Viscoelastic Stress Modeling in Cementitious Materials Using Constant Viscoelastic Hydration Modulus

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Abstract: Viscoelastic stress modeling in ageing cementitious materials is of major importance in high performance concrete of low water cement ratio (e.g. w/c ~0.35) where crack resistance due to deformation restraint needs to be determined. Total stress analysis is complicated by the occurrence of internal stresses due to shrinkage, which requires estimating the stress relaxation effect from tensile creep. This study presents a new and direct methodology for viscoelastic stress analysis based on measurement of the viscoelastic hydration modulus. Autogenous shrinkage, if restrained, creates an internal tensile stress condition which is uniform within a cross section. Autogenous shrinkage stresses develop within the porous hydration products. They are compressive stresses and if restrained by reinforcement a net tensile stress develops. Results show that the viscoelastic hydration modulus is approximately 8000-9000 MPa and is a constant material property. Total stress analysis can now be separated into two components, an elastic stress based on the Young's modulus (typically in the range of 28000-34000 MPa) and a viscoelastic (time-dependent) stress based on measurement of time-dependent strains (creep and shrinkage). The importance of reducing paste content for shrinkage stress control is demonstrated using the Pickett's model.

Keywords: Autogenous shrinkage, High Performance Concrete (HPC), Shrinkage stresses, Modeling viscoelastic effects

1 Introduction

Self-induced tensile stresses develop in concrete if the movements caused by cement hydration reactions are restrained [1]. During early age hydration two active mechanisms are involved in producing these movements, starting with thermal effects which dominate during the first 24-48 hours. Self-desiccation is another consequence of cement hydration as this process consumes water into solid hydration products [2]. As hydration proceeds internal pore drying develops with associated internal stress development from capillary tension in the pores [3]. These stresses transfer to the hydration products as compression and subsequent shrinkage. This type of shrinkage is known as autogenous shrinkage. It is characterized by a uniform volume reduction and at any time is a material property (that is, no moisture gradient), whereas drying shrinkage development is size-dependent and non-uniform. Thermal stresses are relatively short-term acting throughout the concrete composite, the time-dependent shrinkage stresses are internal acting primarily on the porous hydration products.

Autogenous shrinkage is intensified in high performance concrete of low water-cement (w/c) ratio (relative to conventional concrete) due to its generally higher cement content, reduced w/cm, and pozzolanic mineral admixtures. Prior results indicate that cementitious systems containing slag cement produces greater autogenous shrinkage at later ages [4-6]. The early age cracking problem in high performance concrete has become important due to the increased use of these materials [7-10]. The reasons were generally attributed to the higher chemical shrinkage, the finer pore structure, removal of calcium hydroxide as a shrinkage restraint, and a reduction in pore humidity associated with pozzolanic reactions.

The fundamental basis for applying theory of viscoelasticty, developed for polymers, to concrete is the assumed analogy between creep compliance function and stress relaxation modulus [11-13]. The outcome in viscoelastic stress calculations is a reduction in concrete modulus by 50%-75% due to stress relaxation under full external deformation restraint [14-16].

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Recent numerical simulations, using the lattice model for predicting stress development from a mini temperature stress testing machine (TSTM) on small paste specimens, where thermal effects from heat of hydration are minimal, concluded that a poor agreement was obtained between predicted stresses by using three different models for obtaining stress relaxation modulus [17]. The first model was based on an assumption of instant relaxation that internal stresses will cause instant deformation of the micro-porous hydration products. In this model there is no relaxation over time. The second model assumes relaxation based on hydration and it uses an exponential relaxation factor approach [18]. In the last method, stress relaxation modulus is a reduced Young's modulus, since the relaxation is the amount of stiffness lost over time. It was concluded that a better relaxation model is needed.

An intriguing and novel test procedure was used by Bjontegaard [19] who measured the increasing autogenous deformation induced tensile stress development in a fully restrained test (TSTM) combined with parallel measurements of free (i.e. without external restraint) autogenous shrinkage. Although the focus of his study was the early age period (0-7 days) typical results show that once thermal contraction has ceased (typically within 24-48 hours), the two curves of stress and autogenous shrinkage development are parallel with a constant net viscoelastic modulus of about 11000 MPa.

2 Experimental Program

Different mortar and concrete mixes of a 0.35 w/c ratio were prepared in the laboratory according to ASTM C192 and the mix design is listed in Table 1. Raw materials included Type I Portland cement, silica sand and limestone gravel. Cylindrical samples were cast and cured for one day before demoulding. Then they were sealed cured for different ages before the following test procedures were carried out.

