Technische Hogeschool Delft

Afdeling der Weg- en Waterbouwkunde

COASTAL ENGINEERING

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1970

LECTURE NOTES ON

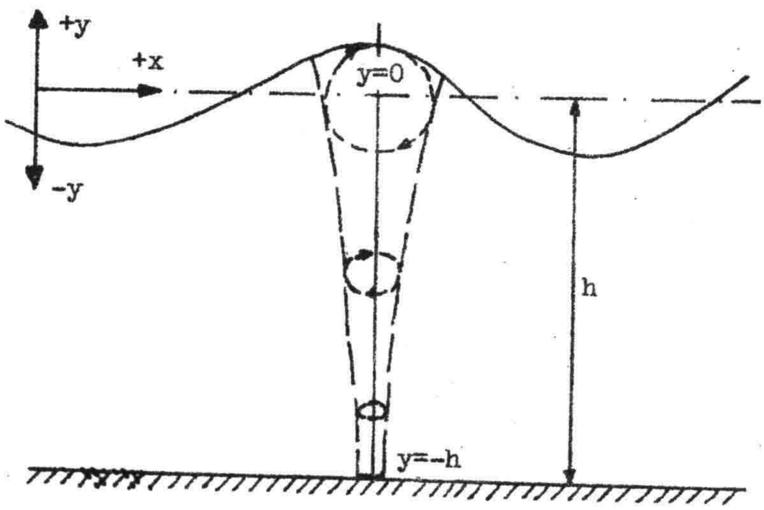
C O A S T A L E N G I N E E R I N G

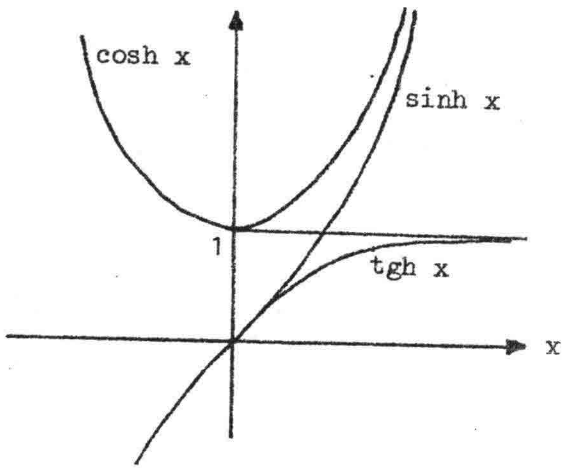
(Prof. dr ir E. W. Bijker)

Coastal Engineering

Under this heading the physical phenomena occurring along a coast and in estuaries will be discussed.
Information on short waves can be found in the lectures on this subject. Some information necessary for understanding this subject will be given here.

The water motion in a progressive wave takes place in closed or almost closed orbits (orbital motion).





Near the surface the orbits are circles and remain so - exponentially decreasing radii - at greater depths when the depth is unlimited. For shallow water the circular orbits change into ellipses.
The horizontal (u) and vertical (v) velocity of the water particles can be written as

$$u = \frac{\omega H}{2} \; \frac{\cosh k(y+h)}{\sinh k h} \; \sin \; (k x - \omega t)$$

$$v = \frac{-\omega H}{2} \; \frac{\sinh k(y+h)}{\sinh k h} \; \cos \; (k x - \omega t)$$

in which the phase velocity $\omega = 2\pi/T$, T = wave period
k = wave number $= 2\pi/L$, L = wave length
H = wave height, h = depth, x and y are coordinates of the point in the zero position.

The axes of the ellipses or the radii of the circles are

$$\text{Hor.:}\ \frac{H}{2} \cdot \frac{\cosh k(y+h)}{\sinh kh}$$

$$\text{Vert.:}\ \frac{H}{2} \cdot \frac{\sinh k(y+h)}{\sinh kh}$$

Between celerity (velocity) of wave propagation(c), wavelength (L) and wave period the following relationships exist:

$$c = L/T = \omega/k$$

From the basic wave theory it follows that

$$c = \sqrt{\frac{g}{k} \cdot \text{tgh}\ kh}$$

For deep water this becomes

$$c = \sqrt{g/k}$$

and for shallow water

$$c = \sqrt{gh}$$

For deep water the following relationships can be written:

$$c^2 = g/k = \frac{\omega^2}{k^2}$$

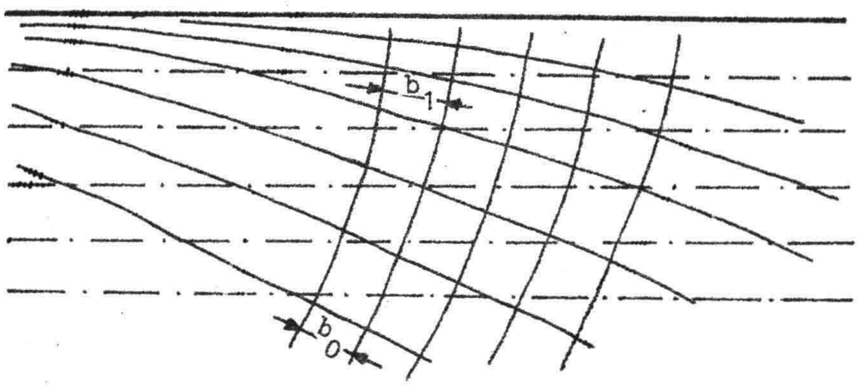
$$\frac{2\pi}{L} = k = \frac{\omega^2}{g} = \frac{(2\pi)^2}{g\ T^2}$$

$$L = \frac{g}{2\pi} \cdot T^2 = 1.56\ T^2$$

$$c = 1.56\ T$$

When the depth is $\frac{1}{2}$ L, $kh = \pi$, and tgh 0.996.
In this case the deviation of c from the deep water value is 0.2 % , which can be ignored.

When a wave train obliquely approaches a coast with decreasing depth ,the wave crests will tend to turn to a direction parallel to the depth contours due to the fact that in shallower water the wavelength is shorter and the celerity of propagation is smaller. This phenomenon is called refraction. The principle of refraction is that the energy of the waves is transported in a direction perpendicular to the wave crests, along the so-called wave orthogonals.



The wave energy per unit of surface can be written as

$$E = 1/8 \ \rho \ g \ H^2, \text{ and}$$

$$E_o/E_1 = b_1/b_o \ , \quad H_1/H_o = \sqrt{b_o/b_1}$$

$\sqrt{b_o/b_1}$ is called the refraction coefficient.

The subscript o indicates deep water, and the subscript 1 indicates the required depth.

When a wave approaches shallow water another phenomenon also occurs.

Due to the continuity the energy flux has to remain constant, so:

$$E_o \cdot c_{go} = E_1 \cdot c_{g1} \quad ,$$

where c_{go} is the velocity of energy propagation in deep water.

$$c_g = m\,c = \tfrac{1}{2}\left[\ 1 + \frac{2\ kh}{\sinh\ 2\ kh}\right] c.$$

So from this follows

$$E_o\ m_o\ c_o = E_1\ m_1\ c_1$$

For deep water $m_o = \frac{1}{2}$, and with this the following relationship is obtained.

$$E_o\ \tfrac{1}{2}\ c_o = E_1\ m_1\ c_1 \text{ , and}$$

$$H_1/H_o = \sqrt{\frac{1}{2}\ \frac{1}{m_1}\ \frac{c_o}{c_1}}$$

The total decrease of a wave approaching a coast obliquely is therefore

$$H_1/H_o = \sqrt{\tfrac{1}{2} \cdot 1/m_1 \cdot c_o/c_1 \quad b_o/b_1}$$

Behind a breakwater or barrier an attenuation of the waveheight occurs. This phenomena, in which energy flux parallel to the wave crest occurs, is called diffraction.

Beach formation

Since the orbital motion is not completely closed, mass transport results. Normally this mass transport is at the bottom directed in the direction of propagation of the waves, and this may result in a transport of bottom material in the direction of wave propagation. (However, this is not quite certain, since in laboratory flumes bedload transport against the direction of wave propagation also has been observed. This seems to be the case if the orbital excursion at the bottom is greater than the ripple length) Due to the transport of the bottom material beaches can be built up.

With regard to the building up of the beaches, the permeability of the beach is also rather important. When the beach is permeable the return flow is smaller and less material will be moved back towards the sea. This may result in much steeper beaches (shingle and coarse sand). Also the reflection will be less in this case. In order of magnitude the reflection of a shingle beach 1:6 will be equal to that of a sand beach 1:20.

When the wave approaches the coast, it will become steeper, and finally it will break or almost break. As these phenomena result in an asymmetric wave form, a mass transport will occur in the upper layers. This is very obvious for breaking waves. This water has to be brought back to sea. This can occur via longshore currents and rip currents and also via an undertow over the bottom. Due to this undertow, which occurs sometimes and due to gravitational effects a certain equilibrium steepness will be developed. The breaking waves can be classified into four different types.

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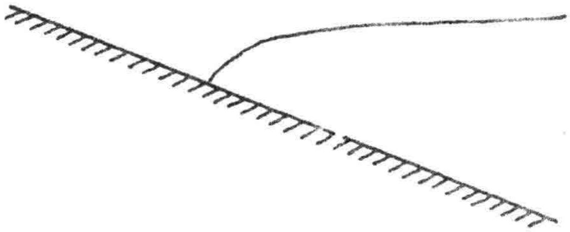
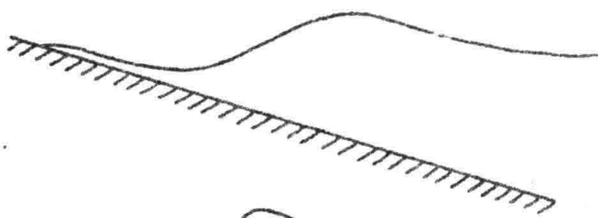
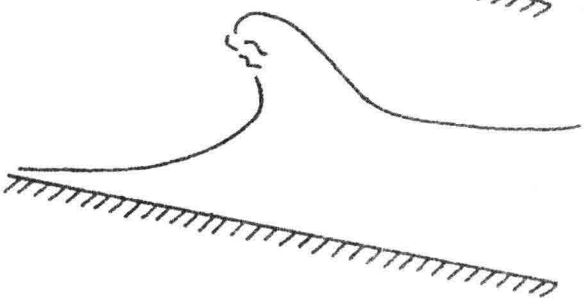


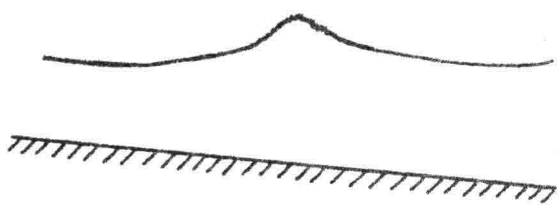
"surging breaker"



"collapsing breaker"



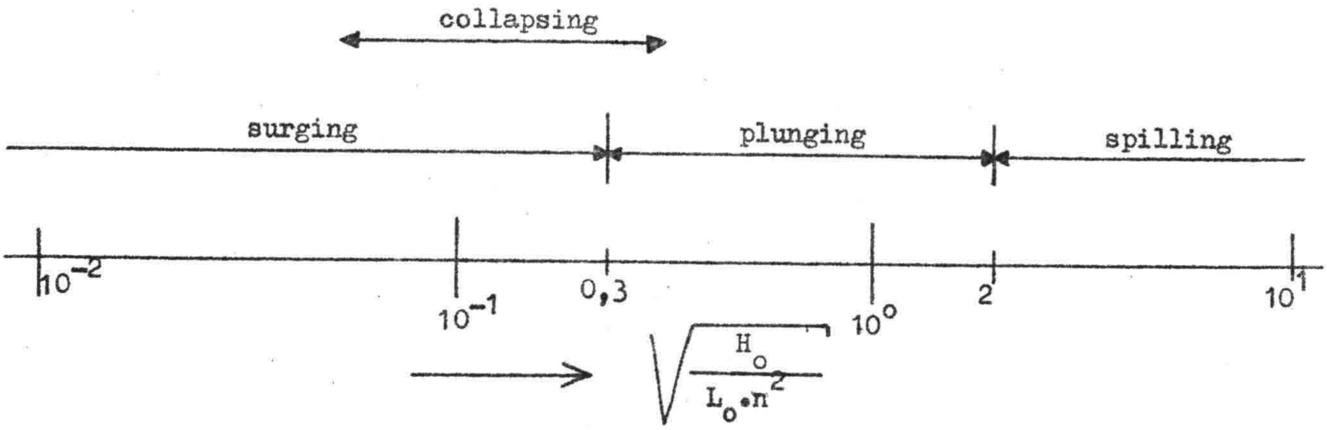
"plunging breaker"



"spilling breaker"

Galvin (C.E.R.C.) has developed several empirical criteria for the classification of the breakers.
(C.J. Galvin: Breaker Type Classification on three laboratory Beaches. Journal Geophysical Research, Vol.73,Nr.12,June 15, 1968, pp. 3651-3659) The various criteria as given by Galvin can be summarized as follows.

1) Outside the refraction zone:

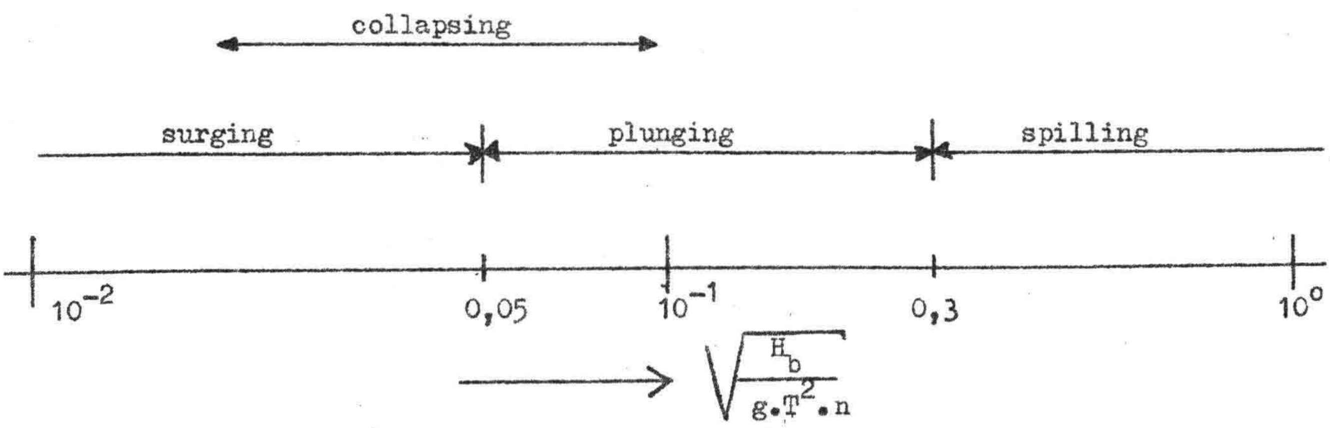


H_o = waveheight in deep water

L_o = wavelength in deep water

n = tangent of the slope

2) Inside the refraction zone:



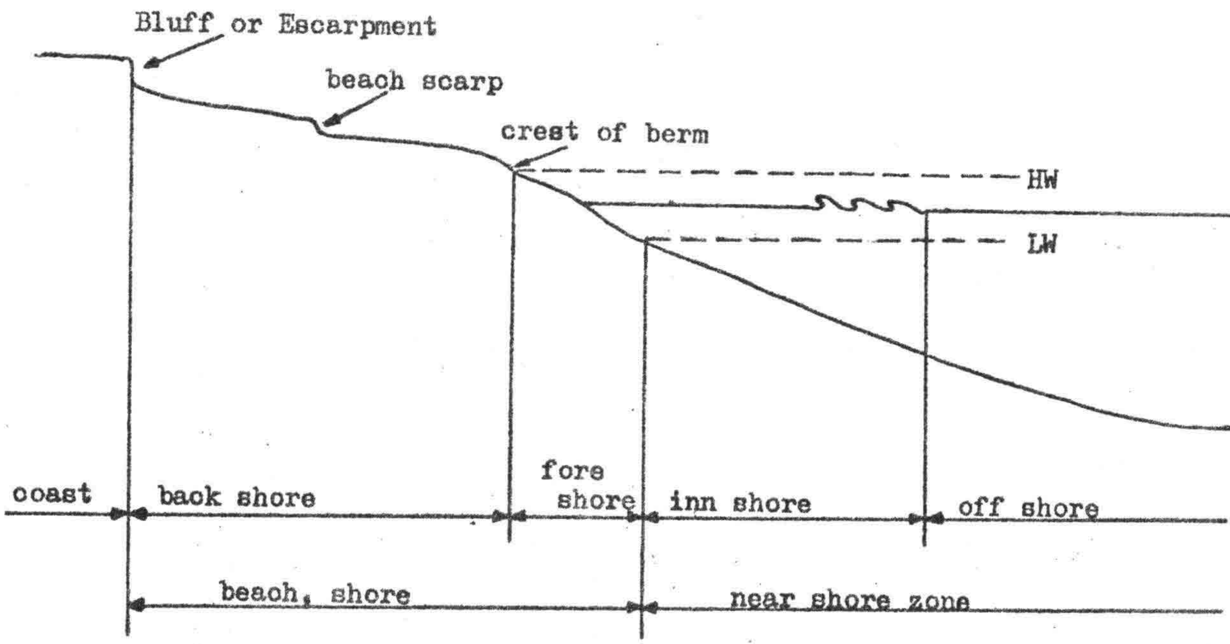
H_o = waveheight in deep water

L_o = wavelength in deep water

n = tangent of the slope

T = wave period

The nomenclature of a beach profile is given in the following sketch:



Due to the wave motion hitting a coast obliquely, a longshore current will be generated. The physical explanation of this longshore current can be threefold.

1) Due to mass transport of the waves (:: v)

2) Due to momentum transport of the waves (:: m v $\longrightarrow$ v^2)

3) Due to energy flux of the waves (:: $\frac{1}{2}$ mv^2 $\longrightarrow$ v^3).

The three approaches are identical in so far that there exists an obliquely directed transport to the coast that results in a longshore transport. This longshore transport would tend to an infinite value if there were not an escape to sea by rip currents or by an evenly distributed undertow. That the longshore current does not increase to infinite values is caused by the bed friction.

Eagleson derived a formula for this current. (P. Eagleson: Theoretical study of longshore currents on a plane beach, M.I.T. Dept. of Civil Engineering, Hydr. Lab., Report N82, 1965). It gives the development of this current with the distance along the coast.
Eagleson based his derivation on the momentum equations.
The ultimate value of this current according to Eagleson is:

$$v_L = \sqrt{\frac{3}{8} \; \frac{g\, H_b^2 m_b}{h_b} \; \frac{\sin\alpha \, \sin\varphi_b \, \sin 2\varphi_b}{f}}$$

in which v_L = value of longshore current velocity,
H_b = breakerheight, h_b = breakerdepth,
m_b = ratio of group velocity c_g to wave celerity =
$= \frac{1}{2}\left[1 + \frac{2\,kh}{\sinh 2\,kh}\right]$, α = beach slope, φ_b = angle of breaker crests and coast line, and f = Darcy Weisbach resistance coefficient = $8g/C^2$, where C = resistance coefficient according to deChezy.

Along sandy coasts this longshore current results in a littoral drift. The principle of this longshore movement of material is that the bed material is stirred up by the waves and transported by the current. Several attempts have been made to derive formulae to compute this littoral drift.

