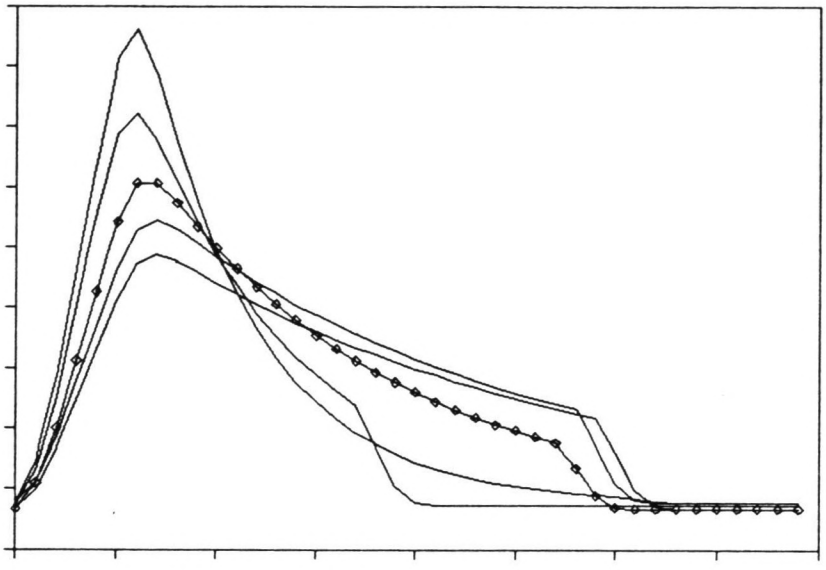
Accuracy of Flood Estimation with the Gama I Unit Hydrograph

August 1991

R.B.H. Huyskens



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ACCURACY OF FLOOD ESTIMATION

with the GAMA I unit hydrograph

PART 1

of M.Sc. thesis

by

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SUMMARY

In this study research is performed into the accuracy of flood estimation. In particular the rivers on the island of Java are considered. For these distinctive rivers, the GAMA I unit hydrograph theory was developed (Sri Harto 1985) for flood estimation.

The GAMA theory defines 3 basic characteristics to represent a unit hydrograph; time of rise (time from the beginning of the storm to the time of peak discharge), peak discharge and base time (total time of run off). For these characteristics, relationships were derived by means of regression analysis using measured data of floods and related catchment characteristics.

In this study the GAMA equations are again derived by means of a program (REGRES) including the calculation of the standard deviations and the correlations of the regression coefficients. These indicate the accuracy of the model.

To include the inaccuracy of the input data a program (SIMCO) is constructed in which the input data (measurements) are extended with stochastic features. It is assumed that all data are normally distributed. A simulation is performed based on the Monte Carlo simulation technique to derive a set of 150 equations for each GAMA equation. The standard deviations and correlations are calculated and indicate the sensitivity of the GAMA equations due to inaccurate measurements.

By means of a fault tree for each GAMA equation the propagation of errors in the derivation is determined in a analytical and numerical way; analytically by means of the mean value approach and numerically by means of Monte Carlo simulation. The influence of the stochastic input data is very large and mainly dued to the variation of the measured unit hydrograph, derived from discharge measurements. The equations for the peak discharge and the base time seem to be very sensitive. Expressed in coefficients of variation the predictions can deviate 200% and 80% respectively (time of rise 25%).

For further investigation only the model accuracy is taken into account; the predictions of the time of rise, peak discharge and base time deviate approximately 20% , 28% and 15% respectively.

For calculation of the total hydrograph another program (HYDROGRAPH) is constructed. This program calculates the total hydrograph for a certain rainfall depth and rainfall distribution. The basic characteristics of the unit hydrograph are assumed to be normally distributed. The program generates the required input data from its distributions. A set of 75 hydrographs can be simulated.

The model is applied to the river Putih in Java. Calculations show that the accuracy of the predicted maximum discharge depends highly on the rainfall distribution. The rainfall depth has no influence on the accuracy.

The model HYDROGRAPH can be applied to every river for which the GAMA equations are valid and shows the accuracy of prediction.

PREFACE

Within the framework of research into the hydrologic and morphologic processes of rivers on volcano slopes an investigation into the accuracy of both hydrologic and morphologic predictions is performed.
This report is the first part of an extended M.Sc. thesis and concerns the accuracy of flood estimation for the rivers on the island of Java.
For this purpose the writer stayed 2 months in Surabaya Indonesia at the Institute for Technology (ITS).

The writer wishes to thank :

- Prof. Dr. M. de Vries for his enthusiastic guidance during the completion of both parts of the writer 's thesis.

- Mr. H.R. Vermeulen for his encouraging guidance and support.

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CONTENTS

1. INTRODUCTION

Hydrologic models for flood estimation can be classified as deterministic, parametric, stochastic or a combination of these.

A deterministic model would be one arrived at through consideration of the underlying physical relationships and would require no experimental data for its application.

A parametric model may be thought of as deterministic in a sense that once model parameters are determined, the model always produces the same output from a given input. On the other hand, a parametric model is stochastic in the sense that parameter estimates depend on observed data and will change as the observed data changes.

A stochastic model is one whose outputs are predictable only in a statistical sense. With a stochastic model, repeated use of a given set of model inputs produces output that are not the same but follow certain statistical patterns.

Because the input for these models mostly have stochastic features it is expectable that the output is also stochastic. From such an output judgement can be given on the accuracy of the model prediction.

The aim of this study is focussed on the accuracy of flood estimation for the rivers of Java by means of the GAMA I unit hydrograph. To determine the accuracy of this empirical, parametric model some stochastic elements are added. These elements find their origin in errors of various kind. The errors include measurements errors, data transmittal errors, processing errors and others. In this analysis it is assumed that the errors are random errors following the normal distribution.

Research on the model derivation is required to locate sources of errors. Therefore the model will be described first. From here the accuracy of the model parameters is determined. For this purpose use is made of the mean value approach next to the monte carlo simulation technique.

Because of its empirical character the accuracy consists of the model accuracy and the accuracy of the measured data, which are used in the model derivation.

Finally the model with its stochastic model parameters is applied to estimate flood with a certain return period for the river Putih in East-Java.

- N : Number of existing rain gauges
- SIM: Symmetry factor
- S : Average main stream slope.

The catchment characteristics are discussed in appendix IX.

2.2.4 Rainfall distribution

For a workable application of the unit hydrograph theory, hourly rainfall data are needed. These are derived from the 24-hourly rainfall data.
To determine the duration of a storm, use is made of "depth-area-duration relationships" by Melchior. Then a hourly distribution is assumed according to ECI (Engineerings Consultants Inc.), which showed good results for Java.

2.2.5 Discharge

The determination of the relationship between the waterlevel and the discharge at a certain gauging site is done by plotting a rating curve. For each catchment a rating curve is determined graphically.
To determine the magnitude of a flood with a specified return period, analysis of flood frequency is needed. Therefore floods for analysis need to be selected.
After inspection of the available record of each catchment the highest water levels are determined. By means of rating curves these water levels are transformed into discharges. Then the series are characterized statistically. After plotting on probability paper, statistical tests of goodness of fit are applied.
The frequency equation which can commonly be applied is:

$$X_T = \overline{X} + K * S \qquad (2\text{-}2)$$

In which

- X_T : Discharge with T years return period [m^3/s]
- $\overline{X}$: Mean discharge [m^3/s]
- K : Frequency factor, depending on the type of the distribution
- S : Standard deviation.

2.2.6 Unit hydrograph

Unit hydrographs can be derived from the selected hydrographs and the corresponding rainfall. A unit hydrograph of 1 mm effective rainfall within 1 hour is selected. Base flow is separated from the total hydrograph by the straight line method. Collins's iterative method is used to derive the unit hydrograph from the selected hydrographs.
For each catchment some unit hydrographs are derived. By averaging those the representative unit hydrograph for each catchment is determined.
Based on the result of that derivation the three basic characteristics of the unit hydrograph could be determined for each catchment. Those are:

- Time of Rise (TR)
- Peak Discharge (QP)
- Base Time (TB).

2.2 Determination of the measured unit hydrograph

2.2.1 Introduction

To determine a unit hydrograph for each catchment, measurements on rainfall and discharges over a considerable long period are required.
The rainfall network in the selected catchments will be discussed in order to check the reliability of the rainfall data. The frequency of rainfall will be determined so that a probability of occurence of a certain rainfall depth can be calculated. The hourly distribution, on which the unit hydrograph is based, will also be discussed.
Measurements on discharge and derivation of the frequency curves are discussed briefly.
Finally the measured unit hydrograph is determined.

2.2.2 Rainfall network

Rainfall depth measured at a certain point is expected to represent the rainfall of a certain area. In tropical area's rainfall varies greatly from place to place. Therefore more rainfall stations are needed. The number of stations required is calculated with Kagan's Method.
With a specified error of 5 and 10% (the error introduced by averaging point rainfall) the number of required rainfall stations is determined. It can be concluded that the number of excisting stations in each catchment is smaller than the minimum demanded by Kagan's principle. Therefore all excisting stations will be used for further analysis.
With the number of excisting stations, Kagan's specified error for each cathment varies from 10 to 60% .

2.2.3 Rainfall frequency

The frequency curve is determined in the following way:

- In a year a maximum rainfall value of a certain station is determined. Then for each catchment the values are averaged.
- This is done for all the years of which data are available.
- After combinations of its statistics, the series are plotted on probability paper. Tests of goodness of fit are applied and a frequency curve is constructed for each catchment.

To estimate the average rainfall for each catchment the maximum rainfall occuring at one rainfall station is multiplied by a reduction coefficient B.
A relationship for B is determined as a function of some catchment characteristics. This is done by means of regression analysis. Therefore measured B values for each catchment are required. To determine these values the average catchment rainfall is derived from the maximum catchment rainfall by means of Thiessen's Polygon. Then B is calculated by dividing the average value by the maximum value. This is done for every catchment and every year. The required B value for regression is averaged over the years of measurements.
The result of the regression analysis:

$$B = 1.5518 * A^{-0.1491} * N^{-0.2725} * SIM^{-0.0259} * S^{-0.0733} \qquad (2\text{-}1)$$

In which:

- A :Catchment area [km^2]

- N : Number of existing rain gauges
- SIM: Symmetry factor
- S : Average main stream slope.

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- Time of Rise (TR)
- Peak Discharge (QP)
- Base Time (TB).

The recession limb of the unit hydrograph is estimated by:

$$Q_t = Q_{max} * e^{-t/k} \qquad (2\text{-}3)$$

In which

Q_t	:	Discharge at t hours after peak discharge [m^3/s]
Q_{max}	:	Peak discharge [m^3/s]
t	:	Time passed since peak discharge [hours]
k	:	Storage coefficient [hours].

With Collin's method the storage coefficient is determined.
Because the Q_t function decreases assymptotically to zero it is stated that from the time TB-1 to TB the Q_t function decreases linearly to zero.
For a graphical presentation see Fig. 2-1.

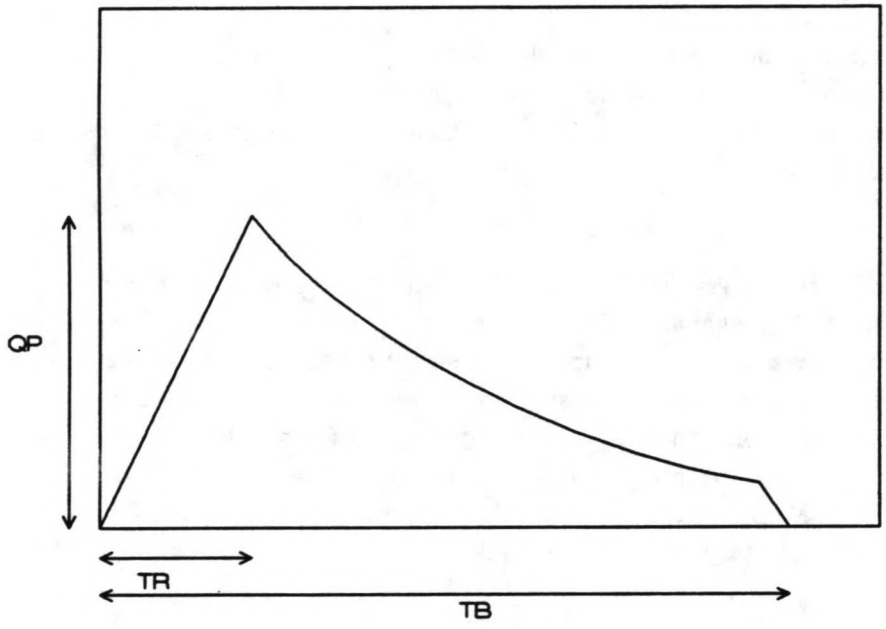

Fig. 2-1 Unit hydrograph characteristics

2.3 Results of the regression analysis

By means of regression analysis relations are found between the catchment characteristics and the flow elements.
The results of this analysis are:

$$TR = 0.43 * \left(\frac{L}{100*SF}\right)^3 + 1.0665*SIM + 1.2775 \qquad (2\text{-}4)$$

$$QP = 0.1836 * A^{0.5886} * JN^{0.2381} * TR^{-0.4008} \qquad (2\text{-}5)$$

3. ACCURACY OF THE MEASUREMENTS

3.1 Introduction

To determine the accuracy of flood estimation by the GAMA I unit hydrograph method (GAMA method) the derivation of it should be analyzed step by step. In every step assumptions are made and errors introduced. From the first assumptions to the final result the propagation of those errors determine the reliability of the method.
In general, the most likely errors in data used for unit hydrograph derivation according to Laurensons and O'Donnel (1969) are those due to :

- Assumption of a uniform loss rate
- Over or under estimation of the total storm rainfall
- Errors in the discharge rating curve, especially due to extrapolation of this curve
- Erroneous base flow seperation
- Lack of synchronization between the rainfall and streamflow record
- Lack of synchronization between the rainfall and streamflow records of different stations on the catchment.

Next, measurements on required data also involve errors, which are mainly caused by:

- Instrumental inaccuracy
- Human observation.

These errors have to be accepted.
The first assumption made in the derivation of the GAMA method is the statement that the data used are assumed to be the best available. From this point this study starts.
The determination of the measured unit hydrograph will briefly be discussed. Because of the many assumptions made on stochastic processes, many errors are involved. Therefore only a few remarks are put to the steps taken. Analysis of the propagation of errors is not made because it would show high rates of variability in the measured unit hydrograph.
Next, the measurement of the catchment characteristics is discussed for this also includes a source of errors.

3.2 Measured unit hydrograph

Rainfall network

The excisting rainfall network in Java is not reliable. The number of excisting rainfall stations is lower than required by Kagan's principle. If this principle may be used in Java then it is shown (Sri Harto 1985) that the specified error for each catchment varies from 10 to 60% of the true value.

Rainfall frequency

The frequency curve constructed for each catchment is dependent upon the accuracy of the rainfall measurements at all the rainfall stations in the catchment. Because all the available stations are used , both manual and automatic, the risk of poor measurement is high.
To average the maximum rainfall a reduction coefficient B is used. This coefficient is related to some catchment characteristics, as is shown in chapter 2. The accuracy of B is dependent upon :

- Accuracy of measured B and catchment characteristics
- Accuracy of the regression.

For every hour the rainfall depth is calculated. The effective rainfall depth is found after substraction of the ϕ-index. For the last 4 hours the ϕ-index exceeds the rainfall depth and the effective rainfall depth is set to zero.
In this case 3 hydrographs can be constructed by multiplying the effective rainfall depths by the peak discharge QP. The sum of the 3 hydrographs plus the base flow QB gives the total hydrograph.
This procedure can easily be pursued by means of a spreadsheet program. A plot of the hydrograph is given in Fig. 2-2.
The maximum discharge seems to be 38.64 m^3/s.

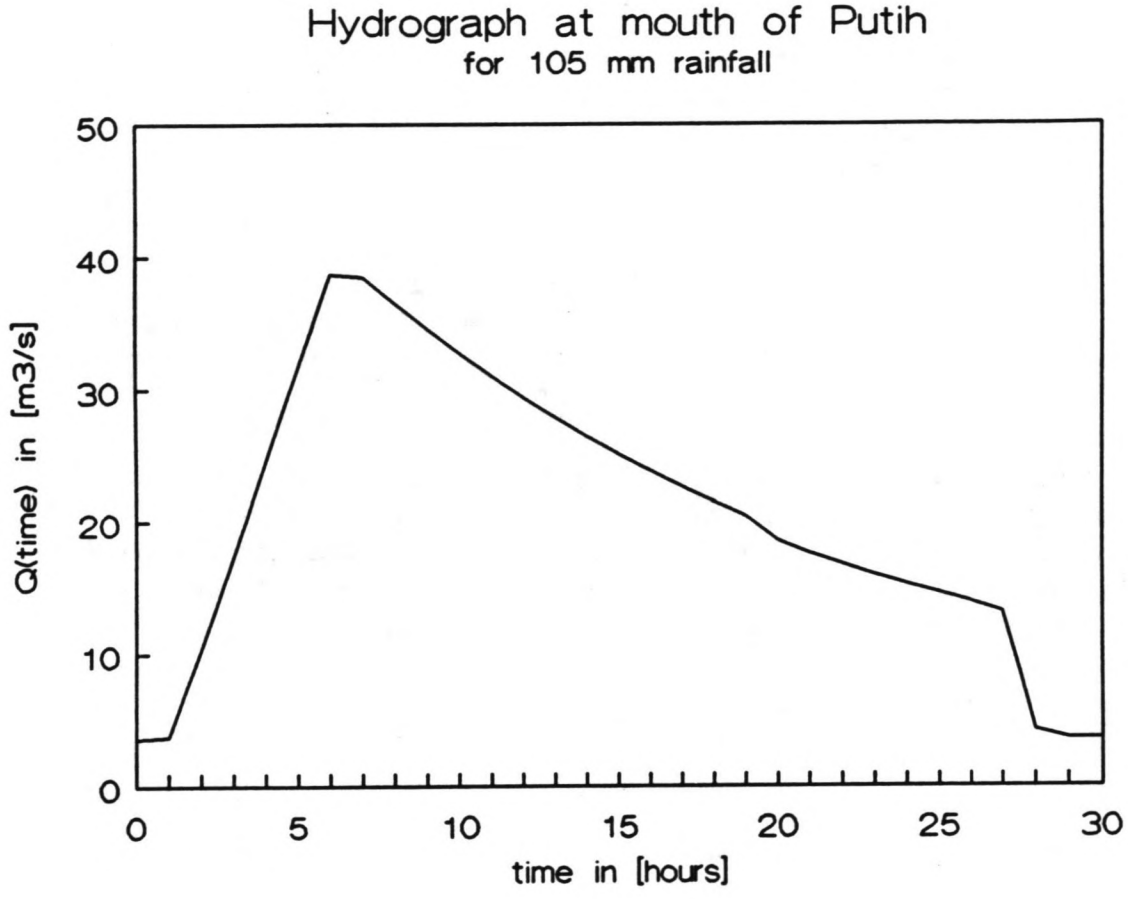

Fig. 2-2 Example of a hydrograph at the mouth (muara) of the kali Putih.

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- Accuracy of measured B and catchment characteristics
- Accuracy of the regression.

The latter showed (Sri Harto 1985) a low multiple correlation coefficient. It is concluded by Sri Harto that B is not useful in estimating the average rainfall of a catchment.
Further research on the accuracy of B will therefore not be performed in this study.

Rainfall distribution

It is very difficult to determine the hourly distribution needed for the unit hydrograph. By assuming several distributions the effect on the hydrograph characteristics can be determined. Then a coefficient of variation can be determined. In this stage this will not be analysed. However the influence of several distributions on the accuracy of the total hydrograph will be discussed in chapter 6.

Discharge

For discharge measurements the same type of errors occurs as in rainfall measurements. Besides the accuracy of the rating curves are of importance.
The frequency equation derived, shows a deviation around the mean discharge.

Unit hydrograph

The selected hydrographs for each catchment are transformed into unit hydrographs by means of Collins's method. The unit hydrograph of the last iteration applied differs less than 5% from the previous iteration result.
For each catchment some 4 to 14 unit hydrographs are derived.
After that the TR, QP and TB are averaged for each catchment. Of interest for the accuracy is the standard deviation of the derived unit hydrographs for each catchment.
The standard deviation is calculated according to "van Soest (1985)" as :

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} \qquad (3\text{-}1)$$

In which

- n : Number of measured values
- x_i : Measured value
- $\bar{x}$: Mean value, from

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Because of the small number of measurements the factor 1/n-1 is used instead of 1/n.
Results of these calculations are shown in Tab. 3-1 .

3.3 Catchment characteristics

To measure the various catchment characteristics use is made of topographic maps (scale 1:50.000) with the obtainable maximum accuracy. Other instruments used are planimeter and chartometer. Errors in measurements are dued to:

- Human observation
- Inaccuracy of topographic maps
- Inaccuracy of the instruments.

The inaccuracy of the maps causes the main errors. It is difficult to estimate a coefficient of variation on the measurements of catchment characteristics. Therefore a value is initially assumed in this study. The influence of different values on the results will be examined.

$$\frac{\delta E}{\delta \alpha} = \sum_{i=1}^{n} 2 s_i \frac{\delta s_i}{\delta \alpha} = 0 \qquad (4\text{-}6)$$

$$\frac{\delta E}{\delta ß} = \sum_{i=1}^{n} 2 s_i \frac{\delta s_i}{\delta ß} = 0 \qquad (4\text{-}7)$$

$$\frac{\delta E}{\delta \delta} = \sum_{i=1}^{n} 2 s_i \frac{\delta s_i}{\delta \delta} = 0 \qquad (4\text{-}8)$$

Taylor series of s_i and $\delta s_i/\delta A$, $\delta s_i/\delta \alpha$, $\delta s_i/\delta ß$ and $\delta s_i/\delta \delta$ at the optimum where E = 0 show with $x' = x + e_x$:

$$s_i(A', \alpha', ß', \delta') = s_i(A, \alpha, ß, \delta) + e_A \frac{\delta s_i}{\delta A} + e_\alpha \frac{\delta s_i}{\delta \alpha} + e_ß \frac{\delta s_i}{\delta ß} + e_\delta \frac{\delta s_i}{\delta \delta} + \ldots$$

$$\frac{\delta s_i}{\delta A}(A', \alpha', ß', \delta') = \frac{\delta s_i}{\delta A}(A, \alpha, ß, \delta) + e_A \frac{\delta^2 s_i}{\delta A^2} + e_\alpha \frac{\delta^2 s_i}{\delta A \delta \alpha} + e_ß \frac{\delta^2 s_i}{\delta A \delta ß} + e_\delta \frac{\delta^2 s_i}{\delta A \delta \delta} + \ldots$$

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Substituting these Taylor series in the equations (4-4) to (4-8) results in:

$$\sum_{i=1}^{n} 2\left(s_i + e_A \frac{\delta s_i}{\delta A} + e_\alpha \frac{\delta s_i}{\delta \alpha} + e_ß \frac{\delta s_i}{\delta ß} + e_\delta \frac{\delta s_i}{\delta \delta}\right) * \left(\frac{\delta s_i}{\delta A} + e_A \frac{\delta^2 s_i}{\delta A^2} + e_\alpha \frac{\delta^2 s_i}{\delta A \delta \alpha} + e_ß \frac{\delta^2 s_i}{\delta A \delta ß} + e_\delta \frac{\delta^2 s_i}{\delta A \delta \delta}\right) = 0$$

4. DERIVATION OF GAMA-EQUATIONS

4.1 Introduction

By means of multiple regression analysis, equations for TR, QP, TB and K are derived. For this purpose computer software is available like SAS and SPSS. Because of some limitations involved in these programs a new program is written in Turbo Pascal 5.5 .This program, called "REGRES"(see appendix I), is based on the Gauss-Newton non-linear regression method, which will be discussed here.
The program has been tested by comparing the calculated regression coefficients of REGRES with those done by SPSS and SAS.
The results will finally be discussed.

4.2 Gauss-Newton non-linear regression

To discuss the Gauss-Newton algorithm the following function will be used for example:

$$Y = A*X1^{\alpha}*X2^{\beta}*X3^{-\delta} \qquad (4\text{-}1)$$

This equation has the same features as the equation for QP.
The dependent variable is Y and the independent variables are X1, X2 and X3. The regression coefficients A, α, β and δ will be calculated for n measured values of the variables.
The error function:

$$E = \sum_{i=1}^{n} \left(Y_i - A*X1_i^{\alpha}*X2_i^{\beta}*X3_i^{-\delta}\right)^2 \qquad (4\text{-}2)$$

In which subscript i represents a measured value.
To minimize the error function the derivatives of the function need to be zero, like:

$$\frac{\delta E}{\delta A}=0,\ \frac{\delta E}{\delta \alpha}=0,\ \frac{\delta E}{\delta \beta}=0 \text{ and } \frac{\delta E}{\delta \delta}=0$$

Assume:

$$s_i = \left(Y_i - A*X1_i^{\alpha}*X2_i^{\beta}*X3_i^{-\delta}\right) \qquad (4\text{-}3)$$

Substitution in (4-2) and the derivatives results in:

$$E = \sum_{i=1}^{n} s_i^2 \qquad (4\text{-}4)$$

$$\frac{\delta E}{\delta A} = \sum_{i=1}^{n} 2s_i \frac{\delta s_i}{\delta \alpha} = 0 \qquad (4\text{-}5)$$

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$$\frac{\delta s_i}{\delta \delta}(A', \alpha', ß', \delta') = \frac{\delta s_i}{\delta \delta}(A, \alpha, ß, \delta) + e_A \frac{\delta^2 s_i}{\delta A \delta \delta} + e_\alpha \frac{\delta^2 s_i}{\delta \alpha \delta \delta} + e_ß \frac{\delta^2 s_i}{\delta ß \delta \delta} - e_\delta \frac{\delta^2 s_i}{\delta \delta^2} + \ldots$$

Substituting these Taylor series in the equations (4-4) to (4-8) results in:

$$\sum_{i=1}^{n} 2\left(s_i + e_A \frac{\delta s_i}{\delta A} + e_\alpha \frac{\delta s_i}{\delta \alpha} + e_ß \frac{\delta s_i}{\delta ß} + e_\delta \frac{\delta s_i}{\delta \delta}\right) * \left(\frac{\delta s_i}{\delta A} + e_A \frac{\delta^2 s_i}{\delta A^2} + e_\alpha \frac{\delta^2 s_i}{\delta A \delta \alpha} + e_ß \frac{\delta^2 s_i}{\delta A \delta ß} + e_\delta \frac{\delta^2 s_i}{\delta A \delta \delta}\right) = 0$$

$$\sum_{i=1}^{n} 2\left(s_i+e_A\frac{\delta s_i}{\delta A}+e_\alpha\frac{\delta s_i}{\delta\alpha}+e_\beta\frac{\delta s_i}{\delta\beta}+e_\delta\frac{\delta s_i}{\delta\delta}\right)*\left(\frac{\delta s_i}{\delta\alpha}+e_A\frac{\delta^2 s_i}{\delta\alpha\delta A}+e_\alpha\frac{\delta^2 s_i}{\delta\alpha^2}+e_\beta\frac{\delta^2 s_i}{\delta\alpha\delta\beta}+e_\delta\frac{\delta^2 s_i}{\delta\alpha\delta\delta}\right)=0$$

$$\sum_{i=1}^{n} 2\left(s_i+e_A\frac{\delta s_i}{\delta A}+e_\alpha\frac{\delta s_i}{\delta\alpha}+e_\beta\frac{\delta s_i}{\delta\beta}+e_\delta\frac{\delta s_i}{\delta\delta}\right)*\left(\frac{\delta s_i}{\delta\beta}+e_A\frac{\delta^2 s_i}{\delta\beta\delta A}+e_\alpha\frac{\delta^2 s_i}{\delta\beta\delta\alpha}+e_\beta\frac{\delta^2 s_i}{\delta\beta^2}+e_\delta\frac{\delta^2 s_i}{\delta\beta\delta\delta}\right)=0$$

$$\sum_{i=1}^{n} 2\left(s_i+e_A\frac{\delta s_i}{\delta A}+e_\alpha\frac{\delta s_i}{\delta\alpha}+e_\beta\frac{\delta s_i}{\delta\beta}+e_\delta\frac{\delta s_i}{\delta\delta}\right)*\left(\frac{\delta s_i}{\delta\delta}+e_A\frac{\delta^2 s_i}{\delta A\delta\delta}+e_\alpha\frac{\delta^2 s_i}{\delta\alpha\delta\delta}+e_\beta\frac{\delta^2 s_i}{\delta\beta\delta\delta}+e_\delta\frac{\delta^2 s_i}{\delta\delta^2}\right)=0$$

This multiplication shows after neglecting the second derivatives and squared parts:

$$\sum_{i=1}^{n}\left(s_i\frac{\delta s_i}{\delta A}+e_\alpha\frac{\delta s_i}{\delta A}\frac{\delta s_i}{\delta\alpha}+e_\beta\frac{\delta s_i}{\delta A}\frac{\delta s_i}{\delta\beta}+e_\delta\frac{\delta s_i}{\delta A}\frac{\delta s_i}{\delta\delta}\right)=0 \qquad (4\text{-}9)$$

$$\sum_{i=1}^{n}\left(s_i\frac{\delta s_i}{\delta\alpha}+e_A\frac{\delta s_i}{\delta\alpha}\frac{\delta s_i}{\delta A}+e_\beta\frac{\delta s_i}{\delta\alpha}\frac{\delta s_i}{\delta\beta}+e_\delta\frac{\delta s_i}{\delta\alpha}\frac{\delta s_i}{\delta\delta}\right)=0 \qquad (4\text{-}10)$$

$$\sum_{i=1}^{n}\left(s_i\frac{\delta s_i}{\delta\beta}+e_A\frac{\delta s_i}{\delta\beta}\frac{\delta s_i}{\delta A}+e_\alpha\frac{\delta s_i}{\delta\beta}\frac{\delta s_i}{\delta\alpha}+e_\delta\frac{\delta s_i}{\delta\beta}\frac{\delta s_i}{\delta\delta}\right)=0 \qquad (4\text{-}11)$$

$$\sum_{i=1}^{n}\left(s_i\frac{\delta s_i}{\delta\delta}+e_A\frac{\delta s_i}{\delta\delta}\frac{\delta s_i}{\delta A}+e_\alpha\frac{\delta s_i}{\delta\delta}\frac{\delta s_i}{\delta\alpha}+e_\beta\frac{\delta s_i}{\delta\delta}\frac{\delta s_i}{\delta\beta}\right)=0 \qquad (4\text{-}12)$$

In matrix notation these equations can easily be solved for e_A, e_α, e_β and e_δ.
The results :

$$e_A=-\frac{1}{\sum_{i=1}^{n}\left(\frac{\delta s_i}{\delta\delta}\frac{\delta s_i}{\delta A}\right)}*\sum_{i=1}^{n}\left(\frac{1}{3}\frac{\delta s_i}{\delta\delta}s_i\right) \qquad (4\text{-}13)$$

$$e_\alpha=-\frac{1}{\sum_{i=1}^{n}\left(\frac{\delta s_i}{\delta\beta}\frac{\delta s_i}{\delta\alpha}\right)}*\sum_{i=1}^{n}\left(\frac{1}{3}\frac{\delta s_i}{\delta\beta}s_i\right) \qquad (4\text{-}14)$$

4.3 Results

The program "REGRES" calculates the values of the regression coefficients in the optimum (Error function minimal), the matrix of covariances cov(z) and the correlation matrix.
Calculations show that the optimum is reached within 30 iterations.
The output for TR, QP, TB and K are shown in Tab. 4-1 to 4-4. For the basic equations see §2.3.

Tab. 4-1 Time of Rise

$$Y = B*\left(\frac{X1}{100X2}\right)^3 + C*X3 + D$$

Calculation of the regression coefficients for TR

Iteration step 17

B	:	0.431906	eB :	0.000001
C	:	1.065065	eC :	-0.000027
D	:	1.283399	eD :	0.000064

sum of squares of the residual: 40.954657

The covariance matrix cov(z)

B	C	D
0.002371	0.001864	-0.008888
0.001864	0.101239	-0.072839
-0.008888	-0.072839	0.135287

The correlation matrix of the parameter estimates

B	C	D
1.000000	0.120321	-0.496269
0.120321	1.000000	-0.622386
-0.496269	-0.622386	1.000000

End of computation

Tab. 4-2 Peak discharge

$$Y = A*X1^{\alpha}*X2^{\beta}*X3^{-\delta}$$

The calculation of the regression coefficients for QP

Iteration step 29

A	:	0.170042	e_A	:	0.000000
α	:	0.636760	e_α	:	0.000000

$$\frac{\delta s_i}{\delta \beta} = -A*X1_i^{\alpha}*X2_i^{\beta}*X3_i^{-\delta}*\ln(X2_i) \qquad (4\text{-}21)$$

$$\frac{\delta s_i}{\delta \delta} = A*X1_i^{\alpha}*X2_i^{\beta}*X3_i^{-\delta}*\ln(X3_i) \qquad (4\text{-}22)$$

The iteration procedure is started in the initial values (A_1, α_1, β_1, δ_1). The values for e_A, e_α, e_β and e_δ are calculated. These values show deviations from the optimum coefficient values. The next iteration step uses the adapted coefficients:

$A_2 = A_1 + e_A$
$\alpha_2 = \alpha_1 + e_\alpha$
$\beta_2 = \beta_1 + e_\beta$
$\delta_2 = \delta_1 + e_\delta$.

