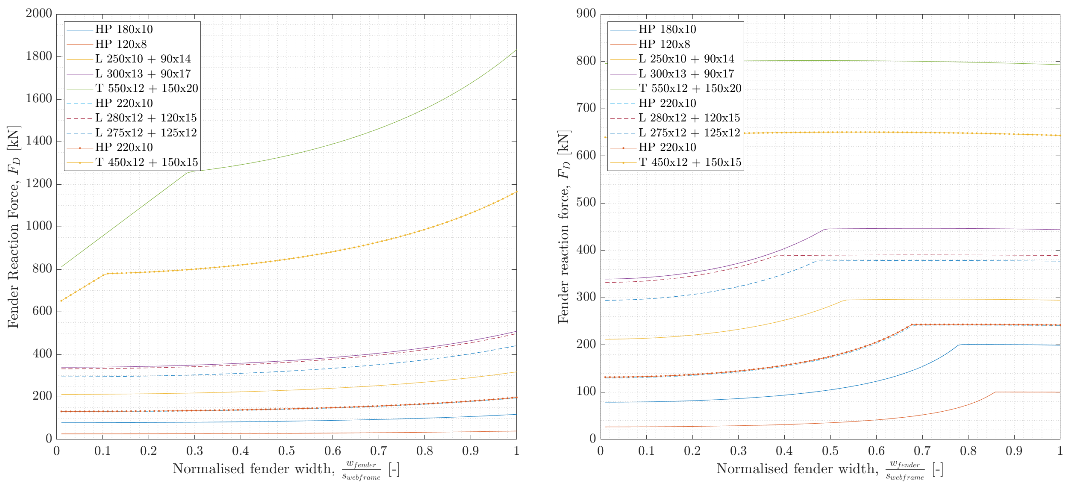
Figure 5.11: Rigidity of stiffener due to concentrated load

5.3. Stiffener Type Sensitivity

In the introduction of this section, the different stiffener types that were used by TNO in their analysis of cylindrical fenders are given. In order to obtain a better insight into the influence of the geometry of the specific stiffener type on the maximum allowable loading, all stiffeners are compared with each other. The web frame spacing, i.e. the length of the stiffener, is noted as another important parameter and therefore the vessel category with the smallest web frame spacing, and the vessel category with the largest web frame spacing are used to cover the total content, i.e. 1995 mm and 5680 mm for respectively the second and fifth tanker category. Eventually, the stiffeners correspond to a specific vessel's structural arrangement, so some comparisons of a stiffener in combination with a specific web frame spacing are not always relevant to make. In subsection 5.3.2, the different stiffeners and the corresponding web frame spacing are compared with one another. Eventually, a generalised formulation is presented in subsection 5.3.3.

5.3.1. Smallest versus Largest Web Frame Spacing

The smallest web frame spacing that is found in section 2.6 for the used vessel categories is 1995 mm. The results in figure 5.12 show that larger stiffeners result in higher maximum allowable loading, which is as one would expect. These figures show that the difference between the maximum allowable loading is very large, since the HP 180x10 has already a maximum allowable force that is twice as high as for the HP 120x8 type stiffener. Furthermore, it is found that large stiffeners in combination with a small web frame spacing result in shear failure for the soft fender as well, noticed by the diagonal part in the curve. For the rigid fender, this shear failure is recognised by a flat or slightly decreasing curve. From this figure, it can be concluded that the specific stiffener geometry in the side hull of vessels entering port can result in much higher allowable fender reaction forces on the ships hull.

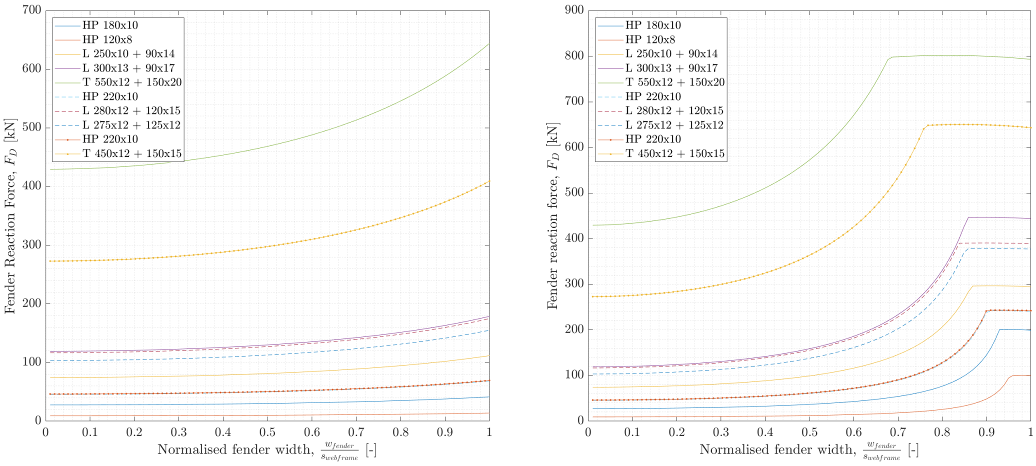


(a) Stiffener type sensitivity for soft fender

(b) Stiffener type sensitivity for rigid fender

Figure 5.12: Smallest considered web frame spacing, i.e. 1995 mm

In figure 5.13 the stiffeners are compared with each other for the largest web frame spacing used in this study, i.e. 5680 mm. In these figures, it is found that the shear stress is not relevant for a soft fender contact area, whilst it was for small fender widths in figure 5.12. Also for the rigid fender contact area, a difference with figure 5.12 can be found. The normalised width at which the shear stress becomes important is higher for all stiffener geometries. Therefore, not only the stiffener geometry but also the web frame spacing is important in order to understand the behaviour of this failure mode.



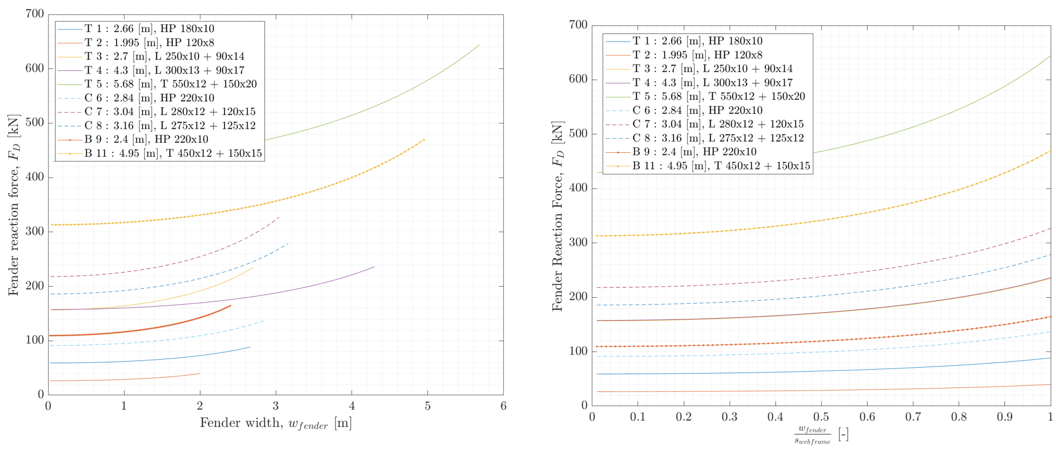
(a) Stiffener type sensitivity for soft fender

(b) Stiffener type sensitivity for rigid fender

Figure 5.13: Largest considered web frame spacing, i.e. 5680 mm

5.3.2. Normalised for Corresponding Web Frame Spacing

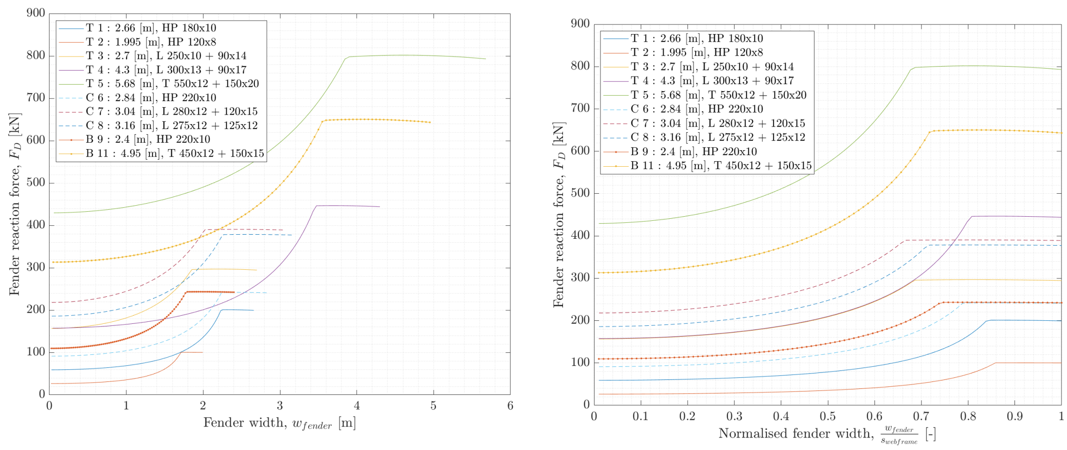
The used stiffeners correspond to a specific web frame spacing. This is presented in figure 5.14a and 5.15a. In order to compare it more easily, all fender widths are divided by the corresponding web frame spacing. This normalised result is shown in figure 5.14b, and 5.15b for respectively uniform pressure and the concentrated loads. From these plots, it is noted that the maximum allowable loading is still highly dependant on the specific vessel type that encounters the fender. The smallest vessels seem the most vulnerable with respect to the stiffeners. Another factor that is not yet implemented in this chapter is the spacing of the stiffeners itself. Since the stiffeners are placed closer together in smaller vessels according to section 2.6 the fender force can be split over multiple stiffeners, and the actual allowable loading increases. Therefore, no direct conclusions can be made from this figure. This will be discussed in chapter 6, the combination of the results.



(a) Absolute fender width, with a maximum equal to web frame spacing

(b) normalised with the corresponding web frame spacing

Figure 5.14: Maximum allowable uniform pressure on a stiffener per vessel category



(a) Absolute fender width, with a maximum equal to web frame spacing

(b) normalised with the corresponding web frame spacing

Figure 5.15: Maximum allowable concentrated loading on a stiffener per vessel category

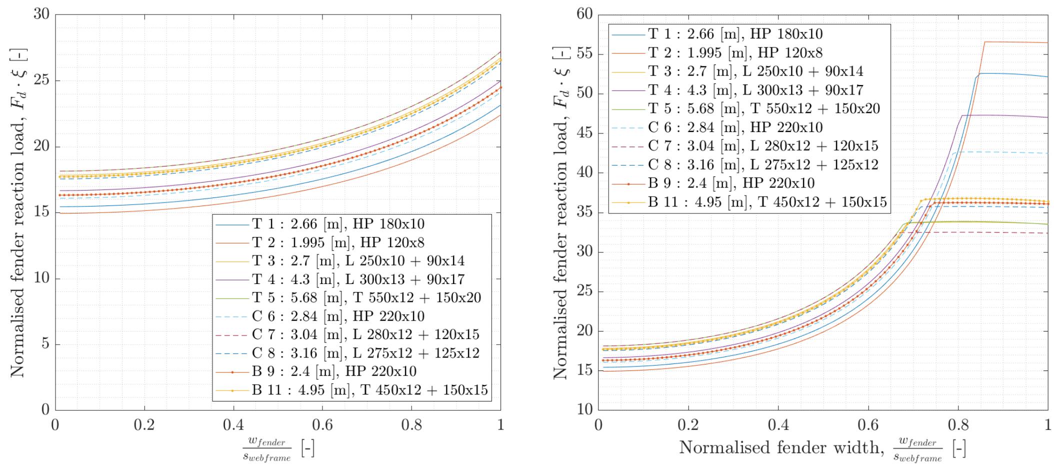
5.3.3. Generalised Formulation

In order to find a general formulation that yields for all different vessel types, and stiffeners, the force is normalised with a coefficient ξ, which is formulated by:

$$\xi = \frac{s_{webframe} \cdot h_{stiffener}}{I_{stiffener} \cdot \sigma_Y} \tag{5.20}$$

This coefficient requires the specific structural lay-out of the vessel to be known. Such as the web frame spacing, $s_{webframe}$, stiffener height, $h_{stiffener}$, yield stress of the material, σ_Y, and the moment of inertia of the plate stiffener combination, $I_{stiffener}$. This moment of inertia includes the specific particulars of the stiffener, as well as the effective width of the plate. In this way, the stiffener spacing is also included in the coefficient.

The maximum allowable forces derived in figures 5.14b, and 5.15b are multiplied with this factor, resulting in figure 5.16. In this figure, a general trend is found for the different vessel types, and the maximum difference between the lower and upper bound is found to be equal to 121.4%. Except for the wide rigid fender contact areas, where $\frac{w_{fender}}{s_{webframe}}$ exceeds 0.73. In this shear stress governed region a maximum difference of 174.3% is found.



(a) Soft fender contact area

(b) Rigid fender contact area

Figure 5.16: Maximum allowable loading on a stiffener per vessel category

The lower boundaries in these figures are used for the proposed design code. This results in a conservative formulation, with a safety factor that reaches a maximum equal to the discussed percentages. The resulting curves are given in figure 5.17.

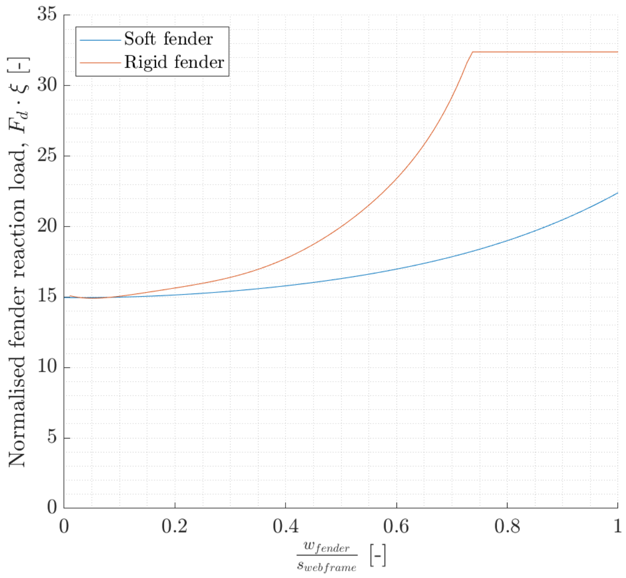


Figure 5.17: Maximum normalised fender reaction force per stiffener

A mathematical formulation of these curves is found by fitting a fourth and sixth-order polynomial, for respectively the soft and rigid fender contact area. The maximum allowable reaction force on one stiffener is defined by:

$$F_{d,Soft}(x,\xi) = \frac{6.182x^4 - 6.073x^3 + 7.791x^2 - 0.4645x + 14.97}{\xi} \tag{5.21}$$

$$F_{d,Rigid}(x,\xi) = min\left(\frac{1483x^6 - 3115x^5 + 2561x^4 - 985.8x^3 + 195.6x^2 - 13.5x + 15.21}{\xi}, \frac{32.4}{\xi}\right) \tag{5.22}$$

In these equations the x is represented by $\frac{w_{fender}}{s_{webframe}}$, and ξ follows from equation (7.2). In the next chapter, these formulations will be extended to a formulation that takes into account multiple stiffeners in the side hull. The specific structural layout of a vessel and the particulars of the stiffener used are not always known for port developers. Therefore, the different vessel types will also be discussed in the next chapter

6 Combination of the Results

The results are discussed per deformation mode in chapter 4 and 5. For every deformation mode, a resulting capacity is obtained for specific fender geometries. The results of the failure modes are evaluated, and the failure mode with the lowest capacity for a certain fender geometry is essential. In this chapter, these results are combined and for one specific grillage, the capacity is given in a graph and a corresponding table for different fender sizes. The grillage used in this chapter represents the largest tanker vessel, T5. Note that the particulars of this vessel are different from the ones used in the previous chapters. This is due to the fact that the described plate and stiffener do not correspond to the same vessel category. The resulting graphs and tables of the other vessel categories can be found in appendix B.

The particulars that correspond to the tanker vessel T5 are given by:

- T 550x12 + 150x20 stiffener type
- 5.68 metres transverse web frame spacing
- 6.44 metres spacing between the decks, i.e. 6 stiffeners
- 0.92 metres stiffener spacing
- 20 millimetres shell plate thickness

In this thesis, it is assumed that the load is equally distributed over the minimum amount of stiffeners corresponding to a certain fender height. Therefore, the allowable loading tends to make jumps at the moment the fender height exceeds the spacing of the stiffeners or a multiple of this spacing. A fender height that is normalised by the stiffener spacing should therefore be rounded down to a whole number. E.g. this means that a fender height of 5 times the fender height results in the same allowable loading on the vessel as a fender height of 5.9 times the stiffener spacing, or every height in between. In all cases, 5 effective stiffeners are considered.

Eventually, the results give both the absolute values, as well as normalised values of the fenders geometry. The appendix consists of normalised values as well, with the intention to compare the different vessel categories with one another. Conclusions on this comparison are made in the next chapter. In this chapter, the absolute values are used to compare the results with examples of current practice, obtained from fender manufacturer Trelleborg AB. The different fender panels that are considered can be found in table 6.1. The different projects are assigned with a specific colour in order to compare the results with the proposed approach from this thesis.

Table 6.1: Currently used fender panels with corresponding dimensions

	Design Reaction Force [kN]	Height [mm]	Width [mm]	Design Hull Pressure [kN/m^2]
Reference Project #1	3280	4000	5500	150
Reference Project #2	3472	7300	3500	150
Reference Project #3	3692	6050	2400	250

In table 6.1, it can be found that reference project 2 exceeds the spacing of the decks, with respect to the height of the fender. This results in a contribution of the decks of the vessel, while the decks are assumed as rigid in this thesis. Therefore, the rigid fender approach results in an infinite high allowable loading on the ship. The soft fender approach assumes a fender height that is equal to the maximum height, for the reference projects that exceed the deck spacing. The different pressure distributions are described per section.

6.1. Soft Fender

Figure 6.1 gives the resulting maximum allowable loading for soft fenders for the fifth tanker vessel type. A detailed overview of this behaviour follows from table 6.2. The plate failure is indicated in the graph as a specific fender height that is equal to a specific number and denoted with a solid line. The stiffener failure is indicated with a dashed line, and a '>' sign in the legend. As said before, the amount of effective stiffeners results in jumps in the graph, and no interpolation is admissible with respect to the height. In order to understand this behaviour more clearly, a three dimensional plot is given in figure 6.2. In this figure, the step-wise increase can be observed in the direction of the normalised fender height. With respect to the fender width, a continuous increase can be observed and therefore interpolation is admissible.

Table 6.2 is used to indicate the maximum allowable hull loading that corresponds to the geometries used in practice. In table 6.2 both the absolute and normalised values of the fender geometry are given. The absolute values are given in order to denote the exact geometries of interest more clearly. The geometry of the reference projects follows from table 6.1. In the table, the maximum force is given in kN. The difference between the plate and stiffener failure is indicated with a horizontal line, where plate failure corresponds to normalised fender heights smaller than or equal to 1 time the stiffener spacing. Different marker colours are used in the table. The orange, purple, and blue markers are used to indicate the reference project following table 6.1. A grey marker is used if the force is the same as for a smaller fender geometry. This is also depicted in the graph by overlapping lines. The overlapping lines, and thus the grey marked values in the table, indicate that the stiffener is the weakest link for wide soft fenders in this specific vessel.

