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Fibre bridging effect on the Paris relation of mode I fatigue delamination in composite laminates with different thicknesses



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ABSTRACT

Fatigue delamination with fibre bridging in composite laminates with different thicknesses was investigated. The experimental results clearly demonstrated fibre bridging had significant retardation effects on fatigue delamination behavior, making it insufficient to use a single Paris resistance curve to determine fatigue crack growth. To address this problem, the coefficients of the Paris relation were correlated to the normalized crack extension $(a - a_0)/L_{pz}$. It was found that the exponent *n* was independent on the normalized crack extension and specimen thickness, whereas the parameter $\log(c)$ bi-linearly decreased with the normalized crack extension and kept constant once fibre bridging became saturation. And the magnitude of $\log(c)$ was independent on specimen thickness. Thus, it was concluded that fatigue delamination behavior and fibre bridging significance were independent on specimen thickness at a given normalized crack extension $(a - a_0)/L_{pz}$. With substitutions of these correlations into the Paris relation, an empirical power law relation was developed to characterize fatigue delamination behavior. And its validation was verified by a comparison between predictions and experiments.

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1. Introduction

A vast number of studies have demonstrated that delamination between neighbored layers of composite laminates was one of the most significant damage in the applications of these materials in engineering [1,2]. Composite laminates are really vulnerable to delamination, due to lack of reinforcement in thickness direction. This failure can be derived from defects in manufacturing process, low/high velocity impact, stress concentration, fatigue loading and so on. Particularly, it is not easy to have real-time monitors on the evolution of this damage, making even urgent requirements on reliable methods in determining delamination growth. Thanks to in-depth understanding of delamination under fatigue loading in the last several decades, damage tolerance philosophy has been increasingly applied in composite structural designs, which can benefit weight reduction as well as increase fuel efficiency.

Indeed, scientific community has paid a lot of attention to delamination growth in the applications of composite laminates under fatigue loading. Methods on fatigue delamination growth

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can be generally divided into four categories [2]. Among this classification, methods based on the fracture mechanics were effective in determining crack propagation behavior in both composites as well as metals. In the framework of the fracture mechanics, fatigue crack growth can be correlated to concepts of the stress intensity factor (*SIF*) *K* and the strain energy release rate (*SERR*) *G*. Particularly, a power law relation, i.e. the well-known Paris relation, has been proposed by Paris et al. [3] and subsequently used by a large number of researchers with necessary modifications in characterizing fatigue crack growth behavior [4–7]. In these relations, fatigue crack growth was determined by the maximum *SERR G*_{max} or the *SERR* range ΔG or combinations of *G*_{max} and ΔG .

According to the Paris relation ant its variations, a series of factors, which can affect interlaminar resistance of composite laminates, have been investigated. Gustafson et al. [4], Hojo et al. [5] and Mall et al. [6] conducted fatigue experiments to explore delamination behavior under different stress ratios. Fatigue delamination can decrease with increase in the stress ratio when da/dN was correlated with G_{max} . However, no significant stress ratio effect was observed in fatigue crack growth in case of plotting da/dN against $\Delta G = G_{max} - G_{min}$. Thus, they suggested taking ΔG as the crack driving force. Similar trend was also observed in a study completed by Shahverdi et al. [7] in case of using G_{max} to represent



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fatigue delamination growth in adhesive bonds under various stress ratios. And R-ratio effect became obscure as using ΔG to interpret fatigue data. Rans et al. [8] recommended using a new definition of SERR range, i.e. $\Delta G = (\sqrt{G_{max}} - \sqrt{G_{min}})^2$, to correctly represent the similarity principle in fatigue data reduction. Khan et al. [9] provided a comparison on different definitions of SERR range in fatigue data reduction. It was reported that the use of $\Delta G = (\sqrt{G_{max}} - \sqrt{G_{min}})^2$ can effectively reflect R-ratio effects on fati-gue crack growth, whereas $\Delta G = G_{max} - G_{min}$ was not a good similitude parameter to determine fatigue delamination. Some scientists paid attention to temperature effects on fatigue delamination behavior. There is was no consensus on its effects in fatigue delamination growth until recently. It was reported that the increase of temperature can decrease interlaminar resistance [10–12]. This meant fatigue crack growth was much faster with an elevated temperature. On the contrary, others observed fatigue crack growth decreased significantly with an increased temperature, attributing to significant ductility of matrix in a high temperature [13,14]. In order to enhance interlaminar resistance, various methods, such as z-pin, stitching, three-dimensional fabric, interlayer/interleaf toughened, have been gradually developed. Delamination behavior of reinforced composite laminates under fatigue loading has been investigated as well. Hojo et al. [15] experimentally investigated fatigue delamination behavior in composite laminates with interlayer/interleaf toughened. They reported that both methods can improve fatigue delamination properties significantly. Cartié et al. [16] investigated fatigue delamination behavior of laminates with different z-pin areal densities. Pegorin et al. [17] recently examined the length scale of z-pin on fatigue delamination resistance. Both research indicated that z-pin was an effective way to prohibit fatigue delamination growth.

