

Wheel-rail interaction at short-wave irregularities

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A.M.D.G.

Voorwoord

Dit proefschrift bevat het resultaat van enkele jaren onderzoek naar diverse aspecten van dynamische wiel-rail interactie ter plaatse van korte oneffenheden in het spoor. Het onderzoek is verricht binnen de vakgroep Railbouwkunde van de faculteit Civiele Techniek en Geowetenschappen van de Technische Universiteit Delft.

Mijn dank gaat uit naar een aantal personen die op diverse wijzen hebben bijgedragen aan de totstandkoming van dit proefschrift.

Dr. Andrei Metrikine, bij wie ik ook afgestudeerd ben, heeft mij in de onderzoeksfase voorafgaand aan dit promotieonderzoek – het betrof een driejarig onderzoek naar de dynamica van slab-track spoorssystemen voor hogesnelheidslijnen en treingeïnduceerde bodemtrillingen – met groot geduld vele nuttige kneepjes van de wiskunde en liefde voor het moeilijke vak golfdynamica in continue systemen bijgebracht. Ing. Nick van den Hurk (Volker Rail) heeft dit universitair onderzoek, in een tijd van economische krapte, zonder concrete tegenprestatie gefinancierd. Beiden ben ik zeer dankbaar voor hun bijdrage aan de totstandkoming van het voortraject van dit onderzoek. Tijdens dit onderzoek heb ik in alle rust de spoorse keuken kunnen verkennen en de bakens kunnen uitzetten voor het in dit proefschrift gepresenteerde onderzoek.

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Boven dit voorwoord en proefschrift schreef ik: ‘A(d) M(ajorem) D(ei) G(loriam)’. Het is een bekende variant op de lijfspreuk van de Benedictijner monniken, degenen die – onder dit motto – pionierswerk verricht hebben t.a.v. de civiele werken in onze Lage Landen: de ontginningen, inpolderingen, waterkeringen en dijken, zonder welke er van infrastructurele werken als spoorlijnen niet eens sprake kan zijn.

Sedert de Verlichting, eind 18^e eeuw, zijn persoonlijke noten en blijken van enthousiasme in een wetenschappelijke publicatie uit den boze geworden en dienen onderzoeksresultaten objectiviteit en reproduceerbaarheid te weerspiegelen. Echter, een voorwoord van een wetenschappelijk proefschrift mag nog altijd deze persoonlijke noten bevatten. Artikel 14.8 van het

vigerende promotiereglement getuigt hiervan expliciet – zij het dat het de nadruk legt op kaderstellende aspecten als ‘beknoptheid’, ‘zakelijke formulering’ en ‘gepastheid’. Paradoxaal genoeg is het voorwoord hierdoor juist vaak de meest gelezen passage van het proefschrift, en er is geen reden waarom dit proefschrift hierop een uitzondering zal vormen. Ik wil dan ook enkele van die noten persoonlijk aanslaan: samen vormen ze een majeur akkoord. De tweede wet van Newton, zij het in een wat vereenvoudigde vorm, heb ik als jeugdig broekje van mijn vader geleerd, en ik herinner mij levendig dat dat bijbrengen bepaald een ondankbare opgave was. De basis van de dynamica was daarmee echter voor mij gelegd. Vele avonden kwam ik in de loop der jaren laat thuis uit Delft, en dan stond mijn moeder erop steevast nog voor de inwendige mens zorg te dragen. De rest van mijn familie, broers en zussen, droeg bij aan het ontspannen van de boog tijdens de avondlijke en geestrijke bijeenkomsten in ons huiselijke zogenaamde ‘stamcafé’ of elders. Mijn dank gaat ook uit naar enkele trouwe vriend(inn)en, meer verwijderde familieleden en kennissen voor hun niet-aflatende bijdrage aan het peil van mijn motivatie gedurende al de afgelopen jaren van studie.

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Contents

Voorwoord	vii
Contents	xi
Nomenclature	xv
1 Introduction	1
1.1 General	1
1.2 Research context and background philosophy	3
1.3 Long-term track behaviour: the importance of energy management	5
1.4 Relevance and actuality of the research	13
1.5 Short-wave track defects; outline and focus of the thesis	15
References	22
2 Elementary types of rolling contact between wheel and rail	27
2.1 General	27
2.2 The contact discontinuity types leading to wheel-rail impact	30
2.3 The wheel-rail contact conditions in relation to track and wheelset deterioration	32
References	33
3 Model for dynamic wheel-rail interaction	35
3.1 Modelling and mathematical description	35
3.2 Solution	38
3.3 Elementary model to describe wheel-rail interface irregularities	44
3.4 The energy input into an irregular track	46
3.5 Model parameters	48

	<i>References</i>	50
4	Short rail irregularities with a broad-band spectrum - rail welds	51
4.1	<i>Introduction</i>	51
4.2	<i>The significance of rail welds in relation to track damage and deterioration</i>	53
4.3	<i>Rail weld geometry assessment: conventional methods and improvement possibilities</i>	60
4.4	<i>Simplified dynamic wheel-rail interaction modelling</i>	65
4.5	<i>Enhanced weld geometry assessment concept: the QI</i>	73
4.6	<i>Statistical properties of first derivatives of rail and rail weld measurements</i>	74
4.7	<i>The relationship between the weld geometry and the dynamic wheel-rail interaction; FEM simulations</i>	77
4.8	<i>The relationship between the weld geometry and the dynamic wheel-rail interaction; analytical investigation</i>	81
4.9	<i>Statistical properties of dynamic wheel-rail interaction at rail welds</i>	98
4.10	<i>The feasibility limit in standardization: high-speed norms</i>	102
4.11	<i>Application: the Dutch rail welding regulations (2005)</i>	105
4.12	<i>Geometrical rail weld assessment in practice, according to the QI-method</i>	108
4.13	<i>Considerations on rail welds in heavy haul, conventional and high-speed lines</i>	109
4.14	<i>Weld geometry, power input into the track and deterioration</i>	112
4.15	<i>Final considerations and conclusions</i>	116
	<i>References</i>	118
5	Wheel flats	123
5.1	<i>General</i>	123
5.2	<i>The stages of wheel flat development and classification</i>	126
5.3	<i>Space and time-domain analysis of a wheel with a flat, rolling on a rigid foundation</i>	127
5.4	<i>Mathematical problem formulation for a rigid foundation</i>	130
5.5	<i>The consequences of the finite contact elasticity and starting plasticity and wear</i>	134
5.6	<i>The influence of the horizontal velocity of the wheel mass and the static axle load</i>	141

5.7	<i>Parametric analysis of the behaviour of the wheel-rail impact force</i>	150
5.8	<i>The stages of wheel flat development: mathematical descriptions of the trajectories and frequency-domain analysis</i>	153
5.9	<i>Dynamic wheel-rail interaction for different wheel flats; modelling and simulation results for a non-rigid track</i>	159
5.10	<i>Discussion; confrontation with reported results from the literature</i>	167
5.11	<i>Experiments with wheel flats: the equivalent rail indentation and its applicability</i>	171
5.12	<i>Experiments with wheel flats: the registration of wheel-rail contact forces</i>	174
5.13	<i>Final remarks and conclusions</i>	178
	<i>References</i>	179
	Summary	183
	Samenvatting	185
	Curriculum Vitae	187

Nomenclature

Latin

c	amplitude; control parameter of the trajectorial curvature for a stage III flat [m]
d	depth of a wheel flat [m]
d_{eff}	effective flat depth [m]
EI	rail bending stiffness
F, F_{dyn}	dynamic component of the wheel-rail contact force [N]
F_{max}	maximum value of the dynamic wheel-rail force or impact force [N]
F_{stat}	static wheelload [N]
f	frequency [Hz]
g	gravitational acceleration [ms^{-2}]
G	transfer function [-]
h	rail height [m]
$H(..)$	Heaviside function
i	imaginary unit
k	wavenumber [m^{-1}]
k_1	primary suspension stiffness [N/m]
k_{f}	rail foundation stiffness [Nm^{-2}]
k_{H}	Hertzian wheel-rail contact stiffness [N/m]
$l, l_{\text{II}}, l_{\text{III}}$	length of a wheel flat, in stages I, II, III [m]
m, m_{w}	unsprung wheel mass [kg]
n	counter
p	vertical linear momentum of the wheel mass [kgms^{-1}]
P	power [W]
R	wheel radius [m]
s	Laplace parameter ($s = \sigma + i\omega$)

t	time [s]
u	vertical wheel gravity centre displacement [m]
V	train speed [m/s]
V_{crit}	critical train speed (with respect to contact loss) [m/s]
w	vertical rail deflection [m]
x	horizontal coordinate [m]
y	vertical coordinate of the gravity centre of the wheel [m]
z	vertical coordinate of the rail geometry (chapter 4); vertical distance between the wheel centre and the wheel-rail contact point [m] (chapter 5)

Greek

$\delta(..)$	Dirac delta-function
ε	linear strain [-]
η	loss factor, accounting for the track elasticity [-]
κ	curvature [m^{-1}]
κ_{eff}	effective curvature [m^{-1}]
λ	lifetime extension factor [-]
θ	angle in the rail geometry along the surface
ρA	distributed rail mass [kg/m]
μ	validation constant [-]
τ	time-scale, related by the train velocity to the degree of wear along the wheel flat [s]
ω	angular velocity of the wheel [rads^{-1}]

1 Introduction

1.1 *General*

Let us consider a train, running at a constant speed along a perfectly homogeneous and flat track on a perfectly homogenous half-space in the gravity field. In a reference system moving along with the train (a convective reference frame), in such an ideal case there are no vibrations in the steady state. This is not true in the transient state, and therefore in reality a non-zero damping in the system is required to reach this steady state. Moreover, this is not true in a stationary reference system, in which the steady-state response field passes as a function of time. Therefore, the steady motion of a passing train is a cause of track and soil vibrations, but not of train vibrations. The steady-state response field may be subsonic, transonic or supersonic, depending on the ratio of the train velocity to the critical wave speeds of the track and the soil structure. In both latter cases, the response field is no longer confined to the vicinity of the moving axle loads and double axially-symmetric in the absence of damping, but spatially unbounded in the absence of damping and only symmetric with respect to the track (a Mach cone is formed on the surface).

In reality, a railway track is not a perfectly homogeneous system in longitudinal direction. Except for the case of continuously embedded rail, the rail is supported periodically by the sleepers. In some types of slab-track, the track is built up from slab sections. For any non-zero train speed, this kind of periodicity yields a periodical excitation of the train-track system. This induces not only track and soil vibrations in a stationary reference system, but also train vibrations in a convective reference frame. The same holds for more randomly distributed track properties, like the soil's Young's modulus.

Therefore, periodicity or random distribution of physical track properties is a second source of soil vibrations, and a first source of train vibrations.

Apart from the fact that no track is perfectly homogeneous, no track is perfectly flat or straight. A third source of dynamic train-track interaction is therefore the presence of geometrical imperfections in the interface between these systems; this interface concerns both the rails and the wheels. The geometrical wavelength spectrum of the wheel-rail interface yields, in combination with a non-zero train speed, a frequency spectrum which excites the train-track system in a convective reference frame. The result is, in a convective reference frame, train vibration along with the generation and radiation of a propagating wave field in the soil. In a stationary reference frame, the same wave field can be registered, but with a frequency-shift for all components: the well-known Doppler Effect. Therefore, irregularities in the wheel-rail interface are a third source of soil vibrations and a second, major source of train vibrations.

As regards the track, the initial railway track geometry has a given wavelength spectrum for a newly-built system. This spectrum is generally measured before operation of a new line and should satisfy given conditions (see e.g. UIC-codes 513 [1.1] and 518 [1.2]). As an example, the geometry of the Dutch high-speed line HSL-South was registered by the Eurailscout UFM-120 and indirectly (via passenger comfort measurement) by a Thalys TGV-PBKA before operation [1.3]. This registration type however commonly does not include the short-wave band (waves shorter than approximately 7 m). This aspect will be returned upon in the following of this chapter.

In the previous, the fact was ignored that not only the track properties in general have a given periodicity, but that the same is valid for the loading: the configuration of axles, bogies and carriages of a train generally leads to a given periodicity of the loading in a stationary reference frame. This is an additional source of track and soil vibrations in a stationary reference frame. However, it is remarked that this is not an essential feature of dynamic train-track interaction, like the other ones. Assuming linear elasticity, the total wave field generated by the passing train, in a stationary reference frame, is just found by superposition of the wave fields by the individual loads with a given time lapse.

1.2 *Research context and background philosophy*

In the previous section, the presence of an initial spectrum of irregularities in the wheel-rail interface was mentioned, along with the fact that in practice this spectrum has to satisfy predefined conditions, at least in the waveband larger than approximately 7 m. A very important aspect of track deterioration, as well as an important motive of maintenance, is the growth of long-wave track irregularities, especially on ballasted tracks. These long- (or longer-) wave irregularities lead to poor vehicle ride and discomfort for passengers. The question can be posed how these waves grow into the track. It can be answered readily, when keeping in mind the different sources of train vibration which were outlined in the previous section: physical inhomogeneities and geometrical irregularities.

In general, short-wave irregularities are not geometrically assessed on new tracks or systematically detected and monitored on existing tracks. This aspect will be discussed more extensively in section 1.4. When the train, which is a multi-body mass-spring system with typical natural frequencies, passes these irregularities, it starts vibrating. The running and simultaneously vibrating train causes fluctuating vertical track forces with different frequency components along the track. This causes the longer-wave irregularities to grow into the track. The above process can be illustrated through the simple case of an insulated rail joint. The – initially straight – joint is loaded by a passing axle. This causes the joint to deflect. This, on its turn, causes a non-zero dynamic axle load component. The result is a local settlement of the ballast bed after some time. Once this is the case, high impact loads occur, the mechanism of which will be discussed in chapter 2 of this thesis. These very high loads lead to a locally rapidly worsening track geometry as well as a high level of train vibrations, which may (when the irregularity has grown to a certain length) even reach the car-body with its low resonance frequency. These vibrations, with different frequency components, which also depend on the actual train speed, lead to varying vertical track forces along the track, which induce non-uniform settlements. This is an explosively worsening situation of track deterioration. This example clearly illustrates how long-wave track irregularities start growing from local short-wave geometrical defects.

Fig. 1.1 shows the relationship between typical wavelengths of some physical or geometrical inhomogeneities which are present in the railway system and the resulting excitation frequencies as a function of the train

speed. Further, the frequency area where significant dynamic amplification in the vehicle-track interaction is mainly of interest as well as the area in which vibration hindrance and train comfort are of main interest is indicated. In the first area, which is obviously related to the short-wave regime of track irregularities, also the phenomena of structural damage to the train-track system and progressive track deterioration enter. This can be easily explained by the fact that the energy of the high-frequency vibrations and waves is absorbed and dissipated in the components of the train-track system. The low-frequency vibrations and long waves cannot be easily absorbed by structural components; they therefore determine the dynamic behaviour of the train and are radiated from the system into the environment. A clear transition between both frequency regimes does not exist, but the aim of Fig. 1.1 is to illustrate a general trend as well as the role of the short-wave contribution in the track geometry spectrum. From a Life Cycle Cost (LCC) perspective, the train-track system is most efficiently managed in such a way that the source of deterioration of the system, if not eliminated, is minimized. This implies that the short-wave contribution of the spectrum of the track geometry must be taken into consideration in an effective track assessment, such as pointed out in e.g. [1.4].

Finally, it is often disregarded that the geometrical track quality also has a significant influence on the economy of the energy-management in the train-track system. If the train-track system is dynamically excited, the system response, consisting of vibrations or propagating waves, is always related to energy loss: radiated travelling waves are energy carriers, and damping of vibrations is related to energy dissipation in structural components (which causes deterioration). Based on the fact that the railway system is a conservative system for which the energy conservation law is valid, this energy is ultimately supplied by the power supply, the overhead wire. One could speak, in analogy to concepts as 'aerodynamic drag' or 'rolling drag', of 'dynamic drag' on tracks with a bad geometry. Although no measurements are available from the literature, it may be therefore postulated that trains running on a deteriorated track have a significantly higher power consumption than trains running on a new track.

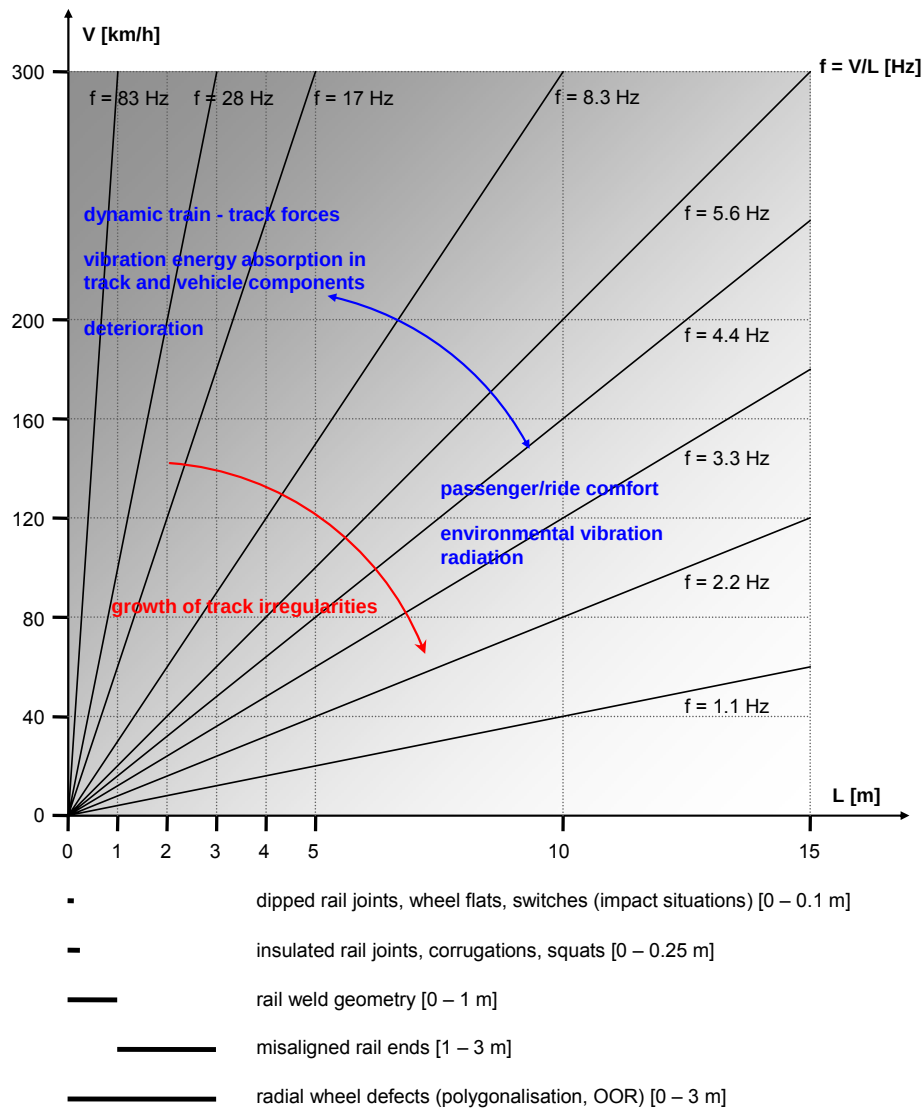


Fig. 1.1 Frequency diagram: coupling of wavelength L and speed V for typical wavelengths (short length-scales) in the train-track system

1.3 Long-term track behaviour: the importance of energy management

In the previous section, the relationship between the energy management in the railway system and long-term track behaviour was touched upon. The subject will be discussed in more detail in this section.

The railway track design is conventionally based on the load spreading principle. A train, running on a railway, is supported in the wheel-rail contact patches by an area with a magnitude in the order of square centimetres. This, in combination with the high train-loads, leads to extremely high contact stresses. The stresses in the wheel-rail contact are among the highest known in

engineering, leading for example to shake-down of the surface layer of newly installed rail [1.5]. These extremely high stresses should be reduced gradually before reaching the subsoil, which should be able to carry the train loads without significant settlements on both a short and a longer term. Therefore, in general two principles are applied: the stiffness of the main 'layers' in the track system decreases from the top to the bottom, and the contact surface between these layers increases in the same direction. The railpad, which is basically used to isolate the rail from the substructure in the high-frequency regime, forms an exception to the decreasing stiffness principle and basically plays a role in the load transfer path. Apart from load spreading, other aspects such as stability against buckling also play a role in the traditional track design.

These conventional track designs focus principally on the *short-term track behaviour*, which is governed by the *stress state* in the system. They do not focus on *long-term track behaviour* and processes such as degradation, which are governed by the *energy management* in the system: without energy dissipation, deterioration of a structural system or its components is excluded, and when this dissipation occurs in an uncontrolled manner, damage is a direct result. The absence of the latter focus is confirmed by the fact that it is common practice to design railway tracks in a *static condition*, accounting for dynamic effects with a dynamic amplification factor for the loads. In such a condition, it is impossible to account for effects induced by the *dynamic*, i.e., both time-dependent and repetitive character of the loading and the response, such as energy dissipation within a load cycle as a function of time, and its distribution over the different track layers. An exception is the design of individual track components, which may be subject to e.g. fatigue calculations. Given the increasing importance of both Life Cycle Cost (LCC) concepts and availability of especially high-speed lines in track management, it is important to be aware of the deterioration process and influencing factors already at the track design stage, and to aim at an optimum track behaviour on both the short and the long term. In the following, this is discussed in some more detail. Fig. 1.2 shows the different layers in a conventional ballasted railway system. Other configurations are possible (baseplates, baseplate pads, slab tracks), but the same principles apply to these variations.

