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Key Points:

- In a graded stream profile concavity and downstream fining are mild
- In an ungraded stream size selective transport can lead to large gradients in slope and grain size
- Channel slope and bed surface texture adjust such that the stream allows for transporting the supplied load

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The graded alluvial river: Profile concavity and downstream fining

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Abstract There has been quite some debate on the relative importance of particle abrasion and grain size selective transport regarding the river profile form and the associated grain size trends in a graded alluvial stream. Here we present new theoretical equations for the graded alluvial river profile that account for the effects of particle abrasion and grain size selective transport in the absence of subsidence, uplift, and sea level change. Under graded conditions we find that abrasion results in a mild profile concavity and downstream fining, whereas under aggradational conditions grain size selective transport can lead to large spatial changes in channel slope and bed surface mean grain size.

1. Introduction

A river channel is generally characterized by a streamwise decrease of the bed slope (i.e., a concave upward profile) and the characteristic grain size of the bed surface sediment (i.e., downstream fining), as represented in Figure 1 [e.g., *Sternberg*, 1875; *Gilbert*, 1877; *Ferguson et al.*, 1998; *Morris and Williams*, 1999a, 1999b]. The identification of the main parameters controlling profile concavity and downstream fining is still a matter of debate. Researchers have indicated various relevant mechanisms: particle abrasion with associated sand and/or silt production [e.g., *Mackin*, 1948; *Snow and Slingerland*, 1987; *Sinha and Parker*, 1996], grain size selective transport, i.e., the difference in mobility between fine and coarse sediment [e.g., *Paola et al.*, 1992; *Gasparini et al.*, 2004; *Fedele and Paola*, 2007; *Miller et al.*, 2014], the streamwise increase in drainage area and water discharge [e.g., *Van Bendegom*, 1967; *Sklar et al.*, 2006], the streamwise decrease in mean size of the sediment added by tributaries [*Mackin*, 1948], an increase in relative base level [e.g., *Parker*, 2004], subsidence [e.g., *Paola*, 1988; *Sinha and Parker*, 1996; *Parker*, 2004], and an outbuilding delta [e.g., *Sinha and Parker*, 1996].

Following *Gilbert* [1877] and *Mackin* [1948], we define the graded or equilibrium river profile as the one that the river approaches when flow, sediment supply, and base level vary around stable values for a long time in the absence of subsidence or uplift. Starting from the equations of mass and momentum conservation, *Van Bendegom* [1967], *De Vries* [1971, 1974] (accessible in *Jansen et al.* [1979]), *Snow and Slingerland* [1987], *Sinha and Parker* [1996], and *Bolla Pittaluga et al.* [2014] find solutions to the graded river profile. Here we extend this approach in that we not only include abrasion [*Snow and Slingerland*, 1987; *Sinha and Parker*, 1996] but also grain size selective transport. For simplicity we consider a two fraction mixture of sand and gravel [e.g., *Paola*, 1988; *Ferguson*, 2003].

2. The Graded Versus the Ungraded River

The traditional view of the relative importance of particle abrasion and grain size selective transport on the river profile is represented by *Mackin* [1948], who follows the *concept of the graded stream* proposed by *Gilbert* [1877], i.e., the concept that a stream, through aggradation or degradation, always tends to create a planform and a channel slope that, over a period of years, will transport exactly the sediment load delivered from upstream. *Mackin* [1948] indicates how in this graded state it is particle abrasion that reduces the bed surface mean grain size in the streamwise direction and that it is grain size selective transport that results in a concave upward profile, as fine grains require a smaller slope to be transported downstream.

Many authors argued against Mackin's ideas as they observed how grain size selective transport is capable of creating strong streamwise sorting and large gradients in channel slope over distances too short for abrasion to be relevant [e.g., *Paola et al.*, 1992; *Ferguson et al.*, 1996, 1998; *Ferguson*, 2003; *Venditti et al.*, 2015].