Martin et al. [18] extended the Paris relation to a new formulation, see Eq. (1), to represent fatigue delamination from threshold to unstable growth regions. It is worth noting that this correlation indeed derived from the observation of fatigue data distribution (in terms of *da/dN* against *SERR*) in the double logarithm scales, rather than physical characterizations [2]. And similar models were subsequently used by other researchers to determine fatigue crack growth behavior in composites and adhesive bonds of full *SERR* range [19–21].

$$\frac{da}{dN} = c_1 (G_{max})^{n_1} \frac{\left[1 - \left(\frac{G_{th}}{G_{max}}\right)^{n_2}\right]}{\left[1 - \left(\frac{G_{max}}{G_c}\right)^{n_3}\right]}$$
(1)

where c_1 and n_i (*i* = 1, 2, 3) are curve-fitting parameters.

It has been reported many times that fibre bridging was a significant shielding mechanism in delamination growth of some kind composite laminates under either quasi-static or fatigue loading [10,22-24]. Particularly, the presence of fibre bridging in the wake of crack front can restrain crack opening and prohibit crack propagation in quasi-static loading. As a result, interlaminar resistance is not constant, but crack extension dependent. It rises with crack growth and becomes plateau after crack extension exceeds a certain level (*R-curve*). According to the quasi-static studies [25-27], the shape of *R-curve* depends on several factors, such as ply orientation, thickness dimension. As fibre bridging was the main reason for the increase of delamination resistance, the differences observed in the *R-curve* shapes demonstrated the significance of fibre bridging was dependent on these factors.

The studies on fatigue delamination with fibre bridging were not as sufficient as these in quasi-static delamination. Hojo et al. [28] and Hwang et al. [29] used Paris-type power law relations to interpret experimental fatigue data. They reported that fibre bridging could retard fatigue crack growth significantly. Similar conclusion was also made by the authors, according to experimental investigations on fatigue delamination with largescale fibre bridging in composite laminates [30–33]. It was found that the Paris resistance curves can decrease shift significantly with fibre bridging development in a log-log plot. This meant that the required SERR of a given da/dN in a long crack was much higher than that in a short crack. As a result, the use of a single Paris resistance curve cannot well represent fatigue delamination behavior. To address this problem, the authors have proposed empirical power law models, by correlating the Paris curve-fitting parameters with crack extension $a - a_0$, to phenomenally determine fatigue delamination with fibre bridging [32,33]. The validation of these methods has been verified via comparisons between experiments and predictions. Other researchers explored fibre bridging effects on fatigue delamination via the bridging laws [34-36]. And methods on how to determine bridging stress vs. crack opening displacement δ have been gradually developed. The results clearly demonstrated that bridging stress in fatigue delamination exponentially decayed with increase of crack opening displacement. In a recent study, Donough et al. [35] applied an inverse method to quantify the bridging laws and proposed a new scaling parameter to represent fatigue delamination under various stress ratios.

Until recently, the Paris relation was still one of most widely used methods in fatigue crack growth studies, even though people have noticed that it was only a phenomenal description of this failure, rather than physical characterization [2]. With this method, it has been reported that many factors can affect the significance of fibre bridging in fatigue delamination of composite laminates [1,2,30,32,33,36]. They are, among others, stress ratio, interface configuration, thickness. It is meaningful to explore the effects of these factors on fibre bridging and to extend the Paris relation with consideration of fibre bridging in fatigue delamination characterization for engineering. In the previous studies [30,32,33], the effects of stress ratio and ply orientation on fibre bridging and fatigue delamination have been carefully examined. However, to the knowledge of the authors, little attention has even been paid into thickness effects. In present study, we therefore tried to experimentally investigate the thickness effects. To achieve this aim, unidirectional double cantilever beam (DCB) specimens with three different thicknesses were manufactured and fatigue tested to explore fatigue delamination behavior as well as fibre bridging significance.

