Submerged Ramjet Intake Modeling

MSc. Thesis Erwin van Wijk



Delft University of Technology, Faculty of Aerospace Engineering TNO Defense, Safety and Security

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Submerged Ramjet Intake Modeling by

Erwin van Wijk

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Defense date:	December 12	
Thesis Committee	Dr. ir. F.F.J. Schrijer	TU Delft
	Ing. R.G. Veraar	TNO
	Dr. ir. B.W. van Oudheusden	TU Delft
	Dr. ir. E. Mooij 4	TU Delft

Preface

After 7 (and a bit more) years I can finally present my thesis. This document covers only the last year of work, mostly performed at TNO Defence and Security. I would like to thank Ronald Veraar of TNO for the oppurtunity to do my thesis project there and for his valuable feedback and advice. To my fellow interns at TNO; it was a pleasure to work with you and watch the monkeys together. Also many thanks to Ferry Schrijer of the aerodynamics department of aerospace engineering for his efforts in advising and supervising.

I owe my family many, many thanks for their continued support throughout the (slightly longer than planned) time of my studies. And finally I would like to express my immense gratitude for the 7 very enjoyable years I have spent at De Bolk. You guys and gals have made being a student a lot more fun that I had expected before coming to Delft.

Summary

A design for a ramjet air intake is theorized where the intake is submerged in the body of the vehicle or projectile propelled by this ramjet. While there is an expected gain in compactness of the propulsion system, there are also a number of drawbacks that may penalize such a design. The initial section of a submerged intake contains an expansion during which the boundary layer grows and the Mach number increases. The former poses problems when the boundary layer interacts with shock waves, which are increased in strength due to the higher Mach number. Stronger shock waves result in a lower total pressure recovery, which reduces engine performance.

To evaluate the performance of proposed submerged intakes in an early design phase, a performance analysis model able to evaluate a range of intake geometries is required. This document details how such a model was developed with the use of available sub-models for specific flow features. A formulation of the method of characteristics suitable for rotational flow was used to compute the parts of the flow field not affected by viscous effects. Boundary layers, both laminar and turbulent, were implemented via integral integral relations that provide the growth of the boundary layer. The effects of shock waves interacting with the boundary layers, both in shock wave reflection on the intake walls and the pressure rise in the shock train, are implemented via experimentally obtained correlations. A wind-tunnel validation of the full integrated intake analysis model could not be performed because the test model could not be completed in time. A conceptual design of a submerged air intake as well as a proposed test program to perform the validation is included.

To investigate the effects that several design variables have on the intake performance, a parametric study was performed with the intake analysis model. It confirmed the initial expectation that an increased expansion angle significantly decreases the total pressure recovery, a measure of efficiency, that is achieved by the intake. Using a gradual compression or multishock compression surface reduces the loss of total pressure due to shock waves. Performance could be further improved by performing further internal contraction via an improved cowl shape.

Nomenclature

Symbols

Symbol	Units	Description
β	deg/rad	Shock wave angle
γ	-	Ratio of specific heats
δ	m	Boundary layer thickness
Δ	-	Transformed boundary layer thickness
Δ	-	Change
δ^*	m	Boundary layer displacement thickness
Δ^*	-	Transformed boundary layer displacement thickness
θ	deg/rad	Deflection angle over a shock wave,
θ	m	Boundary layer momentum thickness
Θ	-	Transformed boundary layer momentum thickness
μ	deg/rad	Mach angle $sin^{-1}(1/M)$
μ	kg/(m s)	Viscosity
ν	m^2/s	Kinematic viscosity
ho	$\rm kg/m^3$	Density
au	Pa	Shear stress
ϕ	deg/rad	Flow angle
ω	deg/rad	Prandtl-Meyer angle
a	m/s	Speed of sound
C_f	-	Skin friction coefficient
Ď	-	Degree of asymmetry
D	m	Diameter
h	J	Enthalpy
g	-	Ratio of enthalpies h/h_0
Η	-	Boundary layer shape factor
H_i	-	Transformed boundary layer shape factor
L	m	Length of shock train
L	m	Interaction length
Μ	-	Mach number
n	m	Distance normal to stream line
р	Pa	Pressure
q	Pa	Dynamic pressure
P_r	-	Pressure recovery factor
\Pr	unit	Prandtl number
R	J/(kg K)	Gasconstant
R	m	Radius
Re	-	Reynolds number
\mathbf{S}	J/(kg K)	Entropy
\mathbf{S}	ĸ	Sutherlands constant

Symbol	Units	Description
S^*	-	Non-dimensional shock strength
Т	Κ	Temperature
u, v	m/s	Velocity components
U	m/s	Velocity in transformed plane
x,y	m	Coordinates
X, Y	-	Transformed coordinates

subscripts

Symbol	Description
A, B, C	Property determined in point/region A, B, C
e	Boundary layer edge
end	End of intake, entry to combustor
ref	Reference condition
rms	Root-mean-square
W	Wall
х, у, Х, Ү	Differentiated w.r.t. the coordinate
0	Stagnation
1, 2,	Station numbers

Abbreviations

- AAM Air-to-Air Missile
- CFD Computational Fluid Dynamics
- MoC Method of Characteristics
- SAM Surface-to-Air Missile
- SWBLI Shock Wave Boundary Layer Interaction

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Chapter 1

Introduction

A large effective range is a useful quality for any weapon system. In the field of air-to-air missiles (AAM) a limiting factor on the range is the amount of fuel and oxidizer available for the rocket motor. A solution to increase the range of an AAM is to replace the rocket motor by a ramjet engine. This was done in the Meteor missile which has a reported maximum range of 300 kilometers under ideal conditions. A ramjet takes air from the atmosphere as oxidizer for the combustion of fuel, allowing a larger amount of fuel to be carried. The air intakes of the Meteor protrude about 10 cm, approximately a body radius as seen in Figure 1.1, from the missile body. These intakes may hamper the integration of the missile in closed weapon bays of fighter planes such as the Joint Strike Fighter.

Aside from AAMs, a ramjet engine may also be employed in extended range artillery. The muzzle velocity of artillery is typically in the range of Mach 2 to 4, sufficiently high for ramjet operation. A part of the explosive charge may be replaced with fuel and nozzle to perform powered flight during a significant part of the trajectory. When the ramjet has burned out at high altitude, the extra speed (or limited deceleration) allows for a long ballistic trajectory which expands the range. Again, form factor is important here, as the shell must fit within the barrel of the artillery gun system.

A ramjet uses shock waves or compression fans to deliver highly compressed air to the combustion chamber. If the design of the weapon system allows it, the intake is placed at the nose of the object. This reduces the frontal area and thus lowers the drag. In the applications mentioned above, the front of the vehicle is allocated to guidance, navigation and control (GNC) systems. As a result the Meteor missile employs the two intakes that can be seen in Figure 1.1. With the increased emphasis on the form factor of the whole vehicle, a theorized design solution is to employ an intake submerged in the vehicle body, which is sketched in Figure 1.2. Indicated in the figure is the projectile body, which bends inward and generates an expansion fan. The flow is directed toward the combustion chamber via a shock wave. Also indicated in Figure 1.2 are the boundary layers of the relevant surfaces and the bleed slot through which the boundary layer on the projectile body is removed.

Even before any analysis is performed it is known that there are two strong drawbacks to this intake geometry. The flow expansion that occurs when the projectile body wall turns inward increases the Mach number of this part of the flow and increases the thickness of the boundary layer. Higher Mach numbers are associated with stronger shock waves and consequently higher loss in total pressure. This reduces the pressure that can be achieved upon entry to the combustion chamber, which is detrimental to the performance of the engine. Thick boundary layers also have a negative impact on the pressure that can be achieved. In the design of the Meteor this problem was reduced by placing the intake some distance away from the boundary layer; a small gap can be seen between the missile body and the intake. In a submerged intake ingestion of the boundary layer cannot be avoided and some kind of boundary layer suction or bleed slot must be employed.



Figure 1.1: MBDA Meteor, taken from mbda-systems.com



Figure 1.2: Sketch of a submerged intake

The gain in form factor may be worth the penalties of reduced total pressure recovery. In order to gain a better insight in these penalties, it is required to analyze a variety of intake designs. Due to the large number of design variables it is not economical to run a detailed CFD simulation over a representative set of designs. A rapid analysis tool that requires little computational effort but produces a sufficiently accurate result would be more useful. In order to attain a sufficiently accurate analysis, it is not enough to only simulate the inviscid flow. A number of viscous phenomena affect the performance of the intake and must be included. For this reason the growth of the boundary layers is predicted, which is in turn used to predict the extent of shock wave boundary layer interactions (SWBLIs) and the pressure rise through the shock train, a system of successive SWBLIs.

Research Goals and Research Questions

The goal of the present research is to obtain this rapid analysis tool for the preliminary design phase of a projectile employing a submerged air intake.

Construct a MATLAB model able to analyze the performance of various design options for a submerged ramjet intake.

To achieve this goal we require numerical or analytical descriptions of several flow phenomena. These sub-models can be combined in a model to analyze a given intake geometry, which is the major component of the rapid analysis tool.

• How can the inviscid flow be decribed?

A large part of the flow will not be affected by viscous effects. This allows for a simplification of the calculations in this region. The method used to calculate this part of the flow must be able to incorporate the effects of shock waves and rotational flow by non-uniform shock wave strength.

• How can the boundary layer growth be calculated?

The boundary layer thickness is known to affect the flow in the intake, and even intake geometry, in several ways. To reduce the extent of the shock wave boundary layer interaction the boundary layer on the projectile body is removed via a bleed slot, sized to remove the whole boundary layer. Shock wave boundary layer interactions and the shock train are influenced by the boundary layer parameters as well. Not only turbulent boundary layers need to be predicted, but laminar boundary layers are present in the intake as well.

• How can the effects of SWBLI be included?

In shock wave boundary layer interaction (SWBLI) the boundary layer parameters may be significantly altered by the sudden pressure increase caused by the shock wave. The effects of boundary layer thickness may be significant on the performance of the intake and must be quantified.

• How can the length and pressure rise over the shock train be calculated?

Due to the presence of the boundary layer, the deceleration to subsonic speeds prior to entry to the combustion chamber will take the form of a shock train; a series of shock wave boundary layer interactions. Of significant importance are the pressure obtained at the combustion chamber entrance and the length of the shock train. The first will affect engine performance, where the latter predicts the moment at which the shock system is expelled from the intake and unstart occurs.

The analysis model will employ engineering models available in literature in order to determine the flow characteristics inside the intake. It will be shown that many of the flow features that are encountered can be accounted for by these approximations sufficiently accurate to give an indication of the submerged intake performance. A full validation of the developed submerged intake analysis model could not be performed in time to be included in this report. It was delayed by the extensive time required in order to create a wind tunnel model and will be performed at a later date.

Structure of Thesis

This thesis is structured as follows; first a theoretical background is established, this concerns an introduction to ramjet intake in general, as well as basic coverage on phenomena such as the shock train. This is followed by the presentation of several sub-models that will be employed in modeling the flow field in Chapter 3. An overview of how these sub-models are integrated into the overall intake model is given in Chapter 4. In the same chapter, the assumptions governing the flow regimes are covered, as well as the limitations of the model. Chapter 5 covers a parametric study performed to establish the effect of several geometric parameters, as well as the effect test conditions on intake performance and the boundary layer on the expansion ramp. Finally, in Chapter 6 the performed work is and obtained results are summarized, and recommendations are made for further developments on the intake analysis model.

Chapter 2

Theoretical Background

This chapter introduces background information on relevant subjects for the present research. First some background on ramjets is given, both in historic and future use of ramjets and an introduction to ramjet intake behavior are presented. Section 2.2 covers the prediction of boundary layer transition. This is followed by Section 2.3 of the phenomenon labeled 'reverse transition' which may occur in the expansion section of the intake. The chapter is concluded by a look at two types of interactions between the boundary layers and shock waves. First as oblique shock wave reflection on the boundary layer in Section 2.4, followed by the shock train phenomenon in Section 2.5.

2.1 Background on Ramjet Intakes

Before flow phenomena and calculation methods will be discussed in detail, it is beneficial to gain an overview of ramjet technology. First a short overview of historic and possible future ramjet propelled vehicles is given. This is followed by a more detailed look into the internal flow dynamics of the intake to indicate which flow behaviour must be modeled in the intake performance analysis model.

2.1.1 Historic and Future use of Ramjets

Although the examples listed in the introduction are high-tech applications, the integration of a ramjet in weaponry is not a new concept. During WWII detailed plans were created for a ramjet powered projectile fired from artillery to achieve a range of 350 km [7]. Ever since, ramjet applications have been continuously tested and several weapon systems have been operational during this period [70].

The first operational system (on western side) to employ a ramjet was the Talos missile used by the US Navy from 1955 to 1980, in both Surface-to-Air and Surface-to-Surface variants. After a solid fuel rocket boosted it to Mach 2.2, the ramjet was activated which allowed cruise flight at Mach 2.7 to strike a target at a maximum range 220 km [70]. The Talos used a conical inlet on the nose and used beam-riding (target indicated from the launch vessel) for guidance.

In 1958 the Bristol Bloodhound entered service in the RAF. It was a Surface-to-Air missile boosted by 4 solid fuel boosters and sustained flight was achieved by two side-mounted ramjets, each with a conical intake. Figure 2.1 shows the Bloodhound on its launch platform, one of the ramjets is clearly visible on the upper side of the missile. The arrangement of the engines at either side of the missile fuselage proved problematic during maneuvers when one intake would be on the leeward side of the fuselage [7]. Such problems were averted in later test bed vehicles with the use of a 'chin' intake on the windward side and bank-to-turn steering [17]. The modern MBDA Meteor employs a similar solution, though with two separate rectangular



Figure 2.1: Bloodhound Surface-to-Air missile, taken from [17]

intakes instead of a continuous 'chin'.

For a full historic overview of ramjet missile development, the work of Fry [17] is recommended.

Further development of ramjet technology lies in the field of supersonic combustion ramjets, or scramjets for short. In a scramjet the air is not decelerated to subsonic speeds prior to the combustion. Due to the high flight speeds, Mach 5 and higher, for scramjet applications deceleration to subsonic speeds would result in temperatures that could affect the structural integrity of the ramjet [14]. Maintaining continuous combustion in a supersonic flow requires different techniques for fuel injection and ignition compared to ramjets, this and the associated combustion dynamics are active fields of research [1, 16].

As ramjets and scramjets are very similar, research has also been undertaken in the requirements for a dual-mode ramjet. In a dual-mode ramjet, the same intake, combustor and nozzle are used. This allow to use the same basic engine in a wide flight regime. This flight regime can extend from Mach 2 to Mach 12, although for such a wide flight regime some variable geometry is required to obtain satisfactory performance [14].

A scramjet will be integrated into a waverider airframe which utilizes shock waves from the forebody as the first compression stage in the (sc)ramjet. A cross section of such a vehicle is sketched in Figure 2.2. The Mach number at the start of the combustor (M2) is below 1 for ramjet operation and above 1 for scramjet operation. Note that also the aft section of the airframe is shaped to form a large nozzle.

Scramjets have been successfully flown on both the X-43 and X-51 experimental vehicles. The latter achieved 210 seconds of powered flight with a top speed of Mach 5.1.



Figure 2.2: Sketch of a dual-mode (sc)ramjet in a waverider vehicle, taken from [14]



Figure 2.3: Types of ramjet intakes, from left to right: external, internal and mixed compression

2.1.2 Intake performance

The main function of the intake is to provide a sufficient mass flow at high pressure at the entrance of the combustion chamber. Mass flow is mostly a function of the intake frontal area, while the achieved pressure strongly depends on the intake geometry. A ramjet utilizes no moving parts to achieve compression. This is done via shock waves or, ideally, via isentropic compression. Shock waves are created by discrete kinks in the surface topology of the ramjet intake, or the alignment of the cowl lip in the flow, as can be seen in Figure 2.3. Over a shock wave, some percentage of the total pressure is lost, with higher losses when the strength of the shock wave increases. Performance of the intake with respect to achieved pressure is often measured with the pressure recovery factor, $P_r = p_{0,end}/p_{0,\infty}$, the fraction of the free stream total pressure that is achieved at the end of the intake, which is the entry to the combustor. In the simplest possible intake, a pitot intake, there is a single normal shock with significant losses in terms of stagnation pressure. As early as 1944 a supersonic intake design optimization study was performed by Oswatitsch [51]. The conclusion was that optimal intake performance, in terms of pressure recovery, was achieved via multiple oblique shocks of equal strength, if ideal isentropic compression is not possible. The pressure recovery is closely related to the turning angle of the flow of the intake. In isentropic compression deceleration to Mach 1 is achieved by gradual deflection of the flow over the Prandtl-Meyer angle of the initial Mach number. Compression via (multiple) oblique shock wave(s) and a normal shock wave requires lower turning angles but is associated with pressure loss. As higher turning angles result in a larger frontal area and thus increased drag, an optimal pressure recovery is thus often compromised in intake design.

In general, intakes are classified as external, internal or mixed compression intakes. This refers to the location where compression occurs, as is shown in Figure 2.3. External compression inlets have ramps or spikes that create shocks or compression waves, and a terminating shock wave at the entrance of the internal duct. In internal compression inlets all shocks or other compression structures occur within a duct, much like an inverted nozzle. A mixed compression intake utilizes both an external compression spike or ramp and a shaped duct to achieve internal compression.

Figure 2.3 shows the intakes at their design Mach number, the point at which they deliver peak performance. Shock waves are focused on the cowl lip and the terminating shock wave is located at the throat, the point of minimal cross sectional flow area of the internal duct. A reduction in Mach number will cause the oblique shock waves to pass in front of the cowl lip, resulting in increased shock wave drag for the vehicle



Figure 2.4: Relation between mass flow rate and total pressure recovery, taken from [28]

and a reduction in mass flow through the intake due to supersonic spillage. Alternatively a higher Mach number than designed will place the shock waves on the inside of the cowl. Some part of the flow will then not encounter the sequence of multiple shock but only a single normal shock, with a loss of pressure as result. A similar effect occurs with the terminating shock wave position via the pressure imposed by the combustor. When this pressure, often called back pressure, is reduced the shock wave moves aft, where the channel diverges and the shock occurs at higher Mach number and pressure recovery is reduced. A high back pressure can eject the shock wave from the intake, causing an unstart of the intake. This is also associated with a strong reduction in mass flow through the intake because subsonic flow is deflected away from the intake. This depicted in Figure 2.4, which shows typical experimental results where the back pressure is increased by closing a throttling valve. Up to the critical point, with the shock at the cowl lip, there is no effect on the mass flow rate. Then the shock is expelled and the mass flow rate drops. Also depicted in Figure 2.4 is the unsteady behavior, labelled as 'buzzing', where the shock jumps in and out of the intake [28]. Figure 2.5 provides an overview of all intake operation conditions. Note that when the normal shock is located in the internal channel, the terminating shock wave will form a shocktrain due to interaction with the boundary layers on the cowl and compression surface. This will be further discussed later in Section 2.5.