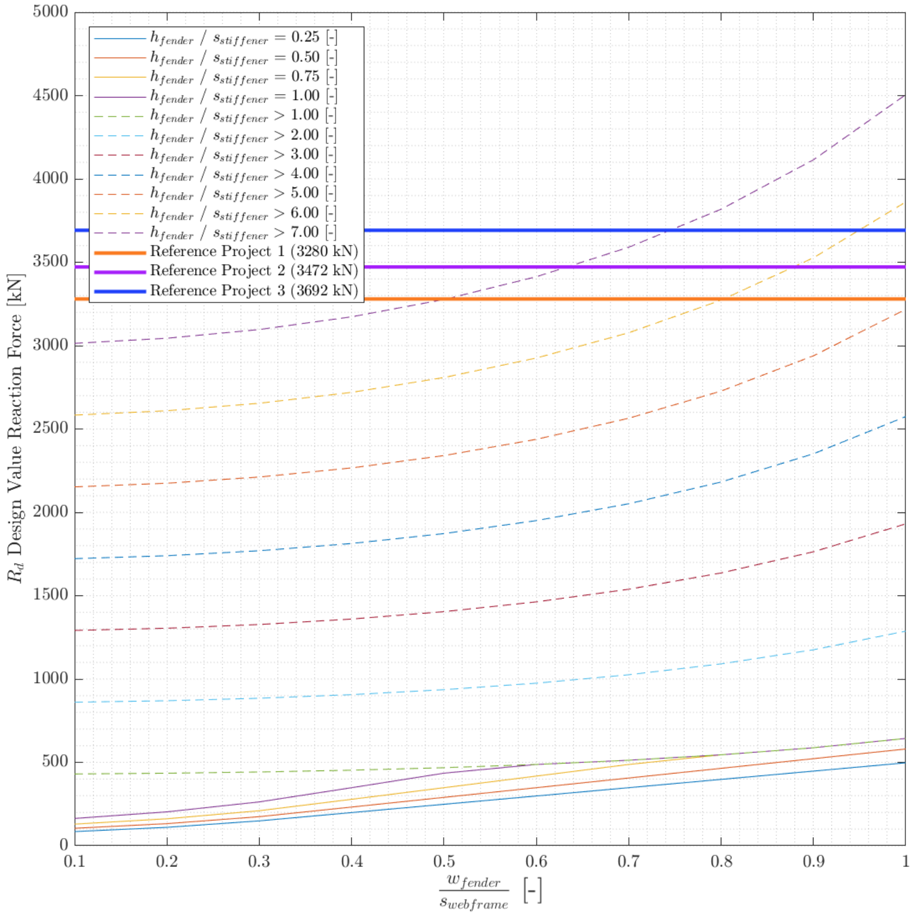


Figure 6.1: Maximum allowable force in kN on the grillage, Tanker 5, due to soft fender contact area

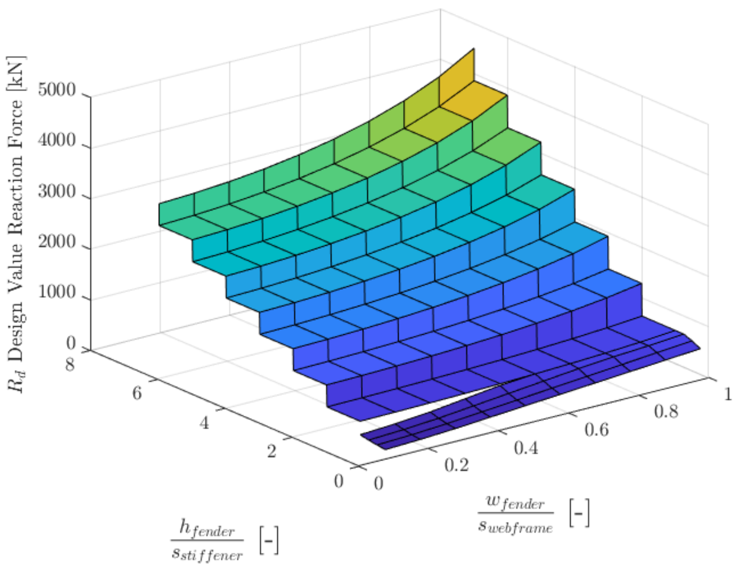


Figure 6.2: Influence of contact area on the maximum allowable loading in 3D

The reference projects from table 6.1 are indicated in table 6.2 with respect to their geometry. Just as was concluded from the graph, an interesting and concerning result is found in table 6.2. The reaction force corresponding to their geometry is lower than indicated in table 6.1. Therefore, it is assumed that for soft fender contacts, this geometry results in stresses in the vessel that exceed the yield stress, whilst it fulfils the current design guidelines. However, the panel in front of the fender is not resulting in a fully uniform distributed pressure on the vessel in reality. Despite that, all assumptions in this method are of a conservative kind, and the particulars of the geometry are rounded down in this table, with respect to the real height and width, the difference in reaction force and capacity is too large to be justified.

Project 2 yields that the panel exceeds the deck spacing, with respect to the height. Therefore the assumption is that the decks will contribute in a way that enlarges the allowable pressure extensively. Furthermore, the order of magnitude of the current and proposed criteria is close for this project. This does not necessarily mean that one of the criteria is right, but the current guidelines could result in acceptable forces following the analyses used in this thesis.

Project 1 and 3 seem to have a possible geometry following the proposed analysis that is assumed to be safe, as discussed by the specific geometries above.

Table 6.2: Maximum allowable force in kN on the grillage, Tanker 5, due to soft fender contact area. Reference projects 1 to 3 are denoted in respectively orange, blue, and purple.

			Fender width									
	Norm. [-]		**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
		Abs. [m]	0.57	1.14	1.70	2.27	2.84	3.41	3.98	4.54	5.11	5.68
Fender height	**0.3**	0.23	85	110	149	199	249	298	348	398	448	497
	0.5	0.46	105	133	174	232	290	348	406	464	522	580
	0.8	0.69	130	162	210	279	348	418	488	545	588	644
	1	0.92	164	203	263	348	435	488	513	545	588	644
	1	0.92	431	435	442	453	468	488	513	545	588	644
	2	1.84	861	870	885	907	936	975	1026	1091	1176	1287
	3	2.76	1292	1305	1327	1360	1404	1463	1539	1636	1763	1931
	4	3.68	1722	1740	1770	1813	1872	1950	2052	2182	**2351**	2575
	5	4.60	2153	2175	2212	2266	2341	2438	2564	2727	2939	3218
	6	5.52	2583	2609	2654	**2720**	2809	2926	3077	3273	3527	3862
	7	6.44	3014	3044	3097	3173	3277	**3413**	3590	3818	4115	4506

6.1.1. Generalised formulation

In subsection 5.3.3, the generalised formulation for stiffeners is given. As discussed in this chapter, the stiffener failure mode is determining the strength of the vessel when the height of a fender exceeds the stiffener spacing. The number of effective stiffeners that correspond to the fender height function as a multiplication factor. This factor is determined by the floored value of $\frac{h_{fender}}{s_{stiffener}}$, which is indicated with the $\lfloor\ \rfloor$ signs. The total maximum reaction force in N, based on a specific height and width of the fender panel, can be determined with equation (6.1). This equation is valid till a maximum width equal to the web frame spacing, and a maximum height equal to the deck spacing. Fenders that exceed either the width or height boundary are considered to be safe, independent on the second geometry particular.

$$R_{d,Soft} = \left\lfloor \frac{h_{fender}}{s_{stiffener}} \right\rfloor \cdot \frac{6.182x^4 - 6.073x^3 + 7.791x^2 - 0.4645x + 14.97}{\xi} \tag{6.1}$$

Where:

$$x = \frac{w_{fender}}{s_{webframe}} \tag{6.2}$$

$$\xi = \frac{s_{webframe} \cdot h_{stiffener}}{I_{stiffener} \cdot \sigma_y} \tag{6.3}$$

6.2. Rigid Fender

Figure 6.3 gives the resulting maximum allowable loading for this vessel type for the rigid fender approach. A detailed overview of this behaviour follows from table 6.3. As discussed in the previous section, this rigid fender approach can not include the geometry of reference project 2 in the table due to the height that exceeds the deck spacing. This project is assumed to be safe in this approach since the decks will account for the full loading. On the other hand, alternative geometries of reference project 2 can be discussed based on figure 6.3.

In figure 6.3 the different failure modes can be distinguished with a solid line for the plate failure, and a dashed line for the stiffener failure. Also, the required reaction force of the reference project can be found by their specific colour. The specific behaviour of the stiffener failure can be explained by the different locations at which the yield stress is reached first. The curve flattens and even slightly decreases at the moment that the shear stress in the web of the stiffener becomes the deterministic stress. The specific width at which this occurs is interesting to compare with the other vessel types in the discussion in chapter 7, based on the results from subsection 5.3.3.

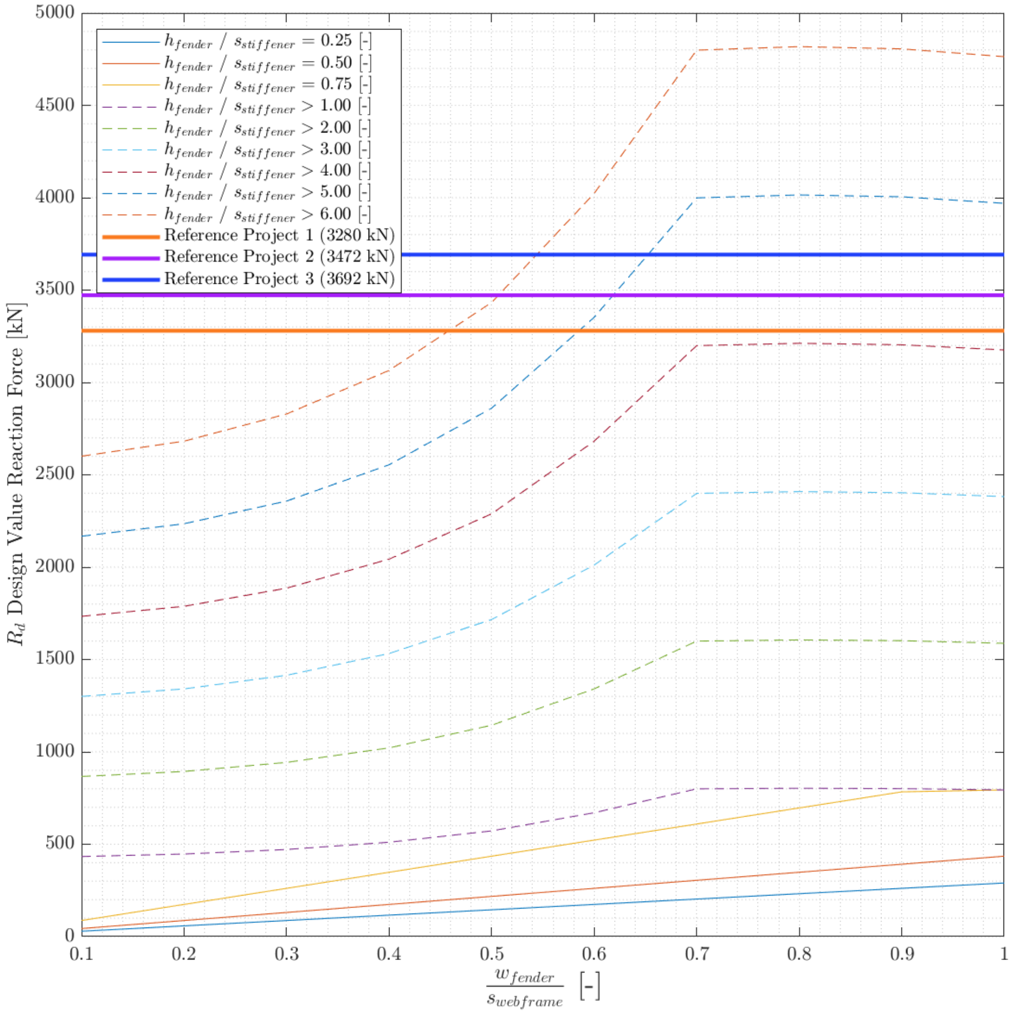


Figure 6.3: Maximum allowable force in kN on the grillage, Tanker 5, due to soft rigid contact area

The same method as discussed in the previous section is used to determine what geometry of fender panel is assumed to be safe. For the specific reference projects the admissible configurations are given by:

1. (a) height > $7 \cdot s_{stiffener} = 6.44$ meters
   width > $0.46 \cdot s_{webframe} = 2.61$ meters
   (b) height > $6 \cdot s_{stiffener} = 5.52$ meters
   width > $0.58 \cdot s_{webframe} = 3.29$ meters
2. (a) height > $7 \cdot s_{stiffener} = 6.44$ meters
   width > $0.51 \cdot s_{webframe} = 2.90$ meters
   (b) height > $6 \cdot s_{stiffener} = 5.52$ meters
   width > $0.62 \cdot s_{webframe} = 3.52$ meters
3. (a) height > $7 \cdot s_{stiffener} = 6.44$ meters
   width > $0.54 \cdot s_{webframe} = 3.07$ meters
   (b) height > $6 \cdot s_{stiffener} = 5.52$ meters
   width > $0.65 \cdot s_{webframe} = 3.69$ meters

In table 6.3 both the absolute and normalised width and height are given. In the table the same colours are used as in table 6.2. It is found that for fenders with small widths, the acceptable reaction force is lower than for the soft fender approach. For larger widths, the allowable forces are found to be higher. This pressure distribution results also in allowable forces that are lower than what the reference projects were designed for. Even larger differences can be found for different geometries with the same surface area. Especially the orange highlighted value, project 1, seems to have an interesting design since a reduction of the width would result in slightly higher allowable forces. Resulting in a possible reduction of materials for the manufacturer.

Table 6.3: Maximum allowable force in kN on the grillage, Tanker 5, due to rigid fender contact area. Reference projects 1 and 3 are denoted in respectively orange, and blue.

			Fender width									
	Norm. [-]		**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
		Abs. [m]	0.57	1.14	1.70	2.27	2.84	3.41	3.98	4.54	5.11	5.68
Fender height	**0.3**	0.23	29	58	87	116	145	174	203	232	261	290
	0.5	0.46	44	87	131	174	218	261	305	348	392	435
	0.8	0.69	87	174	261	348	435	522	609	696	784	794
	1	0.92	433	447	472	511	572	670	800	803	801	794
	2	1.84	867	894	943	1022	1144	1341	1600	1606	1602	1588
	3	2.76	1300	1341	1415	1533	1716	2011	2399	2409	2403	2382
	4	3.68	1734	1788	1886	2043	2289	2682	3199	3212	**3204**	3176
	5	4.60	2167	2235	2358	2554	2861	3352	3999	4015	4005	3970
	6	5.52	2601	2682	2829	**3065**	3433	4023	4799	4818	4806	4764

6.2.1. Generalised formulation

In subsection 5.3.3, the generalised formulation for stiffeners is given. The same approach as discussed in subsection 6.1.1 is used. The total maximum reaction force in N, based on a specific height and width of the fender panel, can be determined with equation (7.1).

$$R_{d,Rigid} = \left\lfloor \frac{h_{fender}}{s_{stiffener}} \right\rfloor \cdot min\left(\frac{1483x^6 - 3115x^5 + 2561x^4 - 985.8x^3 + 195.6x^2 - 13.5x + 15.21}{\xi}, \frac{32.4}{\xi} \right) \tag{6.4}$$

6.3. Comparison to Prior Results

In a previous study regarding cylindrical fenders by TNO [44], the annotation was made that the orientation of the contact surface could result in new conclusions. This study takes this orientation into account since all different square geometries are included. The fairest comparison that can be made with the study presented in this thesis, is the soft fender approach since the research by TNO was focused on rubber cylindrical fenders. TNO's research did assume an equally distributed load along with the line load at the mid-span of the fender, which is equal to the uniform load that is used in the soft fender approach in this study.

It is found that the fender compression force that results in the start of plasticity is in the same order of magnitude as the forces that are found in this thesis. This thesis results in slightly lower forces, which could be explained by the difference in location at which the yield stress occurs first. This difference is found due to the choice of the supports of the longitudinal stiffeners. The prior research resulted in yield stresses in the centre of the stiffener span, due to hinged boundary conditions of the stiffener at the connection with the web frame. This study assumes fully clamped boundary conditions, which results in locations with the highest stress at the connections with the web frame. The choice for a clamped boundary condition in this study was made with the idea that the chosen web frame spacing is the spacing of watertight frames. Therefore, the longitudinals should be welded at the web frame, preventing any rotation of the stiffener which is represented by a clamped end. However, the hinged boundary could also be justified if the stiffeners are not connected to the web frame, and specific holes are cut out in the web frame for the longitudinal stiffener to pass. A vessel's specific lay-out should determine which boundary condition is more realistic for specific cases. It is found that the approach used in this thesis, is more conservative and results in a lower acceptable loading.

The tables 6.2 and 6.3, give the maximum allowable loading in kN for different fender geometries. The same kind of results is obtained for the other vessel types in appendix B. The allowable force in these tables can easily be rewritten to equivalent pressures with the corresponding width and height of the fender panel. This is done in order to make a comparison with the currently used PIANC guidelines [28]. For the vessel type that is discussed in this chapter, the adjusted tables are given in table 6.4, and 6.5, for a soft and a rigid fender contact area respectively.

Table 6.4: Results for tanker T5 of the soft fender approach rewritten to a pressure formulation in kN/m^2

		Fender width									
		0.568	**1.14**	**1.70**	**2.27**	**2.84**	**3.41**	**3.98**	**4.54**	**5.11**	**5.68**
Fender height	**0.23**	652	423	381	381	381	381	381	381	381	381
	0.46	401	254	222	222	222	222	222	222	222	222
	0.69	331	206	179	178	178	178	178	174	167	164
	0.92	313	195	168	167	167	156	140	130	125	123
	0.92	824	416	282	217	179	156	140	130	125	123
	1.84	824	416	282	217	179	156	140	130	125	123
	2.76	824	416	282	217	179	156	140	130	125	123
	3.68	824	416	282	217	179	156	140	130	125	123
	4.60	824	416	282	217	179	156	140	130	125	123
	5.52	824	416	282	217	179	156	140	130	125	123
	6.44	824	416	282	217	179	156	140	130	125	123

Table 6.5: Results for tanker T5 of the rigd fender approach rewritten to a pressure formulation in kN/m[2]

		Fender width									
		0.568	**1.14**	**1.70**	**2.27**	**2.84**	**3.41**	**3.98**	**4.54**	**5.11**	**5.68**
Fender height	**0.23**	222	222	222	222	222	222	222	222	222	222
	0.46	167	167	167	167	167	167	167	167	167	167
	0.69	222	222	222	222	222	222	222	222	222	203
	0.92	829	428	301	244	219	214	217	190	169	152
	1.84	829	428	301	244	219	214	217	190	169	152
	2.76	829	428	301	244	219	214	217	190	169	152
	3.68	829	428	301	244	219	214	217	190	169	152
	4.60	829	428	301	244	219	214	217	190	169	152
	5.52	829	428	301	244	219	214	217	190	169	152

The results from this study show maximum allowable pressures that are only dependent on the width of the fender, since the values in table 6.4 and 6.5 are the same for all heights in combination with one specific width. This behaviour is explained by the fact that an increase in the height of the fender results in a linear increase with the allowable force. Since the same multiplication factor is used, being the number of effective stiffeners, the pressure formulation is constant with respect to the fender height. The maximum allowable pressure shows a decreasing trend for an increased width. This is explained by the fact that the maximum allowable force that can be exerted on the side hull does not scale linearly with the width.