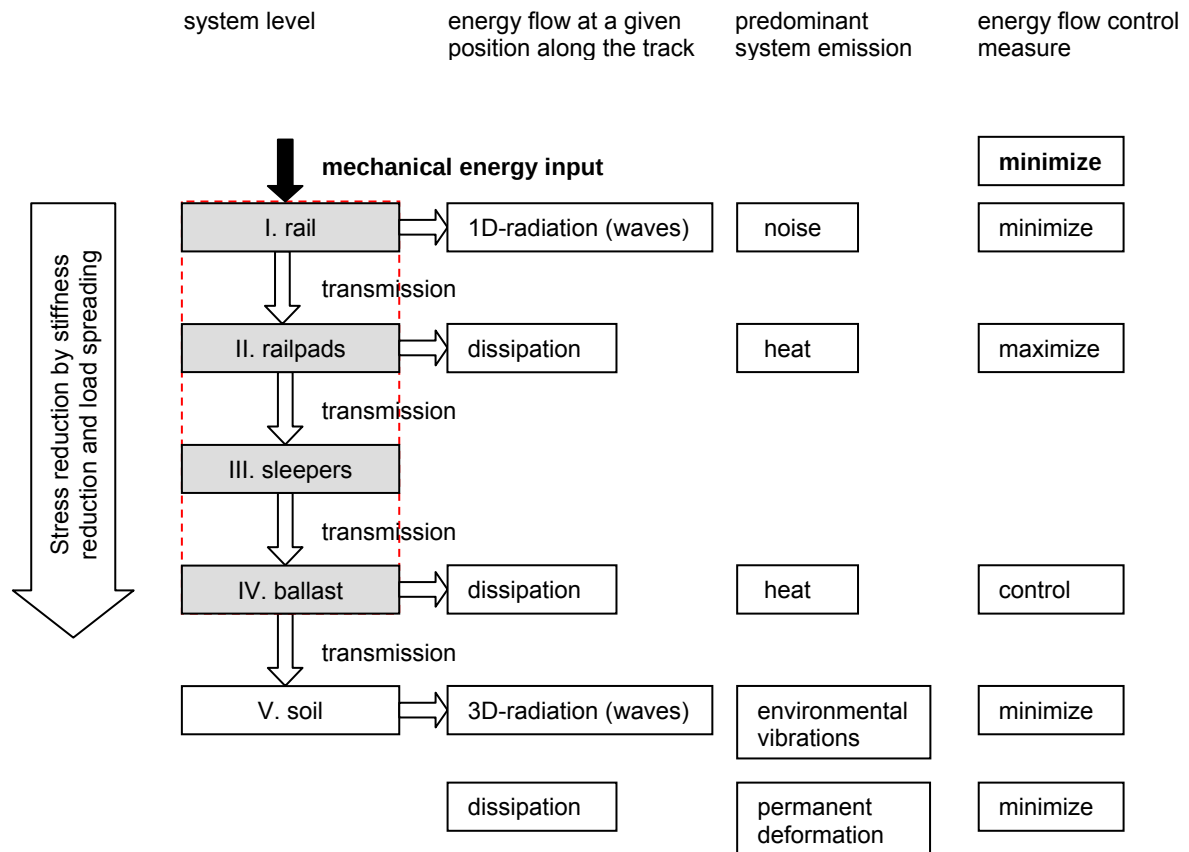


Fig. 1.2 Different levels in a conventional railway system; the double functional principle of load spreading (static, short-term design) and energy management (dynamic, long-term design)

The first column in Fig. 1.2 shows the successive layers of the rail system that is being loaded externally, leading to a given mechanical energy input. The second column specifies the predominant ‘path’ of the energy that is not being transmitted into the next layer. The third column specifies the form in which the energy emission from the system is manifested. The basic principle of the flow chart is the energy conservation law and the fact that the railway is a conservative system. The last column of the chart specifies the measure that should be taken on each level, with a double objective: a) to reduce damage and deterioration and thus to reduce maintenance and extend the lifetime, and b) to avoid hindrance during this lifetime. The mechanical energy input into a purely elastic railway track by a vertical constant axle load travelling without friction at a constant sub-critical speed is zero (the travelling load does not perform work). This is no longer true for a track with non-elastic or physical damping properties. However, the largest contribution to the energy input into a railway track is due to the track irregularities, leading to a time-variant

track loading in a convective reference frame. Therefore, the main control measure displayed in Fig. 1.2 is to minimise the energy input into the total railway system at the wheel-rail interface. The issue of mechanical energy input at this interface and its minimization will be reverted to in sections 3.4 (modelling and mathematical description) and 4.13 (computational and quantitative examples).

It can be observed from Fig. 1.2 that the general or final objective in the energy management is conversion of a given amount of mechanical energy into heat. Transmission of mechanical energy occurs either to the next system level or to the environment where it is dissipated, but in the latter case it causes hindrance (rail: noise, soil: settlements, vibration hindrance). Therefore, the aim is to design the system in such a way that the conversion of the input energy into heat occurs at the proper level and in a controlled manner: the mechanical energy must be dissipated into heat without material damage. Keeping this in mind, in the following each of the system levels will be discussed into some more detail. The path of the energy through the system will be discussed and the control objective for the energy will be formulated, given a predefined energy input coupled to a given track condition.

I) The mechanical energy that is being put into the rail over a given length or at a given position in a given time interval does not remain at this position, but vanishes in time. This occurs due to two mechanisms: wave propagation (waves are energy carriers) into adjacent rail sections and radiation into the underlying structure and the surrounding air (they establish the *flux* over the boundaries of the spatially bounded rail segment), and physical damping. Wave radiation from the rail into the surrounding air becomes manifest as rolling noise (acoustic waves), which may cause hindrance in inhabited areas. It can be combated by application of tuned rail dampers, which absorb and dissipate the energy of the high-frequency vibrations, and by adjusting the railpad properties. The wave propagation type that causes energy transmission into adjacent rail sections depends on the wavelength. For wavelengths that are much larger than the rail height and frequencies that are coupled to these wavelengths via the dispersion relationship (the ‘macro-level’), the rail can be considered as a 1D beam, exhibiting bending and also shear deformation in bending and shear waves. These waves are in general almost fully elastic. For wavelengths that are shorter and the corresponding frequency range (the ‘micro-level’), the rail behaves as a 3D medium. This continuum consists of inhomogeneous

material (suffering e.g. from non-metallic inclusions and non-perfect microcleanliness) and has an anisotropic steel crystalline matrix. Stress waves propagating from the rail surface into the material are diffracted and refracted at all internal inhomogeneities. This may be considered as an important source of progressive rail damage, as can be illustrated on the basis of a simple example from continuum dynamics. When a rod, with free-free boundary conditions, is being excited at one end by an impact load, a longitudinal stress wave with a constant amplitude travels through the rod. At the end of the rod, the stress wave is fully reflected, and during the reflection the stress amplitude doubles. Basically, something similar happens at the interfaces of internal rail material inhomogeneities or defects, such as voids or micro-cracks. At these interfaces the amplitudes of the arriving high-frequency stress-waves are amplified upon reflection and refraction, thus causing a rapid damage progress. It must be realised that especially in the work-hardened rail top layer, the dislocations tend to concentrate at the borders of the crystals or grains, from where the crack nucleation starts. The damage progress (crack propagation, non-fully reversible deformation, accumulation of plastic strain followed by saturation) dissipates mechanical energy and is, together with internal material or particle friction for all wavelengths, referred to as physical damping.

II) It is clear from the above that, in order to avoid progressing rail damage, the railpad should absorb and dissipate as much input energy from the rail as possible in as short a time duration as possible, especially in the high-frequency regime (corresponding to the wavelengths that 'perceive' the rail as a 3D continuum). In current design, the functionality of the railpad is defined from the stiffness point of view: the railpad should distribute the contact load over the sleepers and isolate the rail in the high-frequency regime. In the long-term design perspective, a double functionality could be specified in a general sense of the system layer between the rail and the sleepers: an elastic and a damping functionality. The elastic functionality can be met in terms of material constitutive properties and geometrical properties (one may think of ripples or studs but also of internal voids). The damping functionality can be met in terms of material properties (e.g. the good damping properties of natural cork-rubbers, due to their inhomogeneous material structure, are known [1.6]) and by optimizing the free or contact surface. Systematic attention to these aspects in the railpad design is lacking. To illustrate this, for instance, ripples or studs should not be applied in the

contact surface with the rail. The reason is maximization of the contact area through which the mechanical energy is transferred from the rail to the railpad, and through which the heat is transferred again after transformation into the rail, in order to prevent pad heating. It is remarkable that current railpad specifications, even for slab tracks, do not specify any damping properties, and industrial measurements are often limited to maximum frequencies in the order of 8 Hz, although some scientific publications go up to several hundreds or even 1000 Hz [1.7-1.9] and recently measurement results have been published up to 2500 Hz [1.10].

A physical limitation of the formulated requirement is given by the fact that railpad heating may alter its mechanical properties and even lead to burning. Another limitation is given by the fact that specific high-frequency wave modes in the rail may be confined to e.g. the rail head or web, and thus be virtually isolated from the pad. A further physical limitation of the formulated requirement is given by the geometrical railpad properties. The maximum wavelength that can be integrally and thus efficiently 'trapped' and absorbed can be estimated as the smallest railpad dimension in the horizontal plane. This explains why embedded rail systems are observed to suffer less from high cycle rail fatigue defects: the continuous viscous-elastic foundation can absorb and dissipate the vibratory energy of a larger spectral wavelength band than the conventional discrete railpads.

III) The input energy that travels downward and is not being dissipated by the railpads is transferred to the sleepers. On a macro-scale sleepers can be considered as rigid masses, which therefore cannot dissipate energy. On a micro-scale however the sleeper, like the rail, must be considered as a bounded 3D dispersive medium, in which wave propagation and energy dissipation can occur. It can even radiate acoustic waves into the environment. Energy dissipation within a concrete sleeper leads, apart from particle friction, almost always to structural damage such crack initiation and propagation. It must be remembered that reflection and refraction of internal stress waves lead to amplification of tensile stress amplitudes, which the concrete, unlike the steel, is not able to withstand, especially at the boundaries between the granular matrix and the bonding material. From this viewpoint, the traditional wooden sleeper has a better performance. This is confirmed by the railway practice to apply wooden sleepers on locations where the power input into the track has a particularly significant contribution in the high-frequency regime, such as insulated rail joints. It is indicative that when

concrete sleepers are applied at these positions, cracks tend to start precisely at the bolt holes (see Fig. 2.2). These holes act as stress concentrators: when a high-frequency stress wave arrives and is reflected from the hole boundary, its amplitude basically doubles. Since stress is related to strain and energy is strain-integrated stress, this implies a repetitive significant ‘energy pulse’, which cannot be accommodated elastically by the material and becomes manifest as a rapidly growing crack from the hole surface.

Based on the previous, the requirement in practice should be that the railpad must absorb and dissipate the part of the power spectrum for which the sleeper may behave as a 3D medium, and not transmit this to the sleeper. This frequency regime is again related to the structural sleeper dimensions.

IV) The part of the energy that is not being dissipated in or radiated from the top layers, is transmitted to the ballast bed. Conventional ballast beds absorb a large part of the vibratory energy input into railway systems, due to the high particle friction. However, this energy absorption is mainly uncontrolled: the largest part is not converted into heat in closed load-displacement cycles but leads to ballast settlement and deterioration in the form of abrasion by shearing and breaking of ballast stones, which reduces the particle friction. This problem becomes in particular manifest at the contact area between the bottom of concrete sleepers and the ballast bed. Vibratory energy that is transmitted to the ballast can only be transmitted by compressive stresses: apart from the effect of preloading by the passing train axle, the hard contact between the bottom of the sleeper and the ballast stones cannot withstand tensile stresses. Especially for high frequencies this may lead to repetitive contact loss and contact recovery by impact. This is confirmed by the fact that in practice the interface between concrete sleepers and ballast is often found to be filled with crushed concrete and ballast after a given tonnage. A good solution to this deterioration problem by uncontrolled energy dissipation is the application of an extra control layer: resilient sleeper shoes or undersleeperpads [1.11]. These pads allow for tensile stresses (basically a reduction of the pre-compressive stresses) in the interface between the sleepers and the ballast bed, and furthermore they dissipate the remaining part of the energy of the high-frequency vibrations that have penetrated to this level. Furthermore, they yield an increase of the effective contact area and a lateral ballast stabilisation. These effects reduce ballast accelerations and abrasive wear.

V) The part of the input power spectrum which passes through all track layers is transmitted to the subsoil. In practice it is important to minimize its energy, as it radiated from the track and may cause vibration hindrance in inhabited areas. On its path through the soil, part of the energy of the transmitted vibrations and waves is dissipated and may cause permanent settlements, as well as ballast contamination at the ballast – soil interface.

It has been concluded in the previous that in conventional ballasted tracks only at the level of the railpads energy is dissipated in the system without causing deterioration. This conclusion is even more valid for slab tracks. The long-term behaviour of railway tracks could therefore be improved by adding or improving dissipation mechanisms at other levels and adjusting the ‘path’ of the energy.

Figs. 1.3 through 1.5 show examples of energy dissipation (hysteresis) in cyclic loading of respectively rail steel, railpads and ballast.

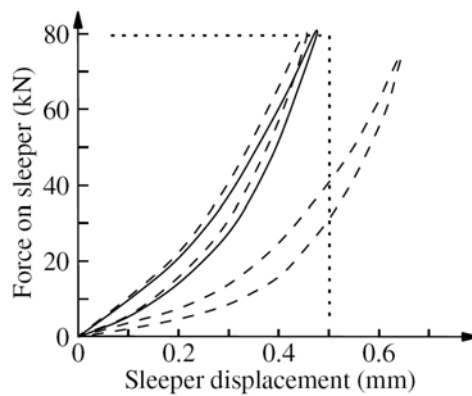


Fig. 1.3 Measured hysteresis in the force-displacement relationship of ballast (from [1.12])

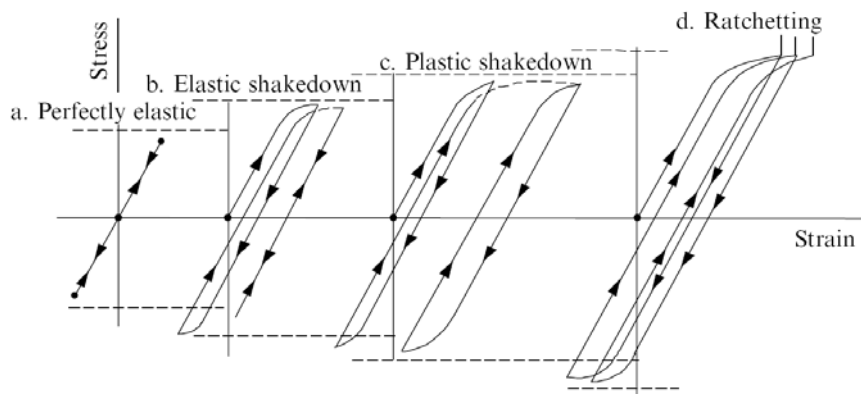


Fig. 1.4 Hysteresis in the stress-strain relationship (stress-controlled) of rail steel (from [1.13])

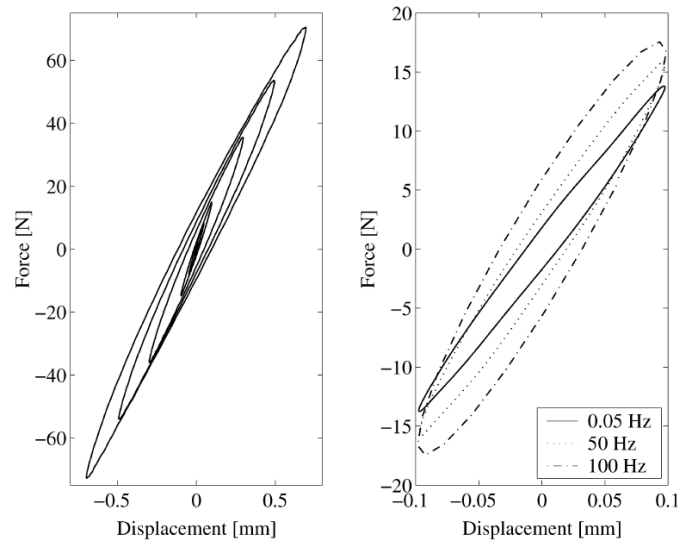


Fig. 1.5 Measured hysteresis loops for a rubber pad at 1 Hz with amplitudes ranging from 0.05 to 0.7 mm (left) and at different frequencies with an amplitude of 0.1 mm (from [1.14]).

1.4 Relevance and actuality of the research

In many countries throughout Europe during the last decade the railway companies have been privatized. The privatization has been accompanied by a split-up between the rail infra or track managers and the train operators. This split-up has led to a shift in the interests: the optimum performance (from both economic and dynamic points of view) for an integral or closed system is different from the optimum for one or more of its separate subsystems, on a lower system level. The track manager has become responsible for the rail infrastructure, which is a subsystem of the integral train-track system, and in this situation also the LCC perspective has become more important. Central questions that can be raised are: what is the optimum way to manage the rail infrastructure? Is it economically more advantageous to realize a given track geometry, and then to perform maintenance at a given interval, or is it better to realize a better initial geometry, and then to reduce the amount of maintenance (see Fig. 1.6)? What is a ‘good initial quality’ and which wavelengths should be included in the assessment? Which are the levels corrective maintenance should comply with? In this LCC optimization of the track sub-system the short-wave irregularities play a major role, as has been explained abundantly in the previous.

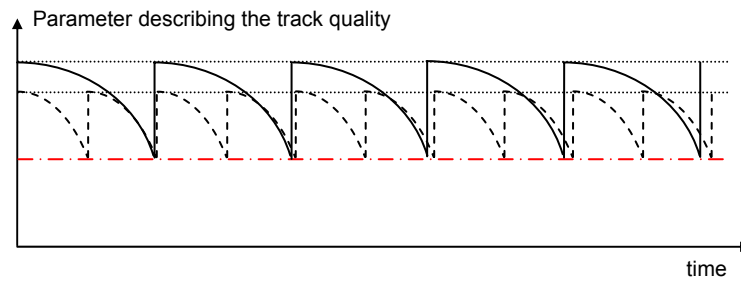


Fig. 1.6 Track quality as a function of time: high initial quality with large maintenance intervals and reduced initial quality with increased deterioration speed and smaller maintenance intervals

In the time before the privatization of the railways, the train-track system was considered as a ‘closed’ system, however without thoroughly considering the individual sub-systems and interactive processes between them. Most attention was given to corrective maintenance of the long wave irregularities in the track (tamping), as these are responsible for vehicle ride and passenger discomfort. The Sperling’s Ride Index, as a measure for train comfort (and which is currently replaced by the standardized ISO weighting) dates back to 1941 [1.15, 1.16]. Railways as a means of public or freight transportation then existed already for about one century, but train speeds were rather limited and no standardized methods for track assessment were in use. However, the growing mechanism of long-wave track irregularities, which is mainly the short-wave contribution to the track irregularity spectrum, was paid only little or non-systematic attention to in the time-span before the privatization.

A second reason why the subject has gained increased attention is the rapid expansion of the high-speed rail network throughout Europe and also outside Europe. These high-speed rail connections must have a very high availability to meet acceptable quality standards for this transport type. As an example, the value for the Dutch HSL-South was set to 99 percent [1.3], to be guaranteed under high penalties by the track manager. These developments ask for new maintenance concepts and the explicit adoption of a LCC perspective with a minimum of maintenance, in which the short-wave contribution to the track geometry spectrum must be paid due attention to (Fig. 1.7).

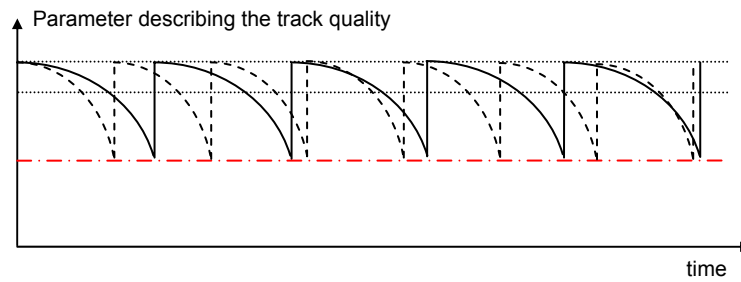


Fig. 1.7 Track quality as a function of time: given initial quality with (solid line) and without (dashed line) elimination of the short-wave contribution to the track geometry wave spectrum

1.5 Short-wave track defects; outline and focus of the thesis

An overview and classification of wheel damage is given in [1.17]. Here, the focus will be placed on the rail and wheel-rail interface geometry in the short wave regime, in as far as this geometry is important in the dynamic wheel-rail interaction in the vertical plane.

Wear of wheels and rails occurs during track use. With respect to the rail it can be basically distinguished into railhead top wear and railhead gauge face wear. On the wheel, hollow wear and flange wear is distinguished. Both wear types occur depending on the transversal contact conditions. The resulting imperfection is in general purely geometrical and occurs in lateral direction; in longitudinal direction in general (i.e. given a constant wheel-rail force) no irregularity results, and therefore it does not affect the dynamic wheel-rail interaction in the vertical plane.

When the rail or wheel steel wear rate is not high enough, the wheel and rail surface rejuvenation is not sufficient and the rail or the wheel surface may become subject to rolling contact fatigue (RCF). Commonly RCF is classified into surface-initiated RCF, sub-surface initiated RCF and deeply initiated RCF.

The question whether RCF is initiated at the rail surface or in the sub-surface is determined largely by the loading conditions. For purely normal loading, the stress state at the surface is nearly hydrostatic; the maximum stress and first yield occur below the surface. Due to tangential traction, the stress at the surface increases, and for a friction coefficient larger than 0.3 first yield occurs at the surface [1.18, 1.19]. In the crack propagation mechanism itself, it must be noted that the lamellar pearlitic steel microstructure is anisotropic and

therefore susceptible to crack branching or propagation after initiation; the crack path will follow the lamellae orientation until brittle failure occurs. This is consistent with the fact that bainitic steels are much less susceptible to crack initiation and flaking RCF than pearlitic steels, independent from the hardness [1.20].