The C.E.R.C. has put all available data together and has come to the following (rough) formula: (Caldwell) :

$$S = 1.4\; 10^{-2}\, H_o^2\, c_o\, K^2 \sin\varphi_b \cos\varphi_b \; ,$$

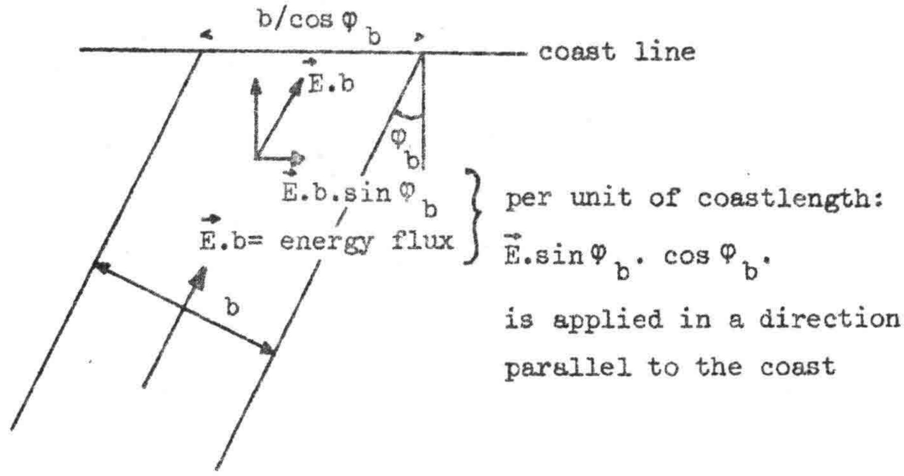
in which S = longshore transport in m^3/s,
H_o = waveheight in deep water in m,

c_o = wave celerity in deep water in m/s,

φ_b = angle of breking waves with coastline,

K = refraction coefficient.

This formula is based on the assumption of energy flux towards the coast which has a component parallel to the coast which is held responsible for the littoral drift.



In the C.E.R.C. formula no influence of grainsize and beach slope is taken into account. Bijker made an attempt to derive a formula where these factors were taken into account and in which it was also possible to include the influence of an extra tidal or sea current, not generated by waves. (E.W. Bijker, Littoral Drift as function of waves and current, Delft Hydr. Publ. Nr. 58, and Proc. Coastal Eng. Conf., London 1968)

Bijker started from the normally applied form of bedload tranpsort formulae:

$$\frac{S}{f(D^{3/2} g^{1/2} \Delta)} = g\left(\frac{\Delta D}{\mu h I}\right),$$

in which: S = transport, D = grainsize,

$\Delta = (\rho_s - \rho_w)/\rho_w$, μ = ripple coefficient =

$$= \left(\frac{C_o}{C_{D90}}\right)^{3/2}$$

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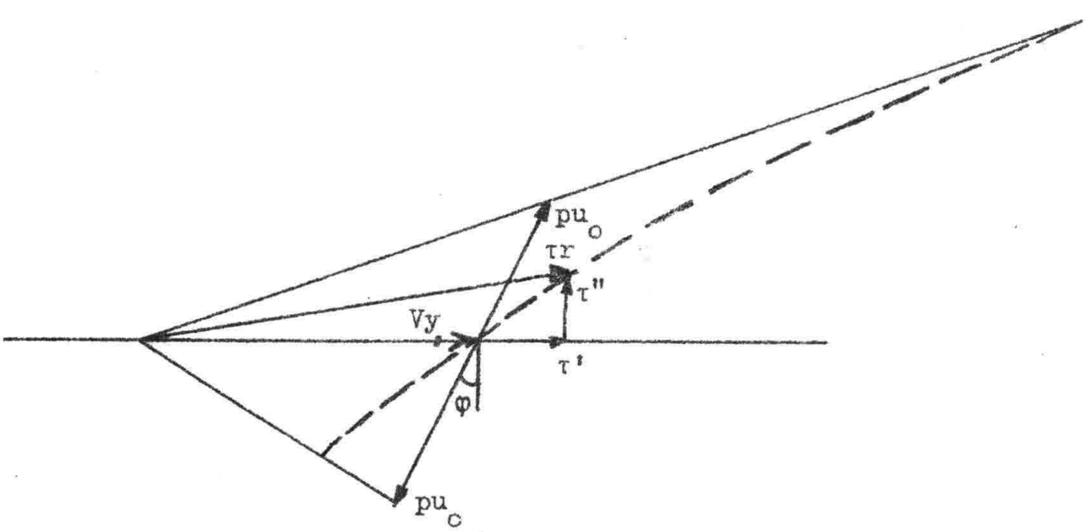
C_o = bed resistance coefficient, C_{D90} = resistance coefficient, due to bed roughness of D_{90},
h = depth and I = energy gradient.

Frijlink suggested writing this formula in the following way:

$$\frac{S}{D(\mu\tau/\rho)^{\frac{1}{2}}} = 5 \exp\left(-0{,}27 \frac{\Delta D \rho g}{\mu \tau}\right)$$

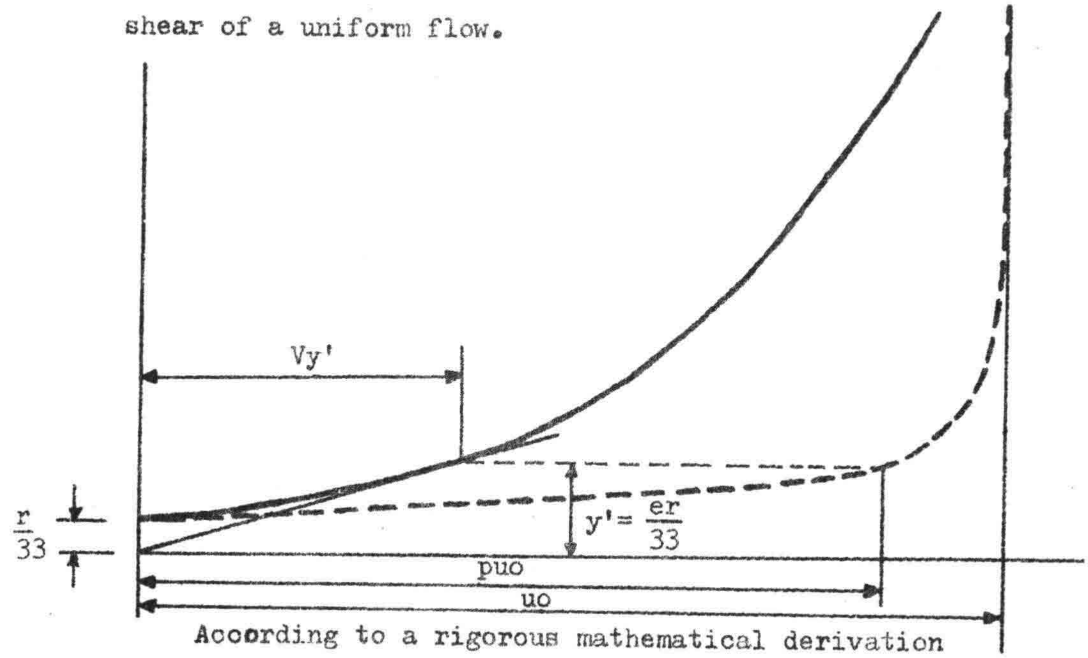
in which τ = bed shear = $\rho g hI = \rho g v^2/C^2$.

The factor before the = can be named the transport factor, and after the = the stirring factor.
For τ in the transport factor the bedshear of the normal longshore current will be introduced. This assumption is based upon the principle that the bed material, once stirred up by the combined action of waves and current is moved by the current.
For τ in the stirring factor, the bedshear resulting from the combined action will be introduced since it may be assumed that this bedshear is a measure for the turbulence. This combined bedshear is indicated on the sketch below.



From computations in which an elliptical integral has to be computed numerically, τ' can be expressed in the form $\tau_c = \rho g v^2/C^2$ and $p\,u_o$, where u_o is the amplitude of the orbital motion at the bed.

The coefficient p indicates the value of the orbital motion at a height of er/33 above the bed, which is a characteristic height for the computation of the bed-shear of a uniform flow.



According to a rigorous mathematical derivation p = f(t, r, H, T). With some simplifications p proves to be constant. From tests in which the increased bed-shear has been measured and after comparison with the computed values of τ', p has been found constant and equal to 0,45 within the limitations of the accuracy of the tests. From the approximate mathematical approach follows p = 0,4 , which seems to be in rather good agreement with the tests.

The resultant bedshear can now be written as

$$\tau_r = \left[1 + \tfrac{1}{2} (\xi \ u_o/v)^2 \right] \tau_c ,$$

in which $\xi = p\, K\, C/\sqrt{g} = 0{,}0575\, C$.

The formula for the bedload transport can now be written as:

$$S = 5\, D \sqrt{\tau_o/\rho}\, \exp(- 0{,}27\, \Delta\, D\, \rho\, g/\mu\, \tau_r)$$

or:

$$S = 5 \sqrt{g}\, D(v/C) \exp \left(- 0{,}27 \frac{\Delta\, D\, C^2}{\mu\, v^2 \left[1 + \frac{1}{2}(\xi\, u_o/v)^2 \right]} \right) .$$

This equation gives, however, only the bedload transport. Especially in these circumstances the transport of material in suspension will be rather important.
In order to compute this, the method as suggested by Einstein in 1950 will be used with some modifications.
Einstein computed the suspended load by integrating the product of velocity and concentration over the height:

$$S_s = \int_a^h v(y)\, c(y).dy$$

The value "a" is a distance above the bed at which the concentration must be known. Then c(y) may be written as:

$$c(y) = c_a \left(\frac{h-y}{y} \cdot \frac{a}{h-a}\right)^z$$

in which $z = w/\kappa v_*$, w = fall velocity of the grains and v_* = bedshear velocity = $\sqrt{\tau/\rho}$ = $v \sqrt{g}/C$,

The crucial point in this derivation is the value of c_a. Einstein determined this value by assuming that the bedload would be transported in a layer of a thickness of some grain diameters just above the bed. From films it became obvious that the original assumption of Einstein did not hold good.
Another assumption has been made now, viz. that the bedload is assumed to be transported in a layer equal to half the ripple height with the virtual bottom assumed half way between crest and trough of the ripples. When the bed roughness is assumed as half the height of the ripples the concentration c_a can be calculated with the assumption that the bedload is transported above the bed in a layer with thickness r - the bed roughness -.
The mean velocity in this layer can be computed to be $\bar{v}_{o-r} = 6.35\ v_*$, so the concentration

$$c_a = S_b/6.35\ v_*\ r.$$

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The formula for the suspended load according to Einstein is now:

$$S_s = 1.83\, S_b\, (I_1 \ln 33h/r + I_2) \quad ,$$

in which

$$I_1 = 0.216\, \frac{(a/h)^{z-1}}{(1-a/h)^z} \int_{a/h}^{1} \left(\frac{1-y/h}{y/h}\right)^z d(y/h)$$

$$I_2 = 0.216\, \frac{(a/h)^{z-1}}{(1-a/h)^z} \int_{a/h}^{1} \left(\frac{1-y/h}{y/h}\right)^z \ln(y/h)\, d(y/h)$$

The integrals are computed by Einstein and given in graph form in his original paper.

It is not certain that this procedure is the best. The fact that a very small bed roughness gives very high values of the suspended load due to the high concentration of c_a gives reason for suspicion. However, results from model tests, and prototype could be computed with this approach with a reasonable degree of accuracy.

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When a littoral drift is interrupted by a mole the beach updrift of this obstruction will accrete. Due to the fact that the littoral drift is interrupted, the coast downdrift of the mole will be eroded.
This erosion is sometimes just as dangerous as the accretion at the other side of the mole or moles.
The form of the beach at both sides can be computed in an approximate way. In order to do this the equations for the movement of the beach have to be derived. To this end two formulae can be used.

1) The C.E.R.C.-formula.
   This formula, $S = 1.4\ 10^{-2} H_o^{\ 2} c_o K^2 \sin\varphi_b \cos\varphi_b$,

can for small values of φ_b be written as

$$S = f(\varphi_b).$$

So $q = dS/d\varphi = S/\varphi$.

2) The more complicated formula of Bijker.
   In this formula,

$$S = 5D \frac{v}{C} \sqrt{g}\ \exp\left\{ -0{,}27 \frac{\Delta\ DC^2}{\mu v^2 \left[1 + \frac{1}{2}\left(\xi \frac{u_o}{v}\right)^2 \right]} \right\}$$

S is for reasonable great value of the transport about proportional with the square of the current.
The longshore current v_L, as generated by the waves can be written as

$$v_L = \sqrt{\frac{3}{8} \frac{g H_b^{\ 2} m_b}{h_b} \cdot \frac{\sin\alpha \ \sin\varphi_b \ \sin 2\varphi_b}{f}}$$

Also in this case the transport can be written for small values of φ_b as $S = f(\varphi_b)$.
So with reasonable approximation also in this case the coastal constant $q = dS/d\varphi$ equals S/φ .
It is of course possible to compute the coastal constant q exactly for any relationship between S and φ.
However, for the following procedure to be discussed the coastal constant should be known as a relatively simple

function of φ.

The littoral drift at any place along the coast can be written as

$$S_x = S_o - q \, {}^{dy}/dx \quad ,$$

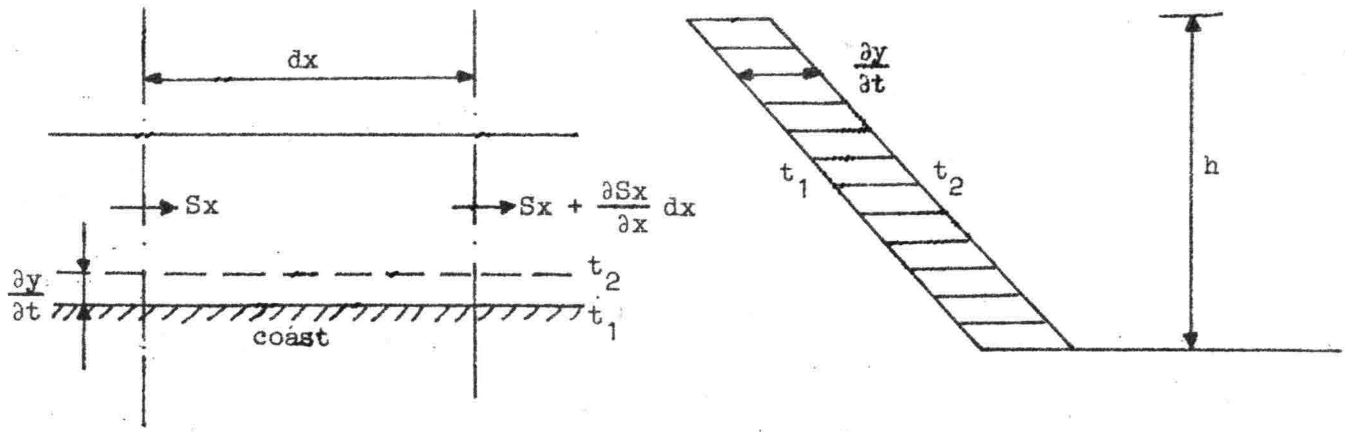
in which S_x = littoral drift at place x, S_o = littoral drift at the undisturbed coast, q= coastal constant and dy/dx = direction of changed coastline with the original coastline as reference.

For the equilibrium in a direction perpendicular on the coast a comparable formula can be written in the form

$$S_y = q_y \, d_\alpha \, ,$$

in which S_y is the off or onshore transport, q_y = dS/dα is the coastal constant for offshore or onshore transport and α = the angle between the actual and equilibrium slope of the innshore.
The approximate method discussed here is of Peluard Considère (R. Peluard-Considère: Essai de Théorie de l'Evolution des Formes de Rivages en Plages de sable et de galets: Quatrième Journées de l'Hydraulique, Paris 13-15 Juin 1954. Les Energie de la Mer, Question III).
In this theory it is assumed that the coast moves forwards with the equilibrium profile. Bakker gives a more sophisticated discussion of the problem in which it is not necessary that the beach grows as an equilibrium profile.
(W.T.J.N.P. Bakker: The dynamics of a coast with a groyne system, Ch. 31, Vol. 1; Proc. 11th Coastal Engineering Conference, London, 1968).

For the computation of the form of the accreting beach updrift of an obstruction, two equations are required, viz the equation of motion (transport equation) and the equation of continuity.



The equation of continuity can be written as

$$(S_x + \frac{\partial S_x}{\partial x} dx)dt - S_x dt = + \frac{\partial y}{\partial t} \cdot h.dx.dt.$$

$$\frac{\partial S_x}{\partial x} - \frac{\partial y}{\partial t} \cdot h = 0$$

The equation of motion is

$$S_x = So - q \quad dy/dx = So(1 - \frac{dy}{dx} / \varphi),$$

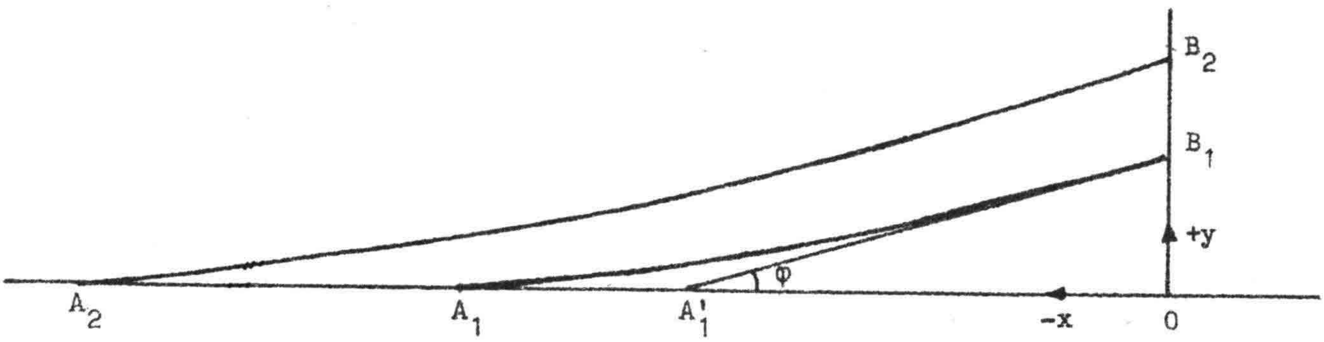
since in this case it is assumed that $q = \partial S/\partial\varphi = So/\varphi$

A combination of the equations of motion and continuity gives us

$$\frac{\partial^2 y}{\partial x^2} = \frac{\varphi h}{S_o} \cdot \frac{\partial y}{\partial t} = \frac{1}{a} \frac{\partial y}{\partial t} ,$$

So : $a = So/\varphi \ h = q/h.$

The coast moves forwards as indicated in the following sketch:



This equation can be solved with the following boundary conditions.

$t=0$ $y=0$ $S_x=S_o$ for all values of x.