The results are graphically presented in figure 6.4. Is noted that indeed the geometry or width highly influences the maximum allowable equivalent pressure. Therefore a general pressure formulation without mentioning the width is not enough to cover the structural behaviour of the vessel. A remarkable result is that the fenders with small widths do comply with the PIANC guidelines. Widths larger than 2.5 metres are found to be unsafe using the current guidelines for this vessel category. In addition, the other vessel types are compared with this equivalent pressure formulation in appendix B. However, the differences that are found are too large to come to a conclusion that covers every vessel type, these plots are given for the different vessel types in appendix B. A new formulation is assumed to be necessary for the other vessel types.

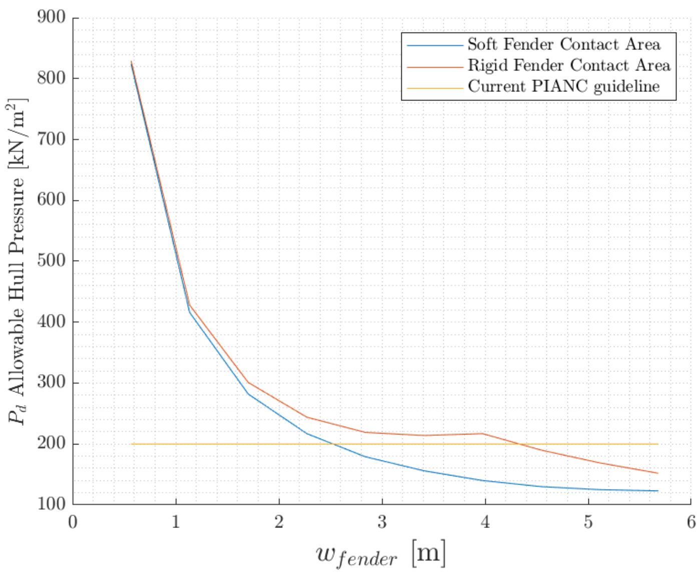


Figure 6.4: Comparison of maximum allowable pressure, once stiffener spacing is exceeded, with the current PIANC guideline [28] for vessel type tanker T5

7 Conclusion and Recommendations

In this thesis, a new approach to determine the maximum allowable loading on a vessel's side hull is proposed. The current approach assumes the vessel as a rigid block and the different stiffness components of a ship are not considered. Furthermore, the current pressure formulation, published in the guidelines on fender design [28], assumes that the exact geometry of a fender panel is not relevant. This study results in a description of the maximum allowable reaction force of the fender for a specific contact geometry. Two pressure distributions are investigated in order to cover the most extreme behaviour of a fender panel. These are a soft fender contact area represented by a uniform pressure, and a rigid fender contact area represented by line loads at the edges of the fender panel.

This chapter concludes the results that are obtained. Firstly general conclusions are made. The second section consists of the scientific recommendations following this study. Lastly, some practical recommendations are given.

7.1. Conclusions

Current design rules for ships lack information on strengthening for berthing operations with external marine fenders. Rules concerning strengthening for tug contact and rules on strengthening for steel fenders integrated into the vessel's side shell are used to make a comparison with the vessel types used in this thesis. From these rules, it is concluded that the used ship types consist of transverse web frames exceeding the minimum thickness by an excessive margin. On the other hand, the plate thickness and plastic section moduli of the longitudinal stiffeners are too small in some cases. Therefore, this study focuses on a grillage consisting of plates and stiffeners in between two web frames and two decks. The web frames and decks themselves are assumed to be rigid. Once a fender's contact area exceeds the spacing between the decks in height or the spacing between the web frames in width, the maximum allowable hull loading is assumed to be not important for the design of a fender panel used for a parallel berthing operation.

In this study, two basic failure modes are distinguished: plate- and stiffener failure. The plate failure is found to only be relevant if the height of the fender's contact area is less than the stiffener spacing of the vessel. Horizontally-oriented V-fenders and cylindrical fenders are often used with a height in this range. These fender types consist of a rubber contact area and are assumed to behave more like the soft fender approach with respect to the pressure distribution. The soft fender approach is also investigated more extensively for plates. A 3D approach described by Timoshenko and Woinowsky-Krieger [40], makes the results reliable for fenders with small widths. For fenders with a contact height smaller than the stiffener spacing, plate failure has to be taken into account.

For fender panels yield that they behave more like rigid contact areas. In practice, these fender panels usually exceed the stiffener spacing in height. Resulting in the stiffener failure becoming the failure mode of interest. For the stiffener failure mode, the resulting bending moment and shear stress in the web of the stiffener are considered. In order to compare the different vessel types, the heights and widths of the contact area are normalised. The results given in Appendix B can be compared with one another, and some general trends can be distinguished.

The results show that rigid fender contact areas do have an optimal width for a specific vessel category. The obtained data indicate that for a rigid fender contact area the shear stress in the stiffeners becomes of interest in between 0.7 and 0.9 times the transverse web frame spacing. This is recognised by a constant maximum allowable loading independent of the width. When the shear stress becomes the failure mechanism of interest, an optimal width of the contact area is found, since a wider panel will not result in a higher maximum allowable loading. It is concluded that larger contact areas are not always resulting in a higher allowable loading, which is contradicting with the current pressure formulation. Due to this finding, the material used in fender panels could be reduced.

Once the fender's contact height exceeds the stiffener spacing, multiple stiffeners contribute to the strength of the vessel's structure. The allowable hull loading on the vessel is assumed to linearly increase step-wise with the number of effective stiffeners. Assuming that the effective stiffeners account for an evenly distributed load. This assumption leads to a finding in which the height of a fender panel can increase the maximum allowable hull loading significantly more per square meter, than the width of a fender panel.

In order to find a general trend for the maximum allowable loading that is valid for all different types of vessels, a factor ξ is introduced. By multiplying the maximum allowable loading with this factor ξ, a general formulation is found. This formulation is valid once the contact area is larger than the stiffener spacing in height. In order to define the factor ξ, it is required to know the specific structural layout of a vessel, such as the specifications of the longitudinal stiffeners that are used.

Differences are found between the allowable loading existing fender panels were designed for, and the results of this study. For these reference projects, it is found that the combination of the used geometry, and the maximum design load of the fender results in plastic deformation within the largest tanker vessel type. One of the reference projects consisted of dimensions that exceeded the height of the grillage, resulting in decks that contribute to the strength. As said, this thesis assumes the deck to be much sturdier than the other structural components, and heights exceeding this spacing are expected to be safe. The two projects that did not exceed the dimensions of the used grillage, resulted in reaction forces that were higher than the maximum allowable loading following this research. For smaller vessel types the two projects also exceeded the deck spacing or web frame spacing. That would therefore result as well in a safe design due to the rigid web frame and deck assumption.

7.2. Scientific Recommendations

The vessel categories used in this study are based on a prior study of TNO [44]. In this study, one of the conclusions was that the vessel type Handysize/Handymax tanker T2, was, and still is, considered as a light structure. However, in this research, the geometry is checked and updated based on actual drawings of vessels in this category. Furthermore, TNO recommended that statistical analyses should not be done based on their report. In general, the recommendation still holds that more data with respect to the so-called scantlings is required to determine the natural variation of the structural layout of vessels.

An additional failure mode was mentioned in chapter 3 in the form of tripping. It would be interesting to include this failure mode since the strength of the structure is reduced once rotation of the stiffeners take place. Moreover, in practical examples, it is observed that transverse web frames of bulk carriers are keen for this failure mode. This specific vessel type has to be further investigated as well since the structural layout of bulk carriers do often not consist of longitudinal stiffeners in between the web frames. These vessels consist mostly of smaller web frames only, and the rigid web frame assumption is therefore no longer valid to use.

In the time span of this thesis not the whole grillage of the vessel was checked within FEA. In the partial analysis of the stiffener, it was already found that FEA resulted in higher allowable forces than the analytical approach. In the simulations of the complete grillage indications of this conclusion were found as well, but no convergence with respect to the mesh size could be obtained. Both time and computational power obstructed the full-size FEA in this project. The analytical formulation is rather useful since it is a conservative approach, and gives a good impression on the behaviour of a vessel's structural members. It is recommended to further investigate this topic with full-scale FEA or even full-scale tests on lateral patch loading.

One of the main topics in the design of fender systems is the energy absorption of the fender system. An interesting aspect that could follow from this study is the energy absorption of the vessel's structure. In this study, the assumption of a rigid vessel is refuted. This is followed by questions about whether the vessel is able to absorb energy as well. This would result in less energy that has to be absorbed by the fender system, and therefore a reduced required reaction force and fender size.

7.3. Practical Recommendations

The practical recommendations of this thesis are focused on the rigid fender approach. The sizes of the contact area that have been investigated show more comparison in practice with rigid fender panel behaviour and are, therefore, more interesting to mention in this section.

This study showed that practical examples are not in line with the proposed criterion when very large fender panels are used. Especially when the web frames inside a vessel have large spacings in between, wide fenders seem to result in high stresses. These wide fenders are used often in order to comply with the current guideline of PIANC WG33 [28], by creating an area large enough to divide the force over. It is recommended to check carefully if the existing fenders with a wide area, are indeed meant to handle the vessel types consisting of a large web frame spacing. Once the fender panel width is larger than the web frame spacing, this study assumes that the maximum allowable hull loading is not important anymore.

The results of this thesis indicate a significant difference with respect to the current guidelines on fender design. For small widths, the current guidelines often result in a conservative recommendation, but for large widths an overestimation is found for the maximum allowable loading on a vessels' hull. Therefore, it is highly recommended to mention a certain fender width to what extent this formulation is valid. More recommended is to consider the result of this thesis, and implement maximum allowable forces on a vessel for a specific fender geometry. The proposed description, and therefore the required analysis, is more extensive, and the specific vessels that berth at the specific facility have to be investigated more carefully.

In this study, it is found that the deck and web frames are much sturdier than the other structural elements in the vessel. It is assumed that once the fender panel exceeds either the deck spacing in height or the web frame spacing in width the allowable hull loading is infinite. For fender panels that are smaller than these particulars, a general formulation of the maximum allowable hull loading is proposed. This formulation requires the specific structural lay-out of the vessel that encounters the berthing facility. For designers of the fenders, these particulars are not necessarily known. If the specific ship's particulars are not known, it is advised to make use of the tables and graphs in appendix B. The particulars of the different reference vessels used in the appendix are given in table 2.3. The results in the appendix can be used as a guideline for vessel types that correspond with this table.

The proposed method to check whether a fender panel does match with the maximum allowable hull loading is summarised in the flow-chart in figure 7.1.

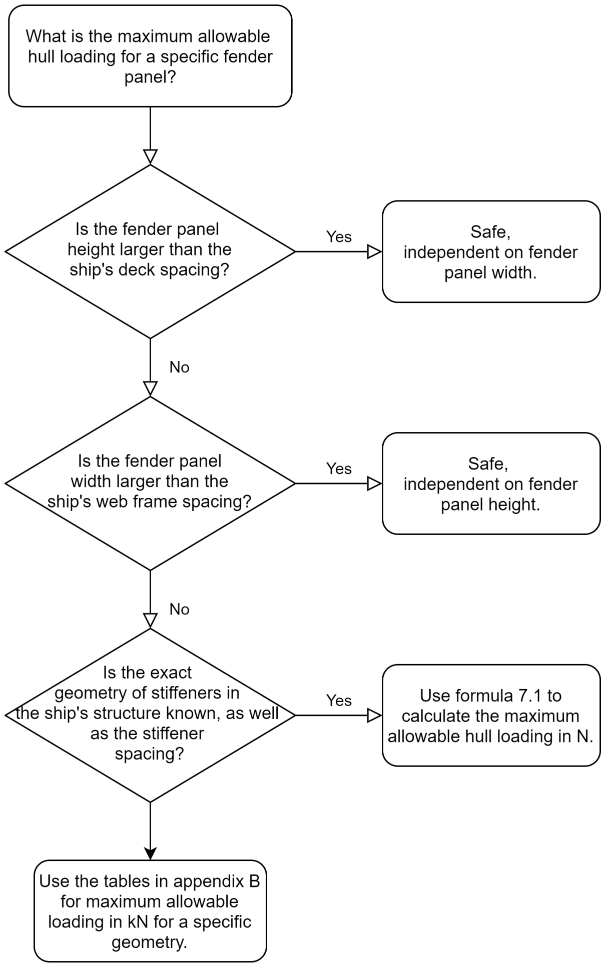


Figure 7.1: Flow chart for the determination of maximum allowable hull loading

Generalised formulation

A formulation for the maximum allowable loading on the vessels' stiffeners is given in chapter 6. The total maximum reaction force in N, based on a specific height and width of the fender panel, can be determined with equation (7.1).

$$R_{d,Rigid} = \left\lfloor \frac{h_{fender}}{s_{stiffener}} \right\rfloor \cdot min\left(\frac{1483x^6 - 3115x^5 + 2561x^4 - 985.8x^3 + 195.6x^2 - 13.5x + 15.21}{\xi}, \frac{32.4}{\xi}\right) \tag{7.1}$$

In this equation h_{fender} is equal to the height of the fender panel in meters, $s_{stiffener}$ is equal to the spacing in between the longitudinal stiffener. The $\lfloor \; \rfloor$ signs indicate that the floored value of $\frac{h_{fender}}{s_{stiffener}}$ should be used. The coefficient ξ is formulated by:

$$\xi = \frac{s_{webframe} \cdot h_{stiffener}}{I_{stiffener} \cdot \sigma_{\gamma}} \tag{7.2}$$

Where the spacing between the web frames is denoted with $s_{webframe}$, $h_{stiffener}$ is equal to the height of the stiffener itself, $I_{stiffener}$ is equal to the moment of the inertia of the stiffener in combination with the effective plate width of the plate in between the stiffeners, and σ_{γ} is equal to the yield stress of the steel that the ship is constructed with.

In general, a specific quay wall is used by various types of vessels. It is non-ideal to account for all different vessels, or even the simplified tables in the appendix of possible vessel categories, that will encounter the fender of interest. However, it is found that smaller vessels do have a smaller stiffener and deck spacing, following the data used in this thesis. Therefore, a safe design that exceeds the deck spacing is obtained more easily for these small vessels than for the larger vessels. It is recommended to design a fender based on the largest vessels that will enter at the specific quay wall, and check afterwards if it does also comply with smaller vessel types.

As mentioned in the first practical recommendation, the wide fenders are used quite often in ports to fulfil the current allowable pressure formulation. In order to comply with the proposed criterion from this study, it is recommended to use fenders with a larger height and a smaller width, in order to comply with the first step in the flowchart. This could result in a reduction of materials and costs and is, therefore, more sustainable.

A

Mathematical Derivations

A.1. Plate: Central Loading with Soft Fender

The 2D approach is based on the beam equations from Roark's formulas for stress and strain [45]. The formulation of a beam that is hinged on one end and guided on the other end is used, as elaborated in section 4.1. Since the distributed load is defined with a length from the opposite direction as used in the standard table superposition is used.

$$M_r = \left(\frac{P_{F,uniform}}{2} \cdot \left(\frac{s_{stiffener}}{2}\right)^2\right) - \left(\frac{P_{F,uniform}}{2} \cdot \left(\frac{s_{stiffener}}{2} - \frac{h_{fender}}{2}\right)^2\right)$$

$$= \frac{P_{F,uniform}}{2} \cdot \left(\left(\frac{s_{stiffener}}{2}\right)^2 - \left(\frac{s_{stiffener} - h_{fender}}{2}\right)^2\right) \tag{A.1}$$

$$P_{F,uniform} = \frac{2 \cdot M_Y}{\left(\left(\frac{s_{stiffener}}{2}\right)^2 - \left(\frac{s_{stiffener} - h_{fender}}{2}\right)^2\right)} \tag{A.2}$$

A.2. Plate: Loading from Edge with Soft Fender

As discussed in section 4.3 the reaction forces in the plate are required to describe the location at which the internal moment is at a maximum. The first reaction force is derived with:

$$M_A = F_B \cdot s_{stiffener} - \left(P_{F,uniform} \cdot \frac{h_{fender}}{2}\right) \cdot \frac{h_{fender}}{4} = 0 \tag{A.3}$$

$$F_B = P_{F,uniform} \cdot \frac{h_{fender}^2}{8 \cdot s_{stiffener}} \tag{A.4}$$

Based on a static approach the second reaction force is obtained.

$$\sum F = F_A + F_B - P_{F,uniform} \cdot \frac{h_{fender}}{2} = 0 \tag{A.5}$$

$$F_A = P_{F,uniform} \cdot \frac{h_{fender}}{2} - P_{F,uniform} \cdot \frac{h_{fender}^2}{8 \cdot s_{stiffener}} \tag{A.6}$$

$$F_A = P_{F,uniform} \cdot \left(\frac{h_{fender}}{2} - \frac{h_{fender}^2}{8 \cdot s_{stiffener}}\right) \tag{A.7}$$

Which is eventually used to describe the maximum moment. A zero shear force in the shear force diagram results in the maximum moment, this is shown in equation (A.8) to (A.10).

$$F_A - P_{F,uniform} \cdot z_{M,max} = 0 \tag{A.8}$$

$$P_{F,uniform} \cdot \left(\frac{h_{fender}}{2} - \frac{h_{fender}^2}{8 \cdot s_{stiffener}}\right) - P_{F,uniform} \cdot z_{M,max} = 0 \tag{A.9}$$

$$z_{M,max} = \frac{h_{fender}}{2} - \frac{h_{fender}^2}{8 \cdot s_{stiffener}} \tag{A.10}$$

A.3. Stiffener: Central Loading with Soft Fender

Following the beam equations from Roark's formulas for stress and strain [45] the moments are described in the ends of the beam. The equations describing a beam which is clamped on one end and guided on the other end are used. Since the distributed load is defined with a length starting from the opposite direction a superposition is used. By a substitution of the yielding moment for M_A, and M_B the maximum allowable pressure formulation is derived. This pressure is used in the report in equations (5.11), and (5.13).

$$M_A = M_Y \tag{A.11}$$

$$\begin{aligned} M_A &= \left(\frac{P_{F,uniform}}{6 \cdot \left(\frac{s_{webframe}}{2}\right)} \cdot \left(\frac{s_{webframe}}{2}\right)^3\right) - \left(\frac{P_{F,uniform}}{6 \cdot \left(\frac{s_{webframe}}{2}\right)} \cdot \left(\frac{s_{webframe}}{2} - \frac{w_{fender}}{2}\right)^3\right) \\ &= \frac{P_{F,uniform}}{6 \cdot \left(\frac{s_{webframe}}{2}\right)} \cdot \left(\left(\frac{s_{webframe}}{2}\right)^3 - \left(\frac{s_{webframe}}{2} - \frac{w_{fender}}{2}\right)^3\right) \end{aligned} \tag{A.12}$$

$$\left(P_{F,uniform}\right)_{A,max} = \frac{3 \cdot M_Y \cdot s_{webframe}}{\left(\left(\frac{s_{webframe}}{2}\right)^3 - \left(\frac{s_{webframe}}{2} - \frac{w_{fender}}{2}\right)^3\right)} \tag{A.13}$$

$$M_B = M_Y \tag{A.14}$$

$$\begin{aligned} M_B = &\left(\frac{P_{F,uniform}}{6 \cdot \left(\frac{s_{webframe}}{2}\right)} \cdot \left(\frac{s_{webframe}}{2}\right)^2 \cdot \left(s_{webframe}\right)\right) \\ &- \left(\frac{P_{F,uniform}}{6 \cdot \left(\frac{s_{webframe}}{2}\right)} \cdot \left(\frac{s_{webframe}}{2} - \frac{w_{fender}}{2}\right)^2 \cdot \left(s_{webframe} + \frac{w_{fender}}{2}\right)\right) \end{aligned} \tag{A.15}$$

$$\begin{aligned} \left(P_{F,uniform}\right)_{B,max} &= \frac{3 \cdot M_Y \cdot s_{webframe}}{\frac{s_{webframe}^3}{4} - \left(\frac{s_{webframe}}{2} - \frac{w_{fender}}{2}\right)^2 \cdot \left(s_{webframe} + \frac{w_{fender}}{2}\right)} \\ &= \frac{24 \cdot M_y \cdot s_{webframe}}{3 \cdot s_{webframe}^2 \cdot w_{fender} - w_{fender}^3} \end{aligned} \tag{A.16}$$

B

Results per Vessel Type

B.1. Tanker 1: Coaster

Soft Fender

Figure B.1 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.1. The stiffener of this vessel is assumed to be of HP 180x10 type, with a transverse web frame spacing of 2.66 metres, a stiffener spacing of 0.65 metres, and a shell plate thickness of 10 millimetres.

