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Cruise range in formation flight

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A new set of analytical range equations (a modification of the traditional Bréguet range equation), suitable for formation flight at transonic flight speeds under realistic operating conditions (constant Mach number and altitude) is derived. Formations of two aircraft of the same type are analyzed to determine the effects of; (1) weight differences between the aircraft, (2) altitude and (3) formation flight range on the potential fuel benefits and the associated optimum Mach number. In case of a weight difference, the lightest aircraft should lead the formation to realize the largest fuel benefits. Overall, fuel savings of 6% to 12% for the total formation can be realized at the expense of a reduction in cruise Mach number from 0.85 to 0.80. The fuel benefit is much less (2% to 8%) when the formation is flown at the original design cruise Mach number. In terms of fuel benefits and Mach number, it is beneficial to fly in formation at higher altitudes. Formation flight step climb procedures are possible but the additional fuel savings are minimal compared to a constant altitude formation flight.

Nomenclature

a_{sl}	=	Speed of sound at sea level conditions, m/s
A	=	Empirical factor dependent on atmospheric pressure
BPR	=	Bypass ratio
C_0	=	Empirical factor, kg/s/N
C_D	=	Drag coefficient
C_D^*	=	Minimum drag coefficient (at positive lift)
C_L^*	=	Lift coefficient at minimum drag
C_M	=	Empirical factor

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c_T = Thrust specific fuel consumption, kg/s/N

F = Net thrust, N

F_0 = Sea level static net thrust, N

g = Gravitational acceleration, m/s^2

G_0 = Gas generator function at sea level

H = Altitude [m]

K_L = Induced drag factor (solo flight)

M = Mach number

$\dot{m}_f$ = Fuel mass flow, kg/s

p = Static atmospheric pressure, N/m^2

q = Dynamic pressure, N/m^2

R = Range, m

T = Temperature, K

T_{sl} = Temperature at sea level, K

V = True airspeed, m/s

W = Aircraft weight, N

W_i = Initial aircraft weight at start of cruise flight segment, N

W_e = Aircraft weight at end of cruise flight segment, N

X = Empirical factor dependent on atmospheric pressure

Z = Empirical factor dependent on atmospheric pressure

γ = Ratio of specific heats (air)

λ = Induced drag reduction factor due to formation flight

θ = Relative temperature

Superscripts

I = Lead aircraft

II = Trailing aircraft

N = Number of aircraft in formation

I. Introduction

In formation flight, trailing aircraft can fly in the upwash region of the wake of the lead aircraft. For a constant lift coefficient this results in a reduction in induced drag [1]. Numerous research studies, ranging from numerical simulation studies to wind tunnel testing and in-flight demonstrations, have highlighted the potential fuel saving benefits of formation flight for commercial aircraft.

Back in 1996, in-flight measurements were performed on two Dornier 28 aircraft in formation flight. Up to 15% fuel burn reduction for the trailing aircraft was measured [2]. Flight test results of two Northrop T-38 Talon aircraft in formation were reported by Wagner et al. [3]. The study revealed that it is possible to achieve an average 8% fuel burn reduction for the trailing aircraft. More recently, NASA conducted flight tests with two F/A-18 aircraft in formation and demonstrated a maximum fuel burn reduction of 18% for the trailing aircraft [4]. In the past years, the Air Force Research laboratory has been investigating the potential benefits for C-17 aircraft flying in formation, in the Surfing Aircraft Vortices for Energy ($AVE) project. Average fuel burn savings for the trailing aircraft in the order of 7-9% are reported in the project [5]-[7]. To analyze the fuel burn saving for various operational conditions, mission simulations were conducted in the SAVE project. For these mission simulations, lookup tables with specific range characteristics in formation were used [7].

Whereas flight tests give accurate measurements of the potential fuel saving benefits, numerical simulations and wind tunnel tests can provide a more insight in the physics governing formation flight. Based on a simulation study, in which incompressible flow conditions were assumed, Ning et al. [8] showed that a reduction of approximately 30% in induced drag can be achieved in a two aircraft formation. Veldhuis et al. [9] used a vortex lattice method to estimate the reduction in induced drag in trimmed flight. Typical values calculated for the reduction in induced drag for the trailing aircraft in a two aircraft formation are 60%. Brahmesfeld and Maughmer [10] used a free-wake, potential flow method to analyze the aerodynamic benefits for high aspect ratio wings flying in formation. They also report a maximum 60% reduction in the induced drag coefficient when the trailing aircraft flies in the optimum location. A comprehensive study based on an Euler solver, in which compressibility effects were included, shows that total formation drag savings of 10% can be achieved when the cruise Mach number is reduced by 1-2% [11]. This is the result of a reduction in induced drag of approximately 50%. This same study also reports that the aerodynamic benefit is smaller than that predicted by incompressible simulations. It can be concluded from the simulation studies that the aerodynamic benefit can be characterized by a reduction factor on the induced drag coefficient.

