

## 21<sup>st</sup> IAHR International Symposium on Ice "Ice Research for a Sustainable Environment", Li and Lu (ed.)

Dalian, China, June 11 to 15, 2012 © 2012 Dalian University of Technology Press, Dalian, ISBN 978-7-89437-020-4

# Novel Ice Induced Vibration Testing in a Large-scale Facility: Deciphering Ice Induced Vibrations, Part 1

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Interest to understand dynamic ice structure interaction and to develop theoretical simulation models has increased significantly lately. However, the background data on both the full-scale or even scale-model measurements is scarce and in some cases incompletely documented. To cover this pitfall the Norwegian University of Science and Technology initiated a project DIIV, Deciphering Ice Induced Vibrations, at the beginning of 2011. The objectives were defined to design and manufacture an adaptable test set-up, to conduct scale-model tests in the EUHYDRALAB Transnational Access Program, and analyze results. Systematic variation of the most important structure and ice parameters allows finding out their contribution to the dynamic ice-structure interaction, and especially to the frequency lock-in vibrations. The measurement program included a test matrix with varying ice mechanical properties, ice velocity, structure waterline diameter, surface roughness, structure compliance, natural frequencies, and as a fresh approach - an analogy to vortex shedding testing - a forced sinusoidal ice crusher movement superposed to the ice velocity. This paper describes the design of the test set-up, instrumentation and calibration, conducted tests, and general findings. Deciphering Ice Induced Vibrations Parts 2 and 3 in this 21<sup>st</sup> IAHR Symposium on Ice present more detailed analysis procedures and some initial results.

#### 1. Introduction

The history of published and analyzed events due to ice-induced vibrations in offshore structures covers soon 50 years. The unpublished history of such events in minor structures is evidently much longer. The first scientific approach to understand ice-induced vibrations was applied to Cook Inlet oil production structures (Peyton, 1968a, b, c; Blenkarn, 1970), thereafter to aids-to-navigation structures in the Gulf of Bothnia (Määttänen, 1975; Engelbrektson, 1977), and also in case of bridge piers (Neill, 1976; Montgomery et al., 1980). Bohai Sea oil and gas production structures have experienced ice-induced vibrations now for 30 years and have been scientifically studied by Xu and Wang (1986), Yue and Li (2003). Intuitively tall and slender offshore structures are considered to be most prone for dynamic ice-structure interaction but in 1986 an unexpected ice induced vibration occurred on a wide and blunt structure, the Molikpaq (Jeffries et al., 1988). Recently, within last 5 years, massive oil production platforms offshore Sakhalin have also encountered dynamic ice-structure interaction. In most of the above cases some full-scale measurement data has been harvested. However, due to difficulties and cost of full-scale measurements, the extent and quality of data has pitfalls, especially in ice properties and ice velocity.

The ice action problems encountered in field initiated scale-model testing to find out the dependencies and importance of ice and structure mechanical properties on the ice-structure dynamic interaction (Määttänen, 1979; Toyama et al., 1983; Tsuchiya et al., 1985; Sodhi, 1991; Izumiyama et al., 1994 and 1997; Kärnä, 2003; Huang et al., 2007). These tests have provided more basic understanding than available limited full-scale data, and have resulted in developing numerical models for dynamic ice-structure interaction. But the problem how to scale up from laboratory tests to full-scale needs still further research.

Ice-induced vibrations in structures occur whenever dynamic ice actions occur. For the three different ice velocity ranges where dynamic action is encountered the ISO 19906 ice load code defines each a different structural response type. Intermittent while ice is failing repeatedly against the structure but the dynamic response has decayed to insignificant level before next ice load build up starts, continuous brittle in which the ice loads varies randomly, and frequency lock-in (FLI) where the ice failures repeat at a frequency close to a natural frequency of the structure. The design of structures for the intermittent or continuous brittle ice failure modes is a straightforward engineering practice. But what is the physical background, and how to model the frequency lock-in has a long history and even a dispute among the ice researchers (Peyton, 1968a, b, c; Blenkarn, 1970; Michel et al., 1977; Neill, 1976; Sodhi, 1988; Määttänen, 1978; Kärnä, 1992).