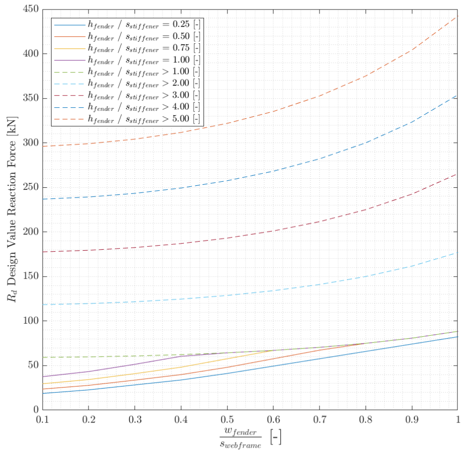


Figure B.1: Tanker 1: Maximum allowable force in kN on the grillage due to a soft fender

Table B.1: Tanker 1: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,27	0,53	0,80	1,06	1,33	1,60	1,86	2,13	2,39	2,66
Fender height	**0,25**	0,16	19	23	28	34	41	49	58	66	74	82
	0,5	0,33	24	28	34	40	48	58	67	75	81	89
	0,75	0,49	30	34	41	48	58	67	71	75	81	89
	1	0,65	38	43	51	60	64	67	71	75	81	89
	1	0,65	59	60	61	62	64	67	71	75	81	89
	2	1,30	118	120	122	125	129	134	141	150	162	177
	3	1,95	178	179	183	187	193	201	212	225	243	266
	4	2,60	237	239	243	249	258	268	282	300	323	354
	5	3,25	296	299	304	312	322	335	353	375	404	443

Rigid Fender

Figure B.2 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.2. The stiffener of this vessel is assumed to be of HP 180x10 type, with a transverse web frame spacing of 2.66 metres, a stiffener spacing of 0.65 metres, and a shell plate thickness of 10 millimetres.

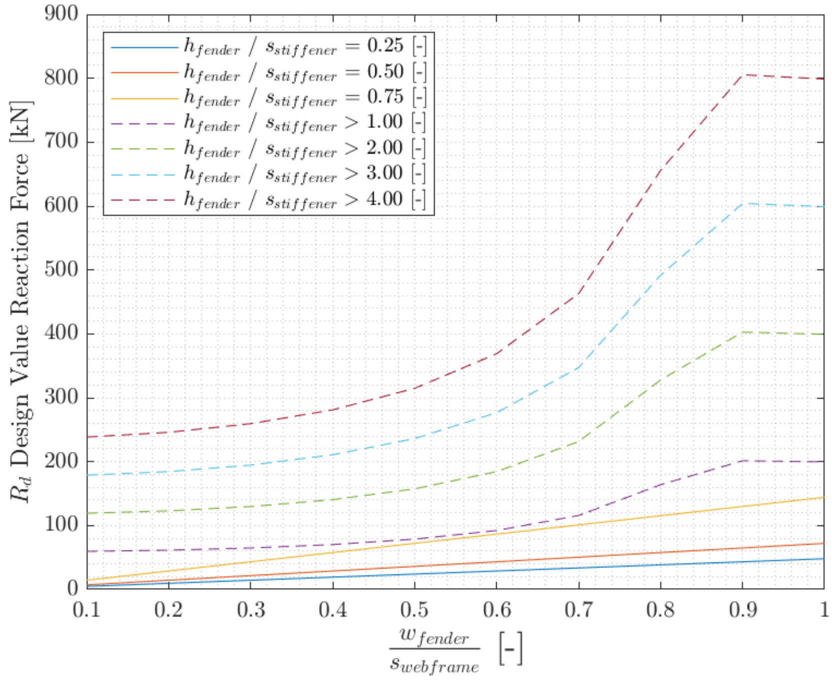


Figure B.2: Tanker 1: Maximum allowable force in kN on the grillage due to a rigid fender

Table B.2: Tanker 1: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,27	0,53	0,80	1,06	1,33	1,60	1,86	2,13	2,39	2,66
Fender height	**0,25**	0,16	5	10	14	19	24	29	34	38	43	48
	0,5	0,33	7	14	22	29	36	43	50	58	65	72
	0,75	0,49	14	29	43	58	72	87	101	115	130	144
	1	0,65	60	61	65	70	79	92	116	164	201	200
	2	1,30	119	123	130	141	157	184	231	328	403	399
	3	1,95	179	184	195	211	236	277	347	492	604	599
	4	2,60	238	246	259	281	315	369	463	656	805	799

Comparison with Current Regulations
In figure B.3 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

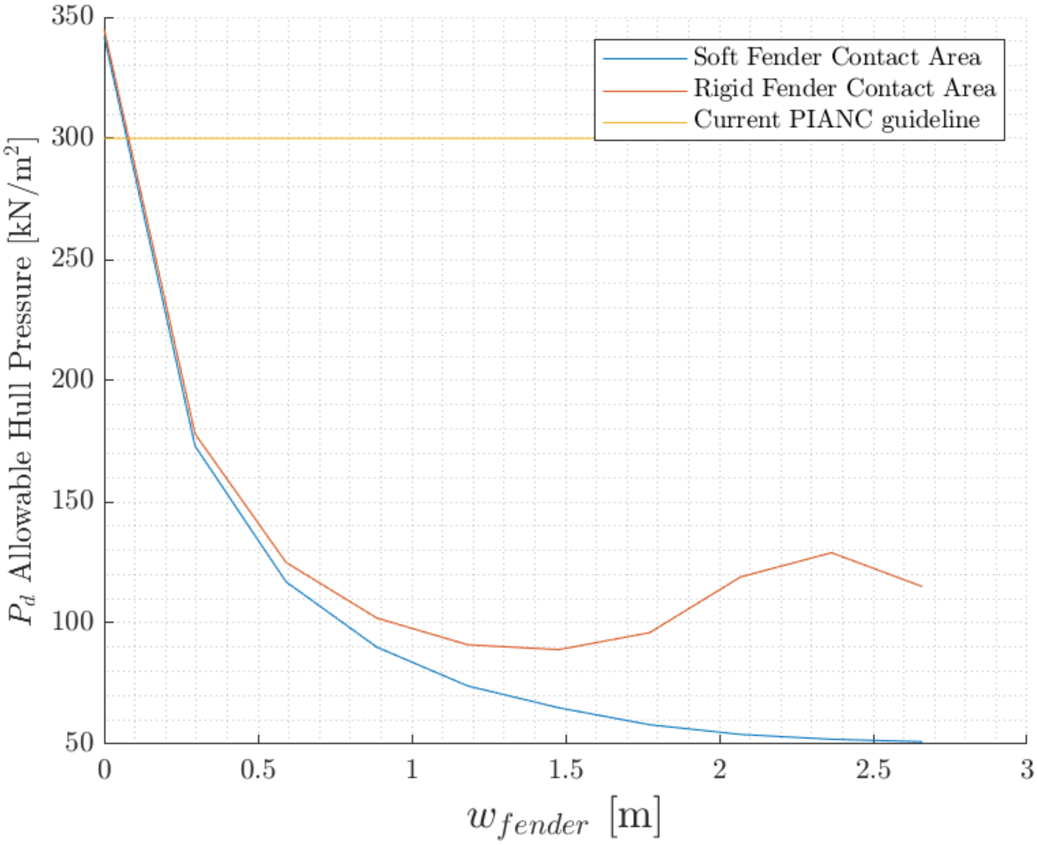


Figure B.3: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.2. Tanker 2: Handysize

Soft Fender

Figure B.4 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.3. The stiffener of this vessel is assumed to be of HP 120x8 type, with a transverse web frame spacing of 1.995 metres, a stiffener spacing of 0.575 metres, and shell plate thickness of 10.5 millimetres.

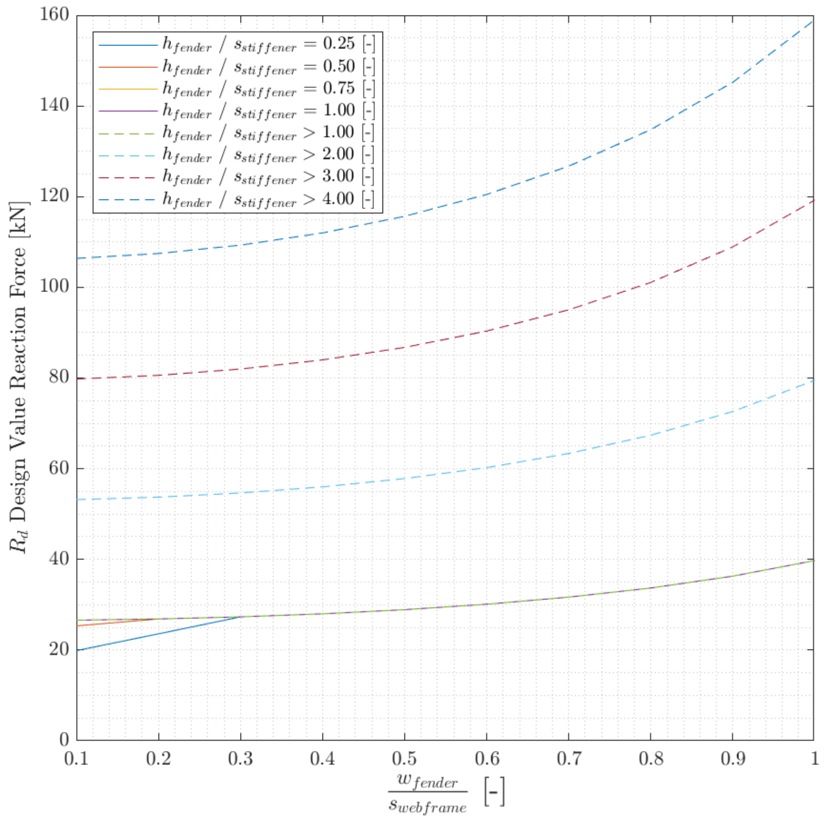


Figure B.4: Tanker 2: Maximum allowable force in kN on the grillage due to a soft fender

Table B.3: Tanker 2: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,20	0,40	0,60	0,80	1,00	1,20	1,40	1,60	1,80	2,00
Fender height	**0,25**	0,14	20	24	27	28	29	30	32	34	36	40
	0,5	0,29	25	27	27	28	29	30	32	34	36	40
	0,75	0,43	27	27	27	28	29	30	32	34	36	40
	1	0,58	27	27	27	28	29	30	32	34	36	40
	1	0,58	27	27	27	28	29	30	32	34	36	40
	2	1,15	53	54	55	56	58	60	63	67	73	80
	3	1,73	80	81	82	84	87	90	95	101	109	119
	4	2,30	106	107	109	112	116	120	127	135	145	159

Rigid Fender

Figure B.5 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.4. The stiffener of this vessel is assumed to be of HP 120x8 type, with a transverse web frame spacing of 1.995 metres, a stiffener spacing of 0.575 metres, and shell plate thickness of 10.5 millimetres.

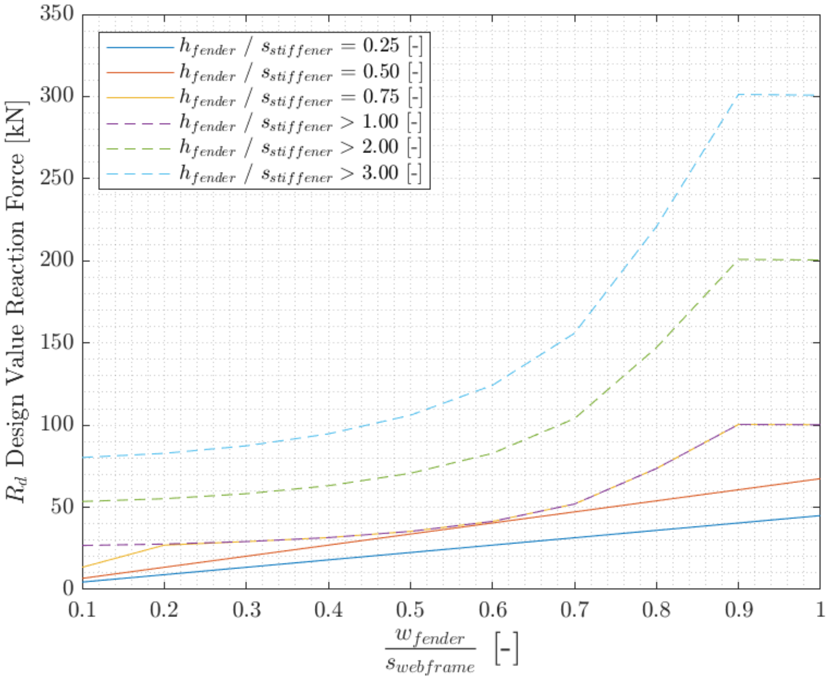


Figure B.5: Tanker 2: Maximum allowable force in kN on the grillage due to a rigid fender

Table B.4: Tanker 2: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,20	0,40	0,60	0,80	1,00	1,20	1,40	1,60	1,80	2,00
Fender height	**0,25**	0,14	4	9	13	18	22	27	31	36	40	45
	0,5	0,29	7	13	20	27	34	40	47	54	61	67
	0,75	0,43	13	27	29	32	35	41	52	74	100	100
	1	0,58	27	28	29	32	35	41	52	74	100	100
	2	1,15	54	55	58	63	71	83	104	147	201	201
	3	1,73	80	83	87	95	106	124	156	221	301	301

Comparison with Current Regulations

In figure B.6 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

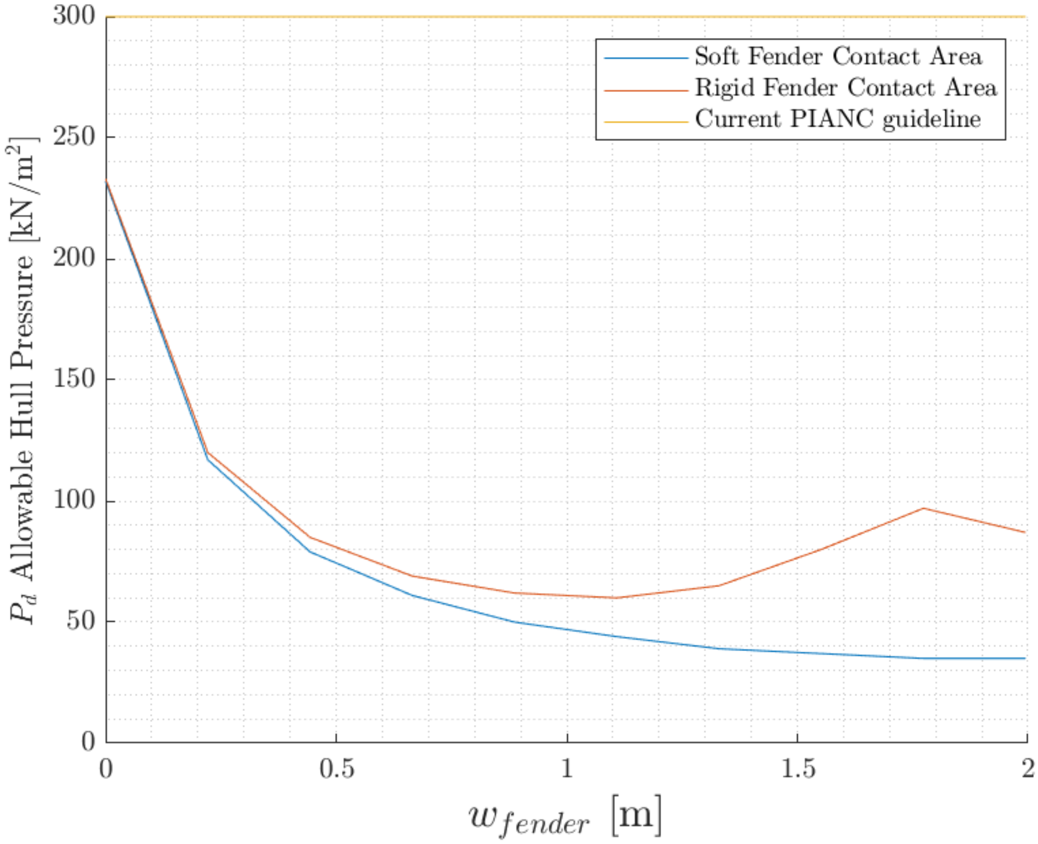


Figure B.6: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.3. Tanker 3: Handymax / Panamax (32m)

Soft Fender

Figure B.7 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.5. The stiffener of this vessel is assumed to be of L 250x10 + 90x14 type, with a transverse web frame spacing of 2.7 metres, a stiffener spacing of 0.7 metres, and a shell plate thickness of 13 millimetres.