The traditional Bréguet range equation, which is of great importance to aircraft performance calculations in the context of aircraft design and operation, is not applicable to formation flight. Kent and Richards [12] investigated the optimal routing for a formation flight case study which involved 210 transatlantic flights. This challenging optimization problem, which requires a large computational effort reveals the need for an analytical range equation applicable to formation flight. In their study, a drag reduction factor was applied on the range equation as a whole and flight at constant altitude and angle of attack was assumed. This is only valid for a flight with a gradual reduction in airspeed. Furthermore, their approach assumes that formation flight also affects the zero lift drag coefficient which is not the case in reality. Xu et al. [1] also investigate aircraft route optimization for formation flight. In their study, the induced drag coefficient of an aircraft in formation flight is calculated beforehand based on an aerodynamic model as a function of various parameters (lift coefficient, position in the formation, formation type). Next, the Bréguet range equation is solved using linear range factors.

In this paper, a new modified form of the Bréguet range equation is presented, which is applicable to formation flight under realistic operating conditions (constant Mach number and constant altitude flight in the transonic speed regime). Since formation flight is done with multiple aircraft, the range equation becomes a set of equations (as large as the formation) with an additional factor to account for the reduction of the induced drag coefficient for the trailing aircraft. The modified set of range equations are intended for two purposes. First, they give a fundamental insight into the various operational factors influencing the fuel benefits of formation flight. Thus, they can be used to determine for which flights it is most beneficial to fly in formation and what the corresponding optimal flight condition is. Second, the analytical set of formation flight range equations can be used for fast computation in large scale flight planning optimization problems.

The paper has the following structure. First, the analytical models required to accurately represent the aerodynamic performance and the propulsion system characteristics of commercial turbofan aircraft at transonic flight conditions are presented. Next, the set of range equations for formation flight at constant altitude and Mach number in the transonic flight regime is derived. Formation flight can be conducted at different altitudes, Mach numbers and cruise distances. Furthermore, the aircraft weight of the lead aircraft may be different than the trailing aircraft. In Section III, a set of realistic formation flight scenarios (combinations of aircraft weight and range) is defined. For each scenario, the fuel benefit and corresponding optimum flight Mach number is determined. This is compared to solo flight performance and formation flight performance at the cruise Mach number used in solo flight. Next, the effect of altitude is investigated. A long range formation scenario in which the potential fuel savings are large is extended by the introduction of a sub-optimal formation step climb flight technique. Finally, a preliminary investigation into the effect of aircraft type on formation flight performance is presented. Overall conclusions and recommendations are made at the end of the paper.

II. Aircraft model

In a practical sense, the largest formation flight fuel savings can be obtained with long range flights. For the current study, a generic large transport aircraft with four turbofan engines is therefore defined. This aircraft has a design range of approximately 10,000 km when flying at 9750 m (32,000 feet) altitude and Mach 0.85. Basic aircraft data are summarized in Table 1. The analytical models representing the aerodynamic and propulsion system characteristics are described in the following two paragraphs.

Variable	Value
Maximum Take-off Weight (MTOW) [kN]	3600
Operational Empty Weight [kN]	1800
Maximum Fuel Weight [kN]	1600
Maximum Payload Weight [kN]	600
Wing surface area [m^2]	525
Cruise Mach number	0.85

Table 1 Basic generic aircraft data

A. Aerodynamics

A three-term parabolic lift drag polar is used to describe the aerodynamics in formation flight. This is required to obtain the sufficient accuracy at transonic Mach numbers [13]. Two-term polars are typically accurate for only a limited range of operating conditions. By definition, the minimum drag coefficient of a two-term lift drag polar occurs at the zero lift condition. In this research, the three-term lift drag polar as used by Torenbeek is used, which includes a parameter that defines the lift coefficient at which the polar has its minimum drag coefficient [13]. The lift drag polar is extended with a factor λ which represents the reduction in the induced drag coefficient due to formation flight when flying in the most optimal location behind the lead aircraft.

$$C_D = C_D^*(M) + \lambda(M) K_L(M)\left(C_L - C_L^*(M)\right)^2 \qquad (1)$$
$$0 \le \lambda \le 1$$

In order to account for compressibility effects, the parameters C_D^*, K_L, C_L^* and λ, are functions of the Mach number. The generic parameters used in this study are summarized in Table 2.

M	C_D^*	K_L	C_L^*
0.30	0.0197	0.085	0.163
0.40	0.0192	0.085	0.163
0.50	0.0187	0.085	0.163
0.60	0.0183	0.095	0.179
0.65	0.0181	0.100	0.186
0.70	0.0172	0.120	0.210
0.75	0.0174	0.133	0.222
0.80	0.0176	0.147	0.232
0.85	0.0184	0.174	0.235

Table 2 Generic three-term lift drag polars

Note that Equation 1 reduces to the more widely used two-term lift drag polar in case $\boldsymbol{C_L^*}$ is set equal to zero. The results presented in the following sections of the paper can therefore also be applied in case only two-term lift drag polars are available.