$t>0$ $x=0$ $dy/dx = +\,tg\,\varphi = +\,\varphi$

$S_x=0$

$x=-\infty$ $dy/dx = 0$

$y=0$

$S_x=S_o=S\varphi$

The angle between the crests of the breaking waves with the undisturbed coastline ($x=\infty$) is φ.
The solution of the differential equation is:

$$y = \frac{\varphi}{\sqrt{\pi}} \left[\sqrt{4\,at}\; e^{-u^2} + x\sqrt{\pi}\;\theta u \right],$$

for small values of φ so that $tg\,\varphi = \varphi$, and with negative values for x.
In this equation:

$u = -x/\sqrt{4\,at}$, $a = S_o/\varphi h$ and

$$\theta u = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-u^2}\, du,$$ which is the probability integral.

$$\theta_u = \frac{2}{\sqrt{\pi}} \left\{ \int_0^\infty e^{-u^2}_u \, du - \int_0^u e^{-u^2} \, du \right\} =$$

$$= 1 - \frac{2}{\sqrt{\pi}} \int_{u\,o} e^{-u^2} \, du \; .$$

The integral $\frac{2}{\sqrt{\pi}} \int_0 e^{-u^2} du$ is tabulated.

This rather complicated equation becomes rather simple for $x = 0$, where

$OB = 2 \varphi \sqrt{at/\pi}$

So $OA' = 2 \sqrt{at/\pi}$, and

Surface $OA'B = 2 \, at \, \varphi / \pi$.

The volume of sand deposited updrift of the mole can be written as:

$$S_o . t = \text{surface } O A_1 B_1 . h \; , \quad \text{so}$$

surface $OA_1B_1 = S_o \; t/h$.

From this follows:

$$\frac{\text{surface } OA_1B_1}{\text{surface } OA'_1B_1} = (S_o \; t/h) . \; \pi/2 \text{ at } \varphi = \pi/2.$$

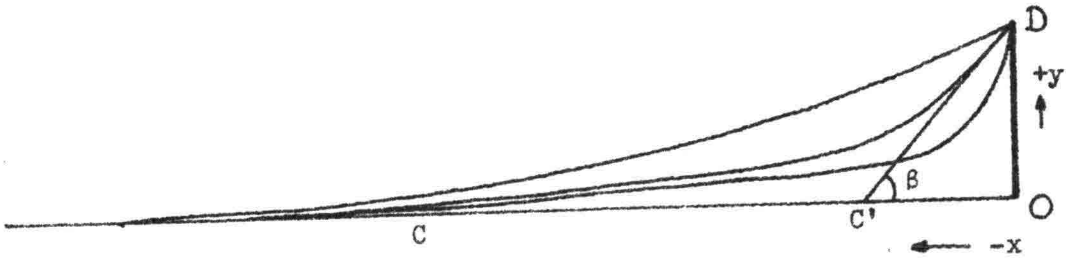
This relationship makes it possible to compute from the length of the obstruction and the angle of approach of the waves the quantity of material which can be stored updrift of this obstruction.
From the equation of the coastline follows the approximation

$$OA = 2.7 \; OA'.$$

With this formulae it is possible to say, when the littoral drift along the undisturbed coast is known, how long it will take before the material will pass around the mole.
When this point is reached the material will start to move around the tip of the mole.

For the movement of the beach the same equations hold true, so the differential equation describing the form of the coastline is again:

$$a \; \partial y^2/\partial x^2 = \partial y/\partial t.$$



The boundary conditions in this case are:

$x=0$ for all values of t : $y = OD =$ constant

$x=\infty$ $S = S_o = S_\varphi$

$t=0$ $y=0$ for all values of x.

$S_{x=0}$ is for $t=0$ directed opposite to the direction of S_o due to the form of the beach with regard to the incoming waves.

When $\beta = \varphi$, the transport around the tip, $S_{x=0}$, will be zero, and later on it will increase in the same direction as S_o.

The solution of the equation is with these boundary conditions:

$$y = OD\ \theta(u) = OD\left[1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du\right]$$

For the transport around the tip of the mole the following equation can be written.

$$S_{x=0} = S_\beta = S_o(1 - \frac{dy}{dx} / \varphi) = S_o(1 - \beta/\varphi).$$

For $\beta < \varphi$ S_β has the same direction as S_o

for $\beta > \varphi$ S_β has a direction opposite to S_o.

The accretion of beach material updrift of the mole is:

increase of surface OCD.h $= S_o - S_{x=0} = S_\varphi - S_\beta$.

$S_\varphi = S_o$ is a constant value.
$S_\beta = f(t)$ since $\beta = f(t)$.

Accretion $= \int (S_\varphi - S_\beta)dt = S_\varphi \int \beta / \varphi \, dt$

The value of β can be calculated from the equation of the shoreline.

$$\beta = dy/dx = \frac{2}{\sqrt{\pi}} \, OD \, \frac{1}{2\sqrt{at}} \, e^{-(\frac{1}{2} x \sqrt{at})^2} =$$

$$= (OD / \sqrt{\pi at}) \exp - (\tfrac{1}{2} x \sqrt{at})^2 .$$

For x=0 $\beta_{x=0} = OD / \sqrt{\pi at}$.

So from this follows:

$$\text{Accretion} = (S_\varphi / \varphi) \int_0^t OD / \sqrt{4 at} \, . \, dt =$$

$$= (S_\varphi / \varphi) \, 2 \, OD \sqrt{t/\pi a} = (S_o / \varphi) \, 2 \, OD \sqrt{t / \pi a}.$$

So surface OD C = accretion / h =

$$(S_o / \varphi h) \, 2 \, OD \sqrt{t/\pi a} = 2 \, a \, OD \sqrt{t/\pi a} = 2 \, OD \sqrt{at/\pi}$$

Surface OD C' $= \frac{1}{2} OD^2 / \beta = \frac{1}{2} OD \sqrt{\pi at}$

From this follows

$$\frac{\text{surface OD C}}{\text{surface OD C'}} = \frac{2 \, OD \sqrt{at/\pi}}{\frac{1}{2} OD \sqrt{\pi at}} = 4/\pi = 1/0.79.$$

In order to be able to transfer from one set of curves (without transport around the tip of the mole) to the other set (with transport around the mole) the two surfaces OAB and OCD must be equal.

So:

$$\pi / 2 \, . \, OA'B = 4/\pi \, . \, OC'D$$

$$\frac{\text{Surface OA'B}}{\text{surface OC'D}} = \frac{8}{\pi 2} = \beta / \varphi \;,$$

when OB = OD and with $\beta = OD \sqrt{\pi \, a t_2}$ and

$$\varphi = \frac{OB}{2} = \sqrt{\pi / a t_1}$$

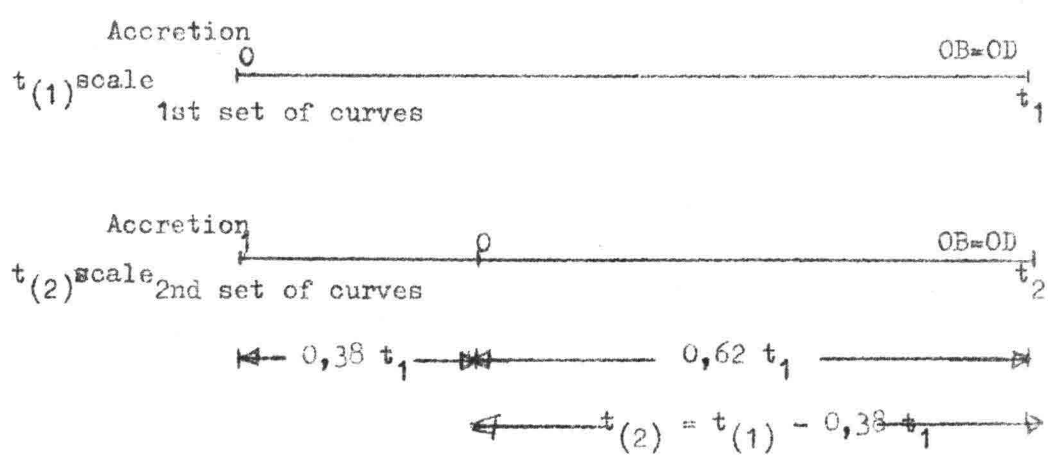
the following relationship between t_1 and t_2 is obtained.

$$\frac{8}{\pi 2} = \frac{OD}{\sqrt{\pi \; a t_2}} \cdot \frac{2}{OB} \sqrt{\frac{a t_1}{\pi}} \;,$$

So $\sqrt{t_1 / t_2} = 4/\pi$, and $t_2 = 0{,}62 \; t_1$.

So $t_1 > t_2$, which could be expected since the accretion according to the second set of curves (t_2) is achieved with sand supply from both sides.
As long as the sand is not passing around the mole a sand transport in a direction opposite to that of S_o occurs around the tip of the mole in the solution for the second set of curves.

According to the two time scales the following pattern is obtained:



From this follows that in the second set of curves, describing the movement of material around the mole, $t_{(2)} = t_{(1)} - 0{,}38 t_1$, in which t_1 is a value of the time axis of the first set of curves, that is of the $t_{(1)}$ scale, for the time required to reach the tip of the mole.

So the transport around the tip of the mole according to the second set of curves can be written as:

$$S_{t_{(1)}} = S_o(1- \beta / \varphi) = S_o \left[1- OB/\varphi \sqrt{\pi\ at_2} \right] =$$

$$S_o \left[1-OB/\varphi\sqrt{\pi\ a(t_{(1)}-0,38\ t_1)} \right] =$$

$$S_o \left[1-OB/\ \varphi\sqrt{\pi a t_1 (\frac{t_{(1)}}{t_1} - 0,38)} \right]$$

With $OB = 2\ \varphi_o \sqrt{at_1/\pi}$ this becomes:

$$S_{t_{(1)}} = S_o \left[1- \frac{2\varphi_o \sqrt{at_1/\pi}}{\varphi_o \sqrt{\pi\, at_1 (\frac{t_{(1)}}{t_1} - 0,38)}} \right] =$$

$$= S_o \left[1- \frac{2}{\pi\sqrt{t_{(1)}/t_1 - 0,38}} \right] =$$

$$= S_o \left[1- \frac{0.638}{\sqrt{t_{(1)}/t_1 - 0,38}} \right]$$

For $t_{(1)} = t_1$ the transport around the tip of the mole is just zero.

This is not in agreement with the above derived formula since:

$$S_{t_{(1)}=t_1} = S_o \left[1- \frac{0.638}{\sqrt{0.62}} \right] > 1$$

This corresponds with the fact that at the moment when the two surfaces are equal, $\beta < \varphi$, and therefore sand has to pass already around the tip of the mole according to the second set of curves.

The formulae has to be corrected by an interpolation of the surfaces of beach accretion according to the two sets of curves.

The results can be summarized as follows

t/t_1	$S_{(x=0)}/S_o$	corrected values
1	0.189	0
1.25	0.315	0.298
1.5	0.397	0.394
2	0.498	0.500
3	0.605	0.607
4	0.665	0.667
5	0.703	0.704

International Course in Hydraulic Engineering - Delft

Lecture notes on

HARBOUR ENTRANCES

Edition 1970

INTRODUCTION

In general, a harbour is a place where ships can load and unload their cargo, and where they can safely enter under almost all conditions of weather and sea.

An important point in the harbour design is the determination of entrance depth.

The depth of the entrance is determined by the following factors:

A) Waterlevel

B) Draught of the ship

C) Movement of the ship } Under keel clearance

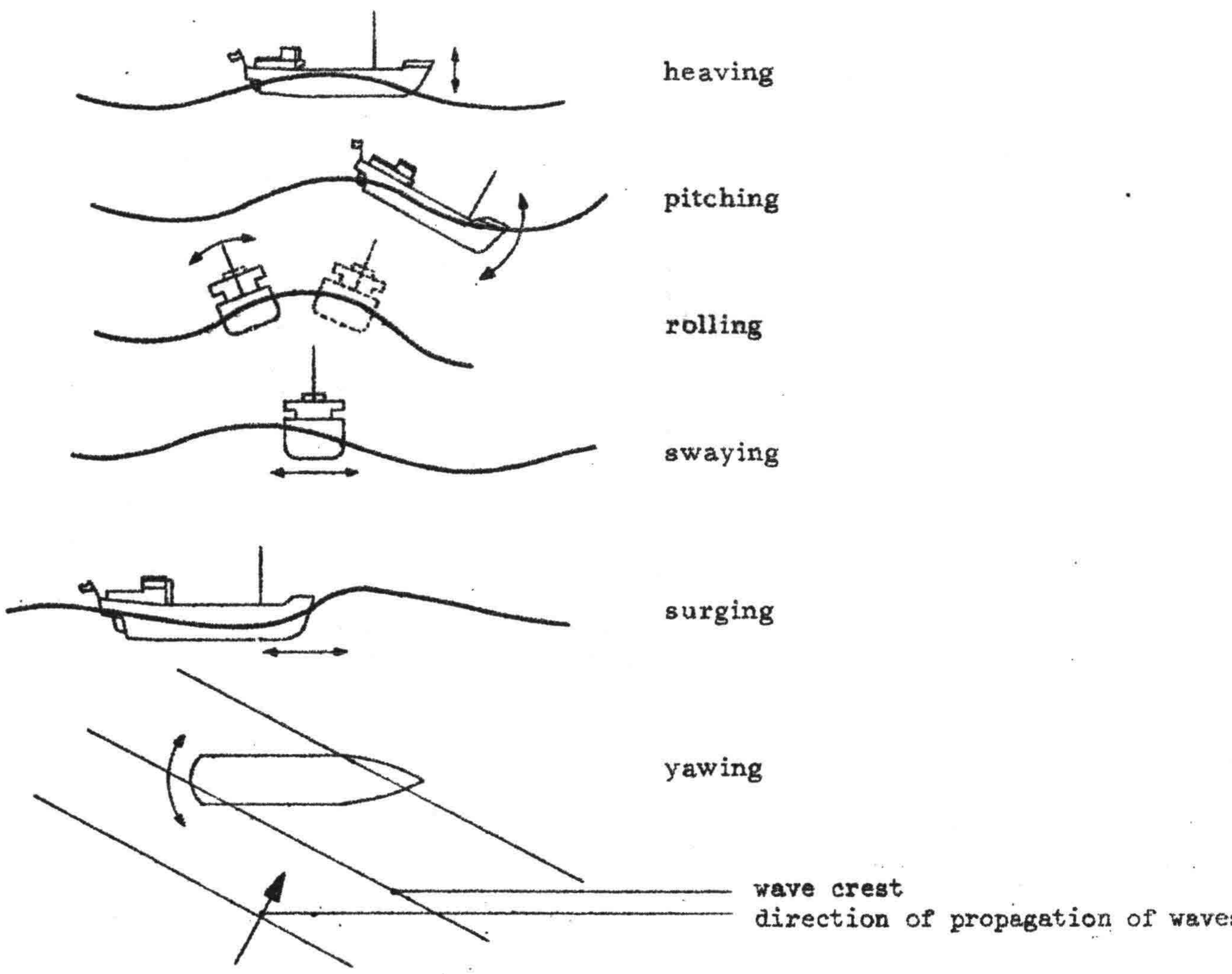
D) Safety margin }

ad A) The waterlevel which is taken into consideration is determined by the frequency with which the ships enter. When the biggest ship is a 200 000 ton oil carrier which enters only once in a few days, it is acceptable that these ships need high water to enter. When it is a ferry service,however, the ferry must be able to enter even at low water spring tide. Of course, a good estimate of the required depth can be determined only after consideration of all factors, including the loss of money when a ship has to wait.

ad B) The draught of the ship is increased by the squat due to the speed of the ship. Sometimes the increase of the draught at the bow is greater than at the stern (this is mostly the case with the carriers which have a great block coefficient) and sometimes it is greatest at the stern. The squat is also determined by the available depth of the fairway. With velocities of four to fifteen knots the squat lays in the order of magnitude of 0.1 to 1.5 m. For the actual determination, tests or recent information from literature will be required.

ad C) Ship movement.

Under influence of the waves the ships will have the following motions:



These motions will increase the depth, depending on the ratio between shipsize and wave motion, with a value up to several meters.
Especially for large carriers with a great beam, the effect of rolling can be very important.

When the depth is limited, the motion of the ship is damped due to the fact that the water between the ships bottom and the sea-bed cannot escape. This is the so-called "cushion effect".

ad D) This safety margin must be small for soft bottoms, medium for sand, and rather large for rock bottoms. In general it varies from - 0.1 or - 0.2 m for soft silt to 1 m for uneven rock bottoms.

For approach channels in shallow water with medium wave motion and for big ships a reasonable guess is that the required depth of the channel is ten to twenty percent more than the draught of the ship.

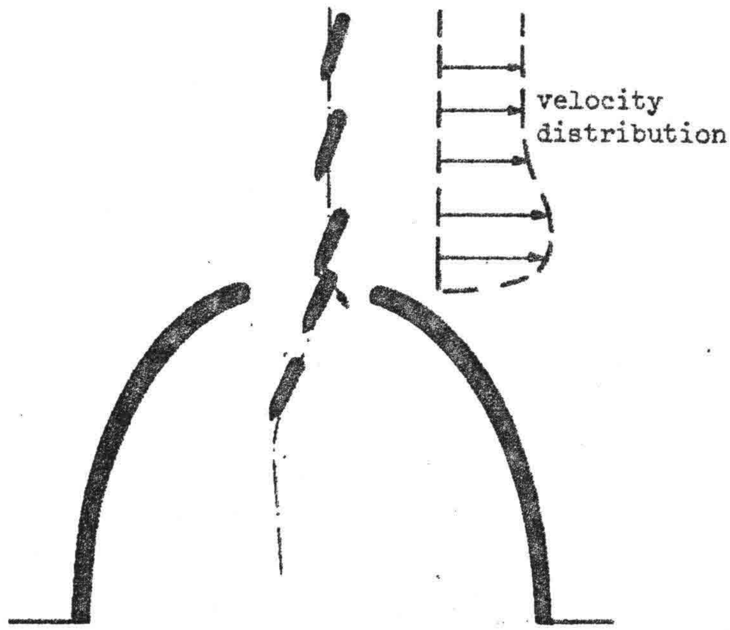
WIDTH OF APPROACH CHANNELS AND HARBOUR ENTRANCES

The width of the approach channel is determined by the hydraulic conditions. Of course a channel with a cross current of importance must be wider than a channel in still water. Also, the number of ships that are expected to sail at the same moment in the channel will determine the width. It is difficult to give fixed data. As a general rule one can state that the path width required by a ship is about 1. 8 times the beam of the vessel. Between two vessels meeting each other a distance of about one beam should be kept between the two paths. In a channel with banks almost up to the waterline, a distance of 1. 5 beams should be kept between the side of the bottom of the channel and the path of the ship (see: C. F. Wicker: Evaluation of present state of knowledge of factors effecting tidal hydraulics and related phenomena. Department of the Army, Corps of Engineers, Chapter X, Design of channels for navigation.

The width of the harbour entrance itself should in principle have the same width as the approach channel just in front of the entrance. However, in a long approach channel a meeting of two ships may be inevitable, whereas this may be avoided in the entrance itself.

On the other hand, touching of the breakwater ends will involve greater damage to ships and possibly give greater hindrance to navigation than the grounding of a vessel in the approach channel.
In order to prevent the ship from completely blocking the channel or the entrance, a width slightly greater than the length of the ship may be used.