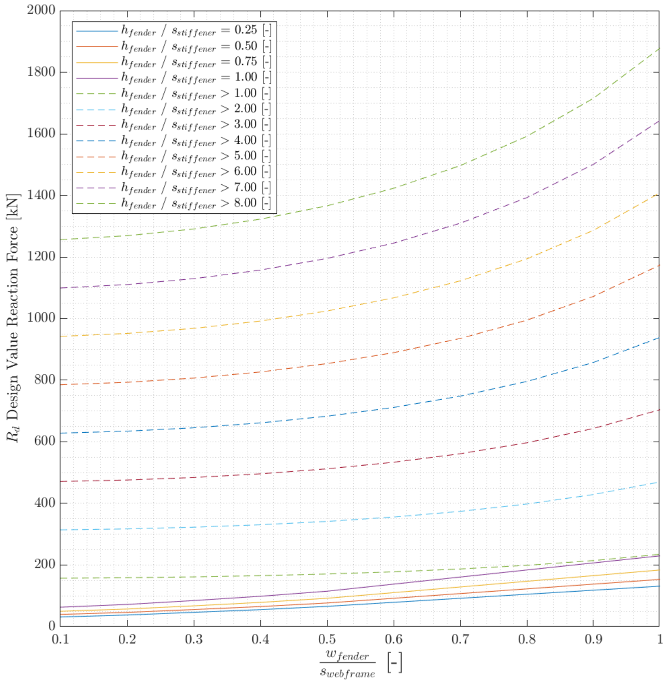


Figure B.7: Tanker 3: Maximum allowable force in kN on the grillage due to a soft fender

Table B.5: Tanker 3: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,27	0,54	0,81	1,08	1,35	1,62	1,89	2,16	2,43	2,70
Fender height	**0,25**	0,18	31	38	46	55	66	79	92	105	118	131
	0,5	0,35	40	46	55	65	77	92	107	123	138	153
	0,75	0,53	50	57	67	79	92	110	129	147	165	184
	1	0,70	63	72	85	98	115	138	161	184	207	230
	1	0,70	157	159	161	165	171	178	187	199	214	235
	2	1,40	314	317	323	331	342	356	374	398	429	470
	3	2,10	471	476	484	496	512	534	561	597	643	704
	4	2,80	628	635	646	661	683	712	748	796	858	939
	5	3,50	785	793	807	827	854	889	935	995	1072	1174
	6	4,20	942	952	968	992	1025	1067	1123	1194	1287	1409
	7	4,90	1099	1111	1130	1157	1195	1245	1310	1393	1501	1644
	8	5,60	1256	1269	1291	1323	1366	1423	1497	1592	1715	1878

Rigid Fender

Figure B.8 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.6. The stiffener of this vessel is assumed to be of L 250x10 + 90x14 type, with a transverse web frame spacing of 2.7 metres, a stiffener spacing of 0.7 metres, and a shell plate thickness of 13 millimetres.

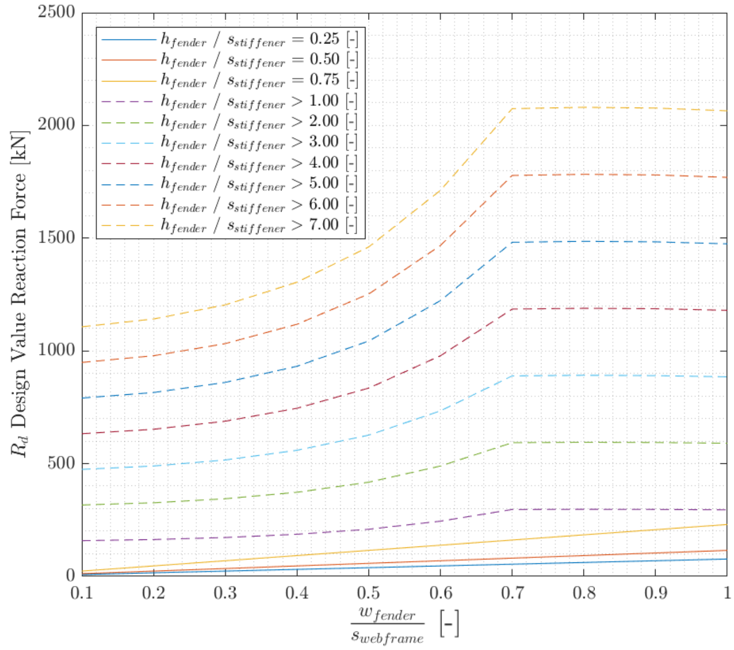


Figure B.8: Tanker 3: Maximum allowable force in kN on the grillage due to a rigid fender

Table B.6: Tanker 3: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,27	0,54	0,81	1,08	1,35	1,62	1,89	2,16	2,43	2,70
Fender height	**0,25**	0,18	8	15	23	31	38	46	54	61	69	77
	0,5	0,35	11	23	34	46	57	69	80	92	103	115
	0,75	0,53	23	46	69	92	115	138	161	184	207	230
	1	0,70	158	163	172	186	209	245	296	297	297	295
	2	1,40	316	326	344	373	417	489	593	594	593	590
	3	2,10	474	489	516	559	626	734	889	892	890	885
	4	2,80	632	652	688	745	835	978	1185	1189	1187	1180
	5	3,50	791	815	860	932	1044	1223	1482	1486	1484	1475
	6	4,20	949	978	1032	1118	1252	1468	1778	1783	1780	1770
	7	4,90	1107	1141	1204	1304	1461	1712	2075	2081	2077	2065

Comparison with Current Regulations

In figure B.9 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

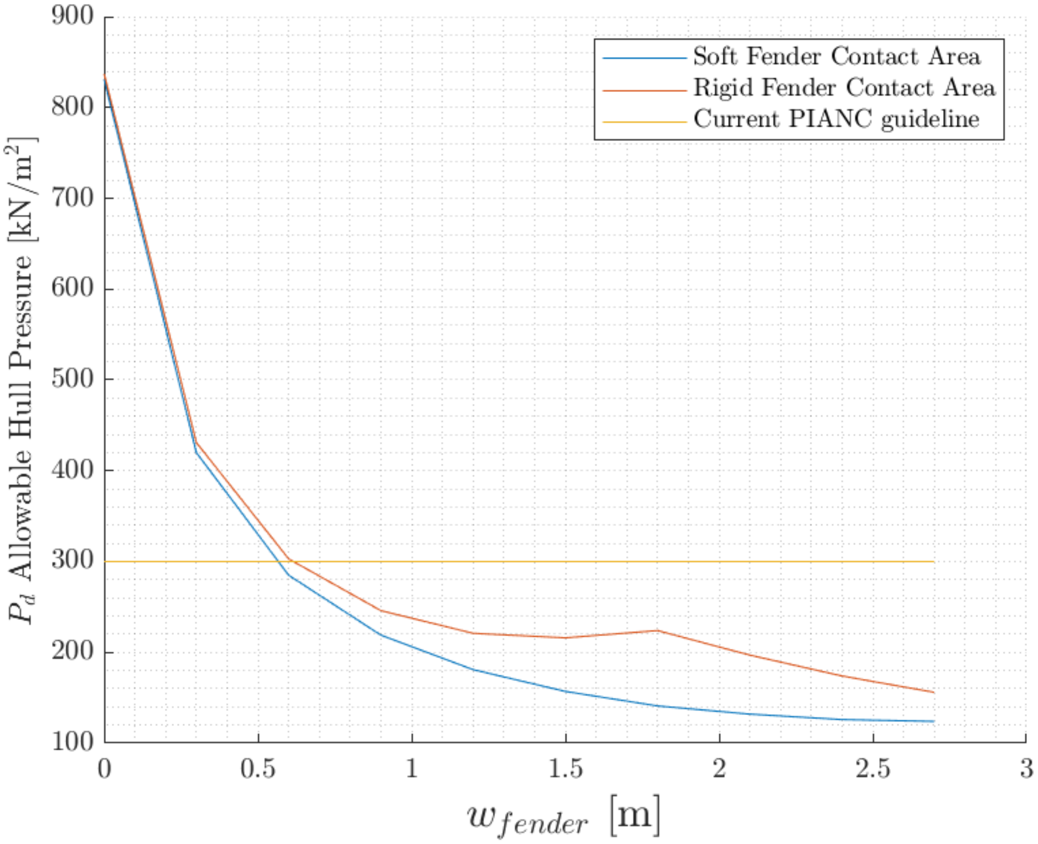


Figure B.9: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.4. Tanker 4: Aframax / Suezmax

Soft Fender

Figure B.10 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.7. The stiffener of this vessel is assumed to be of L 300x13 + 90x17 type, with a transverse web frame spacing of 4.3 metres, a stiffener spacing of 0.855 metres, and a shell plate thickness of 17 millimetres.

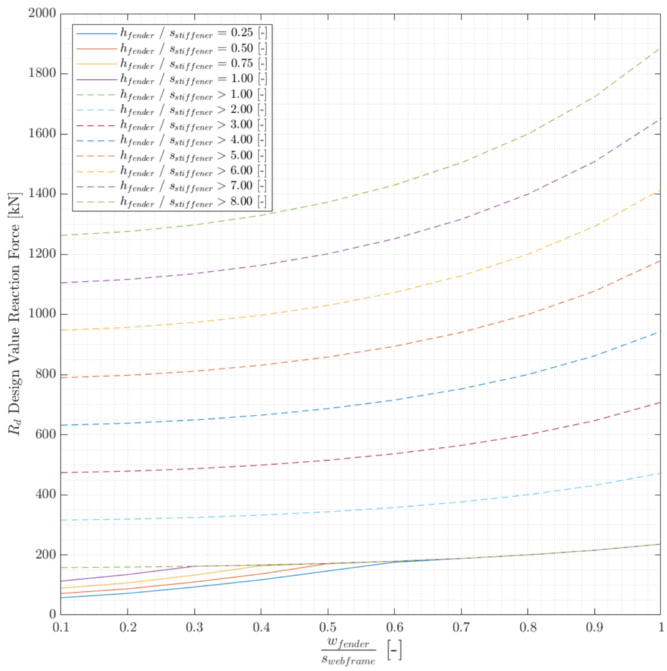


Figure B.10: Tanker 4: Maximum allowable force in kN on the grillage due to a soft fender

Table B.7: Tanker 4: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,43	0,86	1,29	1,72	2,15	2,58	3,01	3,44	3,87	4,30
Fender height	**0,25**	0,21	57	72	93	117	146	176	188	200	215	236
	0,5	0,43	72	87	110	137	171	179	188	200	215	236
	0,75	0,64	89	107	133	164	172	179	188	200	215	236
	1	0,86	113	135	162	166	172	179	188	200	215	236
	1	0,86	158	159	162	166	172	179	188	200	215	236
	2	1,71	316	319	324	332	343	358	376	400	431	472
	3	2,57	474	478	487	499	515	536	564	600	646	708
	4	3,42	631	638	649	665	686	715	752	800	862	944
	5	4,28	789	797	811	831	858	894	940	1000	1077	1180
	6	5,13	947	957	973	997	1030	1073	1128	1200	1293	1416
	7	5,99	1105	1116	1135	1163	1201	1251	1316	1400	1508	1652
	8	6,84	1263	1275	1297	1329	1373	1430	1504	1600	1724	1888

Rigid Fender

Figure B.11 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.8. The stiffener of this vessel is assumed to be of L 300x13 + 90x17 type, with a transverse web frame spacing of 4.3 metres, a stiffener spacing of 0.855 metres, and a shell plate thickness of 17 millimetres.

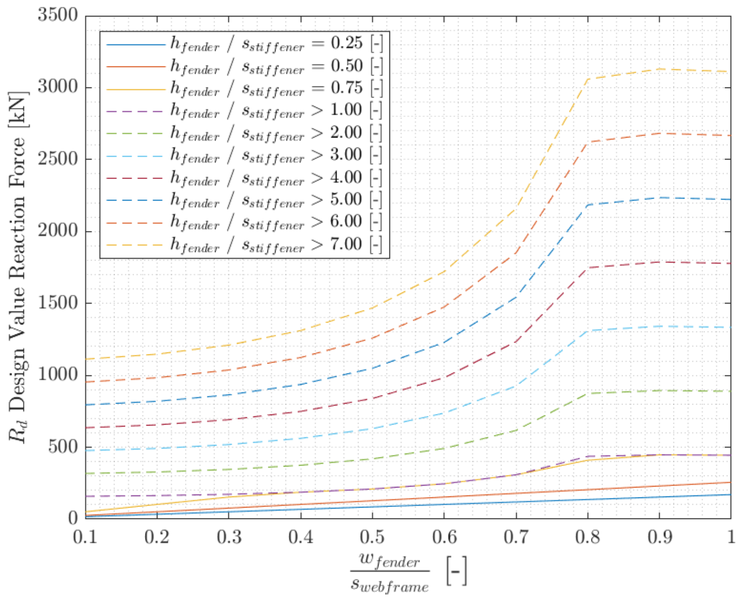


Figure B.11: Tanker 4: Maximum allowable force in kN on the grillage due to a soft fender

Table B.8: Tanker 4: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,43	0,86	1,29	1,72	2,15	2,58	3,01	3,44	3,87	4,30
Fender height	**0,25**	0,21	17	34	51	68	85	102	120	137	154	171
	0,5	0,43	26	51	77	102	128	154	179	205	231	256
	0,75	0,64	51	102	154	187	210	246	308	410	447	445
	1	0,86	159	164	173	187	210	246	308	437	447	445
	2	1,71	318	328	346	375	419	492	617	874	894	889
	3	2,57	477	492	519	562	629	737	925	1311	1341	1334
	4	3,42	636	655	691	749	839	983	1234	1748	1788	1778
	5	4,28	794	819	864	936	1049	1229	1542	2185	2235	2223
	6	5,13	953	983	1037	1124	1258	1475	1851	2622	2682	2667
	7	5,99	1112	1147	1210	1311	1468	1721	2159	3059	3130	3112

Comparison with Current Regulations

In figure B.12 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

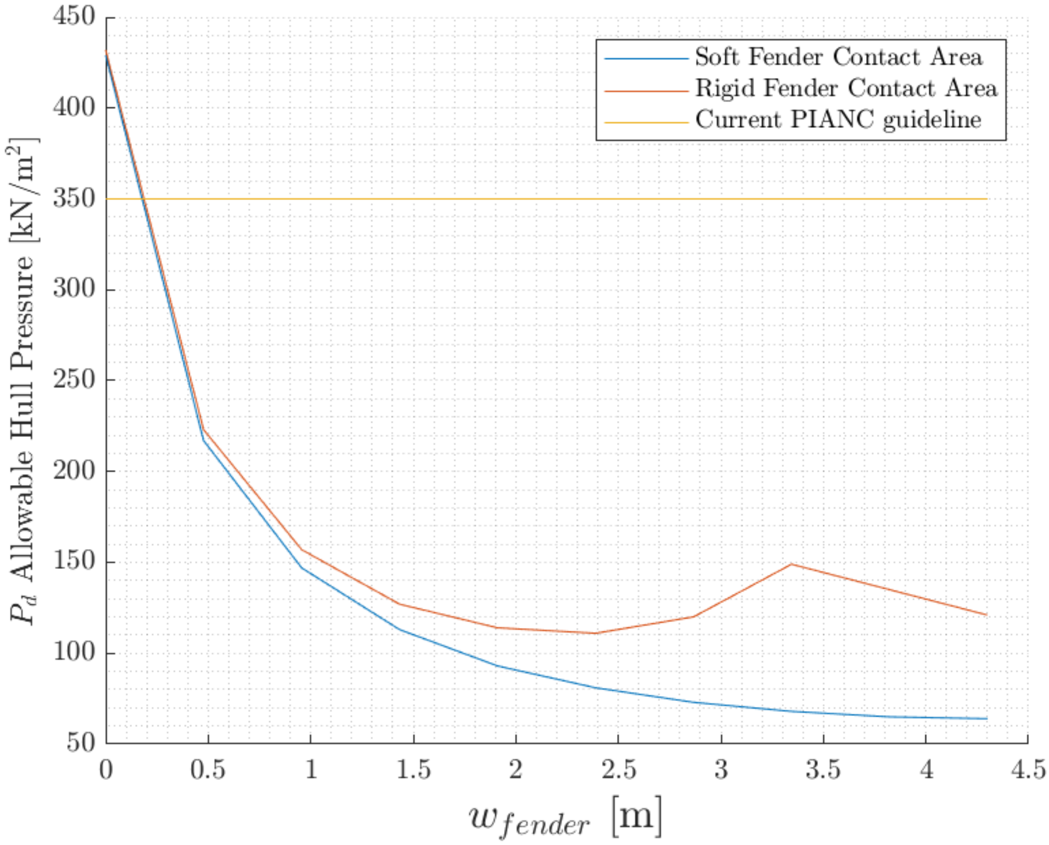


Figure B.12: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.5. Tanker 5: VLCC

Soft Fender

Figure B.13 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.9. The stiffener of this vessel is assumed to be of T 550x12 + 150x20 type, with a transverse web frame spacing of 5.68 metres, a stiffener spacing of 0.92 metres, and a shell plate thickness of 20 millimetres.

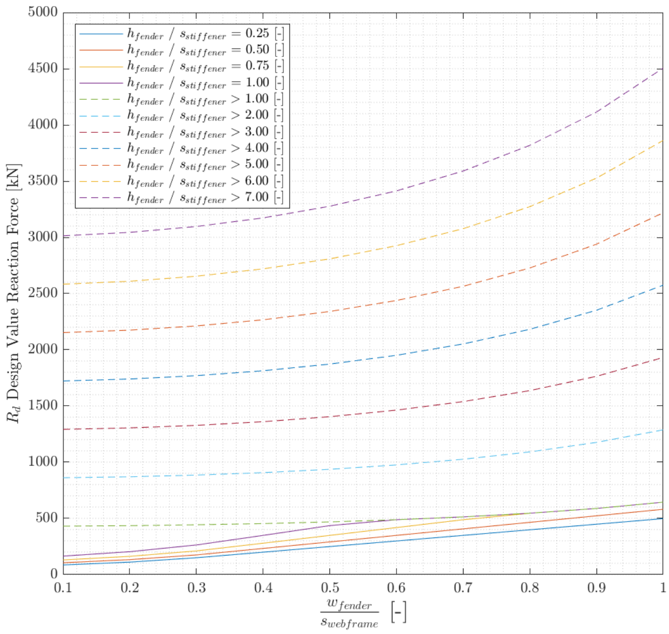


Figure B.13: Tanker 5: Maximum allowable force in kN on the grillage due to a soft fender

Table B.9: Tanker 5: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,57	1,14	1,70	2,27	2,84	3,41	3,98	4,54	5,11	5,68
Fender height	**0,3**	0,23	85	110	149	199	249	298	348	398	448	497
	0,5	0,46	105	133	174	232	290	348	406	464	522	580
	0,8	0,69	130	162	210	279	348	418	488	545	588	644
	1	0,92	164	203	263	348	435	488	513	545	588	644
	1	0,92	431	435	442	453	468	488	513	545	588	644
	2	1,84	861	870	885	907	936	975	1026	1091	1176	1287
	3	2,76	1292	1305	1327	1360	1404	1463	1539	1636	1763	1931
	4	3,68	1722	1740	1770	1813	1872	1950	2052	2182	2351	2575
	5	4,60	2153	2175	2212	2266	2341	2438	2564	2727	2939	3218
	6	5,52	2583	2609	2654	2720	2809	2926	3077	3273	3527	3862
	7	6,44	3014	3044	3097	3173	3277	3413	3590	3818	4115	4506

Rigid Fender

Figure B.14 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.10. The stiffener of this vessel is assumed to be of T 550x12 + 150x20 type, with a transverse web frame spacing of 5.68 metres, a stiffener spacing of 0.92 metres, and a shell plate thickness of 20 millimetres.