B. Propulsion

Fuel flow and thrust are, by definition, related through the thrust specific fuel consumption c_T. Furthermore, the variation in aircraft weight is directly proportional to the fuel flow.

$$\begin{aligned} \dot{m}_f &\equiv c_T F \\ \frac{dW}{dt} &= -\dot{m}_f g \end{aligned} \tag{2}$$

Thrust specific fuel consumption divided by the square root of the relative temperature, to account for altitude effects, is assumed to be a linear function of Mach number [14]. This approximation gives realistic results for the cruise regime, when applied to a limited Mach number and altitude range [14].

$$\begin{aligned} \frac{c_T}{\sqrt{\theta}} &= C_0\left(1 + C_M M\right) \\ \theta &= \frac{T}{T_{sl}} \end{aligned} \tag{3}$$

C_0 and C_M are empirical factors and the parameter $\boldsymbol{\theta}$ is defined as the temperature relative to sea level conditions. For a given aircraft weight, the performance limits (speed and altitude) are determined by the aerodynamic characteristics and the maximum thrust available. An empirical equation introduced by Battel and Young [15] is used to determine maximum turbofan engine thrust available as function of altitude and Mach number.

$$\frac{F}{F_0} = A - \frac{0.377(1+BPR)}{\sqrt{(1+0.82BPR)G_0}} ZM + \left(0.23 + 0.19\sqrt{BPR}\right) XM^2 \tag{4}$$

The empirical factors *A*, *Z* and *X* are functions of the atmospheric pressure. These functions can be found in [15]. So, essentially, this equation only requires the sea level static thrust, atmospheric pressure and bypass ratio as input. Key propulsion system characteristics of turbofan engines are summarized in Table 3.

Variable	Value
Bypass ratio BPR	5
Single engine sea level static thrust F_0 [kN]	270
Number of engines	4
C_M [-]	1
C_0 [mg/s/N]	10

Table 3 Generic aircraft propulsion system data

III. Formation flight at constant altitude and constant Mach number

The basic form of the Bréguet range equation, assuming zero wind conditions, is provided in equation (5). The solution to this equation depends on the flying strategy used. There are various flying strategies possible and solutions to these can be found in standard text books on aircraft performance (e.g. Ohja [16]).

$$R = \int_{W_e}^{W_i} \frac{V}{c_T g} \frac{C_L}{C_D} \frac{1}{W} dW \tag{5}$$

The most fuel efficient cruise flying technique is a gradual climb at constant angle of attack and airspeed. In the real world, commercial aircraft operations are subject to regulations imposed by air traffic control. It is therefore more practical to fly at a constant Mach number and altitude. The optimal cruise climb can then be approximated by several step climbs. Formation flight is even more demanding from an operational perspective, since the trailing aircraft must carefully fly in the optimum position to obtain a reduction in induced drag. The most practical **flying strategy used for formation flight** is therefore one at **constant Mach number and constant altitude**. In this study, it is assumed that the trailing aircraft flies in the optimum location behind the lead aircraft. This optimum location can be determined through computational prediction (CFD), wind tunnel tests or flight-tests and will be different for each combination of aircraft types. In general, the optimal lateral spacing occurs when wing tips overlap 10-25% of their span [10]. The vertical position of the trailing aircraft must be slightly below the lead aircraft with a distance in the range of 0-10% of the wing span [17]. The sweet spot in terms of lateral and vertical position is limited with a radius smaller than 10% of the wing span [18]. The aircraft must therefore be accurately controlled to achieve a drag reduction. There is a large tolerance in the longitudinal separation [4]. Veldhuis et al. use a longitudinal separation of 10 wing spans for civil transport aircraft [9]. The final assumption is that all aircraft in the formation are assumed to be of the same type. The range equations for formation flight using this flying strategy are derived next. The superscript *N* in the equations indicate which aircraft in the formation is evaluated.

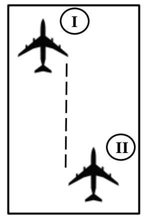

Fig. 1 Definition of lead aircraft and trailing aircraft

There must be vertical equilibrium throughout the complete cruise flight. Assuming the component of the thrust perpendicular to the flight path can be neglected, the following equation for the lift coefficient is obtained:

$$C_L = \frac{W}{qS} \qquad q = \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\gamma p M^2 \tag{6}$$

Equations 1 and 6 can be combined to determine the lift over drag ratio as a function of aircraft weight and this result can be substituted in the range equation (5).

$$R^N = \int_{W_e^N}^{W_i^N} \frac{V}{c_T g q S} \frac{1}{\left(C_D^* + \lambda^N K_L \left(\frac{W^N}{qS} - C_L^*\right)^2\right)} dW^N \tag{7}$$

Airspeed, thrust specific fuel consumption and dynamic pressure are constant since the flight is conducted at constant altitude and Mach number. Thus, they can be placed outside the integral. Next, the following elementary trigonometry rule can be used to solve the integral.

$$\int \frac{dx}{1+ax^2} = \frac{1}{\sqrt{a}} \tan^{-1}\left(x\sqrt{a}\right) \tag{8}$$

The range equation then becomes:

$$R = \frac{V}{c_T g \sqrt{K_L C_D^* \lambda^N}} \left(\tan^{-1}\left(\frac{\sqrt{\lambda^N K_L}\left(C_L^* qS - W_e^N\right)}{qS\sqrt{C_D^*}} \right) - \tan^{-1}\left(\frac{\sqrt{\lambda^N K_L}\left(C_L^* qS - W_i^N\right)}{qS\sqrt{C_D^*}} \right) \right) \tag{9}$$

Since thrust specific fuel consumption is a function of Mach number and the parameters in the three-term lift drag polars as well, this result can be expressed in terms of the Mach number.