2. Material and fatigue experimental program

2.1. Material and specimen geometry

Unidirectional DCB specimens with different thicknesses were manufactured and tested to investigate fatigue delamination behavior with large-scale fibre bridging. The layup sequences for these laminates were designed as $[(0)_{12}/(0)_{12}]$, $[(0)_{16}/(0)_{16}]$ and $[(0)_{24}/(0)_{24}]$, respectively.

The composite laminates were produced by hand-lay-up of thermosetting unidirectional carbon/epoxy prepreg layers of M30SC/DT120 (high strength and modulus carbon fibre/toughened thermosetting epoxy), supplied by Delta-Tech S.p.A, Italy. A 12.7 µm Teflon film was inserted in the middle plane of these laminates during the hand-lay-up process to act as an initial delamination $a_0 = 60$ mm. The laminates were cured in vacuum in an autoclave at a pressure of 6 bars and curing temperature of 120 °C for 90 min. The nominal thicknesses of these composite laminates after curing were 3.75 mm, 5.0 mm and 7.5 mm, respectively. All laminates were C-scanned to detect potential imperfections. These plates were subsequently cut by a diamond saw into 25 mm width beams with 200 mm length. And only these samples

were tested where the C-scan did not reveal any obvious imperfections. A pair of aluminum loading blocks, 25 mm width by 20 mm length with 6 mm thickness, was adhesively bonded onto the specimen at the side of the Teflon insert for load introduction.

One side of DCB specimen was coated with thin typewriter correction fluid to enhance visibility of the delamination front during fatigue test. A strip of grid paper was pasted on the specimen coated side to aid in measuring the delamination propagation length.

2.2. Fatigue testing

All fatigue experiments were performed on a 10 kN MTS machine under displacement control at a frequency of 5 Hz with the same stress ratio R = 0.5 in ambient conditions. A computer controlled digital camera system was employed to automatically monitor crack growth at the maximum displacement with predefined intervals during the test. In displacement controlled tests, fatigue crack growth rate decreases with crack propagation, due to the decrease in the applied load. Thus, the interval was set to be every 100 cycles in the first 5000 cycles, every 500 cycles in the following 15,000 cycles, and every 1000 cycles when the fatigue cycle number exceeded 20,000. The load, displacement and number of cycles were automatically stored in an Excel file every 100 cycles enabling data evaluation after the test. The fatigue experimental set-up is shown in Fig. 1.

To determine fatigue delamination behavior with different amounts of fibre bridging, DCB specimens were repeatedly tested for several times with different applied displacements, but keeping stress ratio the same. Particularly, fatigue delamination growth gradually decreases with the decrease in the *SERR*. This test was manually terminated in case of crack retardation. Subsequently, the test was repeated with increased displacements at the same stress ratio. This sequence was repeated until the maximum displacement capacity of the test machine was reached. With this test procedure, multiple delamination resistance curves were obtained, with each one representing delamination resistance equivalent to a specific fatigue pre-crack length, i.e. delamination length at which that particular fatigue test was initiated.

The Paris relation, see Eq. (2), was applied to interpret fatigue delamination with different amounts of fibre bridging. The SERRs,

i.e. G_{Imax} and G_{Imin} , in fatigue loading were calculated with the Modified Compliance Calibration (MCC) method, recommended in the ASTM D5528-01 standard, see Eq. (3).

$$\frac{da}{dN} = c(\Delta G)^n = c \left[\left(\sqrt{G_{lmax}} - \sqrt{G_{lmin}} \right)^2 \right]^n$$
(2)

$$G_{l} = \frac{3P^{2}C^{(2/3)}}{2A_{1}Bh}$$
(3)

where *c* and *n* are curve-fitting parameters of the Paris relation; *P* is the applied load; *C* is the compliance of the DCB specimen; *B* is the specimen width and *h* is the thickness of the specimen; A_1 is the slope of the curve in the graph where a/h is plotted against $C^{1/3}$.

The 7-point Incremental Polynomial Method, recommended in the ASTM E647-00 standard, was employed to determine the fatigue crack growth rate da/dN.