Deeply induced RCF, often referred to as spalling (for wheels) or deep shelling (for rails), arises from deep material inhomogeneities (up to approximately 10 mm). In the wheel, it may also result from a white layer generation in the tread due to wheel slip. This white layer is particularly susceptible to crack formation. Under cyclic loading (which is often impact loading due to geometrical tread damage) a crack starts to propagate at the trailing edge along the interface between the white layer and the original steel matrix (which is the impact position [1.21, 1.22]), with spalling as a result [1.23, 1.24]. Shelling in rails is often related to the very strong longitudinal stress gradient at the rail surface [1.25-1.27] in combination with high traction forces. In longitudinal direction, in the railhead of new rails residual tensile stresses are present. These tensile stresses are a result of the roller straightening process after cooling down of the newly manufactured rail [1.27]. During the first cycles of axle loading and plastic surface deformation, residual pressure stresses build up at the surface, yielding a strong stress gradient. Moreover, the ductile wheel and rail material strain hardens, as a result of the increasing dislocation density in the atomic steel structure. This occurs both due to mechanical and thermal wheel-rail interaction (the temperature increases to approximately 300 °C for typical conditions and does not yield steel phase transitions), and these effects increase the actual elastic limit at the wheel-rail surface. The maximum value of the Hertzian pressure that causes, in repetitive loading, a material response in the purely elastic regime, after residual stress build-up and work hardening, is called the elastic shake-down limit. If further plastic deformation occurs in each loading cycle, ratcheting occurs, i.e. incremental accumulation of plastic strain. This is the basic RCF mechanism. RCF development is further influenced by non-metallic inclusions and the degree of the steel micro-cleanliness [1.28-1.36]. It is generally counteracted by rail profile reshaping through grinding [1.37-1.40]. RCF cracks that propagate from the rail surface, mostly from the rail gauge corner, are commonly denoted as head checks (Fig. 1.8). They may lead to shelling or flaking of the rail surface, and eventually to rail fracture.



Fig. 1.8 Head checks (left) and RCF damage on a wheel tread (right; from [1.27])

The residual circumferential or hoop stress in the wheel tread is compressive. An area of balancing tension exists in the lower rim. The compressive stresses are a result of rim quenching. This quenching causes the newly manufactured, hot austenitised wheel rim to change into a harder pearlitic microstructure along the surface, simultaneously introducing residual compressive hoop stresses at the tread [1.41, 1.42]. Shakedown upon loading occurs also along the wheel tread, but it does, for the reason explained in the previous, not introduce a strong stress gradient comparable to the one in the rail sub-surface, that may lead to shelling. In the case of wheels however it is important to realize that thermal loads on tread braked wheels may result into local stress reversal [1.43-1.45].

Squats (Fig. 1.9) [1.46, 1.47] are generally counted among the RCF damage family, though RCF then must be interpreted in a larger sense. RCF occurs in principle independently from the material properties, whereas in the case of squats the material properties (microstructural inhomogeneities and cleanliness) seem to play a role. There has been a rapid increase in the number of squat-like damage cases in recent years, especially since the introduction of high-speed lines. Much research work on the causes of squats has been performed, but the initiation mechanism has not been clearly understood yet. Different factors play a role, among which the axle load in combination with the vehicle traction (determining the wheel slip) and the rail material properties seem to be the most important ones. The latter contribution is confirmed by the fact that after removal of squats by rail grinding a de-colored spot remains visible, from which the squat and micro-cracks tend to propagate further into the railhead (Fig. 1.10).



Fig. 1.9 Squats on the rail surface in the contact band



Fig. 1.10 Branching micro-cracks beneath a ground squat at the rail surface

Corrugation [1.52-1.57] (Fig. 1.11), which can be divided into short-pitch corrugation (wavelengths typically ranging from 25 to 80 mm) and long-pitch corrugation, is a wavy wear pattern on the rail surface. Long-pitch corrugation on the rail is comparable to periodic out-of-round of the wheel circumference. An important generation mechanism contributing to short-pitch rail corrugation ('roaring rails') is the so-called 'pinned-pinned rail resonance' [1.56], although also the rail steel type and its mechanical properties play a role [1.57]. Longer corrugation wavelengths may occur due to other resonances in the train-track system. Particularly in rail systems with mono-use (without mixed traffic) and fixed speeds these resonances can manifest themselves easily in rail corrugation.



Fig. 1.11 Rail corrugation or rutting (centre)

Thermal wheel-rail interaction, e.g. wheel slide, can cause abrasive damage to the wheel and the rail; on the wheel this is commonly known as a wheel flat, and on the rail as rail burn. Moreover, it may cause steel phase transitions in the heat-affected zone (HAZ) along the wheel-rail interface. When the steel temperature reaches approximately $800 - 850\text{ }^{\circ}\text{C}$, the pearlitic steel is transformed into austenite. Rapid cooling (e.g. due to heat convection) may cause the austenite to transform into martensite, which is hard and brittle and favours crack pattern formation and crumbling away of material. Therefore, thermally induced material inhomogeneities on their turn mostly lead to geometrical imperfections. This issue was addressed already in the context of wheel spalling.

The rail, unlike the wheel, generally has geometrical imperfections that are manufactured or built-in as such. These are, apart from the fact that rails cannot be manufactured at an infinite length, due to external requirements such as signalling purposes, track switch and expansion possibilities. These include insulated rail joints (Fig. 1.12, left), expansion joints (Fig. 1.13) and transitions in switches (Fig. 1.14). The traditional bolted rail connections nowadays are largely replaced by metallurgical rail welds in continuously welded rail (CWR) (Fig. 1.12, right). A special form of a geometrical irregularity in the rail is the unstraightened rail end, remaining after straightening by the rollers after manufacturing and cooling down [1.58].



Fig. 1.12 Insulated rail joint with wide gap (left) and rail weld (right)



Fig. 1.13 Rail transition in an expansion joint



Fig. 1.14 Switch (left); RCF damage at the transition to the nose (right, zoom-in)

In the previous, different wheel-rail damage types or contact irregularities have been discussed. The short defects in the wheel-rail interface

are extremely important in view of their direct relation with both progressive rail and wheel damage and the deterioration rate of the track as a whole. This is explained as follows: the different frequency components of the power of the dynamic wheel-rail interaction force are dissipated in the vibrations of different track-vehicle system components, and consequently determine their deterioration rate. The issue was addressed systematically in paragraph 1.3. The energy of the excited high-frequency vibrations (in the order 1000 Hz) is partly dissipated in mechanisms such as irreversible deformation and crack propagation in the rail (Fig. 1.15) and the wheelset (e.g. shattering rims or propagating fretting fatigue at the end of the wheel seats [1.59]). The remaining part is either radiated from the system or migrates to lower frequencies and is dissipated in the vibration of train and track components at lower frequencies than those occurring in the wheel-rail interface. Therefore, also other components that dissipate energy are subject to deterioration induced by short wheel-rail irregularities.



Fig. 1.15 Branching and propagating micro-cracks in the railhead, emerging from internal material defects (squat-like defects)

Apart from the brief overview in the previous, it is not the aim of this thesis to address all possible and specific wheel-rail interface irregularity types in the short-wave regime, in the aspects generation mechanism, detection and monitoring (addressed in e.g. [1.60, 1.61]), assessment, prevention, characteristics of dynamic wheel-rail interaction and resulting damage patterns. Many studies addressing different aspects of particular irregularity types have been performed and published in the literature.

The thesis starts with making a general and essential distinction in the different global wheel-rail contact types along the rail. This distinction is of

vital importance for the resulting dynamic-wheel rail interaction as well as the type and damage and deterioration rate that commonly result from them.

Afterwards it addresses two types of wheel-rail interface irregularities that are of great actual importance in the railway sector. The first type is the metallurgical rail weld geometry, which has not been studied extensively before. The second irregularity type that will be considered is the wheel flat, which is a problem as old as the train, and has been studied in the literature, but for which a new theory is presented with respect to the resulting dynamic wheel-rail interaction.

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2 Elementary types of rolling contact between wheel and rail¹

This chapter deals with elementary types of rolling contact between wheel and rail, considering only vertical dynamics and disregarding the lateral motion and dynamics. This is done on a *global* or *macroscopic* scale or level, i.e., the wheel-rail contact is considered as such, without zooming in on phenomena within the contact area. This means that effects such as creep, spin, slip and stick are disregarded, and that it is not the aim to consider aspects on this more specific microscopic level. Theories on these particular aspects of rolling contact have been developed and published in the literature, with major steps performed by Hertz (1882) [2.1], Carter (1926) [2.2], Johnson (1958) [2.3, 2.4], Vermeulen and Johnson (1964) [2.5] and Kalker (1967) [2.6 - 2.8].

2.1 General

In this section, two different types of rolling wheel contact are considered. It is of great importance to distinguish both types, especially when analyzing short-wave irregularities in the wheel-rail interface.

A perfectly circular wheel is considered, rolling on a flat rigid foundation, in the cross-sectional plane perpendicular to the wheel axis. This wheel has a continuous single-point contact in the time domain. The contact point as a function of time forms a straight line. The wheel centre trajectory is

¹ The majority of the content of this chapter is reflected in the following journal publication:

M.J.M.M. Steenbergen. *Modelling of wheels and rail discontinuities in dynamic wheel-rail contact analysis*. Vehicle System Dynamics, 2006, 44 (10), pp. 763-787.

given by a straight line parallel to the foundation, passing through the permanent centre of rotation (the wheel centre). Therefore, the wheel-rail contact force has no contribution originating from the wheel inertia in vertical direction. This changes as soon as the contact point forms no more a straight line in the time domain. This occurs due to any irregularity in the wheel-rail interface, and it is important to note that this irregularity can occur either on the rail, the surface of which is not a perfectly straight line, or on the wheel, the circumference of which is no longer perfectly circular. In the above approach, the finite wheel and rail width as well as geometrical variations in this transverse direction are disregarded.

Now, the contact is considered which no longer forms a straight line in time. It depends on the curvature of this line which contact type occurs. A basic and essential distinction in rolling wheel contact can be made between *continuous single-point contact* and *transient double-point contact* along the wheel circumference. Both situations are illustrated in Fig. 2.1. Irregularities along the wheel-rail interface give rise to *dynamic amplification of the static wheel-load* in the first case. In the second case however they lead to *impact loading*. Both loading types of the wheel-rail system are essentially different. Generally, the frequency content of the contact force spectrum is in the order 100 Hz in the first case, whereas in the second case the frequency range is in the order 1000 Hz. The criterion for transient double-point contact not to occur is that the upward curvature of the wheel-rail interface geometry for the running wheel along the rail may not exceed the circumferential wheel curvature (which is equal to the inverse of its radius). The criterion can also be more readily formulated as follows: the upward curvature of the contact geometry, experienced by the rolling wheel at any time moment, should not exceed the circumferential wheel curvature. It can be observed from Fig. 2.1 that transient double-point contact occurs automatically if an upward kink occurs in the geometry along the rail, even for a minimum angle. This phenomenon is typically the case for dipped rail joints. In reality, this effect can be attenuated to some extent by the finite size and the elasticity of the wheel-rail contact patch, which is never a real 'point'. In these cases however, the wheel-rail contact cannot be described by the Hertzian contact model, which, together with the high-frequency content, makes it hard to determine the occurring force levels, either theoretically or experimentally.

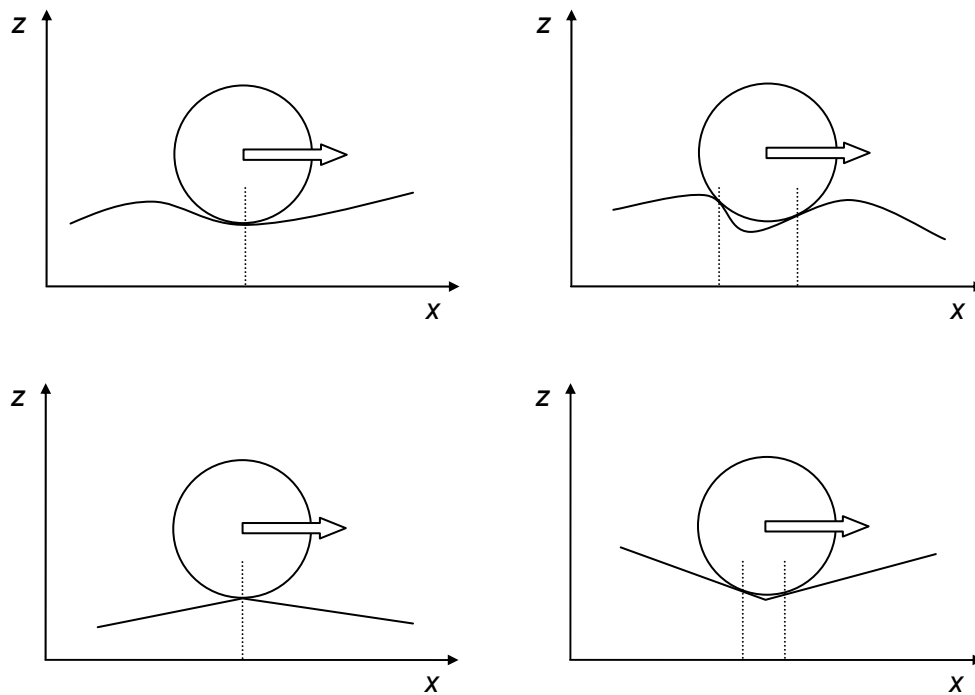


Fig. 2.1 Continuous single-point contact (left) and transient double-point contact (right) for a circular wheel rolling on a substructure

As regards the application of this distinction in the field, transient double-point contact should be avoided as much as possible in railway tracks. The resulting impact loads are detrimental to both the track and the train vehicle (Fig. 2.2). Situations where transient double-point contact typically occurs are dipped insulated or bolted rail joints, severe wheel flats but also noses in switches.

Another reason why potential transient two-point contact situations should be avoided, apart from the resulting high vibration level, is the high noise level that results from them. It is well-known that trains running with wheel-flats are particularly noisy. The same holds for the old, bolted railway tracks, or tracks with badly maintained insulation joints.



Fig. 2.2 Insulated rail joints with transient double point-contact (zoom-in, right: indicator is the interrupted running band on the railhead) and severe track damage (zoom-in, left: broken sleepers until 3 m after the joints)

2.2 *The contact discontinuity types leading to wheel-rail impact*

The fact that impact occurs for two-point contact situations can be readily seen when the kinematical wheel gravity centre trajectory is considered. A two-point contact situation leads to a local discontinuity in the temporal or spatial derivative of this trajectory (see Fig. 2.3).

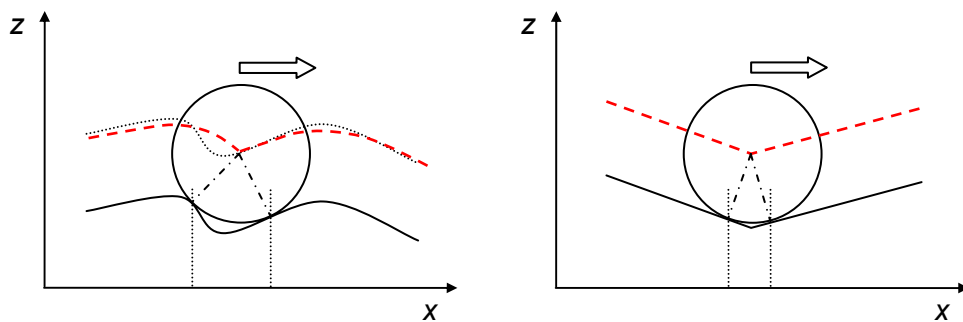


Fig. 2.3 Wheel centre trajectories with discontinuous spatial and temporal derivatives for transient two-point-contact situations, indicating a discrete change in vertical momentum

Keeping in mind that the time-derivative of the trajectory represents the vertical wheel mass velocity, this discontinuity implies that the wheel mass is subjected to a discrete change in vertical linear momentum, which is manifested as an impact in the rolling contact (Fig. 2.4).

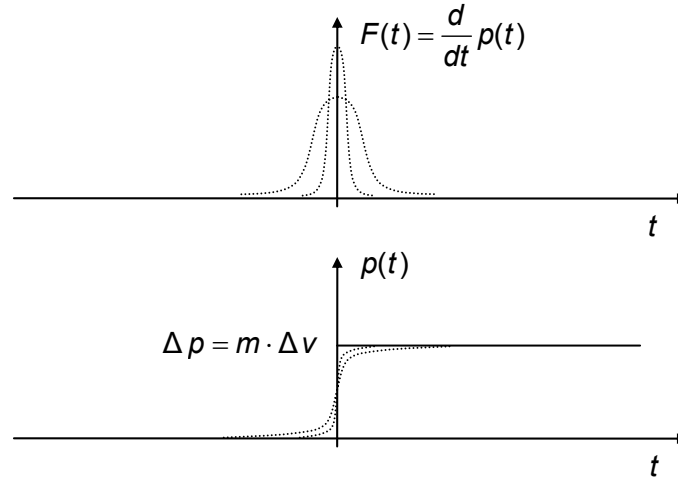


Fig. 2.4 Discrete change in vertical velocity v of the wheel mass m , implying a discrete change in vertical momentum p and manifested as an impact in its time-derivative, the dynamic contact force F ; the attenuating effect of non-zero elasticity in the contact

In Fig. 2.3 (left), it is observed that the wheel centre trajectory is not equal to the longitudinal surface geometry of the supporting structure. This effect must be taken into account when the curvature of the surface geometry has the same order of magnitude as the circumferential wheel curvature (which is the inverse of its contact radius). When the curvature of the surface geometry is much smaller, the effect is negligible.

It is stressed that geometrically induced transient double-point contact is not an exclusive condition of wheel-rail impact occurrence. For example, in the situation in Fig. 2.1 at the bottom, left, the kinematical trajectory of the wheel centre has a discontinuous derivative in space or time. However, no two-point contact occurs. All the same, impact will occur in this case for any non-zero velocity, because of the non-zero wheel inertia, which leads to successive contact loss and recovery. When the surface geometry is smooth, the contact loss becomes velocity-dependent.

In both cases treated above (transient double-point contact and successive contact loss and recovery), the impact arises from the same principle, namely a contact jump along the wheel circumference. However,

the generation mechanism of both impact types is different. The first case may be denoted as geometrically or kinematically induced impact, whereas the second case may be denoted as dynamically induced impact, because the wheel inertia is activated. In the first case, it is sufficient to consider the kinematical trajectory to establish the impact occurrence; in the second case the dynamic (or real) trajectory must be considered. Keeping in mind that in both cases ultimately the wheel-rail contact geometry is responsible for the impact occurrence, we may also speak of *explicitly geometrically determined* and *implicitly geometrically determined* impact situations. For clearness, both situations are illustrated for an elementary case in Fig. 2.5. It can be easily seen that the magnitude or length of the contact jump along the wheel circumference is a measure for the impact magnitude for real wheel-rail contact situations with finite stiffnesses. The distinction will prove to be important in the analysis of wheel flats in chapter 5.

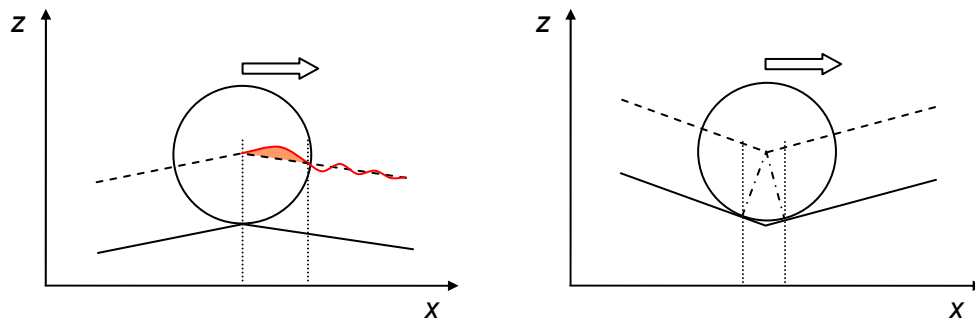


Fig. 2.5 Implicitly geometrically determined impact – successive contact loss and recovery (left) and explicitly geometrically determined impact – transient double-point contact (right). In the first case the dynamic wheel trajectory is shown; in the second case the kinematical trajectory.

From another viewpoint, we may also say that the occurrence of continuous single-point contact in time is a sufficient condition to avoid impact.

2.3 The wheel-rail contact conditions and track and wheelset deterioration

It is often observed in practice that heavily corrugated rails ('roaring rails') or rails being severely affected by squats are particularly noisy during train passage; in fact a severe squat can be heard distinctly. This is an indication that

the wheel-rail contact is not continuous, and that the contact history in the time domain consists of a series of transient contact conditions with impact. These impacts may, dependent on the exact geometry, be explicitly or implicitly geometrically determined, or a combination of both types may occur. These successive impacts lead to extremely rapid crack propagation in the presence of squats or other RCF-defects, and the severe risk of rail fracture. Further, rail fixations may get loose, concrete sleepers may crack and the ballast is locally de-compacted, introducing even more severe non-linearity. This detrimental mechanism is confirmed by practice, and usually grinding measures are being taken at the stage before rails become ‘noisy’.

The noise level that rails produce, being directly related to the contact and loading conditions, therefore can be considered as a measure of their expected lifetime – disregarding corrective maintenance measures such as rail grinding. The same principle can be applied to wheel tires. Because a rail geometry leading to transient double-point contact manifests itself in a severely increased rolling noise level, this also opens possibilities for an easy detection method for critical defects (e.g. severe squats): acoustic detection via microphones in the neighbourhood of the wheel-rail contact of a train running at conventional speed and registering the rolling noise.

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3 Model for dynamic wheel – rail interaction²

3.1 Modelling and mathematical description

In this chapter, an elementary model for dynamic wheel-rail interaction analysis is presented and elaborated, presupposing continuous single-point contact along the wheel-rail interface. The model is shown at the top in Fig. 3.1, in which also some notations are introduced. It consists of a beam, representing the rail, supported by an elastic foundation (Winkler foundation), representing the track, loaded by a moving oscillator, representing a part of the train vehicle and traveling with a speed V along the rail. The unsprung axle mass, the Hertzian contact stiffness and the primary suspension stiffness are included in the oscillator model. The vertical rail geometry is described by $z(x)$; the rail is continuously supported.