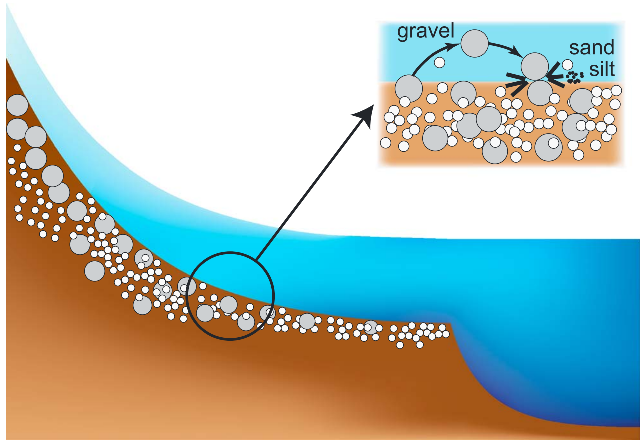

Figure 1. Schematic of the river longitudinal profile: upward concavity and downstream fining.

Here we argue that both seemingly opposing camps are right. The argumentation of *Mackin* [1948] holds under graded conditions, whereas the second group considers ungraded conditions governed by aggradation. The *Ferguson* [2003] numerical experiments illustrate how the strong spatial gradients of an ungraded state reduce to zero while approaching the graded state. Similarly, if the *Paola et al.* [1992] and *Seal et al.* [1997] laboratory experiments were continued under constant feed and flow rates, the slope would have been spatially constant as the flume length makes abrasion irrelevant. Under otherwise similar conditions, the streamwise reduction of both channel slope and grain size are much milder in a graded stream than in an ungraded one [*Mackin*, 1948] and are even negligible in cases where abrasion, tributaries, width changes, temporal variation of the flow, and tides do not play a role.

In the absence of subsidence or uplift, an ungraded reach is adjusting to natural or man-made changes in the boundary conditions and is approaching a graded state. In accordance with the experiments by *Paola et al.* [1992] and *Ferguson* [2003], we consider the gravel-sand transition [e.g., *Yatsu*, 1955; *Sambrook Smith and Ferguson*, 1995] to generally be a slowly downstream migrating gravel front and a characteristic of an ungraded stream. Such an ungraded reach tends to develop toward the graded river profiles described here. Yet subsidence, shoreline progradation, and base level rise can create sufficient accommodation space within the gravel reach for the gravel supplied from upstream such that the gravel-sand transition halts or even retreats.

Kesseli's [1941] argument on how no graded profile in a natural stream can exist as water discharge and other factors are not literally constant is invalid as equilibrium needs to be considered over a period of years [*Gilbert*, 1877; *Mackin*, 1948; *Lane*, 1955], and also numerical runs illustrate that a graded state regarding slope and bed surface texture develops under controlling parameters that vary around stable values [*Wong and Parker*, 2006; *Viparelli et al.*, 2006].

Finally, under conditions with subsidence it is still possible to define a graded state if the sediment supply is sufficient to maintain a constant average bed elevation. Such graded conditions are characterized by downstream fining associated with abrasion and selective transport, responsible for preferential deposition of coarse sediment [*Paola*, 1988].

3. A Model of the Graded River

We set up a morphodynamic model based on the Saint-Venant equations for the flow and the conservation equations of sediment mass at the bed surface for two distinct grain size fractions, sand and gravel. We assume that only gravel particles abrade, which implies that the gravel size slightly reduces with streamwise position. The products of abrasion are sand and silt, and the fraction of abrasion product that is sand (versus silt) is imposed. Further, we consider a situation in which (1) base level is constant; (2) subsidence, uplift, and delta outbuilding are negligible; (3) channel width is imposed and constant with time, and likewise channel curvature and so reach length; (4) the channel is relatively wide; (5) the nondimensional friction coefficient, C_f, is independent of the local flow parameters and grain size; (6) all gravel particles entering the reach at the upstream end have the same size; (7) the annual upstream sediment load is imposed and held constant; and (8) all bed surface sediment has arrived through fluvial sediment transport. We refer to Appendix A for the details of the model.