3. Results and discussion

Unidirectional DCB specimens with three thicknesses were repeatedly tested under fatigue loading with the same stress ratio R = 0.5. These tests can provide enough raw data for the investigation on fatigue delamination behavior in composite laminates. These specimens were distinguished as Sp_24 for 24 layers (h = 3.75 mm), Sp_32 for 32 layers (h = 5.0 mm) and Sp_48 for 48 layers (h = 7.5 mm).

All experimental results were interpreted with the Paris relation and summarized in Fig. 2 in terms of da/dN against the *SERR* range ΔG . The results clearly demonstrate that fatigue delamination is crack scale dependent. With the increase of crack length, the Paris resistance curves significantly shift from left to right and finally converge into a single one in the graph. This means the required ΔG of a given fatigue crack growth rate da/dNincreases with crack propagation and becomes constant after crack length exceeds a certain level, as shown in Fig. 3. According to the previous study [26], the presence of fibre bridging was the main reason for the decrease shift of resistance curves. Particularly, more fibre bridging can be generated in a long crack in comparison with that of a short crack. As a result, these bridging fibres can periodically store and release more *SERR* under cyclic loading, making further decrease shift of the resistance curves. There is a plateau state



Fig. 1. Fatigue experimental set-up.



Fig. 2. Fatigue data interpretation with the Paris relation (a) h = 3.75 mm; (b) h = 5.0 mm; (c) h = 7.5 mm.



Fig. 3. ΔG vs. $a - a_0$ at a given $da/dN = 1 \times 10^{-7}$ m/cycle with crack propagation.

in fatigue delamination of each specimen thickness, reflecting by the coincidence of the most right resistance curves shown in Fig. 2, as well as the plateau value of ΔG illustrated in Fig. 3. In this state, fibre bridging becomes saturation and the new generation of bridging around crack front is comparable to the disappearance of bridging at the end of fibre bridging zone. As a result, the significance of fibre bridging becomes independent on further crack propagation, leading to convergence of the most right resistance curves.

The length of process zone, L_{pz} , is an important parameter in the characterization of resistance increase during crack propagation because of fibre bridging [22,24,27]. This parameter can be defined as the distance between crack front and the end of fibre bridging region for delamination in composite laminates. It has been reported that specimen thickness had important effect on the magnitude of L_{pz} , i.e. it rose with thickness increase. The same trend is also observed in fatigue delamination growth as shown in Fig. 4. Particularly, the value of L_{pz} is 45.9 mm for h = 3.75 mm, 49.6 mm for h = 5.0 mm and 69.6 mm for h = 7.5 mm, respectively.

Suo et al. [22] gave a thorough discussion on the shape of *R*-curve in quasi-static delamination with fibre bridging and proposed a relationship between L_{pz} and specimen thickness *h*,

see Eq. (4). This proportional relation explicitly indicates that the dimension of fibre bridging region is positively related to specimen thickness. Fig. 4 summarizes the values of L_{pz} and $h^{3/4}$ for fatigue delamination growth of different specimens. A strong linear relationship between L_{pz} and $h^{3/4}$ was observed. And a linear curve fitting, with a high determination coefficient R^2 , was conducted to determine this relation. Thus, Eq. (4) seems not only valid for quasi-static delamination, but also useful to evaluate the length of fibre bridging region in fatigue delamination. According to this linear fitting, the values of L_{pz} for specimens with other thicknesses can be evaluated.

$$L_{pz} \propto h^{3/4} \tag{4}$$

In applications of *R-curve* to determine delamination resistance increase under quasi-static loading [22,24,27], the initial delamination resistance (with no fibre bridging) was independent on thickness scale, whereas the interlaminar resistance during delamination propagation (with significant fibre bridging) was related to thickness dimension. Then, one may reasonably ask a question as whether thickness has effects on initial fatigue delaminnation and on fibre bridging significance during fatigue delamination. To answer this question, the results of fatigue delamination with similarly short pre-crack lengths (equivalent to delamination





Fig. 5. Fatigue delamination with different amounts of fibre bridging (a) limited fibre bridging; (b) fully developed fibre bridging.

with limited fibre bridging) and with fully developed fibre bridging were summarized in Fig. 5(a) and (b), respectively. The results shown in Fig. 5(a) indicate that fatigue delamination of different specimen thicknesses is similar more or less. And small dispersion observed in the graph mainly attributes to different amounts of fibre bridging generated during the tests. If there is no fibre bridging, fatigue delamination behavior of different specimen thicknesses should be the same. Further information related to the dispersion will be discussed later. Fig. 5(b) summarizes delamination results with fully developed fibre bridging. All results fall into a narrow band, indicating the same fatigue delamination behavior at the plateau state. As fibre bridging is the main reason for the decrease shift of the Paris resistance curves, the convergence of experimental data derived from different specimen thicknesses means thickness has no or negligible effect on the amount of fibre bridging at the plateau state in fatigue delamination growth.