(1) Compressive strength and split tensile strength were tested on 100 mm × 200 mm cylinders according to ASTM C39 and C496, respectively. Three specimens were used for each age and both the average and individual results were reported.

(2) Static modulus of elasticity was obtained from the stress-strain curve of 300 mm × 600 mm cylinders from the simultaneous measurement of uniaxial compressive load by a static hydraulic system, and linear deformation by a motion capture system (Figure 1).

(3) Autogenous shrinkage was measured on duplicate mortar or paste specimens of 60 mm × 100 mm × 1000 mm where double polystyrene films were used to seal the specimen and an isothermal condition at 20±1 °C was achieved by circulating water through channels embedded into the sides and bottom of the rigs. In addition to the free shrinkage, the restrained shrinkage was measured by the embedment of four symmetrically located rebars in the specimen.

(4) Sealed creep of the concrete mix in compression was measured according to ASTM C512 where the specimens were sealed to achieve a uniform moisture condition during the test.

Table 1 Mix design (kg/m^3)

	cement	gravel	sand	water
Paste	1497	0	0	524
20%agg.	1198	0	528	419
40%agg.	899	0	1055	314
60%agg.	594	936	646	208
70%agg.	450	1093	753	157

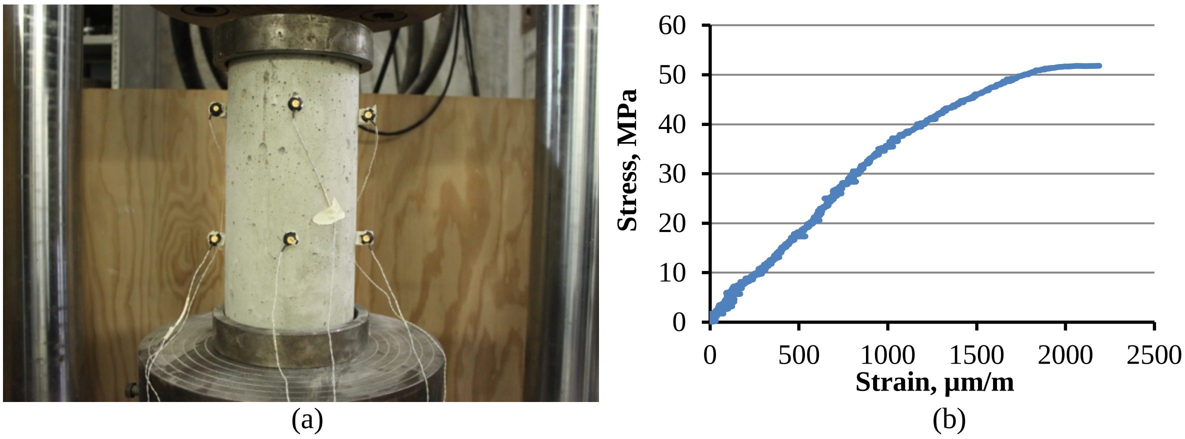


Figure 1 (a) Static modulus measurement by a combination of a hydraulic test system and a motion capture system, (b) measured 7day stress-strain curve of concrete

3 Results and discussions

3.1 Basis for proposed viscoelastic stress-strain analysis of hydration products

Measurements and modeling of autogenous shrinkage of low w/c systems (0.35) for different internal restraint conditions (aggregate particle and steel reinforcement) forms the basis for the proposed activity. Autogenous shrinkage is a form of drying shrinkage, but without shrinkage gradients (Figure 2). A uniform internal stress develops due to the hydration process that proceeds without exchange of water (i.e. sealed curing). Internal stresses increase with increasing hydration resulting in more specimen shrinkage.