As the uncertainty related to predictions of the sediment transport rate is large and a factor 10 difference in predicted transport rates is not exceptional, we compare equilibrium river profiles obtained applying four

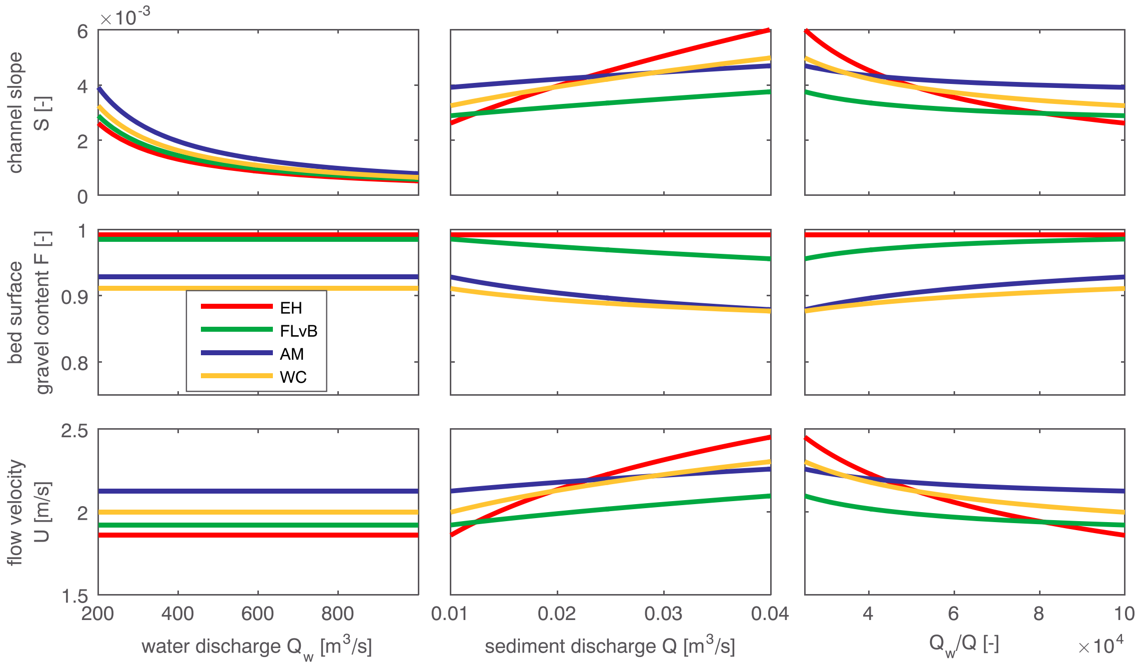

Figure 2. Channel slope, bed surface gravel content, and mean flow velocity for the base case as a function of the water and sediment discharge and the ratio of water to sediment discharge. Predictions are made based on the *Engelund and Hansen* [1967] (EH), *Fernandez-Luque and Van Beek* [1976] (FLvB), *Ashida and Michiue* [1972] (AM), and *Wilcock and Crowe* [2003] (WC) load relations.

load relations: *Engelund and Hansen* [1967] (EH), *Fernandez-Luque and Van Beek* [1976] (FLvB), *Ashida and Michiue* [1972] (AM), and *Wilcock and Crowe* [2003] (WC). Two of these relations do not have a threshold condition for significant transport (EH and WC), thus representing the stochastic nature of sediment transport [*Einstein*, 1950] and two relations account for hiding effects (AM and WC).

The model applies only to equilibrium conditions, and the conservation equations are simplified considering that at equilibrium the river profile is unchanging in time. Application of the EH transport relation then allows for explicit or analytical solutions to the streamwise decrease of both the channel slope and the bed surface mean grain size. For the remaining load relations we find implicit solutions that we solve numerically (Appendix A).

Application of the model to a base case (Appendix A) illustrates how the equilibrium slope adjusts to a change in the single representative water discharge such that the flow velocity (and so bed shear stress) and surface grain size do not change, as the flow velocity and surface size are set by the requirement of transporting the imposed mixed-size load downstream (Figure 2).