As illustrated in Fig. 4, the magnitude of L_{pz} linearly depends on specimen thickness $h^{3/4}$. Incorporating with the results shown in Fig. 5(b), a larger crack growth must be generated in a thicker specimen, in order to reach the saturation of fibre bridging. Due to the difference in the value of L_{pz} , the coincidence of the data at the

plateau state shown in Fig. 5(b) does not mean fibre bridging significance of the same crack extension $a - a_0$ of different specimen thicknesses is identical, before fibre bridging becomes saturation. Particularly, more fibre bridging seems present in the same delamination of a thinner specimen. This can be used to interpret the small dispersion observed in Fig. 5(a). To provide solid evidence to consolidate above discussion, Fig. 6 summaries fatigue delamination results of different specimen thicknesses at the same crack extension $a - a_0$. At a given SERR range ΔG , fatigue delamination in thinner laminates is much slower than that in thicker laminates. Furthermore, if one takes a closer observation on the ΔG increase shown in Fig. 3, the value of ΔG of a thinner specimen is also higher than that of a thicker specimen at the same crack length, which agrees well with the results shown in Fig. 6. According to above discussion, the significance of fibre bridging depends on thickness. Particularly, more bridging fibres can be present in thinner laminates at the same crack length.

From the results shown in Fig. 2, one can conclude that it is insufficient to use a single Paris resistance curve to represent fatigue delamination with large-scale fibre bridging in composite laminates. Particularly, the use of the most left one can cause



Fig. 6. Fatigue delamination with the same crack extension (a) $a - a_0 = 11.6$ mm for h = 5.0 mm, $a - a_0 = 12.5$ mm for h = 7.5 mm; (b) $a - a_0 = 20.9$ mm for h = 3.75 mm, $a - a_0 = 20.4$ mm for h = 7.5 mm.

conservative predictions, whereas the use of the most right one cannot guarantee safety. The resistance curves located inbetween, therefore, must be carefully considered in the characterization of fatigue delamination growth. In addition, one should note that fibre bridging significance is not only related to crack scale, but also depends on specimen thickness before fibre bridging becomes saturation. Thus, crack length and thickness effects on fibre bridging must be well considered in fatigue delamination prediction.

The distribution of the Paris resistance curves shown in Fig. 2 is dominantly determined by two curve-fitting parameters, i.e. *c* and *n*. A single Paris resistance curve can be used to determine fatigue delamination with no fibre bridging, in which these coefficients are constant. However, the results illustrated in Figs. 2 and 6 explicitly demonstrate that these parameters are not constant, but crack scale and thickness dependent. In order to develop an empirical model to represent fatigue delamination with large-scale fibre bridging, a feasible way is to investigate the changes of these parameters with fibre bridging. With substitutions of these relations into the Paris relation, an engineering/empirical method can be developed to determine fatigue delamination with fibre bridging.

4. Bridging effect on the Paris relation

The amount of fibre bridging can be qualitatively represented by crack extension $a - a_0$ [32,33]. And the correlations between fibre bridging and curve-fitting parameters have been investigated for specimens with the same thickness. In present study, fibre bridging significance was represented by the normalized crack extension, i.e. $(a - a_0)/L_{pz}$, in order to take thickness effects into account. Particularly, in the first step, the Paris relation, see Eq. (2), was linearized to investigate the change of exponent *n* with the normalized crack extension via a linear regression analysis. The linearized Paris relation can be given as

$$\log\left(\frac{da}{dN}\right) = n\log(\Delta G) + \log(c) \tag{5}$$

The exponents of each specimen at different normalized crack extensions are summarized in Fig. 7. The exponent *n* remains constant more or less with crack propagation. And the average value of *n* is 12.53 for h = 3.75 mm, 12.73 for h = 5.0 mm and 13.70 for h = 7.5 mm. This indicates fibre bridging has no or negligible effect on the magnitude of exponent in fatigue delamination of



Fig. 7. *n* vs. $(a - a_0)/L_{pz}$

unidirectional composite laminates. Furthermore, the difference in exponents between different specimen thicknesses is not obvious. Therefore, an average value of the exponents, as the dark solid line shown in Fig. 7, can be used in the Paris relation to determine fatigue delamination with large-scale fibre bridging.