NTNU initiated a research project Deciphering Ice Induced Vibrations (DIIV) at the beginning of 2011. The objectives were set to quantify both the ice and the structure physical parameters that control ice-induced vibrations during level ice crushing. The project group designed an instrumented test rig and conducted scale model tests in the facility ARCTECLAB at HSVA in late summer 2011. This paper is the first of the three papers in this IAHR Symposium on Ice and describes the measurement project, test rig design, instrumentation, conducted tests, and overall view of achieved results. The data analysis is still at its initial stage. The other two papers delve into data analysis theory and methods and present first results.

### 2. Scale Model Test Objectives

A test structure for ice-structure dynamic interaction research in scale-model tests shall be able to produce scaled but similar dynamic response to ice loads as a real full-scale structure in nature. Basically this would require the geometric similitude of the real structure. In nature winds or sea currents drive large ice fields to move while in laboratory ice is usually stationary, and the carriage pushes the structure in the ice tank. If the carriage pushes ice against the stationary structure there can be problems with ice sheet integrity and it will be more difficult to make changes to structural properties during testing. As the frequency lock-in vibration is the most severe dynamic loading, the geometric similitude of the model structure is less important than the modal similitude. This means that natural modes of the full-scale and model-scale structures have similar shapes – especially modal amplitudes at ice action points – and have frequencies scaled according to laws of similitude.

During the first phase of dynamic ice-structure interaction cycle the ice load builds up and the displacement velocity of the structure at ice action point is close to ice velocity. As ice failure starts the load reduction is almost instantaneous and the structure begins to accelerate against the ice edge initiating the crushing phase. The real structure has normally many natural modes each reacting to load changes at the speed of its natural frequencies. The higher modes can follow faster the fast ice edge disintegration during the crushing phase, initiate sooner a new contact, and start sooner the next ice load build-up phase. Therefore a single degree of freedom oscillator model cannot follow the fast reduction of ice load and will distort the ice-structure interaction history. Hence the model structure shall have similarly scaled natural modes. Such a model structure – regardless of apparent missing geometric resemblance to the real structure – is having modal similitude and correctly scales dynamic behavior. The natural mode amplitude at ice action point is a measure how easily the mode in question can be excited by the ice load.

The DIIV model structure, Fig. 1, was designed nominally to have about 1:8–1:10 scale ratio with modal similitude to a bottom founded generic offshore structure. The intended model ice thickness of 60–80 mm corresponds to 0.50–0.80 m thick level ice. With three different waterline cylinder diameters - 100, 220 and 400 mm - the model structure can cover a real structure diameter range from 0.8 to 4 m. The model structure support stiffness as well as tuning masses both close to waterline and at the top could be easily adjusted to alter both the natural frequencies and mode shapes.

In addition to the traditional freely developing ice-induced vibrations the DIIV objectives included the Tu Delft proposal of forced sinusoidal movement superposed to the constant ice velocity. This technique originates from vortex induced vibration testing where it has successfully predicted frequency lock-in vortex induced oscillation parameters. By varying the ice velocity, forced movement frequency, and its amplitude allows to determine ice velocity ranges that are likely to induce frequency lock-in vibrations, DIIV paper 3 (Hendrikse et. al, 2012).

## 3. Dynamic Scaling

Scaling laws define how the effect of control parameters changes while changing from full scale to model scale or vice versa. Different physical processes have different scaling laws. In ice tank model testing Froude scaling is a standard procedure. However, it is not applicable for ice

crushing studies due to almost completely missing wave and gravity loads. Dynamic icestructure interaction is mainly based to continuum mechanics: structural dynamics and structural strength, c.f. Cauchy scaling. Even though hydrodynamic and inertia loads are present during crushed ice extrusion and the test structure vibrating partly in the water, their contribution is insignificant compared to level ice crushing loads.