The analysis confirms existing empirical relations by *Rubey* [1952] and *Lane* [1955] in that the equilibrium channel slope, holding other variables constant, increases with decreasing water discharge, with increasing sediment load, and with increasing mean grain size of the supplied sediment. In particular, channel slope scales with the reciprocal of the water discharge (Figure 2), which confirms the relation by, for instance, *Lane* [1955].

The equilibrium bed surface gravel content and so the bed surface mean grain size depend predominantly on the ratio of gravel size to sand size and the local gravel content in the transported load.

4. The Role of Abrasion

In the graded state an increase in the abrasion coefficient corresponds to an increase in both profile concavity and downstream fining (Figure 3). For the load relations that account for hiding effects (AM and WC) we find a somewhat finer bed surface as the gravel is more mobile (larger exposure) and so the bed surface does not need to coarsen as much to be able to transport the supplied gravel downstream. For the load relations without a threshold for significant transport (EH and WC), the abrasion-induced streamwise increase in bed

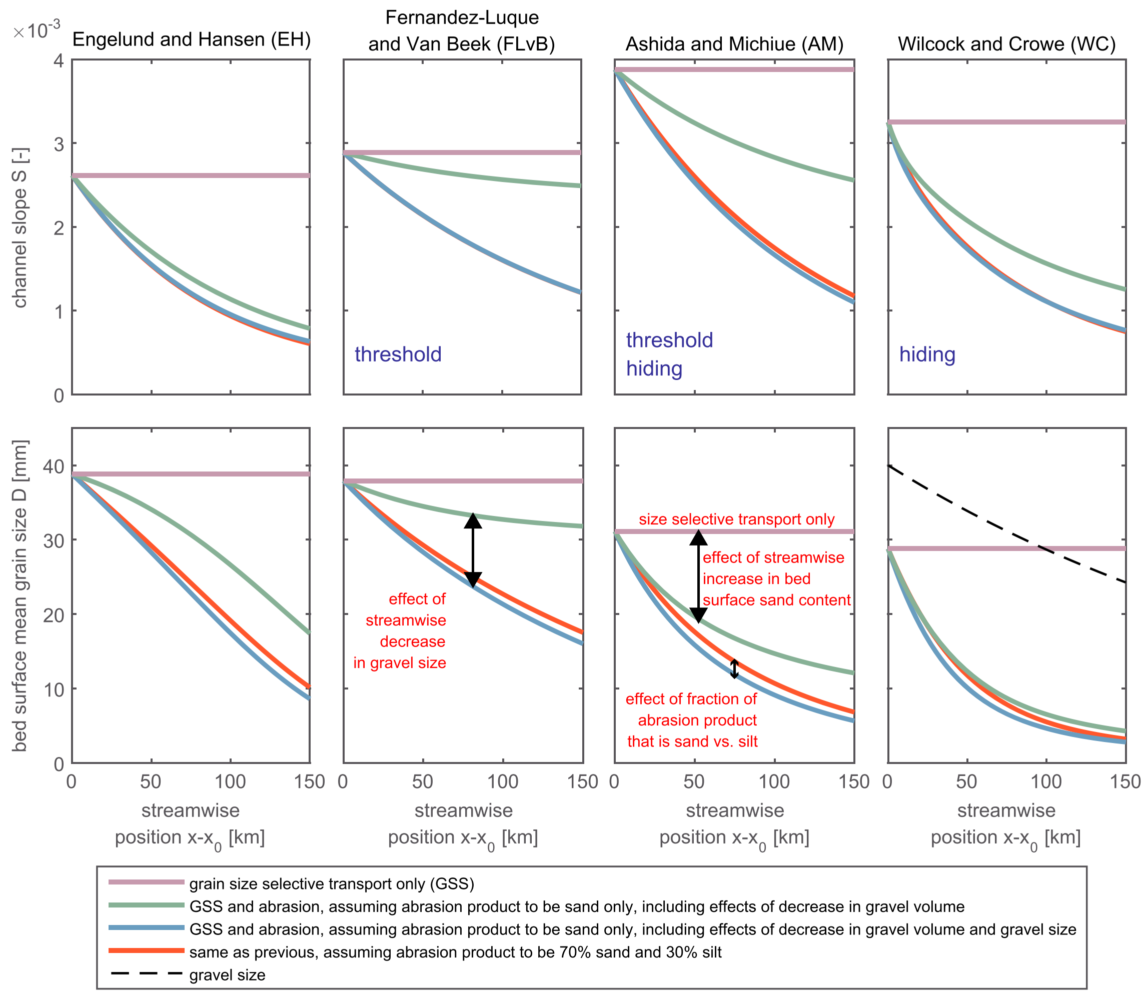

Figure 3. Effects of grain size selective transport and abrasion on the equilibrium channel slope and bed surface mean grain size in the base case. The dashed black line indicates the gravel size that accompanies the blue and red lines.

surface sand content dictates profile concavity, whereas for the threshold-based load relations the decrease in gravel size is of similar importance. The effect of the fraction of abrasion product that is sand (rather than silt) regarding the equilibrium profile is relatively small.