In the second step, linear regression analysis was conducted to determine the coefficient log(c) with the averaged exponent. In a $\log \log (c)$ is the v-intercept of Eq. (5). The decrease shift of the Paris resistance curves can result in decrease of log(c) with crack extension. According to the results shown in Fig. 5(b), this parameter should keep constant at the plateau state for specimens with different thicknesses. The values of log(c) were illustrated in Fig. 8 for different thicknesses. And a bilinear relation can be used to determine the relation between $\log(c)$ and $(a - a_0)/L_{pz}$. The results demonstrate that the magnitude of log(c) at a given normalized crack extension is the same, regardless of thickness dimension. With the fixed exponent, one can conclude that fatigue delamination behavior is thickness independent at the same value of $(a - a_0)/L_{pz}$. This means a single Paris resistance curve can be used to determine fatigue delamination behavior of specimens with different thicknesses at the same value of $(a - a_0)/L_{pz}$. Furthermore, this also indicates that the significance of fibre bridging is the same at a given $(a - a_0)/L_{pz}$. Fig. 9 provides examples of fatigue delamination behavior in specimens Sp_24 and Sp_48 with the same values of $(a - a_0)/L_{pz}$. As expected, all fatigue data overlap each other, demonstrating the same delamination behavior.

According to above discussion, the exponent *n* in fatigue delamination of unidirectional composite laminates remains the same with fibre bridging development. And thickness has little effect on the magnitude of this coefficient. In the plot of $\log(c)$ against $(a - a_0)/L_{pz}$, a bilinear relation can be used to determine the value of $\log(c)$ of different specimen thicknesses. Based on these conclusions and the linear fitting shown in Fig. 4, a new power law relation for fatigue delamination with large-scale fibre bridging in specimens with different thicknesses can be derived as

$$\frac{da}{dN} = c(\Delta G)^n = c \left(\frac{a - a_0}{L_{pz}(h)}\right) (\Delta G)^n \tag{6}$$

In this new relation, the exponent *n* is a constant with fibre bridging development of different specimen thicknesses. The magnitude of coefficient log(c) is a function of both crack extension $a - a_0$ and thickness dimension *h*. With this function, bridging effect on fatigue delamination growth can be phenomenally taken into account. Technically, fatigue delamination behavior of composite laminates with other thickness scales can be determined as well, according to the correlations shown in Figs. 4 and 8.



Fig. 8. $\log(c)$ vs. $(a - a_0)/L_{pz}$.



Fig. 9. Fatigue delamination with the same magnitude of $(a - a_0)/L_{pz}$ (a) $(a - a_0)/L_{pz} = 0.45$; (b) $(a - a_0)/L_{pz} = 0.78$.



Fig. 10. Fatigue delamination predictions of specimen Sp_32 (*h* = 5.0 mm).

5. Method verification

To verify the validation of Eq. (6) in determining fatigue delamination with fibre bridging in composite laminates, this model was used to determine the data sets from specimen Sp_32, as the first attempt to demonstrate its accuracy and reliability. In order to take data scatter into account, the 95% confidence limits of the experimental data were also calculated with the linear regression analysis.

Fig. 10 summarizes the predictions and experimental results in terms of da/dN against ΔG . Acceptable agreements are obtained and the predictions in most cases locate within the 95% confidence intervals. Only fatigue delamination with crack extension $a - a_0 = 37.2$ mm is slightly underestimated. This could attribute to the unavoidable data scatter usually observed in fatigue experiments. It is, therefore, reasonable to make a statement that Eq. (6) is an appropriate method to determine fatigue delamination with fibre bridging in composite laminates. And the effect of fibre bridging on fatigue delamination can be phenomenally represented by the correlation between $\log(c)$ and the normalized crack extension $(a - a_0)/L_{pz}$.

6. Conclusions

Thickness effects on fatigue delamination with large-scale fibre bridging were conducted by experiments on unidirectional DCB specimens with various thickness scales. According to the experimental fatigue data what we already had, the author preferred to draw a conclusion that thickness had important effects on the amount of fibre bridging generated in fatigue delamination. Before fibre bridging became saturation, more significant bridging fibres can be present in thinner composite laminates at a given crack extension $a - a_0$. Furthermore, the length of fibre bridging region was related to thickness. And the correlation, proposed by Suo et al. [22], can be used to determine thickness dependence of the length of fibre bridging region.