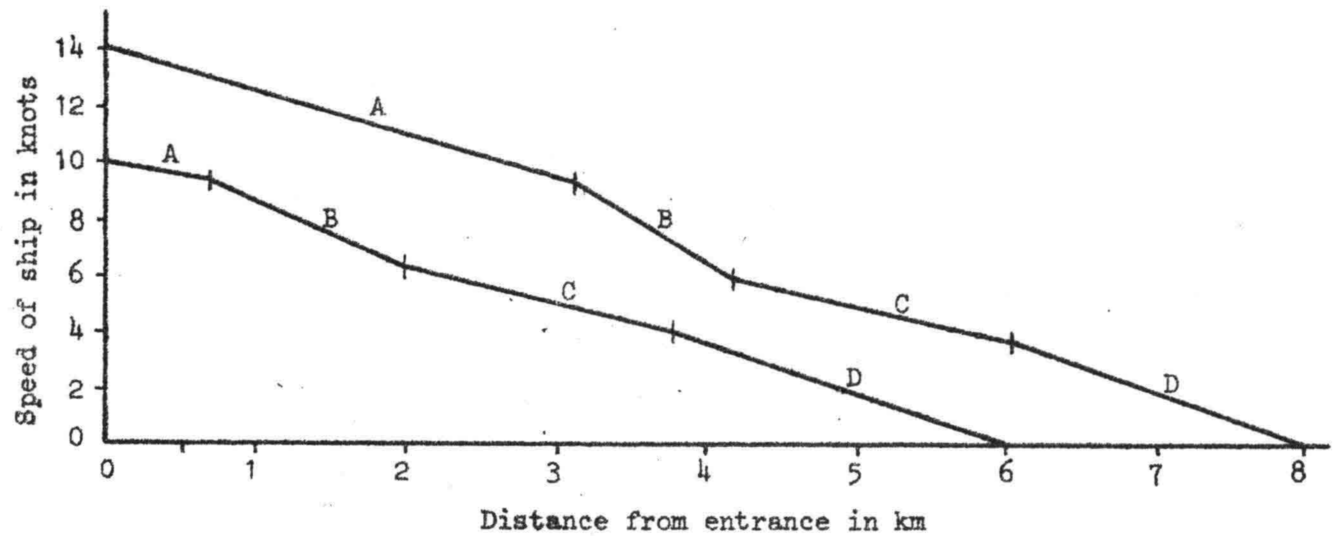
When a ship sails into a harbour and there is a current crossing the approach channel, the ship will follow a course as indicated on the sketch.



Due to the fact that on the moment of actual passing through the entrance, the bow will be in still water whereas the stern will be in the current, a moment will be acting on the ship, forcing it to turn. Sufficient space must be available inside the entrance.

Moreover, as a general rule it can be stated that the approach line of the bigger ships should be as straight as possible.
When a ship has passed the harbour entrance it needs a certain distance to stop. In the case of some wave motion and currents in front of the harbour the minimum velocity with which the ship can enter will be in the order of magnitude of six to ten km. In a harbour it is not possible to give full astern since the ship will then sail to starboard, (when the normal revolution direction of the propeller is clockwise when looking in forward direction). It is therefore necessary that tugboats assist the ship in keeping the right course.

In the following sketch a possible procedure is given for stopping a ship of 100 000 tons in water of a depth of 1.4 * the draught, when sailing with speeds of ten and fourteen knots.



A. Propeller turning free. No power.

B. Full astern. At a velocity of six knots this manoeuvre has to be stopped due to the fact that the ship becomes unsteerable.

C. Propeller turning free. No power. Tugboats take over lines.

D. Half or slow astern with aid of tugboats for steering.

When a 130 000 ton oil carrier sailing at fourteen knots has to make an emergency stop, giving continuous full astern, the stopping length is some 3 or 4 km.

DESIGN CRITERIA

For the design of a harbour project various design criteria have to be adopted.

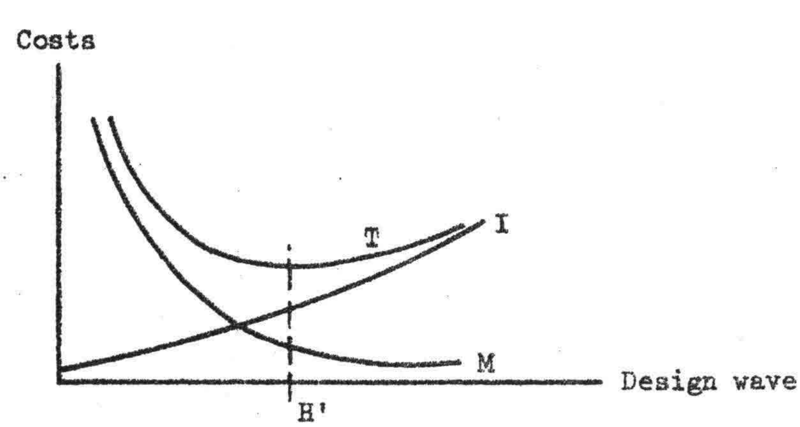
1. For the lay-out of the harbour with respect to geomorphological developments (coastline, depths in the entrance, etc.) conditions which occur more or less regularly are decisive. So in principle the average conditions hold good in this case. How the values describing these conditions have to be chosen exactly is a difficult question for the answering of which a thorough knowledge of the physical phenomena of littoral drift etc. is required.

2. For the lay-out of the harbour with respect to wave penetrations and navigability of the entrance, conditions which occur once or a few times a year are decisive. In this respect the economic loss when ships cannot be loaded or unloaded has to be taken into account.
3. For the design of breakwaters, etc. rarely occurring conditions have to be taken into account, since for example severe damage to a breakwater every year is not acceptable. Such severe damage is acceptable not more than once in ten to fifty years.

   However, this can be regarded only as an order of magnitude since in this way no certainty at all is obtained that the most economical solution is obtained. To this end, the method of optimal design should be applied.

   The principle of this method is that a more seldom occurring circumstance as design criteria, so a higher design wave, results in a stronger, and therefore more expensive construction. However, this construction will suffer less damage and therefore require less maintenance.

   When the initial costs (I), the capitalized maintenance costs (M), and the total costs are set in a diagram against the design wave an optimal can be obtained.



From this, it follows that the design wave should be H' in this case. When a lower wave is chosen, the "regret" will be higher than when a higher wave is chosen. When the data is not very accurate it is therefore better to be a little bit on the safe side, that means that a design

wave equal or somewhat higher than H' should be chosen. This method will be dealt with in detail after the discussion of various types of breakwaters.

Special attention should be given to the probability distribution of the wave heights. Two probabilities should be distinguished.

1. The wave height distribution in a certain period with a "constant" wave height. Any wave motion is a stochastic process and therefore the wave height is never constant. This holds true even for a period of for instance three hours, during which the mean wave height neither increases nor decreases. For almost all prototype conditions the wave heights during a certain period are distributed by a Rayleigh distribution

$$q\left(\frac{H_q}{\overline{H}O}\right) = \pi/2 \, \frac{H_q}{\overline{H}O} \quad \exp \left\{ -\pi/4 \left(\frac{H_q}{\overline{H}O}\right)^2 \right\}$$

In the cumulative form

$$p\left(\frac{H_p}{\overline{H}O}\right) = 1 - \exp \left\{ -\pi/4 \left(\frac{H_p}{\overline{H}O}\right)^2 \right\}$$

in which H_q is the wave height occurring with a probability of q, H_p is the wave height which has a probability of exceedance of p, and $\overline{H}$ is the mean wave height.

For $\overline{H} = H_{50\%}$ any other value $H_{i\,\%}$, so also $H_s = H_{13,5\%}$ can be applied.

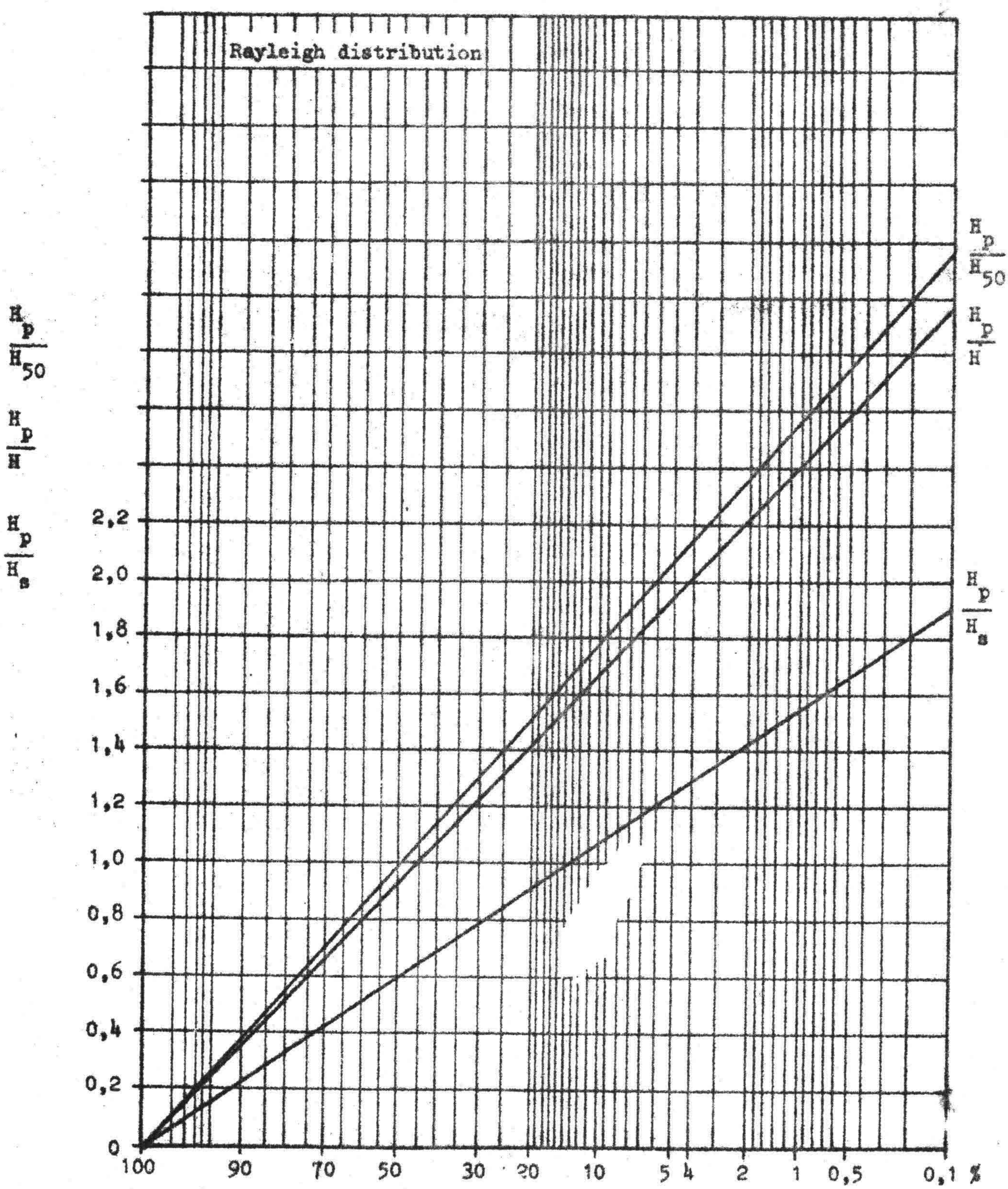
However, the value H/4 in the exponent will have in this case another value. f.i. for Hs the value 2.

The probability of exceedance is

$$p\left(\frac{H_p}{\overline{H}O}\right) = \exp \left\{ -\pi/4 \left(\frac{H_p}{\overline{H}O}\right)^2 \right\}$$

This is indicated in graph form on the following page.

Accordingto this, a wave height which is "constant" during a certain period may be expressed either by $\overline{H}$ $H_{50\%}$, $H_{2\%}$, or by the significant wave height H_s being the mean of the highest one third, so $H_s = H_{13\frac{1}{2}\%}$



Rayleigh distribution
Hp/H50
Hp/H
Hp/Hs
2,2
2,0
1,8
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0
100
90
70
50
30
20
10
5
4
2
1
0,5
0,1 %

The percentage indicates the percentage of exceedance.
This could be called a micro distribution.

2. The various periods with "constant" wave height (characterized by either $H_{50\%}$, H_s or $H_{i\%}$) are also distributed.
According to some probability distribution this could be called the macro distribution. This distribution is normally given in number of occurrences of this condition in times per year. During this occurrence, the wave height will still vary according to the Rayleigh distribution.

The resultant probability of occurrence of individual wave heights can be determined according to the following procedure (see also A. Paape: "Some aspects of the design procedure of maritime structures", SII-5, Int. Nav. Congr. Paris, 1969).
In a period of N successive waves let the frequency (probability) of exceedance of a wave height H_p, be p(H) (the micro distribution). The probability that during a series of N waves the height H_p is indeed exceeded once or more times is the encounter probability E_1.

$$E_1 = 1 - \left[1 - p(H_p) \right]$$

If the series of N successive waves and duration D is characterized by the significant wave height H_s with a probability of $q(H_s)$ times per year and the structure has an assumed life time of L years, the encounter probability of occurrence of H_s during this life time is:

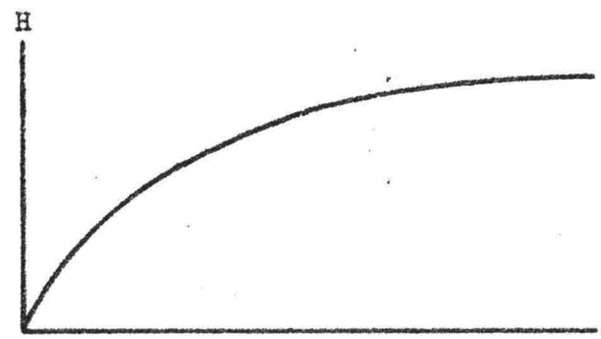
$$E_2 = 1 - \left[1 - q(H_s) \right]^L$$

The encounter probability of a wave height > H_p as a result of the wave condition H_s during the lifetime L is

$$E = E_1 \cdot E_2$$

In this procedure the history of the storm has not been taken into account. Paape discusses in his above mentioned paper an example

in which this history has been taken into account. The difference in the final encounter probability is not very great. When no wave data over a sufficiently long period is available, the waves can be computed from wind data.
When wind blows over a water surface over a certain length (fetch) the following history of the development of the waves will occur.



The wave height will increase with the distance from the startpoint of the wind, and for a certain point also with the time during which the wind has been blowing.
Several formula and graphs have been developed by various investigators, starting with Sverdrup and Munk for deep water waves, and Thysse for deep water and shallow water waves.
From a probability distribution of the wind, the distribution of the waves can be calculated. In this case the macro distribution of the waves will be achieved. This can be done according to the following procedure which has been developed in cooperation with Nicks; a participant in the course '67 - '68.

α = = probability of wind occurring from a certain direction
βj. = probability that a period with a certain wind direction is part of a series of at least $_j$ consecutive periods from the same direction
γ = probability of a wind velocity with a given value.

For a wave height of H_i it has to be calculated how this wave height can be generated, viz.

with velocity v_{i1} during one period and a certain fetch.
with velocity v_{i2} during two periods and a certain fetch.
with velocity v_{i3} during three periods and a certain fetch.
etc.

In this way, the various circumstances in which a wave height can be generated under various conditions of wind velocity, wind duration and fetch are given. Only for the longer duration will the fetch limit the wave height.

Now when the probabilities for the various circumstances of v_{i1}, v_{i2}, v_{i3} etc. are computed and added, the final probability of the said wave height H_i is found. This can be done for various wave heights H_i, and from this data a probability curve for the wave height is found. In this procedure the wave period is not taken into account.

EXAMPLE

Say one hundred periods of wind data are available. For the direction under consideration $\alpha = 0.25$ (25%), so for twenty-five periods the wind blows from the considered direction.
These periods are assembled in the following groups:

6 periods as one single period = 24 % = p_1
6 periods in groups of two periods = 24 % = p_2, so 3 groups
9 periods in groups of three periods = 36 % = p_3, so 3 groups
4 periods in groups of 4 periods = 16 % = p_4, so 1 group

25 periods

In order to obtain a mass distribution curve (accumulative curve) for this probability, the following has to be taken into consideration.

The wave height H_{i3} which is generated by a velocity v_{i3} during three periods is occurring or exceeded during one period in every group of three periods, so the probability is p3/3. This wave height is occurring or exceeded during two periods in every group of four periods, so the probability is 2p4/4.

A wave height H_{i2} occurring after a wind velocity v_{i2} during two periods is occurring during one period in every group of two periods, so the probability is p2/2. During two periods in every group of three periods, the probability is 2p3/3 and during three periods in every group of four periods the probability is 3p4/4.

From this the following table can be drawn up

Probability that wave height H_i is reached or exceeded in groups of j periods

	H_{i1}	H_{i2}	H_{i3}	H_{i4}
single period	$p_1 = 24$	-	-	-
double period	$p_2 = 24$	p2/2 = 12	-	-
triple period	$p_3 = 36$	2p3/3 = 24	p3/3 = 12	-
quadruple period	$p_4 = 16$	3p4/4 = 12	2p4/4 = 8	p4/4 = 4
	100%	48%	20%	4%
	$\beta i1$	$\beta i2$	$\beta i3$	$\beta i4$

The final probability of a wave height H_i from a certain direction reached or exceeded is:

$$\alpha(\beta_{i1} \cdot \beta_{i1} + \beta_{i2} \cdot \gamma_{i2} + \beta_{i3} \cdot \gamma_{i3} + \beta_{i4} \cdot \gamma_{i4}) = \sum_i \alpha\beta_{ij} \, \gamma_{ij}$$

γ_{ij} is the probability of a wind velocity v_{ij} required to generate a wave height H_i after a duration of j periods. In this derivation it has been assumed that the grouping of wind in various consecutive periods is independent of the wind velocity.

One period is the interval between the various readings of the wind velocities. Normally this is three hours. In this case the number of periods during one year is 8 x 365 = 2920.

By this procedure it is possible to compute the probability by which a given wave height H_i is reached or exceeded. By doing this for various wave heights a probability distribution curve for the wave height from a certain direction can be obtained.

BREAKWATERS OR HARBOUR MOLES

The function of breakwaters or harbour moles can be the following:

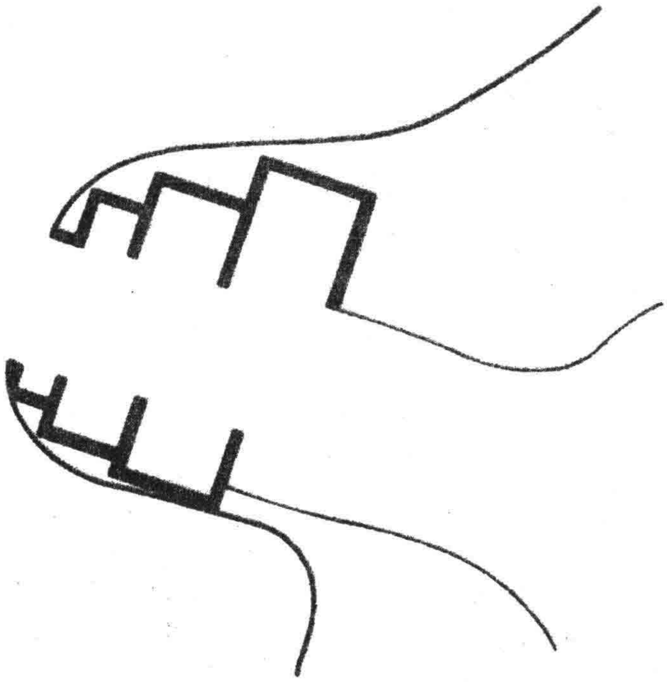
1. Protection against waves (IJmuiden)
2. Guidance of current (Abidjan)
3. Protection against shoaling (IJmuiden, Abidjan, Maracaibo)

The following possibilities exist for 1:

a) Pneumatic breakwater; an air bubble curtain that forces the waves to break.