In each iteration e_A, e_α, e_β and e_δ are calculated. Those values need to converge to zero, the optimum.
The iteration procedure now continues till the values of e_A, e_α, e_β and e_δ are smaller than the desired accuracy.

Of interest for this study are the standard deviations of the regression coefficients. They are estimated by means of the expected mean of sum of squares due to regression, σ^2 . This is the variance of the derived equation due to the regression:

$$\sigma^2 = \frac{1}{n-p} \sum_{i=1}^{n} (Y_i - A^**X1_i^{\alpha^*}*X2_i^{\beta^*}*X3_i^{-\delta^*})^2 \qquad (4\text{-}23)$$

In which

- A^*, α^*, β^* and δ^* are the optimum coefficients
- n is the number of measurements
- p is number of regression coefficients, in this case 4.

The estimation of the matrix of covariances now is defined as (Haan 1977):

$$\mathrm{Cov}(z) = \sigma^2 (A^T A)^{-1} \qquad (4\text{-}24)$$

The variance of z_i is equal to the covariance of z_i with itself and is therefore σ^2 times the i^{th} diagonal element of $(A^TA)^{-1}$. The covariance of z_i with z_j is σ^2 times the i,j^{th} element of $(A^TA)^{-1}$.

The program "REGRES" is based on the theory described. Details of the program are discussed in appendix I.
In case of 3 and 5 regression coefficients to be calculated, like for the TR and TB, the same program can be used after a few adaptations.

4.3 Results

The program "REGRES" calculates the values of the regression coefficients in the optimum (Error function minimal), the matrix of covariances cov(z) and the correlation matrix.
Calculations show that the optimum is reached within 30 iterations.
The output for TR, QP, TB and K are shown in Tab. 4-1 to 4-4. For the basic equations see §2.3.

Tab. 4-1 Time of Rise

$$Y = B*\left(\frac{X1}{100X2}\right)^3 + C*X3 + D$$

Calculation of the regression coefficients for TR

Iteration step 17

B	:	0.431906	eB :	0.000001
C	:	1.065065	eC :	-0.000027
D	:	1.283399	eD :	0.000064

sum of squares of the residual: 40.954657

The covariance matrix cov(z)

B	C	D
0.002371	0.001864	-0.008888
0.001864	0.101239	-0.072839
-0.008888	-0.072839	0.135287

The correlation matrix of the parameter estimates

B	C	D
1.000000	0.120321	-0.496269
0.120321	1.000000	-0.622386
-0.496269	-0.622386	1.000000

End of computation

Tab. 4-2 Peak discharge

$$Y = A*X1^{\alpha}*X2^{\beta}*X3^{-\delta}$$

The calculation of the regression coefficients for QP

Iteration step 29

A	:	0.170042	e_A	:	0.000000
α	:	0.636760	e_α	:	0.000000

ß : 0.207035 $e_ß$: 0.000000
δ : 0.399456 $e_δ$: 0.000000
Residual sum of squares: 874.812248

The covariance matrix cov(z)

A	α	ß	δ
0.007089	-0.008060	0.001575	-0.005151
-0.008060	0.018304	-0.011277	0.007156
0.001575	-0.011277	0.010501	-0.001073
-0.005151	0.007156	-0.001073	0.011150

The correlation matrix of the parameter estimates

A	α	ß	δ
1.000000	-0.707574	0.182491	-0.579346
-0.707574	1.000000	-0.813404	0.500881
0.182491	-0.813404	1.000000	-0.099171
-0.579346	0.500881	-0.099171	1.000000

End of computation

Tab. 4-3 Base Time

$Y = E*X1^{\theta}*X2^{-\varkappa}*X3^{\lambda}*X4^{\nu}$

The calculation of the regression coefficients of TB

Iteration step 34
E : 28.753496 e_E : 0.000085
θ : 0.092441 e_θ : 0.000000
ϰ : 0.118453 $e_\varkappa$: 0.000000
λ : 0.300227 e_λ : 0.000000
ν : 0.852078 e_ν : 0.000008
Residual sum of squares: 418.726940

The covariance matrix cov(z)

E	θ	ϰ	λ	ν
80.631114	0.018228	-0.128597	0.211741	7.044342
0.018228	0.003186	-0.001158	-0.002923	0.004954
-0.128597	-0.001158	0.001119	0.001134	-0.005970
0.211741	-0.002923	0.001134	0.023216	-0.025313
7.044342	0.004954	-0.005970	-0.025313	0.814912

5. ACCURACY OF THE GAMA EQUATIONS

5.1 Introduction

The accuracy of the GAMA equations is dependent upon the accuracy of the various derivation steps, like the accuracy of the regression analysis. Errors made in different stages propagate to the final equation. A way to locate errors and to schematize the derivation process is to construct a fault tree for each equation.
The propagation of errors is determined both analytical and numerical.
The mean value approach is an analytical method which is used in case of analytical relationships. The principles of this method are discussed in appendix IV.
The Monte Carlo simulation technique is an numerical method which is used in case there are no analytical relationships present and is used next to the mean value approach for comparison. The principles of this method are discussed in appendix II.
The Gama equations for QP (Peak Discharge) and TB (Base Time) depend both on TR (Time of Rise). This means that the prediction of TR influences the prediction of QP and TB. Therefore the accuracy of TR prediction is discussed first.

5.2 Time of Rise

5.2.1 Accuracy of regression coefficients (step 1)

The fault tree for TR is included in the fault tree for QP in Fig. 5-1 . Every box indicates the error of the parameter in it. From the roots to the top of the tree the errors are calculated step by step.
By means of regression analysis B,C and D are calculated. So far this is done by assuming that the variables TR ,L ,SF and SIM have deterministic values.
If all 28 measured TR's are explained by the regression line then the regression variance, s^2, would be zero and therefore also the standard deviations of B,C and D.
It is shown in previous chapters that this is not the case and B,C and D are calculated having a mean value and a standard deviation. Besides they are strongly correlated.
It can be stated that these standard deviations are due to the variability of the data used. This will be indicated as error 1 (see also Kevelam 1984).
By assuming stochastic distributions of the variables another kind of standard deviation of B,C and D is introduced. This is indicated as error 2 and is dued to the inaccuracy of the measured variables.

Some remarks on the results:

- It is clear that the regression coefficients derived are slightly different from those derived in the Gama method. This can be explained by the multitude of data, which causes some errors in transmission (typing).
  Besides, the equations for QP,TB and K can be transformed into loglinear equations on which linear regression analysis can be applied, which gives slightly different results.
- It is shown by Haan (1977) that the number of decimal places of measured values influences the calculations of the regression coefficients.
  Because the measurements can not be performed so accurately it is not useful to express the regression coefficients with 4 decimal places.
- It is stated that the covariances and correlations are not influenced by minor differences as discussed.
- The regression coefficients are in general strongly correlated. This means they are not independent.

The standard deviation of the regression coefficients of all equations are derived from the matrices of covariances, resulting in:

- For TR: B (0.43 : 0.0487)
  C (1.0665: 0.3182)
  D (1.2775: 0.3678)
- For QP: A (0.1836: 0.0842)
  α (0.5886: 0.1353)
  β (0.2381: 0.1025)
  δ (0.4008: 0.1056)
- For TB: E (27.4132: 8.9792)
  θ (0.1457: 0.0565)
  $\varkappa$ (0.0986: 0.0335)
  λ (0.2574: 0.1524)
  ν (0.7344: 0.9028)
- For K : F (0.5617: 0.1870)
  ϱ (0.1798: 0.0354)
  τ (0.1446: 0.0531)
  ψ (1.0897: 0.4264)
  ω (0.0452: 0.2161).

5. ACCURACY OF THE GAMA EQUATIONS

5.1 Introduction

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It is shown in previous chapters that this is not the case and B,C and D are calculated having a mean value and a standard deviation. Besides they are strongly correlated.
It can be stated that these standard deviations are due to the variability of the data used. This will be indicated as error 1 (see also Kevelam 1984).
By assuming stochastic distributions of the variables another kind of standard deviation of B,C and D is introduced. This is indicated as error 2 and is dued to the inaccuracy of the measured variables.

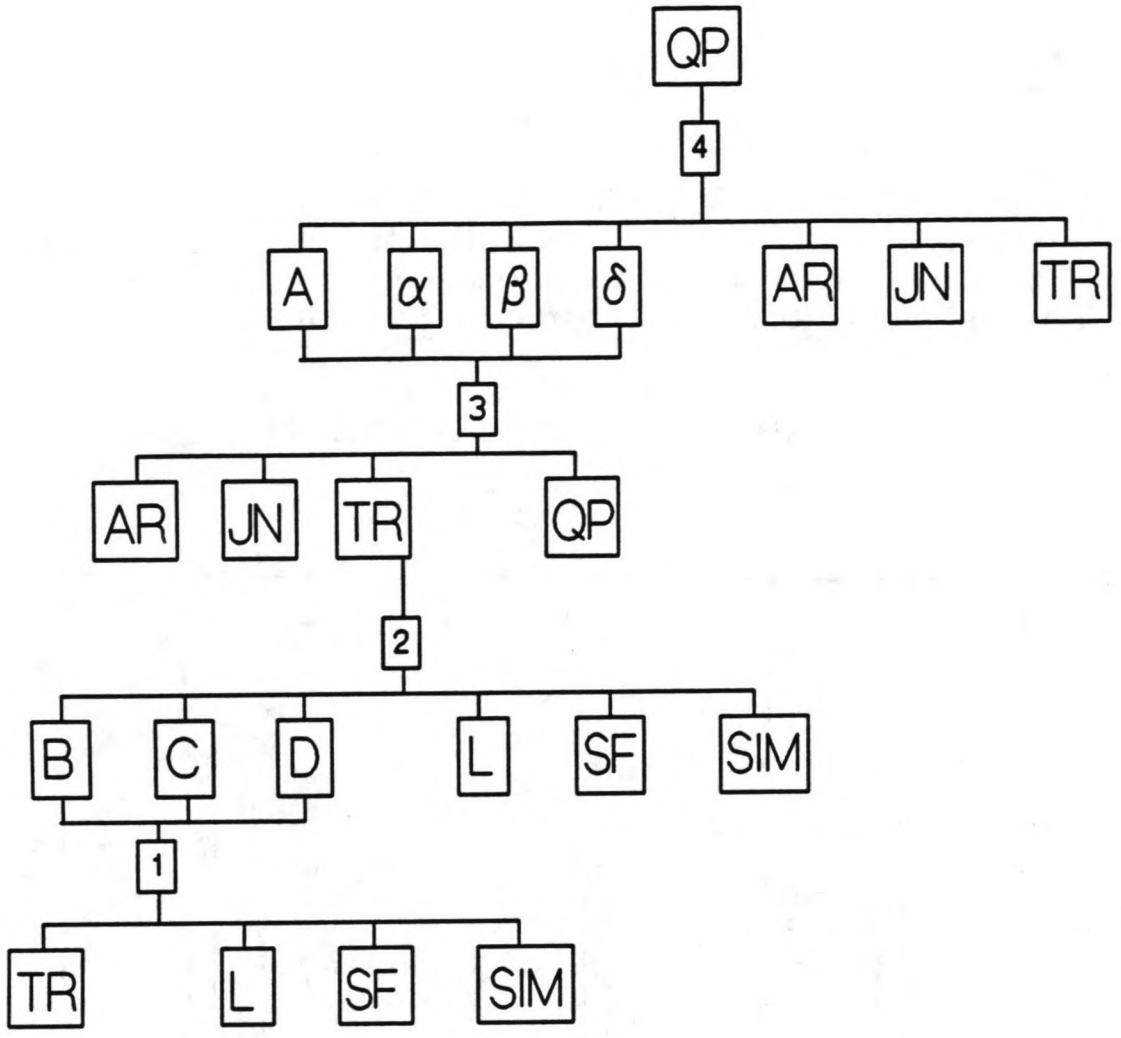

Fig. 5-1 Fault tree for QP

Determination of error 2:

The measured TR's are derived from the UH analysis in §3.2. Table 3.1 shows the mean values and the standard deviations for each catchment.

Research on the distribution of the TR values of each catchment indicates in most cases a normal distribution. In some cases the number of TR values is too small to determine a distribution. It is assumed that a normal distribution represents best the TR.

This is also assumed for the independent variables L,SF and SIM. The variables are independent because the accuracy of measurement of one variable has no influence on the accuracy of another variable. The deviations of these variables are unknown but it is reasonable to assume they are small in comparison with TR.

The regression coefficients are calculated by means of the program "SIMCO". This program generates variables from their distribution and calculates the optimum values for B,C and D.

The generation of variables is based on the Monte Carlo simulation. In appendix III SIMCO is discussed.

It can be seen that the mean values are more influenced than the standard deviations. The latter show very little fluctuation. It is remarkable that the standard deviations tend to decrease as the coefficients of variation increase.
The mean values approximate the GAMA values till 10% variation.
From these figures it is concluded that for the coefficients of variation less than 10% the standard deviations are assumed constant (thus a error of 10% of the measured variable is accepted).
Another point of interest is the distribution of the simulated B,C and D. In case of 150 simulations the values are ranged in successive intervalls. These histograms are shown in Fig. 5-4, in which also the equations for normal distributions are projected. It is reasonable to assume that the regression coefficients B,C and D are normally distributed.

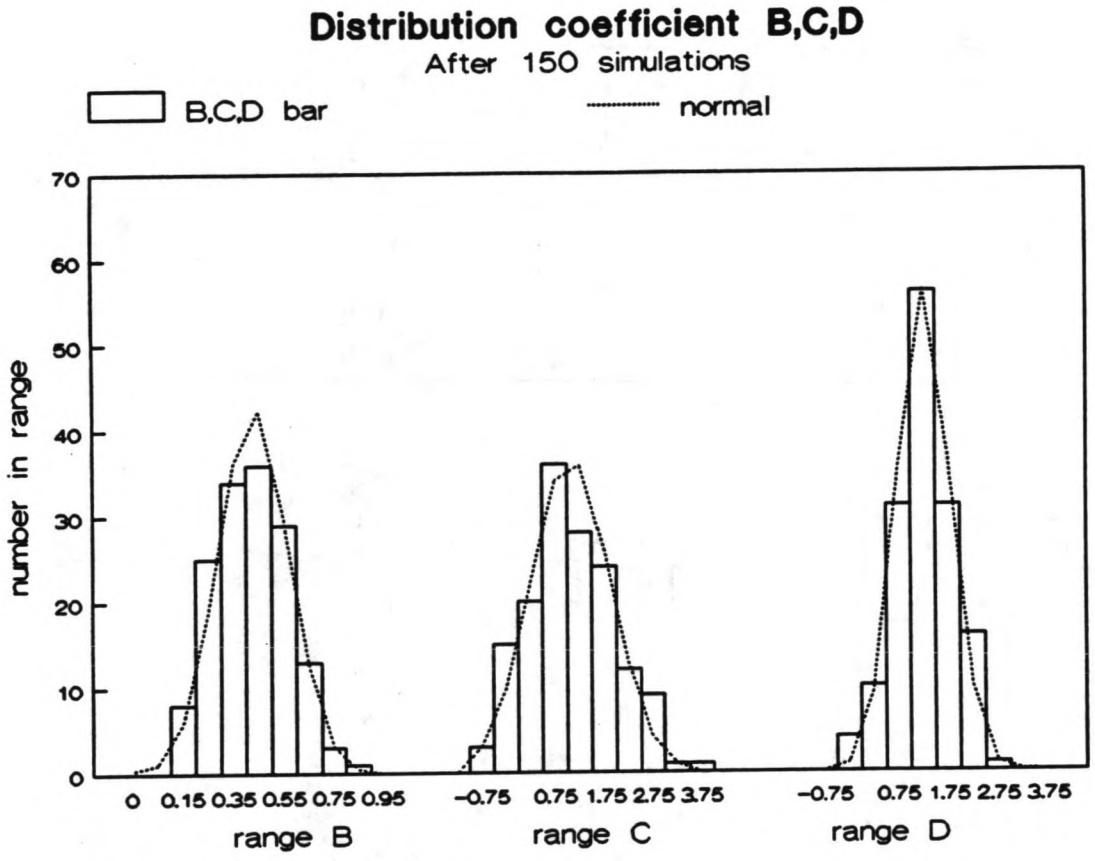

Fig. 5-4 Statistical distribution of B,C, and D

Determination of total error:
The total standard deviation of B,C and D due to error 1 and error 2 is calculated by squared propagation:

$$\sigma_{(1+2)} = \sqrt{\sigma_1^2+\sigma_2^2} \qquad (5\text{-}5)$$

The results are shown in Tab. 5-2.

The influence of V_L,V_{SF} and V_{SIM} on the calculation of the mean values and the standard deviations of the coefficients is shown in Fig. 5-2 and 5-3.

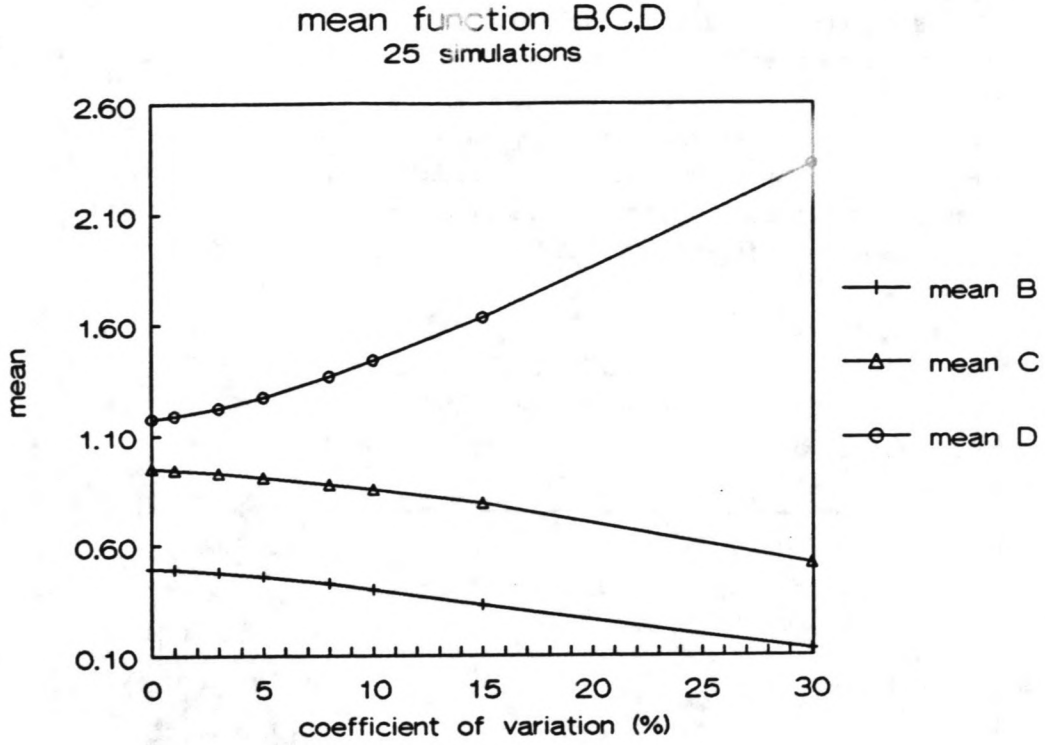

Fig. 5-2 Mean value function of regression coefficients

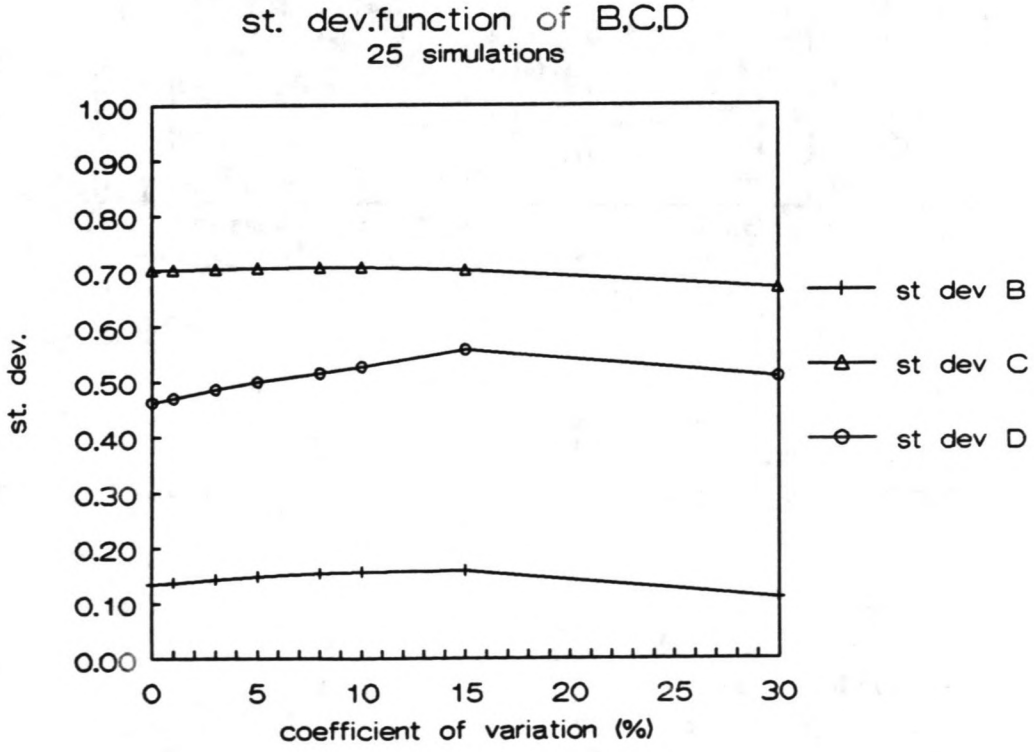

Fig. 5-3 Standard deviation function

It can be seen that the mean values are more influenced than the standard deviations. The latter show very little fluctuation. It is remarkable that the standard deviations tend to decrease as the coefficients of variation increase.
The mean values approximate the GAMA values till 10% variation.
From these figures it is concluded that for the coefficients of variation less than 10% the standard deviations are assumed constant (thus a error of 10% of the measured variable is accepted).
Another point of interest is the distribution of the simulated B,C and D. In case of 150 simulations the values are ranged in successive intervalls. These histograms are shown in Fig. 5-4, in which also the equations for normal distributions are projected. It is reasonable to assume that the regression coefficients B,C and D are normally distributed.

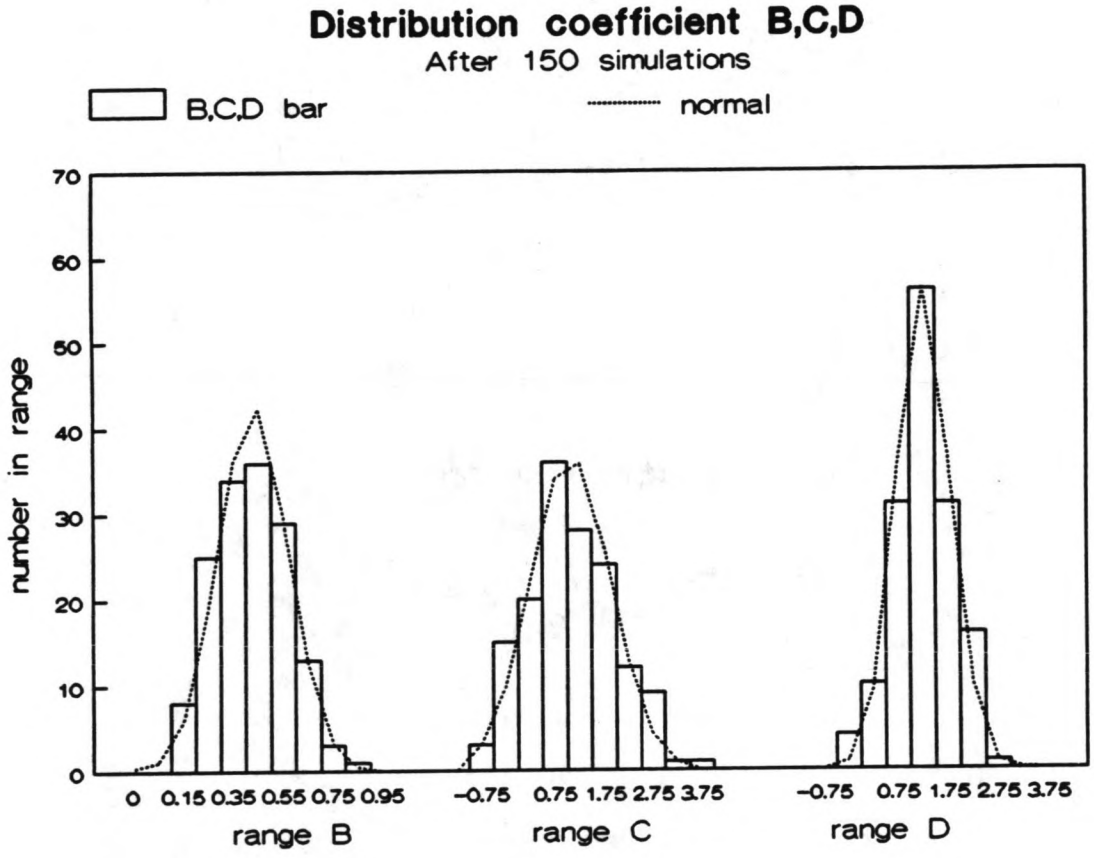

Fig. 5-4 Statistical distribution of B,C, and D

Determination of total error:
The total standard deviation of B,C and D due to error 1 and error 2 is calculated by squared propagation:

$$\sigma_{(1+2)} = \sqrt{\sigma_1^2+\sigma_2^2} \qquad (5\text{-}5)$$

The results are shown in Tab. 5-2.

Tab. 5-2 Total standard deviation

	μ	σ_1	σ_2	σ_{1+2}
B	0.43	0.0487	0.1527	0.1603
C	1.0665	0.3182	0.8761	0.9321
D	1.2775	0.3678	0.5715	0.6796

It is clear that error 2 has the biggest influence on the total standard deviation.
For both error 1 and 2 the covariance and correlation matrix is determined. To calculate the total covariance matrix no analytical solution is available such as 5-5:

$$\sigma_{xy(1+2)} \neq \sqrt{(\sigma_{xy(1)}^2 + \sigma_{xy(2)}^2)}$$

By means of Monte Carlo simulation the total correlation matrix is derived from the correlation matrices of error 1 and 2 in the following way:

- Choleski decomposition of the correlation matrices.
  Because in the Monte Carlo simulation values are randomized from a uniform distribution with $\mu=0$ and $\sigma=1$ the correlation matrices need to be adjusted. This is done by means of the Choleski decomposition (van Kan). The method of Choleski decomposition is discussed in appendix V. Besides a program is written for Choleski decomposition.
- Generation of values for B,C and D.
- Calculation of the correlation matrix.

This procedure is pursued by the program "TOTVAR" (appendix VI) in which the number of simulations is varied from 50 to 2000.
The total correlation matrix after a large number of simulations is:

	B	C	D
B	1	-0.0279	-0.4933
C	-0.0279	1	-0.6849
D	-0.4933	-0.6849	1

To check the program the total variances of B,C and D are simulated and compared with the variances derived from Tab. 5-2. For all three the differences between both results are less than 1% .
The total covariance matrix is derived from the total correlation matrix:

	B	C	D
B	0.0257	-0.0042	-0.0537
C	-0.0042	0.8688	-0.4338
D	-0.0537	-0.4338	0.4619

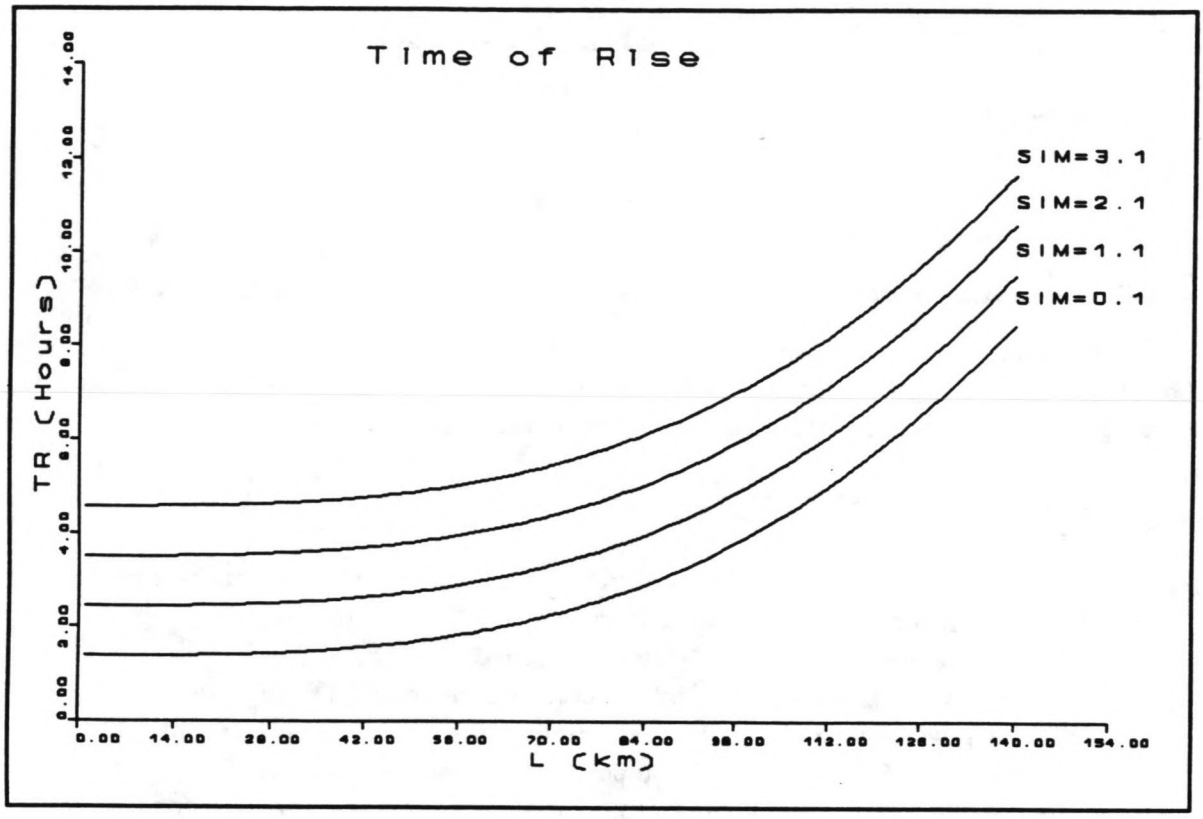

Fig. 5-5 TR as a function of L, for constant SF(=0.55) and various SIM

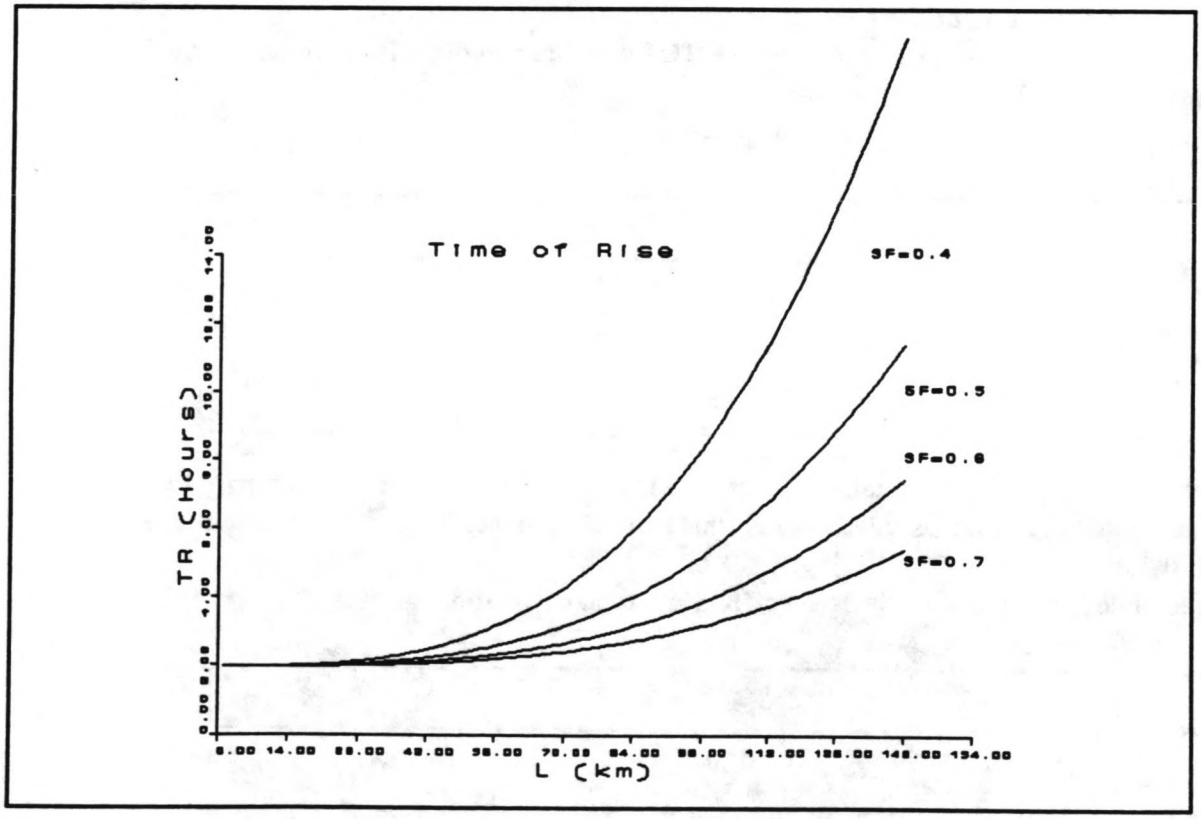

Fig. 5-6 TR as a function of L, for constant SIM(=0.65) and various SF

The variances of L,SF and SIM are replaced by the coefficients of variation as :
$\sigma_X = V_X * \mu_X$.
Again the coefficients of variations are assumed constant.
Substituting the derivatives, the variances and covariances in (5-7) and rearranging the equation gives:

$$\sigma_{TR}^2 = \left(\frac{L}{100SF}\right)^6 * (9B^2V_L^2 + 9B^2V_{SF}^2 + \sigma_B^2) + \left(\frac{L}{100SF}\right)^3 * (2\sigma_{BC}SIM + 2\sigma_{BD}) + SIM^2(\sigma_C^2 + C^2V_{SIM}^2) + 2*SIM*\sigma_{CD} + \sigma_D^2 \qquad (5\text{-}13)$$

Substituting numeric values gives σ_{TR} as function of L,SF and SIM:

$$\sigma_{TR}^2 = 2.87*10^{-14}\left(\frac{L}{SF}\right)^6 - 8.4*10^{-15}\left(\frac{L}{SF}\right)^3 SIM - 1.07*10^{-13}\left(\frac{L}{SF}\right)^3 + 0.87SIM^2 - 0.87SIM + 0.46 \qquad (5\text{-}14)$$

For the 28 catchments used, counts:
$5.5 < L < 137$ [km]
$0.434 < SF < 0.656$
$0.14 < SIM < 3.89$.
Plots are made of TR, σ_{TR} and V_{TR} as function of the stream length L. Fig. 5-5, 5-7 and 5-9 on the next pages show the results in case SF is constant (0.55) and SIM is 0.1, 1.1, 2.1 and 3.1. Fig. 5-6, 5-8 and 5-10 show the results in case SIM is constant (0.65) and SF is 0.4, 0.5, 0.6 and 0.7.