Boundary layer interactions limit the efficiency of an intake. Therefore many intakes utilize a bleed slot that removes the boundary layer on the side of the compression ramp and possibly on the cowl as well. Typically only a few percent of the total mass flow is removed via a boundary layer bleed, but the pressure efficiency achieved by the intake can be increased by up to 25 percent [7, 22, 27, 32]. It has been shown that, as long as the bleed channel is not choked, there is no upstream effect through the boundary layer [27]. This allows for a simplification of the intake analysis model.

The simplicity of the ramjet concept has resulted in the fact that the general behavior characteristics of the ramjet intake as described above have been known for over fifty years [7]. This has not meant that no research has been done since this time, but this has been typically applied to (classified) design studies for military applications [7] or studies into specific flow phenomena. Such flow phenomena are for example



Figure 2.5: Operating modes of a ramjet intake (inviscid), image taken from [45]



Figure 2.6: Mach contours of a ramjet internal flow field, image adapted from [54]

shock wave boundary layer interactions (SWBLI) and the shock train, which are addressed in Sections 2.4 and 2.5 respectively. Figure 2.6 shows the internal flow field of a typical intake. Indicated are the locations where considerable SWBLI can be observed. The location of the shock train is also marked in Figure 2.6.

2.2 Boundary Layer Transition

Transition of the boundary layer is a fundamental flow phenomenon that has been studied for over a century, since the work of Reynolds. Laminar and turbulent boundary layers have drastically different properties regarding skin friction, boundary layer growth, separation and shock wave boundary layer interaction. Hence for accurate results of flow models a prediction of transition location is very important. It has thus received much attention in research, but so far no model which works under all circumstances is available. What complicates the prediction model is that there are many factors that affect transition. In the following section, an overview will be given of the most relevant factors for the intake model, with engineering models



Figure 2.7: The 'roadmap of transition', adapted from [58]

that have been developed throughout the years.

The so-called 'roadmap of transition' is shown in Figure 2.7. It indicates the somewhat oversimplified paths through which a disturbance can create a turbulent boundary layer. We will first focus on relatively small disturbances that grow to turbulence, path A in the roadmap, before we consider effects that lead to turbulence more rapidly.

The first step in the roadmap of transition is receptivity. In the words of Reshotko [55] "By receptivity is meant the means by which a particular forced disturbance enters the boundary layer and the nature of its signature in the disturbance flow." In other words, a disturbance of the free stream will only change the boundary layer if there is a mechanism through which a particular disturbance *can* enact changes on the boundary layer.

Once a disturbance has left its signature in the boundary layer, this signature, in the form of a small oscillation, may be damped out or amplified. Which of these occurs and how fast can be determined with linear stability theory, which follows the growth of a perturbation with the mathematical form:

$$u' = A_0 e^{ia(x-ct)} = A_0 e^{-a_i x} e^{i(a_r - \omega t)}$$
(2.1)

In this equation A_0 is the initial amplitude, a the wave number of the disturbance and ω is the frequency (rad/s) of the disturbance. The sign of the imaginary part of the wave number, a_1 , thus dictates whether the perturbation is stable or unstable. When curves of constant a_i , are plotted on Re_{δ^*} versus ω in Figure 2.8, we can see that at certain combinations of Reynolds number and frequency a perturbation is unstable. Lines of constant frequency over Reynolds number ($F = \omega/Re_{\delta^*}$) indicate that a perturbation can become unstable at some point. A disturbance with a constant frequency may be stabilizing at the early stages of the



Figure 2.8: Stability diagram with lines of constant frequency F versus Reynolds number

boundary layer, where Re_{δ^*} is low and F high. With the growth in boundary layer displacement thickness, F reduces and may enter regions with $a_i < 0$, thus amplifying the disturbance in the boundary layer. The total amplification of the perturbation at a certain location is:

$$A/A_0 = exp\left(\int_{x_0}^x -a_i dx\right) \tag{2.2}$$

The shape and size of the neutral stability curve depends on a large number of variables. For example, it is well known that an adverse pressure gradient is detrimental to the stability of the boundary layer. In a region of pressure rise the region of instability is thus enlarged. An opposite effect is achieved by applying cooling of the wall. The effect of various other parameters are listed in [44] and [4]. When the Mach number of the flow increases beyond ≈ 2.2 a new instability curve appears, corresponding to the so-called Mack-modes. This mode arises due to the large velocity difference within the boundary layer [4].

In linear stability theory, transition is expected to occur at some value of A/A_0 . Independently several researchers proposed transition occuring when this ratio reaches a value of e^N , with N varying between experiment conditions, but in the range 7-10 [43]. Again, this is a problem of many variables, but turbulence in the external flow has been further investigated as dominant effect. It was found that the effect of free stream turbulence, $T = u'_{rms}/\bar{u}$, can be captured in Macks relation $N = -8.43 - 2.4\ln T$ [3]. This relation holds for values of T between 10^{-3} and $2.98 \cdot 10^{-2}$, at lower values the sound of the test environment was more important for the presented data, while the upper limit represents N=0 so that transition happens directly on the critical Reynolds number [3].

Eventually linear stability theory breaks down for two reasons. First is that nonlinear terms are no longer of negligible magnitude. Second is the rise of three-dimensional structures that were not considered in the two-dimensional stability theory. Three-dimensional effects arise from the fact that the Tollmien-Schlichting waves do not grow at an equal rate over the complete spanwise distance of the test plate [73].

The development of disturbance in the boundary layer ends with breakdown into turbulence. Breakdown is a relatively abrupt phenomenon in which the waves that have developed collapse into smaller structures of vortices. This breakdown into turbulence occurs locally and forms turbulent spots [73]. It takes some distance before turbulent spots interact sufficiently to form a turbulent flow over the full span of the plate.



Figure 2.9: Transition on a flat plate, image taken from [73]

This process is represented in Figure 2.9.

It must be noted that the stages of non-linear growth and breakdown occur in a relatively short distance. This explains why methods that utilize only linear theory give accurate predictions of transition location. For typical flat plate experimental conditions, linear theory spans 75 to 85 percent of the distance between start of amplification and transition [3].

Effects of free stream turbulence and sound are a prime cause of differences between flight tests and wind tunnel experiments [59, 60]. An order of magnitude difference in transition Reynolds number can be observed between these cases. This is an additional complicating factor in transferring wind tunnel test results to real flight applications.

In the present application under investigation, the submerged ramjet intake, will probably encounter low values of free stream turbulence, but within the sound field of the projectile. Additionally, the sections where we will see laminar flow are surfaces with a fresh starting boundary layer. Simeonides [60] has produced a correlation of transition location as a function of Mach number, Reynolds number and the bluntness of the leading edge in tunnels with significant sound fields. In the data used to establish the relation of Simeonides,

the Reynolds number based on leading edge radius is used to collapse data on a single trend line. Transition caused by leading edge bluntness appears to dominate when the radius of the leading edge is over 50 micron, in a wide test range of Mach 2 to 7 and Reynolds number per unit length between $3 \cdot 10^6$ and $7 \cdot 10^7$. Although the test conditions are varied, the data presented are gathered in only a handful of tunnels. No information is given on the disturbance environment, i.e. the free stream turbulence and noise levels. Transfer to another environment may thus require some tweaking of the proposed relation.

Many more transition models exist and this text barely scratches the surface of available models to predict the location of transition. There is for example the model by Reda [53] which employs a correlation between momentum thickness Reynolds number and the height of roughness elements on the wall. Also available are models that employ knowledge of velocity fluctuations in the boundary layer, for example by Steelant [64] and more recently by Papp and Dash [52]. These models require very detailed information on the boundary layer and more suited for advanced CFD calculations.

The problem of boundary layer transition has been a challenge for over a century. Reasonable predictions can be made with the use of linear stability theory, but also a correlation of experimental data seems very useful. But a complicating factor is that there are large discrepancies between wind tunnels and actual flight conditions.

2.3 Reverse Transition

In order to submerge the intake into the body of the projectile, the wall must bend inwards. This means that the supersonic flow will expand with a strong drop in pressure. The boundary layer reacts with two results that may be of interest. First is a change in the boundary layer parameters. Most relevant for the application of the intake performance analysis model are the boundary layer thickness and displacement thickness, which both increase significantly [5, 62]. Second is that several indications of turbulence, of which the Reynolds stress is the most important, are strongly altered over the complete height of the boundary layer. Interestingly, Reynolds stress may even change sign when the expansion is strong enough, indicating that the mean flow extracts energy from turbulence [5, 66]. This is a result of stretching of turbulent structures and their subsequent breakdown, at which point some of the energy is transferred back into the main flow [66]. The loss of such fingerprints of turbulence have led to terms as relaminarization and 'reverse transition', i.e. that the boundary layer would regain some properties that are normally associated with a laminar boundary layer.

The first issue raised, the thickening of the boundary layer, can be predicted with the integral boundary layer technique shown in Chapter 3. The issue of relaminanarization is more complex. As mentioned, Reynolds stress, one of the indications of turbulence, changes throughout the boundary layer. The change is a strong reduction in magnitude of the Reynolds stress, even a change of sign if the expansion is strong enough. Amongst the authors who have reported on relaminarization, there is some discussion on what the term entails. In the early work of Narasimha and Sreenivasan [48, 49], it is categorized as the reduction of importance of the Reynolds shear stress in comparison to other terms in the flow equations. Some years later, Smith and Smits [61] caution against the use of the term relaminarization, as not all properties associated with turbulent boundary layers disappear. They report that the mass-flux fluctuations do not change noticeably over the expansion. Arnette et al. [5] do use the term relaminarization, despite the caution of Smith and Smits, as an indication of a sign change in the Reynolds stress and the accompanying sharp reduction of the turbulent kinetic energy.

According to Narasimha and Viswanath [50] relaminarization is expected to occur when $\frac{|\Delta p|}{\tau_w} > 75$, where τ_w denotes the wall shear stress before the expansion.

Though relaminarization over an expansion is covered in many publications, these documents do not describe the boundary layer development over long, say 50 boundary layers thicknesses, distances after when



Figure 2.10: Reynolds stress profiles downstream of a 7 degree centered expansion, taken from [5]

relaminarization should have occurred. This could be very useful information to validate the post-expansion boundary layer development. It is somewhat remarkable that this has not been investigated, Figure 2.10 shows that 20 boundary layers downstream of the expansion the first indications are present that the boundary layer appears to return to the pre-expansion state. However there is no information available past this point on whether the pre-expansion state is reached and if so, the distance required to reached this state. No subsequent research has followed up on this aspect of the work of Arnette et al. A possible cause for the absence of this data is the limited size of wind tunnel test sections. For example the model of Arnette et al. is approximately 20 cm long in the streamwise direction and 5 cm in wall-normal direction. With the requirement of sufficient space above the model to prevent reflection of the expansion fan, the required test section height quickly grows beyond typical test section heights of 15 to 20 cm.

Smith and Smits [61] showed that the streamwise stretching of fluid elements is the main effect that causes the reduction in Reynolds stress. The fact that dilatation is the most important factor supports the comments of Arnette et al. [5], Konopka et al. [35] and Wang et al. [72] that the most laminar-like layers are encountered near the wall. A Prandtl-Meyer expansion over a certain angle causes a relatively greater acceleration at low Mach numbers. Hence the lower and slower layers are accelerated more than the higher layers, causing more stream wise stretching of the turbulent structures.

Some effects, including a change in Reynolds stress, velocity profile, density fluctuations, occur more strongly in some parts of the boundary layer than in others [5, 23, 35, 61, 72]. This serves as a further reminder that relaminarization does not occur over the full height of the boundary layer. Additionally, mass flux fluctuations is one property of turbulence that remains mostly unchanged [61]. These effects are shown in Figures 2.11 and 2.12. Konopka et al [35] note "A large laminar-like sublayer is detected at both



Figure 2.11: Velocity profiles (left) and mass-flux fluctuations (right) of pre-expansion (circles) and postexpansion (squares) boundary layers. Taken from [61]

Mach numbers in which the largest Reynolds stress component reduction occurs, which corresponds to the findings of Arnette et al." The phrase "laminar-like" is perhaps an apt description of the whole problem of relaminarisation, as the resulting flow has some characteristics of a laminar flow in parts of the boundary layer, but remains significantly turbulent in other parts. This may create a challenge for development of the intake model as the limit of applicability of the turbulent boundary layer model may be exceeded. Hence in Chapter 3 the model is validated for this section of the intake model.

2.4 Shock Wave Boundary Layer Interaction

For the creation of the intake performance model we are interested in the interaction between the boundary layer and a shock wave in a compression corner or an incoming oblique shock wave impinging on the boundary layer. Interaction with a normal shock wave is considered irrelevant for the model, as this will occur in a shock train situation, which will be modeled with an empirical relation.

Figure 2.13 shows an oblique shock wave reflecting on a surface with a boundary layer. As the shock enters the boundary layer, the angle of the wave changes due to the locally lower Mach number, and disappears when the sonic line is reached. The pressure rise caused by the shock wave results in a thicker boundary layer, especially of the subsonic region. Information of the pressure rise can be transmitted upstream by the subsonic part of the boundary layer, as a result the thickening starts upstream of where the shock wave would have reached this part of the boundary layer. This initial thickening results in a compression of the supersonic part of the boundary layer, eventually forming a shock wave that propagates into the outer flow. This reflected shock appears earlier than an actual reflection in an inviscid flow. We can speak of a translation by a certain length: the interaction length. A similar effect occurs at compression ramps, as can be seen in in Figure 2.14.

When the shock increases in strength, the pressure rise may be strong enough to result in flow rever-



Figure 2.12: Turbulence intensities (left) and longitudinal Reynolds stresses (right) of pre-expansion (squares) and post-expansion (circles) boundary layers. Taken from [61]



Viscous sublayer

Figure 2.13: Shock wave interacting with a boundary layer, taken from [11]



Figure 2.14: Inviscid (top) and viscous (bottom) shock wave reflection and compression ramp flow, with the interaction length indicated. Taken from [63]

sal within the previously indicated subsonic area. When this happens, the displacement thickness rapidly increases and decreases upstream and downstream of the separation bubble respectively, resulting in two shock waves. These waves will eventually merge with the expansion created at the bubble peak to form a single shock. A similar situation can occur over a compression ramp, in which a lambda shock structure is formed. One foot will be located at the separation point and the other at the reattachment point, as shown in Figure 2.15.

The interaction with shock waves is another point on which laminar and turbulent boundary layers greatly differ. Laminar boundary layers are less resistant to adverse pressure gradients and separate sooner than a turbulent boundary layer. A laminar separation bubble is much larger than a turbulent bubble. For weaker interactions the bubble is very stretched in the streamwise direction but when shock strength increases the vertical size increases rapidly and may even block a significant portion of the internal channel [54].

For the intake model the most important quantities to determine are the size of the interaction region and the effect on the boundary layer properties downstream of the shock. Relations to determine these quantities are discussed in Chapter 3.

2.5 Shocktrain

A shock train is a structure of shock waves and shock wave boundary layer interactions which occurs in ducts through which the flow decelerates to subsonic speeds and pressure increases up to the level at the duct exit. Figure 2.16 shows that if the initial Mach number is just above 1, there is a simple normal shock wave (a). With increasing Mach number the strength of the shock wave increases, and so does the extent of shock wave boundary layer interaction, as seen in (b-c). Further increase of the Mach number causes the bifurcated feet to grow to full oblique shocks with a small normal section in the duct center (d). The boundary layer (and possible separated regions) alternately grow end shrink in size, which results in the flow repeatedly accelerating to supersonic flow speed and again compressed via shock waves. This repeating pattern of shocks has been named a shock train.

The shock train is one part of the entire pseudo-shock region, the viscous deceleration to subsonic velocity, though shock train is often used to refer to the whole region. Figure 2.17 gives a schematic representation of the shock train region and the so-called mixing region behind it. Also represented in this figure is the pressure rise over the pseudo-shock region and a comparison to the inviscid pressure rise via a single normal



Figure 2.15: SWBLI with separation at a corner. Taken from [68]



Figure 2.16: The formation of a shock train, as function of Mach number. Taken from [46]



Figure 2.17: The complete pseudo-shock region with pressure rise indication, grey regions indicate subsonic flow speeds. Taken from [46]

shock. The sequence of shock waves and subsequent accelerations is captured in the pressure on the duct wall. Initially there is a sharp pressure rise, but the slope of the pressure decreases towards the downstream side of the pseudo-shock region.

Determining the length of the pseudo-shock region is very important: if it grows beyond the cowl lip, the entire shock system will be expelled from the intake. This results in unstart or sub-critical operation of the intake as described in Section 2.1.1 of this chapter. Unstart imposes a limit on the pressure that can be reached at the duct exit, which is equal to the combustor entry pressure when speaking in terms of the ramjet propulsion system.

A relation to determine the length of the shock train has been developed experimentally and is presented in Chapter 3. A shock train often results in more pressure loss than via a single normal shock, an effect also presented in Figure 2.17. To achieve a similar pressure, a duct must be impractically long. According to Reinartz et al. [54] 80 % of the normal shock static pressure is achieved in practice. Mahoney [45] reports a further loss of 10 % above the normal shock wave total pressure loss as a rule of thumb for the initial design stage. Presumably more pressure is lost at higher Mach numbers, where the shock train is longer and contains more shock waves.

Chapter 3

Flow Models

This chapter details in depth the techniques that were used in the intake analysis model. First the inviscid flow is covered, this includes a formulation of the method of characteristics suitable for rotational flow with a special function used to determine the Mach number from the Prandtl-Meyer angle. Also covered under inviscid flow is how shock waves are incorporated into the calculation scheme. Section 3.2 details the boundary layer models, for both laminar and turbulent boundary layers, used to determine the growth rate of the boundary layers. This is followed by the description of shock wave boundary layer interactions, again with differing models for laminar and turbulent boundary layers. Finally, in Section 3.4, several correlations are covered that describe the length of the shock train and the pressure rise over the shock train.