Further investigations were conducted to determine the correlations between the Paris curve-fitting parameters and the normalized crack extension $(a - a_0)/L_{pz}$. It was found that the exponent nin fatigue delamination with fibre bridging remained the same, whereas the coefficient $\log(c)$ bilinearly decreased with normalized crack extension. Particularly, the magnitude of $\log(c)$ was thickness independent at a given $(a - a_0)/L_{pz}$. This meant fibre bridging significance in delaminated specimens with the same value of $(a - a_0)/L_{pz}$ was identical. As a result, fatigue delamination in composite laminates with different thicknesses was the same at the same value of $(a - a_0)/L_{pz}$.

An empirical power law relation was proposed to represent fatigue delamination with large-scale fibre bridging of specimens with different thicknesses, according to the investigations on the correlations between the parameters $(\log(c) \text{ and } n)$ and the normalized crack extension $(a - a_0)/L_{pz}$. This new power law relation was subsequently employed to determine fatigue delamination with significant fibre bridging. Really good agreements were obtained between the predictions and the experiments, demonstrating its validation.

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References

- Bak BLV, Sarrado C, Turon A, Costa J. Delamination under fatigue loads in composite laminates: a review on the observed phenomenology and computational methods. Appl Mech Rev 2014; 66: 060803-1-060803-24.
- [2] Pascoe JA, Alderliesten RC, Benedictus R. Methods for the prediction of fatigue delamination growth in composites and adhesive bonds – a critical review. Eng Fract Mech 2013;112–113:72–96.
- [3] Paris PC, Gomez MP, Anderson WE. A rational analytic theory of fatigue. Trend Eng 1961;13:9–14.
- [4] Gustafson CG, Hojo M. Delamination fatigue crack growth in unidirectional graphite/epoxy laminates. J Reinf Plast Compos 1987;6:36–52.
- [5] Hojo M, Tanaka K, Gustafson CG. Effect of stress ratio on near-threshold propagation of delamination fatigue cracks in unidirectional CFRP. Compos Sci Technol 1987;29:273–92.
- [6] Mall S, Ramamurthy G, Rezaizdeh MA. Stress ratio effect on cyclic debonding in adhesively bonded composite joints. Compos Struct 1987;8:31–45.
 [7] Shahverdi M, Vassilopoulos AP, Keller T. Experimental investigation of R-ratio
- [7] Shahverdi M, Vassilopoulos AP, Keller T. Experimental investigation of R-ratio effects on fatigue crack growth of adhesively-bonded pultruded GFRP DCB joints under CA loading. Compos A: Appl Sci Manuf 2012;43:1689–97.
- [8] Rans C, Alderliesten R, Benedictus R. Misinterpreting the results: how similitude can improve our understanding of fatigue delamination growth. Compos Sci Technol 2011;71:230–8.
- [9] Khan R, Rans CD, Benedictus R. Effect of stress ratio on delamination growth behavior in unidirectional carbon/epoxy under mode I fatigue loading. ICCM; 2009.
- [10] Gregory JR, Spearing SM. A fiber bridging model for fatigue delamination in composite materials. Acta Mater 2004;52:5493–502.
- [11] Gregory JR, Spearing SM. Constituent and composite quasi-static and fatigue fracture experiments. Compos A: Appl Sci Manuf 2005;36:665–74.
- [12] Charalambous G, Allegri G, Hallett SR. Temperature effects on mixed-mode I/II delamination under quasi-static and fatigue loading of a carbon/epoxy composite. Compos A: Appl Sci Manuf 2015;77:75–86.
- [13] Coronado P, Argüelles A, Vina J, Mollon V, Vina I. Influence of temperature on a carbon-fibre epoxy composite subjected to static and fatigue loading under mode-I delamination. Int J Solids Struct 2012;49:2934–40.
- [14] Coronado P, Argüelles A, Vina J, Vina I. Influence of low temperatures on the phenomenon of delamination of mode I fracture in carbon-fibre/epoxy composites under fatigue loading. Compos Struct 2014;112:188–93.