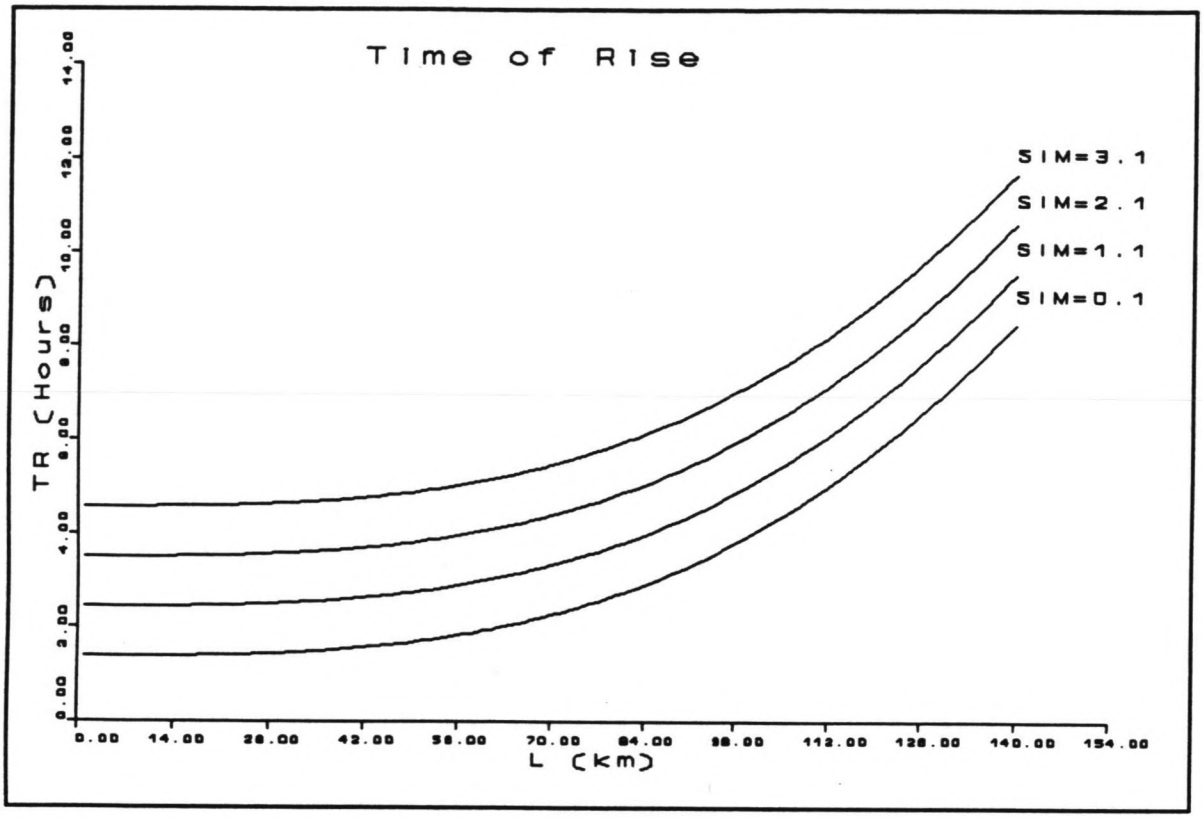

Fig. 5-5 TR as a function of L, for constant SF(=0.55) and various SIM

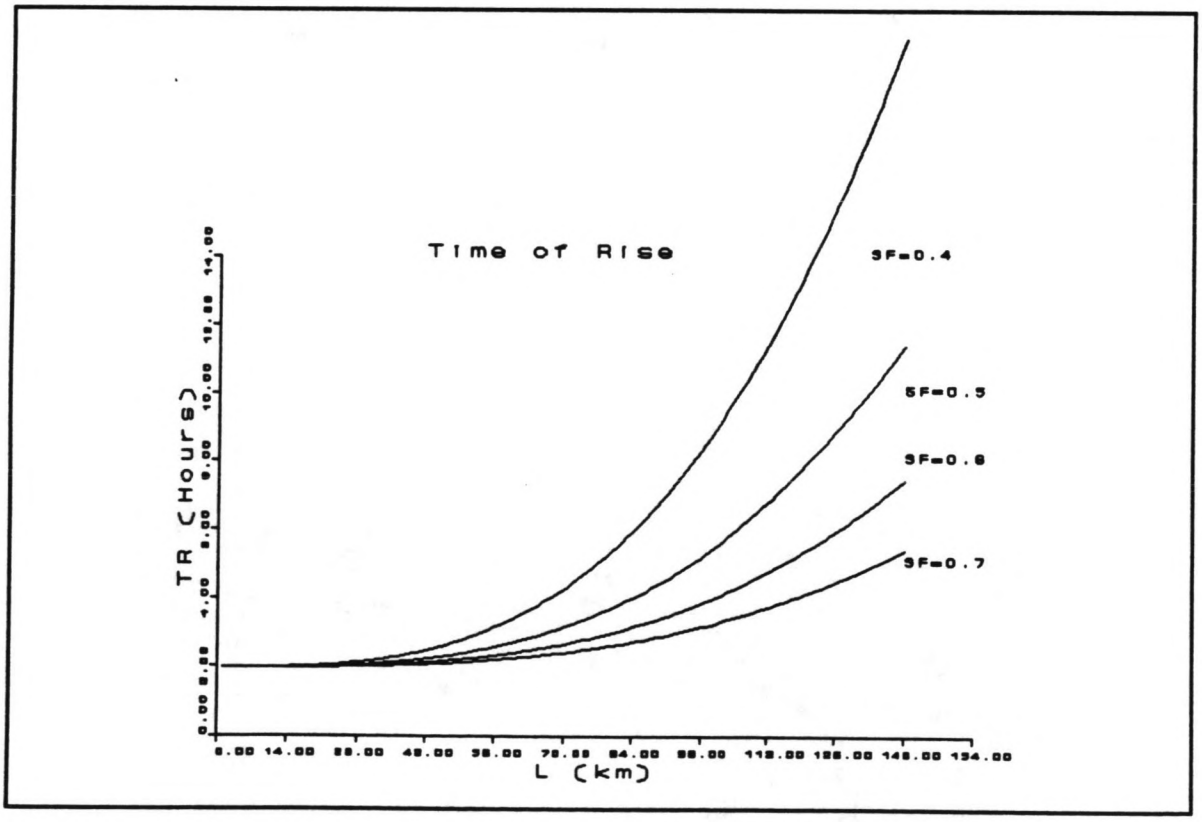

Fig. 5-6 TR as a function of L, for constant SIM(=0.65) and various SF

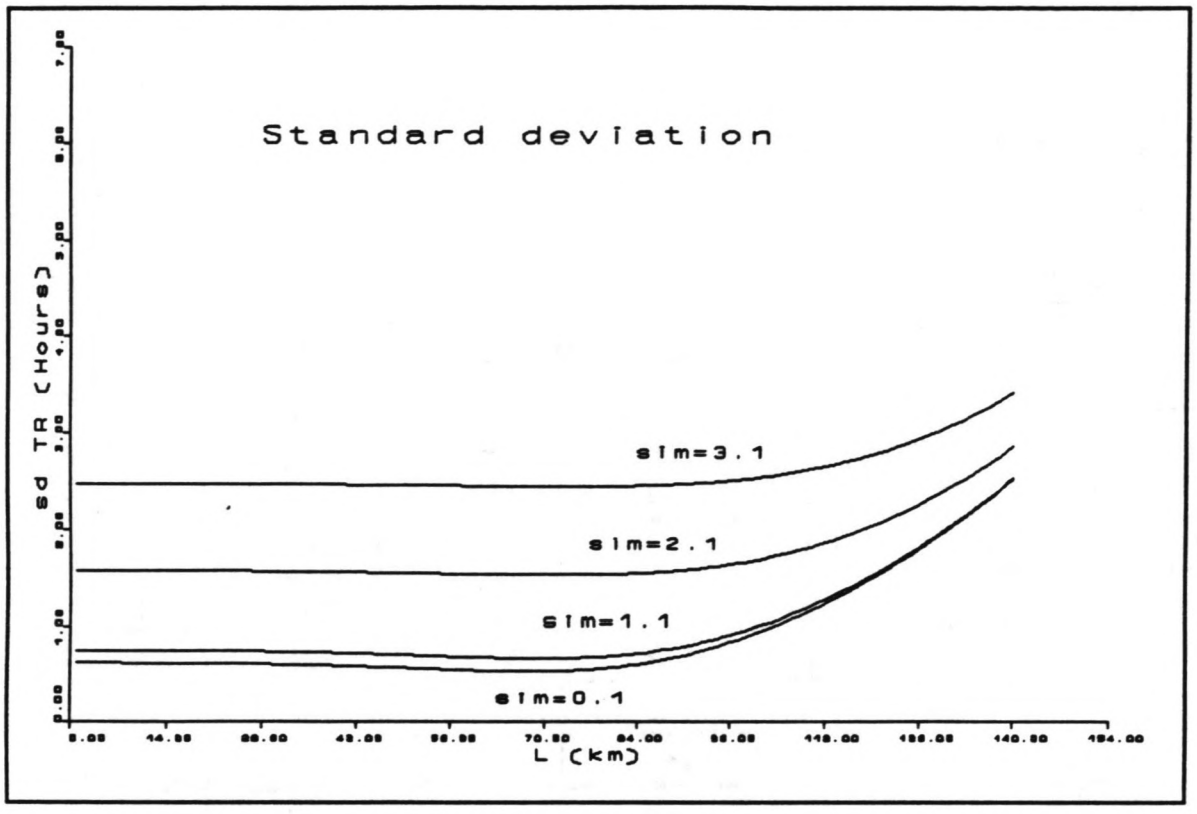

Fig. 5-7 Standard deviation of TR as a function of L, for constant SF(=0.55) and various SIM

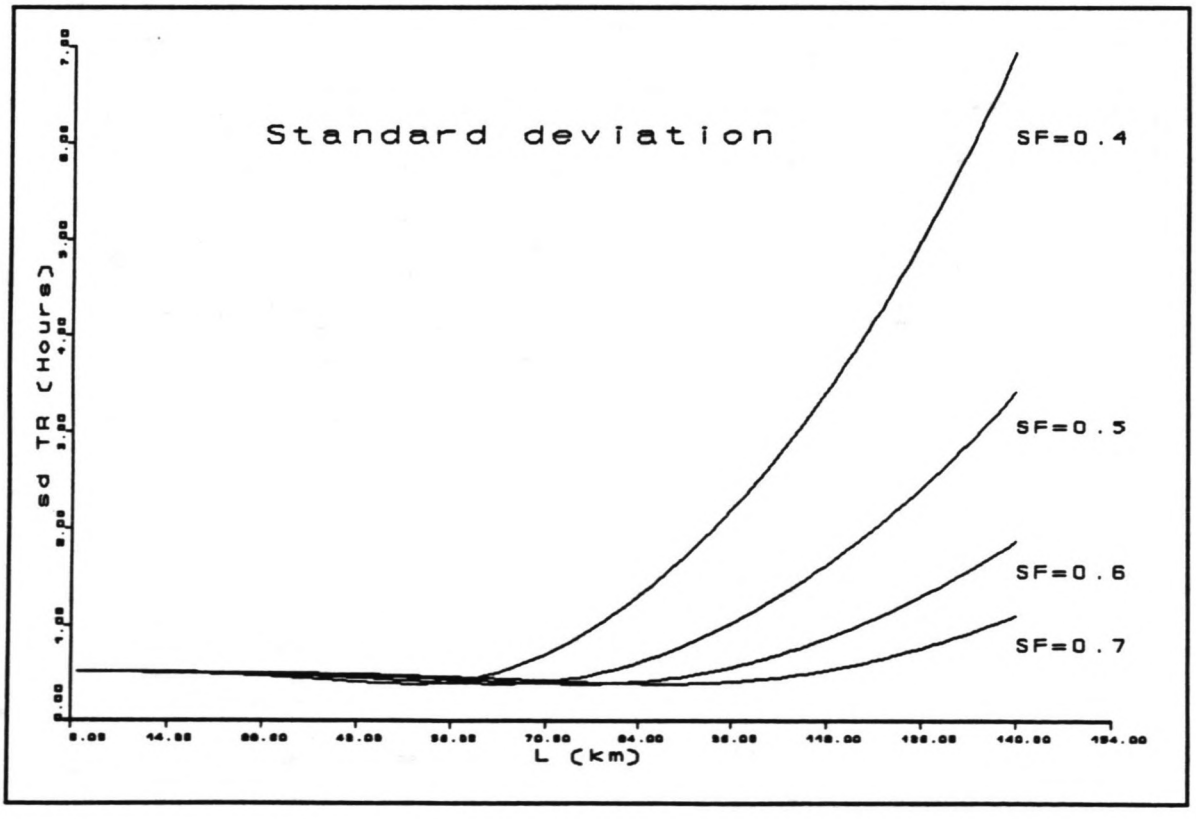

Fig. 5-8 Standard deviation of TR as a function of L, for constant SIM(=0.65) and various SF

Tab. 5-3 Accuracy of TR

Ct	μ_{TR}			V_{TR} (%)			
	GAMA	MC	▲(%)	MV_1	MV	MC	▲(%)
1	1.43	1.45	1.84	23.95	41.88	42.9	2.44
2	6.87	6.75	1.71	15.72	46.26	46.01	0.54
3	3.08	3.05	1.05	13.63	38.8	38.3	1.29
4	2.11	2.10	0.54	14.47	21.64	22.13	2.26
6	3.83	3.86	0.83	9.6	15.84	16.18	2.15
7	2.96	2.98	0.80	10.16	23	22.94	0.26
8	2.80	2.82	0.83	11.09	26.43	26.96	2.01
10	3.63	3.63	0.24	10.43	17.61	18.07	2.61
11	3.45	3.41	1.08	13.28	38.46	39.29	2.16
12	1.82	1.81	0.46	16.28	26.95	26.04	3.38
13	11.70	11.93	1.96	14	31.53	31.67	0.44
14	6.54	6.55	0.22	11.9	25.29	26.72	5.65
15	4.25	4.30	1.25	10.44	19.18	19.57	2.03
16	1.58	1.55	2.13	20.81	35.07	35.35	0.80
17	2.38	2.40	0.55	13.44	20.74	21.36	2.99
18	2.11	2.12	0.53	13.41	23.16	22.95	0.91
19	3.05	3.06	0.32	11.45	18.59	18.86	1.45
20	4.56	4.56	0.11	9.08	16.93	17.47	3.19
21	1.64	1.64	0.35	19.11	31.53	31.9	1.17
22	6.83	6.87	0.66	11.91	25.4	25.83	1.69
23	3.08	3.11	0.74	10.30	24.83	25.68	3.42
24	1.73	1.73	0.32	17.7	28.76	28.81	0.17
25	1.63	1.62	0.52	20.41	34.44	33.82	1.80
26	1.77	1.78	0.24	17.76	28.45	27.66	2.78
27	2.05	2.06	0.36	13.83	22.44	21.68	3.39
28	2.16	2.17	0.50	14.32	21.62	21.75	0.60
29	2.13	2.11	0.71	13.19	20.37	20.3	0.34
30	3.21	3.25	1.28	10.69	16.81	16.64	1.01

MV_1 = model accuracy (error 1 only)
MV = mean value approach
MC = Monte Carlo simulation
▲ = difference between MV and MC.

5.2.3 Monte Carlo simulation

The theory of the Monte Carlo simulation technique is already discussed. In appendix III the program " SIMCO " is printed. This program generates values for L,SF,SIM and B,C,D. Because the variables are independent, generation of values is also independent. The coefficients on the contrary are correlated (see correlation matrix §5.2.1). By applying the Choleski decomposition on the correlation matrix, a matrix is obtained which contains multipliers for the generated values with N(0:1).
SIMCO calculates the mean values, standard deviations and coefficients of variation for TR from histograms.

First the used catchments are simulated. For each catchment 1000 simulations are applied. The simulated mean values for TR and the coefficients of variations are shown in Tab. 5-3 next to the results obtained with the mean value approach.
Of importance for further study is the accuracy of TR due to error 1 only and is printed in the 5^{th} column.

The simulated mean values for TR (μ_{TR}) differ very slightly from the values calculated with the GAMA equation for TR.
For the V_{TR} results are also reassuring.
It can be stated that the mean value approach gives good results; It is correct, for TR, to assume that substituting the mean values of the independent variables gives the mean value for the dependent variable.

Tab. 5-3 Accuracy of TR

Ct	μ_{TR}			V_{TR} (%)			
	GAMA	MC	▲(%)	MV_1	MV	MC	▲(%)
1	1.43	1.45	1.84	23.95	41.88	42.9	2.44
2	6.87	6.75	1.71	15.72	46.26	46.01	0.54
3	3.08	3.05	1.05	13.63	38.8	38.3	1.29
4	2.11	2.10	0.54	14.47	21.64	22.13	2.26
6	3.83	3.86	0.83	9.6	15.84	16.18	2.15
7	2.96	2.98	0.80	10.16	23	22.94	0.26
8	2.80	2.82	0.83	11.09	26.43	26.96	2.01
10	3.63	3.63	0.24	10.43	17.61	18.07	2.61
11	3.45	3.41	1.08	13.28	38.46	39.29	2.16
12	1.82	1.81	0.46	16.28	26.95	26.04	3.38
13	11.70	11.93	1.96	14	31.53	31.67	0.44
14	6.54	6.55	0.22	11.9	25.29	26.72	5.65
15	4.25	4.30	1.25	10.44	19.18	19.57	2.03
16	1.58	1.55	2.13	20.81	35.07	35.35	0.80
17	2.38	2.40	0.55	13.44	20.74	21.36	2.99
18	2.11	2.12	0.53	13.41	23.16	22.95	0.91
19	3.05	3.06	0.32	11.45	18.59	18.86	1.45
20	4.56	4.56	0.11	9.08	16.93	17.47	3.19
21	1.64	1.64	0.35	19.11	31.53	31.9	1.17
22	6.83	6.87	0.66	11.91	25.4	25.83	1.69
23	3.08	3.11	0.74	10.30	24.83	25.68	3.42
24	1.73	1.73	0.32	17.7	28.76	28.81	0.17
25	1.63	1.62	0.52	20.41	34.44	33.82	1.80
26	1.77	1.78	0.24	17.76	28.45	27.66	2.78
27	2.05	2.06	0.36	13.83	22.44	21.68	3.39
28	2.16	2.17	0.50	14.32	21.62	21.75	0.60
29	2.13	2.11	0.71	13.19	20.37	20.3	0.34
30	3.21	3.25	1.28	10.69	16.81	16.64	1.01

MV_1 = model accuracy (error 1 only)
MV = mean value approach
MC = Monte Carlo simulation
▲ = difference between MV and MC.

5.2.4 Kali Putih

For the calculation of the accuracy of TR for several stations in the Kali Putih use is made of previous investigation by the Gadjah Mada university in Yogyakarta. At 5 stations in the river the GAMA hydrograph characteristics are determined.
Now the accuracy of TR is calculated with the mean value approach and Monte Carlo simulation. The calculations are done with a coefficient of variation for L,SF and SIM of 3% and with a number of 10.000 simulations.
The results are shown in Tab. 5-4.

Tab. 5-4 Accuracy of TR of Kali Putih

No	μ		σ		V		
	GAMA	MC	MV	MC	MV_1	MV	MC
1	1.84	1.84	0.49	0.50	0.16	0.27	0.27
2	1.89	1.88	0.48	0.48	0.15	0.25	0.25
3	2.25	2.25	0.51	0.50	0.12	0.23	0.22
4	2.21	2.22	0.40	0.41	0.13	0.18	0.18
5	5.17	5.21	1.20	1.22	0.12	0.23	0.23

For station number 5 , the mouth of the kali Putih, the distribution of TR is derived from the histogram. In Fig. 5-11 the result is shown. The normal distribution is well approximated.

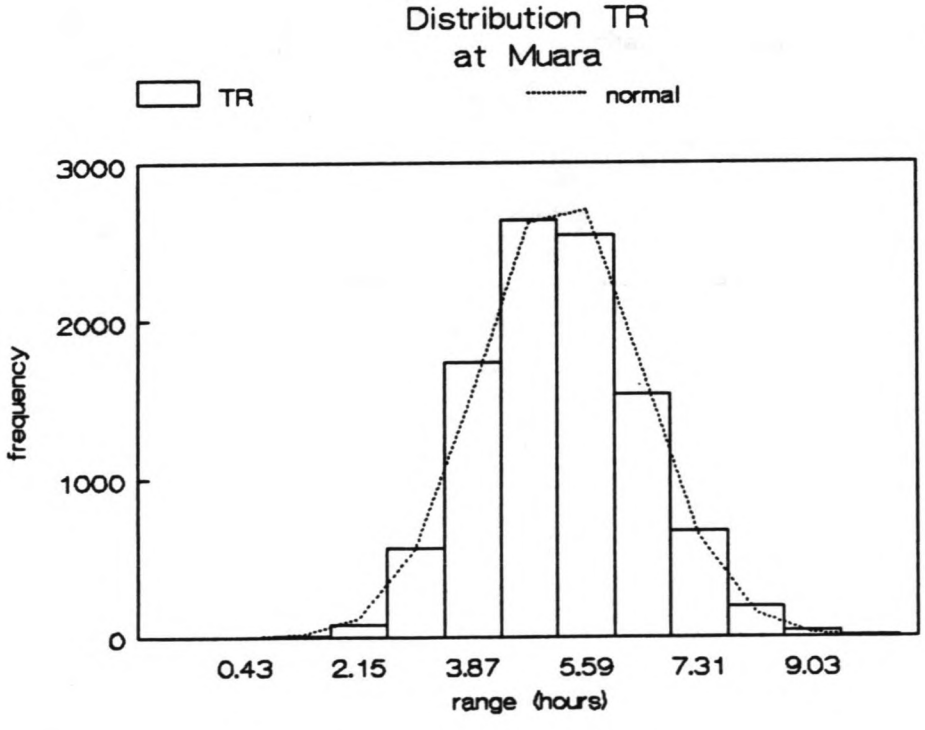

Fig. 5-11 Distribution TR at the mouth of the river

The distribution of A,α,ß and δ is derived from the histograms in case of 150 simulations and shown in Fig. 5-14 and 5-15.

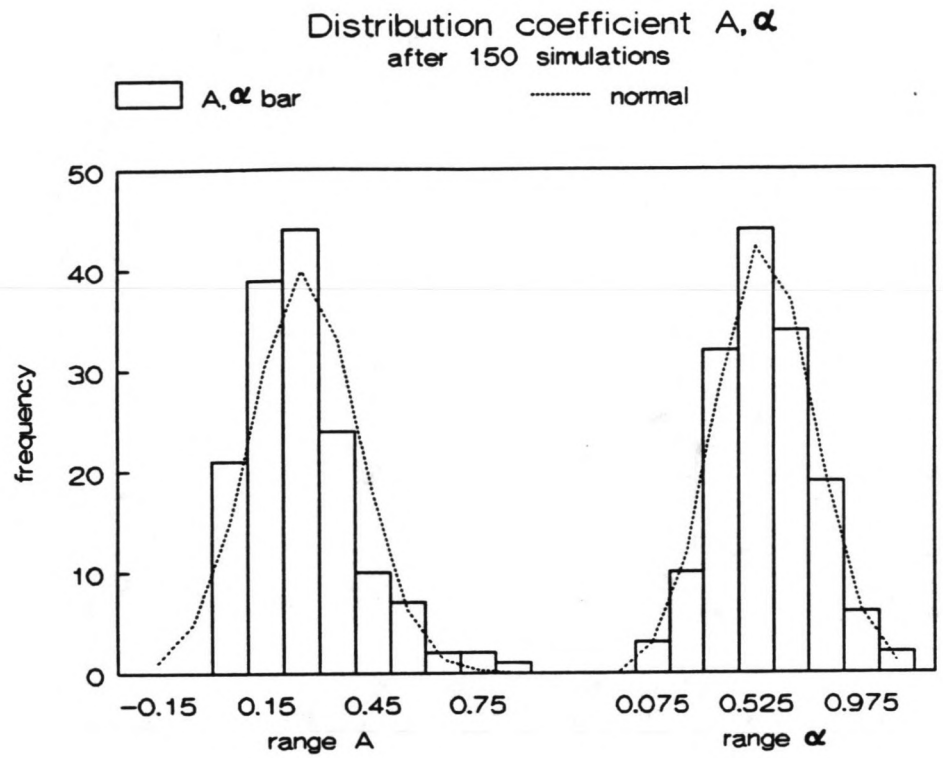

Fig. 5-14 Statistical distribution of A and α

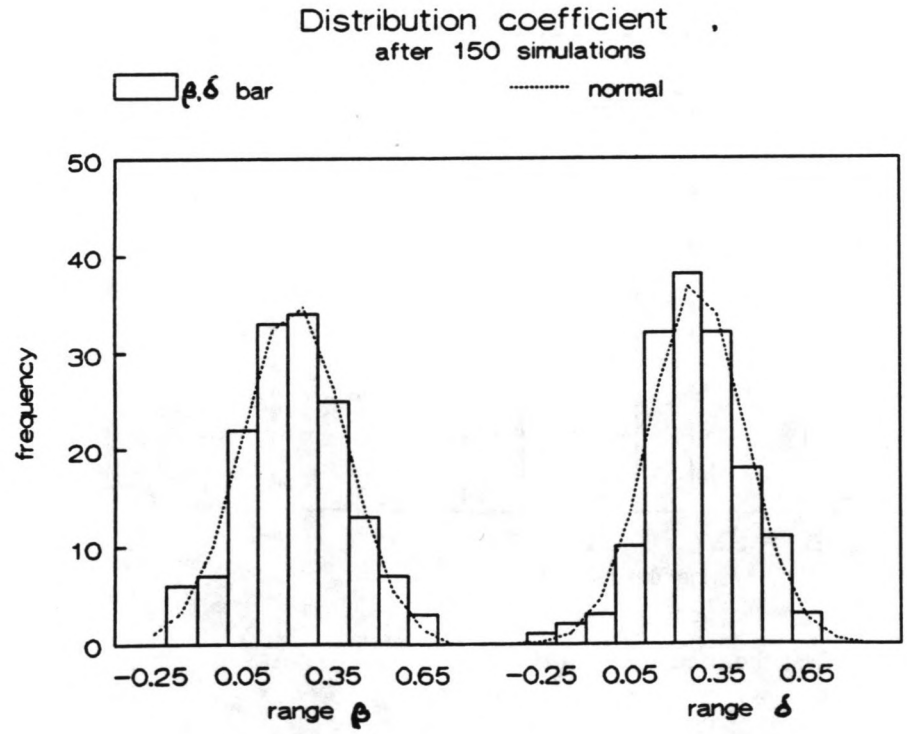

Fig. 5-15 Statistical distribution of ß and δ

The calculations are done for coefficients of variation of 0.03 for AR and JN. For TR the values in table 5.3 are used. These are determined in case of 3% variability of L,SF and SIM. To show the influence of the variation of the independent variables Fig. 5-12 and 5-13 are constructed. Herein only V_{AR} and V_{JN} are varied. For V_{AR} and V_{JN} smaller than 10% the influence on the mean and standard deviation is negligible.