3.1 Method of Characteristics

In this section models for the inviscid flow field will be covered. It will present a formulation of the Method of Characteristics that is applicable to rotational flow, along with techniques that were used to accelerate the speed of the model. These techniques are an inverse Prandtl-Meyer function and a way to determine shock wave angles directly as function of Mach, γ and deflection angle.

3.1.1 Rotational Method of Characteristics

The Method of Characteristics (MoC) is a very powerful technique to compute an inviscid supersonic flow. At the heart of the MoC lies the fact that in a supersonic flow two special lines appear for the gasdynamic equation. Along these lines the partial differential equation is reduced to an ordinary differential equation. These lines are known as characteristics and are located in the flow with orientations $tan(\phi \pm \mu)$, in which ϕ is the flow angle and μ is the Mach angle $sin^{-1}(1/M)$. The characteristics originating from a point A form the limits of the Mach cone. The Mach cone, or wedge in 2D, is the downstream area affected by an object traveling at supersonic speeds. Small perturbations (i.e. sound, but not shock waves) in the flow spread within the limits of this cone.

In homentropic (constant entropy in the flowfield) flow the ordinary differential equations along the characteristics are a function of the Prandtl-Mayer angle and flow direction. As a function of two variables, two characteristics originating from points A and B are required to obtain the flow angle and Mach number at the point C. Figure 3.1 shows these points in relation to each other. Point C is located on the intersection characteristic along $tan(\phi_A - \mu_a)$ originating from point A and the characteristic $tan(\phi_B + \mu_B)$ originating from point B.

When shock waves of non-uniform strength are located in the flow, there will exist gradients in stagnation pressure, or alternatively expressed as entropy, aft of this shock wave. According to Crocco's theorem this means that the flow behind the shock wave is rotational. The first formulation that accounts for gradients in entropy was developed by Ferri in 1946 [15]. Alternative formulations have since been created, such as the one used in the intake model, which was preferred for its concise notation. This formulation was reported by Krasnov [36].

$$d(\omega - \phi) - \frac{dx}{y}l + \frac{dx}{\gamma R}\frac{dS}{dn}c = 0 \quad \text{along } \frac{dy}{dx} = \tan(\phi_A + \mu_A)$$
(3.1)

$$d(\omega + \phi) - \frac{dx}{y}m - \frac{dx}{\gamma R}\frac{dS}{dn}t = 0 \quad \text{along } \frac{dy}{dx} = \tan(\phi_B - \mu_B)$$
(3.2)

With l, m, c, t, ω and $\frac{dS}{dn}$ given by:

$$l = \frac{\sin\phi\sin\beta}{\cos(\phi+\beta)} \tag{3.3}$$

$$m = \frac{\sin\phi\sin\beta}{\cos(\phi - \beta)} \tag{3.4}$$

$$c = \frac{\sin^2 \beta \cos \beta}{\cos(\phi + \beta)} \tag{3.5}$$

$$t = \frac{\sin^2 \beta \cos \beta}{\cos(\phi - \beta)} \tag{3.6}$$

$$\omega = \sqrt{\frac{\gamma+1}{\gamma-1}} \operatorname{atan} \sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \operatorname{atan} \sqrt{M^2-1}$$
(3.7)

$$\frac{dS}{dn} = \frac{S_B - S_A}{|BC|\sin\mu_B + |AC|\sin\mu_A} \tag{3.8}$$

Such that the solutions for point C are given by:

$$\phi_{C} = \phi_{B} + \frac{x_{C} - x_{A}}{2y_{A}} m_{A} - \frac{x_{C} - x_{B}}{2y_{B}} l_{B} + \frac{x_{C} - x_{A}}{2\gamma R} \frac{dS}{dn} t_{A} + \frac{x_{C} - x_{B}}{2\gamma R} \frac{dS}{dn} c_{B} - \frac{1}{2} (\omega_{B} - \omega_{A}) - \frac{1}{2} (\phi_{B} - \phi_{A}) \quad (3.9)$$

$$\omega_C = \omega_B + d\phi_B + \frac{x_C - x_B}{y_B} l_B - \frac{x_C - x_B}{\gamma R} \frac{dS}{dn} c_B \tag{3.10}$$

In these equations ω stands for the Prandtl-Meyer angle. To limit the repeated use of symbols it was chosen to use ω , which appears to be the Russian standard, instead of the western standard ν . S stands for entropy, additional entropy created over a shock can be calculated via $\Delta S = -Rln \frac{p_{0,2}}{p_{0,1}}$. Axisymmetric effects are accounted for by the terms containing 1/y. The variable n denotes the distance between point C and the line of flow through points A or B, as indicated in Figure 3.1. The inclusion of the entropy as an extra variable requires an additional 'characteristic'. This is the stream line that passes through the point C and originates from a point between A and B. Along the stream line the entropy does not change until a shock wave is encountered, how this is coupled with the MoC is covered in Section 3.1.3.

For a point on the wall only a single characteristic is available. Still the flow properties can be determined at that point, as the flow direction is known to be parallel to the wall.

The effects of viscosity are not covered by the Method of Characteristics. The boundary layer on the wall must be incorporated via a different technique, which must supply the displacement thickness. With the



Figure 3.1: Geometric construction for determining point C via the method of characteristics

displacement thickness a virtual surface is defined which defines the flow angle and the wall-bounded entropy.

Due to the fact that MoC provides an exact solution to the gasdynamic equation, the obtained flow field accurately represents reality. Because of this, and the low computational cost, MoC has been used extensively during the early years of computational fluid dynamics. It is still used in present day when the flow is steady, as evidenced by Xie et al. [74].

3.1.2 Inverse Prandtl-Meyer function

The method of characteristics calculates the Prandtl-Meyer angle at point C. The Prandtl-Meyer angle is a function of Mach number and the ratio of specific heats. Unfortunately, there is no exact function that produces Mach number as function of Prandtl-Meyer angle and ratio of specific heats.

While it is possible to approximate the Mach number via interpolation from a dataset, this is not the fastest way. An approximate inverse function was created by Hall[25]. This function is simply:

$$M = \frac{1 + Ay + By^2 + Cy^3}{1 + Dy + Ey^2}$$
(3.11)

In which $y = (\omega/\omega_{\text{max}})^{2/3}$ and $\omega_{\text{max}} = \frac{\pi}{2}\sqrt{\frac{\gamma+1}{\gamma-1}} - \frac{\pi}{2}$ is the maximum Prandtl-Meyer angle. The constants A-E are, for $\gamma = 1.4$, given by Table 3.1. If this method is to be adapted to gases with other ratio of specific heats, the constants A-E must be re-evaluated.

A B C D E 1.3604 0.00962 -0.5127 -0.6722 -0.3278

Table 3.1: Values of the constants A-E for $\gamma = 1.4$


Figure 3.2: Accuracy of the inverse Prandtl-Meyer function

This function is remarkably accurate. As can be seen in Figure 3.2 the error is less than 0.02 % for Mach numbers below 5.

3.1.3 Incorporation of Shock Waves

When characteristics of the same family cross, the MoC breaks down. This would correspond to the formation of a (weak) shock wave in the flow. If the difference in flow properties in the originating points of these characteristics is negligible, all but one of the characteristics can be deleted without significant error [76]. When the difference in flow properties is significant, such as happens when the compression happens via a concave surface, a shock wave of non-negligible strength is formed. In order to compute the flow field in this case, the calculation scheme must be adapted. The situation is sketched in Figure 3.3. A shock is created when characteristics of the same family intersect in point C. The initial section of the shock has an orientation that is the average of the two characteristics. This shock wave angle is used with the shock relations to obtain properties at point E', the ' denoting the downstream side if the shock wave. Point E' is used to compute the flow field behind the shock. Points H-H' and others along the shock wave are then computed following an iterative procedure. Point J is obtained with data from E and I, as if the shock wave did not exist. An assumed wave angle at H and the known wave angle at E yield an intersect with IJ. Interpolation of H and use of the shock relations yield H'. The assumed shock wave angle is then refined until the equation along characteristic G-H' is accurate to within a certain tolerance.

The technique described above works for the case that the shock wave grows in strength, which is known as shock coalescence. If the shock wave remains of equal strength, it is more convenient to decouple the MoC and the shock wave calculation. In that case the shock wave is drawn in small steps, for each step the local flow angle and Mach number is interpolated from the data of the previous flow section. This and the deflection angle are used to compute the orientation of the next section of the shock wave.

The strength of a shock wave may increase via shock coalescence as described above, but a shock wave



Figure 3.3: Grid of characteristics with shock coalescence, a 'denotes a point just downstream of the shock wave



Figure 3.4: Interaction between shock wave and expansion fan at the cowl lip

decreases in strength when it interacts with an expansion fan of the same family. When the intake operates above its design Mach number the cowl lip creates a centered expansion fan which will interact with the shock wave of the compression surface. A sketch of the situation is displayed in Figure 3.4. The initial interaction, when the shock and characteristics of the expansion are of different families (i.e. in opposing directions) the interaction is limited and the deflection remains nearly constant [29] and the simplified approach of the previous paragraph can be used. In same family interaction, occurring after reflecting of the shock wave, the expansion fan and shock wave dissolve each other. Here it is required to keep the coupling between the shock wave determination and MoC, where the deflection angle is re-evaluated at each step to find the best solution that fits the MoC calculation behind the shock wave, in a way similar to the shock coalescence approach.

3.1.4 Shock Wave Angle Determination

The relation between the angle of an oblique shock wave and the deflection angle and Mach number is given by:

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{2+(\gamma-1)M^2\sin^2\beta}{(\gamma+1)M^2\sin^2\beta}$$
(3.12)

This relation does not provide the shock wave angle as an explicit function of the other variables. While it would be possible to generate a data set in order to interpolate, a direct calculation would be a lot faster.

Such a calculation was developed by Hartley [26]. Equation 3.12 can be converted in a cubic equation with $\sin^2 \beta$ as the variable.

$$X^3 + bX^2 + cX + d = 0 (3.13)$$

Where

$$X = \sin^2 \beta \tag{3.14}$$

$$b = -\frac{M^2 + 2}{M^2} - \gamma \sin^2 \theta \tag{3.15}$$

$$c = \frac{2M^2 + 1}{M^4} + \left(\frac{(\gamma + 1)^2}{4} + \frac{\gamma - 1}{M^2}\right)\sin^2\theta$$
(3.16)

$$d = -\frac{\sin^2 \theta}{M^4} \tag{3.17}$$

Then the following parameters can be created

$$Q = \frac{3c - b^2}{9}$$
(3.18)

$$R = \frac{9bc - 27d - 2b^3}{54} \tag{3.19}$$

$$D = Q^3 + R^2 (3.20)$$

When D > 0 the shock wave is detached. For an attached oblique shock wave there are two solutions, which are known as the weak and strong shock waves. Generally the weak shock wave will be formed, unless situations such as a high back pressure force the occurrence of a strong shock [2, 26]. This situation is unlikely to occur. The shock wave angle is then obtained from:

$$\Delta = \begin{cases} 0 \ if R \ge 0\\ 1 \ if R < 0 \end{cases} \tag{3.21}$$

$$\phi = \frac{1}{3} \left(\operatorname{atan} \frac{\sqrt{-D}}{R} + \Delta \right) \tag{3.22}$$

$$\chi = -\frac{b}{3} - \sqrt{-Q} \left(\cos \phi - \sqrt{3} \sin \phi \right) \tag{3.23}$$

$$\beta = \operatorname{atan}\left(\frac{\chi}{1-\chi}\right) \tag{3.24}$$

This method allows to directly compute the shock wave angle for any combination of Mach number, deflection angle and ratio of specific heats, without the use of tables of data and iterative procedures.

3.2 Boundary Layer Prediction

For the implementation of viscous effects into the intake performance analysis model we require a reliable estimation of the boundary layer thickness, displacement thickness and momentum thickness. This section will detail how these properties were determined from integral boundary layer techniques. First an introduction to integral techniques is given, followed by the compressible turbulent and laminar models that were used in the intake performance analysis model.

3.2.1 Integral Boundary Layer Techniques

The internal dynamics of the boundary layer are very complex due to the very different characteristics of the viscosity-dominated regions near the wall to (in the turbulent case) an outer region where strong velocity fluctuations occur due to the presence of turbulence. In order to calculate all the internal dynamics one would require a supercomputer, while even the application of RANS with several assumptions and simplifications with respect to turbulence modeling requires computational times that would be unacceptable for rapid performance analysis. For this reason, and the fact that only the thickness parameters are really of interest, integral boundary layer models were used to predict boundary layer growth.

An integral boundary layer technique gets its name from the fact that the boundary layer equations, conservation of mass and momentum, are integrated from the wall to the edge of the boundary layer. With the no slip boundary condition at the wall and $u = u_e$ and $\frac{\partial u}{\partial y} = 0$ at the top of the boundary layer, the integration yields the Von Karman integral relation [33]:

$$\frac{\partial}{\partial x} \int_0^\delta u^2 dy - u_e \int_0^\delta \frac{\partial u}{\partial y} dy = -\frac{\delta}{\rho} \frac{\partial p}{\partial x} - \frac{\tau_w}{\rho}$$
(3.25)

Where u_e is the velocity at the edge of the boundary layer and τ_w is the shear stress at the wall. Decomposing the term of the pressure gradient using Bernoulli's law and substituting definitions of displacement thickness and momentum thickness results in:

$$\frac{\tau_w}{\rho} = u_e^2 \frac{d\theta}{dx} + (2\theta + \delta^*) u_e \frac{du_e}{dx}$$
(3.26)

Or in a slightly different formulation:

$$\frac{\tau_w}{\rho} = u_e^2 \frac{d\theta}{dx} + \theta (2+H) u_e \frac{du_e}{dx}$$
(3.27)

Where $H = \frac{\delta^*}{\theta}$ is the shape factor. With the externally imposed velocity u_e known from the Method of Characteristics and a initial value for θ , a prediction for the growth of the momentum thickness can be made, given an assumption on τ_w and the development of the shape factor.

These final aspects can be included via several principles. The approach of the turbulent method of Section 3.2.2 is to employ a second equation to track the change in the shape factor and a semi-empirical relation to find the friction on the wall. The laminar technique employs a method similar to the Thwaites correlation, which employs correlation established from numerical and experimental data, to establish the skin friction and change in the shape factor [73].

The above equations are for an incompressible boundary layer. In a compressible boundary layer there exists a strong difference in the temperature close to the wall and near the boundary layer edge. Temperature is closely linked to density and this results in the fact that density can not be taken as constant, which was done to obtain Equations 3.26 and 3.27. Both boundary layer techniques therefore employ a density-based transformation to obtain a compressible version of these two equations.

As will be covered in in Sections 3.2.2 and 3.2.3, the integral boundary layer techniques produce accurate predictions of boundary layer growth at low computational cost. However, the absence of information on the internal dynamics of the boundary layer comes also at the cost of limited insight into, for example, the occurrence of relaminarization during the expansion, which is indicated by a reduction in Reynolds stress.

3.2.2 Turbulent Boundary Layer Model

Let us take a closer look at the model for compressible turbulent boundary layers created by Sasman and Cresci [10]. It is a further development of a method published by Reshotko and Tucker [56]. The following

transformations are applied to the typical x, y coordinate system:

$$X = \int_0^b b(x)dx \tag{3.28}$$

$$Y = c(x) \int_0^y \frac{\rho}{\rho_0} dy \tag{3.29}$$

Where

$$b = (T_0/\bar{T})(T_e/T_0)^{(\gamma+1)/(2\gamma+2)}$$
(3.30)

$$c = (T_e/T_0)^{1/2} \tag{3.31}$$

In the transformed coordinates the integral relation for the momentum thickness is

$$\frac{d\Theta}{dX} + \frac{\Theta}{U_e} \frac{dU_e}{dX} \left[2 + H_i + \frac{1}{\Theta} \int_0^\Delta g - 1dY \right] + j\frac{\Theta}{R} \frac{dR}{dX} = \frac{T_0}{T_e} \frac{\bar{T}}{T_0} \frac{\tau_w}{\rho_e U_e^2}$$
(3.32)

In this equation all capital symbols, Greek and Roman, are properties in the transformed coordinate plane, apart from T and R. H_i is the shape factor in the transformed plane, and R is the radius of the body in axisymmetric applications. The ratio of enthalpies, $g = h/h_0$ appears due to the scaling based on density, which is also a function of temperature. Note that U denotes the velocity in the transformed plane, where the transformation is U = u/c. The transformed momentum equation also holds for axisymmetric flow thanks to the third term in which the factor j is zero for plane flows and 1 in axisymmetric flow. The definitions of the transformed boundary layer properties bear a resemblance to their counterparts in incompressible flows:

$$\Delta^* = \int_0^\Delta 1 - \frac{U}{U_e} dY \tag{3.33}$$

$$\Theta = \int_0^\Delta \frac{U}{U_e} \left(1 - \frac{U}{U_e} \right) \tag{3.34}$$

$$H_i = \frac{\Delta^*}{\Theta} \tag{3.35}$$

The second main equation of the model, the so-called moment-of-momentum equation, is obtained by multiplying the momentum equation with Y before integrating from the wall to the edge of the boundary layer. It is formulated in terms of the transformed shape factor

$$\frac{dH_i}{dX} = -\frac{1}{U_e} \frac{dU_e}{dX} \left[\frac{1}{2} H_i (H_i + 1)^2 (H_i - 1) \right] \left[1 + \frac{2}{(H_i + 1)\Theta} \int_0^\Delta (g - 1) dY - \frac{2(H_i - 1)}{H_i^2 (H_i + 1)\Theta^2} \int_0^\Delta (g - 1) Y dY \right] \\
+ \frac{H_i (H_i^2 - 1)}{\Theta} \frac{\bar{T}}{T_e} \frac{\tau_w}{\rho_e U_e^2} - \frac{(H_i^2 - 1)(H_i + 1)}{\Theta} \frac{\bar{T}}{T_e} \frac{\tau_w}{\rho_e U_e^2} \int_0^1 \frac{\tau}{\tau_w} d\frac{Y}{\Delta} \quad (3.36)$$