$$R = \frac{Ma_{sl}}{C_0\left(1+C_M M\right)g}\frac{1}{\sqrt{C_D^*\lambda^N K_L}}\left[\tan^{-1}\left(\sqrt{\frac{\lambda^N K_L}{C_D^*}}\frac{\left(\frac{1}{2}\gamma pM^2SC_L^* - W_e^N\right)}{\frac{1}{2}\gamma pM^2S}\right) - \tan^{-1}\left(\sqrt{\frac{\lambda^N K_L}{C_D^*}}\frac{\left(\frac{1}{2}\gamma pM^2SC_L^* - W_i\right)}{\frac{1}{2}\gamma pM^2S}\right)\right] \tag{10}$$

This can be rewritten to express the final weight for each aircraft in a formation as a function of formation flight range.

$$W_e^N = \frac{1}{2}\gamma pM^2S\left[C_L^* - \sqrt{\frac{C_D^*}{\lambda^N K_L}}\tan\left\{\frac{RC_0\left(1+C_M M\right)g\sqrt{\lambda^N K_L C_D^*}}{Ma_{sl}} + \tan^{-1}\left(\sqrt{\frac{\lambda^N K_L}{C_D^*}}\frac{\left(\frac{1}{2}\gamma pM^2SC_L^* - W_i^N\right)}{\frac{1}{2}\gamma pM^2S}\right)\right\}\right] \tag{11}$$

The final weight of the lead aircraft can be determined using the range equation with $\boldsymbol{\lambda^I=1}$ since there is no benefit for the lead aircraft. In case of larger formations, the induced drag reduction factor of the third aircraft can be larger than that of the second aircraft. Equation 11 can be rearranged using the following trigonometric identity.

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \tag{12}$$

$$R^N = \frac{Ma_{sl}}{C_0\left(1+C_M M\right)g}\frac{1}{\sqrt{C_D^*\lambda^N K_L}}\left[\tan^{-1}\left(\frac{1}{qS}\frac{\sqrt{\frac{\lambda^N K_L}{C_D^*}}\left(W_i^N + W_e^N\right)}{1+\frac{\lambda^N K_L}{C_D^*}\left(C_L^* - \frac{W_e^N}{qS}\right)\left(C_L^* - \frac{W_i^N}{qS}\right)}\right)\right] \tag{13}$$

The range $\boldsymbol{R}$ of all aircraft for the formation flight cruise segment must be identical. Assuming a two aircraft formation, a relation between the final weight of the lead aircraft and the trailing aircraft can be derived. For the lead aircraft there is no induced drag reduction ($\lambda^I=1$)

$$R^I = R^{II} \tag{14}$$

$$\tan^{-1}\left(\frac{1}{qS}\frac{\sqrt{\frac{K_L}{C_D^*}}\left(W_i^I + W_e^I\right)}{1+\frac{K_L}{C_D^*}\left(C_L^* - \frac{W_e^I}{qS}\right)\left(C_L^* - \frac{W_i^I}{qS}\right)}\right) - \frac{1}{\sqrt{\lambda^{II}}}\left[\tan^{-1}\left(\frac{1}{qS}\frac{\sqrt{\frac{\lambda^{II} K_L}{C_D^*}}\left(W_i^{II} + W_e^{II}\right)}{1+\frac{\lambda^{II} K_L}{C_D^*}\left(C_L^* - \frac{W_e^{II}}{qS}\right)\left(C_L^* - \frac{W_i^{II}}{qS}\right)}\right)\right] = 0 \tag{15}$$

This equation shows the relation between the final weights of both aircraft as a function of the operational condition in terms of Mach number and altitude (combined in dynamic pressure term ***q***) and the initial aircraft weights. Finally, the total fuel consumption for the two aircraft formation can be quantified.

$$W_{fuel} = W_i^I + W_i^{II} - W_e^I - W_e^{II} \tag{16}$$

The optimum Mach number in formation flight is defined as the condition that results in a minimum total fuel consumption.

$$\begin{gathered} \frac{\partial W_{fuel}}{\partial M} = 0 \\ \frac{\partial W_{fuel}}{\partial M} = f\left(\lambda^{II}, W_i^I, W_i^{II}, R\right) \end{gathered} \tag{17}$$

The approach to find the total fuel consumption and corresponding optimum Mach number for a given flight range and altitude can be summarized. First, select a range of Mach numbers from well below the optimum Mach number in solo flight up to the maximum Mach number. Next, specify the initial weight of the lead aircraft and compute its final weight using Equation 11. This can be done iteratively to make sure there is enough fuel onboard (but not too much). Repeat this calculation for the trailing aircraft based on a solo flight to find its initial weight. This is done for safety reasons. The trailing aircraft should be able to reach its destination without formation flight benefits. With the initial weight of the trailing aircraft, calculate its final weight in formation based on Equation 11. Apply equation 16 to find the total fuel consumption. This can be done for all Mach numbers to find the optimum formation flight Mach number.