The model is based on the assumption that the irregularity in the wheel-rail interface is short with respect to the ratio of the train speed to the eigenfrequency of the bogie motion. Under these circumstances (excitation frequencies > 20 Hz), the wheelset motion may be considered as isolated from the bogie and car body motion. Furthermore, comparison in Ref. [3.1] shows that for frequencies higher than approximately 250 Hz the effect of the modelling of the substructure (Winkler foundation or halfspace with sleeper coupling) on the wheel-rail contact force magnitude is negligible.

² The content of this chapter is reflected in parts of the following journal publications:

- M.J.M.M. Steenbergen. *Modelling of wheels and rail discontinuities in dynamic wheel-rail contact analysis*. Vehicle System Dynamics, 2006, 44 (10), pp. 763-787.
- M.J.M.M. Steenbergen. *Quantification of dynamic wheel-rail contact forces at short rail irregularities and application to measured rail welds*. Journal of Sound and Vibration, 2008, 312, pp. 606-629.

The moving-wheel problem, depicted at the top in Fig. 3.1, may be simplified to a stationary wheel problem, according to the assumptions as discussed in the following. Damping mechanisms are not accounted for. The energy input in the rail due to the dynamic contact force generated by the interfacial irregularity is mostly radiated into the rail at the contact point and dissipated in the rail system components, mainly the railpads and the ballast. However, in order to determine local extremes of the dynamic wheel-rail contact force occurring at some point along an irregularity (which is a main focus for application of the model), a relatively short time-scale is sufficient to consider. Damping mechanisms are irrelevant within these small time-scales; they only play a role with increasing time-scales.

The above approach is widely accepted in the investigation of dynamic wheel-rail interaction in the high-frequency regime, see e.g. Refs. [3.2, 3.3]. The stationary wheel model is shown at the bottom in Fig. 3.1. The excitation - as the Hertzian spring contact rolls over the rail geometry $z(x)$ at the velocity V - changes into an excitation $z(t)$ of the Hertzian contact spring at $x = 0$ as a function of time.

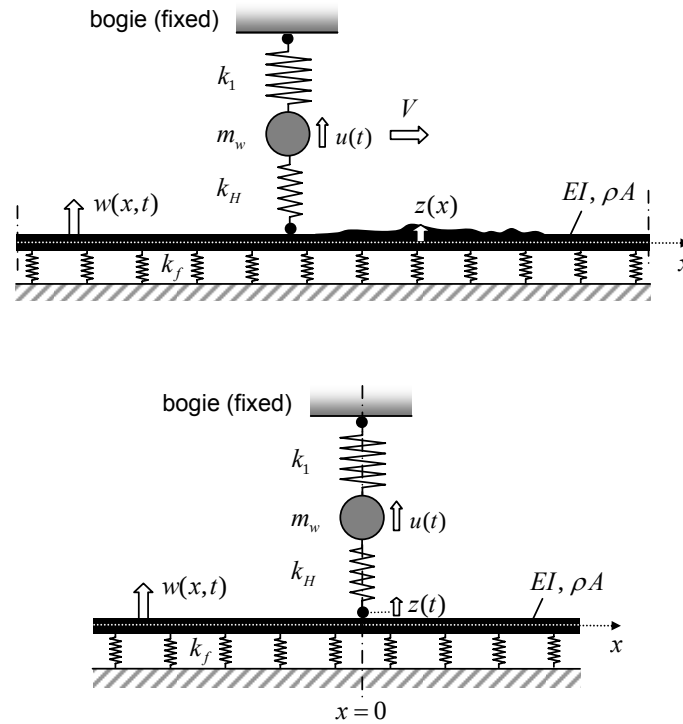


Fig. 3.1 Wheel-rail interaction model with moving wheel and excitation $z(x)$ (top); with stationary wheel and excitation $z(t)$ (bottom)

The model is considered in the frequency-domain. This, along with the fact that the model is rather elementary, puts some severe restrictions: a frequency-domain model does not allow contact loss, and also non-linearity in the contact cannot be accounted for. Furthermore, the individual sleepers are not modelled, rendering impossible an accurate analysis of phenomena in which the sleepers play a role of importance (for the effect of the discrete sleepers on the track receptance, see e.g. [3.3 - 3.6]).

However, this approach also has two important advantages, which correspond to the aim of this thesis. Firstly, it results into rather simple and analytical closed-form expressions for the main involved parameters in the frequency domain (which is equivalent to integral expressions in the time domain), allowing for fast computations for a large number of simulations. Secondly, the frequency-domain approach provides insight in occurring mechanisms, which become manifest in the time domain. Time-domain models or even much more accurate Finite Element models do not provide such insight. This makes the model very suitable for a qualitative investigation of characteristic features of dynamic wheel-rail interaction for different interfacial irregularity types. This will prove advantageous, especially when dealing with wheel flats.

It is, however, extremely important to be aware of the model assumptions during the analysis. For each irregularity type treated in the following chapters, the model properties and its specific limitations will be specifically discussed. When necessary or relevant, references will be made to the literature; often more accurate models have been used for quantitative predictions, however, without explaining or tracing the basic mechanisms.

The following mathematical statement is valid for the model as shown at the bottom in Fig. 3.1:

(a) The equation of motion for the rail, which is modelled as a Euler-Bernoulli beam on an elastic foundation, is given by ($t > 0$):

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} + k_f w(x,t) = -F(t) \delta(x) \quad (3.1)$$

(b) The equation of motion for the wheel system is given by (F represents the contact force in the wheel-rail contact point):

$$m_w \frac{d^2 u(t)}{dt^2} + k_1 u(t) = F(t) \quad (3.2)$$

(c) The contact force in the wheel-rail contact point is given as:

$$F(t) = k_H (-u(t) + w(0, t) + z_c(t)) \quad (3.3)$$

(d) Assuming an infinitely small but non-zero damping in the system, the rail must be undisturbed at $x = -\infty$ and $x = +\infty$ at any time moment (i.e., Sommerfeld's radiation conditions must be satisfied) (3.4)

(e) All initial conditions are assumed zero:

$$u(0) = \dot{u}(0) = 0 \text{ and } w(x, 0) = \dot{w}(x, 0) = 0 \text{ respectively} \quad (3.5)$$

Since the model is linear, the static load and the resulting steady-state rail displacement field are not taken into consideration; only the dynamic contact force and dynamic beam deflections are considered. Further, a linearised Hertzian spring is adopted in the model in order to enable an analytical approach in the frequency domain; this assumption and its validity domain will be discussed in section 3.4.

3.2 Solution

The equation of motion (3.1) of the beam in the t, x domain is written as:

$$\frac{\partial^4 w(x, t)}{\partial x^4} + \frac{1}{a^2} \frac{\partial^2 w(x, t)}{\partial t^2} + b^2 w(x, t) = -\frac{1}{EI} F(t) \delta(x);$$

$$a = \sqrt{\frac{EI}{\rho A}}; b = \sqrt{\frac{k_f}{EI}} \quad (3.6)$$

Laplace-transforming this equation with respect to time, with $w(x, 0) = \dot{w}(x, 0) = 0$, yields:

$$\frac{\tilde{\partial}^4 \tilde{w}(x, s)}{\partial x^4} + \left(\frac{s^2}{a^2} + b^2 \right) \tilde{w}(x, s) = -\frac{1}{EI} \tilde{F}(s) \delta(x) \quad (3.7)$$

in which the transformed variable is indicated by a tilde. Since the rail is undisturbed at $x = \pm\infty$ for any time moment, the solution for the governing variable will be determined by Fourier-transforming Eq. (3.7) with respect to x . The applied Fourier transform is defined as:

$$\bar{a}(k_x) = \int_{-\infty}^{\infty} a(x) e^{ik_x x} dx \quad (3.8)$$

The Fourier-transformed governing equation (3.7) is given by:

$$\bar{w}(k, s) (k^4 + s^2 / a^2 + b^2) = -\frac{1}{EI} \tilde{F}(s) \quad (3.9)$$

with the forward Fourier-transformed variable indicated by a bar. This yields for the solution $\bar{w}$:

$$\bar{w}(k, s) = -\frac{1}{EI} \frac{\tilde{F}(s)}{k^4 + s^2 / a^2 + b^2} \quad (3.10)$$

The inverse Fourier transform of this solution yields the following solution in the x, s domain:

$$\tilde{w}(x, s) = -\frac{\tilde{F}(s)}{2\pi EI} \int_{-\infty}^{\infty} \frac{e^{-ik_x x}}{k^4 + s^2 / a^2 + b^2} dk \quad (3.11)$$

The integral can be calculated by contour integration, adding a semicircle joining $-\infty$ to ∞ and applying respectively Jordan's lemma and Cauchy's residue theorem. The poles of the integrand are equal to the solutions of the equation

$$k^4 + s^2 / a^2 + b^2 = 0 \quad (3.12)$$

The solutions are given as:

$$k_{1,2,3,4} = (\pm 1 \pm i) k_0; \quad k_0 = \frac{1}{\sqrt{2}} \sqrt[4]{\frac{s^2}{a^2} + b^2}, \quad (3.13)$$

with the choice $\text{Im}(k_0) < 0$, $\text{Re}(k_0) > 0$. The individual poles are written as:

$$k_1 = (1 + i)k_0; \quad k_2 = (-1 + i)k_0; \quad k_3 = (-1 - i)k_0; \quad k_4 = (1 - i)k_0 \quad (3.14)$$

Therefore,

$$\text{Im}(k_1) > 0, \quad \text{Im}(k_2) > 0, \quad \text{Im}(k_3) < 0, \quad \text{Im}(k_4) < 0 \quad (3.15)$$

The poles k_1 and k_2 are situated in the upper half-plane and k_3 and k_4 in the lower half-plane of the complex k -plane.

According to Jordan's lemma, $\lim_{R \rightarrow \infty} \int_C f(k) e^{-i k x} dx = 0$ ($x > 0$) if the closing contour C is in the third and fourth quadrant (lower half-plane), and $f(k)$ tends uniformly to zero for increasing radius R of the contour. An analogue statement holds for $x < 0$, yielding a closing contour in the upper half-plane of the complex k -plane. Both closed contours and the poles are shown in Fig. 3.2.

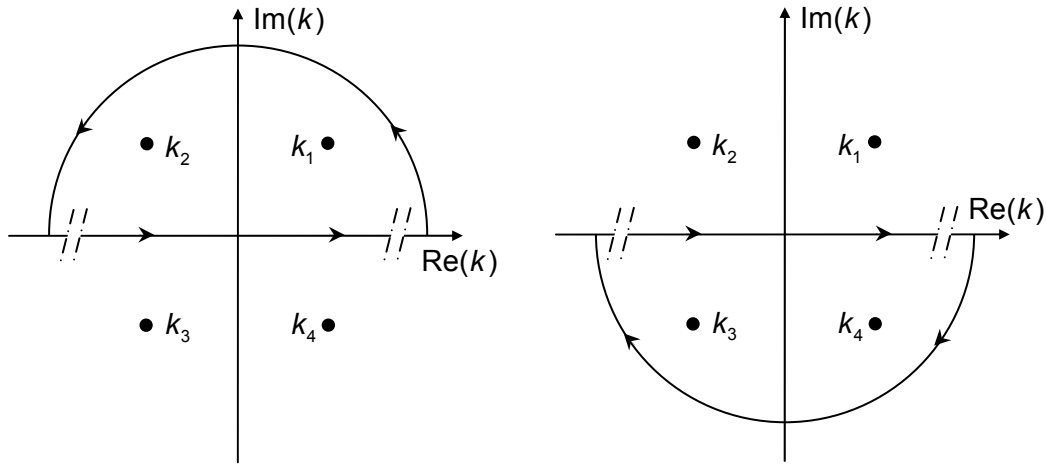


Fig. 3.2 Closed integration contours for $x < 0$ and $x > 0$, respectively

Since the integrand is single-valued, Cauchy's residue theorem can be applied now. First it will be applied for the first contour, $x < 0$; two poles are enclosed by the contour. The residue theorem states:

$$\begin{aligned}
 \int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + s^2/a^2 + b^2} dk &= 2\pi i \sum_{k_1, k_2} \text{Residue of } \frac{e^{-ikx}}{k^4 + s^2/a^2 + b^2} \text{ at } k_{1,2} \\
 &= 2\pi i \sum_{k_1, k_2} \left. \frac{e^{-ikx}}{4k^3} \right|_{k_{1,2}}
 \end{aligned} \tag{3.16}$$

Evaluating this expression with the poles defined by Eq. (3.14) yields:

$$\begin{aligned}
 \int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + s^2/a^2 + b^2} dk &= 2\pi i \sum_{k_1, k_2} \left. \frac{e^{-ikx}}{4k^3} \right|_{k_{1,2}} \\
 &= 2\pi i \left(\frac{e^{-i(1+i)k_0x}}{4((1+i)k_0)^3} + \frac{e^{-i(-1+i)k_0x}}{4((-1+i)k_0)^3} \right) \\
 &= \frac{\pi}{8k_0^3} \left((1-i)e^{(1-i)k_0x} + (1+i)e^{(1+i)k_0x} \right)
 \end{aligned} \tag{3.17}$$

Similarly, the expression for the second closed contour can be found (the contour is in the negative sense; a minus should be added), lower half-plane, $x > 0$:

$$\begin{aligned}
 \int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + s^2/a^2 + b^2} dk &= -2\pi i \sum_{k_3, k_4} \left. \frac{e^{-ikx}}{4k^3} \right|_{k_{3,4}} \\
 &= -2\pi i \left(\frac{e^{-i(-1-i)k_0x}}{4((-1-i)k_0)^3} + \frac{e^{-i(1-i)k_0x}}{4((1-i)k_0)^3} \right) \\
 &= \frac{\pi}{8k_0^3} \left((1-i)e^{(-1+i)k_0x} + (1+i)e^{(-1-i)k_0x} \right)
 \end{aligned} \tag{3.18}$$

If $x < 0$ then $x = -|x|$, whereas if $x > 0$ then $x = |x|$. Therefore, the integral can be evaluated for arbitrary x as:

$$\int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + s^2/a^2 + b^2} dk = \frac{\pi}{8k_0^3} \left((1-i)e^{(-1+i)k_0|x|} + (1+i)e^{(-1-i)k_0|x|} \right) \tag{3.19}$$

Substituting Eq. (3.19) into Eq. (3.11) yields the solution in the s, x domain for the load. This solution is given by:

$$\tilde{w}(x, s) = -\frac{\tilde{F}(s)}{16EI\kappa_0^3} \left((1-i)e^{(-1+i)\kappa_0|x|} + (1+i)e^{(-1-i)\kappa_0|x|} \right); \quad \kappa_0 = \frac{1}{\sqrt{2}} \sqrt[4]{\frac{s^2}{a^2} + b^2} \quad (3.20)$$

Next, expressions are derived for the oscillator, i.e. the wheel mass. Assuming $u(0) = \dot{u}(0) = 0$, its equation of motion, Laplace-transformed with respect to time (the transformed variable is indicated by a tilde), is given as follows:

$$(m_w s^2 + \kappa_1) \tilde{u}(s) = \tilde{F}(s) \Rightarrow \quad \tilde{u}(s) = \frac{\tilde{F}(s)}{m_w s^2 + \kappa_1} \quad (3.21)$$

The coupling between Eq. (3.20) and Eq. (3.21) is established through the interaction force in the time domain according to Eq. (3.3). The Laplace-transform of this expression is given by:

$$\tilde{F}(s) = \kappa_H (-\tilde{u}(s) + \tilde{w}(0, s) + \tilde{\chi}(s)) \quad (3.22)$$

Substitution of the expressions (3.20) for $\tilde{w}(0, s)$ and (3.21) for $\tilde{u}(s)$ yields, after some elaboration, the final expression for the contact force in the Laplace-domain, which is given as (with $s = \sigma + i\omega$):

$$\tilde{F}(s) = \frac{\tilde{\chi}(s)}{1/\kappa_H + 1/(m_w s^2 + \kappa_1) + 1/(8EI\kappa_0^3)}; \quad \kappa_0 = \frac{1}{\sqrt{2}} \sqrt[4]{\frac{1}{EI} (\varrho A s^2 + \kappa_f)} \quad (3.23)$$

The wheel displacement is given by Eq. (3.21). Substitution of Eq. (3.23) yields:

$$\tilde{u}(s) = \frac{\tilde{\chi}(s)}{1 + (m_w s^2 + \kappa_1) \left(1/\kappa_H + 1/(8EI\kappa_0^3) \right)} \quad (3.24)$$

An explicit expression for the rail displacement at the contact point can be found by substituting Eqs. (3.21) and (3.23) into (3.22):

$$\tilde{w}(0, s) = \tilde{\chi}(s) \left(2 - \frac{1}{8EI\kappa_0^3 \left(1/\kappa_H + 1/(m_w s^2 + \kappa_1) \right) + 1} \right) \quad (3.25)$$

The expressions (3.23 – 3.25) must be transformed to the time domain. This inverse Laplace-transform (which is basically an integration over the frequency) will be computed numerically. Because of the fact that the integrand is not single-valued (due to the presence of radicals), the analytical inverse transform, by means of contour integration, implies the more complicated branch cutting. The numerical procedure is explained in the following. The inverse Laplace-transform of $\tilde{F}(s)$ is defined as:

$$F(t) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \tilde{F}(s) e^{st} ds \quad (3.26)$$

Substituting $s = \sigma + i\omega$ with σ some small positive real value, or $ds = i d\omega$, and $-\infty < \omega < \infty$, yields:

$$F(t) = \frac{e^{\sigma t}}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\sigma + i\omega) e^{i\omega t} d\omega \quad (3.27)$$

The integrand in this expression can be elaborated as follows:

$$\begin{aligned} \tilde{F} \cdot e^{i\omega t} &= (\text{Re}(\tilde{F}) + i \text{Im}(\tilde{F})) \cdot (\cos \omega t + i \sin \omega t) \\ &= \underbrace{\text{Re}(\tilde{F}) \cos \omega t - \text{Im}(\tilde{F}) \sin \omega t}_{\text{Real part}} + i \underbrace{(\text{Re}(\tilde{F}) \sin \omega t + \text{Im}(\tilde{F}) \cos \omega t)}_{\text{Imaginary part}} \end{aligned} \quad (3.28)$$

where the imaginary part of the integrand should add up to 0, as the force $F(t)$ is real. This means, $\sin \omega t$ being anti-symmetric and $\cos \omega t$ symmetric with respect to ω , that $\text{Re}(\tilde{F})$ is a symmetric and $\text{Im}(\tilde{F})$ an anti-symmetric function of ω . Thus, the integral (3.27) may be written as follows:

$$F(t) = \frac{e^{\sigma t}}{2\pi} \int_{-\infty}^{\infty} \text{Re}(\tilde{F}(s) e^{i\omega t}) d\omega = \frac{e^{\sigma t}}{\pi} \int_0^{\infty} \text{Re}(\tilde{F}(s) e^{i\omega t}) d\omega \quad (3.29)$$

This last expression will be used for numerical integration of the solutions (3.23 – 3.25) in the Laplace domain.

In summary, the general expressions (in integral form) for the dynamic wheel-rail contact force, the wheel displacement and the rail displacement in

the contact point are given as a function of time, for an excitation function $z(t)$ with Laplace-image $\tilde{z}(s)$, by:

$$F(t) = \frac{e^{\sigma t}}{\pi} \int_0^{\infty} \operatorname{Re} \left(\frac{\tilde{z}(s)}{1/k_H + 1/(m_w s^2 + k_1) + 1/(8EI k_0^3)} e^{i\omega t} \right) d\omega \quad (3.30)$$

$$u(t) = \frac{e^{\sigma t}}{\pi} \int_0^{\infty} \operatorname{Re} \left(\frac{\tilde{z}(s)}{1 + (m_w s^2 + k_1) \left(1/k_H + 1/(8EI k_0^3) \right)} e^{i\omega t} \right) d\omega \quad (3.31)$$

$$w(0, t) = \frac{e^{\sigma t}}{\pi} \int_0^{\infty} \operatorname{Re} \left(\tilde{z}(s) \left(2 - \frac{1}{8EI k_0^3 \left(1/k_H + 1/(m_w s^2 + k_1) \right) + 1} \right) e^{i\omega t} \right) d\omega \quad (3.32)$$

with $k_0 = \frac{1}{\sqrt[4]{2}} \sqrt[4]{\frac{1}{EI} (\rho A s^2 + k_f)}$; $s = \sigma + i\omega$.

3.3 Elementary model to describe wheel-rail interface irregularities

In this section a frequency-domain representation of wheel-rail interface irregularities of arbitrary shape (denoted as $z(x)$) is given, passed at a constant speed V (yielding $z(t)$). With respect to time, a wheel overpassing an irregularity yields a transient excitation and response state, requiring the application of the Laplace-transform (yielding $\tilde{z}(s)$), as has been done in the previous section. The irregularity model will be used later on for the description of measured irregularities. Since any measurement signal is discrete and based on a finite sampling interval, this has to be accounted for. Further, the description will be explicitly expressed in terms of a sequence of discrete slopes; the reason for this will become clear in chapter 4 on rail welds.

The first elementary model which will be considered contains only one such a non-zero slope, on a variable basis (Fig. 3.3, left). The rail surface is horizontal, except for an interval Δx where the rail geometry has a constant slope θ . The slope is chosen to start in the origin.