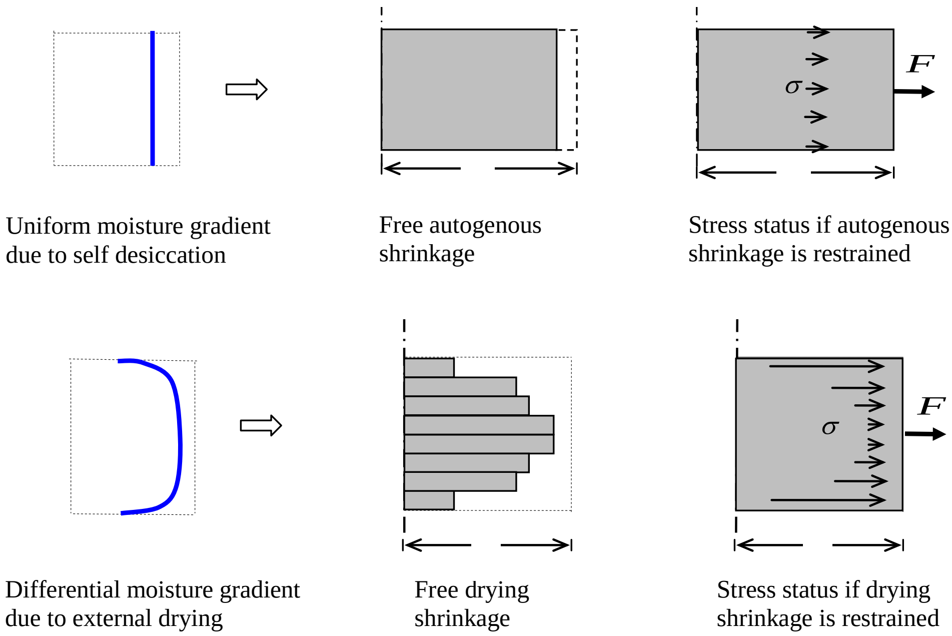


Figure 2 Schematic of stress distribution from autogenous shrinkage and drying shrinkage [20]

The uniform stress condition allows for a straight forward tensile stress analysis when symmetrically placed reinforcement bars are used (Figure 3). Autogenous shrinkage results are shown in Figure 4 versus time. Force equilibrium analysis yields a linear correlation between free shrinkage and rebar restrained shrinkage (Eqs.1-2), from which a constant viscoelastic hydration modulus is obtained based on minimizing error between predicted line (dashed) and measurements (Eq.3 and Figure 5).

$$\varepsilon_s(t)E_sA_s = [\varepsilon_c(t) - \varepsilon_s(t)]E_vA_c \tag{1}$$

Thus,

$$\varepsilon_c(t) = 1 + \frac{\mathrm{A_sE_s}}{\mathrm{A_pE_v}}\varepsilon_s(t) \cong (1 + n_v\rho_s)\varepsilon_s(t) \tag{2}$$

or

$$E_v = E_s\rho_s/(\frac{\varepsilon_c(t)}{\varepsilon_s(t)} - 1) \tag{3}$$

Where

$\varepsilon_c(t)$= free shrinkage of plain mix, $\varepsilon_s(t)$= deformation of steel in reinforced mix,
A_c= area of plain mix, A_s= area of steel, $\rho_s = A_s/A_c$= steel ratio, E_s= steel modulus, E_v= viscoelastic hydration modulus, n_v= E_s/E_v.