IV. Results and discussion

A. Specific range in formation flight

Before the fuel benefits for a complete formation flight can be analyzed, it is worthwhile to first evaluate the benefits for a single flight condition (point performance). The aerodynamic efficiency (*L/D*) of the generic aircraft model is presented as function of Mach number and lift coefficient in Figure 2. These results can be obtained directly from the lift drag polars (Eq. 1). The vertical equilibrium condition for both a low and a high aircraft weight is presented in the figure. The situation where maximum *L/D* is obtained for a given aircraft weight is considered a local optimum in terms of aerodynamic efficiency.

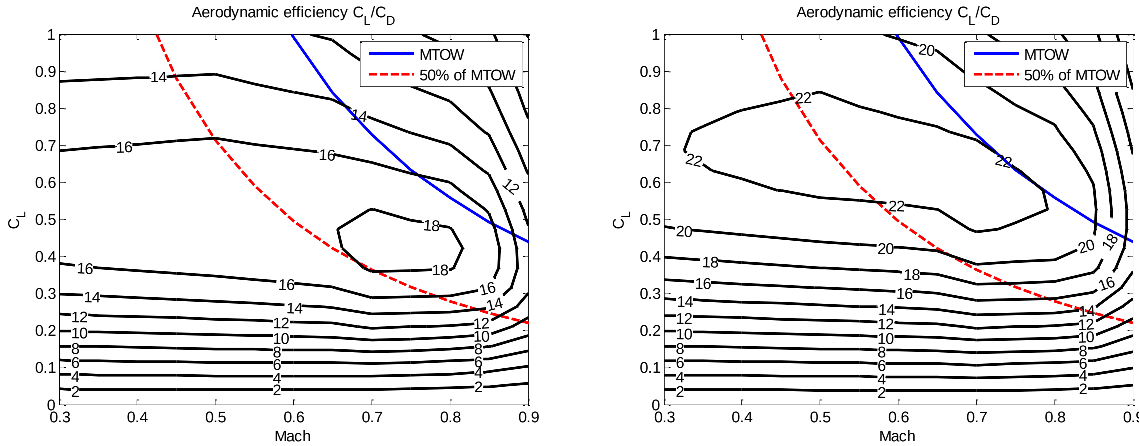

Fig. 2 Aerodynamic efficiency in solo (left) and formation flight (right) (H = 9750 [m], λ^{II} = 0.5)

As expected, Figure 2 demonstrates that the aerodynamic efficiency improves for all Mach numbers and lift coefficients due to formation flight. The optimum Mach number for a fixed aircraft weight in terms of the aerodynamic efficiency decreases. This is expected since the induced drag component of the total drag is larger at lower Mach numbers due to a higher lift coefficient required for vertical equilibrium. It should be noted that the optimum Mach number in formation flight depends on both the optimum Mach number of the lead and trailing aircraft since they have to fly at the same speed.

Range however, does not only depend on the aerodynamic efficiency. The specific range parameter, which includes the contribution of the Mach number and the thrust specific fuel consumption (Eq. 3) is therefore shown in Figure 3. Again, efficiency is improved in formation flight and the optimum flight Mach number for a given aircraft weight is reduced compared to solo flight. This effect is less pronounced compared to the situation where only aerodynamic efficiency if considered. Note that at 50% of MTOW, the vertical equilibrium condition is almost parallel to a contour line of constant specific range, indicating that a large range of Mach numbers results in the same fuel efficiency for the trailing aircraft. Of course one should realize that the total fuel efficiency (both aircraft combined) counts.

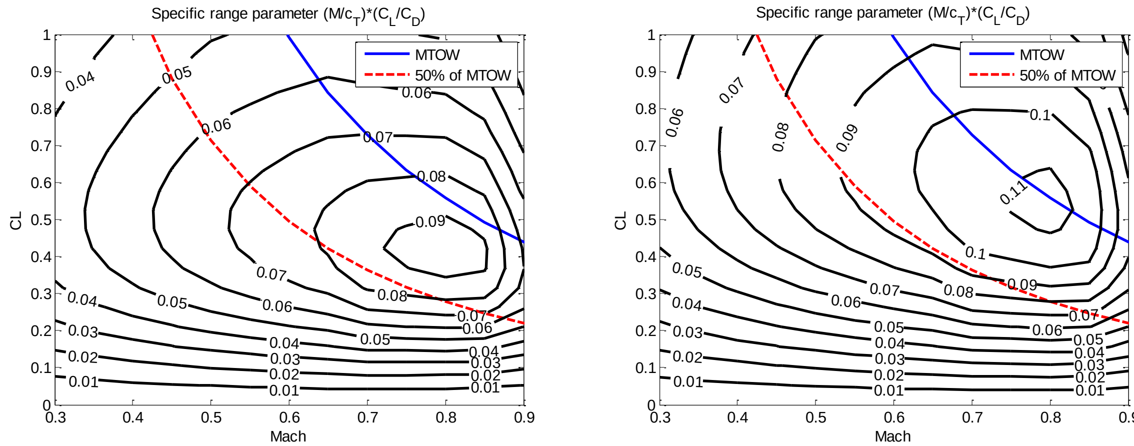

Fig. 3 Specific range in solo (left) and formation flight (right) (H = 9750 [m], λ^{II} = 0.5)

B. Fuel benefits and optimum Mach number in formation flight

In solo flight, the generic aircraft model has a cruise range of approximately 10,000 km when flying at 9750m altitude. In this case, the aircraft starts the cruise phase at 97% of the MTOW and the maximum payload weight is onboard (it is assumed that part of the fuel is consumed in the climb phase). This generic aircraft can also be used for shorter range missions and it is possible to spend only part of a mission in formation. Therefore a large set of different initial aircraft weight combinations and formation flight ranges is possible.