These equations have some terms that are unknown at the moment, which are the friction at the wall and various integrals over the thickness of the boundary layer. With the assumption of a power law for the velocity profile and a Prandtl number equal to one, which is an acceptable approximation for gases, integrals with the enthalpy ratio can be expressed in just the wall value and a relationship of shape factor and transformed momentum thickness. Skin friction is modeled as a function of shape factor and a Reynolds number based on momentum thickness. The semi-empirical relation used was proposed by Ludwieg and Tillmann [41]. This relation was initially created for incompressible flows, but was adapted to a compressible form with use of the coordinate transformations and the reference temperature concept of Eckert [13].

$$C_f/2 = \tau_w/\rho_e U_e^2 = 0.123 e^{-1.561H_i} \left(U_e \Theta/\nu_0\right)^{-0.268} \left(T_e/\bar{T}\right) \left(\bar{\mu}/\mu_0\right)^{0.268}$$
(3.37)

Where \overline{T} is the reference temperature according to Eckert, and $\overline{\mu}$ is the viscosity at that temperature, via Sutherlands law [73]:

$$\bar{T} = 0.5T_w + \left(0.5 - 0.22Pr^{1/3}\right)Te + 0.22Pr^{1/3}T_0$$
(3.38)

$$\mu = \mu_{ref} \left(\frac{T}{T_{ref}}\right)^{3/2} \frac{T_{ref} + S}{T + S} \tag{3.39}$$

Finally the momentum equation is rewritten in a form containing the Reynolds number based on the momentum thickness and is transformed back to the physical x-coordinate. In the original publication this equation contained additional, incorrect, parentheses. The correct form is:

$$\frac{df}{dx} = 1.268 \left[-\frac{f}{M_e} \frac{dM_e}{dx} \left(1 + g_w H_i \right) + A \right]$$
(3.40)

And the final formulation of the moment-of-momentum equation is:

$$\begin{aligned} \frac{dH_i}{dx} &= -\frac{1}{2M_e} \frac{dM_e}{dx} \left[H_i (H_i + 1)^2 (H_i - 1) \right] \left[1 + (g_w - 1) \frac{H_i^2 + 4H_i - 1}{(H_i + 1)(H_i + 3)} \right] \\ &+ \frac{H_i^2 - 1}{f} A \left[H_i - \frac{0.011(H_i + 1)(H_i - 1)^2}{H_i^2} \frac{2}{C_f} \frac{T_0}{\bar{T}} \right] \end{aligned} (3.41)$$

Where f and A are given by:

$$f = (M_e a_0 \Theta / \nu_0)^{1.268} \tag{3.42}$$

$$A = 0.123e^{-1.561H_i} (M_e a_0/\nu_0) \frac{T_e}{\bar{T}} \left(\frac{T_e}{T_0}\right)^3 \left(\frac{\mu}{\mu_0}\right)^{0.268}$$
(3.43)

In order to obtain the physical displacement thickness an inverse transformation must be performed. Sasman and Cresci do give relations to perform this inverse transformation, however it was observed that the relation to determine δ^* yielded results an order of magnitude larger than expected. Because of this, an effort was made to reconstruct the boundary layer velocity profiles in order to establish a transformation relation. How this was done is detailed in Appendix D. It resulted in agreement on momentum thickness to within 1 %

on the momentum thickness, compared to the relation $\theta = \Theta \left(\frac{T_0}{T_e}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$ that was specified by Sasman and Cresci. The obtained boundary layer thickness will be validated later in this section with favorable results. It must be noted that the physical shape factor is significantly larger than the shape factor in the transformed plane. $H_i = 1.3$ results in $H \approx 3$ at Mach 2, and $H \approx 8$ at Mach 4. This near-quadratic dependency on Mach number is related to the rise in temperature of the flow near the wall. This already depends on the square of the Mach number and the decrease in density causes an increase in δ^* while θ decreases, thus amplifying the effect on the shape factor.

When the change in Mach number is discontinuous, as it is around centered expansions, the factor $\frac{dM_e}{dx}$ in Equations 3.40 and 3.41 is infinite. A workaround was found by using the thickness of the boundary layer as the distance over which the expansion is applied. This produced finite gradients which resulted in boundary layer growth comparable to validation data, as will be discussed below.

A recent application of this method was by Xie et al. [74]. They report that the boundary layer was predicted quite accurate, both for a flat plate and a case with a pressure gradient. There is a small overestimation of the boundary layer growth rate on the flat plate. The growth rate is overestimated by about 20 % for the boundary layer thickness and 10 % for the momentum thickness. Over a running length of



Figure 3.5: Comparison between the experimental results of Wang et al. [72] and the Sasman-Cresci model over a gradual expansion surface. Coordinates are taken identical to Figure 3.7, with the start of the expansion at x=0.

approximately 350 mm this results in an overprediction of about 1.5 mm for the boundary layer thickness. Xie et al. further report that validation studies with more detailed CFD show that the mass flow deficit in the boundary layer is lower than expected via the model of Sasman and Cresci and an inviscid calculation via the method of characteristics. According to them, the deficit is about 50 % smaller, however they imply that this may be because they do not iterate the method of characteristics with the boundary layer solution, which causes thicker boundary layers and higher Mach numbers than in reality.

The Sasman-Cresci model has been shown to produce sufficiently accurate results for flat plate and moderate pressure gradients. It is now required to establish it's validity in the strong favorable pressure gradients experienced in the initial section of the intake. The experiment of Wang et al. [72] was simulated with the method of characteristics and the Sasman-Cresci boundary layer model. Figure 3.5 shows the boundary layer growth over a gradual expansion. After a small dip, caused by the initialization of the program, the error in boundary layer thickness nearly vanishes. The presented data is for a Mach 2.95 flow that is gradually expanded over 11 degrees between x = 0 and x = 65 mm. While the expansion in the experiment continues to 16 degrees, no boundary layer thickness data was obtained past x = 65 mm due to limited optical access.

Also the experiments by Arnette et al. [5] were reproduced. In these experiments a Mach 3 flow was expanded with gradual and centered expansion regions over 7 and 14 degrees. The experimental and simulated data are shown in Figure 3.6. The geometry of the test objects is sketched in Figure 3.7. Once again, during the expansion the match between experimental and simulated results is nearly exact. Differences arise when the expansion ends. The Sasman-Cresci model momentarily predicts a sharp rise of the boundary layer thickness and then rapidly changes to a flat plate growth rate. It appears that for the 7 degree gradual expansion



Figure 3.6: Comparison between the experimental results of Arnette et al. [5] and the Sasman-Cresci model over an expansion surface. The expansion ends at $s/\delta_0 = 6$ for the 7 degree gradual expansion and at $s/\delta_0 = 12$ for the 14 degree gradual expansion

about 6 δ_0 are needed after the end of expansion before the boundary layer attains a flat plate growth rate. In the case of the 14 degree expansion this delay places this point outside the measured domain, if it occurs at all. Also included in the figure is the simulated boundary layer for the 7 degree centered expansion. The growth over the expansion occurs instantaneously in the simulation and is not depicted, but it can be seen that at the start of the flat section most of the growth has occurred. The boundary layer thickness between simulation and experiment match well up to $s/\delta_0 = 10$, until the experiment shows sudden extra growth.

When examining Figure 3.6, one must consider several aspects concerning the data that is presented. Thickness of the boundary layer was measured by use of the flatness factor. While the flatness factor is an indication of the state of turbulence/intermittency [57], the implementation by Arnette et al. is a probable source of inaccuracy. Flatness profiles for the 7 degree centered expansion are shown in Figure 3.8. A peak in the flatness profile is taken as the edge of the boundary layer. However the maximum measured value is taken as the peak without curve fitting to establish whether a peak may occur between measured values. As a result δ_0 , the prime scaling unit, was very close to be taken 5% larger, which would re-scale Figure 3.6. Scarcity of measurement locations normal to the wall may also be part of the reason why the boundary layer appears to not grow between s/δ_0 2.8 and 8.4 for the 7 degree centered expansion. Also there is no growth for the gradual 7 degree expansion between s/δ_0 14.4 and 19.2. The limited vertical and stream-wise resolution does not allow for accurate validation of post-expansion growth rate.



Figure 3.7: Sketch of the test geometry of Arnette et al. [5], showing centered (a) and gradual expansion (b). The expansion ends at $s/\delta_0 = 6$ for the 7 degree gradual expansion and at $s/\delta_0 = 12$ for the 14 degree gradual expansion



Figure 3.8: Flatness factor profiles, taken from [5]

3.2.3 Laminar Boundary Layer Model

Laminar boundary layers can be computed with the method of Cohen and Reshotko [9]. A similar coordinate transformation is employed as for the turbulent method:

$$X = \int_0^x \frac{T_0 + S}{T_w + S} \sqrt{T_W / T_0} \frac{p_e a_e}{p_0 a_0} dx$$
(3.44)

$$Y = (T_e/T_0)^{1/2} \int_0^y \frac{\rho}{\rho_0} dy$$
(3.45)

In which S is again the Sutherland constant. These transformations are applied to the boundary layer equations, and the following form of the momentum equation is derived:

$$\frac{d\Theta}{dX} + \frac{U_{e_X}}{U_e} (2\Theta + \Delta^*) = \frac{\nu_0}{U_e^2} U_{Y,w}$$
(3.46)

Which is in turn transformed into

$$-U_e \frac{d}{dX} \frac{n}{U_{e_X}} = 2[n(H_i + 2) + l] = N$$
(3.47)

In which $U = u\sqrt{T_0/T_e}$ and the subscripts X, Y denote differentiation in the transformed coordinate system. This equation uses n and l, which are parameters that relate to pressure gradient and skin friction respectively. They are, in transformed and physical coordinates, given by:

$$n = -\frac{U_{e_X}}{\nu_0}\Theta^2 = -\frac{u_{e_x}\theta}{\nu_w} \left(\frac{T_w}{T_e}\right)^2 \left(\frac{T_0}{T_e}\right)$$
(3.48)

$$l = \frac{\Theta}{U_e} \left(\frac{\partial U}{\partial Y}\right)_w = \frac{\theta}{u_e} \frac{T_w}{T_e} \left(\frac{\partial u}{\partial y}\right)_w$$
(3.49)

n can thus be computed from known quantities, the external flow field and stagnation kinematic viscosity are known, as well as the transformed momentum thickness at the beginning of a step in x-direction. The velocity gradient at the wall is not known, thus determination of l is not so straightforward. For this reason N is introduced. N is a function of n and $S_w (= g_w - 1)$. Cohen and Rethotko present graphs and tabulated data of N for various n and S_w , but no exact relation. In the original article it is advised to approximate N by drawing a line tangent to the graphs. However, with modern available computer power it is possible, and simpler, to perform interpolation of a tabulated data set at marginal costs in terms of computation time.

The friction-related parameter can also be obtained via interpolation of data, and can then be used to compute the velocity gradient at the wall. Determining wall friction is not directly needed for the calculation of the boundary layer characteristics, due to the use of N, but can be used for determining loads on the intake surfaces and validation of the boundary layer model. When the shape factor has also been obtained, the velocity profile can be reconstructed via the method of Pohlhausen [73]. Via a process identical to the one used for turbulent boundary layers, the thickness of the boundary layer and the resulting displacement in physical coordinates is obtained. Momentum thickness can be directly converted to and from the transformed coordinates via the relation $\theta = \Theta \left(\frac{T_0}{T_e}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$.

A complication arises when there is no streamwise velocity gradient, Equation 3.47 has a singularity due to U_{e_X} in the denominator. This can be easily resolved though by imposing an infinitesimal velocity gradient. When doing so the boundary layer obtained from the integral boundary layer method produces a solution that matches the experimental data of Giepman [20]. At 40 mm running length the measured boundary layer thickness is 0.2 mm, where a reproduction with the Cohen-Reshotko boundary layer method predicts 0.18 mm. Similar the friction coefficient was predicted as $6.0 \cdot 10^{-4}$ where Giepman measured $5.5 \cdot 10^{-4}$. Both these predictions deviate roughly 10 % from the actual data, an acceptable error for the current application.

The method of Cohen and Reshotko has been used recently by Ma et al. [42] and before that by McNally [47], who remarked "It is one of the most accurate, programmable general methods available for the laminar case". The latter also achieved displacement and momentum thicknesses accurate to within 10 % for the first 300 mm running distance.

3.3 Shock Wave Boundary Layer Interaction

The effect of shock wave displacement and boundary layer growth due to interaction between boundary layers and shock waves must be predicted. Due to the difference between laminar and turbulent boundary layer profiles, resulting from different resistance to separation, different methods are required. The turbulent method is covered first.

3.3.1 Turbulent SWBLI

A useful model that incorporates both the effects on interaction length and boundary layer thickness of the turbulent boundary layer was presented by Souverein [63]. A sizable dataset of SWBLIs, consisting of both ramp flows and reflecting shock waves, was used to quantify these effects.

A first step is to determine whether a separation bubble is formed. It was found that the rise in pressure over shock was an important factor for separation to occur. In fact, the ratio of pressure rise to pre-shock dynamic pressure (q) proved to be decisive, with only a very minor Reynolds number dependence. This dependence was included in a scaling factor k, which was used to evaluate S^* :

$$S^* = kp/q_e \tag{3.50}$$

$$k = 3.0 \ if \ Re_{\theta} \le 10^4 \tag{3.51}$$

$$k = 2.5 \ if \ Re_{\theta} > 10^4 \tag{3.52}$$

 S^* indicates the strength of the shock wave relative to the incoming flow and will be important for determining the interaction length scale. A separation bubble is formed when S^* is larger than one.

Figure 3.9 shows a schematic overview of the interaction location. Based on mass conversation over a control volume, relations for the interaction length L can be developed. These are:

$$L = \frac{\rho_3 u_3 \delta_3^* - \rho_1 u_1 \delta_1^*}{\rho_2 v_2} \tag{3.53}$$

for the reflecting shock wave, with v_2 denoting the flow speed normal to the wall in the region between the incoming and reflected shock, and for the ramp:

$$L = \frac{\rho_2 u_2 \delta_2^* - \rho_1 u_1 \delta_1^*}{\sin(\theta) \rho_2 u_2}$$
(3.54)

With some trigonometry and mass conservation over a shock wave, a common formulation for L can be obtained:

$$\frac{L}{\delta_{in}^*} = \frac{\sin(\beta - \theta)}{\sin(\beta)\sin(\theta)} \left(\frac{\rho_{out}u_{out}\delta_{out}^*}{\rho_{in}u_{in}\delta_{in}^*} - 1\right)$$
(3.55)

In this equation the subscripts *in* and *out* are used to denote conditions upon entering or exiting the control volume. Finally, L is converted to a non-dimensional form using the displacement thickness and the shock wave angles:

$$L^* = \frac{L}{\delta_{in}^*} \frac{\sin(\beta)\sin(\theta)}{\sin(\beta - \theta)}$$
(3.56)



Figure 3.9: Schematic representation of the interaction region for the reflecting case (a) and the compression corner (b). Here ϕ is used as deflection angle over the shock wave, instead of θ . Image taken from [63].



Figure 3.10: Correlation between non-dimensional quantities shock strength and interaction length, the dashed line is the trend line $L^* = 1.3S^{*3}$. Image taken from [63].

When the dimensionless shock strength and interaction length are plotted against each other, as in Figure 3.10, a power-law trend appears. Souverein fitted a line $L^* = 1.3S^{*3}$ through the data. The data has a significant scatter though, which likely results from the fact that Figure 3.10 covers both ramp flows and shock reflections. It was observed that the trend line Souverein has a significant error for low strength shock reflections. An improved trend line was established based on experimental data from Jaunet [31] and Laurent [37]. This trend line was determined to be $L^* = 2.1S^{*1.9}$ and is presented in Figure 3.11. Also included in the figure is an experimental data point from the work of Van Veen et al. [67].

While slightly different trendlines improve the results of the scaling by Souverein [63], the similarity between ramp flows and reflecting shock waves is remarkable. The similarity between these geometries had been noted long ago and has led to the formulation of the term 'free interactions' by Chapman et al. [8]. It states that instead of the cause of the SWBLI, the strength of the shock and the state of the boundary layer are most important. In the words of Delery and Marvin [12]:"Everything happens as if the flow were entirely determined by its properties at the onset of interaction." The scaling of Souverein shows that the relevant properties at the onset of separation are the boundary layer thickness, Reynolds number based on the momentum thickness and shock geometry and pressure ratio. Differences between ramp flows and reflection of a shock wave is thus limited, but a minor effect does remain.

3.3.2 Laminar SWBLI

Laminar shock wave boundary layer interaction differs significantly from the turbulent case. A very important effect is that laminar boundary layers separate at lower pressure rises. Due to the lower velocity region near the wall, a lower pressure rise is needed to reverse flow direction near the wall. The formed separation



Figure 3.11: Correlation between non-dimensional quantities shock strength and interaction length, improved correlation for shock reflection

bubble has a flat triangular shape, the majority of which is located upstream of where the shock wave strikes the boundary layer. A schematic overview of an interaction is shown in Figure 3.12. Note that the scale can be very deceiving, a bubble can be 50 times longer than tall and as such the compression at the upstream side is very small, only a few degrees [21]. Initially, the boundary layer seems to flow over the bubble relatively undisturbed, until the location where the shock strikes the boundary layer. At this point, the pressure rise initiates a bypass transition mechanism and a fully turbulent state is reached approximately 5 times faster than via natural transition [21].

As with the turbulent boundary layer interaction, we are mostly interested in the size of the interaction region, which is strongly correlated to the pressure rise over the interaction. Figure 3.12 shows a schematic of the interaction, along with the pressure distribution over the interaction. The initial compression causes a pressure rise until a plateau level, where it remains until the shock wave strikes the boundary layer. At the peak of the interaction an expansion fan is created, though significantly weaker than the incident shock. The flow turning at the reattachment point causes compression, usually much stronger than at the separation point.

An estimation of separation bubble size was given by Katzer [34]. As for the turbulent case, first it must be determined whether a separated bubble exists. This was found to occur for a pressure larger than the separation pressure p_s , which is given by:

$$p_s = \frac{p_1}{2} \gamma M_1^2 P_s \left(\frac{C_{f_1}}{\sqrt{M_1^2 - 1}}\right)^{0.5}$$
(3.57)

Where the subscript 1 denotes pre-interaction values and P_s is a constant experimentally determined to be approximately $1.85\sqrt{2}$. Determining P_s is complicated because detecting small separated regions is difficult, so other values deviating \pm 10 % have been reported [24]. Separation can already occur when the post interaction pressure is as low as 1.1 times the initial pressure [21, 34]. Bubble length can then be determined



Figure 3.12: Schematic of laminar SWBLI and pressure over the interaction region, taken from [24]

from

$$\frac{l_B}{\delta_1^*} \frac{M_1^3}{\sqrt{Re_{x_i}/C}} = 4.4 \frac{p_3 - p_s}{p_1} \tag{3.58}$$

In which C is the Chapman-Rubesin constant $C = \frac{\mu_w T_{\infty}}{\mu_{\infty} T_w}$.