The vertical rail surface as a function of the longitudinal coordinate x ($x \geq 0$) is given by ($\tan \theta \approx \theta$ and H denotes the Heaviside step-function):

$$z(x) = \theta x H(\Delta x - x) + \theta \Delta x H(x - \Delta x) \quad (3.33)$$

For a contact point moving at speed V , the expression in the time domain becomes:

$$z(t) = \theta V t H(\Delta x / V - t) + \theta \Delta x H(t - \Delta x / V) \quad t \geq 0 \quad (3.34)$$

The Laplace-image of this expression is given by:

$$\tilde{z}(s) = \frac{\theta V}{s^2} (1 - e^{-s \Delta x / V}) \quad (3.35)$$

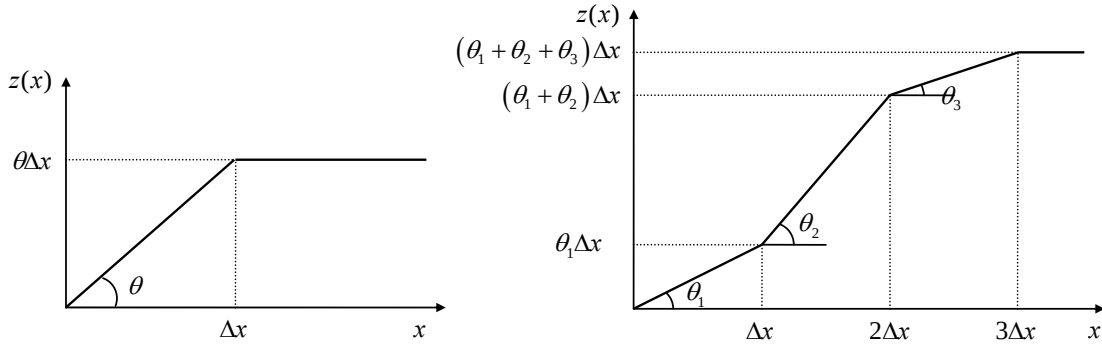


Fig. 3.3 Discrete models to describe longitudinal wheel-rail contact surface irregularities; basic model with linear irregularity (ramp, left) and multi-linear irregularity (right)

A more extended model for description of short irregularities with an arbitrary geometry is shown at the right in Fig. 3.3. The model which is shown in Fig. 3.3 consists of a sequence of three discrete slopes, corresponding to four sampling points, but it can be extended to an arbitrary number of N slopes. As will be established in the following, a spatial or temporal sequence of slopes can be represented as a summation of terms in the Laplace-domain, due to the linearity of the Laplace-transform. This property will allow for a simple derivation of some basic relationships for arbitrary irregularities later on. The vertical surface as a function of the longitudinal coordinate x ($x \geq 0$) is given by:

$$\begin{aligned}
\tilde{\chi}(x) = & \theta_1 x H(\Delta x - x) \\
& + ((\theta_1 - \theta_2)\Delta x + \theta_2 x) H(x - \Delta x) H(2\Delta x - x) \\
& + ((\theta_1 + \theta_2 - 2\theta_3)\Delta x + \theta_3 x) H(x - 2\Delta x) H(3\Delta x - x) + \dots \\
& + (\theta_1 + \theta_2 + \theta_3 + \dots)\Delta x H(x - 3\Delta x)
\end{aligned} \tag{3.36}$$

The expression in the time-domain becomes:

$$\begin{aligned}
\tilde{\chi}(t) = & \theta_1 V t H(\Delta x / V - t) \\
& + ((\theta_1 - \theta_2)\Delta x + \theta_2 V t) H(t - \Delta x / V) H(2\Delta x / V - t) \\
& + ((\theta_1 + \theta_2 - 2\theta_3)\Delta x + \theta_3 V t) H(t - 2\Delta x / V) H(3\Delta x / V - t) + \dots \\
& + (\theta_1 + \theta_2 + \theta_3 + \dots)\Delta x H(t - 3\Delta x / V)
\end{aligned} \tag{3.37}$$

For the three included terms (or slopes) in Fig. 3.3, the Laplace-transform yields:

$$\tilde{\chi}(s) = \frac{V}{s^2} \left(1 - e^{-s\Delta x / V} \right) \left(\theta_1 + \theta_2 e^{-s\Delta x / V} + \theta_3 e^{-2s\Delta x / V} \right) \tag{3.38}$$

This expression can be easily extended for a larger number of slopes. The Laplace-image of a sequence of N discrete slopes becomes:

$$\tilde{\chi}(s) = \frac{V}{s^2} \left(1 - e^{-s\Delta x / V} \right) \sum_{n=1}^N \theta_n e^{-(n-1)s\Delta x / V} \tag{3.39}$$

3.4 The energy input into an irregular track

The importance of the energy management in a railway track has been discussed in section 1.3. In this paragraph some derivations are given to enable computations in this respect in the following chapters.

The energy input into the track due to an arbitrary track irregularity with length L follows from integration of the power input over the period T of passing this irregularity:

$$E_{\text{in}} = \int_0^{T=L/V} P_{\text{in}}(t)dt = \int_0^{T=L/V} F(t) \cdot \dot{w}(0,t)dt \quad (3.40)$$

The requirement, formulated in section 1.3, that the input energy and thus the power input into the track must be minimized, requires a balance between force and displacement. The work performed by the contact load should be minimized. For a low stiffness, the force level is low and the displacement is large. For a high stiffness, the force is high and the displacement is low. Therefore, the stiffness must be optimised. Since this stiffness is dynamic and a function of frequency (even for physical properties independent of the frequency) and, dependent on the modelling, which may account for or disregard wave propagation, also wave-length dependent, its spectrum must be optimised. However, this cannot be done in a general sense, since this spectral optimisation can only be performed for a given geometry. Therefore, ‘the optimum track stiffness spectrum’ does not exist. To give a trivial example, for a perfectly flat geometry the dynamic component of the contact force is zero and does not perform any work, independent from the actual track stiffness spectrum.

According to Eq. (3.40) the power input into the track is computed as the product of the contact force and the resulting displacement:

$$P(t) = F(t) \cdot \dot{w}(0,t), \text{ or } \mathcal{L}\{P(t)\} = \mathcal{L}\{F(t) \cdot \dot{w}(0,t)\} \quad (3.41)$$

in which $\mathcal{L}$ denotes the Laplace transform. Since multiplication in the time domain is equivalent to convolution in the frequency (Laplace-) domain, using the convolution theorem in a general form

$$\mathcal{L}\{f(t) \cdot g(t)\} = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \tilde{f}(p) \cdot \tilde{g}(s-p)dp \quad (3.42)$$

the following expression results for the power input in the Laplace-domain:

$$\tilde{P}(s) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \tilde{F}(p) \cdot s \cdot \tilde{w}(0,s-p)dp \quad (3.43)$$

Using the Laplace-domain expressions for the contact force and the rail deflection, this expression can be transformed to the time domain. Using

directly expressions (3.30) and (3.32), the equation may also be written alternatively in the time domain. Eq. (3.32) yields:

$$\dot{w}(0,t) = \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left(\tilde{\chi}(s) \left(2 - \frac{1}{8EI k_0^3 \left(1/k_H + 1/(m_w s^2 + k_1) \right) + 1} \right) s e^{st} \right) d\omega \quad (3.44)$$

The power which is added to the track during the passage of the irregularity now can be calculated from:

$$\begin{aligned} P_{\text{in}}(t) = & \frac{1}{\pi^2} \int_0^\infty \operatorname{Re} \left(\frac{\tilde{\chi}(s) e^{st}}{k_H^{-1} + (m_w s^2 + k_1)^{-1} + (8EI k_0^3)^{-1}} \right) d\omega \\ & \times \int_0^\infty \operatorname{Re} \left(\left(2 - \frac{1}{8EI k_0^3 \left(k_H^{-1} + (m_w s^2 + k_1)^{-1} \right) + 1} \right) \tilde{\chi}(s) s e^{st} \right) d\omega \end{aligned} \quad (3.45)$$

with $\tilde{\chi}(s)$ according to Eq. (3.39). Eq. (3.40) allows for a calculation of the corresponding energy input.

3.5 Model parameters

Unless indicated differently, the simulations in the following chapters use the representative parameter values indicated in Table 3.1. Representative values have been chosen for the involved parameters. The static wheelload corresponds to the nominal axle load of 225 kN which is allowed in many countries for conventional rail traffic [3.7]. The rail properties of the standard 54E1 profile are used. The value for the rail support stiffness is the one which is used in the design of embedded rail structures [3.8]. The adopted wheel mass corresponds to an unsprung mass of 1800 kg, which is a common value for conventional passenger trains. No damping is present. This is expected to have a negligible influence on the first impact force in the contact, as it is dominated by the wheel and rail inertia, and the damping in the wheel-rail interface is negligible.

Table 3.1 Model parameter values

Parameter	Value
Static wheelload F_{stat}	$112.5 \cdot 10^3 \text{ N}$ (axle load 225 kN)
Wheel radius R_{wheel}	0.46 m
Transversal railhead radius R_{railhead}	0.3 m
Wheel mass (half unsprung mass) m_w	900 kg (ICM-III passenger train)
Primary suspension stiffness k_1	$2 \cdot 10^6 \text{ N/m}$
Rail bending stiffness EI	$4.25 \cdot 10^6 \text{ Nm}^2$ (rail 54 E1)
Rail distributed mass ρA	54.4 kg/m (rail 54 E1)
Rail support stiffness k_f	$50 \cdot 10^6 \text{ N/m}^2$

In reality the wheel-rail contact stiffness, according to the Hertzian theory, is a non-linear quantity. In Eqs. (3.3) and (3.23) a linearization of this stiffness has been used to enable a frequency domain approach. This linearization is applied at the level of the static wheelload, yielding a tangent stiffness. According to [3.7], assuming a circular contact area with a radius equal to the geometrical mean of the elliptical radii and an infinite radius of the transverse wheel profile, an approximate expression for the non-linear Hertzian contact stiffness is given by:

$$k_H = \sqrt[3]{\frac{3E^2 F_{\text{stat}} \sqrt{R_{\text{wheel}} R_{\text{railhead}}}}{2(1-\nu^2)^2}} \quad (3.46)$$

Application of the geometrical parameters according to Table 3.1, with $E = 2.1 \cdot 10^{11} \text{ N/m}^2$ and $\nu = 0.3$ then results into the following expression (SI-units):

$$k_H = \sqrt[3]{8 \cdot 10^{22} F_{\text{stat}} \sqrt{R_{\text{wheel}} R_{\text{railhead}}}} = \sqrt[3]{3 \cdot 10^{22} F_{\text{stat}}} \quad (3.47)$$

The resulting linear stiffness after substitution of F_{stat} will lead to reliable results for dynamic load fluctuations around this level. According to the simplified expression (3.47), the Hertzian stiffness is governed by the third root of the wheelload. Therefore, model results may be considered to be reliable, when the dynamic amplification factor (DAF) does not exceed the value 2, according to the following relationship:

$$k_H(2F_{\text{stat}}) = \sqrt[3]{2} \cdot k_H(F_{\text{stat}}) \approx 1.26 \cdot k_H(F_{\text{stat}}) \quad (3.48)$$

The lower the value for the DAF, the higher is the expected accuracy of the contact force.

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4 Short rail irregularities with a broad-band spectrum – rail welds³

4.1 Introduction

Continuously welded rail (CWR) nowadays is the standard in modern railway tracks. The traditional bolted rail connections are often only still present in the form of insulated rail joints (IRJ) for detection and signalling purposes. The geometry of a bolted rail joint under static train axle loading leads to a high dynamic impact component (Fig. 4.1), which is a source of rapid track deterioration and high noise levels, as pointed out in chapter 2 (see also Refs. [4.1-4.3]), and therefore the welded continuous connection was a significant improvement.

The most common rail welding types are flash butt welding and aluminothermic or thermite welding (Fig. 4.2). The application of the first type is almost fully automated, including the positioning of the rail ends to be joined, and it is therefore often applied in the construction of new tracks.

³ The content of this chapter is reflected in parts of the following journal publications:

- M.J.M.M. Steenbergen, C. Esveld. *Rail weld geometry and assessment concepts*. Proc. Instn. Mech. Engrs. F, Journal of Rail and Rapid Transit, 2006, 220 (3), pp. 257-271.
 - M.J.M.M. Steenbergen, C. Esveld. *Relation between the geometry of rail welds and the dynamic wheel-rail response: numerical simulations for measured welds*. Proc. Instn. Mech. Engrs. F, Journal of Rail and Rapid Transit, 2006, 220 (4), pp. 409-424.
 - M.J.M.M. Steenbergen. *Quantification of dynamic wheel-rail contact forces at short rail irregularities and application to measured rail welds*. Journal of Sound and Vibration, 2008, 312, pp. 606-629.
- and the following articles:
- Michaël J.M.M. Steenbergen, Coenraad Esveld and Rolf P.B.J. Dollevoet. *New Dutch Assessment of Rail Welding Geometry*. European Railway Review, 2005, 11 (1), pp. 71-79.
 - Theo Winter, Peter A.J. Meijvis, Wijnand J.M. Paans, Michaël J.M.M. Steenbergen, Coenraad Esveld. *Track Quality Achieved on HSL-South - reduction of short-wave irregularities cuts life cycle cost*. European Railway Review, 2007, 13 (3), pp. 48-53.

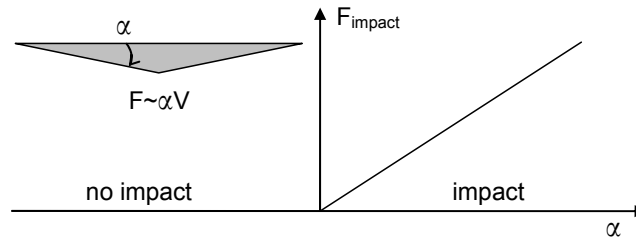


Fig. 4.1 Bi-linear relationship between the dip angle ($\alpha \ll 90^\circ$) of an IRJ and the corresponding impact force (from [4.1])

The second type requires handicraft and is therefore often applied in repair and maintenance work. However, the final grinding of the rail surface is in most cases done manually for both welding types. Only in some cases the welds are being treated by a grinding train, on newly built tracks, yielding a high surface quality (Fig. 4.3).



Fig. 4.2 Flash butt (left) and aluminothermic (right) rail welding on the Dutch HSL-South
(courtesy: Wijnand Paans, BAM Rail)



Fig. 4.3 Manual rail weld grinding (left) and grinding train (Speno) (right) on the Dutch HSL-South

In the literature, several studies have been published on rail welds, most of them dealing with metallurgical or related aspects. These aspects include the mechanical and micro-structural weld properties [4.4], damage types occurring at welds [4.5], testing and inspection methods [4.6], the effects of heat transfer during welding on defect formation [4.7, 4.8], the effects of post-welding heat treatment [4.9], the measurement of residual stresses after welding [4.10, 4.11] and their alleviation [4.12], fatigue crack growth in a weld under residual and temperature stresses [4.13] and the influence of loose sleepers on the bending fatigue of rail welds [4.14]. In Ref. [4.15] attention was paid to the relationship between train loading, weld geometry and weld failure modes. From measurements an approximately linear relationship was found between the train speed (in the interval 25-75 km/h) and the dynamic impact factor (DAF) occurring at welds.

4.2 The significance of rail welds in relation to track damage and deterioration

The rail weld is particularly susceptible to damage formation. This is due to three reasons that have been discussed in a more general context in section 1.4. Firstly, the weld presence leads to a geometrical disturbance along the wheel-rail interface, leading to dynamic axle load variations and dynamic contact stress amplifications. This concerns both normal stresses and tangential stresses and their distribution in the rolling contact. A second reason is the presence of material inhomogeneities along the wheel or rail surface. This includes a non-constant hardness distribution along the rail and micro-structural disturbances, non-metallic inclusions or steel phase differences at the surface or sub-surface. A third reason is the presence of internal material inhomogeneities or clusters of them in the weld. It is remarked that generally none of the previous mechanisms occurs isolated when leading to damage. They interact, significantly increasing the progression speed of existing damage. Therefore, each weld in the rail is a critical spot and must be considered as a potential RCF initiator.

The geometrical irregularity at the rail weld which is present in the majority of the cases can be due to many reasons. These include inaccurate rail end positioning (especially when the alignment is done manually for in-situ welding; normally the rail ends are set up to account for unequal thermal contraction in the rail head and foot), the manual rail surface grinding, and the

influence of inhomogeneous material shrinkage after cooling down in combination with early grinding. The irregularity has a typical length between 0.5 and 1 m. Therefore, in general the rail welds yield a significant contribution to the short-wave part of the track irregularity spectrum. It is important to note however that especially on new tracks the short-wave contribution is almost entirely determined by the rail weld geometry. It was argued already in chapter 1 that this contribution determines the rail damage progress and track deterioration rate, due to the dissipation of different frequency components of the dynamic wheel-rail interaction force power spectrum in the different track-vehicle system components. With a focus on rail welds, this will be discussed in more detail in section 4.4. It can be remarked however that repetitive high-frequency excitations resulting from dynamic wheel-rail interaction at weld irregularities are not only damaging the rail itself, but they are also and especially detrimental for concrete sleepers and rail fastenings; furthermore they are responsible for non-uniform ballast settlement or local decompaction. Considering the wheelset, the presence of rail welds in CWR tracks may cause a periodical amplification of wheel-rail contact stresses, leading to an increase of the wheel or the wheel tire fatigue rate, and the occurrence of RCF (see chapter 1).

When a serious internal material defect is present in a weld, which is not detected by ultrasonic measurement, the weld often breaks at an early stage [4.5], especially in cold winters with high tensile forces in the rail. Therefore, internal material defects are in general not responsible for long-term deterioration at the wheel-rail interface.

The effect of locally varying material properties along the rail surface however is particularly relevant for rail welds. As has been pointed out in the previous, often small and short indentations along the rail surface occur at the centre of rail welds, at the transitions from the weld material to the parent material, the so-called heat-affected zone (HAZ) (Fig. 4.4). This phenomenon is due to shrinkage after grinding and further cooling down, especially when the steel temperature was not low enough at the moment of final grinding. It can be observed particularly for aluminothermic welds in rail repair works, executed under time pressure and often during night and under non-optimal conditions.



Fig. 4.4 Weld and parent material (left: flash butt weld; centre: thermite weld); the heat-affected zone after thermite welding and grinding (right)

The geometrical indentations typically coincide with the local hardness minima along the rail surface in the HAZ at both sides of the weld metal [4.16] (Fig. 4.5).

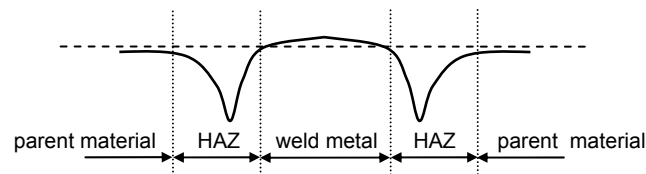


Fig. 4.5 Qualitative behaviour of the steel hardness distribution along the rail surface

The unevenness in the rail surface, resulting in a local fluctuation of the contact stresses, may induce differences in shakedown, work hardening and plastic deformation of the rail surface along the rail [4.17], leading to non-homogeneous wear and a progressively deteriorating geometry. Examples of this phenomenon are shown in Fig. 4.6. In in-situ welding it is almost impossible to avoid these indentations, except for new track construction works, where the grinding can be done separately from the welding.



Fig. 4.6 Deteriorating geometry resulting from non-uniform shakedown and work hardening along a rail weld in the HAZ (left: flash butt weld; centre, right: thermite weld)

In section 4.8, the time-scale of the first peak in the contact force (commonly denoted as P_1) due to a short irregularity in the rail geometry will be shown to range from about 0.5 to 0.75 ms, which corresponds to a length-scale (half wavelength) of about 20-30 mm at a regular line-speed of 140 km/h. At 250 km/h (high-speed lines) this length-scale is about 35-48 mm (the time-scale does not vary significantly). Short-pitch rail corrugations have a wavelength which typically ranges from 25 to 80 mm [4.18]. The corrugation mechanism proposed in [4.19] on the basis of laboratory experiments and initiated by excitation of a ‘contact resonance’ in the wheel-rail interface can be easily excited at a weld. This resonance can lead to non-constant micro-slip and wear in the contact, as well as non-constant shake-down or plastic deformation, causing short-pitch corrugations (Fig. 4.7).



Fig. 4.7 Rail corrugation generated by a squat rail surface irregularity, initiated at a flash butt weld

In Refs. [4.20 - 4.23] a non-constant contact force was indeed shown to have a significant impact on the formation of corrugations. Rail welds therefore are often considered, and also observed in practice, as possible corrugation or

also squat initiators [4.18, 4.24, 4.25]. The latter again is related to the non-constant material properties along the rail surface. In Figs. 4.8 and 4.9 examples are shown of squats on thermite and flash butt welds respectively. Fig. 4.10 (right) shows an example of corrugation and RCF rail damage initiated by the presence of a weld.