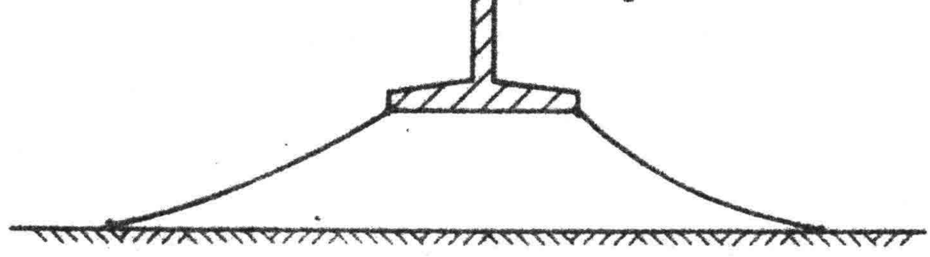
b) Resonant breakwater.

In the harbour entrance basins acting as resonators decrease the height of the wave penetrating into the harbour.

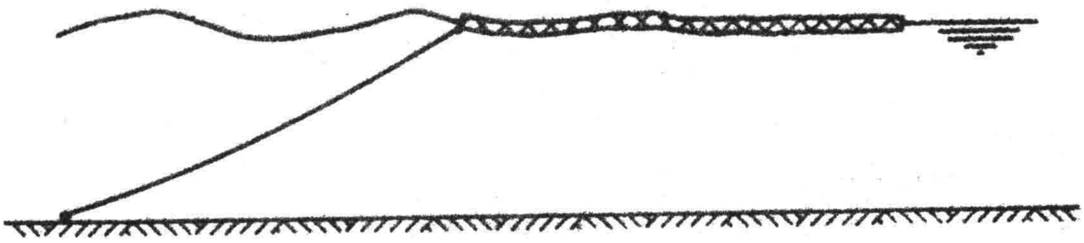


c) Hydraulic breakwater; a waterjet at some distance under the water surface that forces the waves to break.

d) Floating breakwater; which consists of some floating and anchored construction.



e) Hoovering breakwater, which consists of a flexible construction of sufficient size which damps the wave by energy dissipation.

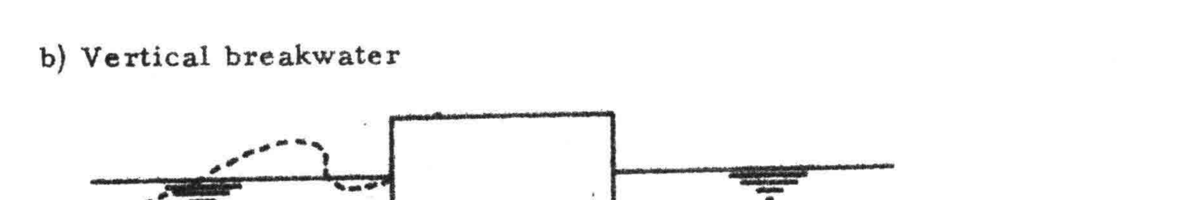


f) Real constructions like dams and walls.

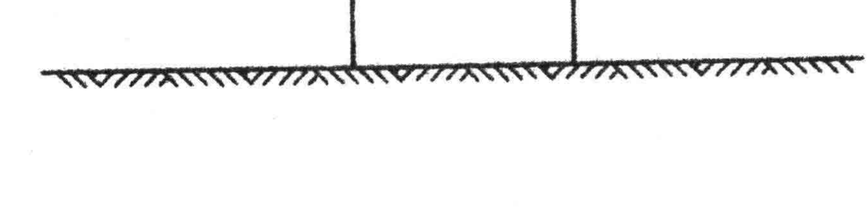
Apart from b and f, most of the above examples which will be discussed later in full, are more or less temporary.

For 2 and 3 only real constructions as mentioned under f can be applied. These breakwaters or moles can be distinguished in three main forms.

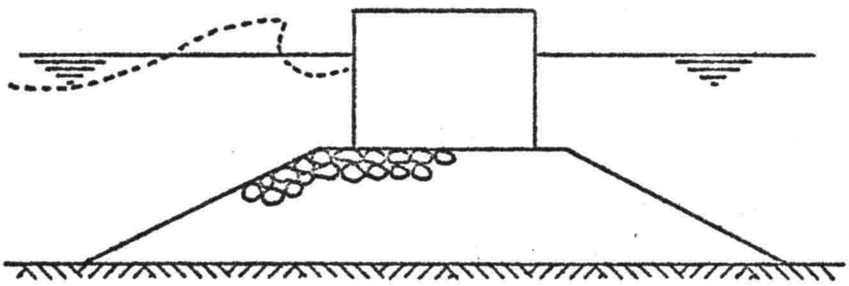
a) Mound breakwater



b) Vertical breakwater



c) Composite breakwater

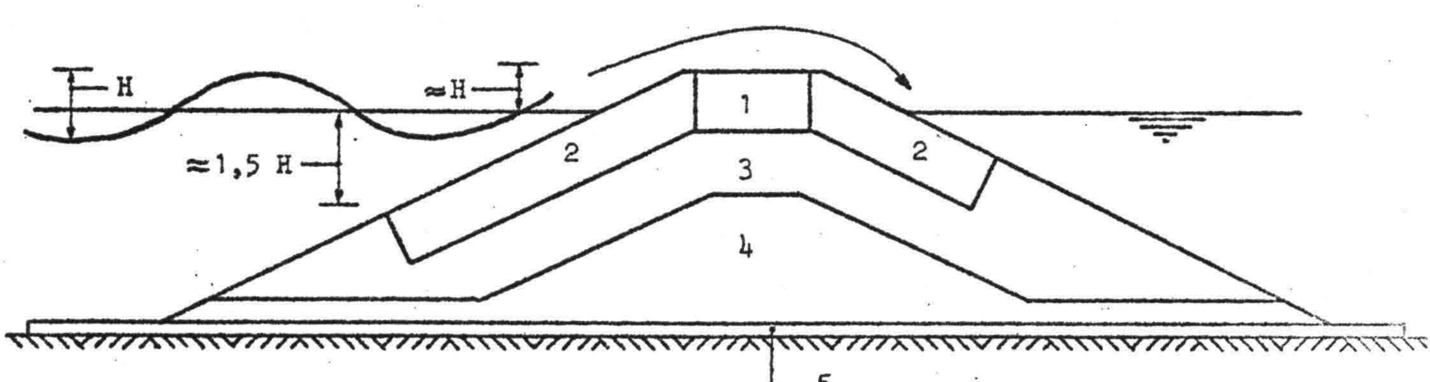


The various advantages and disadvantages will be discussed briefly. In deepwater the mound breakwater needs a great quantity of material. The vertical breakwater is much more sensitive to movements of the subsoil and damages to a vertical breakwater are more disastrous than to a mound breakwater. Against a vertical breakwater very high, short-lasting, forces due to waves breaking against the construction can occur (wave impacts).

The composite breakwater has both the advantages and disadvantages of the mound and vertical type. An extra disadvantage of the composite breakwater is the fact that waves can be forced to break on the slope of the mound and exert very high forces (impacts) on the vertical wall of the superstructure.

CONSTRUCTION OF A MOUND BREAKWATER

A typical cross section of a mound breakwater is the following:



1. Cap construction
2. Armour units

3. Second-class stones
4. Small stones or quarry run
5. Filter layer

(1) and (2) form the protection against the wave attack. The cap construction can be omitted and replaced by the normal armour units. A cap construction is applied if for one reason or another it is necessary that equipment move over the crest of the breakwater.
The armour units (rock blocks or concrete blocks) should be sufficiently heavy that they can resist the wave motion. When the breakwater is rather low, overtopping will occur and the inside will also be attacked. This attack will be - above the water level - just as heavy, or even heavier than at the seaside. The reason for this is the fact that the stability of the inner slope is under attack of overtopping waves more early lost that the stability of the outer slope which is attacked by the waves more perpendicular on the slope. For this reason ten percent damage of the inner slope is assumed to cause a total collapse of the breakwater, whereas the damage of the outer slope can be thirty percent before a total collapse occurs. Only when the breakwater is so high that no serious overtopping occurs, can the harbour side have lighter cover stones. Under the cover layer of heavy stones, smaller and lighter stones can be applied. The only requirement is that these stones (3) cannot be sucked through the cover layer, and that they are sufficiently stable during construction phases. Normally in the core of the breakwater the waste of the quarry (4) will be applied. This has, moreover, the advantage of being almost impermeable for sand. This can be important when the breakwater has to stop a littoral drift.
When the breakwater is constructed on a sandbed, special precautions have to be taken to prevent sand from being moved through the stones. (This can result from wave motion in the body of the breakwater).
To this end, a filter layer has to be placed between the rock blocks and the sand (5). This filter layer can also be a fascine mattress if precautions have been taken that this is sufficiently sandtight. Nowadays it seems also possible to use a woven cloth of synthetic material as a means to prevent the sand from being sucked through the stones. It has to be certain in this

case that the stones cannot pin holes in this cloth. With breakwaters in deep water more layers than indicated can and sometimes have to be applied.
The filter layer is normally extended outside the toe of the breakwater at the seaside, to avoid scouring. In order to prevent the filter layer itself (which acts here as a bottom protection layer or revetment) from being attacked too much by the wave motion in the case of a small waterdepth in front of the breakwater, a layer of heavier stones has to be placed on this filter layer outside the toe of the breakwater.

For the computation of the size of the armour units several formulae have been developed. One of the most well-known is that of Irribarren which has been modified and improved by Hudson to the following form.

$$W = \frac{H^3 \rho}{K \Delta^3 \cot g \alpha}$$

in which

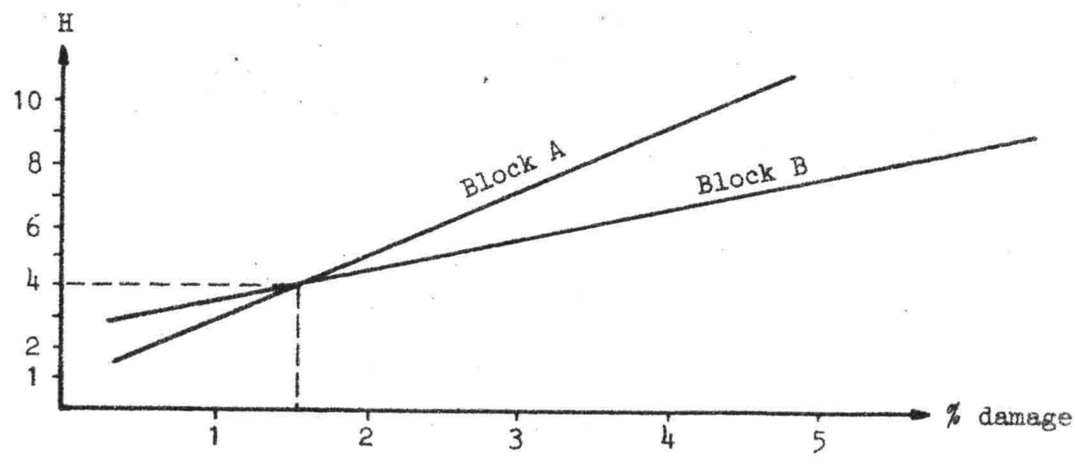
- W = stone weight in ton
- H = wave height in meter
- ρ = density of armour stones in ton/m^3
- Δ = relative density of stone under water = $(\rho - \rho_w)/\rho_w$
- K = coefficient ranging from three for rock to fifteen for artificial units of special form.
- α = angle of slope of breakwater.

This formula can be used to get a first estimate, but never for a definitive design. For a definitive design model tests always have to be employed. In these tests the construction phases can also be studied. In Tech. Rep. 4 of C. E. R. C. data about K values for different circumstances are given.

It has been stated already that the armour units can be rock blocks or concrete blocks. Concrete blocks are used instead of rock blocks when the quarry cannot deliver blocks of sufficient size, or when no rock at all is available at reasonable prices. Blocks of special form are used when, as result of the equipment available, the blockweight is limited. Due to their special form the interlocking of these artificial armour units is greater, and therefore the K value can be assumed higher. Another advantage is the greater porousity which results in a smaller reflection and uprush. So in total a smaller quantity of concrete is used, however, at a higher price per m^3. Various types of artificial armour units are akmos, tetrapods, tribars, modified cubes.

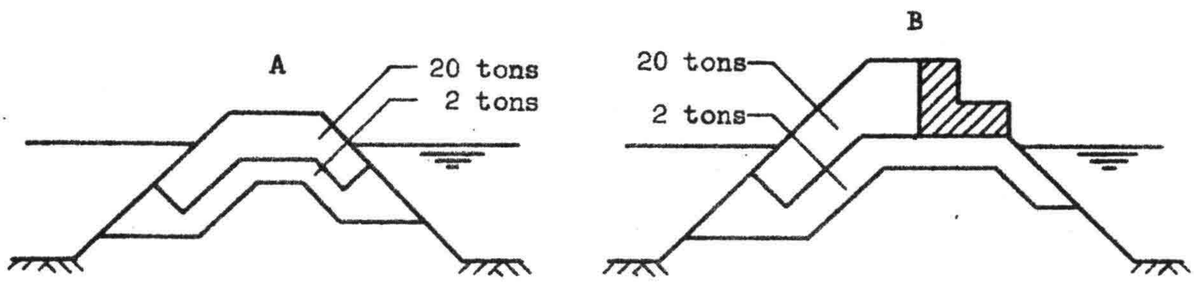
Of very great importance for the design of a breakwater constructed of natural rock, is the quantity and size of stones that the quarry can supply.
So before the actual design of a breakwater starts some trial blasts in the quarry must take place.

Following, is an example of an optimal breakwater design based on the paper of V. d. Kreeke and Paape. For this procedure it is essential that a breakwater which is designed to resist a certain design wave H_{so}, is tested with various wave heights in order to determine the damage which occurs with various wave height > H_{so}. From these tests it becomes clear that one coefficient K cannot always completely characterize a block.
This can be elucidated by the following example. On the graph, the wave height is given on the ordinate, and the damage which results when the breakwater is exposed to this wave height, at certain periods, is given on the abscis. The damage is given as a percentage of the stones which are removed.



For a wave height of four meters both blocks A and B have an equal damage percentage and from that an equal coefficient K. When the waves increase, the damage for block B is greater than for block A, so A should be applied. When the breakwater is tested only for waves smaller than four meters, block B should be chosen since the damage percentage in this case is smaller. From this very simple example the conclusion must be drawn that K values only, cannot be decisive as a design criterium. The increase of damage with increasing wave height has to be the decisive factor.

The following two breakwater cross sections will be discussed now.



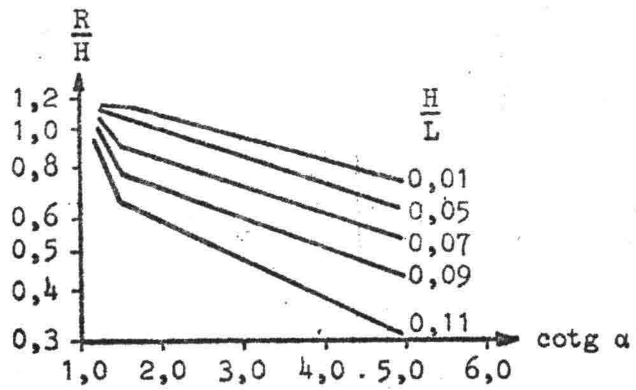
Cross section A

In this cross section considerable overtopping will take place. Damage at inner and outer slopes will be almost equal. From tests it has to be determine how many stones (for instance expressed as a percentage of the total number of stones) may be moved before a total collapse occurs. For the outer slope this will be in the order of magnitude of thirty percent and for the inner slope in the order of magnitude of ten percent.

Cross section B

The monolite construction will give support to the armour units of the outer slope.
The failure criterium for the outer slope is thirty percent damage. When there is considerable overtopping the construction will collaps since the inner slope is barely protected. For overtopping the wave run-up

criterium of Hudson is used, as given in the figure below.



The two cross sections have been designed for a design wave $H_{s_0} = 6$ m, which has a certain probability of occurrence. This states that the storm which is characterized by a significant wave of this height has this probability of occurrence.

In the model test the damage is determined for the following wave intervals.

$$1 < \frac{H_s}{H_{so}} < 1.3 \qquad 1,3 < \frac{H_s}{H_{so}} < 1,45 \qquad 1,45 \leq \frac{H_s}{H_{so}}$$

H_s in this case is the significant wave height of the condition which is adjusted in the wave flume. It is essential in this case that the wave height spectrum in the flume is equal to that in prototype.

For the various wave intervals the damage expressed in repairing costs (ΔW) will be determined and multiplied with the corresponding probability (Δp) of the wave intervals. This is in principle equal to the average yearly repairing costs over an infinite life time of the breakwater.
The total repairing costs, when all damage is repaired immediately after the occurrence, are $\Sigma \Delta p . \Delta W = s$. When the damage is not repaired immediately the damage is:

$$\Delta W_{total\ collapse} \cdot P_{collapse}$$

In some cases the latter procedure will give lower total maintenance costs. This can be, however, only the case when damage at a second event is independent of damage that has occurred earlier. This may be so in the case of a rubble mound breakwater. It is almost never so in the case of a breakwater with a more or less closed surface, for instance a breakwater protected with rock asphalt.

When total collapse occurs it is assumed that repairing costs are equal to the original construction costs. This is probably a rather conservative assumption.

In order to be able to compare the expected damage per year s with the construction costs, the damage costs per year will be capitalized. If interest is added continuously the capitalized value (present value) of a sum s to be paid after t years is: s. exp. $(-\delta\, t/100)$.

Consequently, the capitalized value of the sum of the expected damage costs s to be paid for the life time T of the construction (sum of all present values) is found to be:

$$S = s \int_{0}^{T} \exp(-\delta\, t/100)\, dt$$

in which δ is the rate of interest in percent per year.

For one hundred years

$$S = \frac{100}{\delta}\, s\,(1 - \exp(-\delta)) = 100\, s/\delta$$

For ten years and $\delta = 3.5$ %:

$$S = \frac{100}{\delta}\, s\,(1 - \exp(-\delta/10)) \approx 0.3\ 100\, s/\delta$$

It is normal that for a shorter life time the capitalized maintenance costs are lower. The expected damage repairing costs have to be paid during a shorter time.

In the following tables the various values have been listed.

Table 1.

H_{so} (m)	I	I_{cl}	$1<H_s/H_{so}<1.3$, n=4%			$1.3<H_s/H_{so}<1.45$ n=8%			$H_s/H_{so}>1.45$; collaps		
			Δp	ΔW	ΔpΔW	Δp	ΔW	ΔpΔW	Δp	ΔW	ΔpΔW
4	13900	5280	1.01	420	430	$5.2.10^{-2}$	860	40	$3.8.10^{-2}$	13900	530
5	15220	6600	$1.6.10^{-1}$	530	80	$4.7.10^{-3}$	1060	5	$2.8.10^{-3}$	15220	40
5.5	15900	7280	$6.3.10^{-2}$	580	40	$1.6.10^{-3}$	1160	-	7.10^{-4}	15900	10
6	16540	7920	$2.5.10^{-2}$	630	15	$5.2.10^{-4}$	1260	-	$1.8.10^{-4}$	16540	3

Table 2.