The dynamic equations of motion both in the full and model scales (index m), for a single degree of freedom vibration system as well as for any Eigen mode dynamic behavior of a multi degree of freedom system are

$$F = \kappa \sigma_c h D = kx + d\dot{x} + m\ddot{x}$$
<sup>[1]</sup>

$$F_m = \kappa \sigma_{cm} h_m D_m = p n^2 \kappa \sigma_c h D = k_m x_m + d_m \dot{x}_m + m_m \ddot{x}_m$$
<sup>[2]</sup>

Eqs. [1] and [2] can also be interpreted as matrix equations for a multi degree of freedom system. Here the scale ratio for geometry (size) is n, and for ice crushing strength p (differing from Cauchy scaling), displacement at ice action point is x, with its time derivatives presented by dots above x, while k, d, and m are stiffness, damping and mass. The aspect ratio effect  $\kappa$  does not change with scale if aspect ratio h/D (ice thickness/pile diameter) follows the geometric similitude.

The scale dependence for the parameters in dynamic equations of motion can be deduced term by term from Eqs. [1] and [2]. If the full scale and model structures are manufactured from same material, considering the beam bending as the dominant deformation mode, and geometric similitude in ice crushing, the scale dependence for most important parameters is

stiffness	k ~ n	damping	$d \sim n^2$	
displacement	$x \sim F/k \sim np$	mass	$m \sim n^3$	[3]
velocity	$\dot{x} \sim \omega x \sim p$	frequency	$\omega \sim n^{-1}$	
acceleration	$\ddot{x} \sim \omega^2 x \sim n^{-1} p$	time	t ~ n	

The model dynamic model of the structure can be easiest handled by using modal similitude instead of full geometric similitude.

#### 4. Model Structure Realization

The test structure design was made with an eye on utilizing the experience and infrastructure of ARCTECLAB. The model structure was fixed to the carriage main beam, Fig. 2a. The instrumentation was connected directly to the carriage data logging system. Access to change the structural configuration from one test to another could be accomplished easily and rapidly to minimize changes in model ice properties with time.

The towing carriage is very stiff and massive in relation to the test pile. The elastic modes in the test pile consist both the pile bending and support fixture spring action. Two elastic members – both in bending mode – control the support stiffness of the test pile. First the vertical support beams are fixed to the carriage main beam edges. Then the test pile itself is attached at the center of flat spring steels at the ends of vertical beams. Tuning masses can be added both to the top and

bottom of the test pile, Fig. 1a and 2a. Reducing/increasing the vertical support spacing changes the flexible length of the flat spring steels and makes the structure stiffer/softer. Together with tuning masses a wide range of natural frequencies and different modal shapes can be invoked:

- Design ice load 20 kN
- Stiffness for ice load 0.4 ... 1 kN/mm (3.3 kN/mm with the reinforcing truss)
- Waterline deflection 10 ... 24 mm/10 kN (3 mm with reinforcing truss)
  - Tuning mass 0 ... 240 kg, top and/or bottom
  - $1^{\text{st}}$  natural frequency 5 ... 20 Hz (2x240 ... 0 kg)
  - 2<sup>nd</sup> natural frequency 11 ... 25 Hz (2x240 ... 0 kg)



Figure 1. a) Test structure free vibration set-up

b) Forced movement set-up

For the forced movement tests the same test pile was reconfigured. The vertical supports spacing was reduced to minimum for maximum stiffness, the upper flat spring steel was removed, a reinforcing truss was installed according to the Fig. 1b, and the electrical actuator rod end was attached directly to the pile at the height of removed upper flat spring steel. The actuator produces sinusoidal movement that causes opposite direction movement at the waterline. Thus the main pile is rocking as a rigid body in reference to the lower attachment point ball end bearings.