Figure 4 shows how without calibration the predicted data for the base case fall well within the range of field data on natural streams collected by *Morris and Williams* [1999a, 1999b]. Yet predicted data do not cover those field data that are characterized by large concavity and downstream fining values, as the predicted data (1) do not include the effect of tributaries and (2) cover graded conditions, whereas a considerable part of the field data is likely governed by ungraded conditions and associated stronger concavity and downstream fining values.

5. The Role of Tributaries

River profiles usually deviate from smooth curves due to lateral water and sediment input from hillslopes and tributaries and spatial changes in channel width or bed friction [*Snow and Slingerland*, 1987]. Such effects are captured by the proposed formulations provided that one imposes the lateral input of water, gravel, and sand at specified locations, as well as streamwise changes in channel width and friction. Here a restriction arises due to the fact that the model deals with one gravel fraction (though governed by a downstream decrease in size) and one sand fraction only: a tributary can only add sand or gravel of the same grain size as locally present in the trunk stream.

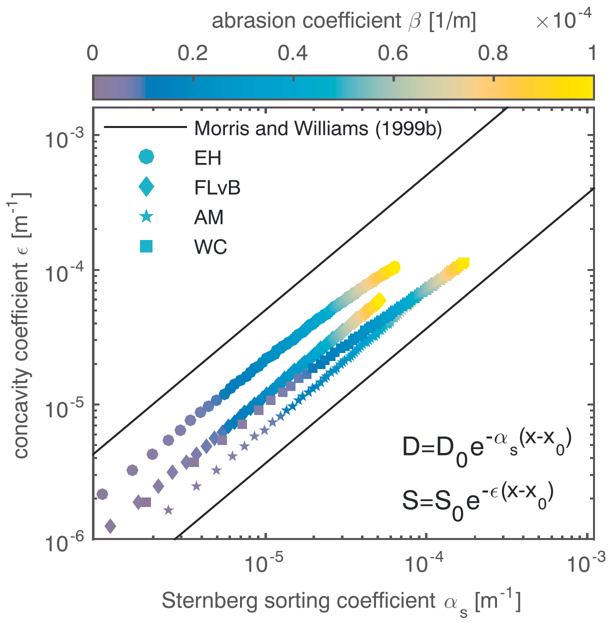

Figure 4. Predicted Sternberg sorting coefficient α_s versus the profile concavity coefficient ϵ (see the exponential relations in the graph), for the base case under a varying abrasion coefficient β. Predicted data are based on an exponential fit as the formulations are not purely exponential. The two black lines indicate the envelope of data on natural streams collected by *Morris and Williams* [1999b].

As the equilibrium bed surface texture does not depend on the water discharge, clear water tributaries add to concavity but not to downstream fining (Figure 5). Depending on their gravel and sand supply rates, tributaries can also result in a streamwise increase in the trunk stream slope as added sediment discharge counteracts the effect of added water discharge (Figures 5b and 5c).