For example, one aircraft flies from Amsterdam to Los Angeles (~9000km) and has a starting weight equal to the MTOW. Another aircraft flies from Amsterdam to New York (~6000km) and since it does not have to fly its design range, it has a lower starting weight equal to ~90% of the MTOW. A flight segment of 5000km between Amsterdam and New York is flown in formation and the other flight segments are flown solo.

The first question to be answered is the following. **Which aircraft should lead the formation?** A formation flight segment with a distance of 2500km is considered to evaluate that aspect. At this relatively short formation flight distance there can be a large difference in the initial aircraft weight. The fuel burn for a formation flight of 2500 km with a heavy aircraft and a light aircraft is provided in figure 4 and compared to the solo flight conditions. The first data set in the figure represents the fuel consumption of aircraft I in solo flight and the second data represents aircraft II in solo flight. One can observe that the optimum Mach number for the heavy aircraft in solo flight is 0.83 and for the light aircraft in solo flight, it equals 0.8. The reference value in the figures is the total fuel consumption when two solo flights are conducted at their respective optimum Mach numbers

$$W_{fuel,reference} = W_{fuel,solo}^{heavy}\Big|_{M=0.83} + W_{fuel,solo}^{light}\Big|_{M=0.80} \tag{17}$$

All data sets in the figure are non-dimensionalized with respect to the reference value. Thus, in solo flight, the heavy aircraft is responsible for 58% of the total reference fuel consumption and the light aircraft for 42%. The third data set shows fuel consumption of the trailing aircraft (II) in formation. For the lead aircraft, a formation flight is the same as a solo flight, apart from the fact that it may be conducted at a different Mach number. Thus, the first data set also represents aircraft I in formation. The summation of the fuel burns of aircraft I and II in formation yields the total formation fuel burn.

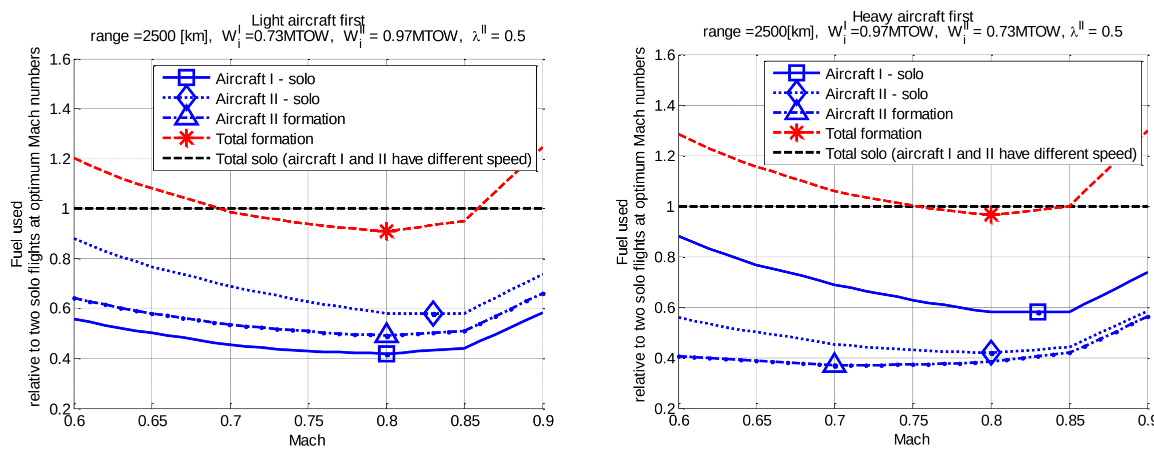

Fig. 4 Fuel used for formation with heavy and light aircraft. (*H* = 9750 m).

In formation the fuel burn of the trailing aircraft is lowered considerably. In formation, both aircraft will have to fly at the same Mach number and thus, each aircraft will fly at a suboptimal speed to achieve the overall minimum fuel burn. Based on this figure, the total fuel benefit of a formation flight compared to two optimum solo flights can be determined. In both scenarios (heavy aircraft first or light aircraft first) significant fuel savings are obtained. With the heavy aircraft as trailing aircraft, total fuel burn is reduced by 11.3%. The fuel burn reduction is 3.6% when the heavy aircraft leads the formation. It can therefore be concluded that the lightest aircraft should lead the formation in order to minimize fuel burn.

To evaluate the different combinations of aircraft weight and range, various scenarios are evaluated. Four formation flight ranges are considered. Four different initial aircraft weights for both the lead and trailing aircraft are considered as well. These weights correspond to a situation with maximum payload and the minimum amount of fuel required to fly the specified range. Reserve fuel and fuel required for the descent and landing is not considered. This suggests that there are 4 (range) x4 (lead aircraft weight) x4 (trailing aircraft weight) = 64 possible scenarios.