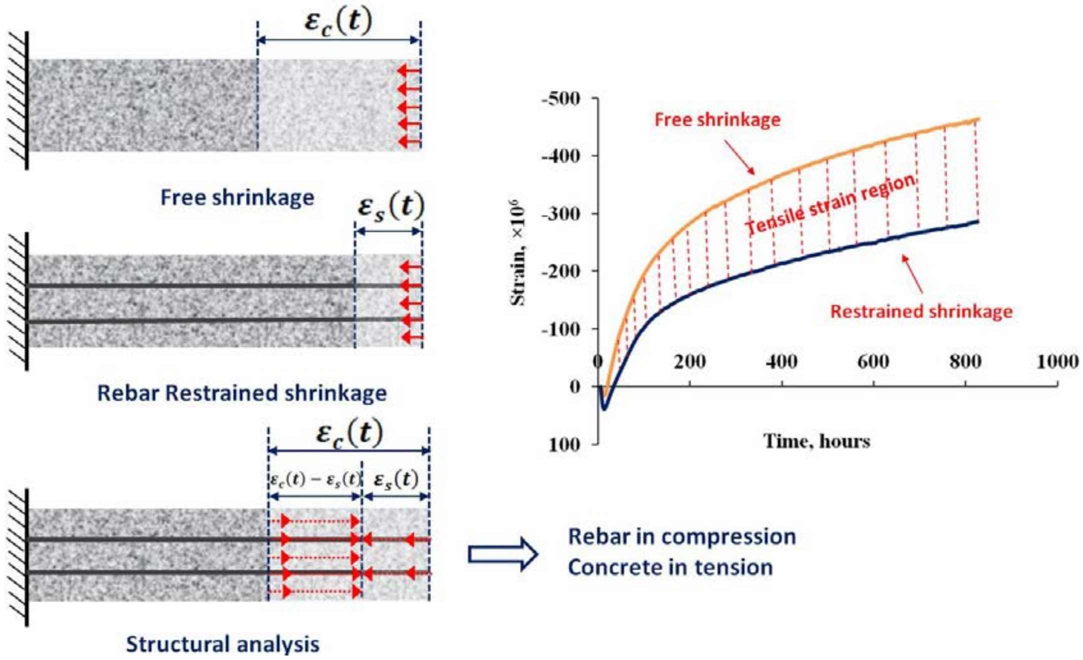
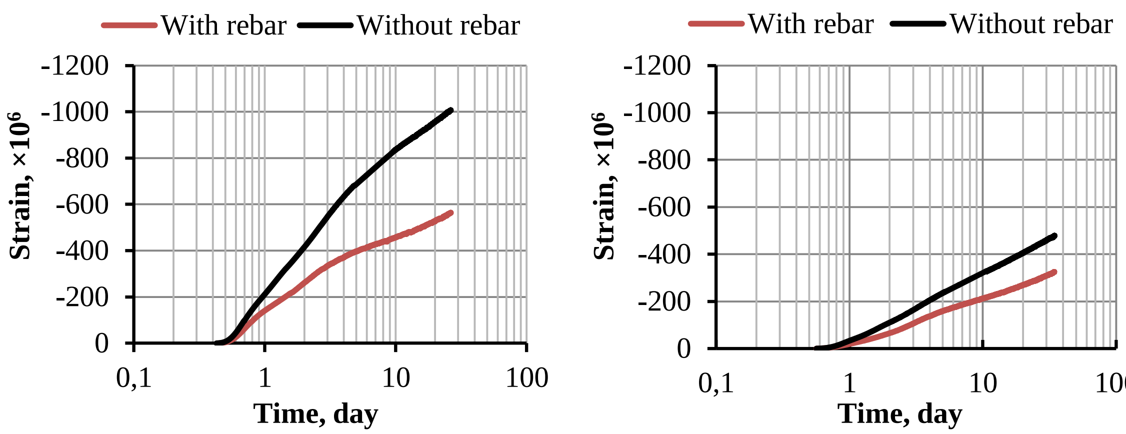


Figure 3 Illustration of free shrinkage and restrained shrinkage by concentrically placed rebars



(a) (b)

Figure 4 Autogenous shrinkage in (a) paste and (b) mortar containing 40% aggregate by volume (0.35 w/c)

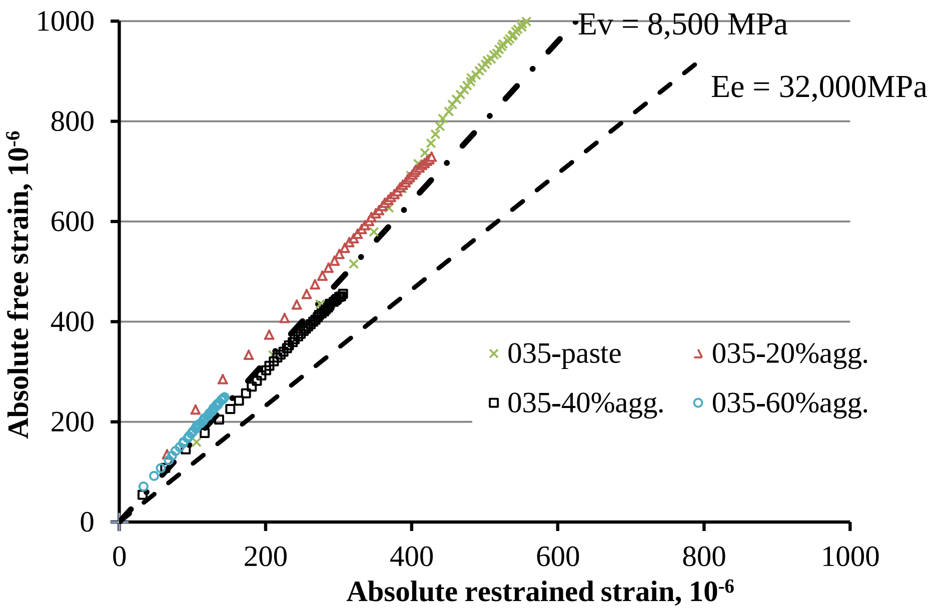


Figure 5 Effect of steel reinforcement on viscoelastic hydration modulus ($\boldsymbol{E_v}$)