Fig. 4.8 Squat initiation on thermite rail welds or in the HAZ (right: accompanied by crack initiation)

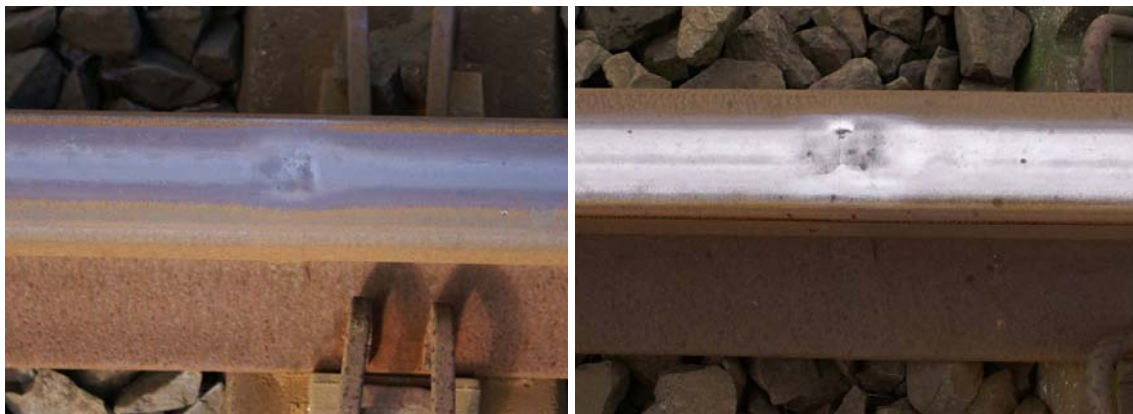


Fig. 4.9 Squat initiation on flash butt rail welds or in the HAZ

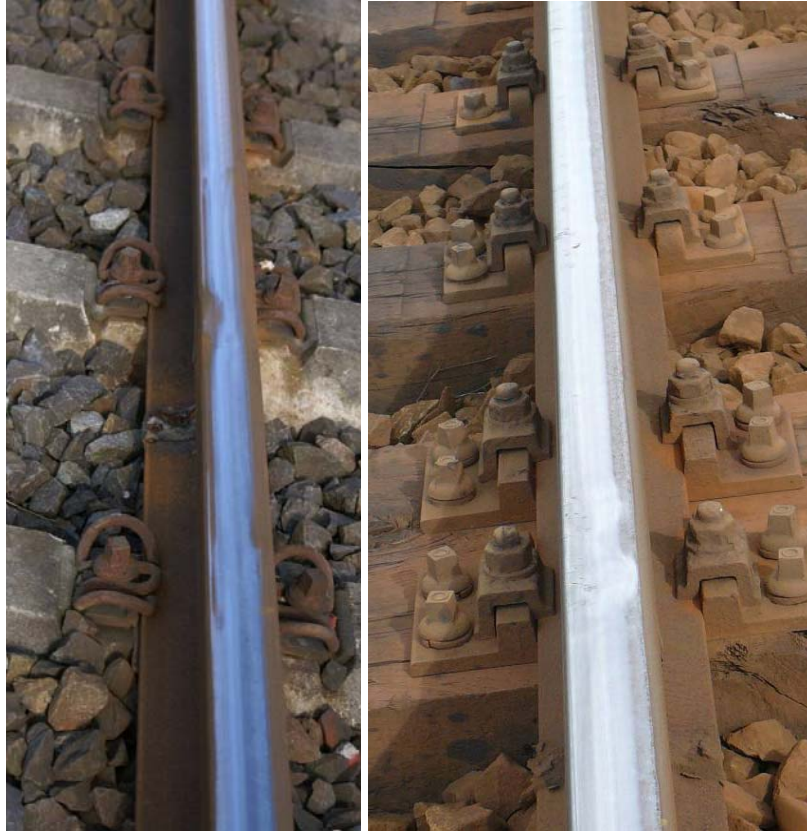


Fig. 4.10 Misaligned rail weld (left) and short-pitch corrugation (accompanied by RCF) initiated by a rail weld (right)

In the previous, the relationship between the presence of irregular rail welds and the damage to the wheelset and track components deriving from mid- or high-frequency (100 – 2000 Hz) interaction and wave propagation into the dispersive rail and wheel media (and other track components) has been considered. This is typically the RCF regime that was also addressed in section 1.5. However, the presence of welding irregularities has also consequences for the bending fatigue life of the rail in the lower frequency regime (Fig. 4.10, left), due to dynamic amplifications of the static axle load (0 – 100 Hz). This fatigue life is generally described by the S - N curve according to the following simple relationship between the number of cycles to failure N (describing the fatigue life) and the stress range $\Delta\sigma$:

$$N = C(\Delta\sigma)^{-m} \text{ or } \frac{\text{Lifetime}_{\text{new}}}{\text{Lifetime}_{\text{orig.}}} = \left(\frac{F_{\text{orig.}}}{F_{\text{new}}} \right)^3; \quad F = F_{\text{stat}} + F_{\text{dyn}} \quad (4.1)$$

where $m = 3$ (up to $N = 5E6$) is used for steel according to EuroCode 3. Thus, the fatigue life can be assumed to be proportional to the 3rd power of the stress range (Fig. 4.11), and the rail bending stress range is proportional to the total wheel load on the rail, which is the summation of its static and dynamic components.

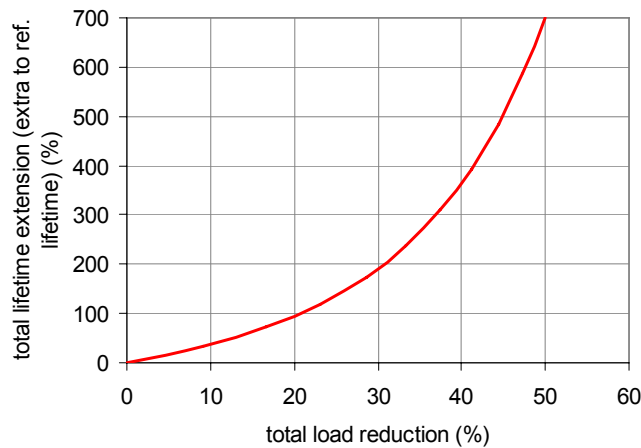


Figure 4.11 Relationship between the total wheel load reduction and the rail weld bending fatigue lifetime extension

As an example, reductions of this dynamic load at welds with factors 1.2 up to 1.7 have been obtained with the employment of grinding trains, after accurate manual pre-grinding, on the Dutch HSL-South [4.26]. Assuming a static wheel-load (half axle-load) of 85 kN (Thalys), these reductions result in an extension of the bending fatigue lifetime of the welded rail with factors 1.2^3 or 1.7 up to 1.7^3 or 4.9. Although these factors only give a general indication, and real factors are expected to be even higher due to the influence in the high-frequency regime, they illustrate the significance of smooth rail transitions, especially from a LCC perspective.

It has been pointed out in the Introduction (chapter 1) that the investigation of the relationship between track geometry and interaction forces is relevant from another viewpoint: railway track assessment is widely based on geometrical requirements without direct relationship to dynamic train-track interaction forces, which is inconsistent from the viewpoint of track deterioration and the resulting maintenance necessity. The aspect of rail weld geometry assessment therefore will be used as a central theme in the following of this chapter.

4.3 Rail weld geometry assessment: conventional methods and improvement possibilities

It is obvious from the previous that it is crucial to realise a vertical weld geometry as close as possible to a straight line. This requires appropriate assessment concepts. Traditionally, the vertical geometry of rail welds is being assessed with the principle of tolerances [4.27, 4.28]. These tolerances are defined for a given basis length, which is commonly 1 m. A common value for the vertical tolerance is 0 – 0.3 mm [4.27]. This method is related to the manual assessment with a steel straightedge in combination with a feeler gauge. A drawback of this method is that it poses no restrictions on the ‘smoothness’ of the rail surface, whereas this shape has a direct relation to the magnitude and the spectrum of the dynamic wheel-rail interaction force. A further drawback of the traditional assessment is the fact that the train speed for the line section in which the weld is made has no influence, whereas its effect on the contact forces is far from negligible.

A typical example resulting from the current rail welding methods (especially aluminothermic in-situ welding) and assessment is shown in Fig. 4.12.

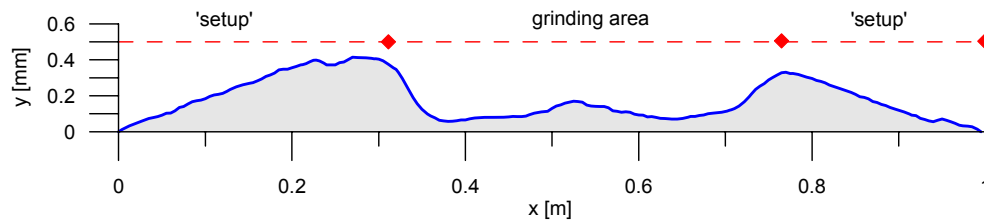


Fig. 4.12 Typical example of the vertical geometry along an in-situ made rail weld, resulting from traditional assessment methods

Both rail ends are ‘set up’ under a small angle in vertical direction (usually the overlift is 1-2 mm on a 1-1.2 m base [4.27]). This is done to account for the fact that thermal contraction of the head occurs later than in the foot, due to their different cross-sectional area. After welding, the resulting ‘top’, as far as it does not fit within the tolerance, is ground off. The result is often far from smooth, and introduces large dynamic amplifications in contact forces.

The development of digital straightedges that sample the vertical rail geometry enables more advanced assessment methods, that do account for the ‘smoothness’ of the rail geometry. An optimum method for geometrical weld assessment would be the evaluation of the spectrum of the contact forces occurring for the weld under consideration. In such a case the spectral amplitudes should not exceed predefined values. This would require the

computation of the dynamic contact force between wheel and rail weld, followed by a spectral analysis in the frequency domain, on the basis of the sampled vertical rail geometry. In case the considered weld does not satisfy the requirements, the weld geometry should subsequently be changed in such a way that it would satisfy the norms. As these norms are expressed in terms of forces and not of geometry, an additional computational effort is required to determine the most efficient way to arrive at an acceptable geometry in an iterative process. In a simplified approach only the dynamic contact force in the time domain should be calculated for the considered weld, and the maximum force should not exceed a predefined value. However, the above methods have two disadvantages which make them unsuitable for practice. In the first place, the wheel-rail contact force magnitude depends to a large extent on the properties and the configuration of the track structure. Therefore, the influence of the local track properties should be accounted for, which makes a uniform assessment, independent of the track type and component properties, impossible. A second problem is of a computational nature. Basically, the solution of the dynamic wheel-rail contact problem in a linear form asks for the solution of a set of second order differential equations. It is not feasible in practice to perform these calculations for each separate rail weld each time it is being measured.

A compromise between theoretical efficiency and practical feasibility, at the cost of exactitude, can be looked for by trying to correlate a parameter describing the weld geometry and the maximum contact force that would occur for a predefined reference track and wheelset. A further, second enhancement is the implementation of the effect of the train speed on the track section where the weld is installed. The objective is then to develop a practical tool for the assessment of the rail weld geometry, by directly relating the vertical rail geometry to the magnitude of dynamic contact forces which occur for that geometry, without having to perform complex dynamic calculations for each separate geometrical measurement.

In order to account for the vertical rail geometry along the weld, it makes sense to correlate the following geometrical parameters to the maximum dynamic wheel-rail contact force:

- i) the extreme value of the 0th derivative – the original measurement signal
- ii) the extreme value of the 1st derivative of the signal
- iii) the extreme value of the 2nd derivative of the signal

Each of these geometrical parameters can be limited by a threshold value, dependent on the train speed, in an assessment procedure.

Method i) is the traditional approach that has been outlined in the previous, and in which vertical tolerances have to satisfy requirements. It can only be enhanced by differentiating on the line section speed.

Before application of the methods i) through iii), it is necessary to realize that the measurement signal obtained with a digital straightedge is a discrete signal, and that therefore the non-zero sampling interval plays a role in the determination of the extreme values in ii) and iii). Therefore, it must be standardized in an assessment procedure. Conventional digital straightedges sample the rail geometry at an interval of 5 mm. This is generally smaller than the length of the wheel-rail contact patch longitudinal axis (ranging approximately between 12 and 20 mm) [4.27, 4.29, 4.30]. Although micro-asperities within the wheel-rail contact patch are relevant for the radiation of noise (see e.g. Ref. [4.31]), they do not play a role in the dynamic wheel-rail interaction. Furthermore, after some train passages the micro-asperities on the rail surface, remaining after final grinding (and depending on the grinding type), are smoothed out by plastic deformation. Finally, each measurement device has a finite accuracy, which introduces 'noise' into the measurement signal. The contribution of noise to the wave-spectrum of the irregularity is especially relevant in the short-wave regime, where the amplitudes generally are very small and the 'noise' becomes dominant. The precise magnitude of all these effects is unknown and would need further investigation; it also depends on the used device. To deal with these effects, in this study the measurement signal obtained by digital devices is subjected to a filtering. This filtering procedure consists of two steps: in the first step a running 5-point average through the signal is determined (this is an averaging over 25 mm), and in a second step each 5th sampling point is taken. The latter choice is also dictated by a practical requirement related to the assessment: it is not feasible to grind micro-irregularities on the railhead with a shorter length-scale than approximately 25 mm. An example of an original measurement signal obtained with a digital straightedge (RAILPROF, based on the eddy current measurement principle) along with the result after application of the filtering is shown in Fig. 4.13.

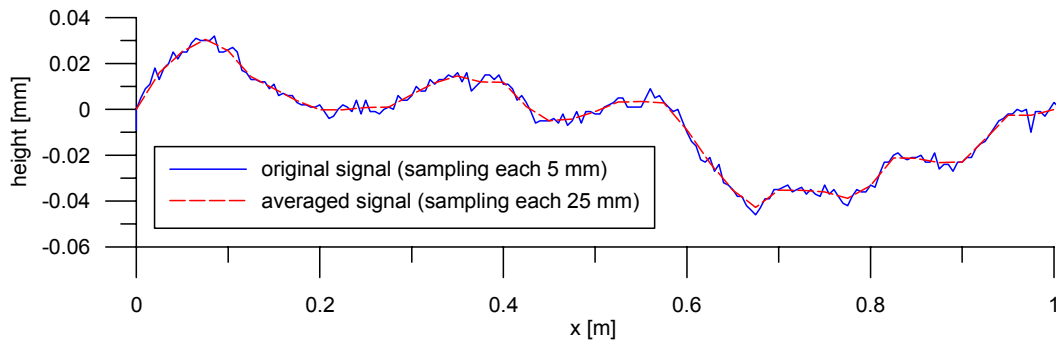


Fig. 4.13 Measured longitudinal rail geometry and averaged signal

In Fig. 4.14 an example is given of the application of both methods ii) and iii) for a weld geometry measurement, based on the filtered measurement signal. The averaged measurement signal and the first and second derivatives are shown, each of them normalised with the respective maximum of its absolute value. It is clear that both methods ii) and iii) provide a good evaluation of the ‘smoothness’ of the wheel-rail interface geometry.

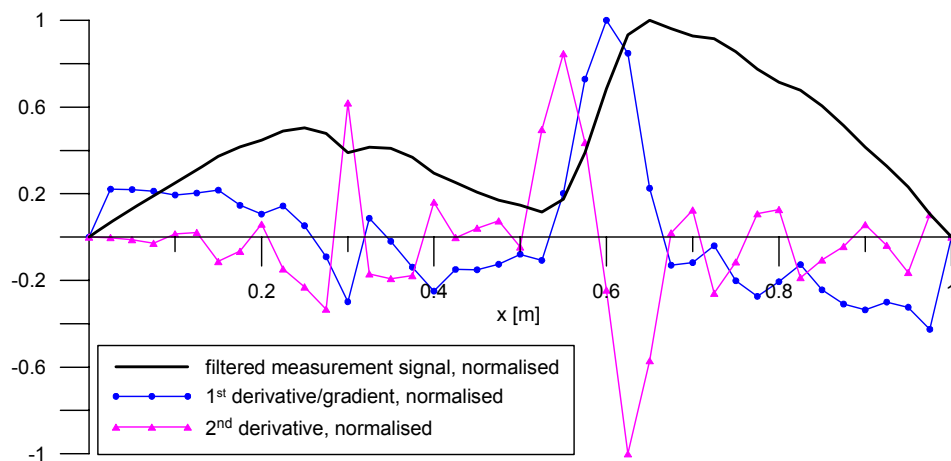


Fig. 4.14 Averaged discrete measurement signal, first and second derivatives, each normalized with their maximum absolute value

In Ref. [4.32] the performance of the methods ii) and iii) has been compared for a sample of actually measured rail welds (72). This comparison lead to observations as discussed and illustrated in the following.

In general, method ii) leads to more moderate values than method iii) (as can also be observed in Fig. 4.14). Method iii) appears to be extremely sensitive to very short irregularities (with a length-scale of a few centimetres) and, on the contrary, not very sensitive for longer irregularities (with a length-

scale in the order of half a metre). This is not true for method ii) (as can also be observed in Fig. 4.14).

In Fig. 4.15, an example is given of an almost perfectly straight weld, however showing small indentations due to non-uniform shrinkage after welding and grinding, as has been discussed in section 4.2. Method iii) is particularly sensitive to these indentations.

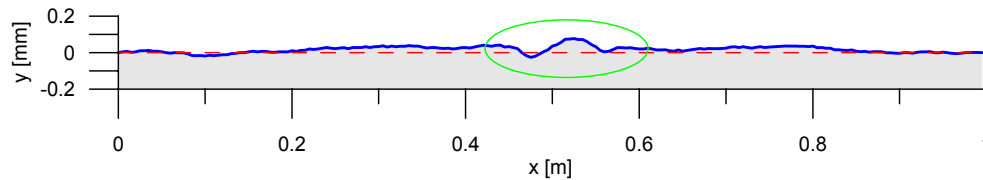


Fig. 4.15 Example of a weld with indentations due to non-uniform shrinkage after welding and grinding

Fig. 4.16 shows a typical example of a weld for which method iii) has a bad performance and ii) a much better performance. The weld geometry is dominated by an irregularity with a longer length-scale and has a very bad quality (maximum height 0.8 mm).

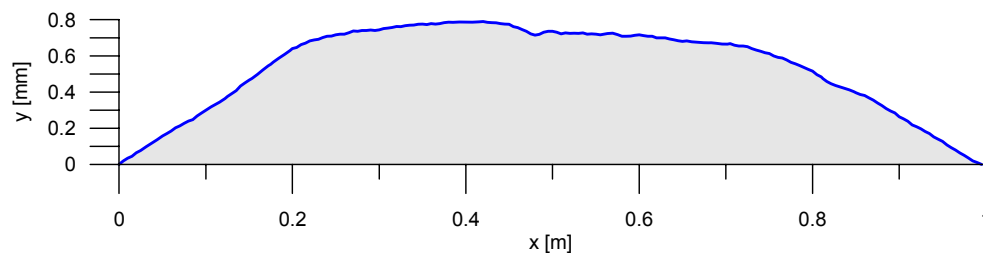


Fig. 4.16 Example of a weld geometry dominated by an irregularity with a longer length-scale

Given the above drawbacks of method iii), method ii), based on first derivatives, and being equally sensitive to short and longer irregularities that can be present in a rail weld, proves to be most appropriate for practical implementation.

In Figs. 4.17 and 4.18, examples are shown of the practical differences between the traditional assessment method according to i) and method ii). The weld in Fig. 4.17, which shows an aggressive step due to a bad alignment of the rail ends before welding, is accepted according to many standards allowing a vertical tolerance of +0.3 mm. However, according to method ii), the weld is rejected according to the large magnitude of the first derivative at

the transition between both rail ends. In Fig. 4.18, an example is shown of an almost perfect weld. However, because it has some negative height coordinates, it is rejected according to traditional standards, which do not allow negative values. The derivatives do not show pronounced peaks and therefore the weld can be accepted according to ii).

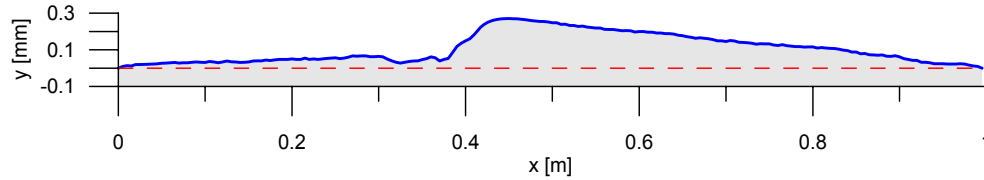


Fig. 4.17 Example of a weld with a pronounced step due to bad rail end alignment

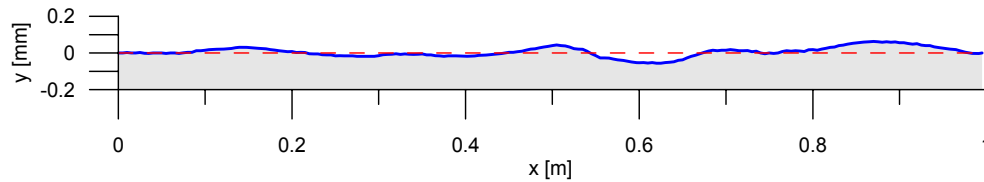


Fig. 4.18 Example of a weld with a ‘smooth’ surface but with negative height coordinates

4.4 *Simplified dynamic wheel-rail interaction modelling*

In the previous section, different rail weld assessment concepts were discussed from a practical viewpoint. A relationship between the geometrical parameter describing the rail ‘smoothness’ and the level of the dynamic wheel-rail interaction force was not established. In order to do so, a theoretical approach is necessary. This will at the same time allow to account for the effect of the train speed.

In Fig. 4.19, simulation results are shown for a train vehicle, passing an artificial irregularity in the track at a speed of 140 km/h. The irregularity is typical for a welding irregularity: it comprises both a smooth irregularity with a longer length-scale (a harmonic wave with a length of 1 m and a top value of 1 mm), and a second non-smooth irregularity with a short length-scale (a triangular peak with a basis of 100 mm and a top of 0.2 mm).