However, an aircraft must have enough fuel onboard to reach its destination, therefore not all scenarios are feasible. Furthermore, in terms of fuel burn it is beneficial to have the heaviest aircraft as trailing aircraft. Therefore only 20 scenarios are feasible. For each scenario, the fuel benefits are calculated for both the optimum Mach number and for a cruise Mach number of 0.85. Results are summarized in Tables 4 and 5.

		Range in formation [km]			
W_i^I / *MTOW*	W_i^{II} / *MTOW*	2,500	5,000	7,500	10,000
0.73	0.73	4.5			
0.73	0.80	6.2			
0.73	0.87	8.5			
0.73	0.97	11.3			
0.80	0.80	5.9	5.0		
0.80	0.87	8.1	7.0		
0.80	0.97	10.9	9.5		
0.87	0.87	7.7	6.7	5.8	
0.87	0.97	10.4	9.1	8.0	
0.97	0.97	9.7	8.6	7.5	6.6

Table 4 Formation flight fuel savings (%) at optimum Mach number

Fuel savings for the formation flight segment between 6.4% and 12.5% can be obtained. The relative fuel savings are largest when there is a large weight difference between the aircraft. The relative fuel saving is fairly constant (approximately 7%) when both aircraft have the same initial weight. It should be noted that these are relative fuel benefits. Obviously the total fuel consumed in a long range flight is much larger than in a short range flight.

		Range in formation [km]			
W_i^I / *MTOW*	W_i^{II} / *MTOW*	2,500	5,000	7,500	10,000
0.73	0.73	2.5			
0.73	0.80	4.1			
0.73	0.87	6.1			
0.73	0.97	8.8			
0.80	0.80	4.0	3.2		
0.80	0.87	5.9	4.9		
0.80	0.97	8.5	7.2		
0.87	0.87	5.7	4.8	4.0	
0.87	0.97	8.2	7.0	5.9	
0.97	0.97	7.7	6.6	5.7	4.9

Table 5 Formation flight fuel savings (%) at Mach = 0.85

At a cruise Mach number of 0.85, fuel savings from 2.5% to 8.8% can be obtained. This is lower than at the optimum Mach number but still considerable. In this case, flight time does not have to be reduced.

C. The effect of altitude

The effect of altitude on the benefits of formation flight is investigated for a scenario where both aircraft have an initial weight at the start of the formation flight segment of 80% of the MTOW. Furthermore, the formation flight segment has a distance of 2500km. This scenario is chosen because the turbofan engines deliver sufficient thrust to fly at realistic Mach numbers at altitudes ranging from 9 km to 11 km for this aircraft weight. Thus, performance limits of the aircraft are not encountered in the analysis. The fuel benefit of formation flight relative to two solo flights is summarized in Figure 5 (left). At each flight altitude, formation flight results in a fuel burn reduction. The optimum flight altitude for solo flights is 9800 m. The fuel used displayed in the picture is relative to this condition. The optimum flight altitude in formation flight is higher, at 10600 m. The fuel burn in the optimum formation flight condition is 6.2% lower than the fuel burn in the optimum solo flight condition. The optimum Mach number at which the maximum fuel benefits is obtained is presented as well (Figure 5 – right). This shows that the optimum Mach number in formation flight increases with altitude from Mach 0.75 at 9km up to Mach 0.8 at 9750m and beyond. For the optimum conditions at 9800 m and 10600, the optimum Mach numbers are identical (Mach = 0.8). Since the temperature decreases up to the tropopause (11 km), the groundspeed of the optimum formation flight is slightly lower (1.1%) than the optimum solo flight

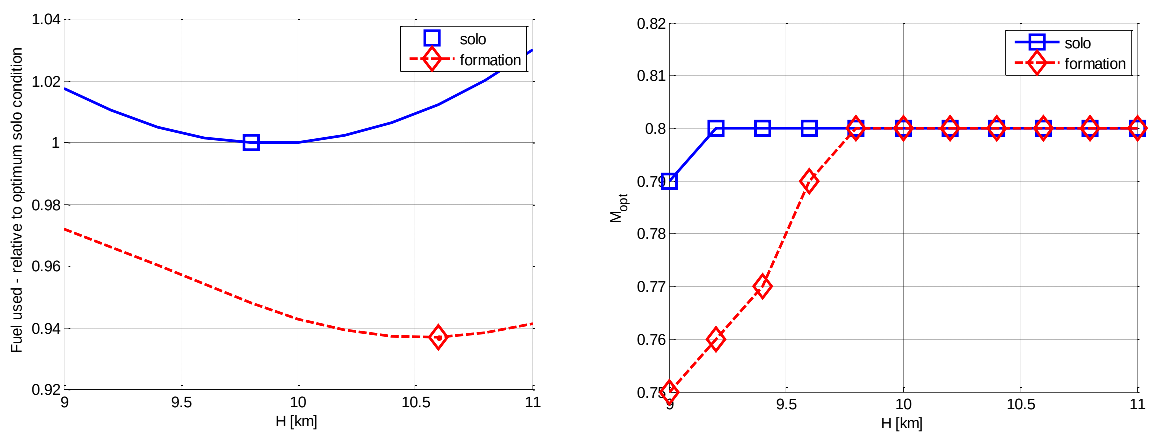

Fig. 5 Fuel used (left) and optimum Mach number (right) as function of altitude. ($W_i^{I} = W_i^{II}$ = 0.8MTOW, λ^{II} = 0.5).