3.2 Pickett model for autogenous shrinkage prediction

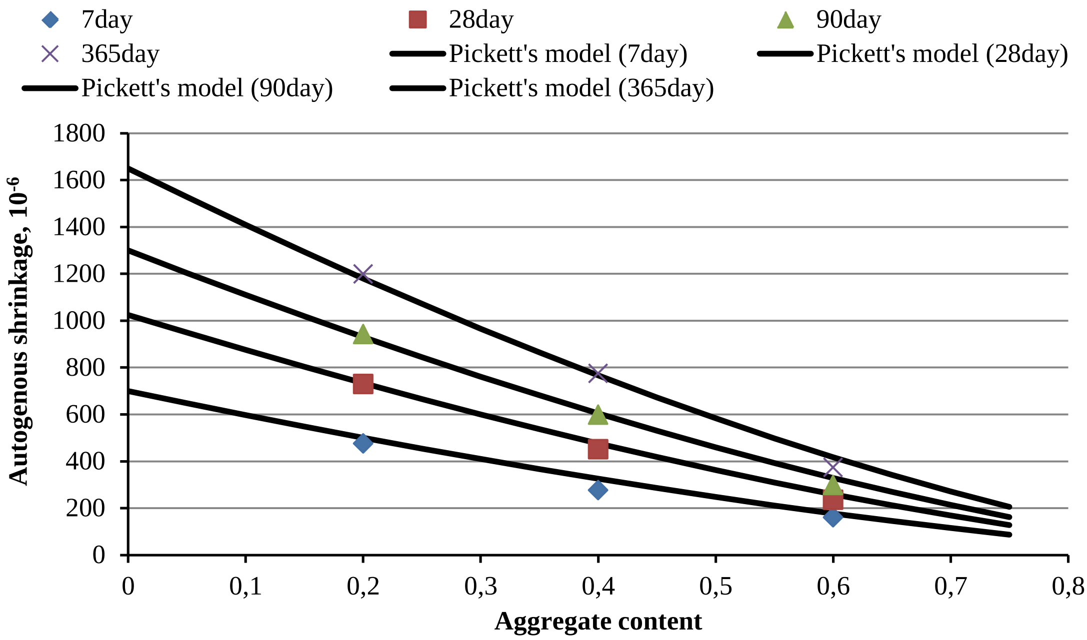


Figure 6 Prediction of shrinkage strain by Pickett's model

The Pickett's shrinkage model is perfect for modeling autogenous shrinkage as it is developed for a uniform paste stress within a cross section [21].

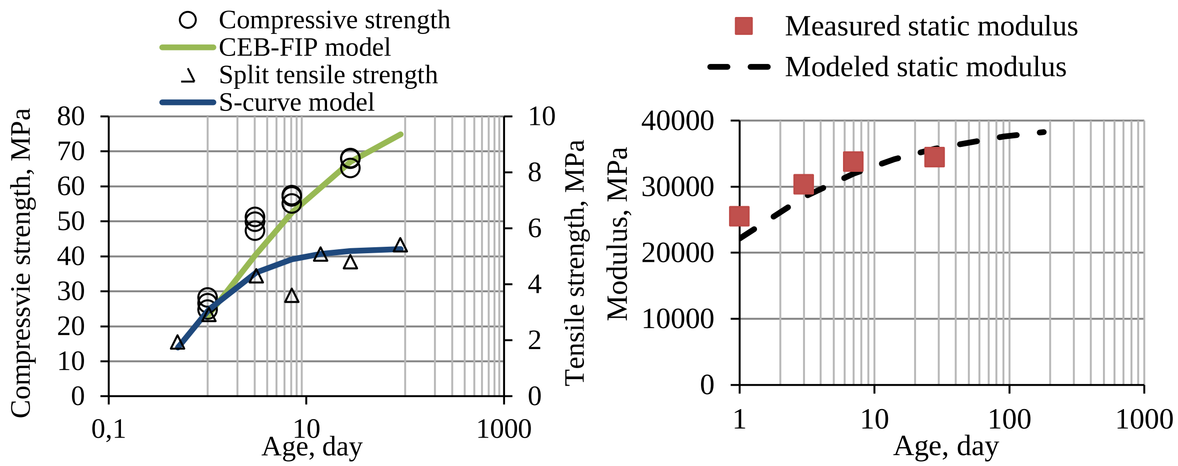
$$\varepsilon_c = \varepsilon_p(1 - V_a)^n \tag{4}$$

Where V_a is relative aggregate volume fraction and n is the shrinkage exponent, a measure of aggregate particle restraining effect.