H_{so} (m)	With repairing partial damage		Without repairing partial damage	
	$s = \Sigma\Delta p.\ \Delta W$	$S = \frac{100}{\delta} s$	s	S
4	1000	30000	530	15900
5	125	3750	40	1200
5.5	50	1500	10	300
6	18	540	3	90

Table 3.

H_{so} (m)	With repairing partial damage			Without repairing partial damage	
	I	S	K	S	K
4	13900	30000	43900	15900	29800
5	15220	3750	18970	1200	16420
5.5	15900	1500	17400	300	16200
6	16540	540	17080	90	16630
6.5	17200	100	17300	20	17220

n = percentage of damage.

All costs are given in Dutch guilders/m^1

I = cost of construction

I_{cl} = cost of cover layer

H_{so} = design wave

ΔW = repairing costs

S = capitalized repairing cost

K = total costs = I + S

With this method, a breakwater has been obtained which has the minimum total cost for the chosen type of cross section.
A most optimal design is, however, also possible in another aspect, viz. a cross section of such a shape that severe damage occurs at various critical places at the same moment. For instance, a breakwater of such a height and shape that collapse of the breakwater is due to complete failure of the outer slope and the inner slope simultaneously.
When overtopping of the breakwater is allowed, the form of the breakwater should be such that the waves will cause ten percent damage at the inner slope while at the outer slope thirty percent damage occurs.
Another approach is to choose the height of the crest so that no overtopping occurs. In that case the inner slope can be covered with much lighter stones. Appreciable overtopping will then result in collapse of the breakwater. This collapse can then be compared with a collapse which results from severe damage (30%) of the outer slope.
In order to determine the failure of the breakwater due to overtopping the information as given by Hudson can be used (see figure on page 20).

For a value of $H_o/L_o = 0.06$, the run-up of the wave will be

$R = 1.35\ H/\sqrt{\text{cotg}\ \alpha}$, in which H is the significant wave height which is in this aspect the design wave height.

For the determination of the damage of the outer slope tests have to be conducted which give the damage in percentages for a certain cross section as a function of the wave height. This wave height will be $1.45\ H_{so}$, for an

armour layer of rock blocks. H_{so} is the design waveheight; that is the waveheight for which no damage occurs.
When both criteria are equal, the values of H_s for the two cases have to be equal. A wave run-up value equal to the height of the crest, h, above the water level, leads to:

$$\frac{h \sqrt{\text{cotg}\ \alpha}}{1.35} = 1.45\ H_{so}$$

For a slope of 1 : 1½

$$h = \frac{1.35 . 1.45}{\sqrt{1.5}}\ H_{so} = 1.6\ H_{so}$$

Another important aspect of the breakwater design is the sequence of the various layers with respect to the possibility of construction. This has to be studied then with knowledge of the quantities of stone of different sizes which can be delivered by the quarry. To this end a trial blast at the proposed quarry site has to be made before the actual design can start.

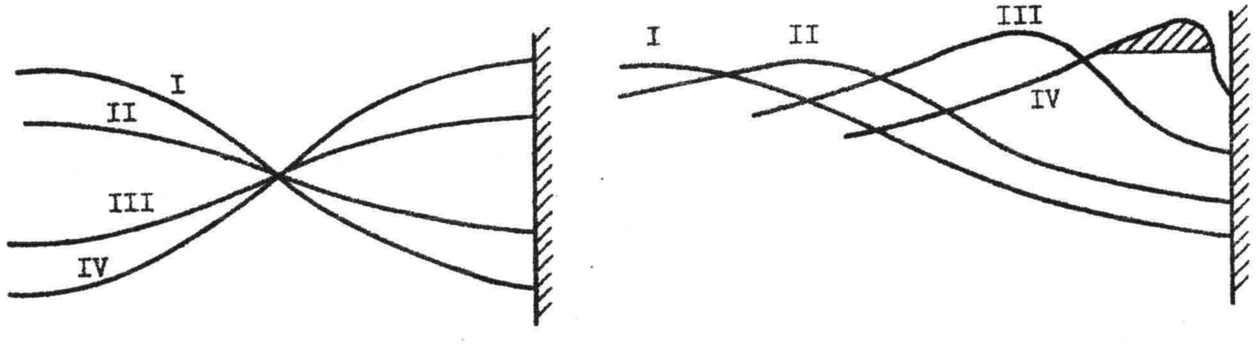
In the following example the development - in history - of the design of the breakwater of IJmuiden will be described. The various cross sections are given on figures 1 through 9 at the end of the lecture notes.

1. Original cross section. Failure occurs due to damage at the harbour side of the crest (inner slope).
2. In order to avoid this, these armour units have been removed. In order to protect the much lighter rock blocks of one to five tons under the first cover layer, these rock blocks have been penetrated with asphalt. Due to the typical lay-out of the moles, waves will reach the inner slope of the mole with crests almost perpendicular on the breakwater ("strijkgolven" in Dutch), resulting in the armour units in the inner slope just below the water surface being attacked.
3. In order to avoid the necessity of penetration, the cap construction of rock asphalt has been continued to a level below water level. When the armour units at the inner slope move (due to the oblique waves) the stability is endangered.

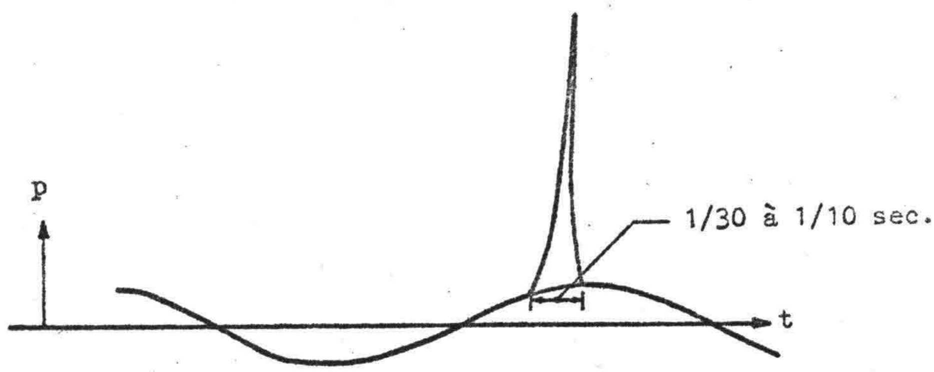
4. In order to overcome these difficulties the inner slope is made completely of rock asphalt. The disadvantage of this solution is, however, that due to pressure differences at both sides of this layer, it can be lifted. This layer therefore has to be of sufficient thickness and weight.
5. For this reason the inner slope is not covered completely with rock asphalt, but only in spots. These spots increase the stability sufficiently without the danger of uplifting.
6. In order to avoid or to decrease the uplift forces the cover of rock asphalt has been extended to the inner and outer slope of the breakwater.
7. Since this breakwater does not suffer from overtopping it can be lowered also.
8. In a later stage of this design development, the crest has again been made higher in order to enable the transport of construction materials over this crest to the cranes standing at the construction area at both sides of the breakwater.
9. This figure shows the savings in material (by the double hatching).

VERTICAL BREAKWATER

For the design of a vertical breakwater the phenomenon of wave impact is of great importance. This impact occurs when a wave crest moves towards the breakwater with a velocity which is almost equal to the wave celerity.
The fundamental difference with a normal standing wave is demonstrated by the following figure



The wave impact force which lasts only a very short time is superimposed on the normal, slowly-varying, pressure of the wave.

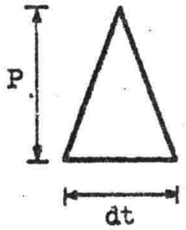


The value of this impact force can be computed in order of magnitude by the following method.

The hatched volume of water hits the breakwater with the wave celerity, c.

This mass is for a wave of 10 sec period and 4 m height about 10(length) x 1.5(height) x 1000(density) = 15 10^3 kg/m'.

When it is assumed that the form of the impact force per m of breakwater length is

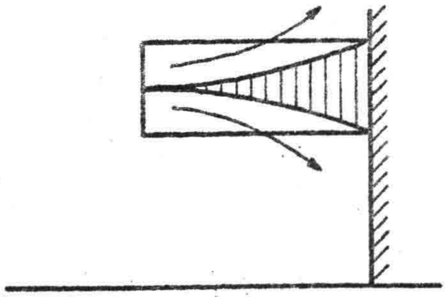


, $K\,dt = \frac{1}{2} P\,dt = d(mv) = 15.\,10^3.\,10$ N.s

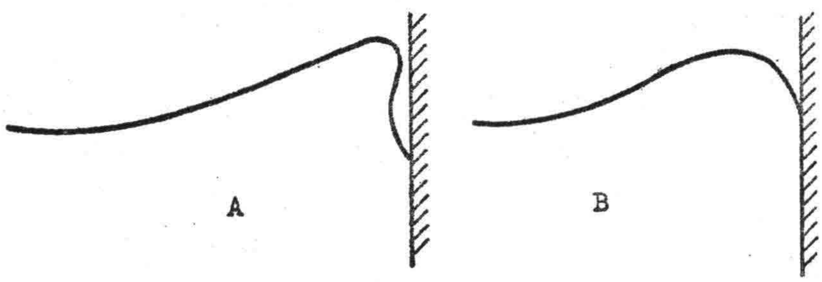
From this follows if dt is 1/10 sec.

$$P = \frac{300.\,10^4}{1.5}\ N/m^2 = 200.\,10^4\ N/m^2$$

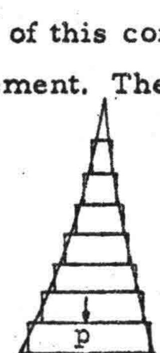
These extremely high values have never been measured. This is probably due to the fact that water escapes in all directions and the value of the mass that is stopped is about one-third of the value assumed here.



When air is enclosed the deceleration goes more slowly, the value of dt is appreciably higher and therefore the value of P is smaller. The two possibilities are shown on the sketch below



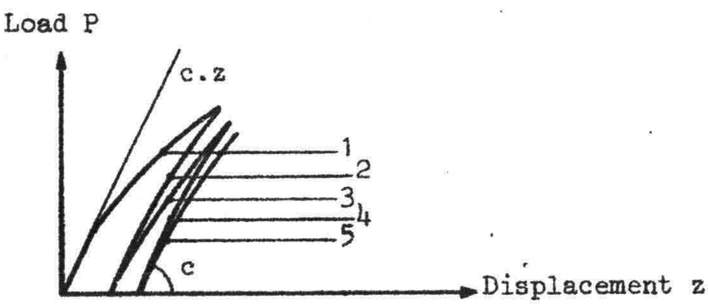
In most cases the wave A is much more impressive since a great spouter occurs when the enclosed air escapes.
To compute the effect of the wave impact forces, the total breakwater and bottom has to be regarded as a mass-spring system. Only the value of the dynamic pressure P will be taken into account. The principle of this computation will be demonstrated for the vertical load and movement. The load will be schematized as a block function.



For the various blocks with value P the movement of the mass-spring system will be analyzed. P is the vertical component of the actual load on the construction.

Due to this load the vertical movement of the breakwater is z. When c is the elasticity constant of the soil the pressure which is exerted by the soil on the construction is z. c. In reality the soil does not behave

as an elastic medium. The force displacement diagram is as shown in the sketch.



It is possible that after some time the curves for loading and unloading will be almost identical.

For a static load the equation of motion of the breakwater in a vertical direction is

$$P + (m_{br} - \rho_w b.h.)g - c.z. = 0,$$

in which m_{br} = mass of breakwater, ρ_w = density of the water, b = width of breakwater, h = depth of water, z = vertical displacement of breakwater, c = elasticity constant of soil.

For a dynamic load the mass-acceleration of the breakwater and the soil moving with the breakwater (m_s) have to be taken into account. The movement of the soil decreases as its distance from the foundation of the breakwater increases. For m_s an equivalent mass is introduced which moves as one rigid volume with the displacement of the breakwater. The energy in this virtual equivalent mass should be equal to the energy of the real mass of the soil.
When only the forces due to the dynamic load are taken into consideration, the equation of motion of the breakwater becomes:

$$P - (m_{br} + m_s)\ddot{z} - c\,z = 0$$

The solution of this equation is

$$z = \frac{P}{c}(1 - \cos \omega t), \text{ in which}$$

$$\omega = \sqrt{c/(m_{br} + m_s)}.$$

In this solution the forces in the ground vary from positive to negative. Of course in normal soil this is not possible, due to the static load of the slowly varying wave forces and the mass of the breakwater.
The force which is exerted by the breakwater on the subsoil is:

$$K = P - m_{br}\ddot{z} = P\left[1 - \frac{m_{br}}{m_{br} + m_s}\cos \omega t\right].$$

The maximum force which is exerted by the breakwater on the bottom is:

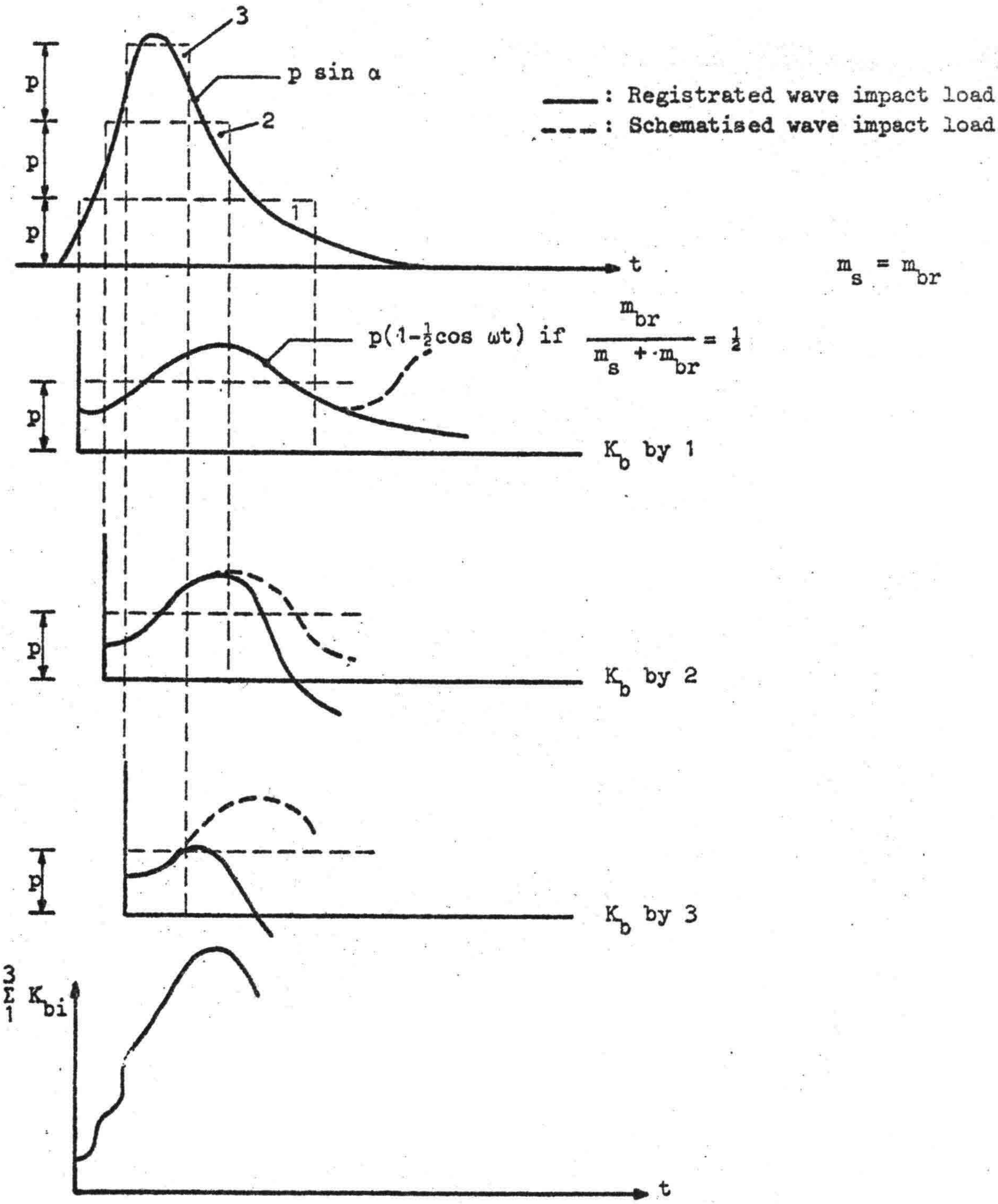
$$P\left(1 + \frac{m_{br}}{m_{br} + m_s}\right)$$

The value $1 + \frac{m_{br}}{m_{br} + m_s} = \chi$ = enlargement factor.

If it is assumed that the duration of the block function P is infinitely long, the following extreme values of χ are possible.

$\chi = 2$, for a great mass of the breakwater as compared with the virtual mass of the soil. This will be the case for rock bottoms.
$\chi = 1$, for a light breakwater, on soft soil, so with a great equivalent mass of the soil as compared with the mass of the breakwater.

The variation in values of ω will be as follows.
large ω for hard soil; great value of c and small value of m_s.
small ω for soft soil, small value of c and rather great value of m_s.


3
p sin α
2
1
p
p
p
t
: Registrated wave impact load
: Schematised wave impact load
$m_s = m_{br}$
$p(1-\frac{1}{2}\cos \omega t)$ if $\frac{m_{br}}{m_s + m_{br}} = \frac{1}{2}$
K_b by 1
K_b by 2
K_b by 3
$\sum_1^3 K_{bi}$
t

For a breakwater the following values for $T = 2\pi/\omega$ have been computed.

hard soil: T = 0. 04 sec.

soft soil: T = 0. 4 sec.

For a real wave impact the following procedure would be followed.

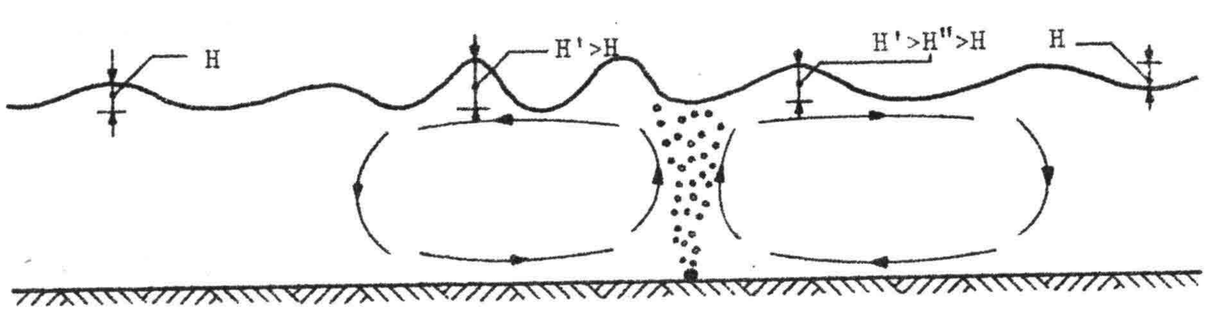
In the first figure the wave impact load as it acts on the construction and as it is schematized is indicated. The second through the fourth figures give the force as exerted by the breakwater on the soil when the factor $\chi = 1.5$.

The last figure gives the total force exerted by the breakwater on the soil.

PNEUMATIC BREAKWATER

The principle of the pneumatic breakwater is the generation of a current against the direction of wave propagation.

Through this current the waves are forced to break and so they will lose their energy. When they are not forced to break, the wave height will become greater in the region with counter current, but this height will again decrease in the region with a current in the direction of propagation of the waves.