Apart from the separation bubble length, relations have been formulated to calculate the deflection at the separation point and the plateau pressure. Hakkinen [24] correlated the pressure coefficient of the plateau to Mach number and skin friction coefficient as:

$$C_{p_{pl}} = 1.65\sqrt{2C_f/\sqrt{M^2 - 1}}$$

Which can be combined with linearized supersonic theory to obtain a flow deflection angle, α , at the separation point [24]:

$$\alpha = \operatorname{atan}\left(0.5C_{p_{pl}}\sqrt{M_1^2 - 1}\right)$$

The experiments performed by Giepman [21] were reproduced in order to check the validity of this SWBLI model. While the flow deflection angle at the separation point could indeed be predicted accurately, as done by Giepman, the relation for bubble length did not provide accurate results. Even when considering that Katzer [34] predicted fully laminar interactions, where in the experiments transition at the shock impingement location was encountered, bubble length is overpredicted by at least 100 % and the error increases with stronger shocks. In the case of strong pressure rise the linearized theory used to predict α no longer holds either.

The separation bubble can reach extreme sizes when the shock strikes just after a strong expansion, as depicted in Figure 3.13. This Figure shows a very similar situation to what happens in the submerged ramjet



Figure 3.13: A large laminar separation bubble due to SWBLI, experiment (top) and simulation (bottom), taken from [54]

intake when the compression shock(s) fall behind the cowl lip. In the simulation depicted in Figure 3.13, which was validated with experiments, a large separation bubble is created on the upper side of the channel. This can be attributed to the laminar state of the boundary layer as the experiment was reproduced with the inclusion of a turbulence producing wire and this caused the separation bubble to be significantly reduced in size. Reinartz et al. [54] note that the large laminar separation bubble is highly undesired due to the significant blockage and the unsteadiness that follows from it.

Another reason that predicting separation bubble size is difficult, is that other currently unquantified factors beside pressure rise appear to have a significant effect. Liu et al. [39] report that the interaction region grows to counter increased wall normal momentum due to increased mass flow rate at the shock impingement location.

As predicting the laminar boundary layer and shock wave interaction can not be predicted accurately, it is fortunate that they do not occur often. It occurs when the compression shock(s) fall behind the cowl lip, which was mentioned above as highly undesired and is a result of flight Mach numbers higher than intended during intake design. The other occurrence of a shock wave interacting with a laminar boundary layer is in the case of 2-shock compression, where a shock wave forms at the kink in the compression surface.

3.4 Shocktrain

In 1973 Waltrup and Billig [69] introduced a relation that approximated the length of the shock train in a cylindrical duct. By independently varying duct diameter, Mach number, Reynolds number and boundary layer momentum thickness, the relation between pressure ratio and shock train length was found to be:

$$\frac{L(M^2 - 1)Re_{\theta}^{0.25}}{\sqrt{D\theta}} = 50\left(\frac{p_{exit}}{p_{inlet}} - 1\right) + 170\left(\frac{p_{exit}}{p_{inlet}} - 1\right)^2$$
(3.59)

This relation is purely a correlation fit to the data of a series of experiments and does not have a basis on the physical flow phenomenon it describes. Some connections between the correlation and the physical flow situation can be inferred though. The quadratic term on the right hand side indicates that the length of the pseudo-shock region increases rapidly when the pressure ratio is increased. This can be related to the slow pressure rise in the mixing region, indicated in Figure 2.17. A smaller duct diameter will decrease the length,



Figure 3.14: Corner momentum thickness as defined by Geerts [18]

as the reduced distance between wall and center line means that the oblique shock waves span a shorter distance. Also the post-shock mixing of the subsonic flow near the wall and the supersonic flow along the center of the channel occurs in a shorter stream-wise distance as well.

Two decades later a modification to this relation for rectangular channels was published by Sullins and McLafferty [65], after further work by Billig [6]. Only two changes were made: the diameter was changed to the channel height and the Reynolds number exponent was reduced to 0.2.

$$\frac{L(M^2-1)Re_{\theta}^{0.2}}{\sqrt{h\theta}} = 50\left(\frac{p_{exit}}{p_{inlet}} - 1\right) + 170\left(\frac{p_{exit}}{p_{inlet}} - 1\right)^2$$
(3.60)

Recent work by Geerts [18, 19] was focused on determining whether the aspect ratio of the duct and boundary layer behavior at the corners had an impact on the shock train length. The effect of aspect ratio was included in the hydraulic diameter, $D_H = \frac{2H_1H_2}{H_1+H_2}$, which replaces the actual duct diameter in the correlation of Waltrup and Billig. A 'corner momentum thickness' is defined via $\theta' = \sqrt{\theta_1^2 + \theta_2^2}$ a situation which is represented in Figure 3.14. Geerts also splits the total shock train length (L_T) into a component along the center line of the duct and a corner flow separation length (L_C) , which is the distance between the first oblique shock starting at the wall and reaching the center of the channel. The resulting relation , solved for L_T , is [19]

$$L_T = \left[50 \left(\frac{p_{exit}}{p_{inlet}} - 1 \right) + 170 \left(\frac{p_{exit}}{p_{inlet}} - 1 \right)^2 \right] \frac{\sqrt{D_H \theta'}}{(M^2 - 1)Re_{\theta'}^{0.25}} + L_C$$
(3.61)

While Geerts reports an improvement in the accuracy of shock train length prediction, there is reason for caution. Geerts tested only 2 ducts configurations at one Mach number and so far no other work using Geerts' correlation has been published. The correlation of Sullins and McLafferty has proven itself over the past 25 years. Also the data of Geerts does not severely deviate from this correlation Although for several data points, especially for lower pressure ration, the correlation does not fit within the experimental margin of error.

The relations above all consider situations with identical boundary layer thicknesses on the bottom and top wall of the channel. However, this does not often occur in practical applications, including the submerged intake considered in this study. Wang et al. [71] tested the effect of (partial) boundary layer removal and the resulting asymmetry between the top and bottom wall. The degree of asymmetry is expressed as:

$$D_{\theta} = \frac{\theta_{max} - \theta_{min}}{\theta_{max}} \tag{3.62}$$

Where min and max refer to the thinnest and thickest boundary layers, respectively. D_{θ} is a value between 0 and 1, where 1 means the absence of a boundary layer on one side of the channel and 0 refers to the perfect symmetrical case. A new correlation was obtained for shock train length:

$$L = \frac{(1+D_{\theta})^{0.3}\sqrt{h\theta}}{(M^2-1)Re_{\theta}^{0.2}} \left[50\left(\frac{p_{exit}}{p_{inlet}} - 1\right) + 170\left(\frac{p_{exit}}{p_{inlet}} - 1\right)^2 \right]$$
(3.63)

This correlation captures the elongation of the shock train due to asymmetry in boundary layer thickness. Again, the correlation is based on limited data, with one series of 8 experiments spanning a Mach range of 1.33-1.85 and unit Reynolds number $3.8-5.1 \cdot 10^7$. The length of the isolator was 10 times the height of the channel and the thickest boundary layer was between 10 and 20 percent of the channel height. These limits are important because the validity of the correlation is limited to similar flow conditions for which it was established.

The efficiency of an intake is generally expressed in terms of the pressure recovery factor $p_{0,e}/p_{0.\infty}$, where $p_{0,\infty}$ is the free stream total pressure $p_{0,e}$ is the total pressure at the end of the intake. However, with the currently available information it is not possible to convert the static pressure to total pressure. A Mach number aft of the shock train is required for that. Lin et al. [38] provided a correlation that fit their data set:

$$M_{exit} = -1.17M^{-1.4} \frac{p_{exit}}{p_{inlet}} + 1.78M^{0.36}$$
(3.64)

Again it should be noted that the data on which this correlation is based is very limited, with a single rectangular isolator tested at M = 1.8 and 2.2. It is therefore not known how this translates to other test conditions and geometries. A circular isolator with identical flow area produced exit Mach numbers that were 20 % lower at no pressure rise, up to 40 % lower at higher pressure ratios. These geometric effects can thus be very significant, and the presented data does not provide sufficient information to incorporate these effects accurately, nor have there been other publications following up on this point of the work of Lin et al. [38].

Chapter 4

Model Integration

This chapter will address various aspects on the integration of the sub-models into the intake performance analysis model. It concerns the assumptions that were made in order to simplify the model, detailed in Section 4.1, and the computational structure of the intake model in the Section 4.2. Limits to the intake performance analysis model, either due to incomplete flow models or applicability to specific situations, are covered in Section 4.3. Finally some examples of the program output are displayed in Section 4.4.

4.1 Assumptions

Steady state The model will only represent a steady state of flight. No effects of gusts and other time-dependent disturbances to the flow will be considered.

Perfect gas with constant specific heats It is assumed that the gas is a perfect gas with temperatureindependent specific heats. This simplifies the model, but the specific heat at constant pressure has increased by over 5% when the temperature exceeds 600 Kelvin. This will not occur in wind tunnel testing, but in real flight it occurs at approximately Mach 2.5 at sea level and Mach 3 at 10 km altitude. These temperatures occur in the slowest part of the boundary layer. The effect is minimal though as the part of the boundary layer where these high temperatures occur is very thin, especially in turbulent boundary layers.

Shock wave and expansion fan interaction When a shock wave and an expansion fan of different families cross, it is assumed that the deflection angle over the shock wave will not change. It was shown by Hillier [30] that the interaction between these phenomena is limited and allows to reduce the complexity of the model.

Boundary layer bleed has no upstream influence The bleed slot is assumed to have no upstream effect. While it is possible that information travels upstream via the subsonic part of the boundary layer, experiments on bleed slots and the channel behind it have shown that this only occurs if the bleed channel is choked [27]. This assumption thus implies that the channel behind the bleed slot is sufficiently wide.

Transition occurs on a specified location As indicated in Chapter 2, transition is a problem affected by so many variables that there is no simple model to accurately predict it with the limited data available. It has therefore been chosen to force transition to occur at a specified point. The location of this point is included as a set distance behind the lip of the compression surface or the cowl, or derived from a Reynolds number of transition. It will be up to the user of the intake model to specify either criterion.



Figure 4.1: Division into computational sections as used in the intake analysis model

Transition occurs instantly In reality the transition from a laminar to a turbulent boundary layer requires some distance over which the shape factor gradually reduces [20]. For the intake performance analysis model this was simplified to instant transition.

Adiabatic wall There is no heat transfer between the boundary layer and the wall. This assumption reduces the complexity of the model by neglecting temperature changes in the projectile body, as well as allowing for a simpler temperature profiles in the boundary layer. Note that the employed methods for the boundary layer can be applied to non-adiabatic walls.

Post SWBLI shape factor It is assumed that the shape factor behind an interaction between shock waves and a turbulent boundary layer is the same as before the interaction. Louman [40] showed that shortly after the interaction the shape factor returns to its pre-shock value.

SWBLI forces transition The strong adverse pressure experienced over a shock is strong enough to force bypass transition. downstream of a laminar SWBLI the boundary layer will thus be turbulent. This is in agreement with the experiments by Giepman [20].

SWBLI perturbations directly coalesce into a single shock A SWBLI causes a set of expansions and shocks which merge some distance above the boundary layer. Because the displacement thickness, the primary scaling unit, is very thin, this coalescence occurs very near the boundary layer. As a result an insignificant amount of the flow passes through the shock-expansion-shock system.

4.2 Model Computational Structure

In Figure 4.1 the division of the intake analysis model into several computational sections is shown. The computation of the expansion region is uncoupled from the compression region. In the compression region, the flow is computed in several blocks. The first block, A, contains the effect of the initial compression via one or multiple oblique shock waves or a gradual compression without a shock at the surface. Also incorporated in block A is the effect of a possible expansion centered at the cowl lip. Finally there are the numbered blocks which are bounded by a reflecting shock wave or the limits of the Method of Characteristics solution.

Each of these sections follow the same basic outline shown in Figure 4.2. For starting a MoC solution an initial data line is required. For the expansion region this is a characteristic with freestream properties and in the internal channel the last characteristic of the previous section. In the region A the initial data line



Figure 4.2: Order of calculations per section

is the downstream side of the shock wave. For internally reflecting shock waves, the shock wave strength is continuously adapted by utilizing a procedure outlined in Section 3.1.3. This procedure combines pre-shock flowfield data, the shock wave equations and the Method of Characteristics in the post-shock flow region. With this approach the varying strength of the shock wave, due to interaction with an expansion fan, can be accurately captured. With the flow properties at the wall known from MoC and, if present, the boundary layer thicknesses and shape factor at the end of the previous block, the boundary layer model is run to calculate the development of the boundary layer. With the boundary layer growth known, the MoC process is repeated. While the second run of the MoC affects flow properties at the wall and thus the boundary layer solution, this effect is negligible. The boundary layer thickness changes by less than one percent over the full distance of the expansion ramp. This is well below the accuracy of the boundary layer model and thus performing more iterations would not add to the accuracy of the intake analysis while increasing run time. If there are no shock waves present, the final characteristic will serve as the initial data line for the next block to be computed. In case there are shock waves, the extent of shock wave boundary layer interaction must be determined before the reflected shock wave is calculated.

Once the flow has been calculated over the complete length of the duct, the shock train script is run. It first gathers the boundary layer momentum thickness along every point on the walls and the Mach number and pressure at the same locations. Then it computes the pressure that would be obtained at the exit of the intake duct if the shock train were to start at that point.

A more complete overview of the model structure and the Matlab scripts and functions is given in Appendix A.

4.3 Limit to Applicability and Accuracy

In a number of cases, flow situations occur for which there was insufficient information available on adequate flow models, or the resulting flow field was deemed too complex for this first version of the intake analysis model. In this section these limits to the intake analysis model are detailed.

First there is the case of compression via multiple shock waves created by the compression surface. For the sake of reducing some time in development, the effect of possible shock-shock interaction is not considered. The user of the intake model must determine whether the location of shock-shock interaction is in a place where it does not interfere with the flow entering the internal part of the intake. A result of shock-shock interaction is a slip line along which entropy and velocity changes discontinuously, which can not be analyzed by the current implementation of the method of characteristics.

A second limit on the intake analysis model is reached when a cowl lip centered expansion fan interacts with the shock wave created by gradual compression. Because of how the intake analysis model currently calls on all lower functions and how the curved shock wave is calculated, the combination with the expansion fan would require extra time to implement. Time constraints did not allow for implementation in the current version.

A weaker point in the model is that the interaction between the laminar boundary layer and a shock wave could not be accurately resolved. While Giepman [20] reported good agreement between his experimental data and relations published by Hakkinen [24] and Katzer [34], these calculations could not be accurately reproduced during the validation process. As stated in Section 3.3.2, the length of the laminar separation bubble deviated by 100 % or more between between experimental data and the reproduced calculation. Fortunately, this problem has limited impact as the laminar boundary layer is very thin and the interaction region with a moderate strength shock measures only a few displacement thicknesses normal to the wall. A more significant effect is that the shock wave causes the boundary layer to transition, resulting in a much more rapid growth of the boundary layer. In an extreme case, well beyond the application range of the scale estimation from available literature, the combination of a laminar boundary layer, an expansion fan and a shock wave, the separation bubble can grow to hundreds of boundary layer thicknesses [54]. This could possibly cause significant blockage of the internal channel. In their chapter on the practice of intake design Goldsmith and Seddon [22] note that the cowl position is often chosen such that the shock falls on the cowl lip at the maximum flight Mach number plus 0.2, in order to avoid problems due to the ingestion of shocks caused by the compression surface. The shock position may also be affected by the angle of attack at which the vehicle flies, but the intake analysis model does not incorporate this in the current version.

As mentioned in Chapter 3, there is some difficulty with accurately determining the total pressure downstream of the shock train. This is because no reliable information on Mach number behind the shock train is available. A rule of thumb by Mahoney [45] is to assume a loss in total pressure of 10 % after the effects of an inviscid normal shock have been applied. However, this is no more than a first approximation based on empirical data on what typical supersonic intakes can achieve. An approach taking into account the flow situation in the isolator would improve accuracy, but such a method is currently lacking.

The correlation used for the shock train pressure rise suffers from a similar problem. Due to the fact it is a correlation based on experimental data, it is only applicable to situations matching the test conditions. These test conditions are listed in Section 3.4 and must be taken into consideration when the shock train correlation is applied.

4.4 Output Produced

Before continuing on to the results of a parametric study in the following chapter, it would be useful to detail the output that is produced during a typical run of the submerged intake performance analysis model.

Figure 4.3 shows an example of an intake with a 2-shock compression surface. This image shows the expansion ramp and the edge of the boundary layer on it in blue, the compression surface and the cowl in black. Also indicated are the shock waves created by the compression surface lip in red, the internal shock waves in green and the cyan line depicts the dividing streamline between captured flow and flow passing over the intake. The dashed vertical line near the end of the internal channel represents the end of the isolator and entry to combustor. This location is used in the shock train pressure rise calculations.

The same plot can be used to display more data on the flowfield. A scatterplot can easily be created to display the data from the MoC calculations, such as in Figure 4.4. In this figure the Mach number is



Figure 4.3: Example output showing the intake geometry, shock waves, boundary layers and the dividing streamline



Figure 4.4: Example output showing the Mach number in the internal channel

indicated. It can be seen that the second shock of the compression surface decelerates the flow from approximately Mach 2.15 to Mach 2. Further deceleration occurs in the internal channel of the intake due to a weak reflecting shock wave and compression via thickening boundary layers. Similar figures can be created for all important flow properties, such as pressure, temperature and flow direction.