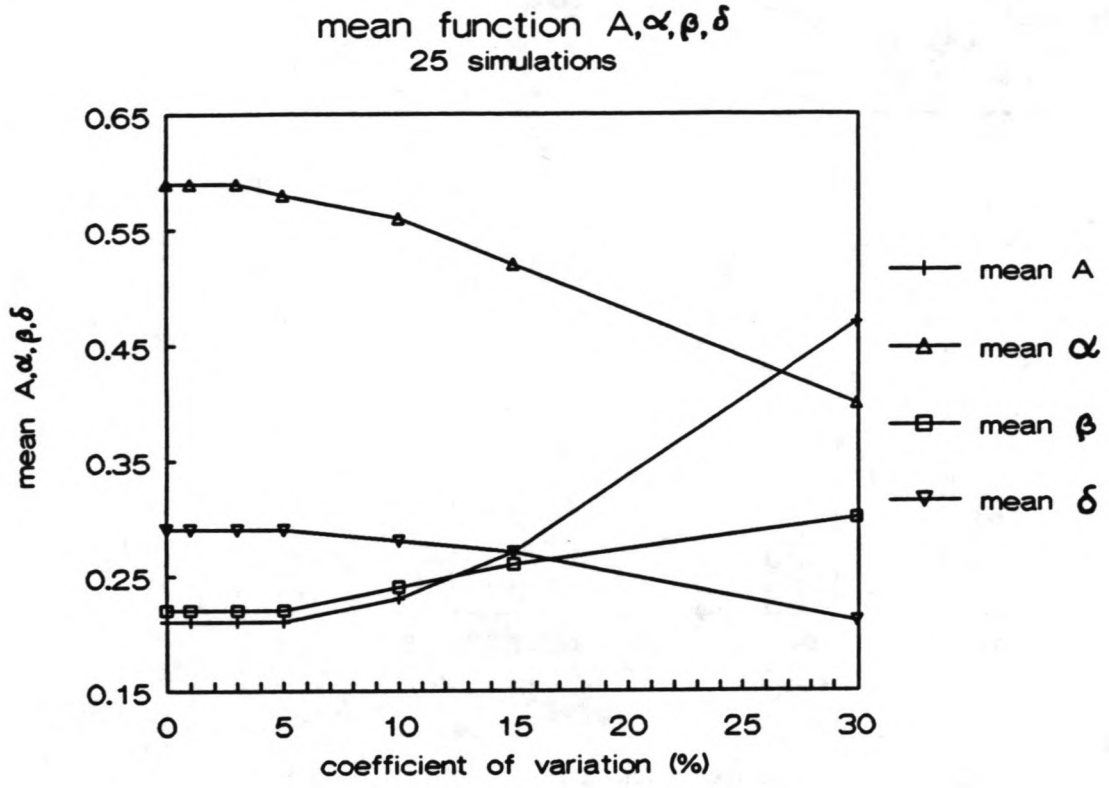

Fig. 5-12 Mean value function of the regression coefficients

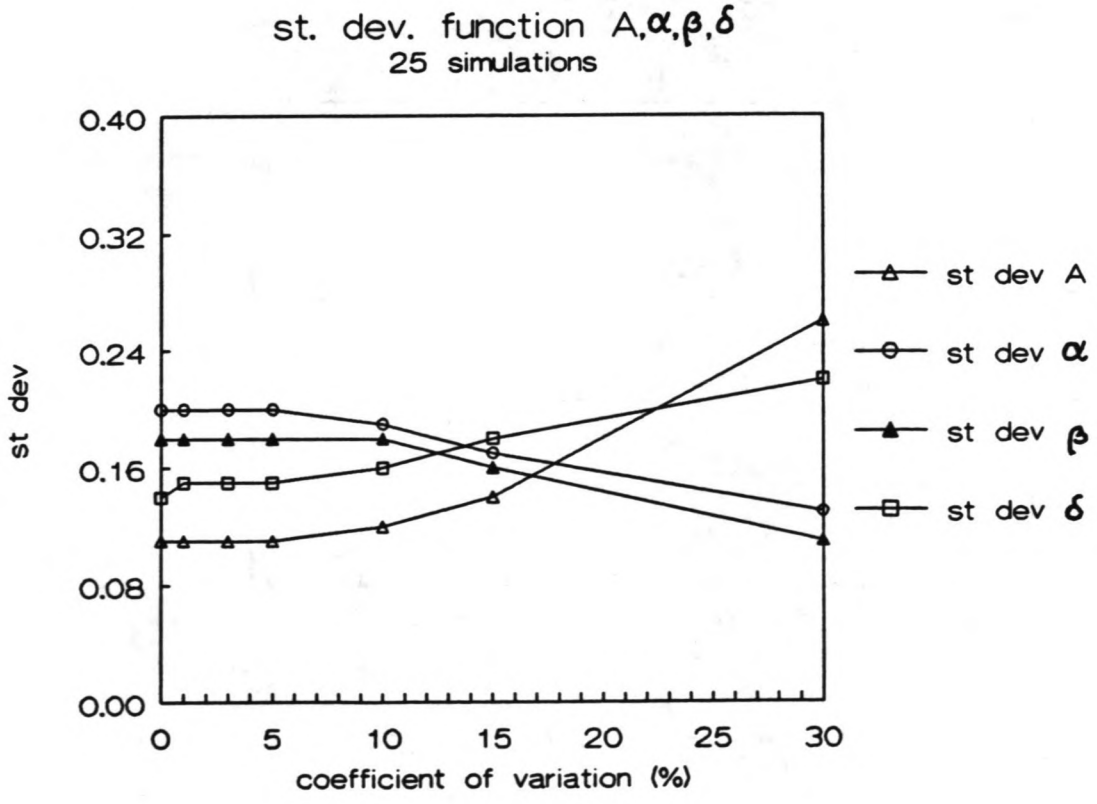

Fig. 5-13 Standard deviation function

The distribution of A,α,ß and δ is derived from the histograms in case of 150 simulations and shown in Fig. 5-14 and 5-15.

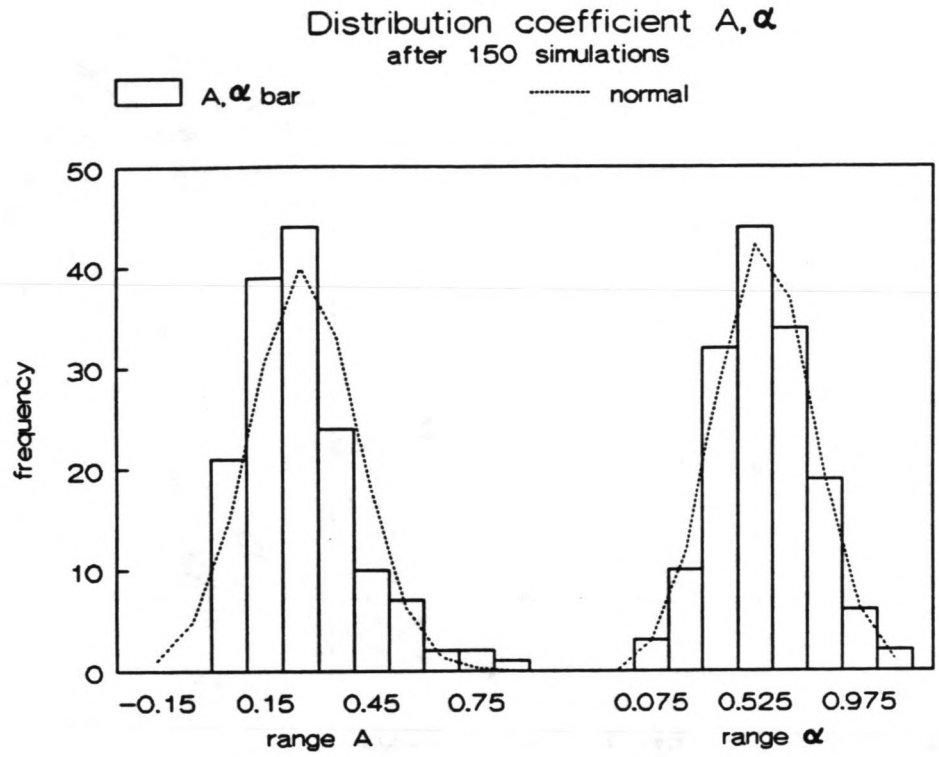

Fig. 5-14 Statistical distribution of A and α

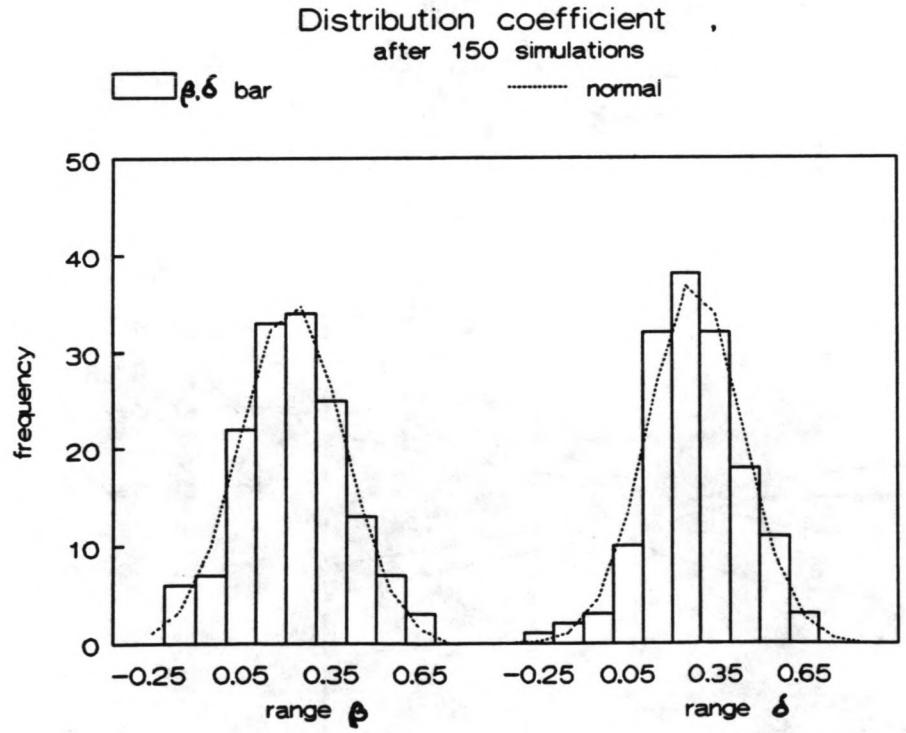

Fig. 5-15 Statistical distribution of ß and δ

Determination of total error:
By means of squared propagation the total error as result of errors 1 and 2 is calculated. The results are shown in Tab. 5-6.

Tab. 5-6 Total standard deviation

	μ	σ_1	σ_2	σ_{1+2}
A	0.1836	0.0842	0.1717	0.1912
α	0.5886	0.1353	0.2104	0.2501
ß	0.2381	0.1025	0.1719	0.2001
δ	0.4008	0.1056	0.2056	0.2311

The total correlation matrix is calculated by means of the program TOTVAR (Appendix VI). After a large number of simulations the correlation matrix is:

	A	α	ß	δ
A	1	-0.6536	0.2077	-0.5162
α	-0.6536	1	-0.8421	0.3448
ß	0.2077	-0.8421	1	0.0545
δ	-0.5162	0.3448	0.0545	1

The total covariance matrix is derived from the correlation matrix and Tab. 5-6 .
The total covariance matrix is:

	A	α	ß	δ
A	0.0366	-0.0313	0.0079	-0.0228
α	-0.0313	0.0626	-0.0421	0.0199
ß	0.0079	-0.0421	0.04	0.0025
δ	-0.0228	0.0199	0.0025	0.0534

5.3.2 Mean value approach

Now different from the procedure pursued in step 2 for the accuracy of TR step 4 for the accuracy of QP is performed along 2 ways; firstly, only error 1 is included and secondly both error 1 and 2 are included.
The equation for QP:

$$QP = A * AR^{\alpha} * JN^{\beta} * TR^{-\delta} \qquad (5\text{-}15)$$

In which:

- AR = $N(\mu_{AR}:\sigma_{AR})$
- JN = $N(\mu_{JN}:\sigma_{JN})$
- TR = $N(\mu_{TR}:\sigma_{TR})$.

Tab. 5-7 Accuracy of QP

Ct	μ_{QP} (m^3/s)			V_{QP} (%)		
	GAMA	MC_1	MC_2	MV_1	MC_1	MC_2
1	3.07	2.96	1.84	25.08	28.19	207
2	5.43	5.45	3.11	24.13	30.91	217
3	7.84	7.70	4.44	20.81	26.25	159
4	27.99	25.34	11.76	20.15	22.89	180
6	24.62	22.14	10.83	19.67	23.17	176
7	21.63	19.40	7.57	18.97	25.16	195
8	18.34	16.55	7.71	17.73	23.12	163
10	38.04	33.72	13.44	18.95	21.02	168
11	10.25	9.60	4.77	23.52	27.14	181
12	4.97	4.50	2.17	22.76	31.13	167
13	28.61	26.19	8.45	20.82	32.44	290
14	56.08	50.06	11.18	20.31	25.67	336
15	18.50	16.76	8.18	19.59	23.46	185
16	21.27	19.19	6.66	19.42	29.92	226
17	50.49	44.79	6.05	19.73	30.23	329
18	19.70	17.40	6.16	19.66	25.46	179
19	53.38	47.25	13.75	20.59	25.80	217
20	50.49	44.20	14.18	19.05	27.40	218
21	11.59	10.88	6.61	21.98	22.83	126
22	58.73	52.72	19.45	20.26	21.66	186
23	24.64	21.89	9.21	18.36	24.72	201
24	7.61	7.00	3.97	19.72	23.30	133
25	24.14	22.43	9.39	22.46	24.96	195
26	13.95	12.81	5.73	19.72	26.14	178
27	14.06	12.52	5.58	19.58	28.26	183
28	24.90	22.22	8.12	19.49	26.32	215
29	26.65	23.49	8.40	19.79	27.10	201
30	54.34	48.43	15.26	20.08	24.74	229

MV_1 = Mixed analytical numerical method for case 1
MC_1 = Monte Carlo simulation for case 1
MC_2 = Monte Carlo simulation for case 2.

In this equation V_{TR} can have 2 values; in case only error 1 is concerned and in case of both errors (values in Tab. 5-3).
Till now only first order Taylor is applied in the mean value approach. Because of the strong non-linearity of the equation for QP and the high variances in the coefficients it is reasonable to assume that higher orders Taylor play a not negligible role.
The second order Taylor is applied in this analysis and the derivation is discussed in appendix VII.
The problem which arises is that parts of the extended equation for V_{QP} are not analytical defined. For example:

$$\frac{\sum_{i=1}^{n}\left[(x_i-\bar{x})^2(y_i-\bar{y})\right]}{n-1}$$

can not be expressed as some kind of (co)variance.
This problem can be solved by means of simulation. The result then is a combined analytical numerical approach. By means of Monte Carlo simulation this second order Taylor is determined. The results of calculation for the 2 cases are shown in Tab. 5-7

5.3.3 Monte Carlo simulation

By means of the program SIMCO the mean values, standard deviations and coefficients of variation for the 28 catchments are calculated. This is done for both situations. The results of simulation are shown also in Tab. 5-7.

For situation 2 simulation shows a large shift from the GAMA mean. This means that the mean value approach may not be applied. Therefore V_{QP} in situation 2 is only simulated. Large values for V_{QP} are found.

For situation 1 the mean value approach may be applied because of the relative small difference in μ_{QP}. However the second order Taylor is included still the V_{QP} differs from the simulations. Satisfying results might be obtained by including more Taylor derivatives.

Tab. 5-7 Accuracy of QP

Ct	μ_{QP} (m^3/s)			V_{QP} (%)		
	GAMA	MC_1	MC_2	MV_1	MC_1	MC_2
1	3.07	2.96	1.84	25.08	28.19	207
2	5.43	5.45	3.11	24.13	30.91	217
3	7.84	7.70	4.44	20.81	26.25	159
4	27.99	25.34	11.76	20.15	22.89	180
6	24.62	22.14	10.83	19.67	23.17	176
7	21.63	19.40	7.57	18.97	25.16	195
8	18.34	16.55	7.71	17.73	23.12	163
10	38.04	33.72	13.44	18.95	21.02	168
11	10.25	9.60	4.77	23.52	27.14	181
12	4.97	4.50	2.17	22.76	31.13	167
13	28.61	26.19	8.45	20.82	32.44	290
14	56.08	50.06	11.18	20.31	25.67	336
15	18.50	16.76	8.18	19.59	23.46	185
16	21.27	19.19	6.66	19.42	29.92	226
17	50.49	44.79	6.05	19.73	30.23	329
18	19.70	17.40	6.16	19.66	25.46	179
19	53.38	47.25	13.75	20.59	25.80	217
20	50.49	44.20	14.18	19.05	27.40	218
21	11.59	10.88	6.61	21.98	22.83	126
22	58.73	52.72	19.45	20.26	21.66	186
23	24.64	21.89	9.21	18.36	24.72	201
24	7.61	7.00	3.97	19.72	23.30	133
25	24.14	22.43	9.39	22.46	24.96	195
26	13.95	12.81	5.73	19.72	26.14	178
27	14.06	12.52	5.58	19.58	28.26	183
28	24.90	22.22	8.12	19.49	26.32	215
29	26.65	23.49	8.40	19.79	27.10	201
30	54.34	48.43	15.26	20.08	24.74	229

MV_1 = Mixed analytical numerical method for case 1
MC_1 = Monte Carlo simulation for case 1
MC_2 = Monte Carlo simulation for case 2.

5.3.4 Kali Putih

For 5 stations in the Kali Putih the QP is determined. Calculations are only performed for case 1, in which only the variability of the data used, is taken into account.
The mean value approach with a simulated second order effect (MV_1) gives better results than the mean value approach without second order infuence. The latter method showed deviations of 20% from the simulated V_{QP}.
Again the calculations are done with coefficients of variation of AR,JN of 3% and a number of 10.000 simulations. The results are in Tab. 5-8.

Tab. 5-8 Accuracy of QP of Kali Putih

No	μ	(m^3/s)	σ	(m^3/s)	V	(%)
	GAMA	MC	MV_1	MC	MV_1	MC
1	0.55	0.53	0.22	0.22	39.25	41.62
2	0.64	0.61	0.26	0.25	40.45	41.81
3	0.66	0.64	0.27	0.27	40.85	42.22
4	0.75	0.71	0.32	0.30	42.29	42.53
5	0.74	0.72	0.38	0.35	51.40	47.69

It seems in case of the Kali Putih that the method MV_1 gives better results than in case of the selected catchments (Tab. 5-7). This is due to the relative small values of the variables; For large values of AR,JN and TR higher order derivatives in the Taylor Series become of importance and in the case of high variances in the coefficients they might increase with regard to first and second order derivatives.
Inspecting the values of V_{QP} it is clear that the lower limit of the 95% confidence interval is bigger than zero. Simulation shows that less than 5% of the total number give negative values for QP.
The histogram of the simulation is shown in Fig. 5-17. The normal distribution is again well approximated.

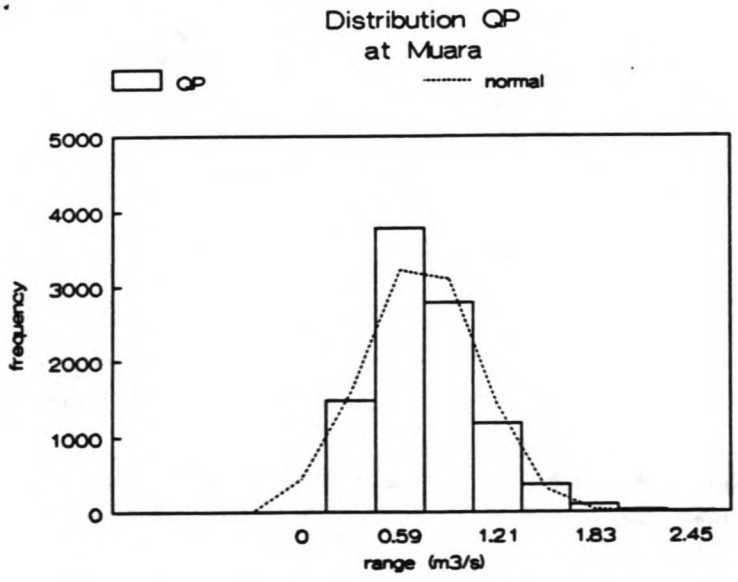

Fig. 5-17 Statistical distribution of QP at the mouth of kali Putih

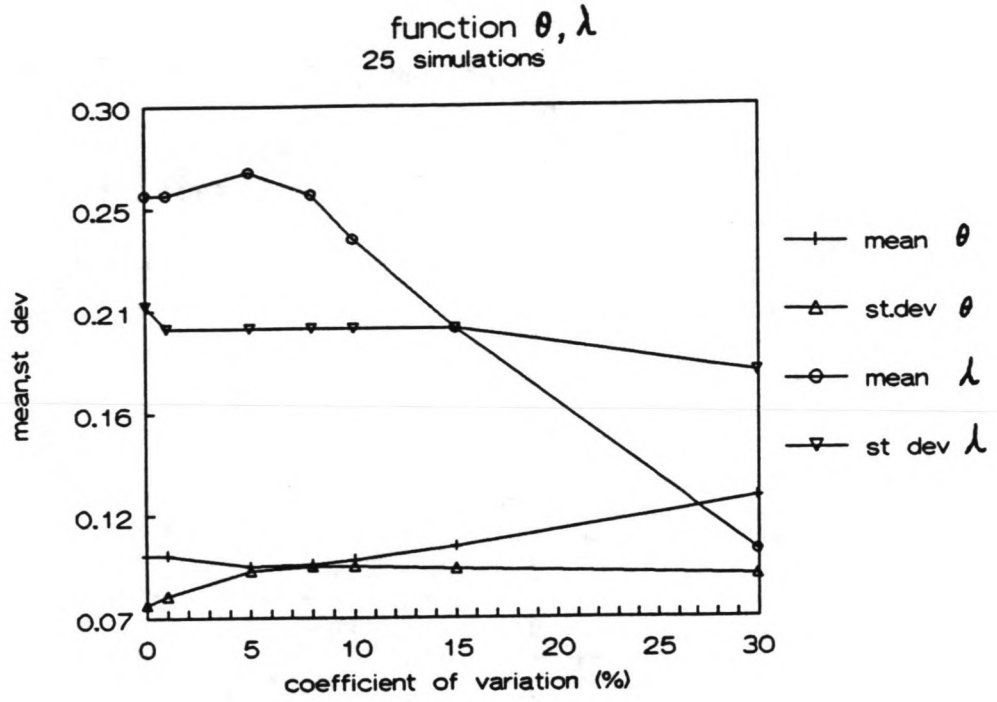

Fig. 5-20 Mean and standard deviation function for θ,λ

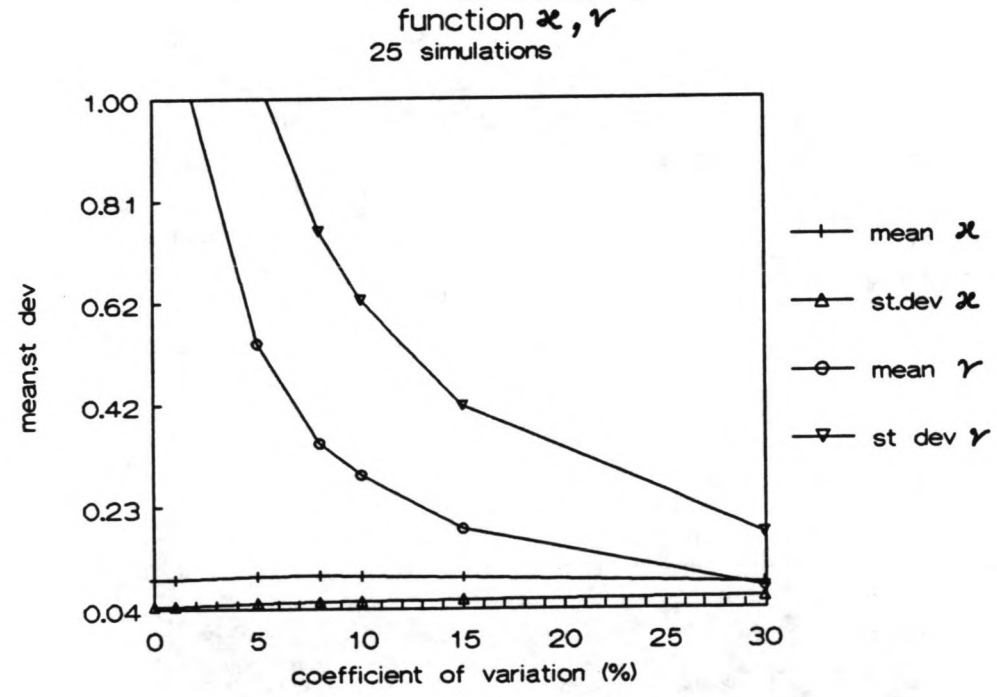

Fig. 5-21 Mean and standard deviation function for ϰ,ν

μ_θ,σ_θ And μ_κ,σ_κ and σ_λ seem to be less influenced by the coefficients of variation than the others. The σ_E and σ_ν even decrease for increasing coefficients of variation.
From these figures it is reasonable to accept an accuracy of measurement of S,RUA and SN of 10% around the mean without influencing the standard deviations (σ^*) of the regression coefficients.

Tab. 5-8 Standard deviation due to error 2

	μ_2	σ_2	μ^*	σ^*
E	29.8710	15.0387	27.4132	15.2906
θ	0.0460	0.0893	0.1457	0.1343
ϰ	0.1336	0.0563	0.0986	0.0665
λ	0.3542	0.2483	0.2574	0.2675
ν	0.4896	1.3177	0.7344	1.3484

Due to the large variation in TR all the mean values of the coefficients (μ_2) except E show a relative large deviation from the GAMA means (μ^*). The standard deviations are adapted (σ^*) and used for further analysis.
The correlation matrix of error 2 in case of 150 simulations:

	E	θ	ϰ	λ	ν
E	1	0.2314	0.6026	0.6564	0.6903
θ	0.2314	1	0.2173	0.1809	0.0504
ϰ	0.6026	0.2173	1	0.6766	0.2125
λ	0.6564	0.1809	0.6766	1	0.0766
ν	0.6903	0.0504	0.2125	0.0766	1

To check the influence of the values for V_S,V_{RUA} and V_{SN} on the regression calculations, Fig. 5-19, 5-20 and 5-21 are constructed.

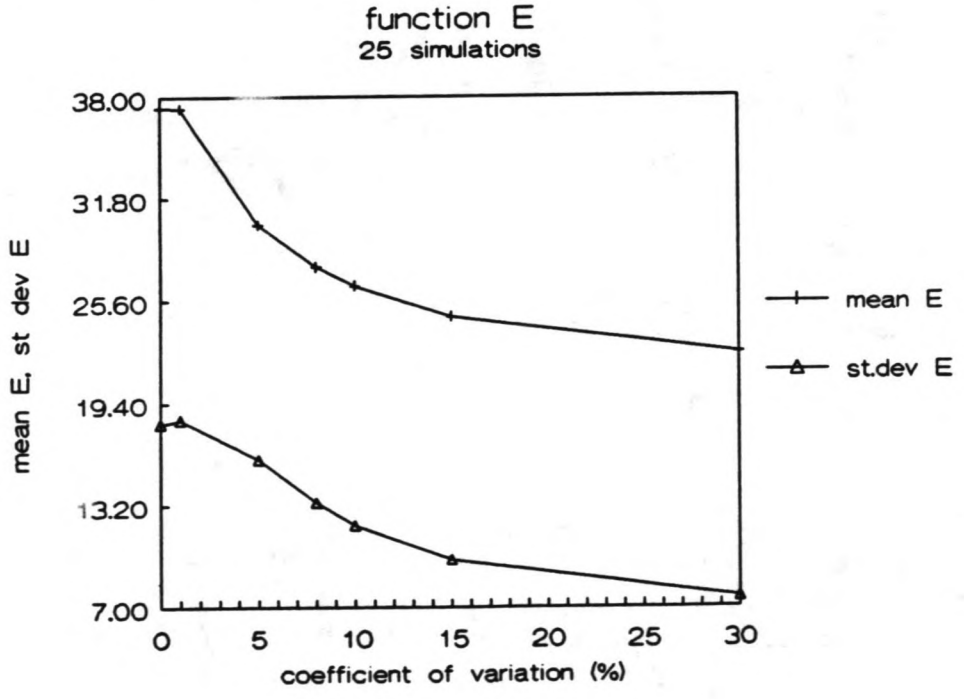

Fig. 5-19 Mean and standard deviation function for E

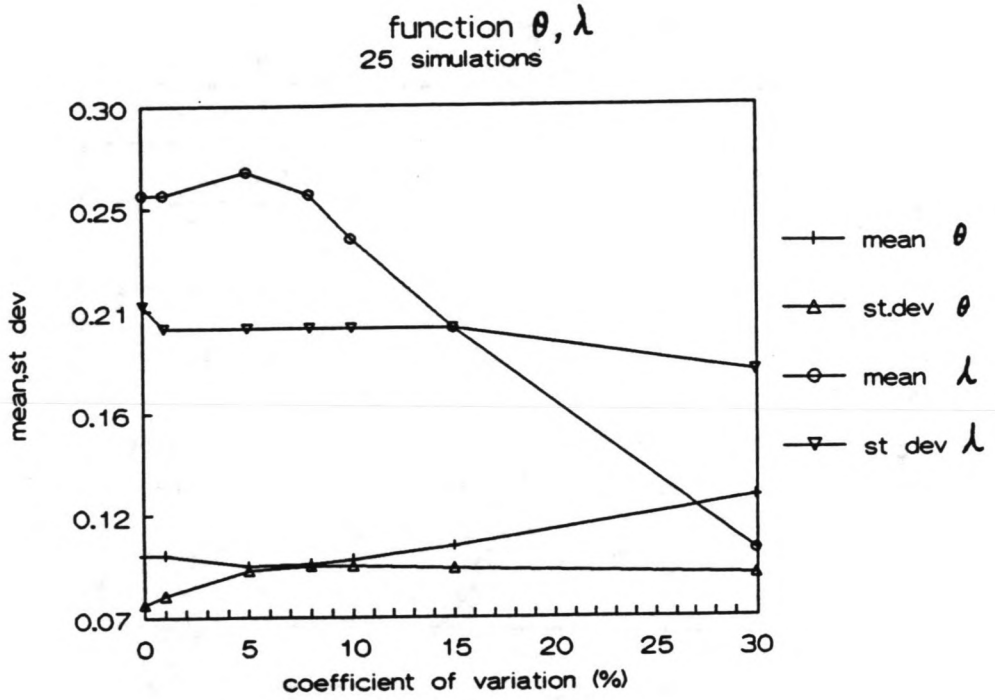

Fig. 5-20 Mean and standard deviation function for θ,λ

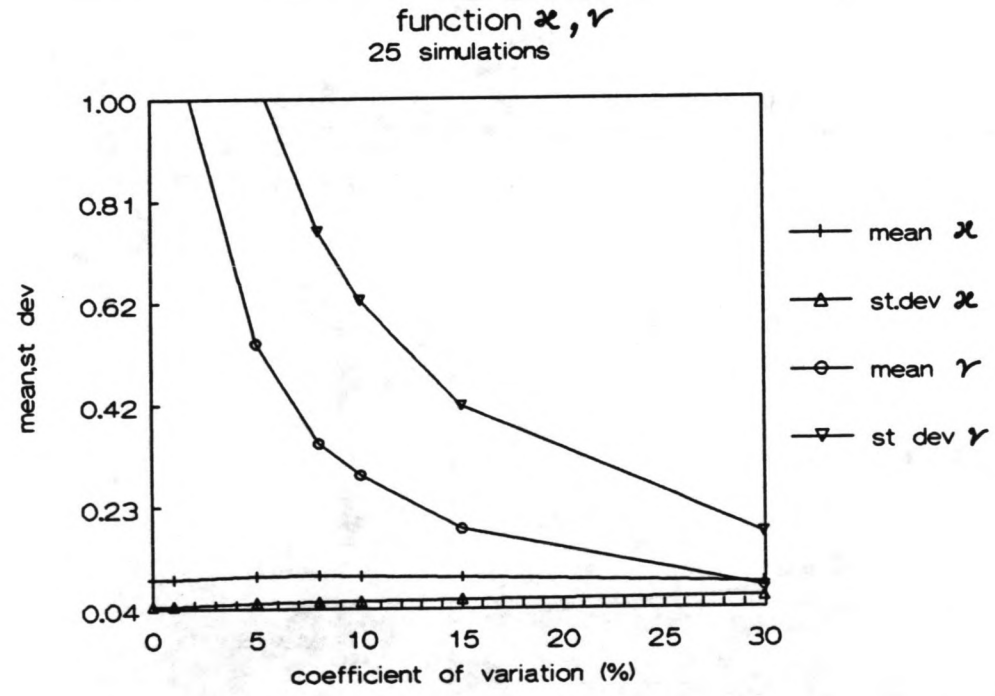

Fig. 5-21 Mean and standard deviation function for ϰ,ν

μ_θ,σ_θ And μ_K,σ_K and σ_λ seem to be less influenced by the coefficients of variation than the others. The σ_E and σ_v even decrease for increasing coefficients of variation.
From these figures it is reasonable to accept an accuracy of measurement of S,RUA and SN of 10% around the mean without influencing the standard deviations (σ^*) of the regression coefficients.

The distributions of the regression coefficients are derived from the histograms in Fig. 5-22, 5-23 and 5-24 ,in which the normal distribution ,as aspected, is well approximated.