In order to calculate this phenomenon, a wave travelling from an area with no current into an area with a current against the direction of wave propagation, will be followed.

$\omega = ck$

$c^2 = \frac{g}{k} \text{ tgh } kh$

$\omega' = c'k'$

$(\frac{\omega'}{k'})^2 = c'^2 = \frac{g}{k'} \text{ tgh } k'h$

It is necessary for both systems that the period related to a fixed coordinate system is equal.

The relationship between ω, c, k and h are physical and related to the medium.

In the case of counter current v, this relationship holds for a coordinate system moving with v. For transformation to a coordinate system which is identical to that of the approaching waves, the relationship $x' = x + vt$ holds.

When this is introduced in the cosine function:

$\chi \cos (k'x' - \omega t)$ this function will be

$$\chi \cos \{k'(x+vt) - \omega 't\} = \chi \cos [k'x - (\omega' - kv)t]$$

So the new ω, with respect to the coordinate system fixed to that of the approaching waves, is $\omega' - k'x$ and must be equal to ω.

We can now write:

$$\omega'^2 = (\omega + k'x)^2 = g\,k' \text{ tgh } k'h$$

For deep water, k'h = 1, so:

$$(\omega + k'v)^2 = gk', \text{ from which follows:}$$

$$k' = \frac{-2\omega v + g \pm \sqrt{(2\omega v - g)^2 - 4 v^2 \omega^2}}{2 v^2}$$

The value of k' can only exist if

$$(2\omega v - g)^2 > 4 v^2 \omega^2,$$

or if:

$v \leq g/4\omega = c_o/4$, with c_o = wave celerity in undisturbed deep water.
So when the current is equal or greater than $c_o/4$ the waves cannot exist, that is, they will be forced to break. In that case, a considerable energy dissipation will take place and the pneumatic breakwater will be effective.
In the case $v \ll c_o/4$, the waves will not break and no energy dissipation and consequently no wave attenuation will occur. This will also occur in the case the waves are longer then expected.

In the case of shallow water the relationship between ω and k' is not so simple and has to be solved by iteration.
The relationship in this case is

$$\omega' = (\omega + k'v)^2 = gk' \text{ tgh } k'h = g'k'.$$

In order to be able to follow the same procedure as for deep water waves g tgh k'h is written as g'. So in this case the requirement for the current velocity v, with respect to ω is

$$v \leq g'/4\omega = (g \text{ tgh } k'h)/4\omega$$

From this equation, v cannot be solved in a simple way since k' is also a function of v.
This relationship is

$$(\omega + k'v)^2 = gk' \text{ tgh } k'h.$$

For very shallow water, that is for values of tgh k'h = k'h the equations become

$$v \leq gk'h/4\omega \qquad \text{and}$$

$$(\omega + k'v)^2 = gk'^2h$$

This can be written as:

$$(gh - 4v^2)^2 = 0.$$

The equations cannot be solved when $v \geq \frac{1}{2} \sqrt{gh} = \frac{1}{2} c_o$, where c_o is in this case the celerity of wave propagation in shallow water without counter current.
When the equations cannot be solved, the wave in the area with counter current is non-existent. The physical meaning of this is that the wave is forced to break.

For circumstances between deep and very shallow water the two equations

$$(\omega + k'v)^2 = g\, k' \text{ tgh } k'h$$

$$v = (g \text{ tgh } k'h)/4\, \omega$$

have to be solved by **iteration**.
This iteration will now be executed for a specific example

h=10m H=3m T=8 sec ω=0.79

(1) L'	(2) $\frac{h}{L'}$	(3) $k'h$	(4) $tgh\ k'h$	(5) $gk'tgh\ k'h$	(6) v	(7) $k'v$	(8) $(\omega + h'v)^2$
70	.143	.90	.72	.65	2.2	.20	.98
60	.67	1.05	:78	.82	2.4	.25	1.08
50	.20	1.26	.85	.99	2.6	.33	1.25
40	.25	1.57	.92	1.44	2.8	.44	1.52
30	.33	2.07	.97	2.03	3.0	.62	1.98

L' is estimated; via a rough computation for deep water $L_o = 1.56\ T^2$, and this wave length must be considerably shorter since the water is shallow and there is a counter current. From L', the values of h/L', k'h, tgh k'h and gk'tgh k'h are calculated. These values are also tabulated in T. R. No. 4. With the second of the two equations, v is now computed, and from this value $(\omega + k'v)^2$ is determined.
According to the first equation this should be equal to gk'tgh k'h.
From comparison between the columns (5) and (8) it follows that $v = 2.92$ m/s meets both equations.

In the foregoing the wave height is not taken into consideration. When this is done, there may occur a situation in which the oncoming wave is already so steep that some shortening of the waves (due to the counter current) will increase the steepness beyond the limit of stability and so the waves will break. For this computation the transport of the energy has to be regarded since the wave height is also changing when the wave comes into the area with a counter current.
The energy transport equation is

$$c_{gr}\ E = c'_{gr}\ E' - v\ E',$$

where $c_{gr'}$ and E' are the values for the group velocity and the energy per unit of surface in the area with counter current.

For deep water the equation becomes:

$\frac{1}{2} c_o E_o = (\frac{1}{2} c' - v)E'$, where the index o indicates the undisturbed wave.

So: $E'/E_o = \frac{1}{2} c_o / (\frac{1}{2} c' - v)$

Since $E = 1/8 \;\rho g h^2$,

$E'/E_o = H^2/H_o^2$.

From this follows:

$$H'/H_o = \sqrt{1/(c'/c_o - 2v/c_o)}$$

$c/c_o = \sqrt{k_o/k'}$, as can be found from the basic formulae for the celerity. From the relationship between ω an k' the following equation can be derived, using the relationship for deep water: $c = g/\omega$.

$$\frac{k'}{k_o} = \frac{g}{\omega^2} \cdot \frac{-2\omega v + g \pm \sqrt{(2\omega v - g)^2 - 4v^2\omega^2}}{2v^2}$$

$$k'/k_o = -c_o/v + (c_o/v)^2 \left[\frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4v/c_o}\right]$$

From this the increase in wave steepness, that is

$s'/s_o = (H'/H_o)(L_o/L') = (H'/H_o)(k'/k_o)$ can be computed as a function of v/c_o.

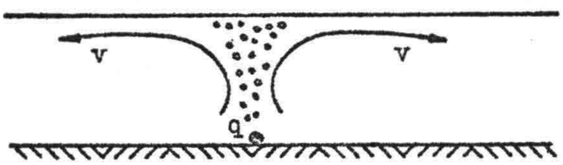
This is a rather lengthy computation which gives only a considerable decrease in the required velocity v for rather high initial steepness. For normal use it is therefore sufficient to use the first described procedure which is based on the possible existence of a wave when it is met by the counter current.

In order to design a pneumatic breakwater, it is necessary to determine the air discharge required to generate the current, which can give the required horizontal velocity. In an empirical way the following formula is obtained:

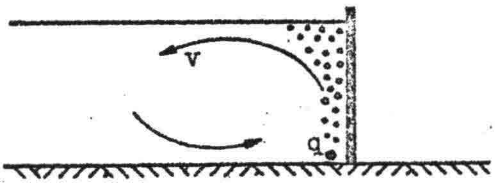
$$v = 1.46 \,(g q_o)^{1/3} \,(1 + h/h_a)^{-1/3}, \text{ in which}$$

v = horizontal velocity of the water at the surface in m/s, h = depth under the surface of the air discharge pipe, h_a = atmospheric pressure in meters water column, q_o = air discharge in $m^3/m/s$ at atmospheric pressure and g = acceleration of earth gravity = 9.81 m/s^2.
In this case the current generated by the air bubbles can flow out in two directions.
This velocity cannot be determined without further precautions from models, since the air bubbles need two meters to reach their equilibrium speed.
This is the reason that the original models in which the air discharge was scaled down according the Fronde scale, gave pessimistic results.
The above mentioned formula can be simplified to:

$q_o = (v/2.6)^3$ for dubble out flow



and $q_o = (v/3.2)^3$ for single out flow



In order to compute the required power for the air plant, the following procedure has to be followed.
For the supply of the air at a depth of h m the air has to be compressed from h_a to h meters water column. When the temperature of the air is assumed to remain constant (which is certainly allowed due to the surrounding water) the product of volume and pressure of the water remains constant.
So: $q_o h_a = q_i (h_a + h_i)$, in which q_o is the air discharge at atmospheric pressure h_a, and q_i = the discharge at a depth of h_i below the surface.

So . $q_i = \frac{q_o h_a}{h_a + h_i}$

The required power is

$$P = \rho g \int_{h_a}^{h} q_i \, dh_i = \rho g \int_{o}^{h} \frac{q_o h_a}{h_a + h_i} \, dh_i$$

$$P = \rho \, g \, h_a \, q_o \ln (h_a + h_i) \Big|_{h_a}^{h}$$

When $h_a \approx 10$ m,

$$P = \rho \, g \, 10 \, q_o \; 2,3 \; \{\log (10 + h) - \log 10\}$$

$$P = 2,3 \; \rho g q_o \; \{\log (10 + h) - 1\} \quad \text{watt}$$

Example

A wave of 8 sec (c = 12,5 m/s in deep water) has to be stopped by a pneumatic breakwater which has an air discharge pipe at a depth of ten meters below the water surface. The required horizontal velocity is v = 3.1 m/s.

For an out flow in two directions the required air discharge is $q = (v/2,6)^3 = 1,7 \; m^2/s$.

When the pneumatic breakwater has a length of 500 m,

$Q = 500 \times 1,7 = 850 \; m^3/s$.

The required power is $P = 850.\,23\;10^4 \,(\log 20 - 1) = 59000$ kw. When this breakwater has to work during 400 hours per year and the price of the electricity is f. 0,07/kwh, the exploitation costs of this breakwater will be with an efficiency coefficient of the plant of 0,7:

59000 . 400 . 0,07/0,7 = f. 2380,000/year.

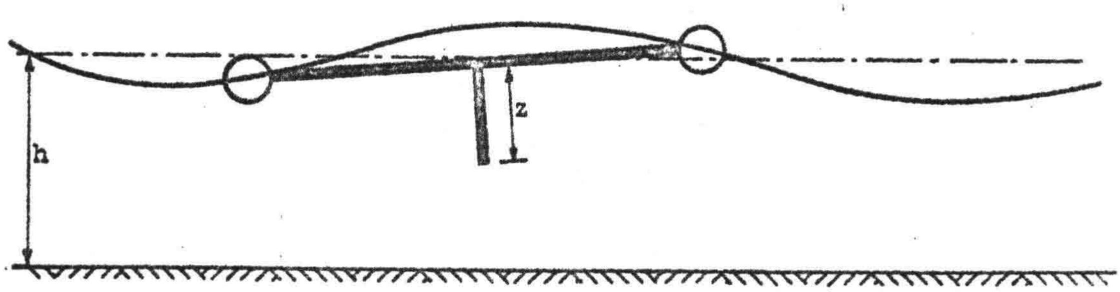
This sum is so high that for normal use these breakwaters are not a feasible proposition.

A solid breakwater of this length will cost about 500x40.000 = f. 20,000.000.

With an interest and attenuation percentage of ten percent the yearly cost of this breakwater will be f. 2.000.000.-, which is in the same order of magnitude.

More information can be found in D. H. L. publ. 42, "Increase of effective working time during operations at sea by means of movable structures", by J. J. Vinjé. In this publication more references are given.

Other temporary means to decrease wave heights are floating breakwaters. A possible solution is the following construction.



Two floats (cylinders) are connected by a construction which supports a vertical screen or bulkhead. When the distance of the cylinders is rather great there will hardly be any movement of the vertical bulkhead and the construction will have its greatest obtainable effect. It is, however, also possible to decrease the distance of the floats to such an extent that, although the screen is moving, it has a sufficient wave damping effect. To this end, the own frequency (period of oscillation) must be appreciably greater than the wave period.
The decrease of the waves can be described for a non-moving screen by

$$H_t/H_i = \sqrt{\frac{2k(h-z) + \sinh 2k(h-z)}{2kh + \sinh 2kh}}$$

in which H_t = transmitted wave height and H_i = incident wave height. This sort of floating construction is named "small width platform". Their damping effect results from the depth of the construction.

Other possibilities are "large width" platforms. In that case the damping effect results from the length of the construction.

There are three different types of this construction:

a) completely flexible, for instance a synthetic folie, pack ice (!), thin oil layer;

b) partly flexible, like a fascine mattress;

c) rigid, for instance a caisson.

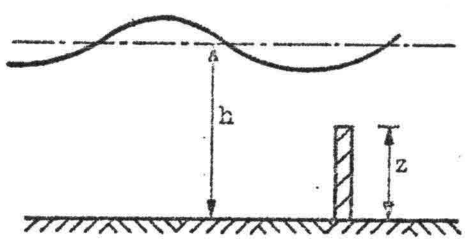
ad a) The attenuation of the wave height is most probably the result of friction between the moving water and the folie.

ad b) In this case the damping effect results from a combination of friction and vertical pressure. For a completely fixed plate the following relationship holds $H_t/H_i = \lambda/(\lambda^2 + \pi^2 w^2)^{\frac{1}{2}}$, in which λ = wave length and w = width of the construction.

ad c) Also in this case the own oscillation period of the caisson determines the damping effect of the construction.

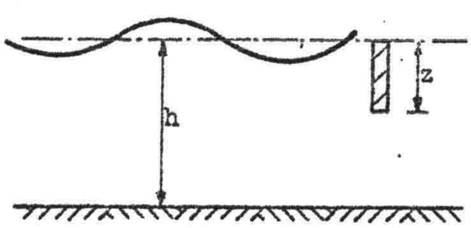
The following two formulae give the wave height attenuation for bulkheads emerging from the bed or from the surface.

For a bottom screen.



$$\frac{H_t}{H_i} = \sqrt{\frac{1 - \dfrac{2k(h-z)}{\sinh 2k(h-z)} + \dfrac{\sinh 2k(h-z)}{\sinh 2k}}{1 + 2\dfrac{kh}{\sinh 2\,kh}}}$$

For a surface screen.



$$\frac{H_t}{H_i} = \sqrt{\frac{2k(h-z) + \sinh 2k(h-z)}{2kh + \sinh 2\,kh}}$$

From a specific example the following results are obtained.

$k = 0.08\ m^{-1}$ $L = 80$ m $h = 10$ m.

$2kh' = 1.6$ $\sinh 0.08 = 0.08$ $\sinh 1.6 = 2.4$.

For a bottom screen this gives $H_t/H_i = 0.6$; so forty percent damping.
For a surface screen this gives

$H_t/H_i = 0.2$ So eighty percent damping.

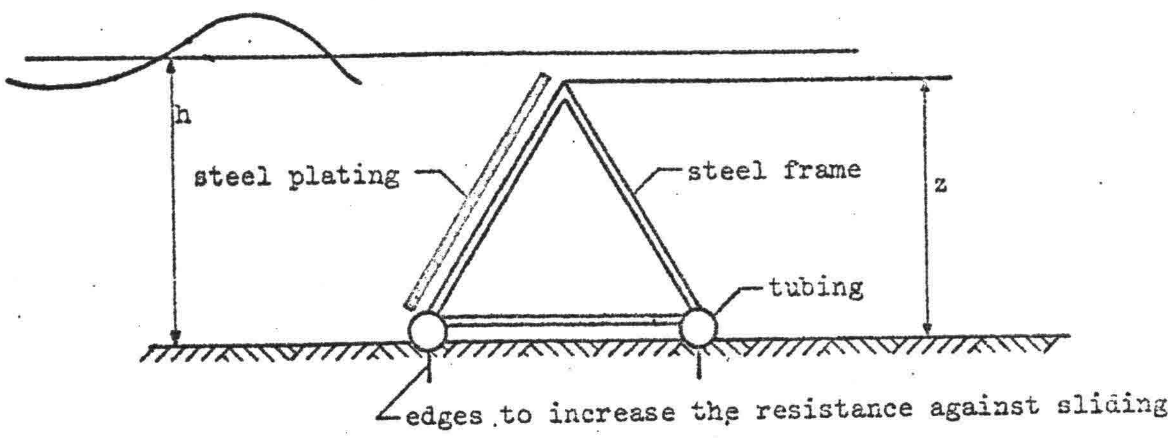
In D. H. L. Publication 42 the following information on damping of waves by slabs is given:

length slab / wave length	H_t/H_i
2	0.8
5	0.5
10	0.2

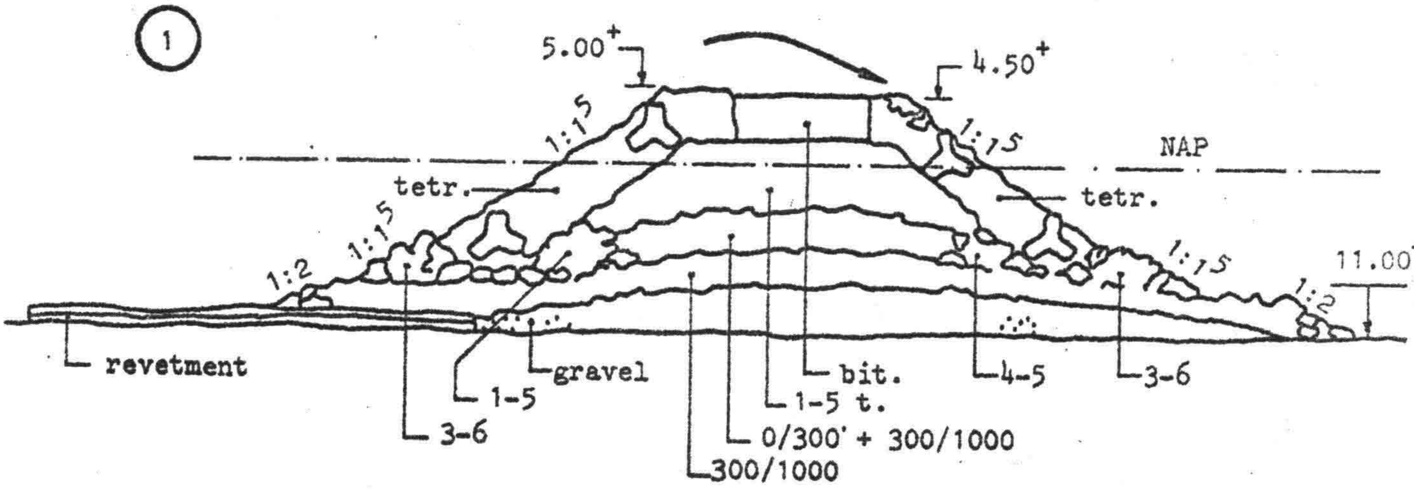
Another type of flexible breakwater is the hoovering breakwater. These are bags filled with water. The damping effect is probably caused by a wave which is generated in the bags and which is not in phase with the incoming surface wave.

Fixed constructions, other than normal breakwaters, are arrays or rows of piles, and movable, partly submerged constructions.
Pile rows are not very effective since the energy that is transmitted is proportional with the relative opening between the piles. So when the distances between the piles is equal to their diameter, the transmitted energy is fifty percent. Since the wave energy is proportional with H^2, the wave attenuation is only 0.7.

Movable constructions which are placed on the bed can have the following form



In D. H. L. Publication 42 results are given about the effect of these constructions.
Only when the top of the construction almost reaches the surface is the effect appreciable; that is, about fifty percent.