For simplicity we assume in our examples that bed friction does not vary with surface grain size. It is difficult to prescribe how friction changes with streamwise position as moving downstream the finer bed surface sediment likely reduces skin friction yet increases form drag due to bed forms. The effects of multiple grain size fractions beyond one gravel fraction (with a streamwise decrease in size) and one sand fraction require further study using a numerical model but are not expected to change the main findings of the current analysis.

The proposed formulations can straightforwardly be combined with empirical relations between, for instance, drainage area and channel width or water discharge [e.g., *Hack*, 1957; *Tucker and Bras*, 1998] to find drainage area based relations for the graded channel slope and bed texture.

6. Conclusions

We present a model based on physical conservation laws that accounts for abrasion and grain size selective transport and provides a solution to the graded profile of alluvial rivers. In the absence of subsidence, uplift,

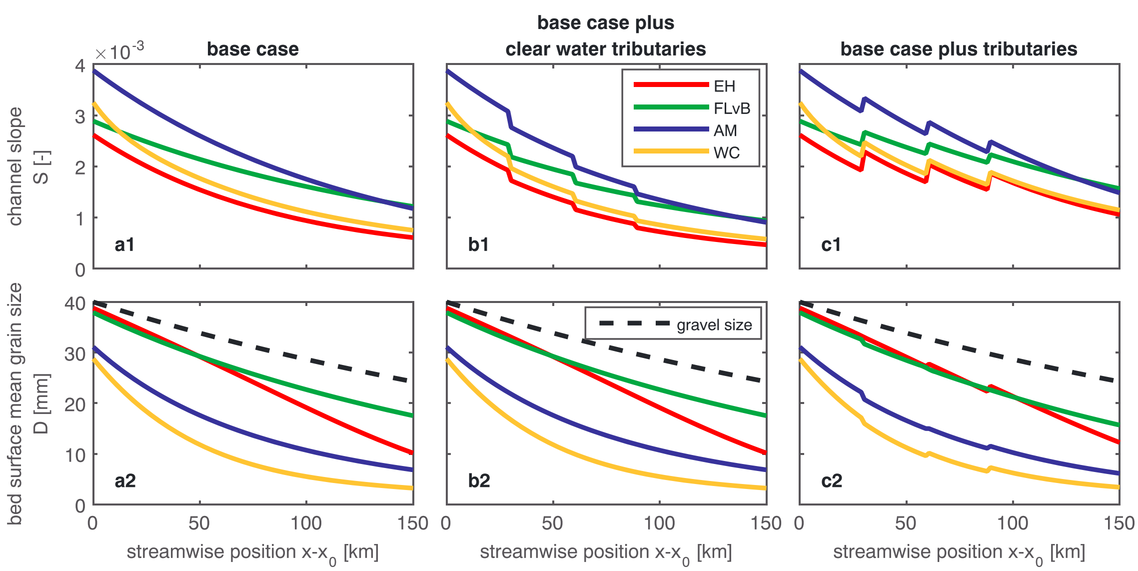

Figure 5. Effects of tributaries on the equilibrium channel slope and bed surface mean grain size for (a) the base case, (b) the base case plus clear water tributaries, and (c) the base case plus tributaries delivering gravel and sand. At the tributary locations (at 30, 60, and 90 km from the upstream end) we impose a stepwise streamwise increase in gravel load of $2 \cdot 10^{-3}$ m^3/s, in sand load of $2 \cdot 10^{-3}$ m^3/s, in water discharge of 20 m^3/s, and channel width of 20 m.

and sea level change, the graded profile is characterized by a mild profile concavity and downstream fining. The controlling parameters are the yearly averaged upstream gravel and sand loads, the water discharge, and the base level. This graded river state is a special case of topographic steady state in which the vertical crustal speed tends to zero. More complex steady state patterns can be obtained when the vertical crust speed or changes in relative base level are accounted for.