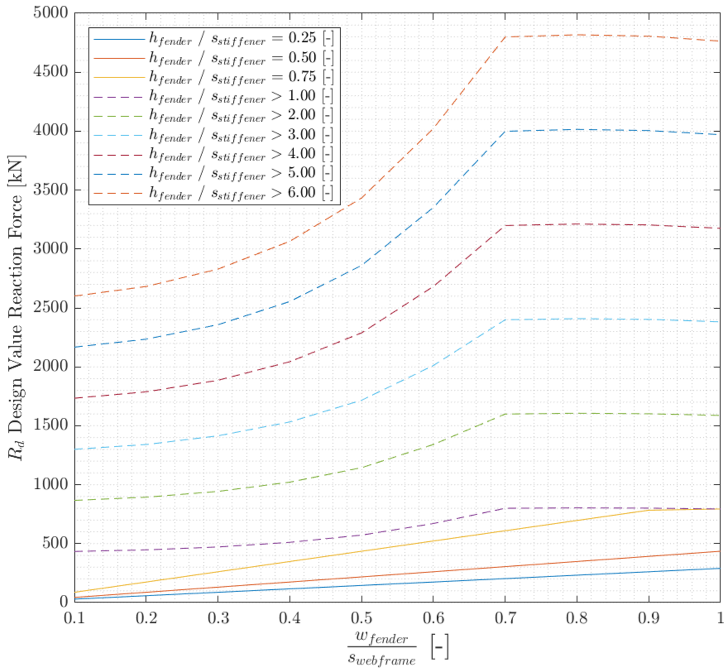


Figure B.14: Tanker 5: Maximum allowable force in kN on the grillage due to a soft fender

Table B.10: Tanker 5: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,57	1,14	1,70	2,27	2,84	3,41	3,98	4,54	5,11	5,68
Fender height	**0,3**	0,23	29	58	87	116	145	174	203	232	261	290
	0,5	0,46	44	87	131	174	218	261	305	348	392	435
	0,8	0,69	87	174	261	348	435	522	609	696	784	794
	1	0,92	433	447	472	511	572	670	800	803	801	794
	2	1,84	867	894	943	1022	1144	1341	1600	1606	1602	1588
	3	2,76	1300	1341	1415	1533	1716	2011	2399	2409	2403	2382
	4	3,68	1734	1788	1886	2043	2289	2682	3199	3212	3204	3176
	5	4,60	2167	2235	2358	2554	2861	3352	3999	4015	4005	3970
	6	5,52	2601	2682	2829	3065	3433	4023	4799	4818	4806	4764

Comparison with Current Regulations

In figure B.15 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

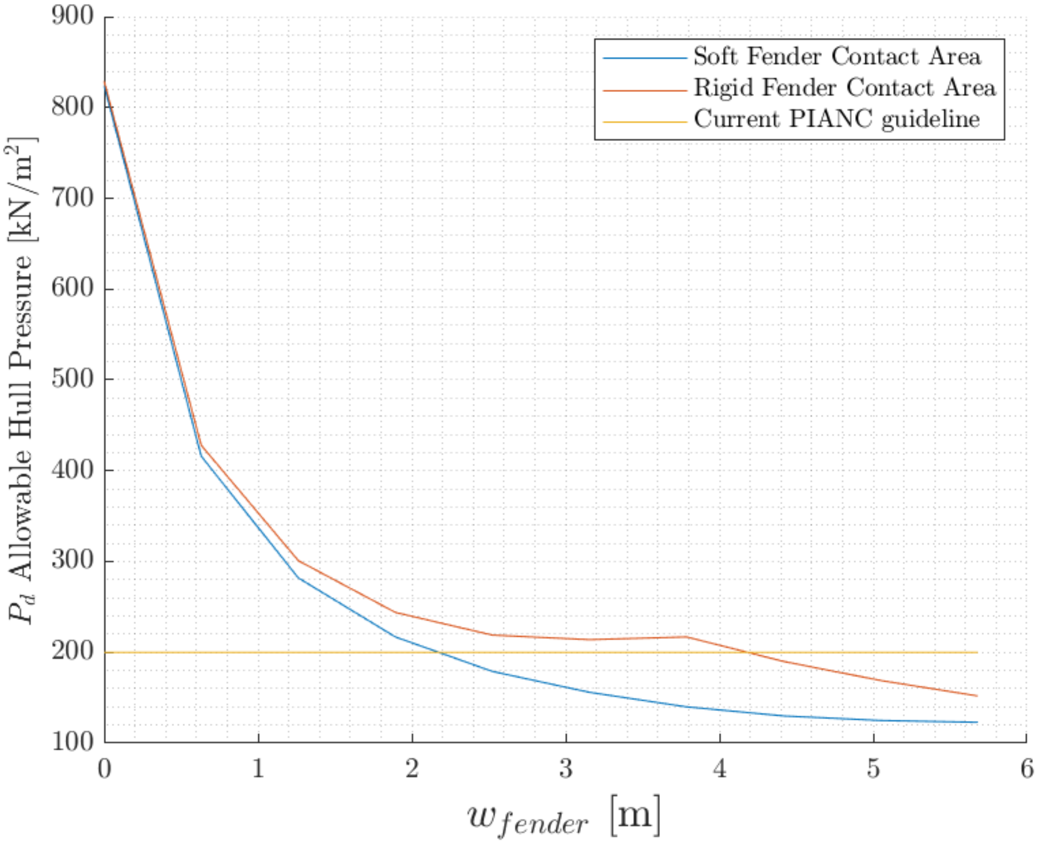


Figure B.15: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.6. Container 6: Coaster / Feeder

Soft Fender

Figure B.16 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.11. The stiffener of this vessel is assumed to be of HP 220x10 type, with a transverse web frame spacing of 2.84 metres, a stiffener spacing of 0.55 metres, and a shell plate thickness of 15 millimetres.

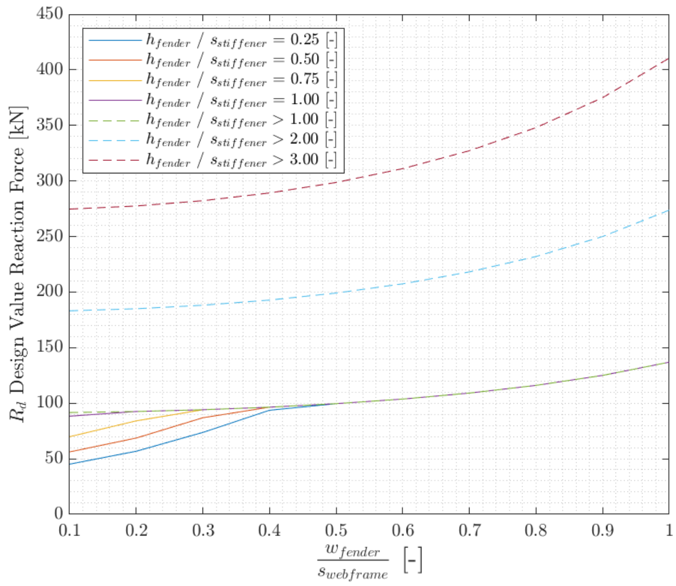


Figure B.16: Container 6: Maximum allowable force in kN on the grillage due to a soft fender

Table B.11: Container 6: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,28	0,57	0,85	1,14	1,42	1,70	1,99	2,27	2,56	2,84
Fender height	**0,25**	0,14	45	57	74	94	100	104	109	116	125	137
	0,5	0,28	56	69	87	96	100	104	109	116	125	137
	0,75	0,41	70	84	94	96	100	104	109	116	125	137
	1	0,55	88	92	94	96	100	104	109	116	125	137
	1	0,55	92	92	94	96	100	104	109	116	125	137
	2	1,10	183	185	188	193	199	207	218	232	250	274
	3	1,65	275	277	282	289	299	311	327	348	375	411

Rigid Fender

Figure B.17 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.12. The stiffener of this vessel is assumed to be of HP 220x10 type, with a transverse web frame spacing of 2.84 metres, a stiffener spacing of 0.55 metres, and a shell plate thickness of 15 millimetres.

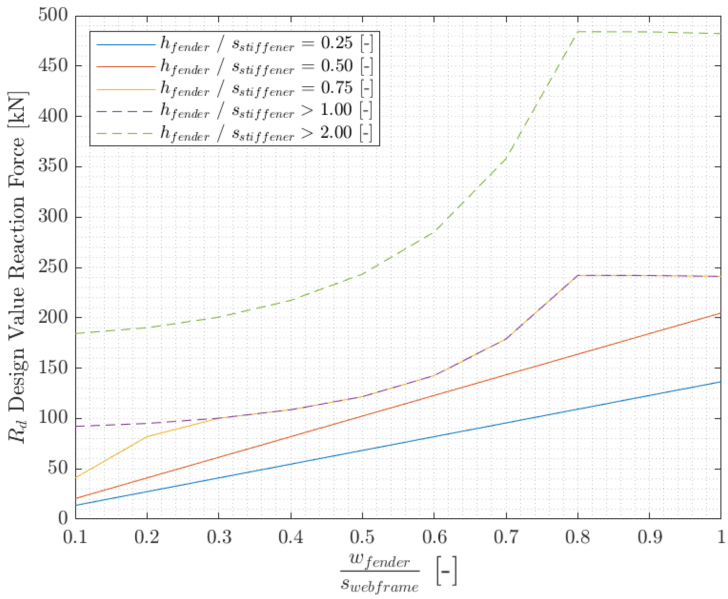


Figure B.17: Container 6: Maximum allowable force in kN on the grillage due to a soft fender

Table B.12: Tanker 6: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,28	0,57	0,85	1,14	1,42	1,70	1,99	2,27	2,56	2,84
Fender height	**0,25**	0,14	14	27	41	55	68	82	96	109	123	137
	0,5	0,28	20	41	61	82	102	123	143	164	184	205
	0,75	0,41	41	82	100	109	122	143	179	242	242	241
	1	0,55	92	95	100	109	122	143	179	242	242	241
	2	1,10	184	190	201	217	243	285	358	484	484	482

Comparison with Current Regulations

In figure B.18 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

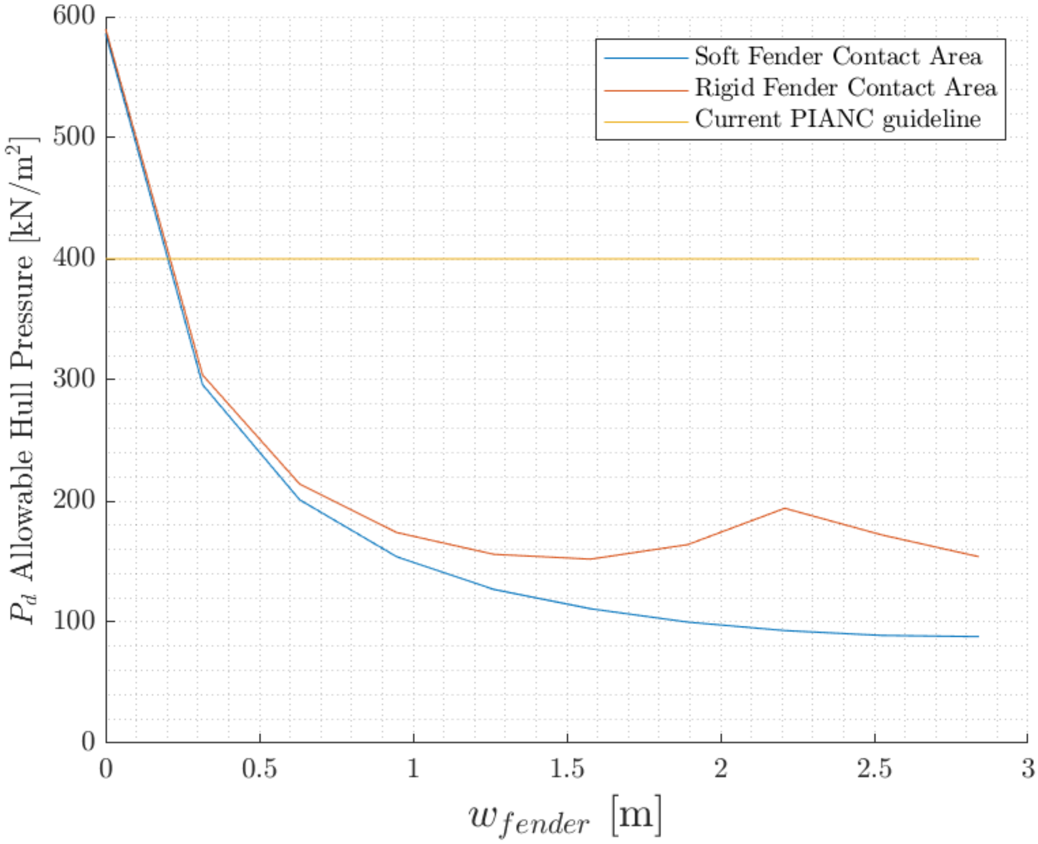


Figure B.18: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.7. Container 7: Panamax (32m)

Soft Fender

Figure B.19 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.13. The stiffener of this vessel is assumed to be of L 280x12 + 120x15 type, with a transverse web frame spacing of 3.04 metres, a stiffener spacing of 0.86 metres, and a shell plate thickness of 18 millimetres.

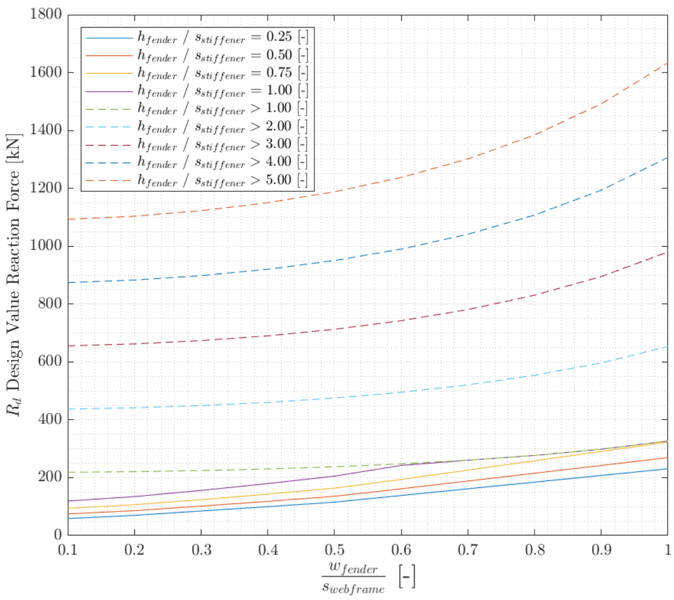


Figure B.19: Container 7: Maximum allowable force in kN on the grillage due to a soft fender

Table B.13: Container 7: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,30	0,61	0,91	1,22	1,52	1,82	2,13	2,43	2,74	3,04
Fender height	**0,25**	0,22	59	70	85	100	115	138	162	185	208	231
	0,5	0,43	75	86	102	118	136	162	188	215	242	269
	0,75	0,65	94	107	124	143	164	194	226	258	291	323
	1	0,86	119	135	156	180	205	242	260	277	298	327
	1	0,86	219	221	225	230	238	248	260	277	298	327
	2	1,72	437	442	449	460	475	495	521	554	597	653
	3	2,58	656	662	674	690	713	743	781	831	895	980
	4	3,44	874	883	898	920	950	990	1041	1108	1194	1307
	5	4,30	1093	1104	1123	1150	1188	1238	1302	1384	1492	1634
	6	5,16	1311	1325	1347	1381	1426	1485	1562	1661	1790	1960
	7	6,02	1530	1545	1572	1611	1663	1733	1822	1938	2089	2287
	8	6,88	1748	1766	1796	1841	1901	1980	2083	2215	2387	2614
	9	7,74	1967	1987	2021	2071	2139	2228	2343	2492	2685	2941

Rigid Fender

Figure B.20 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.14. The stiffener of this vessel is assumed to be of L 280x12 + 120x15 type, with a transverse web frame spacing of 3.04 metres, a stiffener spacing of 0.86 metres, and a shell plate thickness of 18 millimetres.

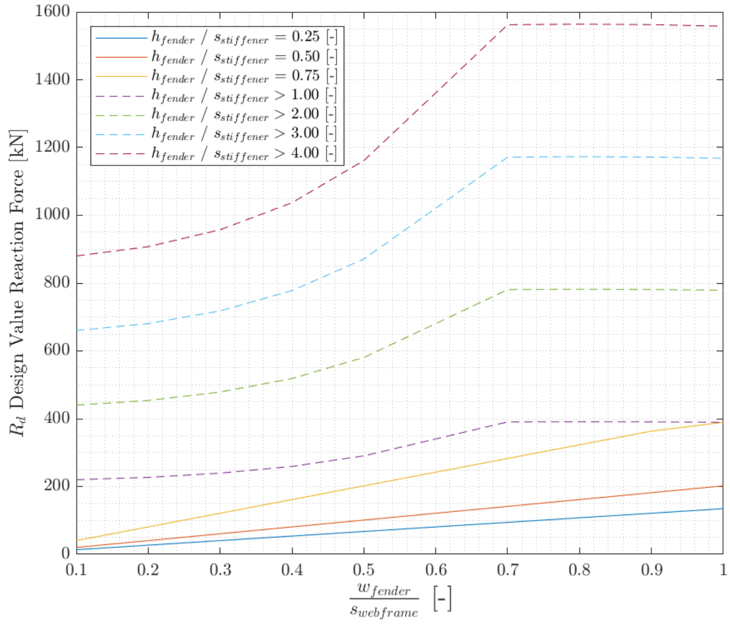


Figure B.20: Container 7: Maximum allowable force in kN on the grillage due to a soft fender

Table B.14: Tanker 7: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,30	0,61	0,91	1,22	1,52	1,82	2,13	2,43	2,74	3,04
Fender height	**0,25**	0,22	13	27	40	54	67	81	94	108	121	135
	0,5	0,43	20	40	61	81	101	121	141	162	182	202
	0,75	0,65	40	81	121	162	202	242	283	323	363	389
	1	0,86	220	227	239	259	290	340	391	391	391	389
	2	1,72	440	454	479	519	581	681	781	782	781	779
	3	2,58	660	681	718	778	871	1021	1172	1173	1172	1168
	4	3,44	880	908	957	1037	1162	1361	1562	1564	1563	1558
	5	4,30	1100	1134	1197	1297	1452	1702	1953	1955	1953	1947
	6	5,16	1320	1361	1436	1556	1743	2042	2343	2346	2344	2337
	7	6,02	1540	1588	1676	1815	2033	2382	2734	2737	2734	2726
	8	6,88	1760	1815	1915	2074	2323	2723	3124	3128	3125	3116

Comparison with Current Regulations

In figure B.21 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

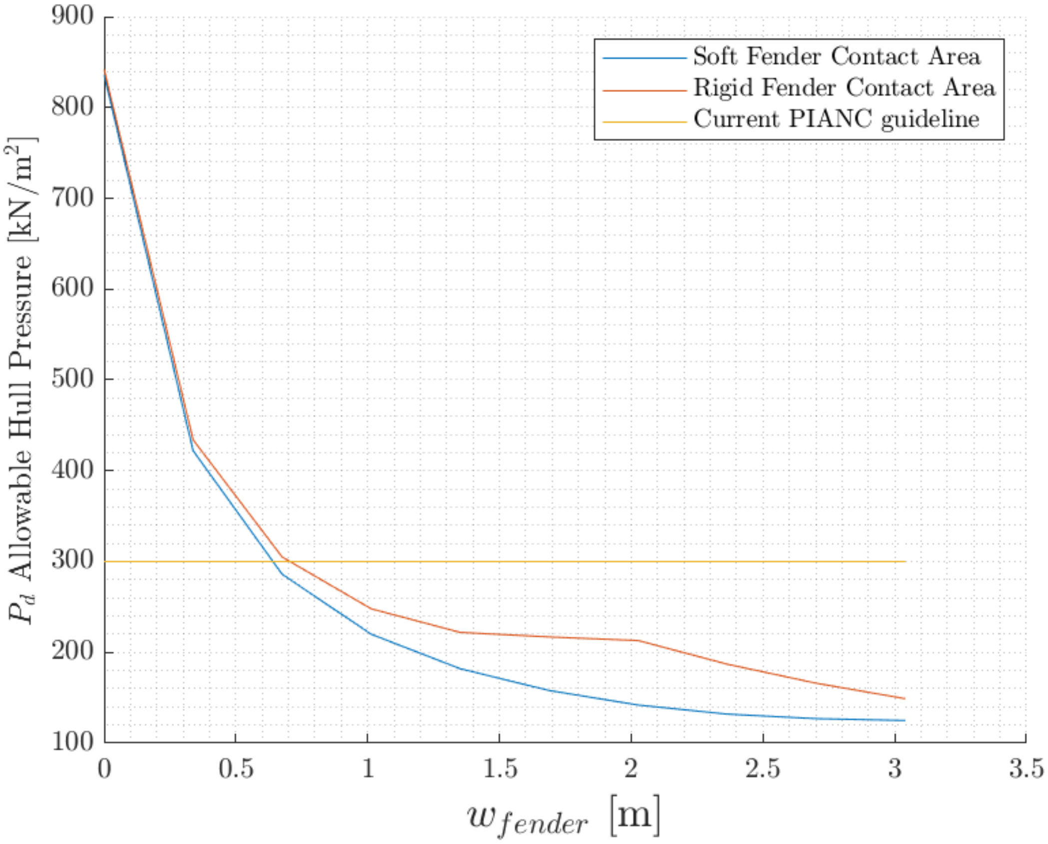


Figure B.21: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.8. Container 8: ULCV

Soft Fender

Figure B.22 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.15. The stiffener of this vessel is assumed to be of L 275x12 + 125x12 type, with a transverse web frame spacing of 3.16 metres, a stiffener spacing of 0.85 metres, and a shell plate thickness of 18 millimetres.