1
5.00+
4.50+
1:1.5
1:1.5
NAP
tetr.
tetr.
1:1.5
1:1.5
1:2
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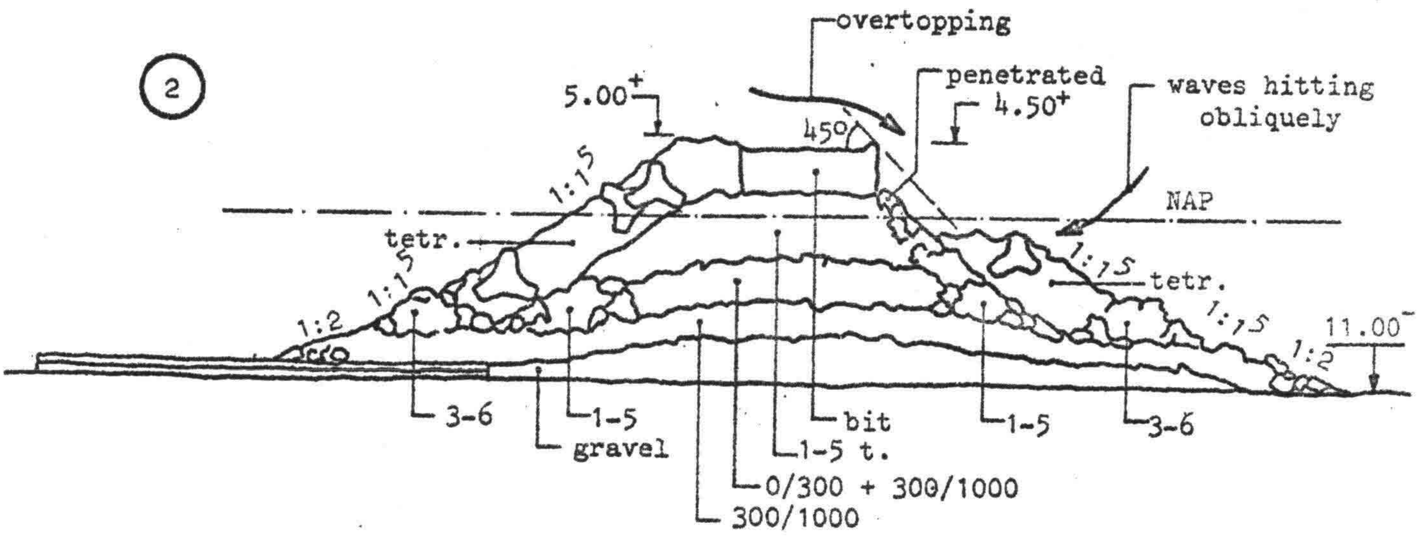
11.00-
revetment
gravel
bit.
4-5
3-6
1-5
1-5 t.
3-6
0/300 + 300/1000
300/1000


overtopping
penetrated
waves hitting obliquely
2
5.00+
4.50+
45°
1:1.5
NAP
tetr.
1:1.5
1:1.5
tetr.
1:1.5
1:2
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3-6
1-5
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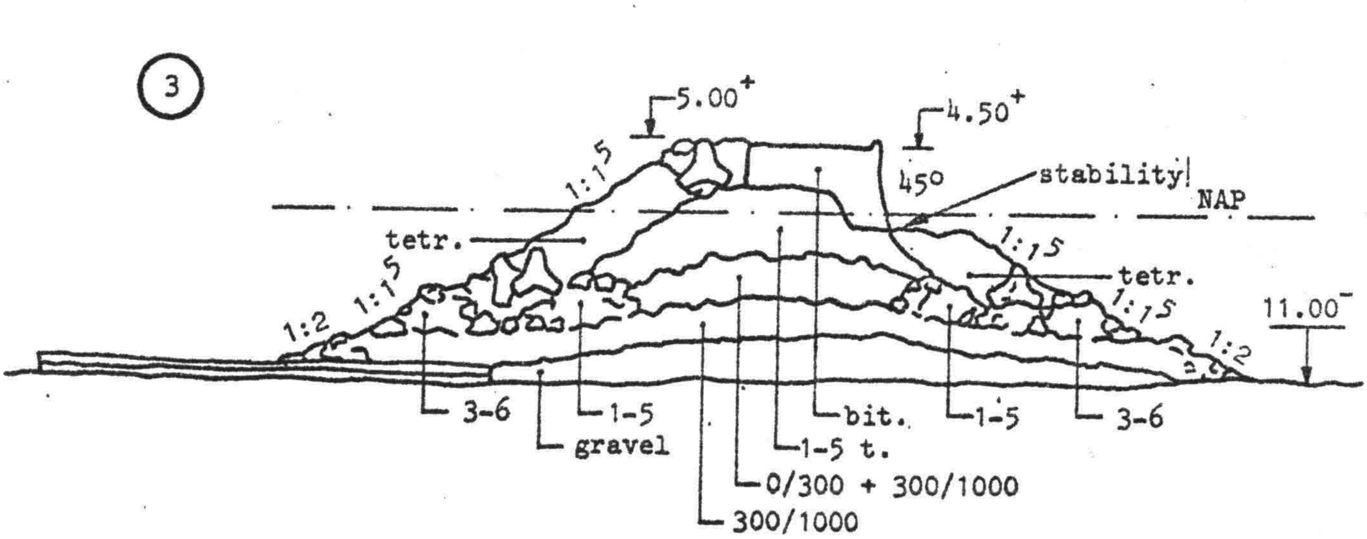
1-5
3-6
gravel
1-5 t.
0/300 + 300/1000
300/1000


3
5.00+
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stability!
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1:1.5
tetr.
1:1.5
1:1.5
1:2
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3-6
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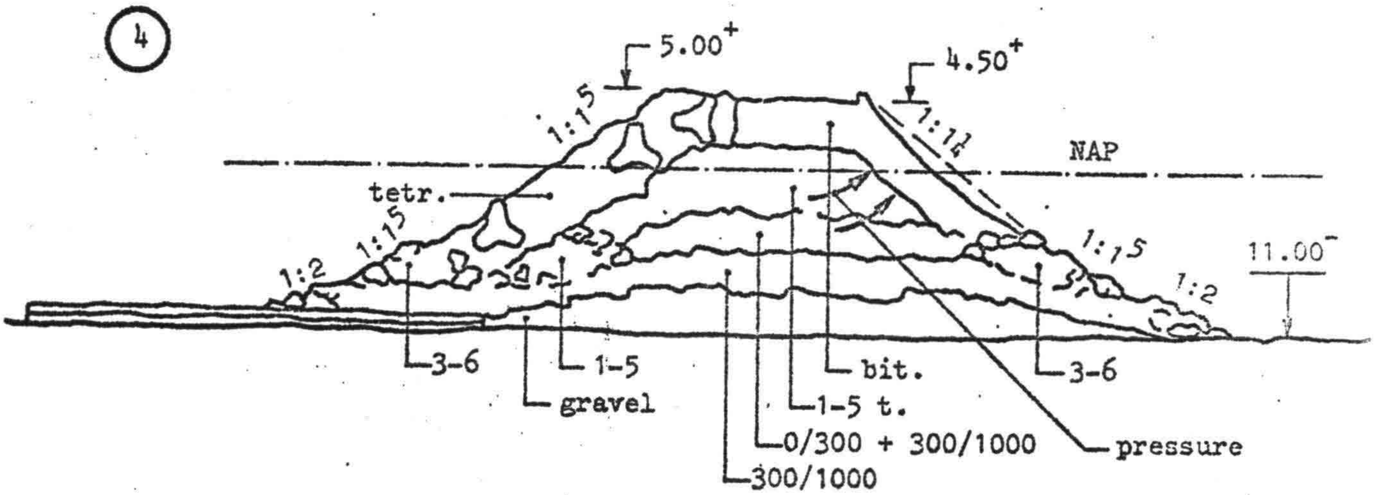
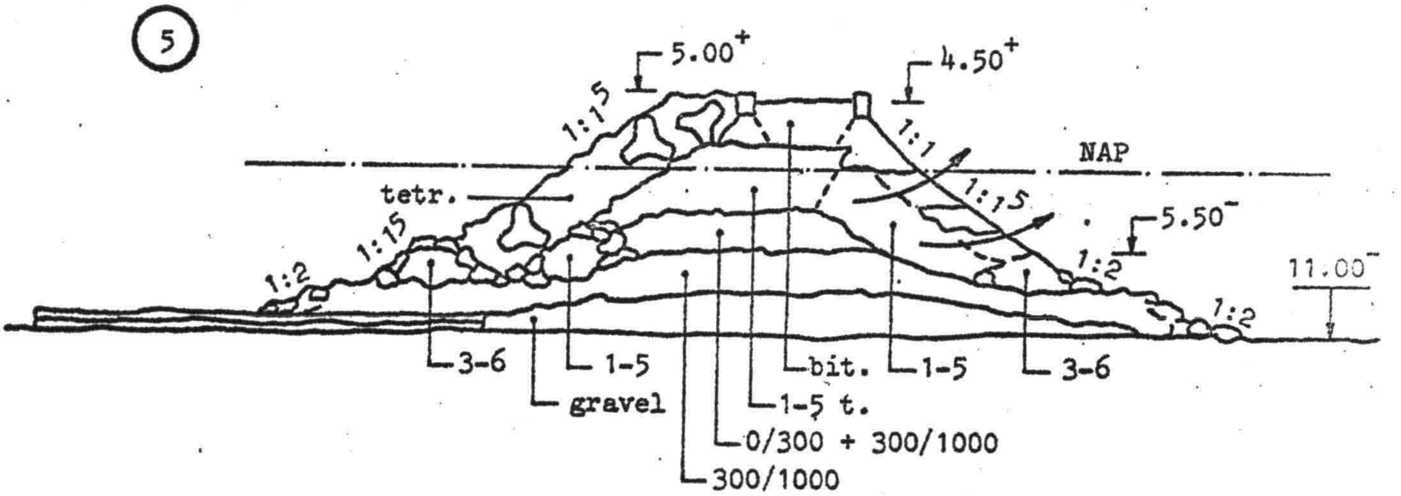
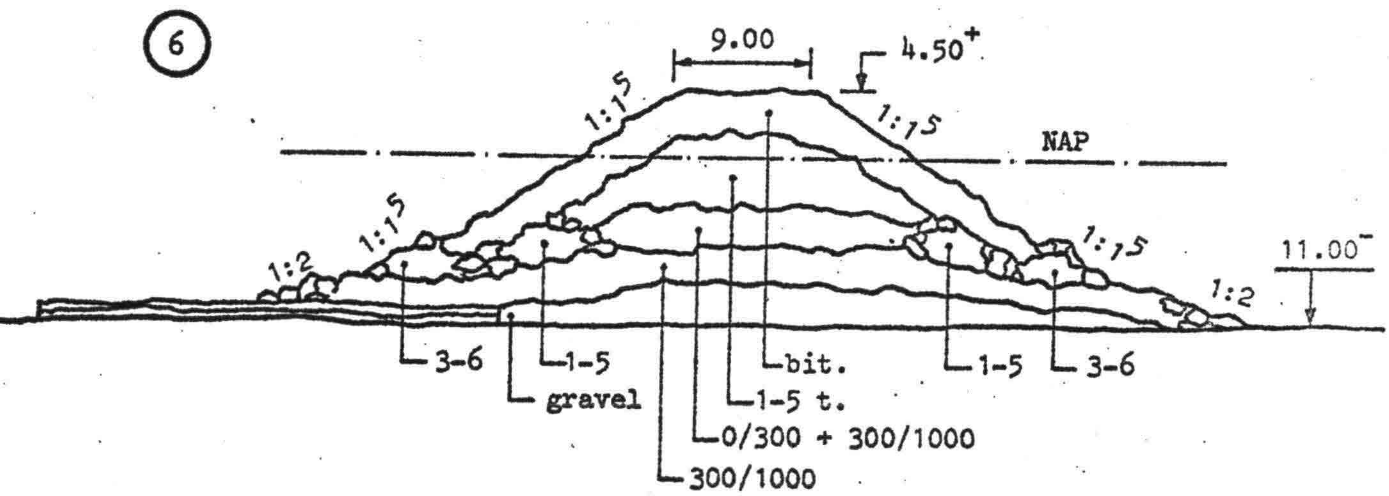
1-5 t.
0/300 + 300/1000
300/1000


4
5.00+
4.50+
1:1,5
1:1¼
NAP
tetr.
1:1,5
1:1,5
1:2
11.00−
1:2
3-6
1-5
gravel
bit.
1-5 t.
0/300 + 300/1000
300/1000
3-6
pressure


5
5.00+
4.50+
1:1,5
1:1
NAP
tetr.
1:1,5
1:1,5
5.50−
1:2
1:2
11.00−
1:2
3-6
1-5
gravel
bit.
1-5 t.
0/300 + 300/1000
300/1000
1-5
3-6


6
9.00
4.50+
1:1,5
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NAP
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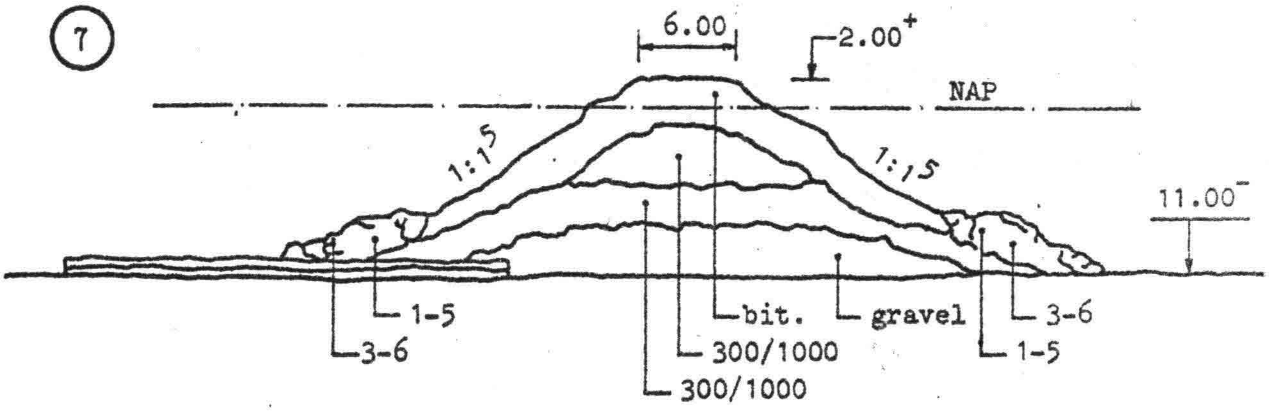
0/300 + 300/1000
300/1000
1-5
3-6


7
6.00
2.00+
NAP
1:1,5
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3-6
bit.
300/1000
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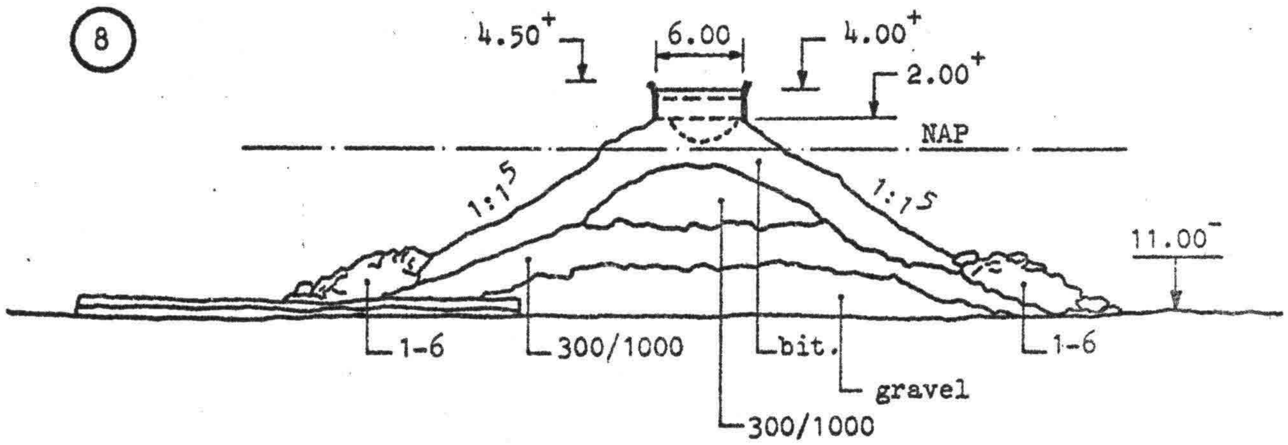
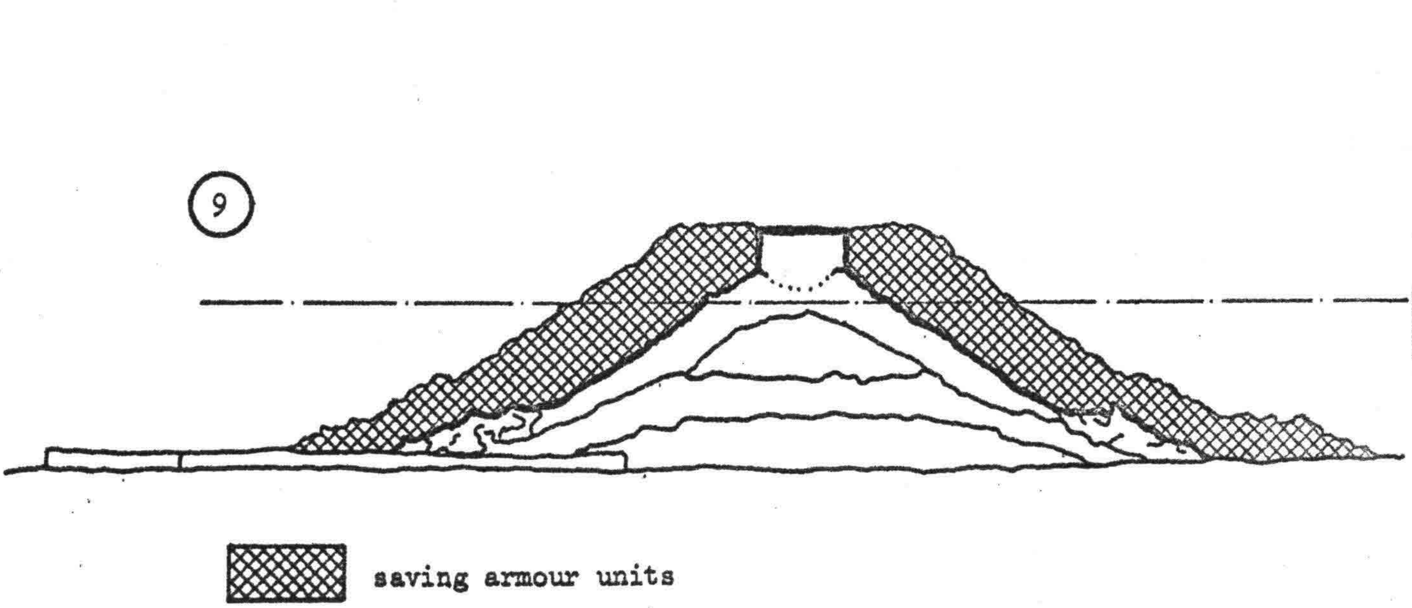
gravel
3-6
1-5


8
4.50+
6.00
4.00+
2.00+
NAP
1:1,5
1:1,5
11.00-
1-6
300/1000
bit.
gravel
300/1000
1-6


9
saving armour units