In a graded river the channel slope has adjusted such that it provides the flow velocity and bed surface texture required to transport the mixed-size load supplied from upstream. The representative water discharge does not affect the equilibrium representative flow velocity, bed shear stress, and bed surface texture. As such, in the graded state clear water tributaries increase profile concavity but not downstream fining. The bed surface texture required to transport all of the mixed-size load supplied from upstream results in the commonly present mobile armor.

Imposing channel width and reach length, we find one solution to the graded state. In natural streams channel width and curvature can adjust to changing conditions and in that case there likely is a range of equilibria for which the channel is able to transport the load supplied from above.

Appendix A: The Mathematical Model

The one-dimensional model of the graded river is based on the *Saint-Venant* [1871] equations and the conservation equations for the mass of two distinct sediment modes (gravel and sand) at the bed surface [*Hirano*, 1971]. Following *Cui* [2007], we add abrasion-related terms representing the loss of gravel to sand and silt to the Hirano equations. To this end, we introduce a nondimensional abrasion coefficient, β^*, to describe the fraction of gravel volume V_g lost per unit strike [*Parker et al.*, 2008; *Chatanantavet et al.*, 2010]:

$$\left(\frac{\Delta V_g}{V_g}\right)_{\text{strike}} = -\beta^* \tag{A1}$$

Since a gravel particle strikes the bed once per unit distance saltation or hop length (L_{salt}), we find

$$\frac{\mathrm{d}V_g}{\mathrm{d}x} = -\beta V_g, \qquad \beta = \frac{\beta^*}{L_{\text{salt}}} \tag{A2}$$

where β is the abrasion coefficient. For values of the abrasion coefficient we refer to *Kodama* [1994]. Based on the equivalent spherical gravel size D_g for a gravel particle of volume V_g, the gravel size at x, D_{gx}, decreases exponentially with x [e.g., *Parker et al.*, 2008; *Chatanantavet et al.*, 2010]:

$$D_{gx} = D_{g0} e^{-x_D^*} \tag{A3}$$

where the subscript 0 indicates the upstream end of the considered reach and $x_D^* = 1/3\beta(x - x_0)$. The number of gravel particles that strike the bed per unit area and time equals E_{salt}/V_g where E_{salt} is the volume entrainment rate of saltating gravel particles per unit bed area. The loss of volume per strike, including striking gravel particle and stricken gravel particles, now equals $(1+F)\beta^* V_g$. Thus, the rate of volume loss of bed and bedload gravel particles per unit area and time is

$$\frac{E_{\text{salt}}}{V_g}(1+F)\,\beta^* V_g = (1+F)\,\beta q_g \tag{A4}$$

as the gravel transport rate is given by $q_g = E_{\text{salt}} L_{\text{salt}}$, where q_g denotes the volume of transported gravel per unit width and time. The right-hand term in equation (A4) is the gravel loss term that we add to the gravel-related Hirano equation. The by-product of each strike is sand and silt, and the latter moves as wash load. Consequently, the rate of added sand volume per unit time and area resulting from gravel abrasion equals $k_{ss}(1+F)\,\beta q_g$, where k_{ss} is the fraction of abrasion product that is sand (versus silt).

Under graded conditions and a steady water discharge, the Saint-Venant equations reduce to $Q_w = \mathrm{BUH}$ and the backwater equation, where Q_w denotes the water discharge, B channel width, U flow velocity and H is the flow depth. The backwater equation is further simplified to the normal flow equation as spatial changes are very small. In the absence of subsidence and uplift the conservation equations for the gravel and sand mass reduce to

$$\frac{dq_g}{dx} = -\kappa\beta q_g \tag{A5}$$

$$\frac{dq_s}{dx} = k_{ss}\kappa\beta q_g \tag{A6}$$

where q_s denotes the volume of transported sand per unit width and time, and $\kappa = 1 + F$ which implies $1 \leq \kappa \leq 2$. For simplicity we assume that κ is constant, which implies that κ (just as β) is independent of x. Equation (A5) and the boundary condition $q_{g0} = p_{g0}q_0$ then yield

$$q_g = p_{g0}q_0 e^{-x^*_\kappa} \tag{A7}$$

where $x^*_\kappa = \kappa\beta(x - x_0)$ and p_g is the gravel content in the load ($p_g = q_g/q$ where $q = q_g + q_s$). Then, from equations (A6) and (A7) and the boundary condition $q_{s0} = q_0 - p_{g0}q_0$, we find

$$q_s = q_0 - p_{g0}q_0 + k_{ss}p_{g0}q_0\left(1 - e^{-x^*_\kappa}\right) \tag{A8}$$