The test structure was designed to have at least two distinct natural modes to better simulate a real bottom founded full-scale structure dynamic behavior. The adjustable support stiffness by flat spring steels free span together with tuning masses allows a wide natural frequency and mode shape control. Natural mode amplitude squared at waterline is directly proposal to the ice load potential to excite frequency lock-in vibrations (Blenkarn, 1970). In the DIIV test structure this is well achieved, Fig. 2b.



Figure 2. a) Test structure installed on the carriage b) Typical natural mode shapes

## 5. Instrumentation

The objectives of instrumentation were to measure ice load, ice crushing cylinder displacement and velocity from the measured data. The installation of direct ice load measuring load cells for different size of cylinders was problematic. And in anyhow, the true ice force with load cells will have contribution due to the cylinder and pile inertial loads. Therefore an indirect method based on frequency response function was chosen. Dynamic response is measured at different locations of the test pile. The dependence on the true ice load can be solved by making dynamic calibration with a known (= measured) load function at the location and direction of the real ice load. Then the frequency response function can be calculated and used to solve the true ice load history. Whenever the test structure mechanical properties are changed the dynamic calibration have to be done for each different configuration.

The test pile was furnished with two laser displacement transducers, one accelerometer, and two x-direction and one y-direction strain gauge bridges. From each but y-axis strain gauges the x-axis true ice load can be solved. Hence there is a five times redundancy to calculate the true ice load history. However, the accelerometer accuracy is poor for static or close to zero frequency ice load components. The ice velocity is that of the ice tank carriage velocity. The schematic locations of transducers are depicted in Fig. 1a.

Tactile sensor was installed now for the first time on a cylindrical structure to study the crushing pressure distribution at different phases of an ice-structure dynamic interaction cycle. The objectives were to find out both the vertical and circumferential pressure distributions, and crushing pressure synchronization. The used sensor sensing area  $416 \times 156$  mm<sup>2</sup> is divided into a

grid of 52 columns and 44 lines, a total of 2288 individual sensing elements. It covered 216 degrees frontal sector around the 220 mm cylinder, e.g. wider than the active circumferential contact area. The sensor was protected from direct ice abrasion by a 0.5 mm thick aluminum foil, and it was waterproofed by silicon. Fig. 3 gives an example of recorded line like contact during crushing, color code in kPa. Note that only a sector about 180 degrees is loaded from the total sensor width of 220 degrees.



Figure 3. Contact line during crushing.

#### 6. Dynamic Calibration

A dynamic calibration for frequency response function based measurements can be accomplished with different loading types. The criterion to select an acceptable method is simple. The calibration load shall provide enough energy to the structure at the whole expected frequency range of the to be measured ice load. Typically ice load has a significant average - zero frequency - load level with superposed sudden variations due to ice failures that require higher frequencies. Impact hammer is easy and good for higher frequencies but by nature often inadequate for the zero frequency. The now used step relaxation calibration load fulfills both the low and high frequency criteria.

The step relaxation dynamic calibration was – and had to be – conducted to each different structural configuration, e.g. for each changed structural stiffness, tuning masses, and different ice crushing cylinder. The dynamic calibration also observes naturally the added mass of water due to the submerged part of the pile lower end cylinder that crushes ice. In praxis the step relaxation load was measured at the ice action point by a calibrated load cell. Glider aircraft towing 10 kN safety fuses were loaded to the direction of ice load by steel wire rope connected to a manual lever and pulleys. By slowly increasing the load, the safety fuse attached to the load cell breaks suddenly, and a pure step relaxation load is introduced. This dynamic calibration method was predictable, fast and easy to use. Dynamic calibration data analysis mathematical background and examples of results are presented in DIIV paper 2 (Nord et al., 2012).