- [15] Hojo M, Matsuda S, Tanaka M, et al. Mode I delamination fatigue properties of interlayer-toughened CF/epoxy laminates. Compos Sci Technol 2006;66:665–75.
- [16] Cartié DDR, Laffaille JM, Partridge IK, Brunner AJ. Fatigue delamination behavior of unidirectional carbon fibre/epoxy laminates reinforced by Z-fibre pinning. Eng Fract Mech 2009;76:2834–45.
- [17] Pegorin F, Pingkarawat K, Daynes S, Mouritz AP. Influence of Z-pin length on the delamination fracture toughness and fatigue resistance of pinned composites. Compos B: Eng 2015;78:298–307.
- [18] Martin RH, Murri GB. Characterization of mode I and mode II delamination growth and thresholds in AS4/PEEK composites. ASTM STP 1990;1059: 251–70.
- [19] Shivakumar K, Chen H, Abali F, Le D, Davis C. A total fatigue life model for mode I delaminated composite laminates. Int J Fatigue 2006;28:33–42.
- [20] Chen H, Shivakumar KN, Abali F. Application of total fatigue life model to T700 carbon/vinyl ester composite. Compos B: Eng 2008;39:36–41.
- [21] Shahverdi M, Vassilopoulos AP, Keller T. A total fatigue life model for the prediction of the R-ratio effects on fatigue crack growth of adhesively-bonded GFRP DCB joints. Compos A: Appl Sci Manuf 2012;43:1783–90.
- [22] Suo Z, Bao G, Fan B. Delamination R-curve phenomena due to damage. J Mech Phys Solids 1992;40:1–16.
- [23] Spearing SM, Evans AG. The role of fiber bridging in the delamination resistance of fiber-reinforced composites. Acta Metall Mater 1992: 2191–9.
- [24] Jacobsen TK, SΦrensen BF. Mode I intra-laminar crack growth in compositesmodelling of R-curves from measured bridging laws. Compos A: Appl Sci Manuf 2001;32:1–11.
- [25] Shokrieh MM, Heidari-Rarani M. Effect of stacking sequence on R-curve behavior of glass/epoxy DCB laminates with 0//0 crack interface. Mater Sci Eng, A 2011;529:265–9.
- [26] Yao L, Alderliesten R, Zhao M, Benedictus R. Bridging effect on mode I fatigue delamination behavior in composite laminates. Compos A: Appl Sci Manuf 2014;63:103–9.
- [28] Hwang W, Han KS. Interlaminar fracture behavior and fiber bridging of glassepoxy composite under mode I static and cyclic loading. J Compos Mater 1989;23:396–430.
- [29] Hojo M, Ochiai S, Aoki T, Ito H. Mode I fatigue delamination for CF/PEEK laminates using maximum-energy-release-rate constant tests. J Soc Mater Sci Jpn 1995;44:953–9.
- [30] Yao L, Sun Y, Zhao M, Alderliesten RC, Benedictus R. Stress ratio dependence of fibre bridging significance in mode I fatigue delamination growth of composite laminates. Compos A: Appl Sci Manuf 2017;95:65–74.
- [31] Yao L, Alderliesten R, Zhao M, Benedictus R. Discussion on the use of the strain energy release rate for fatigue delamination characterization. Compos A: Appl Sci Manuf 2014;66:65–72.

- [32] Yao L, Alderliesten R, Benedictus R. The effect of fibre bridging on the Paris relation for mode I fatigue delamination growth in composites. Compos Struct 2016;140:125-35.
- [33] Yao L, Sun Y, Alderliesten R, Benedictus R, Zhao M. Fibre bridging effect on the
- [35] Yao L, Sun Y, Alderhesteri A, Benedictus A, Zhao M. Fibre bridge election the Paris relation for mode I fatigue delamination growth in composites with consideration of interface configuration. Compos Struct 2017;159:471–8.
 [34] Stutz S, Cugnoni J, Botsis J. Studies of mode I delamination in monotonic and fatigue loading using FBG wavelength multiplexing and numerical analysis. Compos Sci Technol 2011;71:443–9.
- [35] Donough MJ, Gunnion AJ, Orifici AC, Wang CH. Scaling parameter for fatigue delamination growth in composites under varying load ratios. Compos Sci Technol 2015;120:39–48.
- [36] Farmand-Ashtiani E, Cugnoni J, Botsis J. Effect of large scale bridging in load controlled fatigue delamination of unidirectional carbon-epoxy specimens. Compos Sci Technol 2016;137:52–9.