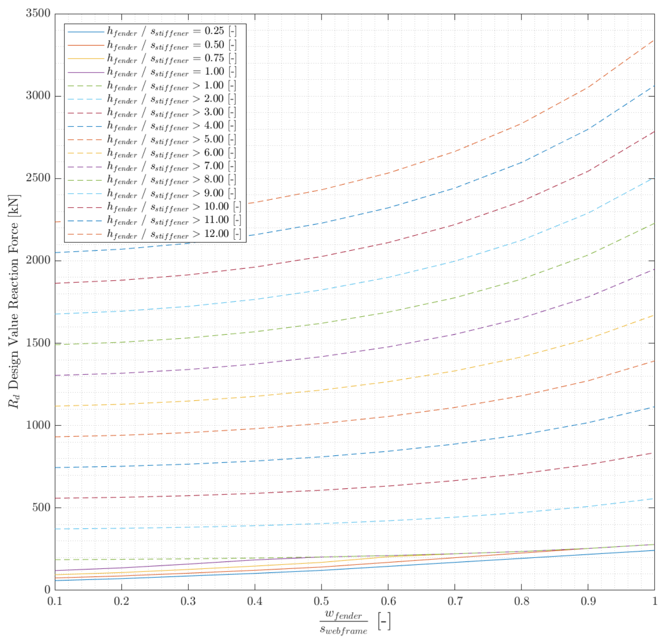


Figure B.22: Container 8: Maximum allowable force in kN on the grillage due to a soft fender

Table B.15: Container 8: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,32	0,63	0,95	1,26	1,58	1,90	2,21	2,53	2,84	3,16
Fender height	**0,25**	0,21	59	71	87	103	121	146	170	194	218	243
	0,5	0,43	75	87	104	122	142	170	198	226	254	279
	0,75	0,64	95	108	127	147	170	204	222	236	254	279
	1	0,85	120	136	159	185	203	211	222	236	254	279
	1	0,85	186	188	191	196	203	211	222	236	254	279
	2	1,70	373	376	383	392	405	422	444	472	509	557
	3	2,55	559	565	574	589	608	633	666	708	763	836
	4	3,40	745	753	766	785	810	844	888	944	1018	1114
	5	4,25	932	941	957	981	1013	1055	1110	1181	1272	1393
	6	5,10	1118	1129	1149	1177	1216	1266	1332	1417	1527	1672
	7	5,95	1304	1318	1340	1373	1418	1477	1554	1653	1781	1950
	8	6,80	1491	1506	1532	1570	1621	1688	1776	1889	2035	2229
	9	7,65	1677	1694	1723	1766	1824	1900	1998	2125	2290	2507
	10	8,50	1864	1882	1915	1962	2026	2111	2220	2361	2544	2786
	11	9,35	2050	2071	2106	2158	2229	2322	2442	2597	2799	3065
	12	10,20	2236	2259	2298	2354	2431	2533	2664	2833	3053	3343

Rigid Fender

Figure B.23 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.16. The stiffener of this vessel is assumed to be of L 275x12 + 125x12 type, with a transverse web frame spacing of 3.16 metres, a stiffener spacing of 0.85 metres, and a shell plate thickness of 18 millimetres.

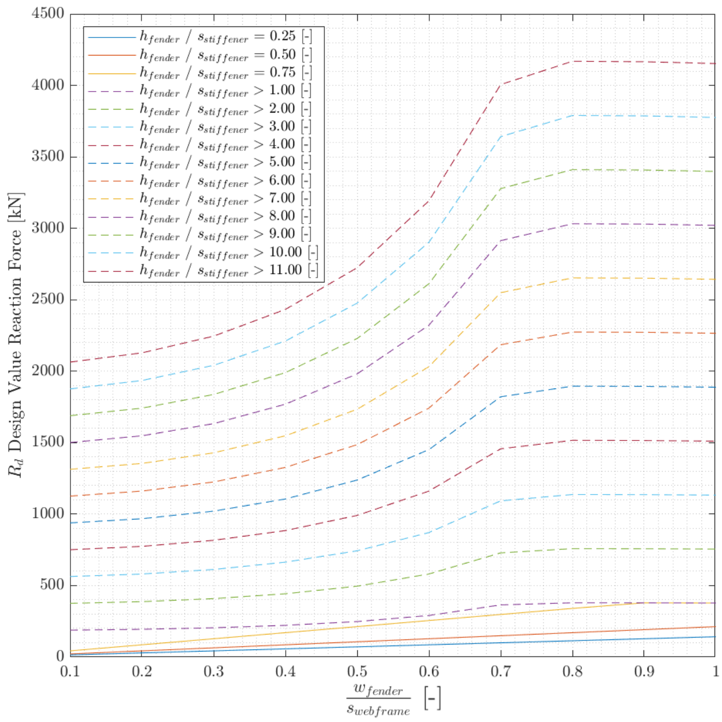


Figure B.23: Container 8: Maximum allowable force in kN on the grillage due to a soft fender

Table B.16: Tanker 8: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,32	0,63	0,95	1,26	1,58	1,90	2,21	2,53	2,84	3,16
Fender height	**0,25**	0,21	14	28	42	57	71	85	99	113	127	142
	0,5	0,43	21	42	64	85	106	127	149	170	191	212
	0,75	0,64	42	85	127	170	212	255	297	340	379	378
	1	0,85	188	193	204	221	248	290	364	379	379	378
	2	1,70	375	387	408	442	495	580	728	758	757	755
	3	2,55	563	580	612	663	743	871	1093	1137	1136	1133
	4	3,40	750	774	816	884	991	1161	1457	1516	1515	1510
	5	4,25	938	967	1021	1106	1238	1451	1821	1895	1894	1888
	6	5,10	1126	1161	1225	1327	1486	1741	2185	2274	2272	2265
	7	5,95	1313	1354	1429	1548	1734	2031	2549	2653	2651	2643
	8	6,80	1501	1548	1633	1769	1981	2322	2913	3032	3030	3021
	9	7,65	1688	1741	1837	1990	2229	2612	3278	3411	3408	3398
	10	8,50	1876	1935	2041	2211	2476	2902	3642	3790	3787	3776
	11	9,35	2064	2128	2245	2432	2724	3192	4006	4169	4166	4153

Comparison with Current Regulations

In figure B.24 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

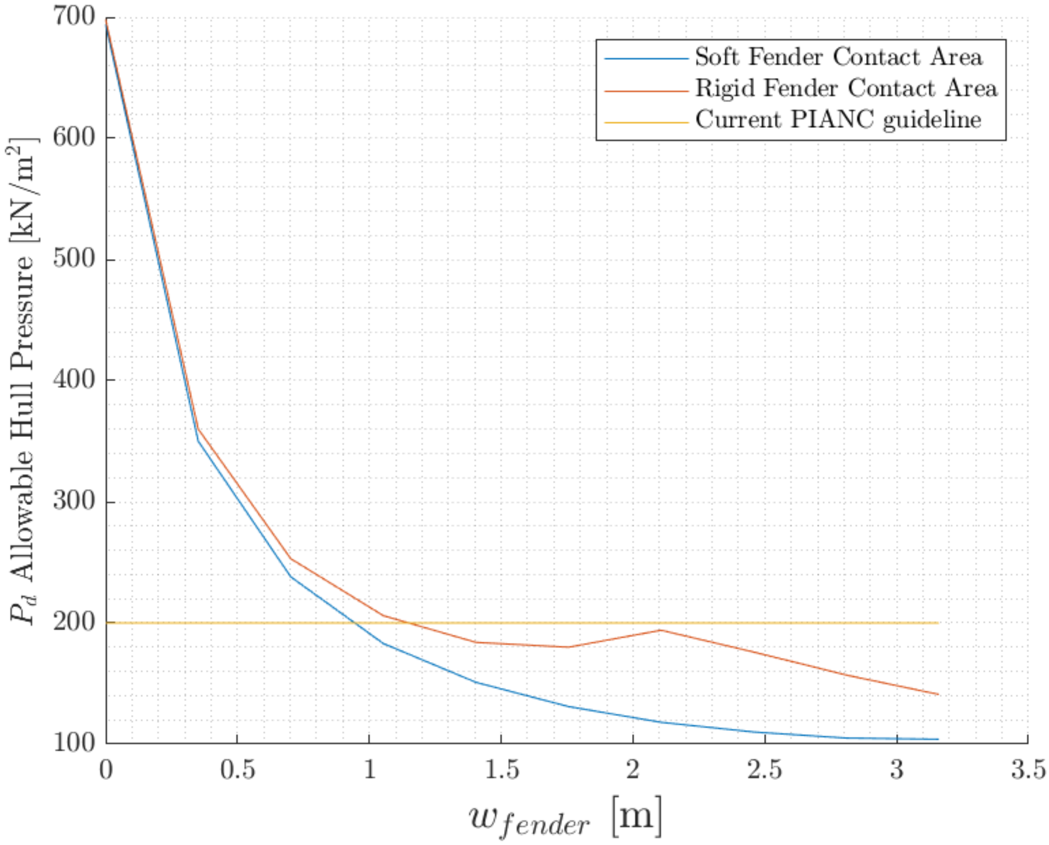


Figure B.24: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

B.9. Bulk 9: Coaster / Handysize

Soft Fender

Figure B.25 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.17. The stiffener of this vessel is assumed to be of HP 220x10 type, with a transverse web frame spacing of 2.40 metres, a stiffener spacing of 0.8 metres, and a shell plate thickness of 13 millimetres.

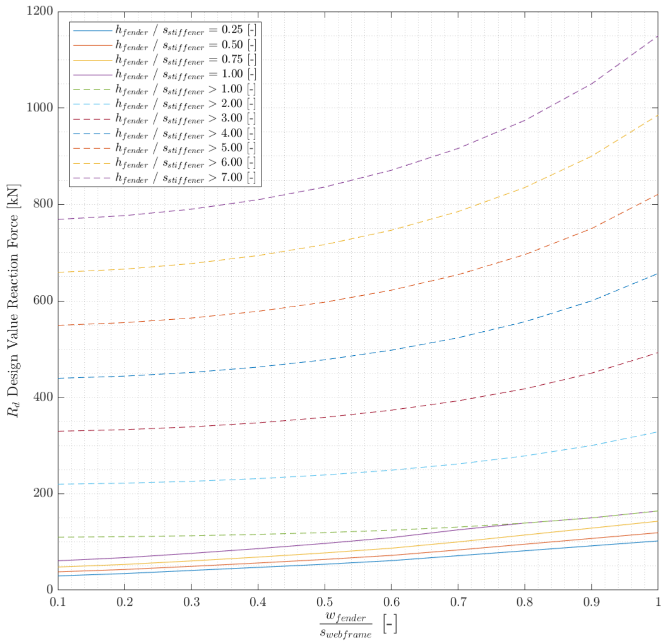


Figure B.25: Bulk 9: Maximum allowable force in kN on the grillage due to a soft fender

Table B.17: Bulk 9: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,24	0,48	0,72	0,96	1,20	1,44	1,68	1,92	2,16	2,40
Fender height	**0,25**	0,23	30	34	41	47	54	61	71	82	92	102
	0,5	0,46	38	43	49	56	64	72	83	95	107	119
	0,75	0,69	48	53	61	69	77	87	100	114	129	143
	1	0,92	61	67	76	86	97	109	125	139	150	164
	1	0,92	110	111	113	116	119	124	131	139	150	164
	2	1,84	220	222	226	231	239	249	262	278	300	328
	3	2,76	330	333	339	347	358	373	393	418	450	493
	4	3,68	439	444	451	463	478	498	523	557	600	657
	5	4,60	549	555	564	578	597	622	654	696	750	821
	6	5,52	659	666	677	694	717	747	785	835	900	985
	7	6,44	769	777	790	810	836	871	916	974	1050	1150

Rigid Fender

Figure B.26 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.18. The stiffener of this vessel is assumed to be of HP 220x10 type, with a transverse web frame spacing of 2.40 metres, a stiffener spacing of 0.8 metres, and a shell plate thickness of 13 millimetres.

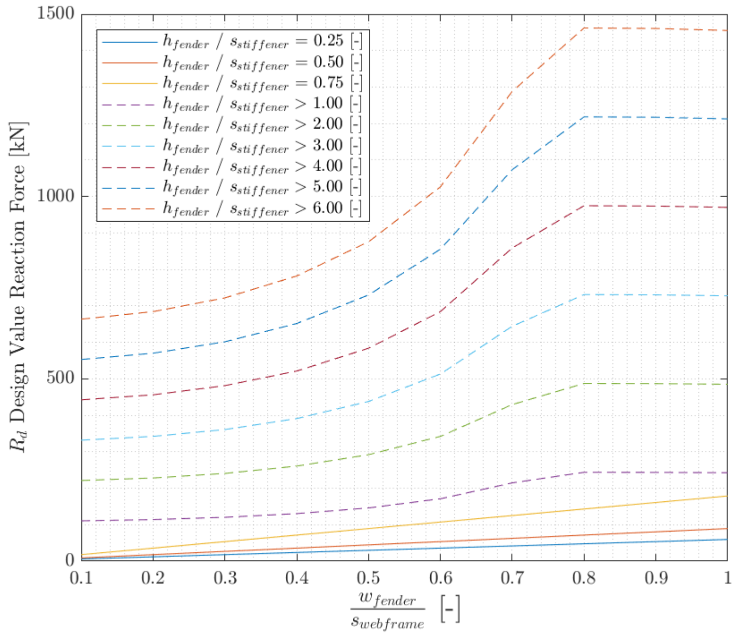


Figure B.26: Bulk 9: Maximum allowable force in kN on the grillage due to a soft fender

Table B.18: Tanker 9: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,24	0,48	0,72	0,96	1,20	1,44	1,68	1,92	2,16	2,40
Fender height	**0,25**	0,23	6	12	18	24	30	36	42	48	54	60
	0,5	0,46	9	18	27	36	45	54	63	71	80	89
	0,75	0,69	18	36	54	71	89	107	125	143	161	179
	1	0,92	111	114	120	130	146	171	215	244	243	243
	2	1,84	221	228	241	261	292	342	429	487	487	485
	3	2,76	332	342	361	391	438	513	644	731	730	728
	4	3,68	442	456	481	521	584	684	859	975	974	970
	5	4,60	553	570	602	652	730	855	1073	1218	1217	1213
	6	5,52	664	684	722	782	876	1026	1288	1462	1461	1455

Comparison with Current Regulations

In figure B.27 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

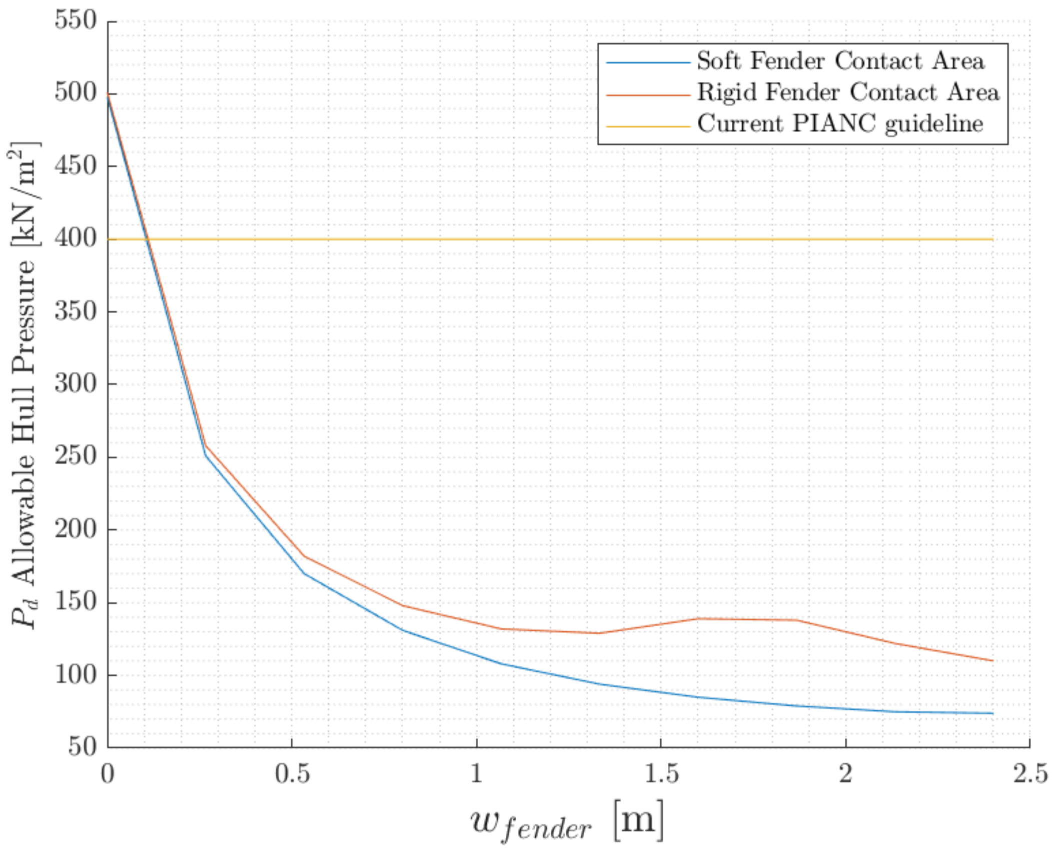


Figure B.27: Maximum allowable pressure in kN/m² dependant on the width of the fender

B.10. Bulk 11: Capesize / VLBC

Soft Fender

Figure B.28 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.19. The stiffener of this vessel is assumed to be of T 450x12 + 150x15 type, with a transverse web frame spacing of 4.95 metres, a stiffener spacing of 0.844 metres, and a shell plate thickness of 18 millimetres.