## 7. Model Ice

The tests were carried out in the 78 m long and 10 m wide HSVA Large Ice Tank. The water depth is 2.5 m with a deep section of 5 m at the end of the basin ( $10 \times 12$  m). Six columnar-

grained level ice sheets were produced in NaCl doped water (water salinity -6.8%) by seeding. During ice growth process at  $-22^{\circ}$ C air temperature, water saturated with air is charged under high pressure through perforated tubes fixed at the bottom of the ice tank. When the pressure drops, tiny air bubbles with 200–500 µm diameters rise upwards and are embedded in the growing ice sheet. Due to air inclusions the ice gets an opaque color and behaves more brittle when it fails, Evers and Jochmann, 1993.

After ice growth phase the ice tank air temperature was raised in the DIIV tests depending on target model ice properties to -10--3°C. Test temperature decision is normally based on cantilever index bending strength tests, and seldom on more time consuming uni-axial compressive strength tests. Vertical and horizontal thin sections of model ice under polarized light indicate the fine columnar crystal structure, Fig. 4.





Horizontal thin section (horizon 13 mm from top)

Figure 4. Vertical and horizontal ice thin sections with air inclusions

To verify the ice uni-axial crushing strength dependence on strain rate ice samples were cut just after one test at different locations in the ice tank (21, 32, 43, and 55 m) and uni-axial compressive strength tests were conducted at different strain rates Fig. 5a, in the cold lab at -2°C, the actual ice sheet centre thickness temperature during the tests. The data indicates an expected average decreasing trend with strain rate. However, the strength reduction is only about 15 %, significantly less, only about a quarter what has been measured e.g. with Cook Inlet sea ice at wide temperature range from -5 to -20°C, Fig. 5b (Peyton, 1968a, b, c), or in laboratory (Wu. et. al., 1976).

The model structure vertical support beams were bolted to the towing carriage main beam in the carriage framework as shown in Fig. 2. The carriage pushing capacity is 50 kN, speed range 1–3000 mm/s. During the tests both constant acceleration and constant velocity tests were conducted up to 350 mm/s. Low constant acceleration provides gradually increasing pseudo constant ice velocity. In praxis this gives time enough for over ten test structure natural mode vibration cycles to build up frequency lock-in vibration amplitude at almost constant ice velocity. This way the possible frequency lock-in velocities can be detected and chosen for more detailed testing at constant velocity. The traditional testing using step increase for constant velocities can miss a possible frequency lock-in velocity between the steps.



**Figure 5.** Ice strength vs. strain rate, a) Model test ice  $-2^{\circ}$ C, b) Sea ice,  $-5 - -20^{\circ}$ C (Peyton, 1968a, b, c).

#### 8. Completed Tests and General Findings

DIIV scale-model tests utilized six ice sheets, a total of over 360 m model structure crushing in ice. Four ice fields were used for free ice-induced vibration testing and two for the forced movement testing. Ice thickness varied from 40 to 85 mm. To promote frequency lock-in vibrations the plan was to conduct tests with as cold and strong as possible ice without exceeding test rig design limits. As model ice bending strength had a poor correlation on crushing strength we adopted a procedure to make first a short load level calibration run. Based on this the structural configuration was changed if needed to avoid overloading. A total of 13 velocity sweep tests with different natural mode combinations were conducted at 2 or 3 mm/s<sup>2</sup> constant acceleration, starting at about 20 mm/s and stopping to about 300 mm/s velocity. The raw measurement data showed only indications on occurring frequency lock-in. Three such cases were chosen for constant velocity test runs.