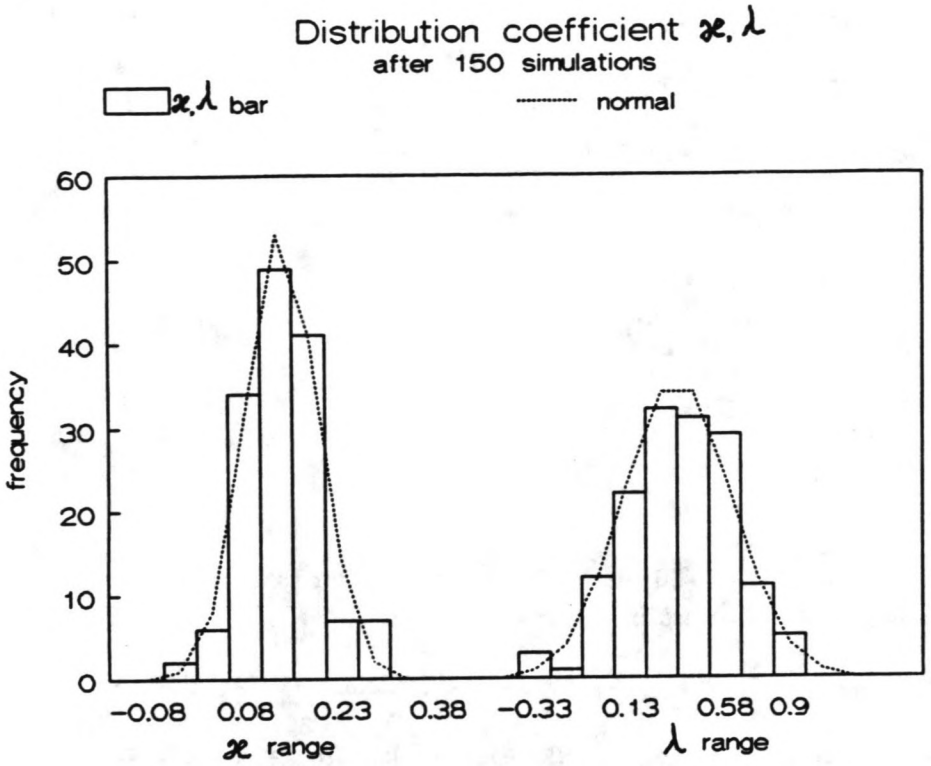

Fig. 5-22 Statistical distribution of $\varkappa$ and λ

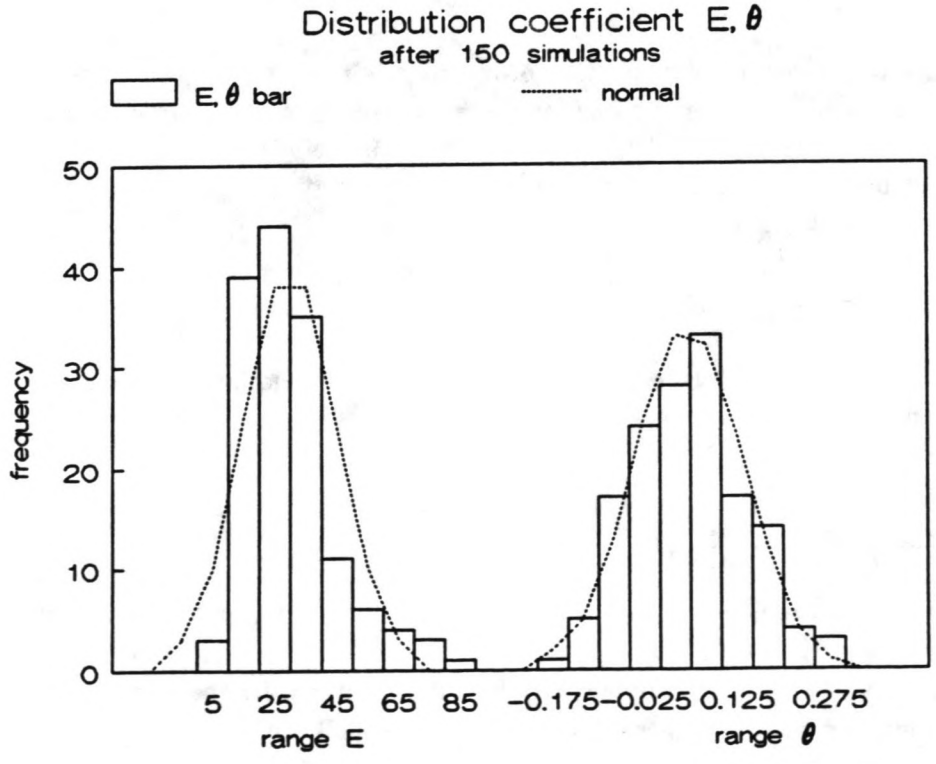

Fig. 5-23 Statistical distribution of E and θ

The derivatives of TB:

$$\frac{\delta f}{\delta TR} = E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\theta*TR^{-1}$$

$$\frac{\delta f}{\delta S} = -E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\kappa*S^{-1}$$

$$\frac{\delta f}{\delta RUA} = E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\lambda*RUA^{-1}$$

$$\frac{\delta f}{\delta SN} = E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\nu*SN^{-1}$$

$$\frac{\delta f}{\delta E} = TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}$$

$$\frac{\delta f}{\delta \theta} = E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\ln(TR)$$

$$\frac{\delta f}{\delta \kappa} = -E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\ln(S)$$

$$\frac{\delta f}{\delta \lambda} = E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\ln(RUA)$$

$$\frac{\delta f}{\delta \nu} = E*TR^{\theta}*S^{-\kappa}*RUA^{\lambda}*SN^{\nu}*\ln(SN)$$

The coefficient of variation for TB, V_{TB}, is derived from (5-20) after substituting the derivatives and the (co)variances. The result:

$$V_{TB}^2 = V_{TR}^2*\theta^2 + V_S^2*\kappa^2 + V_{RUA}^2*\lambda^2 + V_{SN}^2*\nu^2 + V_E^2 + V_\theta^2(\theta\ln(TR))^2 + V_\kappa^2(\kappa\ln(S))^2 + V_\lambda^2(\lambda(RUA))^2 +$$
$$V_\nu^2(\nu\ln(SN))^2 + \frac{2}{E}\ln(TR)\,\sigma_{E\theta} - \frac{2}{E}\ln(S)\,\sigma_{E\kappa} + \frac{2}{E}\ln(RUA)\,\sigma_{E\lambda} + \frac{2}{E}\ln(SN)\,\sigma_{E\nu} - 2\ln(TR)\ln(S)\,\sigma_{\theta\kappa} +$$
$$2\ln(TR)\ln(RUA)\,\sigma_{\theta\lambda} + 2\ln(TR)\ln(SN)\,\sigma_{\theta\nu} - 2\ln(S)\ln(RUA)\,\sigma_{\kappa\lambda} - 2\ln(S)\ln(SN)\,\sigma_{\kappa\nu} +$$
$$2\ln(RUA)\ln(SN)\,\sigma_{\lambda\nu}$$

(5-27)

This equation only includes taylor series of the first order. Like in the case of QP, higher orders might be of importance. The second order can be calculated by means of simulation. To give an indication of the amount of work if higher order Taylor is included the number of derivatives to be calculated is:

		TB	QP
-	1st order:	9	7
-	2nd order:	45	28
-	3rd order:	154	78.

For TB also only the second order will be simulated. The results of calculation for the 2 cases are shown in table 5.10.

5.4.3 Monte Carlo simulation

The simulation procedure is performed for both cases. The number of simulations is 1000. The results are included in Tab. 5-10.
Comparing the results of V_{TB} with the results of V_{QP} in Tab. 5-7 it is clear that for V_{TB} MV_1 is better approximated by MC_1 than for V_{QP}. Next also for TB the simulated mean values show a negative trend for increasing variances (error 1 and 2).

The total covariance matrix is derived from Tab. 5-9 and the correlation matrix. The total covariance matrix is:

	E	θ	ϰ	λ	ν
E	314.4274	0.4666	0.4402	2.8729	21.1661
θ	0.4666	0.0212	0.0006	0.0033	0.0133
ϰ	0.4402	0.0006	0.0056	0.0129	0.0108
λ	2.8729	0.0033	0.0129	0.0948	-0.0021
ν	21.1661	0.0133	0.0108	-0.0021	2.6332

5.4.2 Mean value approach

Step 4 in the fault tree of Fig. 5-18 is calculated along 2 ways, as in the case of QP ; firstly, only error 1 and secondly both error 1 and 2 are included.
The equation for TB:

$$TB = E * TR^{\theta} * S^{-\varkappa} * RUA^{\lambda} * SN^{\nu} \qquad (5\text{-}25)$$

In which:

- TR = $N(\mu_{TR} : \sigma_{TR})$
- RUA = $N(\mu_{RUA} : \sigma_{RUA})$
- S = $N(\mu_{S} : \sigma_{S})$
- SN = $N(\mu_{SN} : \sigma_{SN})$

$N(\mu : \sigma_1 : \sigma_2)$

- E = N(27.4132: 8.9792 : 17.7321)
- θ = N(0.1457: 0.0565 : 0.1457)
- ϰ = N(0.0986: 0.0335 : 0.0745)
- λ = N(0.2574: 0.1524 : 0.3079)
- ν = N(0.7344: 0.9028 : 1.6227)

$\sigma_{E\theta}$ = (0.0182: 0.4666) $\sigma_{\theta K}$ = (-0.0012: 0.0006)
$\sigma_{E\kappa}$ = (-0.1286: 0.4402) $\sigma_{\theta\lambda}$ = (-0.0029: 0.0033)
$\sigma_{E\lambda}$ = (0.2118: 2.8729) $\sigma_{\theta\nu}$ = (0.0050: 0.0133)
$\sigma_{E\nu}$ = (7.0445:21.1661) $\sigma_{\kappa\lambda}$ = (0.0011: 0.0129)
$\sigma_{\lambda\nu}$ = (-0.0253:-0.0021) $\sigma_{K\nu}$ = (-0.0060: 0.0108).

The variance of TB is expressed as:

$$\sigma_{TB}^2 = \sigma_{TR}^2\left(\frac{\delta f}{\delta TR}\right)^2 + \sigma_S^2\left(\frac{\delta f}{\delta S}\right)^2 + \sigma_{RUA}^2\left(\frac{\delta f}{\delta RUA}\right)^2 + \sigma_{SN}^2\left(\frac{\delta f}{\delta SN}\right)^2 + \sigma_E^2\left(\frac{\delta f}{\delta E}\right)^2 + \sigma_\theta^2\left(\frac{\delta f}{\delta \theta}\right)^2 + \sigma_\kappa^2\left(\frac{\delta f}{\delta \kappa}\right)^2 + \sigma_\lambda^2\left(\frac{\delta f}{\delta \lambda}\right)^2 +$$
$$\sigma_\nu^2\left(\frac{\delta f}{\delta \nu}\right)^2 + 2\sigma_{E\theta}\left(\frac{\delta f \delta f}{\delta E \delta \theta}\right) + 2\sigma_{E\kappa}\left(\frac{\delta f \delta f}{\delta E \delta \kappa}\right) + 2\sigma_{E\lambda}\left(\frac{\delta f \delta f}{\delta E \delta \lambda}\right) + 2\sigma_{E\nu}\left(\frac{\delta f \delta f}{\delta E \delta \nu}\right) + 2\sigma_{\theta\kappa}\left(\frac{\delta f \delta f}{\delta \theta \delta \kappa}\right) + 2\sigma_{\theta\lambda}\left(\frac{\delta f \delta f}{\delta \theta \delta \lambda}\right) +$$
$$2\sigma_{\theta\nu}\left(\frac{\delta f \delta f}{\delta \theta \delta \nu}\right) + 2\sigma_{\kappa\lambda}\left(\frac{\delta f \delta f}{\delta \kappa \delta \lambda}\right) + 2\sigma_{\kappa\nu}\left(\frac{\delta \delta f}{\delta \kappa \delta \nu}\right) + 2\sigma_{\lambda\nu}\left(\frac{\delta f \delta f}{\delta \lambda \delta \nu}\right) \qquad (5\text{-}26)$$

Tab. 5-10 Accuracy of TB

Ct	μ_{TB} (hours)			V_{TB} (%)		
	GAMA	MC_1	MC_2	MV_1	MC_1	MC_2
1	20.04	18.89	15.67	18.21	19.60	119
2	43.29	39.99	41.94	11.57	14.31	92
3	30.96	28.92	27.33	11.26	12.46	84
4	28.92	27.17	24.21	10.93	12.59	94
6	35.94	33.83	34.22	10.59	11.57	80
7	32.61	30.58	28.63	11.15	11.43	83
8	31.02	29.09	26.60	11.98	15.01	87
10	33.63	31.77	30.96	10.76	10.87	73
11	30.76	28.86	28.25	11.99	11.81	72
12	26.89	25.25	22.85	12.52	14.52	75
13	49.20	45.95	51.06	11.60	12.34	97
14	36.81	34.53	33.75	11.79	11.99	82
15	39.22	36.89	37.05	11.03	12.15	88
16	25.46	23.87	21.48	12.32	13.30	70
17	28.28	26.89	25.12	10.51	10.71	72
18	26.87	25.38	22.50	12.34	15.32	83
19	31.02	29.14	27.81	10.65	11.50	78
20	33.22	31.22	29.95	11.53	12.83	81
21	27.19	25.47	23.69	11.99	13.89	76
22	36.90	34.78	34.64	13.52	12.89	78
23	32.25	30.53	29.55	11.92	12.49	71
24	27.56	25.76	23.49	10.97	13.07	80
25	28.46	26.60	24.64	11.26	12.89	71
26	30.41	28.62	26.41	12.09	12.25	81
27	34.40	32.74	33.13	12.67	14.35	68
28	34.94	32.89	32.14	12.10	13.81	86
29	34.47	32.48	32.45	11.02	13.10	80
30	34.36	32.37	31.65	10.83	12.92	78

MV_1 = Mixed analytical numerical approach for case 1
MC_1 = Monte Carlo Simulation for case 1
MC_2 = Monte Carlo Simulation for case 2.

5.4.4 Kali Putih

For 5 stations in the Kali Putih the accuracy of the Base Time is determined. Using the results of previous investigations (Gadjah Mada University 1986) and previous chapters of this study, calculations are performed in case of error 1 (variability of the measured data only). The number of simulations is again 10,000 and the coefficients of variation of S,RUA and SN again 3%. The results are shown in Tab. 5-11 .

Tab. 5-11 Accuracy of TB of Kali Putih

No	μ	(hours)	σ	(hours)	V	(%)
	GAMA	MC	MV	MC	MV	MC
1	22.54	21.34	4.30	3.80	19.09	17.76
2	24.37	22.91	3.42	3.25	14.02	14.14
3	24.77	23.27	3.32	3.35	13.42	14.37
4	25.31	23.80	3.35	3.33	13.23	13.99
5	28.03	26.45	3.85	3.79	13.74	14.28

For station number 5 at the mouth of the river the distribution of TB is derived from the histogram as shown in Fig. 5-25.

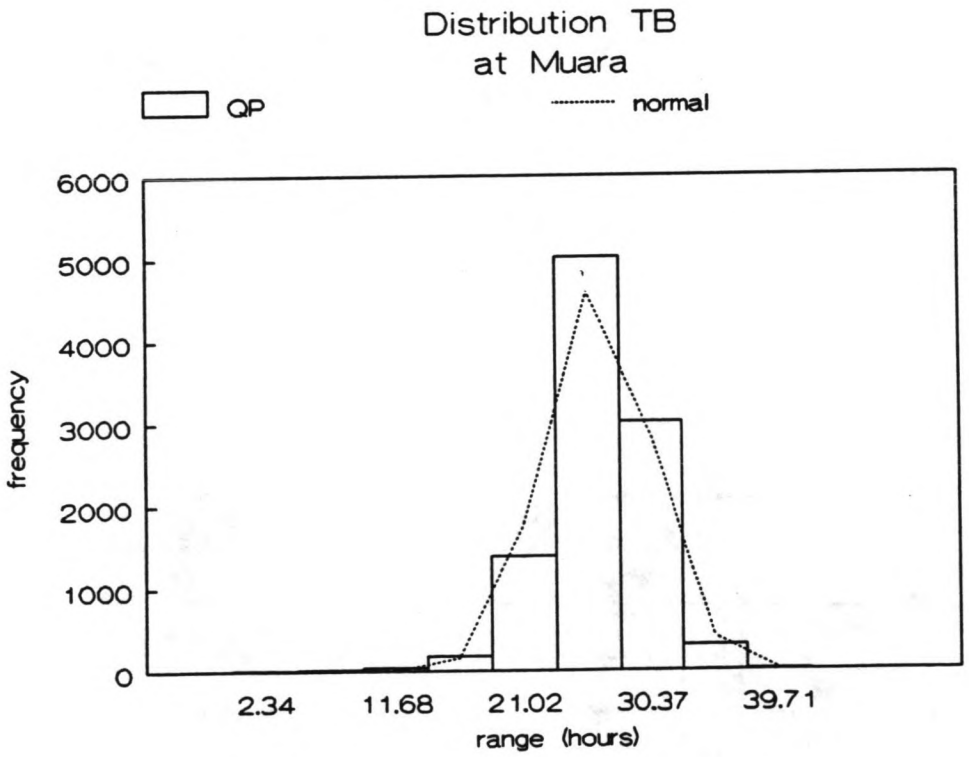

Fig. 5-25 Statistical distribution of TB at the mouth of the Putih

5.5 Conclusions

In case of strong non-linear relationships as for QP and TB it is proved that the mean value approach for accuracy calculations does not give satisfying results. By including higher order Taylor Series satisfying results might be obtained. But for equations with a large number of variables the amount of work to calculate the accuracy increases tremendously.
Simulation by means of the Monte Carlo method showed to be a good substitute in case of such relationships.
Regression analysis is very sensitive for inaccurate measurements. If the data used for regression are not measured accurately the results are not reliable. In case of QP and TB high coefficients of variation are obtained if inaccuracy of the measurements is taken into account (error 2).
If only the variability of the data is used then the coefficients of variation for QP and TR are bigger than for TB.
So far calculations are done for measurements on the catchment characteristics with a variation of 3%. The influence of higher variation, thus less accurate measurements, on the accuracy of TR,QP and TB at the mouth of the kali Putih (muara=mouth) is shown in Fig. 5-26 till 5-28.
In these figures the maximum coefficient of variation accepted is 10%. Higher values would influence the accuracy of the regression coefficients.
First TR is investigated. The Mean value approach is used to show the influence of increasing coefficients of variation of the variables. From Fig. 5-26 it can be seen that the symmetry factor SIM has no influence. The main stream length L and the source frequency SF have equal influence.

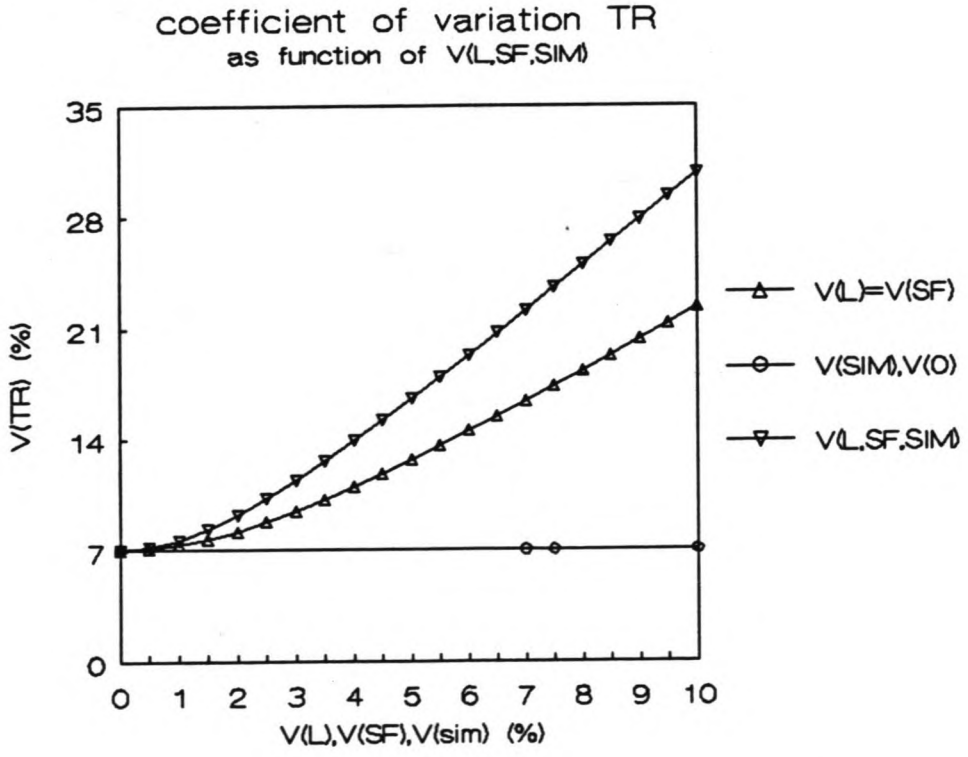

Fig. 5-26 Influence of the accuracy of the variables on TR

For QP and TB values are simulated. If all measurements are 100% accurate the coefficients of variation are determined by the accuracy of the regression coefficients of both TR and QP, and TR and TB ,V(0) in Fig. 5-26 and 5-27 respectively. Increasing variation has little in-

fluence. Between the data points lines are drawn. This is done for illustration; there is no relationship between the data points.
Different from the case of TR the regression coefficients of the QP and TB determine the accuracy.

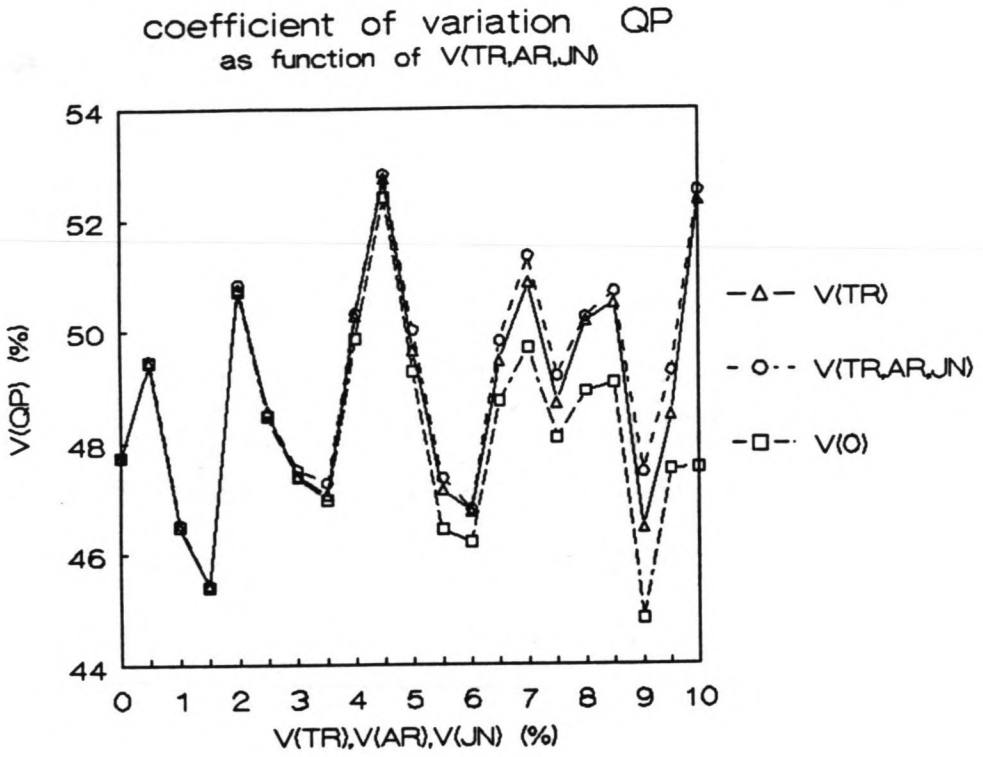

Fig. 5-27 Influence of the accuracy of the variables on QP

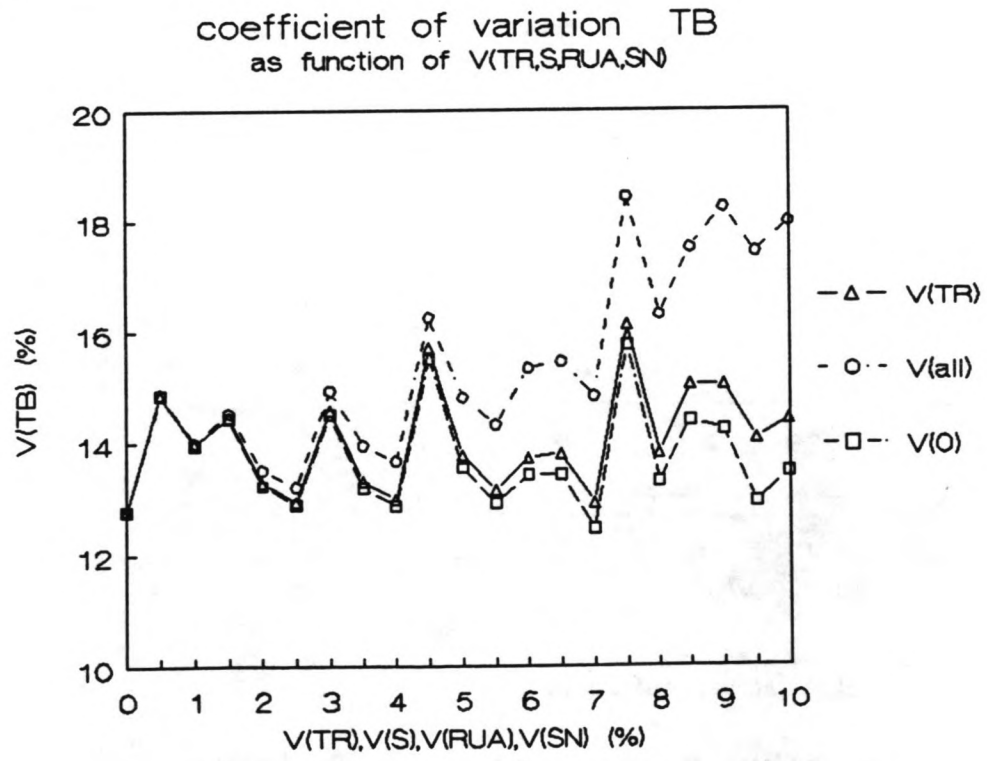

Fig. 5-28 Influence of the accuracy of the variables on TB

To visualize the accuracies of TR, QP and TB in one figure, 200 unit hydrographs are calculated for the station at the mouth of the kali Putih. This is done in case of the maximum allowable coefficient of variation, 10%, of the catchment characteristics. In Fig. 5-29 the simulated QP 's as function of the time are marked, next to the TB 's.
If the QP is too small, it is possible that the unit depth is not equal to 1 mm rainfall depth for $K \rightarrow \infty$ .22% of 200 simulated QP 's is too small and therefore not included in the figure.

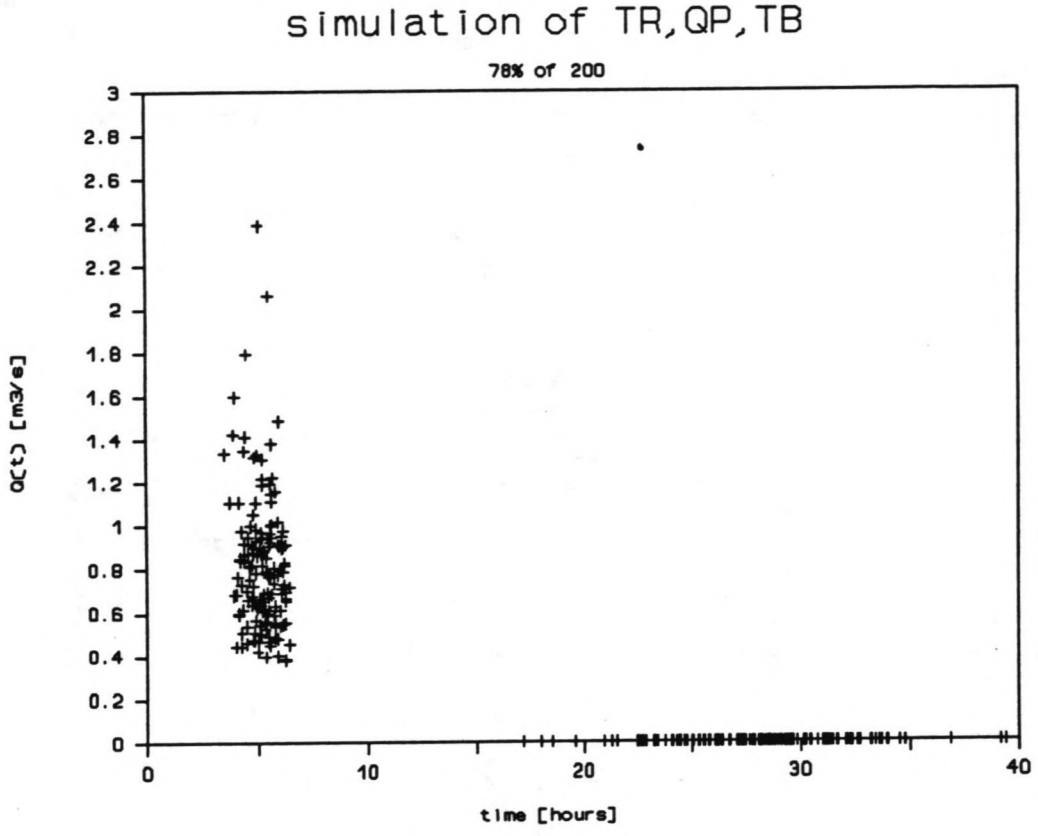

Fig. 5-29 Scatter diagram of simulated unit hydrograph characteristics for the mouth of the Putih

6. ACCURACY OF FLOOD ESTIMATION FOR KALI PUTIH

6.1 Introduction

In the previous chapters the GAMA I unit hydrograph is analyzed. For the three unit hydrograph characteristics TR,QP and TB coefficients of variation are determined to indicate the accuracy of the unit hydrograph.
The unit hydrograph is the base for the flood estimation. Other factors or input for the flood estimation are the rainfall depth with a certain return period, the rainfall losses (ϕ-index), the hourly distribution and the base flow.
All these factors have stochastic features. So the accuracy of the total hydrograph is determined by the accuracy of these factors. To visualize the influence a fault tree is constructed (Fig. 6-1).
In this chapter the factors of importance will briefly be discussed. A total hydrograph for a station in the Kali Putih is simulated from the distributions TR,the catchment characteristics and the factors mentioned above.

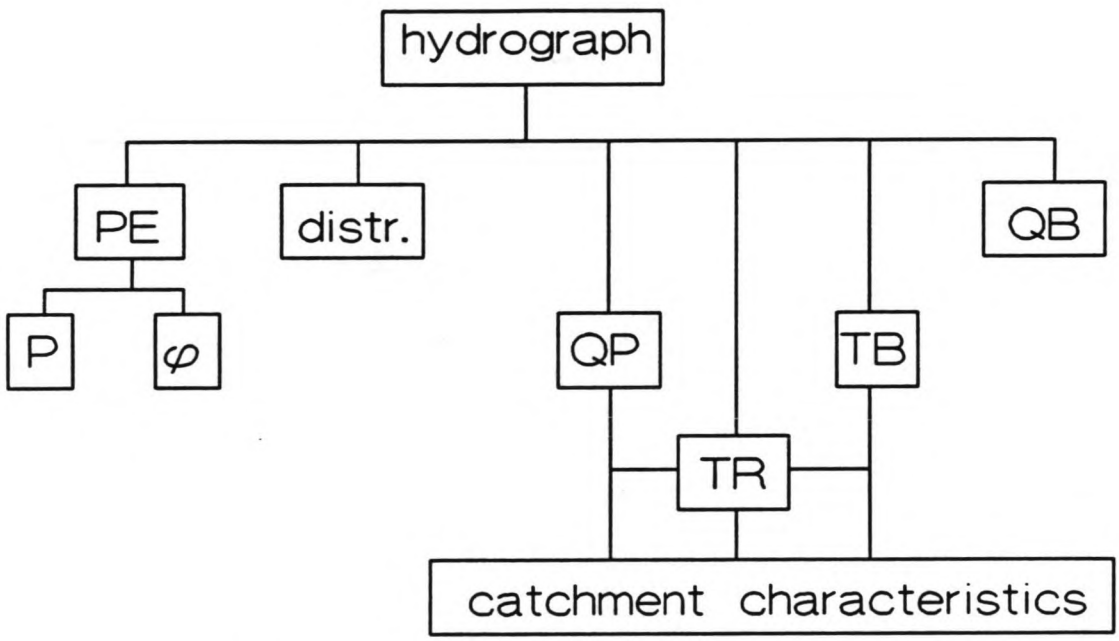

Fig. 6-1 Fault tree for the total hydrograph

6.2 Rainfall

The main input in the estimation of floods is the rainfall depth with a certain return period. By means of frequency analysis a value for the rainfall depth is derived from the frequency curve. The frequency curve is a cumulative distribution curve, e.g. Log Pearson, Log Normal, Normal, which fits best to the available rainfall data.
As already mentioned in §3.2 the available data are subject to different sources of errors. Therefore it is difficult to determine the accuracy of the rainfall data. An extra dificulty is that the frequency curves of the catchments can differ from eachother. So for every cathment a study needs to be made of the accuracy of the rainfall prediction.