Free shrinkage results for different paste volume fractions can be fitted using the Pickett model with an exponent n ~ 1.5 in this case (Figure 6). This model is a powerful tool for evaluating effect of paste content (1-V_a) on concrete shrinkage.

3.3 Total stress in HPC due to full shrinkage restraint

Based on the measurement and modelling of the compressive strength, split tensile strength (Figure 7(a)) and static modulus (Figure 7(b)) of concrete mix, the predicted elastic and viscoelastic shrinkage stress development from a full deformation restraint and sealed cured specimen is shown in Figure 8. The shrinkage stresses calculated with a constant viscoelastic hydration modulus are significant and increasing over time, thus reducing the crack resistance of HPC. Typically a failure limit of 1% is used. This corresponds to an allowable stress/strength ratio of 0.56 [22].



(a) (b)

Figure 7 Measurement and prediction on (a) compressive and split tensile strength and (b) static modulus

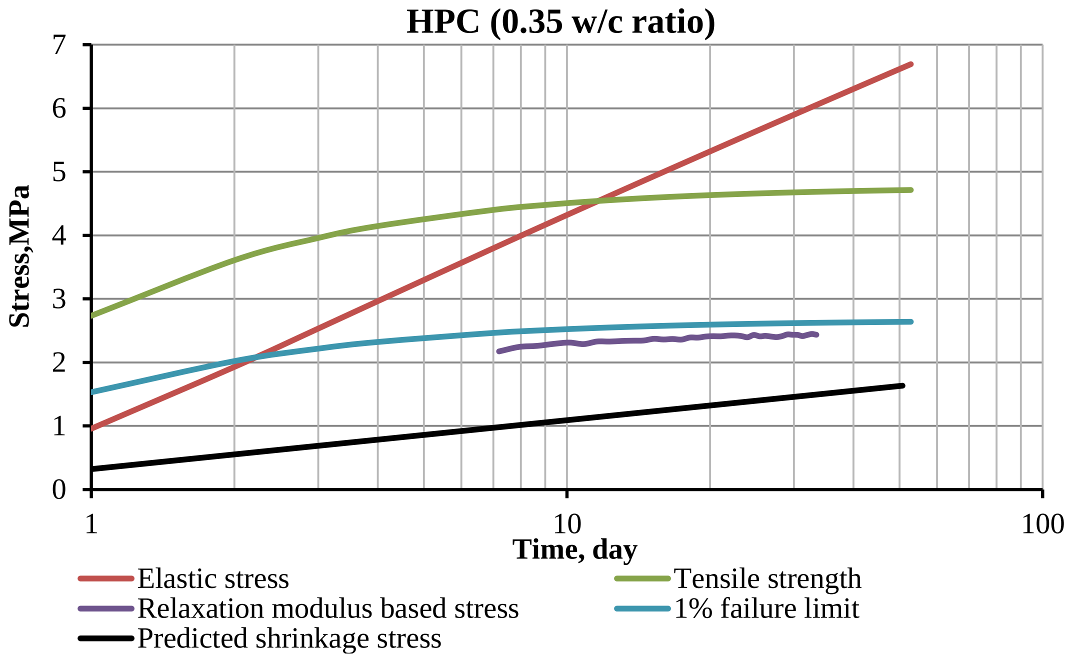


Figure 8 Self-desiccation stress development in high performance concrete (w/c =0.35) subjected to full shrinkage restraint based on different stress prediction methods

This methodology replaces the need for relaxation modulus calculations using either the so-called analogy between creep compliance and relaxation modulus, or effective modulus method or viscoelastic modeling.

4 Conclusions

Results from this study demonstrate that shrinkage stresses can be predicted using the hydration modulus which is obtained from autogenous shrinkage measurements.

This methodology replaces the need for complicated creep and relaxation modulus analysis.

The importance of reducing paste content for shrinkage stress control is demonstrated using the Pickett's model.

5 References

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