Forced movement test were conducted at varied constant ice velocities with superposed sinusoidal velocity that made the ice crushing cylinder velocity vary from zero to double the ice velocity. At 100 mm/s ice velocity it was learnt that the forced movement actuator was slipping, e.g. the movement centre position shifted. The open loop velocity control could not maintain the pure sinusoidal movement as the direction of actuator movement changed from pushing to pulling while the ice action remained to the same direction. The actuator control parameters adjustments helped only insignificantly due to the available short time to avoid model ice properties changing. The purchase of a closed loop displacement control unit to the actuator for the next test day was not either possible before DIIV time slot in the facility ARCTECLAB would end. Therefore the rest of forced movement tests were completed at 10 mm/s ice velocity increments and limiting the maximum ice velocity below 100 mm/s. Regardless of this some slipping in the actuator occurred which makes the data analysis more complicated.

At the initial stages of data analysis the general observations of the DIIV scale model tests confirm the known velocity dependencies in ice crushing against a vertical structure at low and high ice velocities. The velocity range between these extremes indicates frequency lock-in vibrations but not as clearly as what have been measured or observed earlier in full-scale ice action events and in scale-model tests. The added damping and added mass maps based on forced movement data analysis support these observations (Hendrikse et al., 2012). More understanding is expected after the energy balance integrations through full vibration cycles have

been realized. Both these steps pave way towards improved numerical model predictions on frequency lock-in vibrations. Even though the small reduction of model ice strength with ice velocity made frequency lock-in vibrations marginal in these tests, the data can be used to calibrate numerical models and to predict possibility and the expected level of sustained frequency lock-in vibrations at larger ice strength reduction with increasing velocity.

The test structure fulfilled the expectations, simple structure, two distinct natural modes, easy to change structural configuration, and also adaptable to forced movement testing. Test structure attachment to the carriage was simple, same as connecting the instrumentation to the carriage data logging system. The model ice was repeatable even though index bending strength correlation to ice crushing strength does not always provide good estimate to the crushing strength. Ice failure modes against the cylinder and crushed channel through the ice field had similar appearance that has been learnt in full scale. The routine to carry through tests helped to accomplish up to two ice fields in a week with multiple structural configurations.

## 9. Conclusions

In the NTNU DIIV scale model measurements controlling parameters were varied systematically to find out the physical background on the rise of frequency lock-in vibrations during dynamic ice-structure interaction. The varied properties of the structure were its stiffness, natural modes, the diameter of the cylinder crushing the ice, cylinder surface roughness, and the effect of forced movement. The model ice thickness and crushing strength could be varied only minimally. The most important - ice velocity - was varied to sweep through the frequency lock-in velocity range from that causing low ductile ice failure mode to high brittle ice failure mode.

The model test structure was designed to have two distinct natural modes that could be adjusted at relatively large range to present a typical bottom-founded offshore structure with scale ratio about 1:10. The structure configuration change was designed fast and easy. The structure could also be configured to be very rigid while providing forced sinusoidal velocity superposed to the constant ice velocity.

The model test structure instrumentation was based for indirect ice load measurement with five times redundancy. Indirect method requires dynamic calibration for each different structural configuration to measure the relevant frequency response function that is used to solve the true ice load function by de-convolution.

Constant low acceleration velocity sweep – pseudo constant ice velocity – was used to locate frequency lock-in ice velocity ranges. Due to saline model ice less brittle behavior than in full scale no persistent frequency lock-in was recorded. Many short time ice load histories, over 20 repeating vibration cycles, were captured. The yet unfinished data analysis will delve into the energy balance during such vibration cycles to find out if there has been enough energy input into the structure to overcome the internal damping and other related energy losses to keep FLI alive. The forced movement data analysis attacks the same problem in a different way by plotting stability charts for parameter combinations to produce FLI.

#### Acknowledgments

The work described in this publication was supported by the European Community's 7<sup>th</sup> Framework Programme through the grant to the budget of the Integrated Infrastructure Initiative HYDRALAB-IV, Contract no. 261520. The authors would like to thank the Hamburg Ship Model Basin (HSVA), especially the ice tank crew, for the hospitality, technical and scientific support and the professional execution of the test programme in the Research Infrastructure ARCTECLAB.

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