In this stage it is not of importance to know if a storm with a return period of 30 years gives 100 mm or 130 mm rainfall. Of importance is the influence of the amount of rainfall depth on the accuracy of flood estimation irrespective the return period. This influence can be checked by changing the rainfall depth in the input of the program "HYDROGRAPH", which will be explained in §6.5.

6.3 ϕ-index

The ϕ-index is the amount of water loss due to infiltration, depression storage, evatransporation, interception and other phenomena that prevent rain water to be transported. Sri Harto 1985 states that the ϕ-index is dependent upon catchment characteristics such as catchment area A and source frequency SN. By means of regression analysis a functional relationship is derived. The result:

$$\phi = 10.4093 - 3.859 * 10^{-6} * A^2 + 1.6985 * 10^{-13} \left(\frac{A}{SN} \right)^4 \qquad (6\text{-}1)$$

In which ϕ in mm/hour.
This equation has a low coefficient of correlation. So estimating the ϕ-index with this equation gives inaccurate results. Realizing this, the number of decimal places of the regression coefficients in (6-1) should be reduced in order to avoid apparently accurate results.
So it is for practical application recommended to reduce equation (6-1), like:

$$\phi = 10.4 - 3.9 * 10^{-6} * A^2 + 1.7 * 10^{-13} \left(\frac{A}{SN} \right)^4 \qquad (6\text{-}2)$$

Because SN is almost constant for all considered catchments the influence of increasing catchment area can be shown graphically. In Fig. 6-2 ϕ is plotted as function of A and for SN=0.73.
From this figure it can be seen that the ϕ-index is constant for small catchment area's (A<200 km^2).

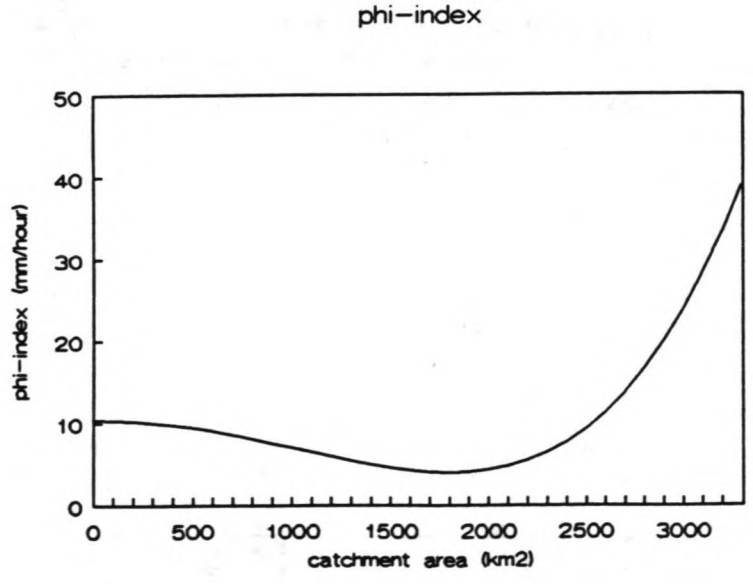

Fig. 6-2 Rainfall losses related to catchment area

It is assumed that the ϕ-index is constant during the storm. In reality the losses are usually higher at the beginning of a storm than at the end. So at the beginning of the storm the ϕ-index is overestimated and at the end of the storm underestimated (Laurensons and O'Donnel 1969).
It is difficult to find reliable values for the rainfall losses. Because of its generally small values it is less important to know the exact values . In this study no further attention is paid to the accuracy of the ϕ-index. In HYDROGRAPH the ϕ-index is calculated according to equation (6-1) in which A and SN are generated from their distributions.

6.4 Base flow

Base flow, QB, is the flow in streams in the absence of rain. The existing flow in streams is sustained by ground water discharge and a small amount of depletion of channel storage. The magnitude of the groundwater discharge is roughly proportional to the ground water storage. Besides, it depends also on the catchment permeability, length of streams and the penetration of streams into the aquifer.
The amount of recharge is in proportion to the catchment area. The larger the area the higher the recharge. The total length of streams is expressed as the length per unit area, or as the drainage density D.
The magnitude of the measured QB of each catchment is obtained by drawing depletion curves as an enveloping straight line on semi-logarithmic paper, log Q versus t.
Again regression analysis is applied to find a relationship between QB and A,D. The equation calculates a constant value for QB:

$$QB = 0.4751 * A^{0.6444} * D^{0.9430} \qquad (6\text{-}3)$$

Also for QB the accuracy of the regression coefficients is small because of a low coefficient of correlation. Because the QB is roughly estimated no further research will be performed on its accuracy. In HYDROGRAPH the QB will be calculated with generated values for A and D.

6.5 Hydrograph

The accuracy of the total hydrograph can be calculated by means of simulation. The program "HYDROGRAPH" is based on the Monte Carlo method (see appendix VIII). The input characteristics, as mentioned previously, are generated from their distributions. HYDROGRAPH calculates a number of hydrographs according the GAMA I method. The hydrographs are stored in a m*n matrix, in which m the number of simulated hydrographs and n the time of discharge (see appendix VIII). By transforming the output to a spreadsheet matrix operations can be performed.
For the Kali Putih calculations are done at the mouth. The input for calculation:

- rainfall depth with return period of 30 years : 105 mm
- storm duration : 7 hours
- hourly distribution: 10.8% 53.3% 13% 9.2% 5.6% 5.3% 2.8%
- catchment characteristics:

A km^2	L km	S	SIM	SF	SN	JN
32.76	24.4	0.047	0.2320	0.1197	0.6667	1

- Time of Rise TR : μ = 5.17 hours, V = 0.12
- lag time K : 16.4 hours (initial value).

The results of calculation are shown in Fig. 6-3 in which 4 of the 75 simulated hydrographs are plotted next to the hydrograph calculated with the mean values (see also Fig. 2-2).
The hydrographs have remarkable sharp peaks. This is caused by the shape of the unit hydrograph and the hourly distribution.

How to determine the accuracy from the hydrographs?
It is not possible to average the discharge of all the hydrographs for each hour of discharge, because the obtained mean hydrograph wouldn't enclose 105 mm anymore. For further analysis, like the influence on the accuracy of morphological predictions, all the hydrographs have to be taken into account.

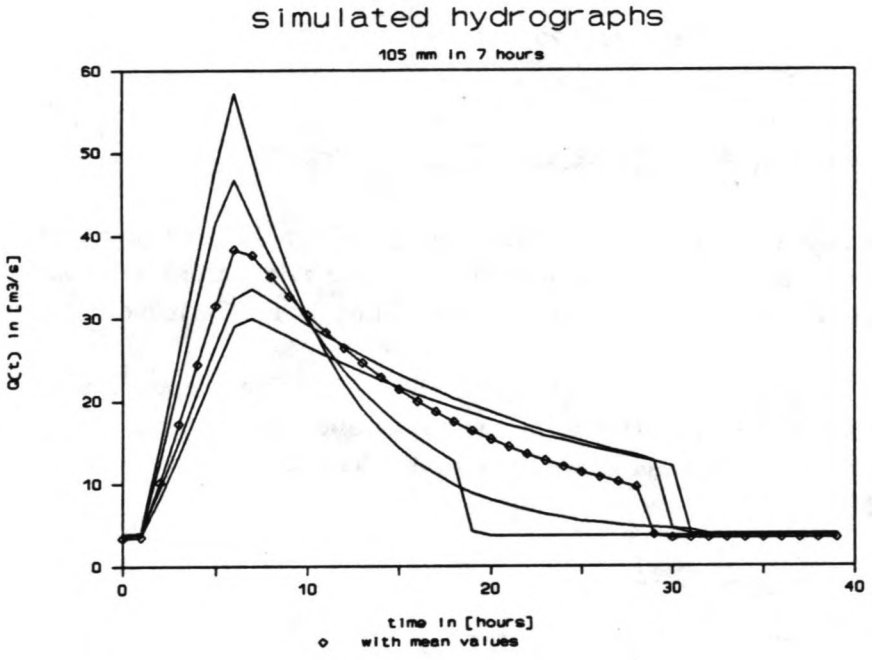

Fig. 6-3 A number of simulated hydrographs for 7 hourly storm

However characteristics of the hydrographs can be discussed. For the maximum discharge QP^*, the time to maximum discharge TR^* and total time of run off TB^* it shows:

	TR^* (hours)	QP^* (m^3/s)	TB^* (hours)
μ	6.39	41.46	28.46
σ	0.69	14.78	4.28
V	10.8 %	35.6 %	15.0 %

For QP^* the distribution is shown in figure 6.4.

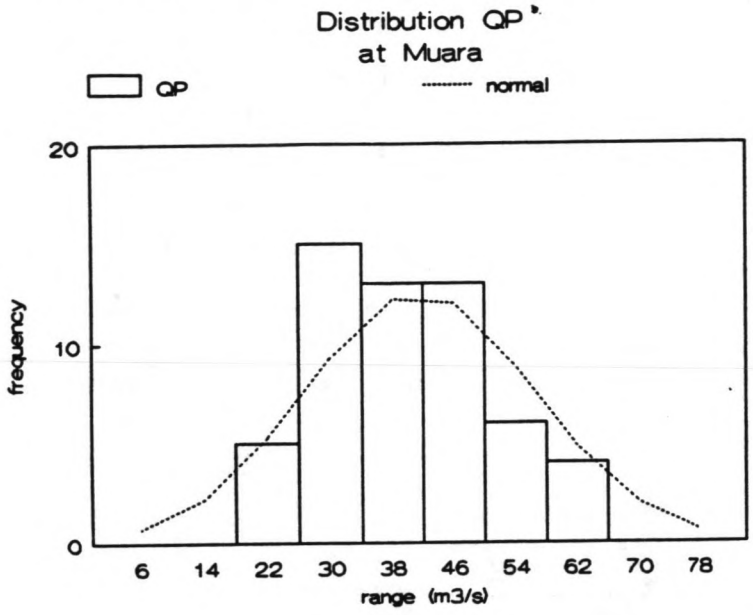

Fig. 6-4 Statistical distribution of maximum discharge at the mouth

Calculations with varying rainfall depth showed a linear change of the QP^* and therefore the coefficient of variation remained constant. This is due to the linear relationship between Q(t) and PE (effective rainfall). So the accuracy of flood estimation is not influenced by the rainfall depth.
Calculations with a different hourly distribution however give remarkable results. The hourly distribution as recommended by Sri Harto (1985) for a storm duration of 9 hours is : 24% 26% 17% 11% 7% 5% 4% 3% 3%. The hydrographs have a less peaky shape and the maximum values are flattened (see Fig. 6-5).

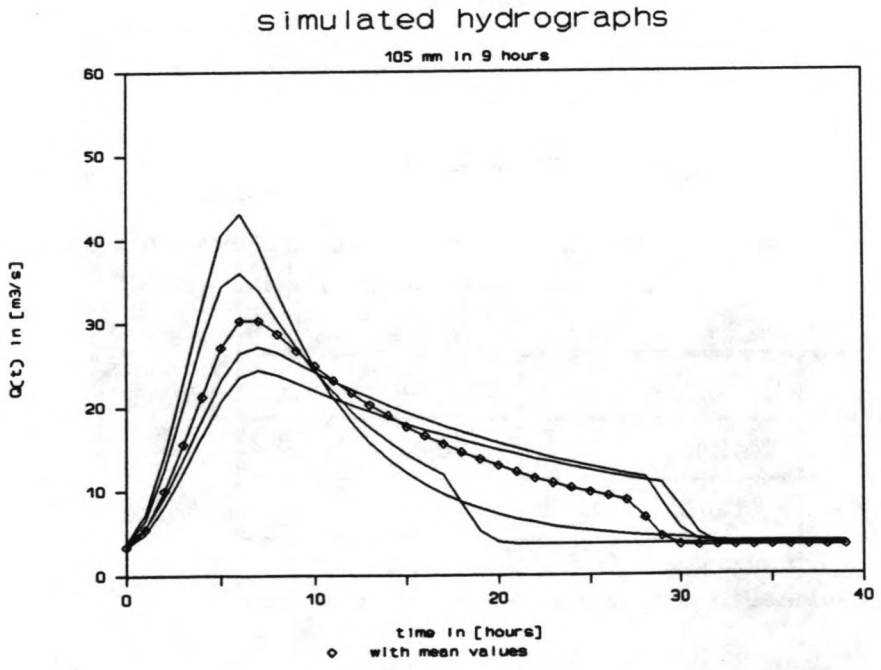

Fig. 6-5 A number of simulated hydrographs for 9 hourly storm

Examination of the hydrograph characteristics show:

	TR^* (hours)	QP^* (m3/s)	TB^* (hours)
μ	6.58	32.32	29.26
σ	0.83	10.16	4.28
V	12.6 %	31.4 %	14.6 %

It is clear that flood estimation is more accurate in case of the latter distribution. In general this means that for large varying rainfall distributions flood estimation is less accurate.

7. CONCLUSIONS AND RECOMMENDATIONS

The empirical, parametric model ,the GAMA I unit hydrograph, for flood estimation for the rivers on Java is extended with stochastic features. The model HYDROGRAPH can be used to calculate the accuracy of the flood estimation.

The following conclusions are drawn from this study:

- To apply non-linear regression analysis for the derivation of equations, accurately measured data are required. If this is not the case then the obtained equations are not reliable.

- The Monte Carlo simulation technique is a reliable method for calculating the accuracy of (non-)linear relationships.

- The GAMA equations can be simplified by reducing the number of decimal places of the regression coefficients. Calculations showed that the differences of the results are negligible in comparison with the accuracy of the model.
  The following equations can be used:

$$TR = 0.43 * \left(\frac{L}{100SF} \right)^3 + 1.07 * SIM + 1.28$$

$$QP = 0.18 * AR^{0.59} * JN^{0.24} * TR^{-0.4}$$

$$TB = 27.41 * TR^{0.15} * S^{-0.1} * RUA^{0.26} * SN^{0.73}$$

Further topics of investigation in future studies can be:

- Application of the model HYDROGRAPH to catchments of which records of measured floods are available.

- Improvement of the rainfall network in Java.

- Investigation of the rainfall distribution.

- Extension of the model HYDROGRAPH by including the accuracy of the rainfall depth.

8. REFERENCES

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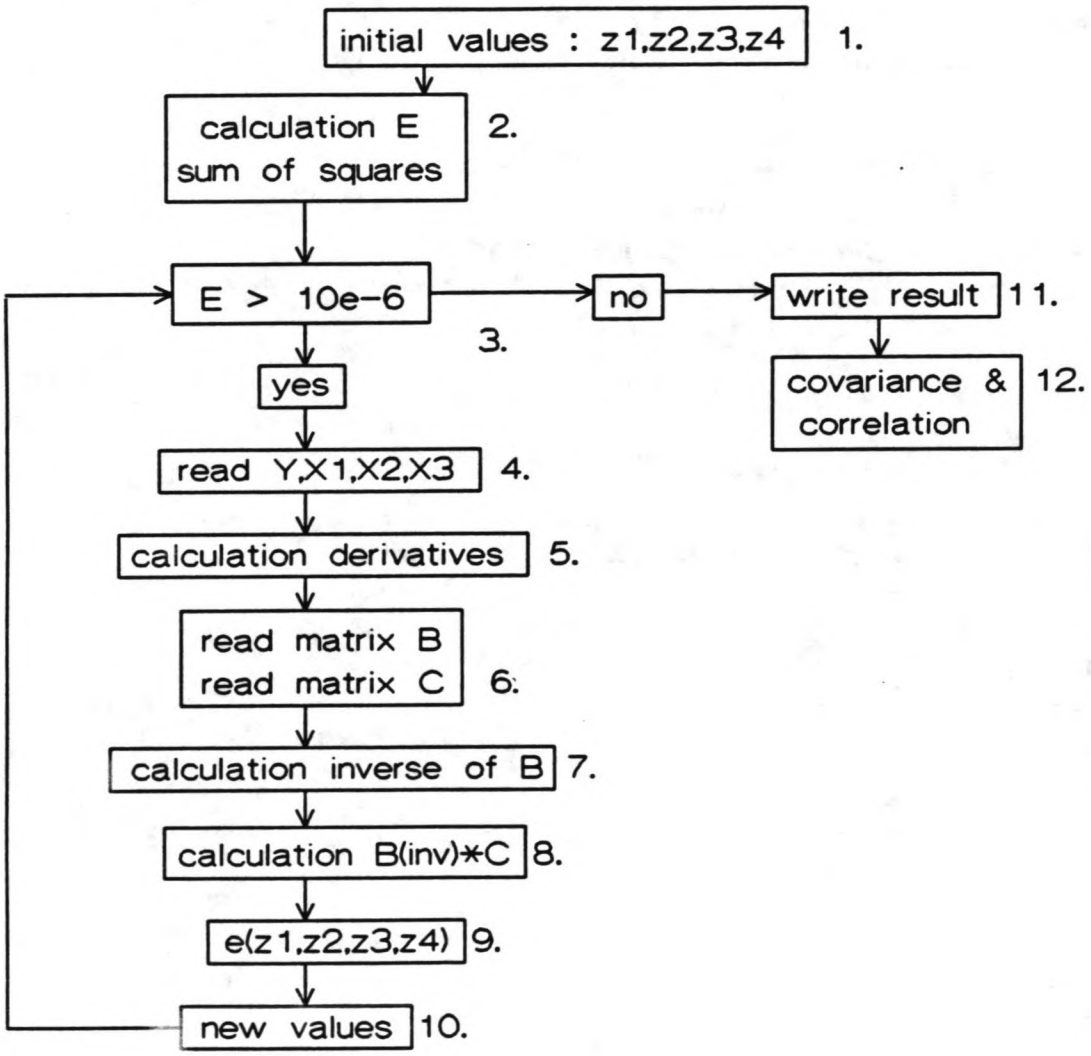
APPENDIX I

NON-LINEAR REGRESSION

The program REGRES is written in order to calculate the regression coefficients of linear and non-linear relationships. This program is of a classical type, which means that the input and output are stored in files. This makes REGRES user unfriendly. But then software like SPSS and SAS, which are more user friendly, already excists.
On the other hand the structure of REGRES makes it easy to make adaptions for other purposes, like extensions with stochastic processes.
This simple program is discussed by means of the following scheme:



Step 1 : To start the iteration procedure initial values for the regression coefficients are defined. In case of the equation for TR: B,C,D; and QP: A, α, ß, δ; where the latter is expressed in the program as z1,z2,z3,z4.

Step 2 : The error function and the difference with the previous iteration step is calculated.

Step 3 : Criterium for E to continue the iteration procedure.

Step 4 : Reading the variables from the input file. For QP(Y), X1(AR), X2(JN), X3(TR) are read.

Step 5 : Calculation of the derivatives: $\delta Y/\delta z1, \delta Y/\delta z2$ etc.

Step 6 : Matrix B is equal to the matrix $(A*A^T)$.
Matrix C is equal to the matrix (A^T*s) as mentioned in §4.2.

Step 7 : Procedure "Inverse" in REGRES calculates the inverse matrix of B.

Step 8 : Procedure "Matrixmultiplication" in REGRES calculates: $(A*A^T)^{-1}*(A^T*s)$.

Step 9 : The deviations from the optimum are determined.

Step 10: Procedure "Newvalues" in REGRES assigns new values to z1,z2,z3,z4.

Step 11: Procedure "Result" in REGRES writes the optimum coefficients if the accuracy criterium of the iteration is met.

Step 12: Finally the matrices for the covariances and correlations is determined.

Explanation of the parameters used in REGRES:

- z1,z2,z3,z4: Regression coefficients
- Y: Dependent variable
- X1,X2,X3: Independent variables
- sdY: Standard deviation of Y
- dz1: Derivative of Y to z1
- ez1: Deviation of z1
- sum: Sum of squares due to regression
- ms: Mean square :sum/n-4
- corr: Correlation matrix.

```
program REGRES(input,output);
type matrix = array [1..10,1..10] of real;
type vector = array [1..10] of real;
var z1,z2,z,z3,z4,P,SI,Y,sdY,X1,X2,X3,sdX3,dz1,dz2,dz3,dz4: real;
   ez1,ez2,ez3,ez4,sum,E,E1,ms: real;
   b,corr:matrix;
   c,d: vector;
   i,j,k,l,m,n,q,iter: integer;
   r,f: text;
   s,t,u,v,w:real;

{inverse of matrix}

procedure inverse (s,t,u,v,w: real);
begin
    for i:=2 to 4 do
    begin
        s:=b[i,1]/b[1,1];
        for j:=1 to 8 do b[i,j]:=b[i,j]-s*b[1,j];
    end;
    for i:=1 to 1 do
    begin
        t:=b[i,2]/b[2,2];
        for j:=1 to 8 do b[i,j]:=b[i,j]-t*b[2,j];
    end;
    for i:=3 to 4 do
```

```
        begin
            t:=b[i,2]/b[2,2];
            for j:=1 to 8 do b[i,j]:=b[i,j]-t*b[2,j];
        end;
        for i:=1 to 2 do
        begin
            u:=b[i,3]/b[3,3];
            for j:=1 to 8 do b[i,j]:=b[i,j]-u*b[3,j];
        end;
        for i:=4 to 4 do
        begin
            u:=b[i,3]/b[3,3];
            for j:=1 to 8 do b[i,j]:=b[i,j]-u*b[3,j];
        end;
        for i:=1 to 3 do
        begin
            v:=b[i,4]/b[4,4];
            for j:=1 to 8 do b[i,j]:=b[i,j]-v*b[4,j];
        end;
        for i:=1 to 4 do
        begin
            w:=b[i,i];
            for j:=1 to 8 do b[i,j]:=b[i,j]/w;
        end;
    end;

    {matrixmultiplication}

    procedure matrixproduct (b:matrix; c:vector);
    begin
        for i:=1 to 4 do
        begin
            d[i]:=0;
            for j:=5 to 8 do d[i]:=d[i]+b[i,j]*c[j];
        end;
    end;

    {clearing matrices}

    procedure clearmatrix;
    begin
        for i:=1 to 4 do
        begin
            for j:=1 to 8 do b[i,j]:=0;
        end;
        for i:=5 to 8 do c[i]:=0;
    end;

    {read matrices}
```

```
procedure readb;
begin
    b[1,1]:=b[1,1]+sqr(dz1); b[2,2]:=b[2,2]+sqr(dz2);
    b[3,3]:=b[3,3]+sqr(dz3); b[4,4]:=b[4,4]+sqr(dz4);
    b[1,2]:=b[1,2]+dz1*dz2; b[1,3]:=b[1,3]+dz1*dz3;
    b[1,4]:=b[1,4]+dz1*dz4; b[2,3]:=b[2,3]+dz2*dz3;
    b[2,4]:=b[2,4]+dz2*dz4; b[3,4]:=b[3,4]+dz3*dz4;
    for i:=1 to 4 do
    begin
        for j:=1 to 4 do b[j,i]:=b[i,j];
    end;
    b[1,5]:=1; b[2,6]:=1; b[3,7]:=1; b[4,8]:=1;
end;

procedure readc;
begin
    c[5]:=c[5]+dz1*SI;
    c[6]:=c[6]+dz2*SI;
    c[7]:=c[7]+dz3*SI;
    c[8]:=c[8]+dz4*SI;
end;

{new values}

procedure newvalues(d:vector);
begin
    ez1:=-(1/3)*d[1]; if abs(ez1)<0.000001 then ez1:=0;
    ez2:=-(1/3)*d[2]; if abs(ez2)<0.000001 then ez2:=0;
    ez3:=-(1/3)*d[3]; if abs(ez3)<0.000001 then ez3:=0;
    ez4:=-(1/3)*d[4]; if abs(ez4)<0.000001 then ez4:=0;
end;


procedure result;
begin
    writeln (f);
    writeln (f,'Iteration step ',iter);
    writeln (f,'A    :',z1:10:6,'eA    :',ez1:10:6);
    writeln (f,'alfa :',z2:10:6,'ealfa :',ez2:10:6);
    writeln (f,'beta :',z3:10:6,'ebeta :',ez3:10:6);
    writeln (f,'delta:',z4:10:6,'edelta:',ez4:10:6);
    writeln (f,'Residual sum of squares: ',sum:10:6);
    writeln (f); writeln (f);
end;

{correlation of parameters}

procedure correlation;
begin
    writeln (f,'The covariance matrix cov(z)');
```

```
    writeln (f);
    write (f,'   A      alfa       beta      ');
    writeln (f,'delta');
    writeln (f);
    ms:=sum/(n-4);              {mean square}
    for i:=1 to 4 do
    begin
        for j:=1 to 4 do
        begin
            b[i,j]:=b[i,j+4];
            b[i,j]:=b[i,j]*ms;
            write (f,b[i,j]:10:6);
        end;
        writeln (f);
    end;
    writeln (f); writeln (f);
    write (f,'The correlation matrix of the ');
    writeln (f,'parameter estimates');
    writeln (f);
    write (f,'   A      alfa       beta      ');
    writeln (f,'delta');
    writeln (f);
    for i:=1 to 4 do
    begin
        for j:=1 to 4 do
        begin
            corr[i,j]:=b[i,j]/(sqrt(b[i,i]*b[j,j]));
            write (f,corr[i,j]:10:6);
        end;
        writeln (f);
    end;
end;


{main program}

begin
    assign (r,'b:insimqp.pas');
    assign (f,'a:outcmqp.pas');
    rewrite (f);
    writeln (f);
    write (f,'The calculation of the regression ');
    writeln (f,'coefficients for QP');
    writeln (f);
    z1:=0.10; z2:=0.5; z3:=0.25; z4:=0.35;
    k:=0;
    ez1:=0; ez2:=0; ez3:=0; ez4:=0;
    E:=1; E1:=0;
    iter:=1;
    while (abs(E)>0.000001) and (iter<=40) do
```

```
    begin
        z1:=z1+ez1; z2:=z2+ez2; z3:=z3+ez3;
        z4:=z4+ez4;
        sum:=0;
        clearmatrix;
        reset (r);
        readln (r,n);
        readln (r);
        for q:=1 to n do
        begin
            readln (r,Y,sdY,X1,X2,X3,sdX3);
            P:=exp(z2*ln(X1))*exp(z3*ln(X2))*exp(-z4*ln(X3));
            SI:=Y-z1*P;
            dz1:=-P;
            dz2:=-z1*P*ln(X1);
            dz3:=-z1*P*ln(X2);
            dz4:=z1*P*ln(X3);
            readb;
            readc;
            k:=k+1;
            sum:=sum+sqr(SI);
        end;
        inverse (s,t,u,v,w);
        matrixproduct (b,c);
        newvalues (d);
        E:=E1-sum;
        E1:=sum;
        iter:=iter+1;
    end;
    result;
    correlation;
    writeln (f);
    writeln (f,'End of computation');
    close (f);
end.
```

APPENDIX II

MONTE CARLO SIMULATION

Due to the increasing technological development of personal computers with increasing memory and decreasing calculation time simulation techniques have become more and more popular.
Simulation techniques are mainly applied to check the results of analytical calculations on stochastic processes. If no analytical methods are available simulation has proved to be a reliable alternative.
The Monte Carlo simulation method makes use of the possibility to generate random values from a uniform distribution. This is offered by all nowadays computer languages. In this case Turbo Pascal 5.5 .
It is possible to generate values from any distribution. For example, if a number of generated values from the random generator is added then according to the central theorem a normal distributed stochast y is obtained, for which counts:

$$y = \frac{\left(\sum_{i=1}^{N} X_u\right) - \frac{N}{2}}{\sqrt{\frac{N}{12}}}$$

In which

- X_u is uniformly distributed $0<X_u<1$
- N is number of generations.

The stochast y has a normal distribution with a mean of 0 and a standard deviation of 1 (N(0:1)).
To obtain a value with any mean and standard deviation the following transformation is applied:

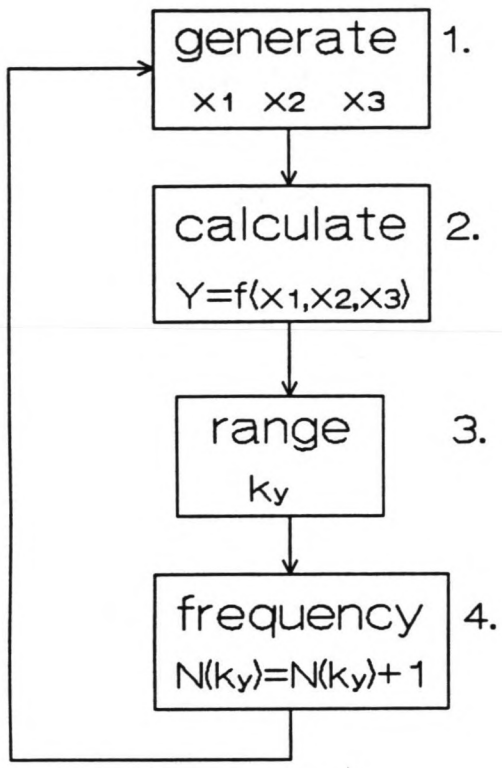
$$X_N = \sigma * y + \mu$$

Substituting y gives:

$$X_N = \sigma\left(\frac{\left(\sum_{i=1}^{N} X_u\right) - \frac{N}{2}}{\sqrt{\frac{N}{12}}}\right) + \mu$$

For other distributions than the normal one, similar procedures are available (Vrouwenvelder and Vrijling 1987).
The application of the Monte Carlo similation will be explained by means of an example .
Assume the function Y as function of the independent variables X1,X2,X3 ; Y=f(X1,X2,X3).
The independent variables have a normal distribution with means μ_1,μ_2 and μ_3 and standard deviations σ_1,σ_2 and σ_3.
Aim of simulation is to determine the mean and standard deviation of Y.
Assume that x_1,x_2 and x_3 are generated values from the distributions of respectively X_1,X_2 and X_3 .

The Monte Carlo simulation proceeds as follows:



Step 1 : The generation of values x_1,x_2 and x_3 from the distributions of X_1,X_2 and X_3. For each generated variable N times a X_u has to be determined. The higher N the more a normal distribution is approximated. A good approximation is achieved for N=10.

Step 2 : Calculation of the dependent variable Y with the generated x_1,x_2 and x_3.

Step 3 : Determination of the range in which the value for Y is located.

Step 4 : If the range for Y is found the frequency in this range is upgraded with one.

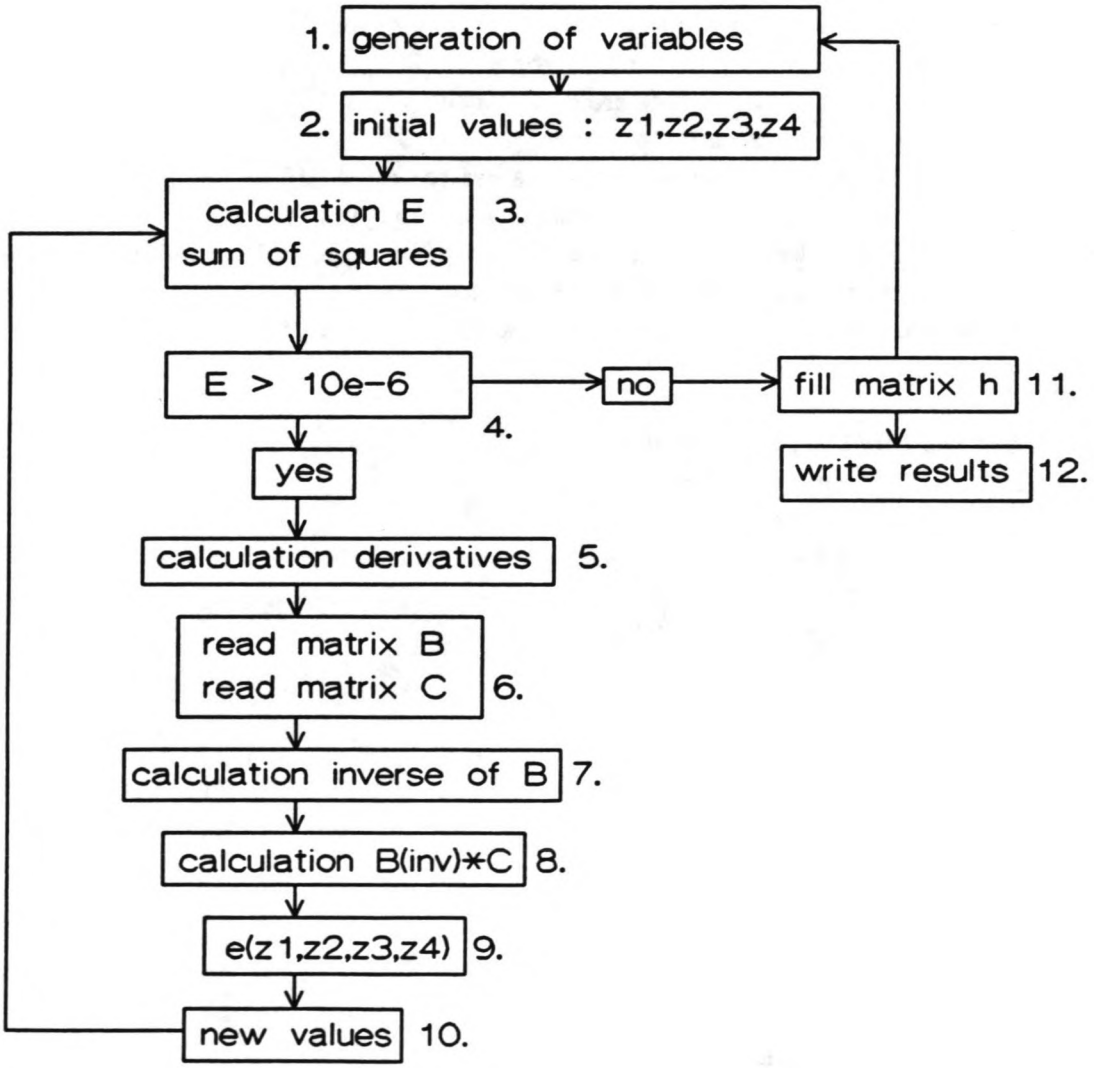
The result of the simulation is a histogram of Y values. From this histogram the mean and standard deviation can easily be calculated.