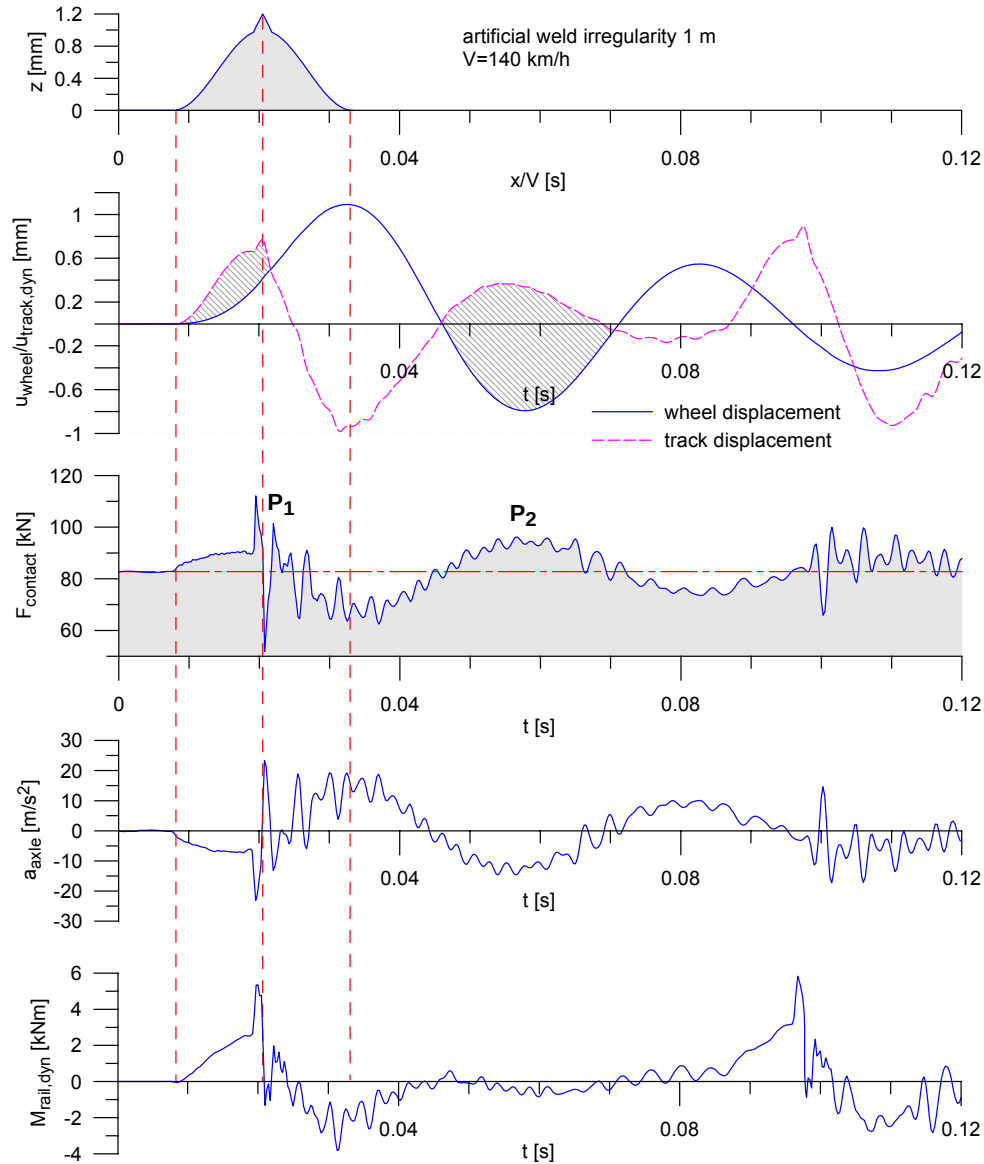


Fig. 4.19 Dynamic response of the wheel-rail system to an artificial rail weld with both a long and short length-scale irregularity

The results in Fig. 4.19 have been obtained from finite element simulation with the DARTS-NL package [4.33]. An overview of the model is given in Fig. 4.20. The rail is divided into beam elements with a length of 10 mm and a nodal distance of 5 mm; therefore each element has 3 vertical coordinates. The irregularity is centred in the middle of a sleeper span, where the weld is made in practice. The time-step in the calculations is taken as the ratio of the element size to the train velocity (a reduction of this time-step by 50 percent leads to deviations in the results within a margin of 3 percent). A large number of sleeper bays are included, with non-reflective boundaries at both

ends, though it is remarked that the number of sleeper bays is irrelevant, given the very local effect of the weld irregularity.

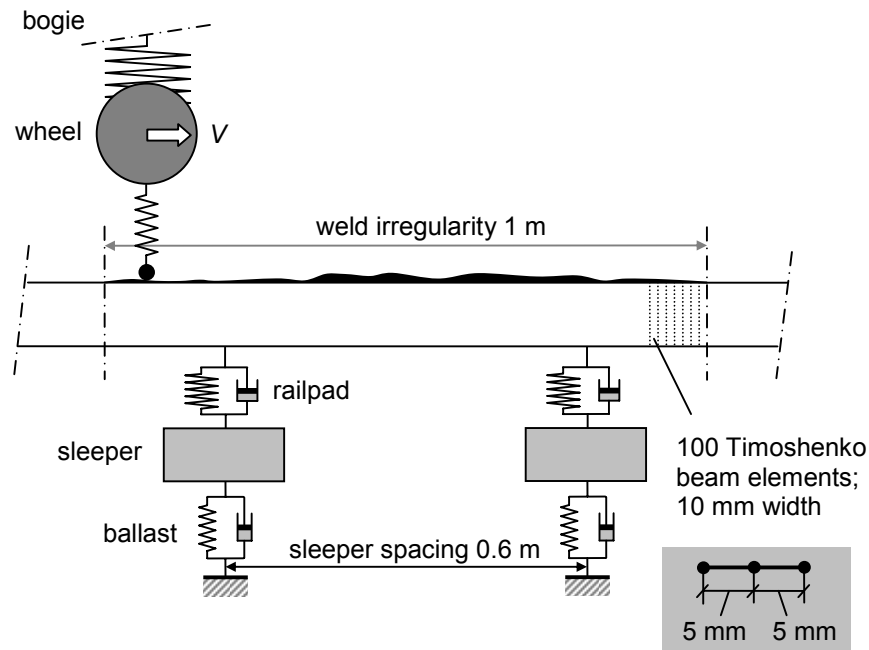


Fig. 4.20 FE model for simulation of the wheel-rail interaction at rail weld surface irregularities

The following parameter values have been adopted in the calculation: a wheel mass (half un-sprung mass) of 970 kg; a sleeper mass of 300 kg; the 54E1 rail profile is used; the primary suspension stiffness equals $1.8 \cdot 10^6$ N/m (per wheel), the rail-pad stiffness $1.2 \cdot 10^9$ N/m and the ballast stiffness $30 \cdot 10^6$ N/m per sleeper. The latter value is rather small, to account for the generally bad compaction of the ballast underneath the sleepers close to the irregularity. The package uses a non-linear Hertz contact model.

In Fig. 4.19 the following quantities are shown as a function of time:

- the geometry of the irregularity z
- the dynamic component of the vertical wheel displacement of the first wheel of a passing bogie, defined positive in upward direction and calculated in a convective reference frame moving along with the wheel
- the dynamic component of the displacement of the rail/track, calculated in a static coordinate system with its origin at the centre of the irregularity and defined positive in downward direction
- the wheel-rail contact force for the concerned wheel (the dynamic force is superimposed on the static value), in a convective reference frame

- the axle box acceleration, in a convective reference frame
- the dynamic component of the bending moment in the rail, in a fixed coordinate system at the centre of the irregularity

From Fig. 4.19, a number of observations can be made. They are as follows:

- the track/rail follows the vertical irregularity quasi-instantaneously; especially for the short-length irregularity this is clearly visible. The wheel displacement shows a delay in response relative to the track. It does not show any influence of the short peak in the irregularity, whereas this peak is reproduced almost exactly in the rail displacements.
- the maximum dynamic contact force is determined by the irregularity with the shortest length-scale. However, the corresponding force peaks (P_1 forces) are rather narrow, and the corresponding amount of energy is rather small. These high-frequency peak forces damage mainly the rail itself, as can be observed also from the dynamic bending rail moments. The related energy input will vanish by wave propagation in the rail and dissipation in – mainly – the railpads.
- the highest energy of the dynamic contact force is contained in the ‘carrier frequency’, which can be clearly observed. This carrier frequency is related to the delayed reaction of the wheel mass on the overall track stiffness to the irregularity (the P_2 force). The relatively large energy contained in the relatively low carrier frequency is mainly responsible for ballast bed deterioration, as it is not efficiently dissipated in the rail and the railpads. Due to the P_2 force often a difference in ballast settlement or compaction can be observed in practice: the ballast bed at a few metres of a weld irregularity with a long length-scale is often well consolidated, because the sleepers are pressed into the ballast by all passing axles.

In a global sense it can be said that components of irregularities which have a longer length-scale than the wheel radius or the sleeper span (0.5 – 0.6 m) damage the ballast-bed, whereas components with shorter length-scales damage the railhead and rail (Fig. 4.21).

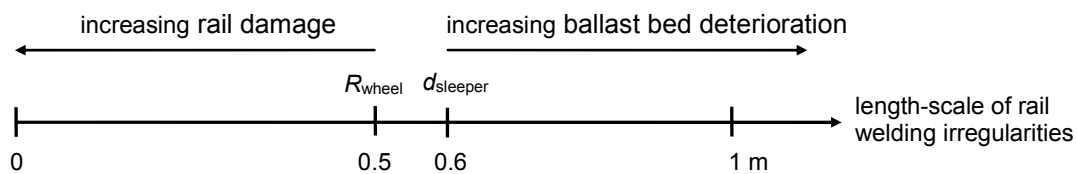


Fig. 4.21 Types of deterioration of the track system dependent on the length-scale of rail welding surface irregularities

As has been concluded in the previous, short irregularities in a welding irregularity lead to P_1 forces, whereas longer irregularities lead to P_2 forces. This effect occurs mainly due to the difference in inertia between the unsprung wheel mass and the equivalent track mass (Fig. 4.22): assuming an equivalent track mass comprising two fully effective concrete sleepers of each 300 kg and four meters of effective rail mass (60 kg/m; 60E1 profile) the equivalent track mass can be estimated at 850 kg. This is less than fifty percent of the unsprung axle mass, which is approximately 1800 kg for conventional trains. The response of the wheel to the short excitation therefore shows a delay relative to the response of the track (rail and sleepers). The P_1 - force, which is a quasi-instantaneous amplification of the wheel-rail contact force, originates from the reaction of the track and is mainly determined by the rail properties (bending stiffness and inertia). The P_2 - force results from the reaction of the wheelset on the track stiffness to the excitation. Its magnitude is determined mainly by the unsprung wheel mass and the equivalent track stiffness; the time-scale of the peak is larger due to the relatively low eigenfrequency of the dominating mass-spring system. In the case of occurrence of P_1 forces, P_1 and the subsequent P_2 force are interrelated, i.e. the magnitude of P_2 is determined by P_1 .



Fig. 4.22 Unsprung vehicle mass (wheelset) on a ballasted railway track

For the reasons explained in the previous, the wheel mass may be considered as fixed in vertical direction during the time interval that the wheel passes the weld irregularity, in a very simplistic model that focuses on the maximum value of the wheel-rail contact force (Fig. 4.23, left). As the Hertz contact is far more stiff than the equivalent track stiffness, it is assumed to be rigid in the model (Fig. 4.23, right). The combination of both the previous

assumptions will allow for a reduction of the system description from a differential formulation to an algebraic formulation, directly relating the rail geometry to the contact force. Fig. 4.23 (right) shows the track – wheelset model resulting from the above assumptions, disturbed by the weld irregularity $z(x)$ moving at train speed V , and valid during the time interval of passage of the weld.

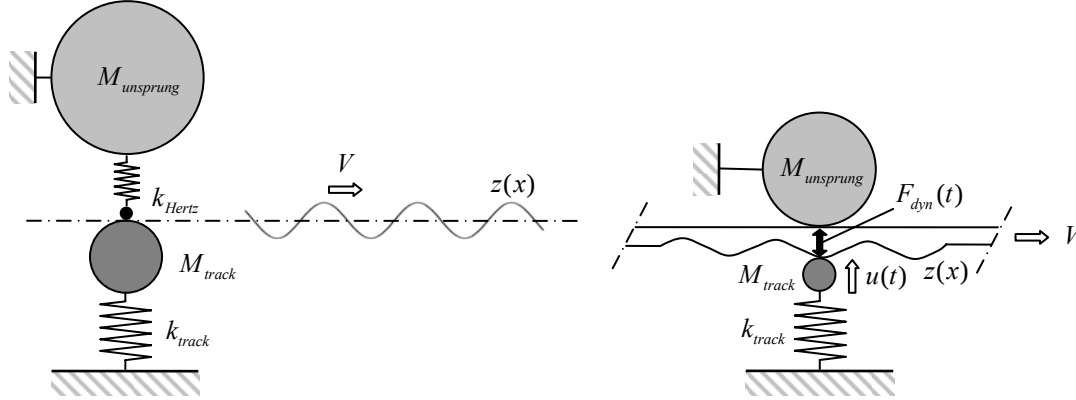


Fig. 4.23 Simplified wheel-rail interaction models and disturbance by a rail irregularity

The equivalent track mass was assumed to be forced to follow the irregularity, or $u(t) = z(t)$, which means that the degree of freedom of the system is prescribed by the excitation. The equation of motion of the system in Fig. 4.23 (right) is then simply given by:

$$M_{track} \ddot{z}(t) = k_{track} z(t) \quad (4.2)$$

or, the system is described by a simple algebraic equation in terms of z instead of a differential equation (or a set of differential equations) in terms of z and u . The dynamic contact force in the model of Fig. 4.23 (right) is then directly proportional to the excitation as a function of time:

$$F_{dyn}(t) \sim M_{track} \ddot{z}(t) \sim k_{track} z(t) \quad (4.3)$$

Using the transform $z_{,tt} = V^2 z_{,xx}$ from the temporal to the spatial domain and introducing a dimensionless calibration factor, the maximum dynamic contact force may be written as:

$$F_{dyn, \max} = \beta M_{track} V^2 \left| z_{,xx} \right|_{\max} \quad (4.4)$$

It is possible to express the force (4.4), which is expressed now in terms of the maximum second derivative of the measurement signal, in terms of the maximum first derivative of the signal. For a discrete signal, a relationship exists between the maximum first derivative and maximum second derivative, under the condition that the magnitude of one of both is subject to a limitation. The relationship however is not unique, having a certain bandwidth, as will be shown in the following.

The first derivative of the measurement signal at sample position i is defined according to:

$$dz_i / dx = (z_{i+1} - z_{i-1}) / 2d; \quad (4.5)$$

the second derivative at the same sampling position is defined according to:

$$d^2 z_i / dx^2 = (z_{i+2} - 2z_i + z_{i-2}) / 4d^2. \quad (4.6)$$

In accordance with method ii) from section 4.3, a limitation of the first derivative of the measurement signal is assumed: $|dz_i / dx|_{\max} = \psi$. The limits of the second derivative are then given by:

$$\frac{d^2}{dx^2} z_i = \frac{1}{2d} \left(\frac{z_{i+2} - z_i}{2d} - \frac{z_i - z_{i-2}}{2d} \right) = \frac{\pm\psi \pm \psi}{2d} = \pm \frac{\psi}{d} \quad (4.7)$$

or

$$\left| \frac{d^2 z_i}{dx^2} \right|_{\max} = \frac{1}{d} \left| \frac{dz_i}{dx} \right|_{\max} \quad (4.8)$$

The relationship is illustrated in Fig. 4.24. The actual correlation between the maximum absolute first and second derivatives for a sample of 239 rail weld measurements is illustrated in Fig. 4.25. It is clear that the correlation is rather good. Substitution of (4.8) into Eq. (4.4) yields:

$$F_{\text{dyn, max}} = \beta M_{\text{track}} V^2 \frac{1}{d} \left| \frac{dz_i}{dx} \right|_{\max} \quad (4.9)$$

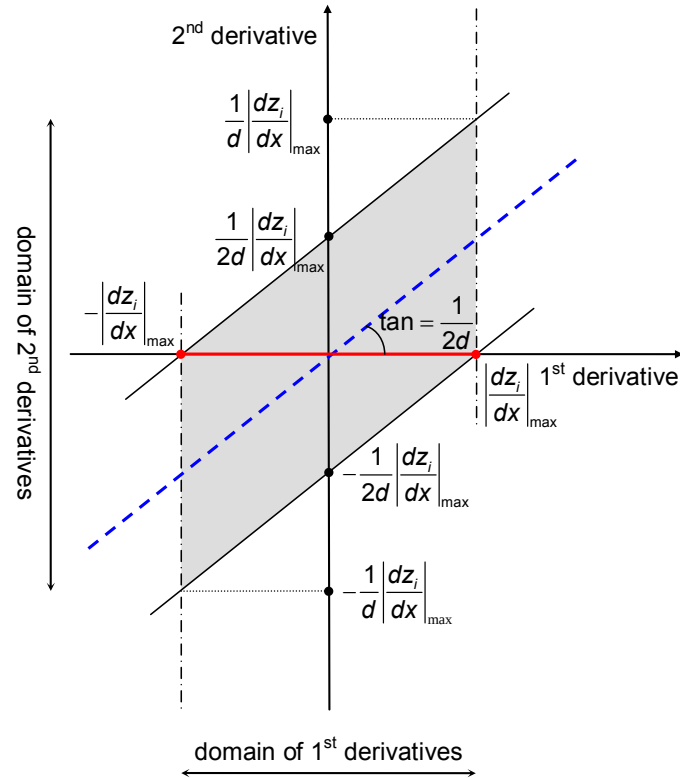


Fig. 4.24 Implicit limitation of the second derivative by limitation of the first derivative of a discrete signal

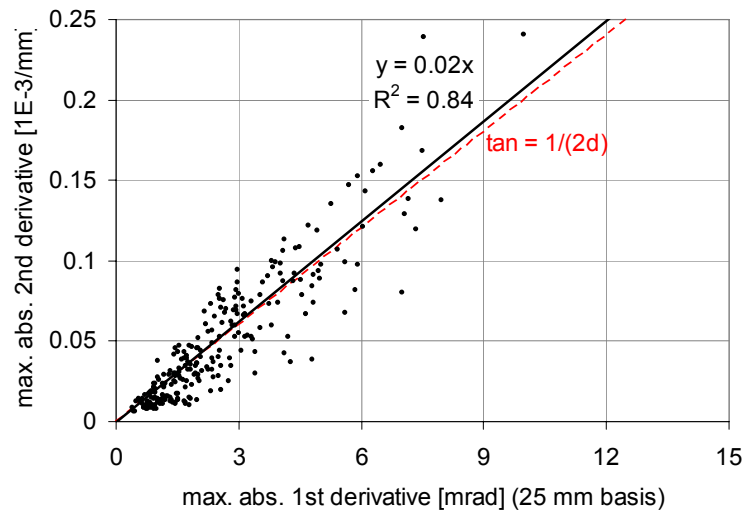


Fig. 4.25 Actual correlation between the maximum absolute first and second derivatives of a sample of 239 welds measurements (derivatives on 25 mm basis)

Therefore, a theoretical correlation is shown to exist between the maximum absolute first derivative of a discrete excitation signal and the maximum absolute value of the contact force. However, the sense of such a relationship decreases with decreasing sampling interval, because its bandwidth grows to

infinity. It is important to note that the relationship vanishes completely in the limit, for a continuous signal, and is therefore non-physical. In Eq. (4.9) the full bandwidth was used to determine the maximum (in Fig. 4.24, the boundaries of the ‘band’ were used). From a statistical point of view it can be argued that the discrete maximum first derivatives have a distribution which is, over a large part of the first derivative domain, close to normal with zero mean (as will be shown in the following section 4.6). Therefore, a zero bandwidth will be more realistic when maximum contact forces for a large number of welds are considered. Using the central relationship in Fig. 4.24 with zero bandwidth yields:

$$F_{\text{dyn, max}} = \beta M_{\text{track}} V^2 \frac{1}{2d} \left| \frac{dz_i}{dx} \right|_{\text{max}} \quad (4.10)$$

4.5 *Enhanced weld geometry assessment concept: the QI*

On the basis of the findings of sections 4.3 and 4.4, and mainly Eq. (4.10), a concept can be elaborated for the assessment of rail weld geometry, both accounting for the ‘shape’ of the rail surface irregularity, by means of its maximum first derivative, and the train speed on the line section where the weld is made. Combination of both parameters can give a good indication of the level of the dynamic force occurring at rail welds, which can be limited in a standardization procedure. The introduced simplifications however necessarily are compensated for by an accuracy reduction: the norm value of the force has a distribution with a non-zero bandwidth.

For assessment purposes, the dimensionless rail weld Quality Index (QI) is introduced. It is defined as the actually measured maximum absolute first derivative (on 25 mm basis) normalised with a norm value, which can be defined differently for different values of the line section speed:

$$\text{QI} = \frac{\left| \frac{dz_i(x)}{dx} \right|_{\text{max, actual}}}{\frac{dz_i}{dx}_{\text{norm}}} \quad (\leq 1: \text{acceptance}; >1: \text{rejection}) \quad (4.11)$$

The norm values $d\zeta/dx_{\text{norm}}$ for different line section speeds should be established such that the dynamic force, according to Eq. (4.10), is constant. The concept discussed in this section will be elaborated both qualitatively and quantitatively in the following sections.

4.6 Statistical properties of first derivatives of rail and rail weld measurements

Fig. 4.26 shows the cumulative distribution and the normal probability test for the discretised first derivatives (not their extreme values) for a sample of 110 rail weld measurements (thus, given the discretisation basis of 25 mm and 200 sample points per weld, 4.400 derivatives are displayed).

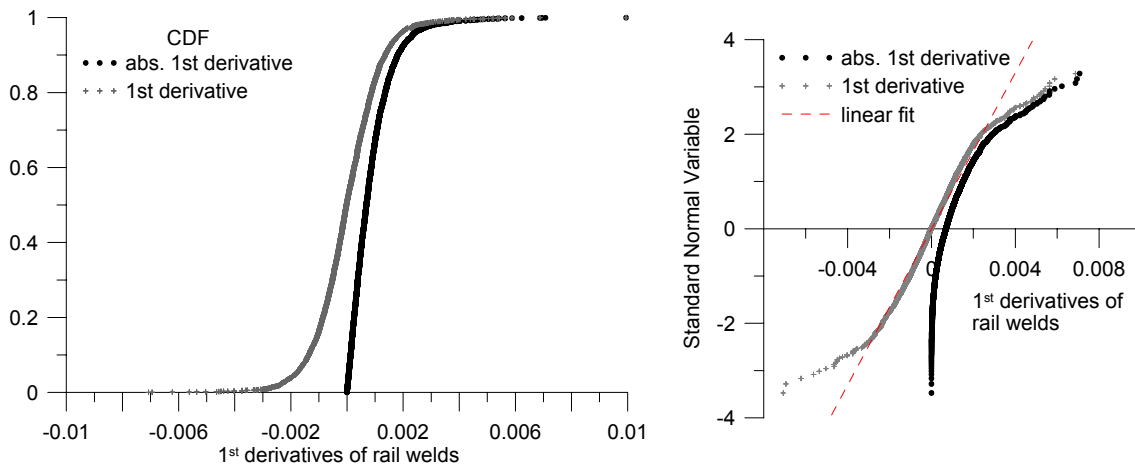


Fig. 4.26 Cumulative distribution of the first derivative of the rail weld geometry and its absolute value, derived from measurements (left) and normal probability test (right)

Fig. 4.27 shows the normal and the lognormal probability test for the maximum absolute first derivative for the set of 110 rail weld measurements.