A second group of output consists of plots of wall-bounded properties. This includes the expected temperature and pressure on the walls and the boundary layer growth, shown in Figure 4.5. Indicated in this plot are the overall boundary layer thickness (δ) , displacement thickness (δ^*) and momentum thickness (θ) on both the compression surface and the cowl. Very noticeable is the strong rise in the growth rate after transition to the turbulent state, which occurs at $x \approx 0.16$ due to a shock wave on the compression surface and via natural transition on the cowl at $x \approx 0.24$. Effects of SWBLI can also be observed in in the boundary layer thickness. While this test case has relatively weak shock waves, small perturbations can be seen at $x \approx 0.24, x \approx 0.32$ and $x \approx 0.38$ in the compression surface boundary layer thickness.

The final graphical output of relevance concerns the shock train. In Figure 4.6 this data is presented graphically. It shows the relation between the static pressure at the isolator exit and the start point of the shock train, according to the correlation of Wang et al. [71]. Also the expected total pressure that can be obtained is indicated. For comparison Figure 4.6 also shows the input total pressure and the static pressure



Figure 4.5: Example output showing the growth of the boundary layer

that would be attained by an inviscid normal shock at the cowl lip. The pressure achieved via a shock train is always lower than via an inviscid shock, due to viscous effects [22]. It can also be observed that the gradient of the shock train correlation is very shallow near the cowl lip, implying that a small rise in back pressure can rapidly push the shock structure out of the intake internal channel. This unstart of the intake is accompanied by a drastic loss in mass flow.

Aside from several data plots that are created, all data mentioned above are also stored in data structs and can be easily retrieved for more detailed study.



Figure 4.6: Example output showing the static and total pressure recovery

Chapter 5

Parametric Study

There are a large number of design variables present in the submerged intake. The usefulness of the created analysis model can now be proven by determining the effect of several of these design variables. A parametric study was performed in order to obtain information that can be used to direct initial design of a submerged intake. This chapter shows the result of this parametric study, defining the effect of each variable on the obtained mass flow rate, static and total pressure delivered by the submerged air intake. Mass flow rate is expressed as 'net capture height' to easily compare with the frontal area of regular intakes, which capture an undisturbed flow. The 'net capture height' accounts for the airflow removed via the boundary layer bleed slot. Static and total pressure are expressed as ratios over their free stream values.

The basis intake design around which the parametric study was performed is discussed in Appendix B and depicted in Appendix C. If not specified otherwise, simulations were performed at Mach 2 with stagnation pressure 1 MPa and stagnation temperature 280 K, corresponding to anticipated wind tunnel test conditions in the validation study.

Figure 5.1 indicates the geometric parameters that are varied during this parametric study. This image shows the two-shock compression surface, with two parameters regarding the first section indicated as well.

First we look at the effects that the expansion ramp parameters and Mach number have on the boundary layer growth. Then the performance effects are determined of the intake geometry for the three types of compression surfaces.

5.1 Expansion Ramp

First the effects of varying geometric and flow parameters on the expansion ramp boundary layer are investigated. These parameters are the turning radius and angle of the ramp, and the Mach number before the expansion. It is important to know the boundary layer thickness in order to properly size the bleed slot between the expansion ramp and the compression surface.

At the start of the expansion the boundary layer has a transformed shape factor of 1.3 and a physical momentum thickness of 0.045 mm. This corresponds to a 5 mm thick boundary layer for Mach 2, corresponding to the expected conditions during the validation test. The obtained thickness data is presented as ratios δ_2/δ_1 , δ_2^*/δ_1^* , θ_2/θ_1 , where the station number 1 indicates the start of the expansion and 2 denotes the point on the expansion surface nearest to the compression surface lip.



Figure 5.1: Parameters investigated during the parametric study indicated in the assembly of the wind tunnel model

Expansion Radius

In Table E.1 the effect of the radius of the expansion region on the boundary layer is presented. It shows that the radius itself of the expansion has very little effect, but because the total running length of the ramp increases with increasing radius the boundary layer has a longer distance to grow and is thicker at the end of the expansion ramp. δ , δ^* and θ have different growth ratios because the shape factor and Mach number change, which affect the relations between these three thicknesses. For a sudden expansion, $\mathbf{r} = 0$, the adaptation to the boundary layer method covered in Chapter 3 produces data that fit well with other small expansion radii. The x-coordinate of the ramp at the cowl lip y-coordinate is also reported. This is an indication of the running length of the boundary layer and it is not surprising that the boundary layer becomes thicker with increased running length. More surprising is the slight decrease in the shape factor H_i , this most probably a Reynolds number effect which will be covered in the following section.

Expansion Angle

The angle of the expansion ramp strongly affects the boundary layer growth, as depicted in Table E.2. An increase in Mach number has a strong effect on the ratio between momentum thickness and the total boundary layer thickness. As an expansion over a greater angle increases the final Mach number, this means thicker boundary layers for larger angles. In order to provide a useful comparison on the effect of this design variable on the rest of the intake, the depth of the intake was kept the same. Shallower angles require a longer running length to reach this depth, for the very shallow angles this is so significant that this is where the thickest boundary layers are found. During the expansion the shape factor of the boundary layer reduces. After the expansion is over the shape factor slowly develop towards a value depending on the Reynolds number based on boundary layer momentum thickness, a correlation of this is captured in Figure 5.2.

Reynolds Number

The effect of the Reynolds number based on the starting momentum thickness was also investigated, with the obtained data presented in Table E.17 and Figure 5.3. It can be observed that for very low Reynolds numbers $(5 \cdot 10^3)$, corresponding to thin boundary layers, the growth rate is two to three times larger than for the expected test condition $(Re_{\theta} \approx 6 \cdot 10^4)$. This is closely related to the square root connection between momentum thickness and the running length of the boundary layer, $\theta \sim \sqrt{x}$. A thin boundary layer will first grow rapidly, but this growth will slow down as the boundary layer develops. This is reflected in the data with reduced slope of the boundary layer parameters as the Reynolds number increases. Again the trend of a reduction is shape factor as Reynolds number increased is observed.



Figure 5.2: Correlation between shape factor and momentum thickness (δ_2 in the notation of the source) based Reynolds number, taken from [75]



Figure 5.3: Boundary layer parameters on the expansion ramp as function of Re_{θ} at the start of expansion



Figure 5.4: Boundary layer parameters on the expansion ramp for $M_{\infty} = 2$ (left) and $M_{\infty} = 3$ (right)

Wind Tunnel vs. Atmospheric Flight

It is also of interest to know how the wind tunnel experiments will relate to actual intakes in atmospheric flight. Atmospheric flight will be associated with much higher stagnation temperatures, a variable closely related to viscosity and the growth of boundary layers. Table E.18 presents the data obtained for this part of the parametric study. It can be observed that an increase in the flight altitude will reduce the boundary layer growth rate over the expansion ramp, although this is also associated with a reduction in Reynolds number, which would increase growth rate according to the previous section. The stagnation temperature is used in multiple instances in the boundary layer model, not only for determination of the viscosity. It appears that the temperature has a stronger effect than the Reynolds number on the boundary layer in the considered data range.

Freestream Mach Number

Finally there is the effect of the free stream Mach number to be investigated. Table E.3 summarizes the results of a 14 degree expansion ramp with varying initial Mach number. A more detailed look at the boundary layer data, presented in Figure 5.4, shows that there are two stages of the boundary layer development. First there is the expansion stage followed by a flat plate stage, with the transfer between the two at $x \approx 0.05$ where the expansion curve ends. The flat plate stage is characterized by steady development with near linear growth of the thickness parameters and a slow change in the transformed shape factor. The development of θ during the expansion depends strongly on the initial Mach number. For $M_{\infty} = 2$ there is first a small reduction in θ and the total growth at the end of the expansion stage is only 3 %, whereas for $M_{\infty} = 3$ 37 % growth has been achieved.

The other boundary layer thicknesses are determined from a function of Mach number and shape factor. Higher Mach numbers result in a larger temperature difference between the free stream and the wall, which causes an even stronger increase of the displacement thickness and total boundary layer thickness. The change in shape factor is also very rapid during the expansion stage, but only changes slowly on the flat section. As mentioned in the previous section, the shape factor seems to approach a Reynolds number dependent value.



Figure 5.5: The effect of the expansion angle and the obtained static and total pressure

5.2 Flat Compression Surface

In this section we will investigate the effect of several parameters on the performance of the intake. This section concerns the flat compression surface, which has no associated parameters itself. On this simple geometry the effects of more general intake related parameters are evaluated. These parameters are the expansion angle, depth of the intake and length of the isolator (the internal constant cross-section area channel). Figure 5.1 shows these parameters in the intake wind tunnel model.

Expansion Angle

The effect of the expansion angle on the intake performance was tested first. The compression surface was placed at 2 cm depth in all cases, meaning that the expansion ramp increases in length with decreasing expansion angle. The effect this has on the expansion ramp boundary layer was further investigated earlier in this chapter. The cowl bleed slot width is kept at 35 mm and the cowl is placed such that the shockwave passes just in front of the cowl lip. Results are displayed in Table E.4 and Figure 5.5. In this context the subscript 2 is used to denote the end of the isolator.

It can be observed that there is a strong dependency between the expansion angle and the resulting static and total pressure. This shows the inherent problem of the submerged intake, the expansion increases Mach number and as a result shock losses are higher. A simple pitot intake at Mach 2 would have a total pressure ratio of 0.649, when accounting for the expected 10 % viscous loss, which is almost reached at 4 degree expansion. A shallower expansion ramp also means a little more mass captured by the intake, about 10 % more at the 4 degrees than at 14 degrees. At these shallow angles the height of the internal channel is practically identical to the frontal area of a regular intake

In the determination of the static pressure some unexpected behavior was observed. In Figure 5.6 we see that there is a large variation between x=0.28 and 0.33, where an almost flat profile was expected that fits with the remainder of the graph. The most probable cause appears to be that the very thin laminar boundary



Figure 5.6: Shock train output for 8 degree expansion. An unexpected strong pressure rise is present before x=0.33, the location of transition of the cowl boundary layer

layer causes a singularity in the shock train equation, sometimes even exceeding the expected total pressure. This behavior can occur unexpectedly and thus requires that the user pays attention to the produced data. At lower expansion angles this effect is more pronounced. Table E.4 shows both the highest static pressure ratio and the ratio obtained just after the location of transition. In the pressure data presented it is clear that a larger expansion angle results in a lower static pressure. This is related to the loss of total pressure associated to higher Mach numbers an stronger shocks. The Mach number downstream of the compression shock is nearly identical to the free stream Mach number, 1.96 for the 14 degree expansion-compression and asymptotically approaching Mach 2 for weak a expansion and recompression.

Depth

The depth of the intake is the second parameter investigated. Depth is a parameter that has a strong effect on the size of the whole intake, not only for the length of the expansion ramp but also for isolator length. The pressure rise strongly depends on the ratio between isolator length and depth. For this test the isolator length has been kept at 10 times the depth, as it is in the baseline design.

Table E.5 presents the data obtained from the intake analysis model. It is obvious that a larger intake captures a larger amount of airflow, but now we observe a constant relation between depth and the capture height, for this case about 90 %. Also there is no significant effect on the total pressure obtained. For static pressure the unexpected behavior mentioned in the previous section surfaces again for depths of 2 cm and smaller. For larger depths the effect on pressure is small.

Isolator Length

As mentioned in the previous section, isolator length/depth has an effect on the pressure rise through the shock train. In this section that effect is investigated using the base depth of 2 cm and varying the isolator length. The results of this investigation are displayed in Table E.6.

As expected the mass captured and total pressure recovery do not change because the geometry upstream of the isolator remains unchanged. The static pressure increases significantly, even beyond the expected total pressure $(p_2/p_{\infty} > 4.5)$ for the very long isolator. How is this possible? This is a case in which the limits of the the shock train correlation and the total pressure rule of thumb are approached. In practical applications the isolator lenght is 5 to 10 times the height, because the increase in pressure is limited for the cost associated with a longer intake. The correlation for the shocktrain is thus applied outside the range for which data was available. Also the determination of the total pressure recovery is based on a rule of thumb and does not account for the flow situation in the internal part of the intake.

Cowl Lip Position

The location of the cowl lip, especially with respect to the shock wave originating from the compression surface, will also impact intake performance. In Table E.7 and Figure 5.7 the results of a varying cowl lip position are presented. These data were obtained for an expansion angle of 9 degrees, as difficulties were encountered due to the very strong expansion near the cowl lip with the base design of 14 degrees. For this geometry the shock on lip condition is reached with the cowl lip at x=0.2224 m. Negative values on the x-axis of Figure 5.7 mean that the shock wave is captured inside the cowl and there is a centered expansion at the cowl lip. It can be observed than when the shock wave is in front of the cowl, no changes occur in the total pressure factor and mass flow rate. When the cowl is moved forward an expansion fan is created where the Mach number is higher and thus the shock losses in total pressure are increased. And obviously narrowing the inflow gap decreases the amount of mass flow entering the internal channel of the intake.

The static pressure sharply increases when the shock wave is behind the cowl lip. This is once again due to the singular behavior with very low boundary layer momentum thickness, increased with the higher Mach number just behind the cowl lip. It must be stated once again that the applicability of the shock train relation in this situation is questionable, due to very thin (laminar) boundary layers and the highly disturbed flow field. Shifting the cowl in the downstream direction there is a slight decrease in the obtained static pressure. This can be attributed to thicker boundary layers due to a longer running length of the compression surface.

Mach Number

The effect of the Mach number on intake performance is the last test performed with the flat compression surface. In order to capture the effect of only the Mach number, the cowl is placed such that the shockwave is just in front of the cowl lip. The results obtained during this study are presented in Table E.8 and Figure 5.8. There is a trend of an increase in the static pressure ratio that can be obtained with increased Mach number, though this increase is not monotonic and seems to level off above Mach 2.5. A monotonic decrease is present in capture height, roughly 15 %, when the Mach number increases from 2 to 3. A more significant decrease is present in the total pressure recovery ratio. While any intake performs worse at higher Mach numbers due to increased shock losses, the submerged intake loses even more performance due to the additional acceleration. Compared to a pitot intake, also incorporating the 10 % viscous loss rule-of-thumb, this submerged intake loses approximately another quarter of attainable total pressure at Mach 3.



Figure 5.7: The effect of the cowl lip position with respect to the shock wave on the obtained static and total pressure



Figure 5.8: The effect of Mach number on static pressure and net capture height (left) and total pressure recovery (right)

Boundary Layer Transition Location

As has been mentioned the effect of the transition location affects the static pressure obtained by the shock train. Table E.9 shows the result of varying the length until the boundary layer transition. 5 cm is the 'standard' value used in the rest of this chapter. The singularity behavior observed in Figure 5.2 is stronger when the boundary layer remains laminar for longer distances, resulting in a strong increase between transition distance 5 and 6 cm. The change in the pressure ratio that can be obtained at the cowl boundary layer transition location is much smaller. If the distance until transition is increased from 2 to 6 cm, the pressure reduction is only 6 %, which can in part be attributed to a shorter remaining length of the isolator. The net capture height is unaffected while total pressure is slightly decreased with a longer distance until transition. A thicker displacement thickness, due to early transition, causes slight additional compression resulting in this higher total pressure.

5.3 Two-shocks Compression Surface

The 2-shocks compression surface has geometric variables that can be optimized for better performance. We can vary the angle of the initial surface in order to reduce the shock strengths and associated shock losses. Also the length of the initial section can be changed. After these effect have investigated, we also look at the performance in higher Mach numbers and varying cowl positions.

Angle Initial Section

Table E.10 and Figure 5.9 show the effect of varying the angle of the initial section on intake performance. In all cases the cowl was placed just behind the second shock wave. All performance properties peak when the shock waves have nearly equal strength. Maximum mass flow and total pressure were obtained with the first plate at -7 degrees. Both of these quantities are also higher than for the single shock compression. Mass flow is higher because in the current design and test Mach number there is some distance between the shock waves near the cowl. This area with downward flow directs more flow into the internal part of the intake. The increase in the obtained total pressure recovery is because multiple weak shocks result in a lower loss of total pressure. It has been known since the 1940s that the best results would be obtained via a series of equal strength shocks [51], so it is encouraging that the model predicts a similar result. The gain in total pressure with respect to the flat compression surface is 9 %. Static pressure also peaks for -7 degrees. The singular behavior covered in the previous section does not occur strongly in this case, nor in in the remainder of this chapter and will hence be omitted in subsequent tables. The approximate 10 % difference between the static pressure ratios can be attributed to boundary layer growth and effectively a shorter isolator.

Length Initial Section

The length of the initial surface is the second geometric parameter related to the 2-shocks compression surface. As can be seen in Table E.11, this length appears insignificant with respect to intake performance. It must be emphasized that this is for two shock waves that do not interact with each other as shock wave interaction is not included in the model. There is a slight decrease in static pressure due to a slightly thicker boundary layer on the compression surface, which affects the pressure rise via the shock train.

Cowl Position

Table E.12 summarizes the effects of the cowl position on intake performance. For both shock waves in front of the cowl lip, it can be observed that total pressure recovery and capture height are (practically) constant. This is identical behavior to the flat compression surface case. A slight change in static pressure can be covered by the thickening of the boundary layer due to a longer buildup. When the cowl lip is in front of the shock wave, and thus causes an expansion fan, a part of the flow experiences a stronger shock wave due to a



Figure 5.9: The effect of the first section angle on the obtained static and total pressure

higher local Mach number. As a result the total pressure recovery reduces, though only with about 2 percent in the most extreme case. As the expansion at the cowl lip is only 5 degrees, the effect on total pressure is smaller than in the single shock compression. A similar slight reduction can be observed in the captured mass flow. A stronger effect , a rise of approximately 10 %, is obtained in the static pressure ratio. This is because the reflection of the shock wave on the cowl results in a significantly larger static pressure at the start of the shock train. As the shock train is affected by the pressure ratio this pressure rise is amplified, despite the thicker boundary layer due to shock wave boundary layer interaction.

Mach Number

Table E.13 summarizes the effect of free stream Mach number on the intake performance. It shows a similar strong reduction in total pressure recovery as was seen for the flat compression surface case, though always 10-20 % higher due to compression via 2 weaker shocks. Also capture height follows the same trend direction observed earlier, with a monotonic decrease when the free stream Mach number increases. The decrease is less severe though, with only 5 % compared to the 15 % decrease in the flat compression case. There is a notable rise in the static pressure ratio with increasing Mach number. The stronger shock waves due to a higher Mach number are responsible for this.