For a reliable result the number of simulations needs to be large; The larger the number of simulations the more accurate the prediction of mean and standard deviation.

APPENDIX III

SIMULATION OF REGRESSION COEFFICIENTS

The program SIMCO is derived from the program REGRES which is already discussed. REGRES is extended with the possibility to generate variables from normal distributions according to the Monte Carlo method.
In this study it is shown that the dependent and the independent variables have stochastic features.
In SIMCO also the input and output are stored in seperate files, which is not quite user friendly.
So the program SIMCO is developed with the possibility to randomize variables from its distributions and calculate the regression coefficients.
The program consists of a main part and some procedures.
The structure of the program is visualized in the following scheme:



Most steps are already explained in appendix 1 for the program REGRES. Nevertheless all steps in SIMCO are briefly discussed.

Step 1 : There are 28 catchments and for each catchment the variables are generated from a normal distribution.
In case of the equation for QP this means that a quartet (QP,AR,JN and TR) is generated for each catchment. In the program these are expressed as Y,X1,X2 and X3 respectively.

Step 2 : To start the iteration procedure initial values for the regression coefficients are defined. In case of the equation for QP: A,α,β,δ, which are expressed in the program as z1,z2,z3 and z4.

Step 3 : The error function and the difference of it with the previous iteration is calculated.

Step 4 : If the difference is smaller than the desired iteration accuracy, e.g. 10E-6, then the iteration procedure is ended.

Step 5 : Calculation of the derivatives: $\delta Y/\delta A$, $\delta Y/\delta\alpha$ etc.

Step 6 : Matrix B is equal to the matrix $(A*A^T)$.
Matrix C is equal to the matrix (A^T*s) as mentioned in §4.2.
Procedures "readb" and "readc" in the program.

Step 7 : The inverse of the matrix B is calculated.
Procedure "inverse" in program.

Step 8 : The matrices are multiplied as $(A*A^T)^{-1}*(A^T*s)$.
Procedure "matrixmultiplication" in program.

Step 9 : The deviations e_A,e_α,e_β and e_δ are determined.
and 10 Procedure "newvalues" in the program.

Step 11: Matrix h is filled with the optimum values for the coefficients. If the number of generations is 150 then the dimensions of h are 150*4 in case of Qp.

Step 12: The matrix h is written to the output file. It is also possible to write the result of each generation, like is shown §4.3, to the output file.
Procedure "coefficients" and for the latter "result" in the program.

Explanation of the parameters used in SIMCO:

- l : Number of generations
- z1 : Regression coefficient
- SI : Deviation of regression s_i in §4.2
- Y : Dependent variable (e.g. QP)
- X1 : Independent variable
- dz1 : Derivative of Y to z1
- ez1 : Deviation z1
- sum : Sum of squares due to regression.

```
program SIMCO(input,output);
const l:integer = 150;
type matrix = array [1..200,1..10] of real;
type vector = array [1..10] of real;
var z1,z2,z,z3,z4,P,SI,Xu,Y,sdY,X1,X2,X3,sdX3,dz1,dz2,dz3,dz4: real;
   ez1,ez2,ez3,ez4,s,t,u,v,w,E,E1,sum: real;
   b,g,h:matrix;
   c,d: vector;
   i,j,k,max,n,q,iter,gen: integer;
   r,f: text;
```

```
{inverse of matrix}

procedure inverse (s,t,u,v,w: real);
begin
    for i:=2 to 4 do
    begin
        s:=b[i,1]/b[1,1];
        for j:=1 to 8 do b[i,j]:=b[i,j]-s*b[1,j];
    end;
    for i:=1 to 1 do
    begin
        t:=b[i,2]/b[2,2];
        for j:=1 to 8 do b[i,j]:=b[i,j]-t*b[2,j];
    end;
    for i:=3 to 4 do
    begin
        t:=b[i,2]/b[2,2];
        for j:=1 to 8 do b[i,j]:=b[i,j]-t*b[2,j];
    end;
    for i:=1 to 2 do
    begin
        u:=b[i,3]/b[3,3];
        for j:=1 to 8 do b[i,j]:=b[i,j]-u*b[3,j];
    end;
    for i:=4 to 4 do
    begin
        u:=b[i,3]/b[3,3];
        for j:=1 to 8 do b[i,j]:=b[i,j]-u*b[3,j];
    end;
    for i:=1 to 3 do
    begin
        v:=b[i,4]/b[4,4];
        for j:=1 to 8 do b[i,j]:=b[i,j]-v*b[4,j];
    end;
    for i:=1 to 4 do
    begin
        w:=b[i,i];
        for j:=1 to 8 do b[i,j]:=b[i,j]/w;
    end;
end;

{matrixmultiplication}

procedure matrixmultiplication (b:matrix; c:vector);
begin
    for i:=1 to 4 do
    begin
        d[i]:=0;
        for j:=5 to 8 do d[i]:=d[i]+b[i,j]*c[j];
    end;
```

```
end;

{clear matrices}

procedure clearmatrix;
begin
    for i:=1 to 4 do
    begin
        for j:=1 to 8 do b[i,j]:=0;
    end;
    for i:=5 to 8 do c[i]:=0;
end;

{read matrices}

procedure readb;
begin
    b[1,1]:=b[1,1]+sqr(dz1); b[2,2]:=b[2,2]+sqr(dz2);
    b[3,3]:=b[3,3]+sqr(dz3); b[4,4]:=b[4,4]+sqr(dz4);
    b[1,2]:=b[1,2]+dz1*dz2; b[1,3]:=b[1,3]+dz1*dz3;
    b[1,4]:=b[1,4]+dz1*dz4; b[2,3]:=b[2,3]+dz2*dz3;
    b[2,4]:=b[2,4]+dz2*dz4; b[3,4]:=b[3,4]+dz3*dz4;
    for i:=1 to 4 do
    begin
        for j:=1 to 4 do b[j,i]:=b[i,j];
    end;
    b[1,5]:=1; b[2,6]:=1; b[3,7]:=1; b[4,8]:=1;
end;

procedure readc;
begin
    c[5]:=c[5]+dz1*SI;
    c[6]:=c[6]+dz2*SI;
    c[7]:=c[7]+dz3*SI;
    c[8]:=c[8]+dz4*SI;
end;

{new values}

procedure newvalues(d:vector);
begin
    ez1:=-(1/3)*d[1]; if abs(ez1)<0.000001 then ez1:=0;
    ez2:=-(1/3)*d[2]; if abs(ez2)<0.000001 then ez2:=0;
    ez3:=-(1/3)*d[3]; if abs(ez3)<0.000001 then ez3:=0;
    ez4:=-(1/3)*d[4]; if abs(ez4)<0.000001 then ez4:=0;
end;
```

```
{write results of 1 simulation}

procedure results;
begin
    writeln (f); writeln (f);
    writeln (f,'Iteration step ',iter);
    writeln (f,'number of catchments ',k);
    if sum<10000 then
    begin
        writeln (f,'A    : ',z1:10:6,ez1:10:6,d[1]:10:6);
        writeln (f,'alfa : ',z2:10:6,ez2:10:6,d[2]:10:6);
        writeln (f,'beta : ',z3:10:6,ez3:10:6,d[3]:10:6);
        writeln (f,'delta: ',z4:10:6,ez4:10:6,d[4]:10:6);
        writeln (f);
        writeln (f,'Sum of squares : ',sum);
    end
    else writeln (f,'Convergence criterium not met');
end;

procedure coefficients;
begin
    writeln (f); writeln (f);
    writeln (f,'Coefficients of TR of ',gen,' generations');
    writeln (f,'    A        alfa      beta     delta');
    for i:=1 to l do
    begin
        for j:=1 to 6 do write (f,h[i,j]:10:6,' ');
        writeln (f);
    end;
end;



{main program}

begin
    assign (r,'b:insimqp.pas');
    assign (f,'b:outsimqp.pas');
    rewrite (f);
    writeln (f);
    writeln (f,'The simulation of the regression coefficients of QP');
    writeln (f); writeln (f);
    for gen:=1 to l do
    begin
        reset (r);
        readln (r,n);
        readln (r);
        for i:=1 to n do
        begin
            readln (r,Y,sdY,X1,X2,X3,sdX3);
```

```
    Xu:=0;
    for j:=1 to 10 do Xu:=Xu+random;
    Y:=sdY*((Xu-10/2)/sqrt(10/12))+Y;
    if Y<0 then Y:=abs(Y);
    Xu:=0;
    for j:=1 to 10 do Xu:=Xu+random;
    X1:=0.03*X1*((Xu-10/2)/sqrt(10/12))+X1;
    if X1<0 then X1:=abs(X1);
    Xu:=0;
    for j:=1 to 10 do Xu:=Xu+random;
    X2:=0.03*X2*((Xu-10/2)/sqrt(10/12))+X2;
    if X2<0 then X2:=abs(X2);
    Xu:=0;
    for j:=1 to 10 do Xu:=Xu+random;
    X3:=sdX3*((Xu-10/2)/sqrt(10/12))+X3;
    if X3<0 then X3:=abs(X3);
    g[i,1]:=Y; g[i,2]:=X1; g[i,3]:=X2; g[i,4]:=X3;
end;
z1:=0.18; z2:=0.56; z3:=0.24; z4:=0.4;
ez1:=0; ez2:=0; ez3:=0; ez4:=0;
E:=1; E1:=0;
max:=40;
iter:=1;
while (abs(E)>0.000001) and (iter<max) do
begin
   z1:=z1+ez1; z2:=z2+ez2; z3:=z3+ez3; z4:=z4+ez4;
   sum:=0;
   k:=0;
   clearmatrix;
   for q:=1 to n do
   begin
      Y:=g[q,1]; X1:=g[q,2]; X2:=g[q,3];
X3:=g[q,4];
      P:=exp(z2*ln(X1))*exp(z3*ln(X2))*exp(-z4*ln(X3));
      SI:=Y-z1*P;
      dz1:=-P;
      dz2:=-z1*P*ln(X1);
      dz3:=-z1*P*ln(X2);
      dz4:=z1*P*ln(X3);
      readb;                {read matrices}
      readc;
      k:=k+1;
      sum:=sum+sqr(SI);
   end;
   inverse (s,t,u,v,w);
   matrixmultiplication (b,c);
   newvalues (d);
   E:=E1-sum;
   E1:=sum;
   iter:=iter+1;
```

```
        end;
        for j:=1 to 4 do h[gen,j]:=0;
        if sum<10000 then
        begin
            h[gen,1]:=z1; h[gen,2]:=z2; h[gen,3]:=z3;
         h[gen,4]:=z4; h[gen,6]:=sum;
        end;
        {results;}
    end;
    coefficients;
    writeln (f); writeln (f);
    writeln (f,'End of computation');
    close (f);
end.
```

APPENDIX IV

MEAN VALUE APPROACH

This analytical method calculates the mean and the standard deviation of a dependent variable Y, which is a function of independent variables $X_1 ... X_n$; $Y=f(X_1 .... X_n)$.
The assumption made in this approach is that if the mean values of $X_1 ... X_n$ are substituted in the equation for Y then the mean value for Y is obtained. This is true for linear relationships and for non-linear relationships this method is an approach.
An advantage of this method is that the relative influence of errors on the final error can easily be determined.

If the function of Y is linear in $X_1 .... X_n$, for example
$Y = aX_1 + bX_2$ then :

$$\mu(Y) = a\mu(X_1) + b\mu(X_2) \text{ and}$$
$$\sigma^2(Y) = a^2\sigma^2(X_1) + b^2\sigma^2(X_2) + 2ab*\varrho(X_1X_2)*\sigma(X_1)\sigma(X_2).$$

In which $\varrho(X_1X_2)$ is the correlation coefficient.
With $\sigma(X_1X_2) = \varrho(X_1X_2)*\sigma(X_1)\sigma(X_2)$ the equation for $\sigma^2(Y)$ becomes:

$$\sigma^2(Y) = a^2\sigma^2(X_1) + b^2\sigma^2(X_2) + 2ab*\sigma(X_1X_2).$$

If the function of Y is not linear in $X_1 .... X_n$, for example
$Y = X_1*X_2$ then an approximation based on Taylor series is used:

$$Y = f(\overline{X_1}, \overline{X_2}) + (X_1 - \overline{X_1}) \frac{\delta f}{\delta X_1} (\overline{X_1}, \overline{X_2}) + (X_2 - \overline{X_2}) \frac{\delta f}{\delta X_2} (\overline{X_1}, \overline{X_2})$$

In which $(\overline{X_1}, \overline{X_2})$ are the mean values of the independent variables.

For Y it counts:

$$\mu(Y) = f(\overline{X_1}, \overline{X_2})$$

$$\sigma^2(Y) = \left[\frac{\delta f}{\delta X_1} (\overline{X_1}, \overline{X_2}) * \sigma(X_1) \right]^2 + \left[\frac{\delta f}{\delta X_2} (\overline{X_1}, \overline{X_2}) * \sigma(X_2) \right]^2 + 2 * \frac{\delta f}{\delta X_1} (\overline{X_1}, \overline{X_2}) * \frac{\delta f}{\delta X_2} (\overline{X_1}, \overline{X_2}) * \sigma(X_1X_2)$$

The coefficient of variation is defined as:

$$V^2(Y) = \frac{\sigma^2(Y)}{\mu^2(Y)}$$

If X_1 and X_2 are not correlated then $\sigma(X_1,X_2) = 0$ and for the coefficient of variation is found:

$$V^2(Y) = V^2(X_1) + V^2(X_2)$$

This equation is known as the squared propagation of errors.

APPENDIX V

CHOLESKI-DECOMPOSITION

The Choleski decomposition is a variant of the LU decomposition and applicable to positive definite matrices.

The correlation matrix R is positive definite when:

(1) the matrix R is symmetric

(2) for every $x \in R^n \setminus \{0\}$ counts : $(x,Rx)>0$.

$(x,Rx) = \Sigma x_i y_i$ for i=1 to n.

First the LU decopmposition is applied to the correlation matrix R.

By means of Gauss-elimination the correlation matrix R is transformed into a triangle matrix U:

$$U = \begin{pmatrix} r_{11} & r_{12} & r_{13} & .. & .. & .. & r_{1n} \\ 0 & r_{22}^{(1)} & r_{23}^{(1)} & .. & .. & .. & r_{2n}^{(1)} \\ 0 & 0 & r_{33}^{(2)} & r_{34}^{(2)} & .. & .. & r_{3n}^{(2)} \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & .. & .. & .. & .. & r_{nn}^{(n-1)} \end{pmatrix}$$

At the same time multipliers $m_{j,i}$ are calculated which are stored in a triangle matrix L:

$$L = \begin{pmatrix} 1 & 0 & 0 & .. & .. & .. & .. \\ m_{21} & 1 & 0 & 0 & .. & .. & .. \\ m_{31} & m_{32} & 1 & 0 & 0 & .. & .. \\ .. & .. & .. & .. & .. & .. & 0 \\ m_{n1} & m_{n2} & .. & .. & .. & m_{n,n-1} & 1 \end{pmatrix}$$

In which

$$m_{j1} = \frac{a_{j1}}{a_{11}} , \quad j=2,3,...n$$

$$a_{jk}^{(1)} = a_{jk} - m_{j1} a_{1k} , \quad k=1,..n$$

And

$$m_{j2} = \frac{a_{j2}^{(1)}}{a_{22}^{(1)}} , \quad j=3,4,...n$$

$$a_{jk}^{(2)} = a_{jk}^{(1)} - m_{j2} a_{2k}^{(1)} , \quad k=2,...n$$

The matrix U can be written as U=D*U' with D=diag (a_{11},$a_{22}^{(1)}$, ...,$a_{nn}^{(n-1)}$) and U':

$$U' = \begin{pmatrix} 1 & \frac{a_{12}}{a_{11}} & .. & .. & .. & .. & \frac{a_{1n}}{a_{11}} \\ 0 & 1 & \frac{a_{23}^{(1)}}{a_{22}^{(1)}} & .. & .. & .. & \frac{a_{2n}^{(1)}}{a_{22}^{(1)}} \\ .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & .. & .. & .. & .. & 1 \end{pmatrix}$$

Thus, R = L*D*U' and because R is symmetric also $R = R^T = (L*D*U')^T = U'^T*D*L^T$.
Because R is also positive definite with positive values in D it is allowed to write T^2=D. besides it is demonstrable that $U'=L^T$ and therefore $R = L*D*L^T$.
Substituting T^2 gives $R = L*T^2*L^T = (LT)*(LT)^T$.
With G = L*T:
$R = G*G^T$.
This procedure is called the Choleski decomposition.

Because the calculation of the G matrix is quite complicated for large R matrices a program is written.
The output of the program is also printed.

```
program CHOLESKI (input,output);
type matrix = array [1..10,1..10] of real;
var i,j,n,k:integer;
   h,L,G,T: matrix;
   r,f: text;

begin
    assign (r,'b:inLU1qp.pas');
    reset (r);
    assign (f,'b:outLU1qp.pas');
    rewrite (f);
    writeln (f,'The correlation matrix of QP coefficients');
    writeln (f);
    writeln (f,'  A     alfa     beta    delta ');
    writeln (f);
    readln (r,n);
    for i:=1 to n do
    begin
        for j:=1 to n do
        begin
            T[i,j]:=0; h[i,j]:=0; G[i,j]:=0; L[i,j]:=0;
        end;
    end;
    for i:=1 to n do
    begin
        for j:=1 to n do
```

```
        begin
            read (r,h[i,j]);
            write (f,h[i,j]:8:4);
        end;
        readln (r);
        writeln (f);
    end;
    writeln (f);
    writeln (f,'Matrix G of Choleski');
    writeln (f);
    for j:=1 to n-1 do
    begin
        i:=j+1;
        while (i<=n) do
        begin
            L[i,j]:=h[i,j]/h[j,j];
            for k:=1 to n do h[i,k]:=h[i,k]-L[i,j]*h[j,k];
            i:=i+1;
        end;
    end;
    for i:=1 to n do
    begin
        T[i,i]:=sqrt(h[i,i]);
        L[i,i]:=1;
        end;
    for i:=1 to n do
    begin
        for j:=1 to n do
        begin
            G[i,j]:=0;
            for k:=1 to n do
            begin
                G[i,j]:=G[i,j]+L[i,k]*T[k,j];
            end;
            write (f,G[i,j]:8:4);
            end;
        writeln (f);
    end;
    writeln (f,'End');
    close (f);
end.
```

output for QP:

The correlation matrix of QP coefficients

A	alfa	beta	delta
1.0000	-0.6536	0.2077	-0.5162
-0.6536	1.0000	-0.8421	0.3448
0.2077	-0.8421	1.0000	0.0545
-0.5162	0.3448	0.0545	1.0000

Matrix G of Choleski

1.0000	0.0000	0.0000	0.0000
-0.6536	0.7568	0.0000	0.0000
0.2077	-0.9333	0.2930	0.0000
-0.5162	0.0098	0.5831	0.6272

End

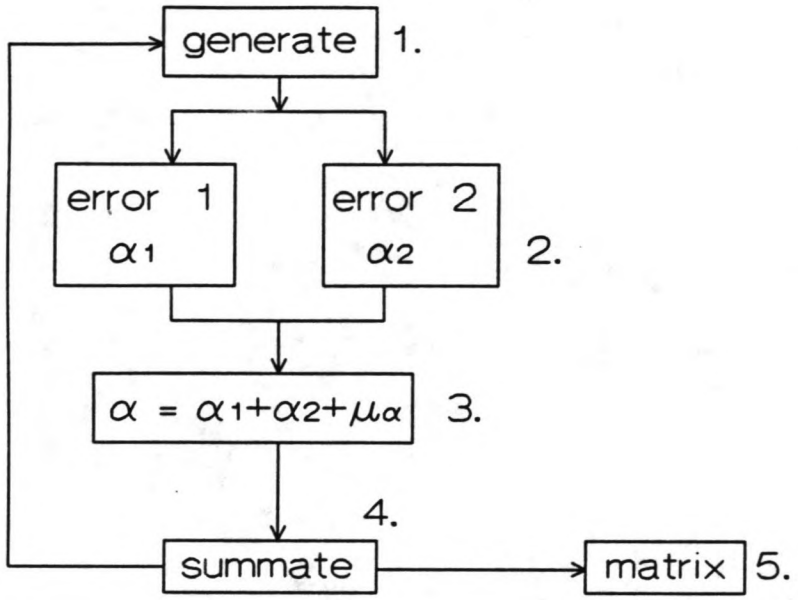
APPENDIX VI

TOTAL CORRELATION

The program TOTALCOR calculates the total correlation matrices for different number of simulations. The input for the program are the G (Choleski) matrices of error 1 and 2.

TOTALCOR pursues the scheme below.

The number of simulations is varied between 50 and 2000. The results on the correlations are shown in the figures below. These figures show that the correlations with low numbers of simulations (till 500) vary stronger than others.



Step 1 : Generation of random numbers from random generator

Step 2 : Multiplication with G matrix of error 1 and 2, which gives values for α_1 and α_2 for α of QP

Step 3 : Simulated values for the coefficients as $\alpha=\alpha_1+\alpha_2+\mu_\alpha$

Step 4 : Summation of the values ($\Sigma(\alpha)^2$, $\Sigma(\alpha\beta)$,$\Sigma(\alpha\delta)$ etc).

Step 5 : Calculation of the correlation matrix if the number of simulations is reached.

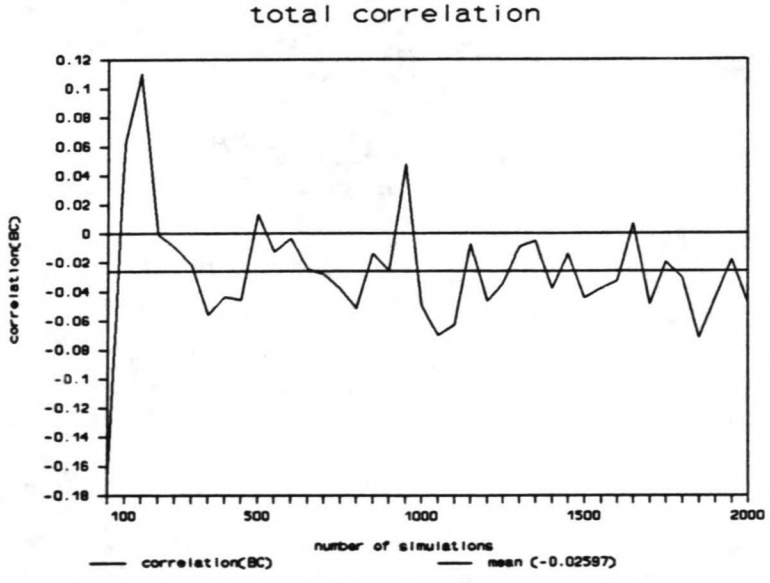
```
program TOTALCOR (input,output);
type matrix = array [1..200,1..10] of real;
type vector = array [1..10] of real;
var A,al,be,de,A1,al1,be1,de1,A2,al2,be2,de2,dA1,dA2,dal1,dal2,dbe1:real;
   dbe2,dde1,dde2,Am,alm,bem,dem,den:real;
   Xu: real;
   i,j,l,n,sim,max:integer;
   sum,h,v: matrix;
   g,y: vector;
   r,f: text;

begin
   assign (f,'b:outtotq.pas');
```

```
rewrite (f);
write (f,'The total correlation matrix of QP ');
writeln (f,'coefficients and variances');
writeln (f,'after simulation');
writeln (f);
writeln (f,'number of simulations');
writeln (f);
writeln (f,'    A     alfa     beta     delta');
writeln (f);
{mean values}
Am:=0.1836; alm:=0.5886; bem:=0.2381; dem:=0.4008;
{standard error 1}
dA1:=0.0842; dal1:=0.1353; dbe1:=0.1025; dde1:=0.1056;
{standard error 2}
dA2:=0.1717; dal2:=0.2104; dbe2:=0.1719; dde2:=0.2056;
max:=50;  n:=1;
while (max<2050) do
begin
    for i:=1 to 4 do
    begin
        g[i]:=0;
        for j:=1 to 4 do sum[i,j]:=0;
    end;
    for sim:=1 to max do
    begin
        for l:=1 to 8 do
        begin
            Xu:=0;
            for j:=1 to 10 do Xu:=Xu+random;
            y[l]:=((Xu-10/2)/sqrt(10/12));
        end;
        A1:=dA1*y[1];
        al1:=dal1*(-0.7076*y[1]+0.7066*y[2]);
        be1:=dbe1*(0.1825*y[1]-0.9684*y[2]+0.17*y[3]);
        de1:=dde1*(-0.5793*y[1]+0.129*y[2]+0.7718*y[3]+0.2285*y[4]);
        A2:=dA2*y[5];
        al2:=dal2*(-0.6463*y[5]+0.7631*y[6]);
        be2:=dbe2*(0.2177*y[5]-0.9333*y[6]+0.2855*y[7]);
        de2:=dde2*(-0.5061*y[5]-0.0448*y[6]+0.6221*y[7]+0.5957*y[8]);
        A:=A1+A2+Am;
        al:=al1+al2+alm;
        be:=be1+be2+bem;
        de:=de1+de2+dem;
        sum[1,1]:=sum[1,1]+sqr(A);
        sum[2,2]:=sum[2,2]+sqr(al);
        sum[3,3]:=sum[3,3]+sqr(be);
        sum[4,4]:=sum[4,4]+sqr(de);
        sum[1,2]:=sum[1,2]+A*al; sum[2,1]:=sum[1,2];
        sum[1,3]:=sum[1,3]+A*be; sum[3,1]:=sum[1,3];
        sum[1,4]:=sum[1,4]+A*de; sum[4,1]:=sum[1,4];
```

```
            sum[2,3]:=sum[2,3]+al*be; sum[3,2]:=sum[2,3];
            sum[2,4]:=sum[2,4]+al*de; sum[4,2]:=sum[2,4];
            sum[3,4]:=sum[3,4]+be*de; sum[4,3]:=sum[3,4];
            g[1]:=g[1]+A;
            g[2]:=g[2]+al;
            g[3]:=g[3]+be;
            g[4]:=g[4]+de;
        end;
        writeln (f,max);
        for i:=1 to 4 do
        begin
        {variances}
            v[n,i]:=(sum[i,i]-(sqr(g[i]))/max)/(max-1);     {variance matrix}
            for j:=1 to 4 do
            begin
                den:=sqrt((sum[i,i]-(sqr(g[i]))/max)*(sum[j,j]-(sqr(g[j]))/max));
                h[i,j]:=(sum[i,j]-(g[i]*g[j])/max)/den;
                write (f,h[i,j]:10:6);
            end;
            writeln (f);
        end;
        writeln (f);
        max:=max+50; n:=n+1;
    end;
    writeln (f);
    writeln (f,'End');
    close (f);
end.
```

Result of calculation:



APPENDIX VII

TAYLOR SERIES

The variance of a dependent variable $Y = f(X_1,X_2,X_3..X_n)$ can be expressed as a function of the variances of the independent variables $X_1,X_2,X_3..X_n$ and the derivatives.
Use can be made of the Taylor series. For linear relationships Taylor series give exact solution for the variances of Y. For non-linear relationships Taylor series approximate the exact solution for the variance of Y, so called linearization.
In general, Taylor series of the first order give satisfying results. In case of large variances of the independent variables and multiple differential relationships higher order Taylor have to be included in the linearization.
Assume the equation:

$$Y = A*X_1^{\alpha}*X_2^{\beta}$$

Taylor series to approximate Y_i gives:

$$Y_i = f(\overline{X_1},\overline{X_2},\overline{\alpha},\overline{\beta}) + (X_{1i}-\overline{X_1})\left(\frac{\delta f}{\delta X_1}\right) + (X_{2i}-\overline{X_2})\left(\frac{\delta f}{\delta X_2}\right) + (\alpha_i-\overline{\alpha})\left(\frac{\delta f}{\delta \alpha}\right) + (\beta_i-\overline{\beta})\left(\frac{\delta f}{\delta \beta}\right) +$$

$$\frac{1}{2}\left[(X_{1i}-\overline{X_1})\left(\frac{\delta}{\delta X_1}\right) + (X_{2i}-\overline{X_2})\left(\frac{\delta}{\delta X_2}\right) + (\alpha_i-\overline{\alpha})\left(\frac{\delta}{\delta \alpha}\right) + (\beta_i-\overline{\beta})\left(\frac{\delta}{\delta \beta}\right)\right]^2 + \qquad \text{(VII-1)}$$

$$R_3(\overline{X_1},\overline{X_2},\overline{\alpha},\overline{\beta},X_{1i},X_{2i},\alpha_i,\beta_i)$$

In which the derivation in the mean values of the variables.
Mean Value Approach assumes:

$$\overline{Y} = f(\overline{X_1},\overline{X_2},\overline{\alpha},\overline{\beta}) \qquad \text{(VII-2)}$$

Substituting (VII-2) in (VII-1) and summing the squared equation gives:

$$\sum_{i=1}^{n}(Y_i-\overline{Y})^2 = \sum_{i=1}^{n}\Bigg((X_{1i}-\overline{X_1})\left(\frac{\delta f}{\delta X_1}\right) + (X_{2i}-\overline{X_2})\left(\frac{\delta f}{\delta X_2}\right) + (\alpha_i-\overline{\alpha})\left(\frac{\delta f}{\delta \alpha}\right) + (\beta_i-\overline{\beta})\left(\frac{\delta f}{\delta \beta}\right) +$$

$$\frac{1}{2}\Bigg[(X_{1i}-\overline{X_1})^2\left(\frac{\delta^2 f}{\delta X_1^2}\right) + (X_{2i}-\overline{X_2})^2\left(\frac{\delta^2 f}{\delta X_2^2}\right) + (\alpha_i-\overline{\alpha})^2\left(\frac{\delta^2 f}{\delta \alpha^2}\right) + (\beta_i-\overline{\beta})^2\left(\frac{\delta^2 f}{\delta \beta^2}\right) +$$

$$2(X_{1i}-\overline{X_1})(X_{2i}-\overline{X_2})\left(\frac{\delta^2 f}{\delta X_1 \delta X_2}\right) + 2(X_{1i}-\overline{X_1})(\alpha_i-\overline{\alpha})\left(\frac{\delta^2 f}{\delta X_1 \delta \alpha}\right) + 2(X_{1i}-\overline{X_1})(\beta_i-\overline{\beta})\left(\frac{\delta^2 f}{\delta X_1 \delta \beta}\right) +$$

$$2(X_{2i}-\overline{X_2})(\alpha_i-\overline{\alpha})\left(\frac{\delta^2 f}{\delta X_2 \delta \alpha}\right) + 2(X_{2i}-\overline{X_2})(\beta_i-\overline{\beta})\left(\frac{\delta^2 f}{\delta X_2 \delta \beta}\right) + 2(\alpha_i-\overline{\alpha})(\beta_i-\overline{\beta})\left(\frac{\delta^2 f}{\delta \alpha \delta \beta}\right)\Bigg]\Bigg)^2$$

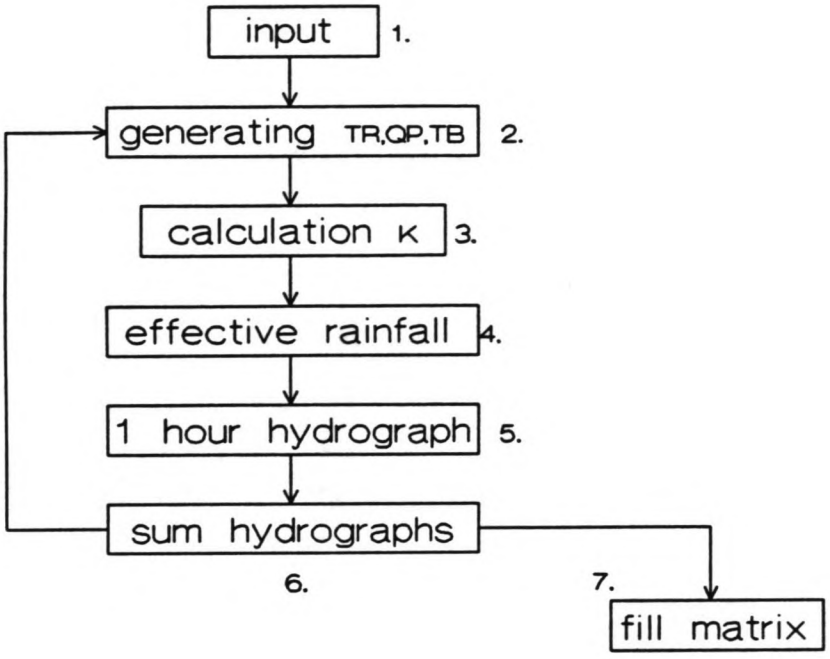
(VII-3)

Dividing both sides of equation (VII-3) by the number of values n gives the variance of Y as function of the (co)variances of X_1,X_2,α and ß and other (analytical undefined) combinations.