Since altitude has a positive effect on fuel benefit and Mach number, it could make sense for long range flights to conduct a **sub-optimal step climb procedure**. It is called sub-optimal because the altitude change in a step climb is restricted by the engine performance limits of the heaviest aircraft in the formation. Hence, the lightest aircraft cannot necessarily climb to the altitude that would be most efficient. Thus, not only the flight Mach number is a compromise between the aircraft in the formation but also the flight altitude. The sub-optimal step climb profile for a long range formation flight is presented in Figure 6. Also displayed in the figure is a constant altitude flight at the highest altitude possible at the start of the flight. The altitude in that case is limited by the engine thrust available. In this sub-optimal formation step climb, the formation climbs from approximately 9km altitude to 10.75km altitude. The fuel consumption of this formation step climb is only 0.5% lower than the constant altitude formation flight. Since additional fuel will be burnt in the climb segments, which is not calculated, the additional benefit is negligible. It is therefore not recommended to perform sub-optimal formation flight climb procedures.

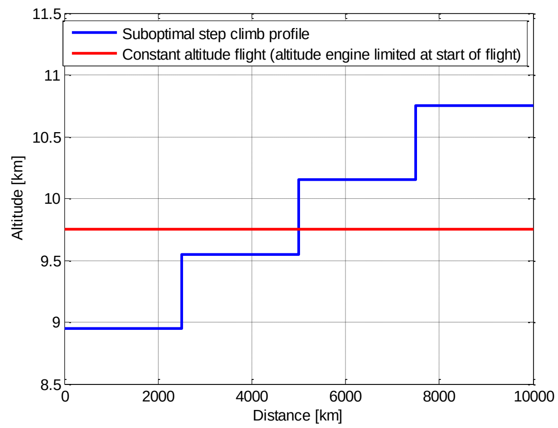

Fig. 6 Suboptimal formation flight step climb profile. ($W_i^I = W_i^{II} = 0.97MTOW$, $\lambda^{II} = 0.5$).

D. The effect of aircraft type

So far, aircraft of the same type were considered and the effects of weight, Mach number, range and flight altitude were investigated. It is also possible to perform a formation flight with another aircraft type that has different aerodynamic and propulsion system characteristics. Furthermore, it is even possible to combine different aircraft types in a single formation. To investigate the effect of aircraft type on fuel burn savings in formation flight, a sensitivity study can be performed. For this, the partial derivatives of the aerodynamic coefficients and the propulsion system coefficients should be determined either analytically or numerically. The sensitivity of the fuel savings to aerodynamic and propulsion system characteristics is presented in Table 6. The reference is a long range formation flight of 10,000km executed at the optimum flight Mach number (see Table 4).

Reference	C_0 *(+10%)*	C_M *(+10%)*	λ *(+10%)*	C_D *(+10%)*	K_L *(+10%)*	C_L^* *(+10%)*
6.6%	6.3%	6.5%	5.9%	6.0%	6.9%	5.9%

Table 6 Sensitivity of fuel savings to aircraft characteristics (Range = 10,000km)

The empirical propulsion system coefficients are increased independently with 10%. This means that the thrust specific fuel consumption increases by 10% or that the sensitivity to the Mach number increases. This does not affect the aerodynamic situation but it does affect the fuel flow and thereby it has a small influence on the fuel saving which is relative. Changing the bypass ratio or sea level static thrust would mainly influence the maximum flight altitude. Altitude effects are discussed in the preceding section. An increase in one of the aerodynamic coefficients by 10% has a larger effect than a change in the propulsion system as expected. The fuel savings in that scenario are still comparable to the reference. A 10% change in aerodynamic coefficients is considered huge from a design perspective. Note that a 10% change in the induced drag reduction factor has the largest impact. Interestingly, this is also the parameter with most uncertainty. Furthermore, literature shows that this factor is very sensitive to the position in which the trailing aircraft flies. Based on the limited sensitivity study, it is expected that the results presented are also valid for other aircraft types. The key parameter that must be identified accurately is the induced drag reduction factor.

V. Conclusions and recommendations

A modified range equation, suitable for formation flight at transonic flight speeds is introduced. Due to operational constraints, the flight strategy is one of constant Mach number and altitude. The equations are based on three-term lift drag polars and a thrust specific fuel consumption model which is a linear function of Mach number. An induced drag reduction factor is part of the equations to account for the formation flight benefit of the trailing aircraft. Several conclusions can be drawn from the equations.

- In case of a weight difference, the lightest aircraft should lead the formation to achieve the largest fuel benefits.
- Fuel savings for the complete formation of 6% -12% can be achieved when the cruise Mach number is lowered (typically from 0.85 to 0.80)
- At the design cruise Mach number of 0.85, formation flight still results in fuel savings. However, these are considerably lower at values ranging from 2.5% to 8.8%
- Altitude has a positive effect on the fuel benefit and on the corresponding optimum Mach number.
- The optimum Mach number and altitude in formation flight depend on the characteristics of both aircraft in the formation

- For long range formation flights, a sub-optimal step climb procedure is possible. The additional benefit compared to constant altitude formation flight is however very small.
- The analytical range equations are suitable for use in flight planning optimization problems.

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