Wind Tunnel vs. Atmospheric Flight

Table E.19 compares the intake performance in representative wind tunnel and atmospheric flight conditions. No effect on the total pressure recovery and the capture height is observed, as was expected because stagnation temperature and pressure do not affect the inviscid flow. The flight conditions do affect the static pressure ratio. As has been remarked in the section on the expansion ramp boundary layer, temperature has a strong effect on the boundary layer growth and viscosity. Both the viscosity and the momentum thickness of the boundary layer have a strong effect on the pressure rise over the shock train. It appears that the stagnation pressure does not affect the shock train significantly, as the pressure ratio change is 7 % between the atmospheric conditions, which is close to the 6.5 % change between the wind tunnel simulations with constant stagnation pressure.

5.4 Gradual Compression

Gradual compression is the final possible design option for a compression surface that can be analyzed with the intake performance model. This compression surface has one primary design variable, the radius of the compression region.

Compression Radius

Table E.14 shows the effect of the compression radius on the performance of the intake, with the cowl lip fixed at x=0.21 m. The effect on the pressure recovery can be explained by the fact that with a larger compression radius the shock forms at a greater distance from the compression surface and thus a lower fraction of the air passes through the shock wave. Near the surface the compression is isentropic and thus the air enters the intake at free stream Mach number, and a higher Mach number results in higher total pressure loss further downstream in the intake. It is only a minor effect though, less that 3 % in the tested range of radii. A similar marginal effect is observed in the captured mass flow rate. The static pressure appears to peak at a radius of approximately 0.2 m. Presumably the effect of lower shock losses at longer compression radii are compensated by a thicker boundary layer due to a longer length prior to entry into the internal channel.

Mach Number

Table E.15 summarizes the effect of the Mach number on the performance of the gradual compression intake. The cowl lip was placed at x=0.22 m, just downstream of the shock wave at M=2.5. As the intake performance analysis model can not analyze an overspeeding gradual compression intake, no higher Mach numbers are reported in Table E.15. The total pressure obtained reduces with increasing Mach number, at a similar rate to the two-shock compression surface. The increase in static pressure is lower compared to the two-shock intake. Surprising is that the Mach number has no effect on the mass flow capture height, this may be related to the location of the cowl lip which is fixed and not translated to match the position of the shock wave as was done previously.

Cowl Position

Table E.16 presents data on the effect of the cowl position on the intake performance. None of the variables change by more than a few percent, but there is an increase in total pressure and captured mass flow when the cowl is placed further aft. This is a somewhat contradictory result as the extra air has passed through the shock wave and should therefore have lost some of its total pressure. Static pressure appears to peak near cowl lip position 0.215, presumably because the same effect that increases the total pressure boosts static pressure as well, until a thicker boundary layer on the compression surface results in a reduced pressure rise via the shock train.

5.5 Mixed Compression Intake

During the parametric study it was observed that the total and static pressure can be increased by an internal system of oblique shock waves. Though in the simplified geometry considered previously this can only be achieved by moving the cowl lip forward, which is accompanied by several drawbacks mentioned earlier in this chapter. This can be countered by an improved design of the cowl, which is sketched in Figure 5.10. The lip of the cowl is angled downward to create an oblique shock wave which will reflect in the internal channel of the intake. Eventually the cowl surface curves in order to form a constant area throat, this results


Figure 5.10: Sketch of the mixed compression intake

in an expansion fan. Each shock wave increases the static pressure and lowers the Mach number at lower losses in total pressure than via a normal shock wave or shock train.

The mixed compression intake is currently only implemented in combination with the two shock compression surface. This shape of the cowl can be described with three parameters. First there is the angle of the cowl lip, which strongly affects the strength of the shock wave formed. The second parameter is the length of the first part of the cowl, which affects the cross-sectional area of the isolator as well as the relative position of the shock waves and the expansion fan. Also the radius of the curve can be varied, which also has some influence on the isolator depth.

There was insufficient time to perform a complete analysis on the performance effects of the cowl shape. However the first data obtained showed that significant performance increase can be obtained with the adapted cowl shape. Due to the reduction in Mach number with very little total pressure loss, the attainable total pressure recovery increases from 0.626 to 0.739, an increase of 18%. A smaller internal channel also means that the isolator length/depth ratio is increased, if the total intake length remains unchanged, and thus the static pressure at the end of the isolator is higher as well.

Chapter 6

Conclusion and Recommendations

This thesis summarizes the work performed for developing a performance analysis model for a ramjet air intake submerged in a vehicle body. A study of relevant literature was performed to find suitable sub-models for various flow phenomena that are encountered in the intake. The intake performance analysis model was used to perform a parametric study to determine the effects of several design variables on the mass flow and pressure recovery obtained by the intake.

6.1 Conclusion

The literature review resulted in a number of sub-models that were suitable for implementation in the intake performance analysis model. Integral boundary layer techniques, one for laminar and one for turbulent boundary layers, were employed to determine the boundary layer growth. These models were shown to be able to produce accurate results, including the expansion region where some characteristics of the turbulent boundary layer can radically change.

The integral boundary layer techniques require the external, inviscid flowfield to impose boundary conditions in terms of velocity, temperature and pressure. The inviscid flowfield was computed via the Method of Characteristics, in a formulation that can incorporate rotational flow originating from non-uniform shock waves. An inverse Prandtl-Meyer function was used to increase the speed of the model, instead of interpolating from a tabulated data set.

Interactions between shock waves and a turbulent boundary layer are incorporated via a correlation that provides an indication of shock wave displacement due to the presence of the boundary layer. For laminar boundary layer interactions a model to estimate the interaction length was found, but this model could not be validated with the available data. For the case of a laminar shock wave boundary layer interaction just downstream of a cowl lip centered expansion a large and unpredictable separation bubble may be formed. The user of the intake performance analysis model is advised to avoid this situation.

The final sub-model required is a correlation that describes the pressure rise over the shock train, a system of SWBLIs caused by an imposed high pressure at the downstream end of the intake. This correlation accounts for the effects of Mach number, Reynolds number and the asymmetry between the boundary layers on the compression surface and the inside cowl wall.

The created intake performance analysis model was used to determine the performance impact of a number of design variables. As expected, the initial expansion is detrimental to the total pressure recovery at the combuster entrance. Total pressure recovery is improved by the use of (partly) isentropic compression or multiple oblique shock waves. These shock waves may arise from the compression surface or the cowl lip in a mixed compression design. The obtained static pressure is dominated by the effect of increased the isolator length, which allows for a longer pseudo-shock region. Finally, the mass flow through the internal channel was comparable to an external intake with identical cross-sectional area, after accounting for the flow diverted through the bleed slot.

6.2 Recommendations

A first step that must be taken is to perform the proposed validation test. This would affirm the accuracy of the complete intake performance analysis models, complementary to the validation of only the sub-models that has been performed.

As indicated at various points during the parametric study, sometimes contradictory results were obtained with respect to the total and static pressure, where the static pressure exceeded the total pressure. This highlights the problem that they are determined from an experimentally obtained correlation (static pressure) and a rule-of-thumb based on the performance of other intakes (for total pressure). Also it is questionable whether the shock train correlation can be applied to the flow perturbations, e.g. shock waves and/or expansions, that may be present in the intake internal channel. A description taking into account the actual flow situation situation would increase the accuracy of the intake performance model.

A good model for laminar SWBLI is also absent. While the laminar boundary layer is very thin, and thus the size of the interaction is usually limited. As has been stated, a very large separation bubble may be formed in the interaction between a cowl lip centered expansion, a shock wave and a laminar boundary layer. It is now impossible to predict the existence and size of this bubble, due to an absence of sub-models related to this case.

Another point of recommendation is to further develop the intake performance analysis model to be able to analyze more types of geometry, most notably in the form of further implementation of the mixed compression intake. The first results obtained for the mixed compression intake were very promising and optimization could further improve performance. The continued development should also be directed to improve the stability and user friendliness of the model. Improved stability refers to the problem that errors do occasionally occur, resulting in a terminated run of the model. Especially the analysis of intakes with a gradual compression surface is complicated by this issue. The user friendliness could be improved, for example, by using a graphical user interface to determine test geometry and conditions instead of editing values in multiple scripts. While this works for the developer of the model, the learning curve for a new user might be too steep, despite the included documentation.

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Appendix A

Model Flowcharts

In this appendix the inner workings of the intake model will be discussed in more detail than performed in Chapter 4. First the general structure is covered before some individual functions and scripts are detailed.

Figures A.1 through A.5 represent the structure of various parts of the model. In order to indicate the type of files scripts are denoted with rectangles around the script name and functions are encircled. Additionally there are rectangles with a dashed line, which indicate an action that is not covered by a function or a script.

Figure A.1 shows how the main script, named *Skeleton.m*, is organized. There are two scripts that are called early on. These are *input.m* and *geometry.m*. In *input.m* the flight or test conditions are specified. This includes the properties of the gas, such as γ , Prandtl number and the specific gasconstant, the free stream Mach number and associated stagnation pressure and temperature. Additionally input information required is the boundary layer momentum thickness at the start of the computational domain, and the distance from the leading edge at which the boundary layer transitions to a turbulent state. In *geometry.m* the geometry of the intake is created from a number of parameters. These include (but not limited to) the angles of the expansion ramp and cowl and compression plate, depth of the compression plate within the projectile body, radii of the gradual expansion and compression curves.

In the next stage of the model one of two scripts is activated, depending on the geometry of the expansion ramp. The two scripts, *expansion_sharp.m* and *expansion_gradual.m*, are nearly identical, as can be seen in Figure A.2. They only differ in the respect that in the case of a "sharp" expansion a centered fan is created. This required a slightly different structure of the Method of Characteristics function, and a way to deal with the factor $\frac{dM}{dx}$ in the boundary layer solution. A boundary layer thickness was used as a length scale to determine the Mach number gradient, which yielded acceptable results.

A similar construction was used for the flow in the compression region. This is depicted in Figures A.3 and A.4. These figures are much more complex than their counterpart for the expansion region. Due to interaction with the cowl lip multiple outcomes can occur. In *compression_singleshock.m* and *compression_multishock.m* shock waves originate from the lip of the compression surface and the kink on the surface for the multishock case. When the final shock falls behind the cowl lip, an expansion centered at the cowl lip is created. This expansion causes a deflection of the shock wave, which is computed in *shocktrace_type3.m*. The flow field behind this curved shock can be computed with MoC_comp_A , with iteration with the boundary layer in this section. Additionally the boundary layer inside the cowl is computed between the cowl lip and the point where the shock wave reaches the cowl. This is the start point of the internal block structure, with in this case internally reflecting shock waves.

When the shock waves defining the start of region A (see Chapter 4) pass in front of the cowl lip, region



Figure A.1: Flowchart of main script Skeleton.m



Figure A.2: Flowcharts of the scripts used to compute the expansion region

A encompasses the location of the cowl lip. Depending on the alignment of the flow and the cowl lip an expansion fan or shock wave may be created. This part of the scripts is represented in Figure A.4.

Finally there is $compression_gradual.m$, which differs slightly from the others because the function $MoC_comp_gradual.m$ determines both the shock wave shape and the flow behind it. When the shock wave passes in front of the cowl lip, this is no problem. However an interaction between this shock wave and the expansion fan of the cowl lip is too complex to implement in the current version of the model. As a result an error will be displayed when this occurs.

All of the compression scripts end with a reference to 'internal blocks'. This is not a separate script or function, but are an order of function calling that is used to calculate the internal flow of the intake. They all follow the procedure outlined in Chapter 4, and is repeated here in Figure A.5 with all function names.

Functions

MoC_exp.m, **MoC_exp_sharp.m** These functions compute the flow field around the expansion ramp. Two different functions are required as the centered expansion requires that extra characteristics are implemented.

BL_expansion.m, BL_expansion_sharp.m The boundary layer also requires a slightly different treatment as the sharp expansion is characterized with a infinite Mach gradient. A solution was found by using a boundary layer thickness as a characteristic length.

shocktrace.m, **shocktrace_type3.m**, **shocktrace_internal.m** In these functions the locations of shock waves are determined along with the flow properties behind the shock wave, which serve as the start of the subsequent method of characteristics section. 3 functions are required because of the different conditions in which they are applied. *shocktrace.m* is used to determine the shock wave(s) originating from the shape of the compression surface. *shocktrace_type3.m* determines the curved shock that is created by the interaction with a cowl lip centered expansion. Finally *shocktrace_internal.m* is used for shock waves in the internal channel. Where the first two functions assume that the shock has a constant deflection, as discussed in Chapter 4, in *shocktrace_internal.m* the deflection angle is constantly re-evaluated. This is necessary to determine the interaction between the shock and an expansion of the same family.

MoC_comp_A.m, MoC_cowllip_exp.m, compression_gradual.m These functions are used in the early stage of the compression phase. They are made specifically for certain flow situations, which were discussed earlier in this chapter.

BL_compression_start.m, **BL_continue.m**, **BL_postshock.m** The first of these functions starts a laminar boundary layer and predicts its growth. In this same function the boundary layer transitions to a turbulent state after a specified distance. $BL_continue.m$ is used to continue the boundary layer in a subsequent flow region unless these regions are separated by a shock wave. In that case $BL_postshock.m$ must be used.



Figure A.3: Structure of the scripts that compute flow in the compression region, part 1 of 2



Figure A.4: Structure of the scripts that compute flow in the compression region, part 2 of 2



Figure A.5: Flowchart of calculation of the internal flow

Appendix B Validation Tests

While the sub-models have been validated as much as possible, limited by available data in literature, the full integrated model must be validated as well. Preparations were made to perform a test in the ST-15 tunnel in the High Speed Laboratory of the Delft University of Technology. Due to the long lead time before the wind tunnel model was available, the validation tests could not be performed within the framework of this thesis project. The lead time consisted of a period in which the geometry of the model was determined, for which a first rudimentary version of the model had to be developed. This required reliable assessment of boundary layer thickness to size the bleed slot, as well as a determination of the shock wave locations. The second part of the lead time consisted of the detailed design phase for integration of pressure sensors and design of support structures for placement in the wind tunnel.

In this chapter the designed wind tunnel model elements are presented, as well as the proposed test schedule and measuring techniques.

B.1 Wind Tunnel Model

The wind tunnel model consists of 3 separate parts (not including the required supports): a ramp over which the flow expands downward, a flat plate on the top of the internal channel, known as the intake cowl. Finally there is the compression surface, where the flow is deflected in the direction of the internal channel and subsequent combustion chamber. 3 compression surfaces were designed; a flat plate yielding a single shock compression ramp, a a two-shock compression ramp in order to reduce pressure losses by using weaker shocks. The final compression surface has a curved section in the front, again in order to reduce pressure losses, this time by isentropic compression of least part of the incoming flow. Figure B.1 shows how the parts of the model are placed relative to each other. Not shown is the required support structure to hold the model in place and free from vibrations when installed in the ST-15 wind tunnel.

The dimensions in Figure B.1 were determined by considering the total wind tunnel test section length, approximately 40 cm, and the area of optical access. With the dimensions as shown, the aft part of the expansion ramp is within the window area to allow PIV measurements on the boundary layer. Also in the visible area are the leading edges of the compression surface and the cowl, as well as the upstream part of the internal channel to observe shock reflection on the cowl and compression surface. The bleed slot has been sized at 35 mm horizontal distance between the expansion ramp will be a few millimeters below the compression lip. This was determined for a Mach 2 test with an incoming boundary layer thickness of 5 mm, which is known from previous experience with the geometry upstream of the intake model. Due to the uncertainty associated in the boundary layer prediction it is desired that the compression surface can be moved a few centimeters downstream if the boundary layer is thicker than expected. Alternatively, the



Figure B.1: Schematic representation of the wind tunnel model, showing the three main components and the window of the wind tunnel

compression lip may be moved upstream in order to investigate the effect of the compression surface scooping up (a part of) the boundary layer. In the assembly presented in Figure B.1, the compression surface and the cowl were placed in such a way that the shock waves would fall on the cowl lip for the planned test condition. In order to emulate the effects of Mach numbers higher or lower than the design Mach number, the cowl can also be moved to place the shock wave behind or in front of the cowl lip. As the ST-15 tunnel utilizes nozzle blocks for either Mach 2 or Mach 3, it would not be possible to emulate effects of moderate overspeeding, at Mach 2.2 for example.

More detailed drawings of the individual components can be found in Appendix C.

B.2 Test Schedule

In this section various tests are proposed to validate both the sub-models as well as the complete submerged intake performance model.

It is wise to first perform PIV measurements on the expansion ramp, without a cowl or compression surface in place. Without these two elements present, the whole ramp can be illuminated by the laser. While there is no optical access to the first part of ramp, the growth and internal characteristics of the boundary layer can be tracked in the range 15-50 boundary layer thicknesses downstream of the onset of expansion. This will provide valuable data to validate the Sasman-Cresci boundary layer model in the expansion region.

For validation of the location and angle of the shock waves caused by the compression surface, Schlieren photography is the preferred choice. PIV could theoretically also be employed, though illumination and equipment placement will be complicated with the presence of the cowl and compression surface. Schlieren photography will also provide direct information on whether the internal channel of the model has supersonic flow and on the location of the cowl lip with respect to the leading edge shock wave. PIV could supply the same information, but only after processing of the images, which would be detrimental to the speed at which the tests can be performed.

SWBLI can, at least for the present application, be sufficiently validated via Schlieren as well, as it can be used to observe shock wave angle and viscous displacement. PIV could provide more detailed information in internal dynamics of the interaction, but this is not the focus of the validation test.

The final sub-model to be validated is the shocktrain, but this will be difficult due to the boundary layer present on the wind tunnel side walls. On the tunnel walls the boundary layer thickness is significantly larger than on the compression surface or the cowl. As the boundary layer momentum thickness is an important factor for the shocktrain length, it is possible that data obtained may not be suitable for validation of the computer model, which ignores the effect of side walls. The data obtained comes from pressure sensors placed along the center line of the wind tunnel models. Pressure sensors allows determining the start of the shock train and the pressure rise throughout the length of the isolator. A shock train is only created when a sufficiently high back pressure is present. This is usually achieved by restricting the outflow area of the isolator. The valve or plug element required to achieve this is not included in Figure B.1.

The wind tunnel model is designed to be integrated in a setup that was already available. This setup is designed to operate with the Mach 2 nozzle block of the ST-15 tunnel. As a result the test Mach number can not be varied. In order to represent the various conditions with shock waves impinging on the cowl lip, passing in front of it or being ingested, the cowl element must be moved with respect to the compression surface.