APPENDIX VIII

HYDROGRAPH

Application of the GAMA I unit hydrograph theory concerns the determination of the unit hydrograph characteristics TR,QP and TB. These characteristics are normally distributed with a mean value and a standard deviation. By means of simulation values from these distributions are generated. The program "HYDROGRAPH" is based on this principle (see next page).
For better understanding the following scheme is constructed:



-Step 1: The input for the program is read from an external file (see next page), and contains the rainfall depth P, the duration of the storm, the hourly distribution and the catchment characteristics of importance for QP and TB.

-Step 2: Values for TR,QP and TB are generated from their distributions (TR directly and QP,TB indirectly because they depend on TR).

-Step 3: Fitting the unit hydrograph to a an area of 1 mm rainfall depth evenly distributed over the area. This is done by varying the lag time K.
K_{max} is set to 60 to prevent instability, which can occur if the iteration procedure does not converge. Hydrographs with K=60 are deleted.

-Step 4: The effective rainfall per hour is determined after substraction of the losses.

-Step 5: The hydrograph for each hour is constructed by multiplying the effective rainfall depth with the unit hydrograph ordinates.

-Step 6: Summation of all hydrograps of one storm, which is 1 simulated total hydrograph.

-Step 7: Ordinates of each simulated hydrograph are stored in a matrix for later matrix operations.

input file:

```
105          {rainfall depth}
7            {duration of storm}
0.108 0.533 0.13 0.092 0.056 0.053 0.028 {distribution}
5            {number of stations}
No AR   JN   TR     S   RUA    SN    D   VTR   K
5 32.76 1  5.1671 0.04684 0.4237 0.6667 0.74 0.12 16.3765
```

```
program HYDROGRAPH(input,output);
const l:integer = 75;
type matrix1 = array [1..80,1..40] of real;
type matrix2 = array [1..80,1..10] of real;
type vector = array [1..20] of real;
var  QP,QB,Qt,TB,TR,T,X1,X2,X4,X5,X6,X7,VC,VTR,K,Xu,area1,area2,area3,E :real;
    z1,z2,z3,z4,z5,z6,z7,z8,z9,sdz1,sdz2,sdz3,sdz4,sdz5,sdz6,sdz7:real;
    sdz8,sdz9,P,phi:real;
    Ctch,sim,time,i,j,n,duration:integer;
    r,f:text;
    G,H,UH:matrix1;
    test:matrix2;
    v,distr,PE:vector;

begin
    assign (r,'a:inputhg.pas');
    assign (f,'a:outputhg.pas');
    rewrite (f);
    for i:=1 to l do
    begin
        for j:=1 to 40 do
        begin
            UH[i,j]:=0; H[i,j]:=0;
        end;
    end;
    VC:=0.10;
    for sim:=1 to l do
    begin
        for i:=1 to duration do
        begin
            for j:=1 to 40 do G[i,j]:=0;
        end;
        reset (r);
        readln (r,P);
        readln (r,duration);
        for i:=1 to duration do read (r,distr[i]);
        readln (r);
        readln (r,n); readln (r);
        readln (r,ctch,X1,X2,TR,X4,X5,X6,X7,VTR,K);
        z1:=0.1836; sdz1:=0.0842; z3:=0.2381; sdz3:=0.1025;
```

```
z2:=0.5886; sdz2:=0.1353; z4:=0.4008; sdz4:=0.1056;
z5:=27.4132; sdz5:=8.9792; z8:=0.2574; sdz8:=0.1524;
z6:= 0.1457; sdz6:=0.0565; z9:=0.7344; sdz9:=0.9028;
z7:= 0.0986; sdz7:=0.0335;
for i:=1 to 16 do
begin
    v[i]:=0;
    Xu:=0;
    for j:=1 to 10 do Xu:=Xu+random;
    v[i]:=(Xu-10/2)/sqrt(10/12);
end;

z1:=sdz1*v[1]+z1;
z2:=sdz2*(-0.7076*v[1]+0.7066*v[2])+z2;
z3:=sdz3*( 0.1825*v[1]-0.9684*v[2]+0.1700*v[3])+z3;
z4:=sdz4*(-0.5793*v[1]+0.1290*v[2]+0.7718*v[3]+0.2285*v[4])+z4;
z5:=sdz5*v[5]+z5;
z6:=sdz6*( 0.0360*v[5]+0.9994*v[6])+z6;
z7:=sdz7*(-0.4282*v[5]-0.5986*v[6]+0.6770*v[7])+z7;
z8:=sdz8*( 0.1548*v[5]-0.3457*v[6]+0.1209*v[7]+0.9176*v[8])+z8;
z9:=sdz9*( 0.8690*v[5]+0.0660*v[6]+0.3159*v[7]-0.3639*v[8]+0.0908*v[9])+z9;

X1:=VC*X1*v[10]+X1;          X5:=VC*X5*v[14]+X5;
X2:=round(VC*X2*v[11]+X2);   X6:=VC*X6*v[15]+X6;
TR:=abs(VTR*TR*v[12]+TR);    X7:=VC*X7*v[16]+X7;
X4:=VC*X4*v[13]+X4;

QP:=abs(z1*exp(z2*ln(X1))*exp(z3*ln(X2))*exp(-z4*ln(TR)));
TB:=z5*exp(z6*ln(TR))*exp(-z7*ln(X4))*exp(z8*ln(X5))*exp(z9*ln(X6));
QB:=0.4751*exp(0.6444*ln(X1))*exp(0.9430*ln(X7));
phi:=10.4903-3.859e-6*sqr(X1)+1.6985e-13*exp(4*ln(X1/X6));
T:=TB-TR-1;
area1:=0.5*QP*3.6*TR/X1;
E:=1;
while (abs(E)>0.0001) do
begin
    area2:=(3.6*QP*K-3.6*QP*K*exp(-T/K))/(X1);
    area3:=(0.5*3.6*QP*exp(-T/K))/(X1);
    E:=1-area1-area2-area3;
    if E<0 then K:=0.9*K else K:=1.1*K;
    if abs(K)>60 then
    begin
        E:=0;
        K:=60;
    end;
end;
time:=1;      {time 0}
while (TR>(time-1)) do
begin
    Qt:=(time-1)*(QP/TR);        {in m3/s}
```

```
            UH[sim,time]:=Qt;
            time:=time+1;
        end;
        while ((TB-1)>(time-1)) do
        begin
            Qt:=QP*exp(-(time-1-TR)/K);
            UH[sim,time]:=Qt;
            time:=time+1;
        end;
        UH[sim,time]:=0;
        test[sim,1]:=K; test [sim,2]:=area1+area2+area3;
        for i:=1 to duration do
        begin
            PE[i]:=distr[i]*P-phi;
            if PE[i]<0 then PE[i]:=0;
        end;
        for i:=1 to duration do
        begin
            for j:=1 to time do
            begin
                G[i,j+i-1]:=PE[i]*UH[sim,j];
            end;
        end;
        for i:=1 to 40 do
        begin
          H[sim,i]:=0;
          for j:=1 to duration do H[sim,i]:=H[sim,i]+G[j,i];
          H[sim,i]:=H[sim,i]+QB;
        end;
    end;
    writeln (f);
    writeln (f,'Simulation of Hydrograph');
    writeln (f);
    for i:=1 to l do
    begin
       if test[i,1]<60 then
     begin
        for j:=1 to 40 do
              begin
                 write (f,H[i,j]:4:1,' ');
              end;
              writeln (f);
     end;
    end;
    writeln (f);
    for i:=1 to l do writeln (f,'K=',test[i,1]:8:4,'area=',test[i,2]:5:4);
    writeln (f);
    writeln (f,'End');
    close (f);
end.
```

Output file

```
Simulation of hydrograph
   0    1    2    3    4    5    6    7    8    9   10   11   12   13   14
 2.6  4.2  7.7   12 16.4 19.1 19.7 19.5   19 18.6 18.1 17.7 17.3 16.9 16.5
 3.4 10.4 25.2 43.6 62.5 81.3 73.4 45.8 22.6 11.2  6.5  4.7  3.9  3.6  3.5
 3.2  4.2  6.4    9 11.8 14.5 17.1 18.5 18.7 18.4   18 17.6 17.3 16.9 16.5
 3.6  4.9  7.8 11.4 15.1 18.9 22.2 23.7 23.4 22.4 21.3 20.3 19.4 18.5 17.6
 3.3  4.8  8.2 12.3 16.5 20.5 22.5 22.7   22 21.3 20.7   20 19.4 18.8 18.2
 2.9  4.6    8 12.3 16.7 21.1   25 26.7 26.2 24.9 23.6 22.4 21.3 20.2 19.2
 3.9  5.6  9.3 13.9 18.5 23.2 26.4 27.2 26.5 25.3 24.2 23.1 22.1 21.2 20.2
 3.4  5.2  9.1   14 18.9 23.9 26.8   27 25.8 24.5 23.2   22 20.8 19.8 18.8
 3.5  5.5  9.8 15.1 20.5 25.2 27.3 27.1   26 24.8 23.7 22.6 21.6 20.7 19.8
 4.1  5.5  8.6 12.4 16.4 20.3 23.1 24.1 23.8 23.2 22.5 21.8 21.2 20.6   20
   3  5.9 11.9 19.3 26.9 32.6 33.7 31.3   28 24.9 22.2 19.9 17.8   16 14.4
 3.3    5  8.6   13 17.5 22.1 26.6 29.1   29 27.7 26.4   25 23.8 22.6 21.5
 3.3  5.7 10.9 17.3 23.9 30.5   35 35.1 32.1 28.3 24.8 21.9 19.3 17.1 15.2
 3.5  5.4  9.4 14.4 19.5 24.6 28.3 29.4 28.5 27.1 25.7 24.3 23.1 21.9 20.8
 3.6  5.3  8.8 13.1 17.6 22.1 25.5 26.7 26.3 25.3 24.3 23.3 22.4 21.5 20.7
 3.4  5.9 11.1 17.6 24.2 30.9 35.8 36.8 34.8 31.9 29.1 26.6 24.4 22.3 20.5
 3.6  9.3 21.4 36.3   47 48.7 44.1 38.1 32.8 28.3 24.5 21.3 18.6 16.3 14.3
 3.9  7.2 14.3 23.1 32.1 40.6 43.1 39.4 33.8 28.8 24.6 21.1 18.3 15.8 13.8
 2.8  4.1  6.8 10.1 13.5 16.9 18.9 19.3   19 18.6 18.3 17.9 17.5 17.2 16.9
 3.6  5.5  9.7 14.9 20.2 25.6 30.9   34 34.1 32.4 30.4 28.5 26.8 25.1 23.6
 2.9  6.1   13 21.6 30.3 38.1 40.4 37.7 33.5 29.4   26 22.9 20.3   18   16
 3.5  6.9 14.1 23.1 32.2 37.6 37.4 34.1 30.4 27.1 24.1 21.6 19.3 17.4 15.7
 3.3  5.3  9.8 15.2 20.8 26.4 30.7 31.6   30 27.7 25.4 23.4 21.5 19.9 18.3
 3.9    6 10.5   16 21.7 27.4 31.6 32.8 31.9 30.3 28.7 27.2 25.8 24.5 23.2
 2.8  4.2  7.2   11 14.9 18.7 22.1 23.3 22.5 20.8 19.2 17.8 16.5 15.3 14.1
 3.1  7.8 17.9 30.3   43 50.7 49.4 42.6 35.3 29.1 24.1   20 16.8 14.1   12
 3.7  6.6 12.9 20.7 28.7 34.3 35.8 34.8 33.1 31.4 29.8 28.2 26.8 25.4 24.2
   3  5.6   11 17.7 24.6 28.8   29   27 24.6 22.4 20.4 18.6   17 15.6 14.3
 3.7  6.5 12.6 20.2 27.9 34.4 36.1 33.9 30.4 27.1 24.3 21.8 19.5 17.6 15.9
 3.4  4.8  7.8 11.4 15.2 18.9 21.1 21.3 20.8 20.1 19.4 18.8 18.2 17.6 17.1
 2.4    4  7.2 11.3 15.4 18.3 19.1 18.8 18.1 17.4 16.8 16.2 15.6   15 14.5
 3.1    7 15.4 25.7 36.4 41.5 39.7 34.6 29.5 25.1 21.5 18.5   16 13.8 12.1
 3.8  5.5  9.2 13.8 18.5 23.1 26.2 26.9 26.4 25.6 24.8   24 23.2 22.5 21.8
 3.4  4.9  8.2 12.3 16.5 20.7 23.6 24.4 23.8 22.9   22 21.1 20.3 19.5 18.7
 3.2  5.6 10.6 16.8 23.1 29.5 35.8 38.2 35.6 30.8 26.2 22.4 19.2 16.5 14.3
 3.2  5.8 11.3 18.1 25.1 31.9 34.4 32.3 28.6 25.1 22.1 19.5 17.3 15.4 13.7
 3.4  6.1 11.7 18.7 25.9 33.1 38.7   40 37.9 34.8 31.9 29.2 26.7 24.5 22.5
 3.3  5.3  9.6   15 20.5   26 30.1 30.7 28.8 26.2 23.7 21.5 19.5 17.8 16.2
 3.5  6.2 11.9 19.1 26.4 33.7 37.3 36.1 32.6   29 25.8   23 20.6 18.4 16.6
 2.9  5.2 10.2 16.3 22.6 28.9 35.1   37 33.9 28.9 24.4 20.6 17.5   15 12.9
 3.5  5.2  8.8 13.4   18 22.4 24.8   25 24.4 23.6 22.8 22.1 21.4 20.7   20
 3.5  5.8 10.8 16.9 23.2 27.9 29.2 28.1 26.3 24.5 22.8 21.3 19.9 18.6 17.4
 3.4  6.6 13.4 21.8 30.5 39.1 45.6 45.7 40.7 34.5 29.2 24.7 21.1   18 15.5
 3.7  6.2 11.5 18.1 24.8 31.6 35.4   35 32.2 28.9   26 23.4 21.2 19.1 17.3
 3.5  4.5  6.8  9.5 12.4 15.2   18 20.2 21.1 21.1 20.9 20.6 20.3   20 19.7
 3.4  5.5 10.1 15.7 21.5 26.1 27.7 27.1 25.7 24.3   23 21.8 20.7 19.6 18.6
 3.1  5.1  9.2 14.2 19.4 24.6 27.8   28 26.4 24.4 22.6 20.9 19.3 17.9 16.6
 2.8  4.3  7.6 11.7 15.9   20 24.2 26.7 26.7 25.2 23.4 21.7 20.2 18.7 17.4
 3.5  6.3 12.3 19.6 27.1 34.6 38.1 36.4 32.3 28.1 24.5 21.5 18.9 16.6 14.7
 3.2  5.8 11.2 17.9 24.8 31.7   38 38.5 33.4   27 21.7 17.6 14.4 11.9   10
 3.9  5.3  8.5 12.4 16.4   20 21.8   22 21.5   21 20.4 19.9 19.4 18.9 18.4
 3.8  5.8  9.8 14.9   20 25.2 29.5   31   30   28 26.1 24.3 22.7 21.2 19.8
 3.7  6.1 11.2 17.5   24 29.3 31.3 30.4 28.6 26.7   25 23.4 21.9 20.5 19.3
 2.8  4.7  8.9   14 19.3 24.5 27.3 27.1 25.5 23.8 22.2 20.7 19.3   18 16.9
 3.9  6.9 13.2   21   29   37 43.9 45.5 42.3 37.5 33.2 29.4 26.1 23.2 20.7
 2.9  5.9 12.4 20.4 28.5 36.7 42.6 42.5   38 32.6 27.8 23.8 20.4 17.6 15.2
 3.1  5.2  9.7 15.3   21 24.5 25.1 24.3 23.1   22   21   20 19.1 18.3 17.4
```

15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
16.2	15.8	15.4	15.1	14.8	14.4	14.1	13.8	13.5	13.2	9.2	4.8	2.9	2.6	2.6
3.5	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
16.2	15.9	15.5	15.2	14.9	14.6	14.3	14	13.7	13.5	13.2	9.4	5.3	3.5	3.2
16.8	16.1	15.4	14.7	14.1	13.5	12.9	12.4	11.9	11.4	10.9	10.5	10.1	9.8	9.4
17.6	17.1	16.5	16	15.6	15.1	14.6	14.2	13.8	13.4	13	12.6	9.1	5.2	3.5
18.2	17.3	16.5	15.7	14.9	14.2	13.5	12.9	12.3	11.8	11.2	10.8	10.3	7.4	4.4
19.4	18.6	17.8	17.1	16.4	15.7	15.1	14.5	13.9	13.4	12.9	12.4	12	11.6	8.6
17.8	16.9	16.1	15.3	14.6	13.9	13.3	12.7	12.1	11.6	8.4	5	3.6	3.4	3.4
18.9	18.1	17.3	16.6	15.9	15.3	14.7	10.4	5.8	3.8	3.5	3.5	3.5	3.5	3.5
19.5	18.9	18.4	17.9	17.4	17	16.5	16.1	15.7	15.3	14.9	14.5	14.1	13.8	13.4
13	11.8	10.7	9.8	8.9	8.2	7.6	7	6.5	6.1	5.7	5.4	4.4	3.5	3.1
20.5	19.5	18.6	17.7	16.9	16.1	15.4	14.7	14.1	13.4	9.5	5.3	3.6	3.3	3.3
13.6	12.2	11	9.9	9	8.2	7.6	5.8	4.1	3.4	3.3	3.3	3.3	3.3	3.3
19.8	18.8	17.9	17	16.2	15.4	14.7	14	13.4	12.8	12.2	11.7	11.2	10.7	10.3
19.9	19.1	18.4	17.7	17	16.4	15.8	15.2	14.7	14.2	10.1	5.8	3.9	3.6	3.6
18.8	17.3	16	14.8	13.7	12.7	11.8	11	10.2	9.6	9	8.4	8	7.5	5.9
12.7	11.3	10.1	9.1	8.3	7.6	7	6.4	5.3	4.1	3.7	3.6	3.6	3.6	3.6
12.2	10.8	9.6	8.7	7.9	7.2	6.6	6.2	5.8	5.5	5.2	5	4.8	4.6	4.5
16.5	16.2	15.9	15.6	15.3	15	14.7	14.5	14.2	13.9	9.8	5.1	3.1	2.8	2.8
22.2	20.8	19.6	18.5	12.6	6.5	3.9	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
14.3	12.8	11.5	10.4	9.4	8.5	7.8	7.1	6.6	6.1	5.7	5.3	5	4.7	4.5
14.1	12.8	11.7	10.7	9.8	9	8.3	7.7	6	4.3	3.6	3.5	3.5	3.5	3.5
16.9	15.7	14.5	13.5	12.5	11.7	8.3	4.9	3.5	3.3	3.3	3.3	3.3	3.3	3.3
22.1	21	20	19	18.1	17.2	16.4	15.7	15	10.7	6.1	4.2	3.9	3.9	3.9
13.1	12.2	11.4	10.6	9.9	9.3	8.7	8.2	7.7	7.3	6.9	6.5	6.2	5.9	5.6
10.3	8.9	7.8	6.9	6.2	5.6	5.1	4.7	4.4	4.2	4	3.8	3.7	3.6	3.5
23	21.9	20.8	19.8	18.9	18	17.1	11.9	6.4	4	3.7	3.7	3.7	3.7	3.7
13.2	12.1	11.2	10.4	9.6	8.9	8.3	7.8	7.3	6.9	6.5	6.1	5.8	4.7	3.6
14.4	13.1	11.9	8.5	5.2	3.9	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7
16.5	16	15.5	15	14.6	14.1	13.7	13.3	12.9	12.5	12.2	11.8	11.5	11.2	10.9
14	13.5	13	12.6	12.1	11.7	11.3	10.9	10.6	10.2	9.9	9.6	6.9	3.9	2.6
10.6	9.4	8.3	7.5	6.7	6.1	5.6	5.2	4.9	4.6	4.3	4.1	3.9	3.6	3.2
21.1	20.5	19.8	19.2	18.7	18.1	17.6	17	16.5	16.1	15.6	15.2	10.8	6.1	4.1
18	17.3	16.7	16.1	15.5	14.9	14.4	13.8	13.4	12.9	12.4	12	11.6	11.2	10.9
12.4	10.9	9.6	8.6	7.7	6.9	6.3	5.8	5.4	5	4.7	4.5	4.3	4.1	3.9
12.3	11	10	9	8.2	7.6	7	6.5	6	5.6	5.3	5	4.8	4.1	3.5
20.7	19.1	12.8	6.5	3.8	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
14.8	13.5	12.4	11.4	10.6	9.8	9.1	8.4	7.9	7.4	5.7	4.1	3.4	3.3	3.3
14.9	13.5	12.2	11.1	10.2	9.3	8.6	8	7.4	6.9	6.5	6.1	5	4	3.5
11.1	9.7	8.5	7.5	6.7	6.1	5.5	5.1	4.7	4.4	4.1	3.9	3.7	3.6	3.5
19.4	18.8	18.2	17.7	17.1	16.6	16.1	15.6	15.2	14.7	14.3	10.2	5.7	3.8	3.5
16.3	15.3	14.4	13.5	12.7	12	11.3	10.7	10.1	9.6	7.2	4.7	3.6	3.5	3.5
13.4	11.7	10.3	9.1	8.1	7.3	5.7	4.1	3.5	3.4	3.4	3.4	3.4	3.4	3.4
15.8	14.4	13.1	12	11.1	10.2	9.5	8.8	8.2	7.7	7.2	6.8	5.6	4.3	3.8
19.4	19.2	18.9	18.7	18.4	18.1	17.9	17.7	17.4	17.2	16.9	16.7	16.5	16.3	11.5
17.6	16.7	15.9	15.1	14.4	13.7	13	12.4	11.9	11.3	10.9	10.4	9.9	9.5	9.2
15.5	14.4	13.4	12.5	11.7	10.9	10.3	9.6	9.1	8.5	8.1	7.6	7.3	6.9	5.4
16.2	15.1	14.1	13.2	12.3	11.5	10.8	10.2	9.5	6.9	4.1	2.9	2.8	2.8	2.8
13.1	11.7	10.5	9.5	8.6	7.9	7.2	6.7	6.2	5.9	5.5	5.2	5	4.8	4.6
8.5	7.3	6.4	5.7	5.1	4.7	4.4	4.1	3.9	3.6	3.4	3.3	3.2	3.2	3.2
17.9	17.5	17	16.6	16.2	15.8	15.4	15.1	14.7	14.4	14	13.7	13.4	13.1	12.8
18.6	17.4	16.3	15.3	14.4	13.6	12.8	9.3	5.6	4.1	3.8	3.8	3.8	3.8	3.8
18.1	17	16	15.1	14.2	13.4	12.7	12	11.4	10.8	8	5.1	3.9	3.7	3.7
15.8	14.8	13.9	13	12.2	11.5	10.8	10.2	9.6	9.1	8.6	8.2	6	3.8	2.9
18.5	16.6	15	13.5	12.3	11.2	10.2	9.4	8.7	8.1	7.5	7.1	6.7	6.3	6
13.3	11.6	10.2	9	8	7.2	6.5	5.9	5.5	5	4.7	4.4	4.2	4	3.8
16.7	15.9	15.2	14.6	13.9	13.4	12.8	9	5	3.3	3.1	3.1	3.1	3.1	3.1

30	31	32	33	34	35	36	37	38	39	max Q m3/s
2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	19.7
3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	81.3
3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	18.7
9.1	6.9	4.7	3.7	3.6	3.6	3.6	3.6	3.6	3.6	23.7
3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	22.7
3.1	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	26.7
5.5	4.1	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	27.2
3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	27
3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	27.3
9.9	6	4.3	4.1	4.1	4.1	4.1	4.1	4.1	4.1	24.1
3	3	3	3	3	3	3	3	3	3	33.7
3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	29.1
3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	35.1
9.9	9.5	9.1	7	4.6	3.6	3.5	3.5	3.5	3.5	29.4
3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	26.7
4.2	3.5	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	36.8
3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	48.7
4.4	4.2	4	3.9	3.9	3.9	3.9	3.9	3.9	3.9	43.1
2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	19.3
3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	34.1
4.2	4.1	3.6	3.1	2.9	2.9	2.9	2.9	2.9	2.9	40.4
3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	37.6
3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	31.6
3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	32.8
5.4	4.3	3.3	2.8	2.8	2.8	2.8	2.8	2.8	2.8	23.3
3.4	3.3	3.2	3.1	3.1	3.1	3.1	3.1	3.1	3.1	50.7
3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	35.8
3.1	3	3	3	3	3	3	3	3	3	29
3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	36.1
10.6	7.9	4.9	3.6	3.4	3.4	3.4	3.4	3.4	3.4	21.3
2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	19.1
3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	41.5
3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	26.9
8	4.9	3.6	3.4	3.4	3.4	3.4	3.4	3.4	3.4	24.4
3.8	3.7	3.6	3.6	3.5	3.5	3.4	3.3	3.2	3.2	38.2
3.3	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	34.4
3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	40
3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	30.7
3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	37.3
3.4	3.3	3.1	3	2.9	2.9	2.9	2.9	2.9	2.9	37
3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	25
3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	29.2
3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	45.7
3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	35.4
6.2	3.8	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	21.1
8.8	6.7	4.5	3.5	3.4	3.4	3.4	3.4	3.4	3.4	27.7
3.9	3.2	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	28
2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	26.7
4.4	4.1	3.7	3.6	3.5	3.5	3.5	3.5	3.5	3.5	38.1
3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	38.5
12.5	12.2	12	11.7	8.7	5.5	4.1	3.9	3.9	3.9	22
3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	31
3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	3.7	31.3
2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	27.3
5.7	5	4.3	4	3.9	3.9	3.9	3.9	3.9	3.9	45.5
3.7	3.3	3	2.9	2.9	2.9	2.9	2.9	2.9	2.9	42.6
3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	25.1

APPENDIX IX

CATCHMENT CHARACTERISTICS

WF : **Width factor**, which is defined as the ratio of the width of the catchment measured at 3/4 and at 1/4 of the stream length (L) measured from the gauging site (see figure IX-1).

$$WF = \frac{WU}{WL}$$

In which

WF: Width factor [-]
WU: Width of the catchment at the point 3/4 L from the gauging site [km]
WL: Width of the catchment at the point 1/4 L from the gauging site [km].

SIM : **Symmetry factor**, presented as

$$SIM = \frac{RUA*WU}{WL} = RUA*WF$$

In which

SIM: Symmetry factor [-]
RUA: Relative upstream area of catchment [-].

SF : **Source factor**, defined as

$$SF = \frac{\Sigma L_1}{\Sigma L_u}$$

In which

SF: Source factor [-]
ΣL_1: Total length of the first order streams [km]
ΣL_u: Total length of streams of all order [km].

SN : **Source frequency**, defined as

$$SN = \frac{\Sigma N_1}{\Sigma N_u}$$

In which

SN: Source frequency [-]
ΣN_1: Total number of segments of the first order streams [-]
ΣN_u: Total number of stream segments [-].

JN : **Total number of stream junctions**, defined as the number of segments of the first order streams minus one (ΣN_1-1).

S : **Average main stream slope [-].**

L : **Main stream length [km].**

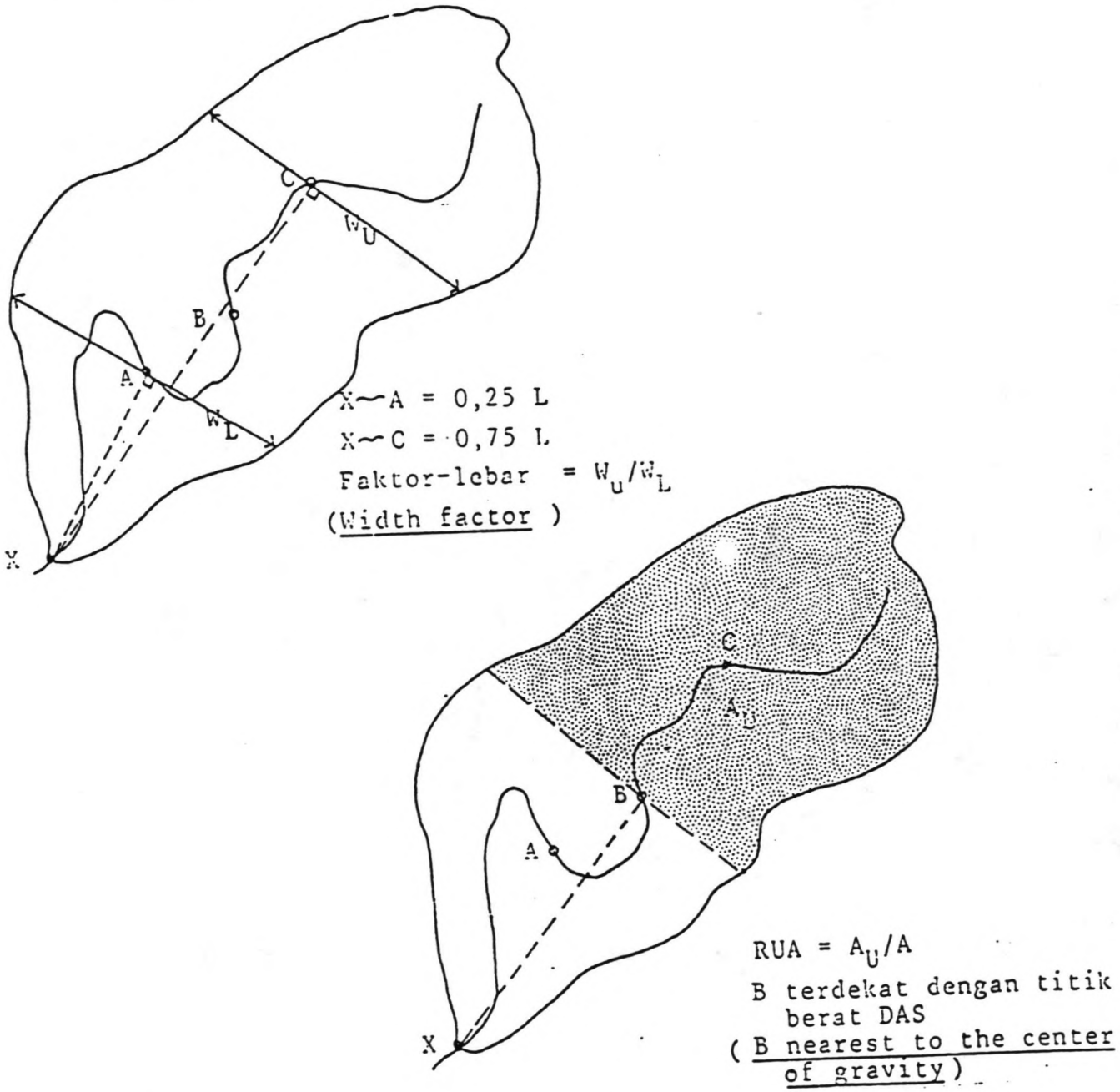


Fig. IX-1 Diagram of determining the width factor and the relative upstream area