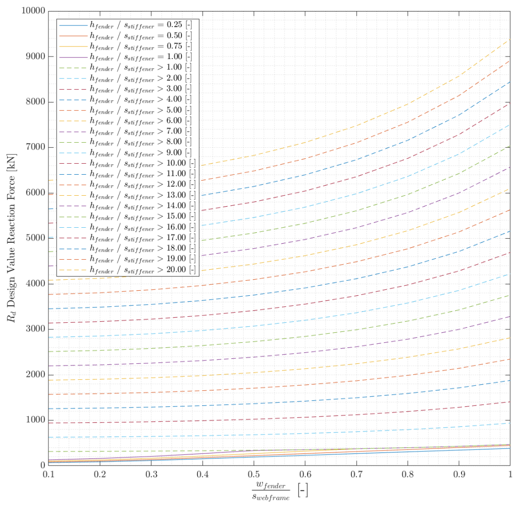


Figure B.28: Bulk 11: Maximum allowable force in kN on the grillage due to a soft fender

Table B.19: Bulk 11: Maximum allowable force in kN on the grillage due to a soft fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,50	0,99	1,49	1,98	2,48	2,97	3,47	3,96	4,46	4,95
Fender height	**0,25**	0,21	68	87	116	153	191	230	268	306	345	383
	0,5	0,42	84	105	136	179	223	268	313	357	402	447
	0,75	0,63	104	128	164	214	268	322	374	398	429	469
	1	0,84	131	161	206	268	335	355	374	398	429	469
	1	0,84	314	317	323	330	341	355	374	398	429	469
	2	1,69	628	634	645	661	683	711	748	795	857	939
	3	2,53	942	951	968	991	1024	1066	1122	1193	1286	1408
	4	3,38	1256	1268	1290	1322	1365	1422	1496	1591	1714	1877
	5	4,22	1569	1585	1613	1652	1706	1777	1870	1988	2143	2346
	6	5,06	1883	1902	1935	1983	2048	2133	2243	2386	2571	2816
	7	5,91	2197	2219	2258	2313	2389	2488	2617	2784	3000	3285
	8	6,75	2511	2537	2580	2644	2730	2844	2991	3181	3428	3754
	9	7,60	2825	2854	2903	2974	3072	3199	3365	3579	3857	4223
	10	8,44	3139	3171	3225	3305	3413	3555	3739	3977	4285	4693
	11	9,28	3453	3488	3548	3635	3754	3910	4113	4374	4714	5162
	12	10,13	3767	3805	3870	3966	4095	4266	4487	4772	5143	5631
	13	10,97	4081	4122	4193	4296	4437	4621	4861	5170	5571	6100
	14	11,82	4394	4439	4515	4626	4778	4977	5235	5567	6000	6570
	15	12,66	4708	4756	4838	4957	5119	5332	5609	5965	6428	7039
	16	13,50	5022	5073	5160	5287	5460	5688	5983	6363	6857	7508
	17	14,35	5336	5390	5483	5618	5802	6043	6356	6760	7285	7977
	18	15,19	5650	5707	5805	5948	6143	6399	6730	7158	7714	8447
	19	16,04	5964	6024	6128	6279	6484	6754	7104	7556	8142	8916
	20	16,88	6278	6341	6450	6609	6826	7110	7478	7954	8571	9385

Rigid Fender

Figure B.29 gives the resulting maximum allowable loading for this vessel type. The detailed overview of this behaviour follows from table B.20. The stiffener of this vessel is assumed to be of T 450x12 + 150x15 type, with a transverse web frame spacing of 4.95 metres, a stiffener spacing of 0.844 metres, and a shell plate thickness of 18 millimetres.

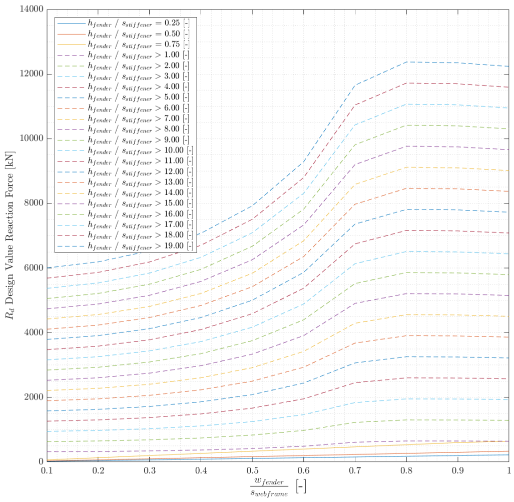


Figure B.29: Bulk 11: Maximum allowable force in kN on the grillage due to a soft fender

Table B.20: Tanker 11: Maximum allowable force in kN on the grillage due to a rigid fender

			Fender width									
	Norm. [-]		**0,1**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**1**
		Abs. [m]	0,50	0,99	1,49	1,98	2,48	2,97	3,47	3,96	4,46	4,95
Fender height	**0,25**	0,21	22	45	67	89	112	134	156	179	201	223
	0,5	0,42	33	67	100	134	167	201	234	268	301	335
	0,75	0,63	67	134	201	268	335	402	469	536	603	644
	1	0,84	316	326	344	372	417	489	613	651	650	644
	2	1,69	632	652	688	745	834	978	1227	1302	1300	1288
	3	2,53	948	978	1031	1117	1251	1466	1840	1954	1950	1932
	4	3,38	1264	1303	1375	1490	1668	1955	2454	2605	2600	2576
	5	4,22	1580	1629	1719	1862	2086	2444	3067	3256	3249	3220
	6	5,06	1896	1955	2063	2235	2503	2933	3680	3907	3899	3864
	7	5,91	2212	2281	2406	2607	2920	3422	4294	4558	4549	4508
	8	6,75	2528	2607	2750	2979	3337	3910	4907	5209	5199	5152
	9	7,60	2844	2933	3094	3352	3754	4399	5521	5861	5849	5796
	10	8,44	3160	3259	3438	3724	4171	4888	6134	6512	6499	6440
	11	9,28	3476	3585	3782	4097	4588	5377	6747	7163	7149	7084
	12	10,13	3792	3910	4125	4469	5005	5866	7361	7814	7799	7729
	13	10,97	4108	4236	4469	4842	5423	6355	7974	8465	8449	8373
	14	11,82	4424	4562	4813	5214	5840	6843	8588	9116	9099	9017
	15	12,66	4740	4888	5157	5586	6257	7332	9201	9768	9748	9661
	16	13,50	5056	5214	5500	5959	6674	7821	9815	10419	10398	10305
	17	14,35	5372	5540	5844	6331	7091	8310	10428	11070	11048	10949
	18	15,19	5688	5866	6188	6704	7508	8799	11041	11721	11698	11593
	19	16,04	6004	6192	6532	7076	7925	9287	11655	12372	12348	12237

Comparison with Current Regulations

In figure B.30 the derived forces are rewritten to a pressure formulation in order to make a fair comparrisson with the current guidelines from PIANC WG33.

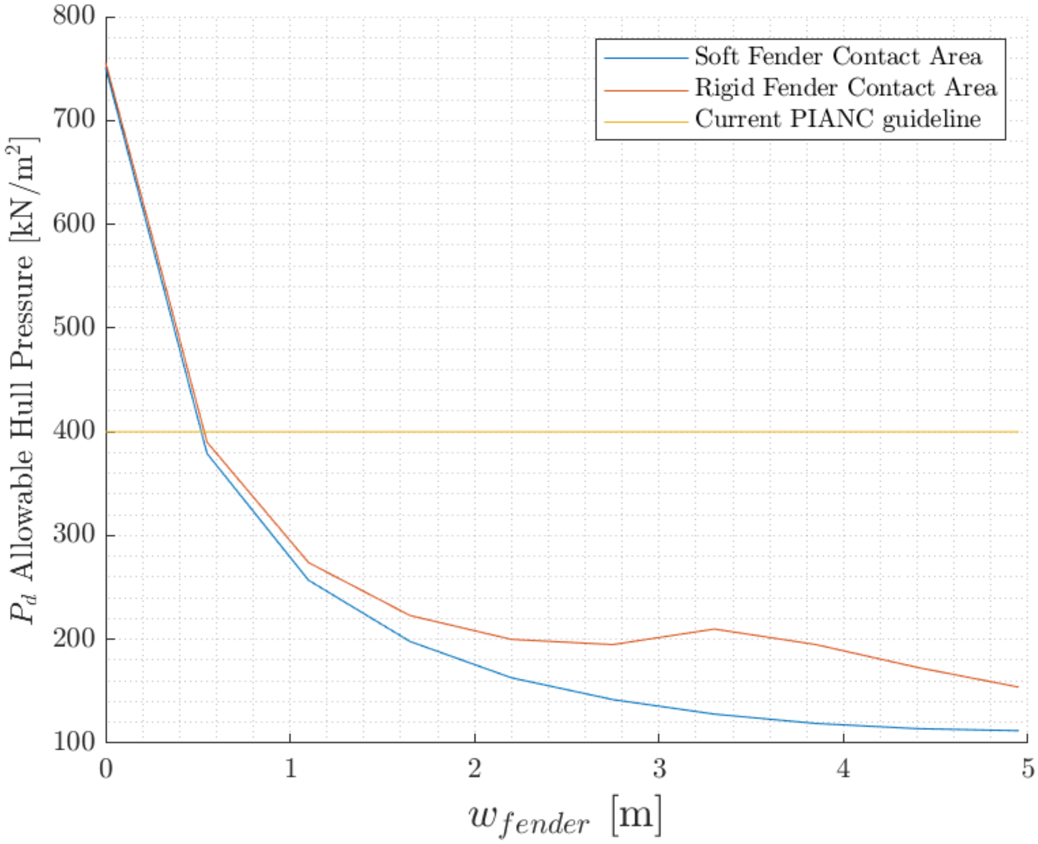


Figure B.30: Maximum allowable pressure in kN/m^2 dependant on the width of the fender

Bibliography

[1] Ansys® Academic Research Mechanical, Release 19.2.

[2] Adamchak and C. John. Design equations for tripping of stiffeners under inplane and lateral loads. No. DTNSRDC-79/064 Final Rpt. Technical report, 1979.

[3] G. Antolloni, S. Carbonari, F. Gara, C. Lorenzoni, and A. Mancinelli. Simple Physical Models to Simulate the Behavior of Buckling-Type Marine Fenders. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 143(1):04016014, 1 2017. ISSN 0733-950X. doi: 10.1061/(asce)ww.1943-5460.0000360.

[4] E. J. Broos, M. P. M. Rhijnsburger, A. W. Vredeveldt, and W. Hoebee. The safe use of cylindrical fenders on LNG, Oil and Container Terminals. Panama City, 2018. Pianc-World Congress.

[5] Bureau Veritas. Rules and Regulations for the classification of steel ships. URL https://marine-offshore.bureauveritas.com/nr467-rules-classification-steel-ships.

[6] W. Cai, L. Zhu, and F. Wang. Plastic deformation of ship plate subjected to repeated patch loads at different locations - Elastoplastic numerical analysis and design equation. *Marine Structures*, 76, 3 2021. ISSN 09518339. doi: 10.1016/j.marstruc.2020.102901.

[7] Coastal Development Institute of Technology. Guidelines for Design and Testing of Rubber Fender Systems. 2019.

[8] C. Daley, G. Hermanski, M. Pavic, and A. Hussein. Ultimate Strength of Frames and Grillages Subject to Lateral Loads - an Experimental Study. In *10th International Symposium on Practical Design of Ships and Other Floating Structures*, 2007.

[9] DNV GL. Rules for classification: Ships — DNVGL-RU-SHIP Pt.3 Ch.10., 7 2019.

[10] D. Faulkner. A review of effective plating for use in the analysis of stiffened plating in bending and compression. *J. of Ship Research*, 19(1):1–17, 1975.

[11] M. J. Gudmunsen and J. Morris. Strength of Side Shell Structure to Resist Quayside Fender Loads - Report No. CSD 89 /24. Technical report, Lloyd's Register of Shipping - Class Computational Services Group, 1989.

[12] M. J. Gudmunsen and J. Morris. A Container Ship's Ability to Resist Quayside Fender Loads - Report No. CSD 92/38. Technical report, Lloyd's Register of Shipping - Class Computational Services Group, 1993.

[13] R. Hibbeler. *Mechanics of Materials*. Pearson, 9 edition, 2014. ISBN 978-981-06-9436-4.

[14] L. Hong and J. Amdahl. Plastic design of laterally patch loaded plates for ships. *Marine Structures*, 20(3):124–142, 7 2007. ISSN 09518339. doi: 10.1016/j.marstruc.2007.05.003.

[15] F. Hughes and M. Ma. Elastic Tripping Analysis of Asymetrical Stiffeners. *Computers & Srrucrures*, 60(3):369–389, 1996.

[16] O. Hughes. Design of Laterally Loaded Plating Concentrated Loads. *Journal of Ship Research*, 27(4):252–264, 1983.

[17] O. Hughes and J. B. Caldwell. Plate Bending. In *Ship Structural Analysis and Design*, chapter 9. Society of Naval Architects and Marine Engineers (SNAME), 2010.

[18] O. F. Hughes. Design of Laterally Loaded Plating - Uniform Pressure Loads. *Journal of Ship Research*, 25(2):77–89, 1981.

[19] O. F. Hughes and J. K. Paik. 1.2.2.2 Departure from Simple Beam Theory. In *Ship Structural Analysis and Design*. Society of Naval Architects and Marine Engineers (SNAME), 2010. ISBN 978-0-939773-78-3. URL https://app.knovel.com/hotlink/khtml/id:kt008RR6N1/ship-structural-analysis/departure-from-simple.

[20] International Association of Classification Societies. UR S12 Side Structures in Single Side Skin Bulk Carriers - Rev.5 May 2010.

[21] R. Iversen, W. Bruin, B. Phan, and J. Pyun. A Proposed Rational Approach to Design of Fenders and Supporting Structures in the United States. In *Ports 2019*. American Society of Civil Engineers, 2019.

[22] N. Jones. Review of the Plastic Behavior of Beams and Plates. *International Shipbuilding Progress*, 19(218), 1972. ISSN 0020868X. doi: 10.3233/isp-1972-1921801.

[23] L. Konieczny and M. Bogdaniuk. Design of transversely loaded plating based on allowable permanent set. Technical report, 1999.

[24] J. Liu, K. Shen, H. Ding, and J. Diao. Contrast test on fender forces between flexible and rigid berthing systems. In *Applied Mechanics and Materials*, volume 155-156, pages 1184–1188, 2012. ISBN 9783037853771. doi: 10.4028/www.scientific.net/AMM.155-156.1184.

[25] Lloyd's Register of Shipping. Rules and Regulations for the classification of ships. URL https://www.lr.org/en/rules-and-regulations-for-the-classification-of-ships/.

[26] A. T. Metzger, J. Hutchinson, and J. Kwiatkowski. Measurement of marine vessel berthing parameters. *Marine Structures*, 39:350–372, 12 2014. ISSN 09518339. doi: 10.1016/j.marstruc.2014.10.001.

[27] F. Orlin. Reliability-Based Assessment for Fender Systems. Technical report, 2020.

[28] PIANC WG33. Guidelines for the Design of Fenders Systems: 2002.

[29] G. L. Putra, M. Kitamura, and A. Takezawa. Structural optimization of stiffener layout for stiffened plate using hybrid GA. *International Journal of Naval Architecture and Ocean Engineering*, 11(2):809–818, 7 2019. ISSN 20926790. doi: 10.1016/j.ijnaoe.2019.03.005.

[30] M. Rhijnsburger and A. Vredeveldt. Ship hull response during fender impact. Technical report, TNO, Delft, 5 2017.

[31] Rob-In-Transit. VLCC Block Sections [Photograph], 2010. URL https://www.flickr.com/photos/30109215@N07/5100648300.

[32] A. Roubos, L. Groenewegen, and D. J. Peters. Berthing velocity of large seagoing vessels in the port of Rotterdam. *Marine Structures*, 51:202–219, 1 2017. ISSN 09518339. doi: 10.1016/j.marstruc.2016.10.011.

[33] A. Sawczuk. Large deflections of rigid-plastic plates. In *Applied Mechanics*. 1966. doi: 10.1007/978-3-662-29364-5{_}26.

[34] Shibata Fender Team. Cylindrical Fenders Product Information, . URL https://www.shibata-fender.team/files/content/products/Cylindrical_Fenders/Product_Information_Cylindrical_Fenders.pdf.

[35] Shibata Fender Team. Shibata Fender Team, . URL https://www.shibata-fender.team/en/.

[36] Shibata Fender Team. SPC Fenders Product Information, . URL https://www.shibata-fender.team/files/content/products/SPC-Cone-Fenders/Product_Information_SPC_Fenders.pdf.

[37] Shibata Fender Team. V Fenders Product Information, . URL https://www.shibata-fender.team/files/content/products/V_Fender/Product_Information_V_Fenders.pdf.

[38] Shibata Fender Team. V Fenders Reference, . URL https://www.shibata-fender.team/en/references/references-detail/boudewijnkanaal-zeebrugge-beligium.html?application=General%20Cargo%20Terminals&fender_type=V%20Fenders&country=Belgium.

[39] T. Shibue, T. Kajiya, R. Shiba, T. Hayami, T. Sawai, M. Ohmasa, N. Hirokawa, and K. Kato. Effect of the Cross-Sectional Shape on the Buffering Performance of Hard Rubber. In *International Ocean and Polar Engineering Conference*, 2014. ISBN 9781880653913.

[40] S. Timoshenko and S. Woinowsky-Krieger. *Theory of Plates and Shells*, volume 2nd Edition. 1959. ISBN 0-07-064779-8.

[41] S. P. Timoshenko and J. M. Gere. *Theory of elastic stability*. McGraw-Hill, 1961. ISBN 0-07-Y85821-7.

[42] T. von Kármán, E. E. Sechler, and L. H. L. H. Donnell. The Strength of Thin Plates in Compression. *Trans. ASME*, 54(2):53–undefined, 1932.

[43] A. W. Vredeveldt. Memorandum: Structural analyses for LNG carriers mooring at the LBBR Gate Terminal, 4 2016.

[44] A. W. Vredeveldt and M. P. Rhijnsburger. Ship side - cylindrical fender contact characteristics, a systematic investigation R10427 Final report. Technical report, TNO, Delft, 12 2019.

[45] W. C. Young, R. G. Budynas, and A. M. Sadegh. *Roark's Formulas for Stress and Strain*. 2012. ISBN 9780071742481.

[46] Z. Yu, J. Amdahl, and Y. Sha. Large inelastic deformation resistance of stiffened panels subjected to lateral loading. *Marine Structures*, 59:342–367, 5 2018. ISSN 09518339. doi: 10.1016/j.marstruc.2018.01.005.

[47] L. Zhu, W. Cai, P. A. Frieze, and S. Shi. Design method for steel deck plates under quasi-static patch loads with allowable plastic deformations. *Marine Structures*, 71, 5 2020. ISSN 09518339. doi: 10.1016/j.marstruc.2019.102702.