It would also be interesting to see what the effect would be of reducing the bleed slot width. If it would turn out that a part of the boundary layer from the expansion can be scooped up with no adverse effects, this would be very useful to know. A direct consequence would be that the channel diverting the bleed air can be reduced in size, freeing precious internal space.

Appendix C Wind Tunnel Model Drawings

A preliminary sketch of the wind tunnel model elements are presented here. The sketches are preliminary as they do not include elements required for supporting the model vibration free in the wind tunnel, nor the internal space needed for the pressure sensors. Also thicknesses of the cowl and compression surfaces, and the length of the expansion ramp are not fixed yet. The relevant dimensions that are fixed are indicated in Figures C.1 through C.4.



Figure C.1: Expansion ramp



Figure C.2: Flat compression surface



Figure C.3: 2-shock compression surface



Figure C.4: Gradual compression surface

Appendix D

Boundary Layer Transformation to Physical Coordinates

The boundary layer calculation methods that were covered in Chapter 3 calculate the momentum thickness in a transformed plane. In order to obtain the boundary layers thicknesses in the physical coordinates a transformation must be performed. It was shortly touched upon in Chapter 3 on how this transformation was performed, but as this is such a significant part of the boundary layer determination, additional coverage was deemed useful. First the Pohlhausen profiles, which are used for laminar boundary layers, are covered. This is followed by a short comment on the turbulent power law profile. Then the approach used to perform the inverse transformation is discussed. Finally some results showing the effect of the Mach number are presented.

Pohlhausen Laminar Velocity Profile

In the early 20th century a formulation of the velocity profile in the laminar boundary layer was created by Pohlhausen. It approximates the velocity profile with a fourth order polynomial function:

$$\frac{u}{u_e} = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4$$

In which η is the wall normal coordinate y/δ . In order to determine the coefficients a_i some boundary conditions are required. The first boundary conditions are obviously u = 0 at y = 0, which directly eliminates a_0 , and $u = u_e$ at $y = \delta$. A second set of boundary conditions is that the velocity profile matches smoothly with the external velocity. This requires that the derivatives $\frac{\partial u}{\partial y}$ and $\frac{\partial^2 u}{\partial y^2}$ are equal to zero at the edge of the boundary layer. Pohlhausen expressed a solution for a_i in terms of a single shape parameter λ :

$$a_1 = \frac{\lambda + 12}{6}, \quad a_2 = -\frac{\lambda}{2}, \quad a_3 = \frac{\lambda - 4}{2}, \quad a_4 = \frac{6 - \lambda}{6}$$

This shape parameter can have a value between -12 and 12, the former indicates the moment of separation, because it results in $\frac{\partial u}{\partial y} = 0$ at the wall. The limit $\lambda = 12$ indicates where the velocity in the boundary layer exceeds the velocity outside the boundary layer. λ can be correlated to a shape factor, incompressible or in a transformed plan, between 3.5 ($\lambda = -12$) and 2.25 ($\lambda = 12$). Incorporating the effects of compressibility is covered later in this chapter.

Turbulent Velocity Profile

The turbulent boundary layer model used employs a power law approximation, coupled to the shape factor. In the transformed plane the velocity profile is approximated as $\frac{U}{U_e} = \left(\frac{Y}{\Delta}\right)^{N_i}$, where the exponent follows

from Hi = 2Ni + 1.

Transformation into Physical Plane

The boundary layer models employed a transformation based on the density gradient in the boundary layer. This simplified the integral boundary layer layer equations, but an inverse transformation must be performed to retrieve the physical displacement thickness. The transformed y-coordinate, Y, is obtained via:

$$Y = \sqrt{T_e/T_0} \int_0^y \frac{\rho}{\rho_0}$$

And the transformed velocity is $U = u\sqrt{T_0/T_e}$. Using the velocity profiles, Pohlhausen for the laminar and the power law for the turbulent boundary layer, the transformed boundary layer characteristics Δ^* and Θ are determined in relation to Δ . With the velocity transformation a physical velocity can be obtained. This is used to determine the temperature, and with it the density, throughout the boundary layer. The temperature is not the stagnation temperature, due to the temperature gradient there is an outward transport of energy and at the wall the so-called adiabatic recovery temperature is obtained. Adiabatic because of the assumption that there is no heat transfer between the wall and boundary layer. The temperature can be determined from:

$$T = T_e + R \frac{u_e^2 - u^2}{2c_p}$$

Wherein R is the recovery factor that is $Pr^{1/3}$ for the turbulent boundary layer and $Pr^{1/2}$ for the laminar boundary layer. The wall temperature is thus slightly higher when the boundary layer is turbulent, because the turbulent properties deliver kinetic energy from high in the boundary layer rapidly towards the wall.

Temperature is used to determine density with the assumption of constant pressure through the boundary layer. This is then used to get the physical y coordinate by inverting the transformation. Now that the velocity and density as function of height are known, the physical displacement thickness and momentum thickness can be obtained via their well-known definitions.

The article describing the turbulent boundary layer method provided algebraic relations to obtain δ^* and θ directly, however it was found that δ^* was an order of magnitude larger than expected. For this reason this boundary layer reconstruction effort was made. It was found that the relation presented in Chapter 3 for θ was an exact match, demonstrating the reliability of the performed reconstruction.

For the reconstruction of the turbulent boundary layer initially the method of Reshotko and Tucker [56] was used. This method employs a more algebraic approach in reconstructing the boundary layer, however it was found that the steps outlined above produced identical identical results. The method employed here is simpler to implement and can, thanks to increases in computational power since 1957, produce boundary layer profiles on a representative set of Mach numbers and shape factors in seconds.

Results

Figures D.1 and D.2 depict typical results obtained by the inverse transformation. It can be observed that at very low Mach numbers both parameters, boundary layer thickness and shape factor, are identical to their transformed counterparts, as was to be expected because $\sqrt{T_e/T_0} = 1 = \frac{\rho}{\rho_0}$ for low speed flows. As the Mach number increases, the difference increases very rapidly. This is caused by the density ratio ρ/ρ_0 that causes deformation between the physical and transformed distance to the wall. The decreased density near the wall also increases the displacement and momentum thickness, due to the lower mass flow near the wall.



Figure D.1: Boundary layer thickness as function of incompressible shape factor and Mach number



Figure D.2: Boundary layer actual shape factor $(H = \delta^*/\theta)$ as function of Mach number, transformed shape factor $Hi = \Delta^*/\Theta = 1.3$

Appendix E

Parametric Study Data

Radius [m]	δ_2/δ_1	δ_2^*/δ_1^*	θ_2/θ_1	x ₂ [m]	Hi_2
0	1.7074	1.3922	1.1098	0.0802	1.1618
0.04	1.7044	1.3716	1.0983	0.0851	1.1618
0.08	1.7267	1.3844	1.1091	0.09	1.1611
0.12	1.7478	1.3945	1.1178	0.0949	1.1601
0.16	1.7685	1.4033	1.1256	0.0999	1.1589
0.2	1.7889	1.412	1.1333	0.1048	1.1578
0.24	1.8092	1.4204	1.1407	0.1097	1.1567
0.28	1.8293	1.4288	1.1482	0.1146	1.1557
0.32	1.8493	1.437	1.1555	0.1195	1.1546
0.36	1.8692	1.4453	1.1628	0.1244	1.1537
0.4	1.8889	1.4536	1.1701	0.1293	1.1527

Table E.1: Effect of expansion ramp turning radius on expansion ramp boundary layer

Angle [deg]	M ₂	δ_2/δ_1	δ_2^*/δ_1^*	$\theta_2/ heta_1$	x ₂	Hi_2
14	2.549	1.7889	1.4119	1.1333	0.1048	1.1578
12	2.463	1.6714	1.3651	1.1391	0.1151	1.1706
10	2.38	1.6251	1.3341	1.1565	0.1309	1.183
8	2.299	1.5984	1.3226	1.191	0.1563	1.1938
6	2.221	1.6317	1.3415	1.2562	0.2008	1.2012
4	2.144	1.7822	1.4255	1.3917	0.293	1.203
2	2.054	2.3008	1.7428	1.7937	0.5762	1.1994

Table E.2: Effect of expansion angle on the expansion ramp boundary layer

Table E.3: Effect of free stream Mach number on the expansion ramp boundary layer

M_1	M_2	δ_2/δ_1	δ_2^*/δ_1^*	θ_2/θ_1	Hi ₂
2	2.549	1.7889	1.4119	1.1333	1.1578
2.1	2.67	1.8465	1.4636	1.1654	1.1567
2.2	2.792	1.9045	1.5161	1.1974	1.1554
2.3	2.917	1.9635	1.5688	1.2288	1.1541
2.4	3.043	2.0223	1.6215	1.2596	1.1527
2.5	3.172	2.0828	1.6754	1.2901	1.1512
2.6	3.301	2.1442	1.7315	1.3206	1.1496
2.7	3.432	2.2068	1.7888	1.3510	1.1480
2.8	3.565	2.2706	1.8448	1.3812	1.1462
2.9	3.700	2.3358	1.8986	1.4105	1.1443
3	3.836	2.4021	1.9527	1.4392	1.1423

Table E.4: The effect of expansion angle on intake performance (flat compression surface)

Exp. angle [deg]	$\max p_2/p_{\infty}$	p_2/p_∞ at the location of cowl boundary layer transition	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
14	4.103	3.466	0.576	1.785
12	4.412	3.646	0.604	1.865
10	5.096	3.865	0.624	1.923
8	5.353	3.893	0.641	1.960
6	5.480	3.986	0.644	1.981
4	5.550	3.998	0.648	1.994

Table E.5: The effect of intake depth on intake performance (flat compression surface)

Depth [cm]	$\max p_2/p_{\infty}$	p_2/p_{∞} at the location of cowl boundary layer transition	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]	capture/depth
1	4.385	3.020	0.577	0.892	0.892
1.5	4.663	3.155	0.577	1.338	0.892
2	4.104	3.351	0.576	1.785	0.893
2.5	3.897	3.469	0.577	2.231	0.892
3	3.821	3.538	0.577	2.678	0.893
3.5	3.842	3.598	0.578	3.124	0.893
4	3.878	3.532	0.578	3.571	0.893

Isolator length/height	$\max p_2/p_{\infty}$	p_2/p_∞ at the location of cowl boundary layer transition	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
7.5	3.652	2.885	0.576	1.785
10	4.104	3.351	0.576	1.785
12.5	4.504	3.745	0.576	1.785
15	4.865	4.091	0.576	1.785
17.5	5.197	4.405	0.576	1.785
20	5.505	4.693	0.576	1.785

Table E.6: The effect of isolator length on intake performance (flat compression surface)

Table E.7: The effect of cowl	ip position on intake	performance (flat	compression surface)
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Displacement cowl [cm]	$\max p_2/p_{\infty}$	p_2/p_{∞} at the location of cowl boundary layer transition	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
-1	5.959	4.404	0.601	1.751
-0.6	5.957	4.103	0.623	1.878
-0.4	5.968	4.227	0.626	1.901
-0.2	6.073	4.367	0.628	1.923
design condition	5.247	3.718	0.631	1.944
0.2	5.154	3.692	0.631	1.944
0.4	4.613	3.668	0.631	1.944
0.6	4.485	3.646	0.631	1.943
0.8	4.399	3.627	0.631	1.943
1	4.315	3.606	0.631	1.943

Table E.8: The effect of free stream Mach number on intake performance (flat compression surface)

М	Cowl lip x [m]	$\max p_2/p_{\infty}$	p_2/p_∞ at the location of cowl	Net capture	$p_{0,2}/p_{0,22}$	p_0 loss compared
		$ \lim_{p \ge 1/p \propto}$	boundary layer transition	height [cm]	$P0,2/P0,\infty$	to pitot intake [%]
2	0.1911	4.104	3.351	1.785	0.576	11.22
2.1	0.1946	4.123	3.450	1.764	0.533	12.16
2.2	0.1982	4.187	3.540	1.741	0.491	13.14
2.3	0.2019	4.215	3.633	1.717	0.450	14.28
2.4	0.2056	4.235	3.697	1.691	0.410	15.65
2.5	0.2093	4.276	3.783	1.663	0.373	16.95
2.6	0.2131	4.252	3.806	1.634	0.338	18.38
2.7	0.2169	4.258	3.824	1.603	0.306	19.74
2.8	0.2208	4.269	3.855	1.570	0.276	21.27
2.9	0.2247	4.264	3.877	1.536	0.248	22.96
3	0.2287	4.253	3.901	1.501	0.225	23.85

Table E.9: The effect of transition location on intake performance (flat compression surface)

Xtr [cm]	$\max p_2/p_{\infty}$	p_2/p_{∞} at the location of cowl boundary layer transition	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
2	3.682	3.484	0.578	1.785
3	3.699	3.451	0.578	1.785
4	3.817	3.413	0.577	1.785
5	4.104	3.351	0.577	1.785
6	4.815	3.274	0.576	1.785

Initial section angle [deg]	$\max p_2/p_{\infty}$	p_2/p_{∞} at the location of cowl boundary layer transition	Net capture height [cm]	$p_{0,2}/p_{0,\infty}$
-11	3.494	3.192	1.886	0.616
-9	3.605	3.279	1.924	0.629
-7	3.662	3.320	1.936	0.633
-5	3.647	3.310	1.920	0.626
-3	3.629	3.260	1.880	0.612
-1	3.546	3.174	1.821	0.591

Table E.10: The effect of the initial section angle on 2-shock intake performance

Table E.11: The effect of initial section length on 2-shock intake performance

Section length [mm]	$\max p_2/p_{\infty}t$	Net capture height [cm]	$p_{0,2}/p_{0,\infty}$
30	3.647	1.920	0.627
35	3.647	1.920	0.626
40	3.639	1.920	0.627
45	3.633	1.919	0.627

Table E.12: The effect of cowl lip position for a 2-shock intake

Displacement cowl [cm]	$\max p_2/p_{\infty}$	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
1	3.567	0.626	1.915
0.8	3.585	0.626	1.916
0.6	3.604	0.626	1.917
0.4	3.624	0.626	1.918
0.2	3.645	0.626	1.919
0 [design condition]	3.667	0.627	1.920
-0.2 [shockwave behind cowl]	4.076	0.624	1.910
-0.4	4.080	0.623	1.902
-0.6	4.083	0.621	1.895
-0.8	4.081	0.62	1.887
-1	4.078	0.618	1.880
-1.2	4.073	0.617	1.872
-1.4	4.068	0.615	1.865
-1.6	4.062	0.613	1.857

Table E.13: The effect of free stream Mach number on the 2-shock intake

Mach number	$\max p_2/p_{\infty}$	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
2	3.457	0.631	1.912
2.1	3.697	0.590	1.909
2.2	3.859	0.548	1.901
2.3	4.023	0.508	1.894
2.4	4.151	0.469	1.88
2.5	4.311	0.432	1.869
2.6	4.428	0.397	1.857
2.7	4.526	0.364	1.843
2.8	4.612	0.333	1.828
2.9	4.750	0.305	1.815
3	4.893	0.279	1.800

Radius	$\max p_2/p_{\infty}$	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
0.12	3.582	0.664	2.058
0.14	3.600	0.662	2.051
0.16	3.636	0.66	2.044
0.18	3.724	0.659	2.035
0.2	3.733	0.656	2.023
0.22	3.648	0.654	2.010
0.24	3.660	0.652	2.000
0.26	3.613	0.65	1.993
0.28	3.598	0.65	1.989
0.3	3.578	0.649	1.986

Table E.14: Effect of radius on the gradual compression intake

Table E.15: Effect of free stream Mach number on the gradual compression intake

Mach number	$\max p_2/p_{\infty}$	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
2	3.734	0.662	2.046
2.1	3.994	0.621	2.054
2.2	3.986	0.580	2.058
2.3	4.075	0.538	2.053
2.4	4.213	0.498	2.051
2.5	4.335	0.462	2.057

Table E.16: Effect of cowl displacement on the performance of the gradual compression intake

Displacement cowl [cm]	$\max p_2/p_{\infty}$	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
design condition	3.696	0.652	2.005
0.5	3.679	0.654	2.012
1	3.735	0.656	2.023
1.5	3.769	0.659	2.033
2	3.736	0.662	2.046
2.5	3.696	0.666	2.059
3	3.703	0.669	2.072

Table E.17: The effect of momentum thickness Reynolds number on expansion ramp boundary layer

$Re_{\theta} \cdot 10^4$	δ_2/δ_1	δ_2^*/δ_1^*	θ_2/θ_1	Hi ₂
0.50	4.74	4.93	3.8	1.231
2.00	2.187	2	1.579	1.193
4.00	1.876	1.55	1.238	1.168
6.00	1.79	1.411	1.133	1.158
8.00	1.748	1.346	1.084	1.153
10.00	1.723	1.309	1.056	1.151
12.00	1.706	1.286	1.038	1.149

Flight altitude [m]	Re per unit length $\cdot 10^8$	$Re_{\theta} \cdot 10^4$	$T_0[K]$	$p_0[Mpa]$	δ_2/δ_1	δ_2^*/δ_1^*	$\theta_2/ heta_1$	Hi_2
0	0.4573	2.08	518.67	0.793	1.758	1.478	1.179	1.172
5000	0.2849	1.30	460.17	0.423	1.742	1.517	1.205	1.181
10000	0.16745	0.76	401.67	0.207	1.724	1.567	1.237	1.191
Wind tunnel	1.3187	6.00	280	1	1.79	1.411	1.133	1.158
Wind tunnel	0.809	3.68	400	1	1.7755	1.439	1.153	1.164
Wind tunnel	0.5749	2.62	520	1	1.765	1.462	1.168	1.1685

Table E.18: Comparison between wind tunnel and atmospheric flight on the expansion ramp boundary layer

Table E.19: Comparison between wind tunnel and atmospheric flight on the performance of the 2-shock intake

Flight altitude [m]	Re per unit length $\cdot 10^8$	$T_0[K]$	$p_0[Mpa]$	$\max p_2/p_{\infty}$	$p_{0,2}/p_{0,\infty}$	Net capture height [cm]
0	0.4573	518.67	0.793	3.434	0.627	1.918
5000	0.2849	460.17	0.423	3.315	0.628	1.916
10000	0.16745	401.67	0.207	3.192	0.629	1.916
windtunnel	1.3187	280	1	3.727	0.626	1.92
windtunnel	0.8086	400	1	3.581	0.627	1.919
windtunnel	0.5749	520	1	3.486	0.627	1.918