The formulations in equations (A7) and (A8) are generic (i.e., not specific to a load relation), and the gravel and the sand load need to fulfill these relations in the graded state.

We now set equations (A7) and (A8) equal to the gravel and sand transport rates predicted using the fractional form of the load relations by *Engelund and Hansen* [1967] (EH), *Fernandez-Luque and Van Beek* [1976] (FLvB), *Ashida and Michiue* [1972] (AM), and *Wilcock and Crowe* [2003] (WC). Except for the EH load relation, we find implicit solutions to the graded river profile that we solve numerically. The fractional form of the EH transport relation reads

$$q_i = F_i \frac{G}{D_i} U^n, \qquad G = \frac{0.05C_f^{3/2}}{(Rg)^2} \tag{A9}$$

where $n = 5$, the subscript i indicates either gravel or sand, F_i the volume fraction content of size fraction i at the bed surface, R the submerged density, and g is the gravitational acceleration. Using the EH relation, we find the following explicit relations for channel slope, S, bed surface gravel fraction, F, flow depth, H, and flow velocity, U:

$$S = \frac{C_f B^{2/n}}{gQ_w}\left(\frac{D_s\mu}{G}Q_0\right)^{3/n} \tag{A10}$$

$$F = \frac{D_{gx}}{D_s\mu}p_{g0}e^{-x^*_\kappa} \tag{A11}$$

$$H = \frac{Q_w}{B^{4/n}}\left(\frac{G}{D_s\mu}\frac{1}{Q_0}\right)^{1/n} \tag{A12}$$

$$U = \left(\frac{D_s\mu}{G}\frac{Q_0}{B}\right)^{1/n} \tag{A13}$$

where Q_0 denotes the sediment discharge at the upstream end of the reach and μ is given by

$$\mu = 1 - p_{g0} + k_{ss}p_{g0}\left(1 - e^{-x^*_\kappa}\right) + \frac{D_{gx}}{D_s}p_{g0}e^{-x^*_\kappa}. \tag{A14}$$

The equilibrium mean grain size of the bed surface sediment, D, is then computed from

$$D = D_{\text{ref}}2^{\psi}, \qquad \psi = F\psi_g + (1 - F)\psi_s \tag{A15}$$

where $D_{\mathrm{ref}} = 1$ mm, ψ the equilibrium mean grain size on ψ scale, and ψ_g and ψ_s are the gravel and sand size on ψ scale ($\psi_g = \log_2(D_{gx}/D_{\mathrm{ref}})$ and $\psi_s = \log_2(D_s/D_{\mathrm{ref}})$, this definition makes the grain size on ψ scale conveniently nondimensional).

The stationary equations (A5) and (A6) hold when applying *Hirano* [1971] under graded conditions as well as for vertically continuous mass conservation models [e.g., *Parker et al.*, 2000; *Blom and Parker*, 2004]. The resulting solutions to the graded river profile are therefore not restricted to applying the *Hirano* [1971] conservation equations.

Parameters of the base case are as follows: g=9.81 m/s^2; R=1.65; D_{g0}=0.04 m; D_s=0.001 m; $\beta = 10^{-5}$ m^{-1}; $\kappa = 1.5$; $C_f = 0.008$; $Q_w = 200$ m$^3/s$; $Q_0 = 1 \cdot 10^{-2}$ m$^3/s$; $p_{g0} = 0.75$; $k_{ss} = 0.7$; $B = 100$ m.

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