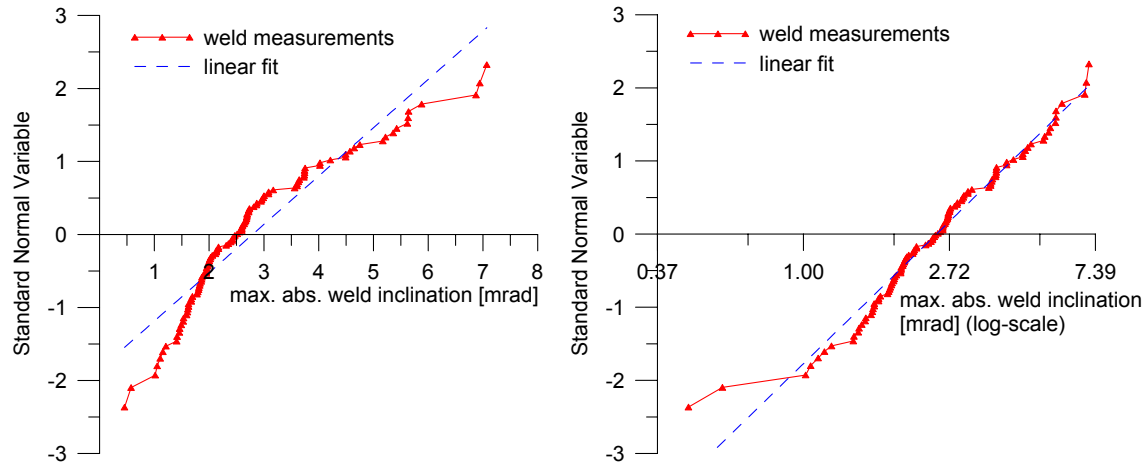


Fig. 4.27 Normal (left) and lognormal (right) approximation of the probability of the maximum absolute first derivatives of 110 rail weld measurements

For comparison, a sample of 100 measurements has been taken on new straightened longrail [4.34]. Fig. 4.28 shows the cumulative distribution and the normal probability test for the first derivatives (not their extreme values) of these 100 rail segments (thus, 4.000 derivatives are displayed).

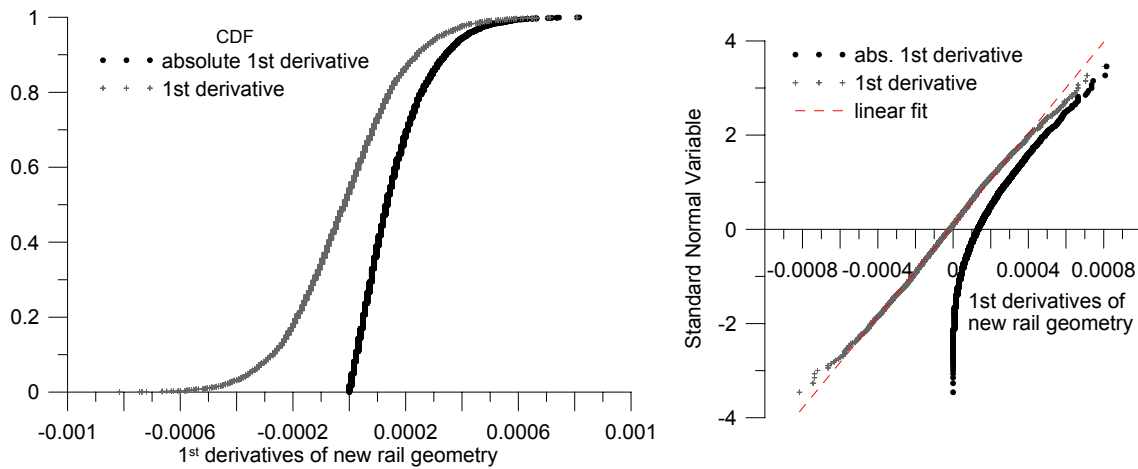


Fig. 4.28 Cumulative distribution of the first derivative of new vertical rail geometry and its absolute value (left) and normal probability test (right)

Fig. 4.29 shows the normal and the lognormal probability test for the maximum absolute first derivative for the set of 100 rail measurements.

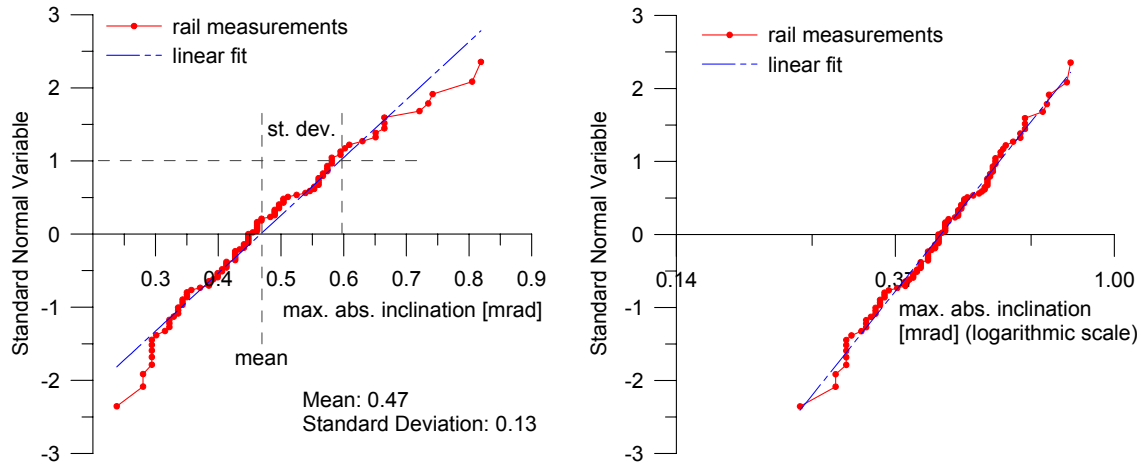


Fig. 4.29 Normal (left) and lognormal (right) approximation of the probability of the maximum absolute first derivatives of 100 rail segments

Comparing Figs. 4.28 and 4.29, it is obvious that the maximum absolute first derivative of the rail geometry is very well approximated by a lognormal distribution, since the first derivative itself has an almost perfect normal distribution: the linear fit passes perfectly through the origin (indicating a zero mean value). The maximum of a perfectly normally distributed random variable has a type I – Gumbel distribution, which is an exponential distribution. Taking the absolute values apparently only has a slight influence in this case.

Comparison of Figs. 4.26 and 4.27, for the weld statistics, shows similar but not identical features. The first derivative of the weld geometry can be well approximated as normally distributed within the interval $[-3, 3 \text{ mrad}]$. Also in this case the linear fit passes through the origin. Only the tails at both sides show significant deviations from this normal distribution. This is explained as follows. The considered welds have been assessed according to a norm based on tolerances. Probably, the tails show the data points at the coordinates along the rail where the geometry did not satisfy the requirements, and where the normal, random distribution was ‘disturbed’. This is confirmed by the rail measurements in Fig. 4.28, where these ‘tails’ do not occur. The effect of the deviating tails on the distribution of the maximum absolute first derivative of the rail welds however appears to be rather limited; it can be reasonably well approximated as lognormal.

4.7 *The relationship between the weld geometry and the dynamic wheel-rail interaction; FEM simulations*

The FE model introduced in section 4.4 has been used to investigate the quantitative relationship between the rail weld geometry and the dynamic wheel-rail response. The simulations have been performed for a sample of 239 rail weld measurements; a CDF of the maximum absolute inclinations of these welds (the filtered signals have been used, with 25 mm basis) is shown in Fig. 4.30. The model parameters have been used as defined in section 4.4, except for the ballast stiffness, which was taken as $78 \cdot 10^6$ N/m per sleeper, which is a conventional value. The simulations have been performed for different train velocities: 40, 80, 140 (conventional lines) and 300 km/h (high-speed lines) respectively.

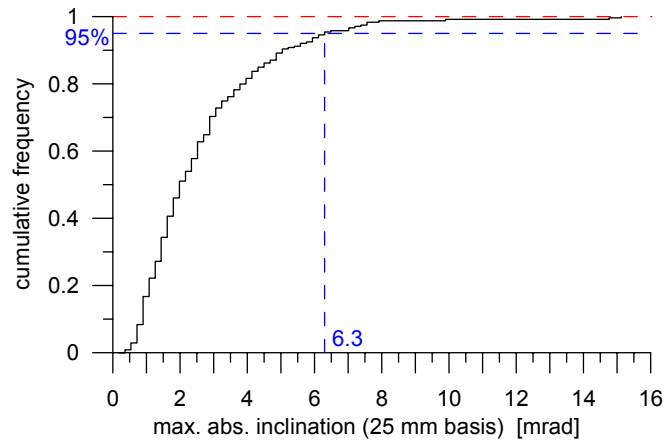


Fig. 4.30 Cumulative distribution of the maximum absolute inclinations of the sample of 239 rail welds, used for FEM simulations

These simulations have been reported by Steenbergen and Esveld in [4.35] and [4.36]; only some final results will be reproduced and briefly discussed here. The relationship between the parameters describing the weld quality (peak deviation, maximum absolute first derivative and also the maximum absolute second derivative) and the parameters describing the dynamic wheel-rail response (maximum dynamic wheel displacement and rail deflection, maximum dynamic contact force, maximum dynamic rail bending moment and shear force) was investigated by linear regression analysis.

The maximum wheel displacement showed a very good correlation with the peak deviation of the weld geometry. Further, the relationship between the wheel displacement and the peak deviation could be

approximated as directly proportional; only at 300 km/h the growth of the wheel displacement was shown to be less than directly proportional. The correlation between maximum wheel displacements and maximum absolute first derivatives was found to be very low for all velocities.

The maximum dynamic rail deflection showed an almost perfect correlation with the vertical peak deviation of the weld geometry, except for 40 km/h. Furthermore, the relationship between maximum dynamic rail deflections and the peak deviations of the weld geometry could be assumed directly proportional in a good approximation (again, except for 40 km/h). This justifies the global assumption that was made in the modelling of the wheel-rail contact in section 4.4, according to which the track was forced to follow the vertical rail irregularity quasi-instantaneously.

Maximum dynamic wheel-rail contact forces generally showed a good correlation with the maximum absolute first derivative. This correlation was found to increase with the speed. The correlation of the maximum forces with the peak deviations of the weld geometry on the contrary was found to be extremely poor for all velocities.

Generally, the maximum dynamic rail bending moments were shown to correlate much better with the gradient than with the peak deviation; again the correlation improved with the velocity. The same was found for maximum dynamic rail shear forces.

The variables involved in the dynamic wheel-rail interaction in general were found to behave similarly as a function of both maximum first and second derivatives. This was due to the good correlation between maximum first and second derivatives of the weld measurements (Fig. 4.25).

In Fig. 4.31 at the left, the relationship between the maximum dynamic wheel-rail contact force and the maximum absolute first derivative (on 25 mm basis; the filtered measurement signal is used) is shown for the different velocities for which simulations were performed. For each velocity, a linear best fit through the origin is displayed, resulting from the FEM simulations (thus, scatter is not being accounted for).

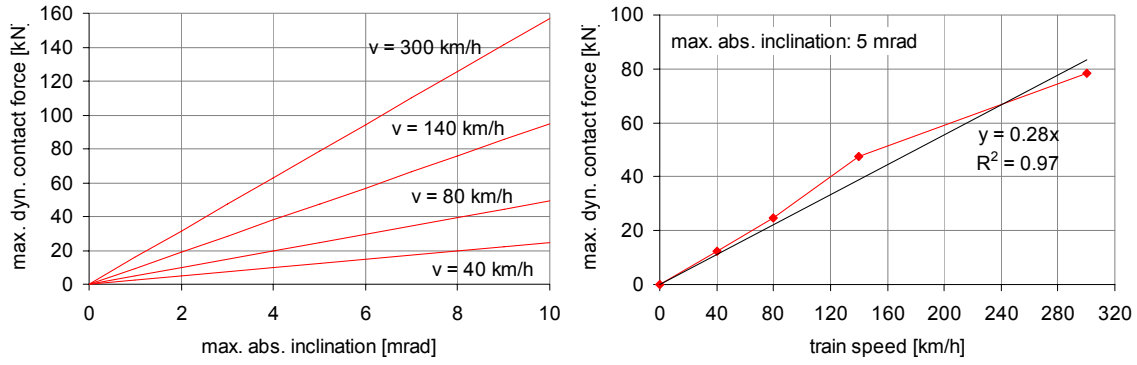


Fig. 4.31 Dependence of the dynamic wheel-rail contact forces at rail welds on the train velocity, found from FEM simulations

Fig. 4.31 shows at the right the relationship between the contact force and the train speed for a predefined maximum absolute first derivative, which is taken as 5 mrad (on 25 mm basis). It is observed that the influence of the velocity is not quadratic, as was predicted from Eq. (4.9). In general the relation between the force and the speed can be well approximated as linear, yielding the following adapted relationship:

$$F_{\text{dyn, max}} = \beta M_{\text{track}} V \frac{1}{d} \left| \frac{dz_i}{dx} \right|_{\text{max}} \quad (4.12)$$

In this expression, the dimensions are no longer consistent, implying a dimension-bearing validation coefficient. This issue will be addressed in section 4.8. The linear relationship between the maximum dynamic contact force (or the DAF) and the train speed is confirmed experimentally by the results in [4.15]. Validation of the linear relationship (4.12) between the maximum dynamic wheel-rail contact force, the train speed and the maximum absolute first derivative on the basis of the FEM results summarised in Fig. 4.31 leads to the following approximate expression, in terms of the variables V [m/s] and $\left| dz_i / dx \right|_{\text{max}}$ [mrad] (on 25 mm basis):

$$F_{\text{dyn, max}} = 0.22 \cdot V \cdot \left| dz_i / dx \right|_{\text{max}} \text{ [kN]} \quad (4.13)$$

The formula is valid in the speed regime 0 – 300 km/h. It is observed that a very similar result was found, both from simulations and experiments, by Jenkins et al. in 1974 [4.37] for the peak forces occurring at dipped rail joints. Results were reflected in the well-known Jenkins formula for the calculation

of P_1 and P_2 forces. They found these peak forces to be governed by the dip angle and the train speed; furthermore the relationship between maximum dynamic contact forces and both variables proved to be linear in a good approximation. Jenkins' results for rail joints, together with the results obtained for rail welds, are shown in the graph of Fig. 4.32. In this figure, the dynamic peak forces that occur for rail joints and welds are depicted as a function of the product of the maximum absolute inclination and the train speed; $a \approx \tan a$ is defined as the angle of a dipped rail end with the horizontal for rail joints, and as the maximum absolute inclination (on 25 mm basis) of the geometry for rail welds. It is clear that the force level for rail welds is significantly lower than for rail joints; the reduction factor is approximately 3. This is partly due to the quasi-static rail deflection under axle loading, which is much larger for a jointed rail (behaving as a hinged beam) than for a welded rail. This quasi-static deflection increases the dip angle at the moment of wheel passage [4.1].

The dynamic force cannot grow unlimitedly with the train speed. A trend to tail off between 140 and 300 km/h is already visible in Fig. 4.31. At higher speeds phenomena like contact loss also may start to play a role and cause asymptotic behaviour.

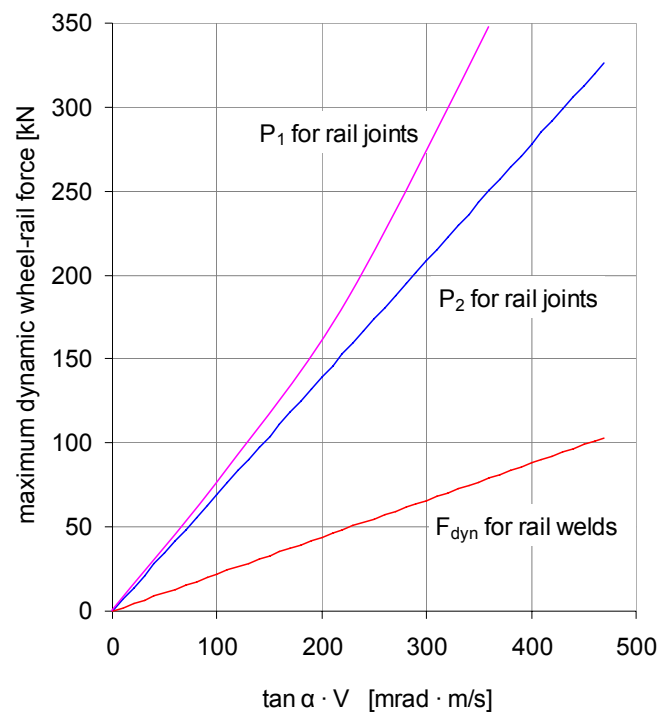


Fig. 4.32 Maximum dynamic wheel-rail forces for rail welds and rail joints, as a function of the product of the maximum inclination (25 mm basis) or the dip angle and the speed

4.8 *The relationship between the weld geometry and the dynamic wheel-rail interaction; analytical investigation*

In the previous section, FEM results from [4.35] of dynamic wheel-rail interaction for measured welds were discussed. However, FEM simulations have a number of disadvantages. These include the sensitivity of quantitative results in the high-frequency regime (500-2000 Hz) to filtering and damping (see also [4.38, 4.39]), the validation of artificial damping and the simulation time needed for large measurement samples. Further, system features that are at the basis of trends cannot be investigated properly with a time domain FEM – approach. In this section the dynamic wheel-rail interaction at rail welds is therefore investigated with an analytical approach, based on the simplified wheel-rail interaction model presented in chapter 3. The problems connected to FE modelling are avoided, but the method also has a disadvantage: because of the fact that the model is simplified and its linearity, it is not suitable for quantitative predictions with a high level of accuracy. The aim of this chapter is therefore not to compute quantitative results with a high accuracy level, but to clarify the basic mechanisms in dynamic wheel-rail interaction due to short disturbances in the interface, to establish theoretical relationships, and to investigate and compare trends and their backgrounds. Furthermore, the general formulations will allow for an application of the results of the investigation on any wheel-rail interfacial irregularity with a broad-band spectrum.

The use of the model of chapter 3 to analyse the dynamic wheel-rail contact in the presence of rail welds is based on the following assumptions:

1. The irregularity of the rail surface is short compared to the ratio of the train speed to the eigenfrequency of the bogie motion. Under these circumstances ($f > 20$ Hz), the wheelset motion may be considered as isolated from the bogie and car body motion. Generally, this holds for welding irregularities, which have a length-scale of about 1 m. In this case, only the unsprung vehicle mass plays a role.
2. The considered irregularities lead to continuous single-point contact in time. Transient two-point-contact between wheel and rail leads to completely different mechanisms in the wheel-rail-interaction, as has been shown in chapter 2, and is therefore avoided. This is realistic for modern undamaged rail welds (but not for rail joints, like insulated rail joints or bolted connections).

3. The rail is modelled as continuously supported. This is a significant simplification, which will especially affect the model results for welding irregularities having a predominant wavelength that is comparable to the sleeper distance (the pinned – pinned rail resonance is around 1050 Hz, see [4.40]). However, this simplification is justified by the following reasons. As has been stated already, it is not the primary aim of this section to predict quantitative results with a high level of accuracy, but to investigate basic features that can explain trends. Moreover, rail welding irregularities generally have a broad-band spectrum and do not exhibit any preferred wavelength. Therefore, the broad spectrum of the excitation has an averaging effect on peaks in the track receptance. This effect becomes even stronger when trends are considered for a large number of measured welds. Finally, it was shown in section 4.4 that the shortest waves in the rail irregularity generate the highest peak contact forces. This is because the inertia of the system dominates the response. Since these peak forces, and not the time histories, are mainly of interest in this study, the rail support is less relevant. For a comparison of receptance functions of continuously and discretely supported rails, and the effect of the sleeper positions, see e.g. [4.38, 4.41-4.43].

Substitution of Eq. (3.39) into Eq. (3.30) for the contact force yields an explicit integral expression for this force, for an arbitrary rail irregularity. For a single ramp (with one discrete slope, as depicted in Fig. 3.3 (left) the expression becomes:

$$F(t) = \frac{\theta V}{\pi} \int_0^{\infty} \operatorname{Re} \left(\frac{(1 - e^{-s\Delta x/V})/s^2}{1/k_H + 1/(m_w s^2 + k_1) + 1/(8EI k_0^3)} e^{\sigma t} e^{i\omega t} \right) d\omega \quad (4.14)$$

For the sequence of N discrete slopes, the expression for the contact force may be written as follows:

$$F(t) = \frac{1}{\pi} V \sum_{n=1}^N \theta_n \int_0^{\infty} \operatorname{Re}(f_n(s)) d\omega \quad (4.15)$$

in which:

$$s = \sigma + i\omega, \quad f_n(s) = \frac{(1 - e^{-s\Delta x/V}) e^{-(n-1)s\Delta x/V}}{1/k_H + 1/(m_w s^2 + k_1) + 1/(8EI k_0^3)} \frac{e^{st}}{s^2} \quad (4.16)$$

It follows from Eq. (4.14) that a theoretical perfectly linear relationship exists between the slope of a ramp and the dynamic contact force in the time domain. The relation between the contact force and the velocity cannot be extracted explicitly from Eq. (4.14), but it will be shown later on that also this relationship is perfectly linear. The same can be observed for the rail and wheel displacements in the time domain, according to Eqs. (3.31) and (3.32).

It is noted that the time history of expression (4.15) consists of a linear superposition of terms (4.14) giving the force occurring for a single ramp (as is expected for a linear system). This means that the component of the force for each discrete slope θ_n as a function of time is proportional to θ_n . However, the maximum of Eq. (4.15) is only reached for $\theta_{\max}$ if the time functions (the results of the integration over the frequency) in the separate terms do not interact. This is not necessarily the case, and for an arbitrary geometry a mutual influence must be expected, especially as the sampling basis Δx approaches zero. Therefore, a perfectly linear relation between $F_{\max}$ and $\theta_{\max}$ does not exist. The bandwidth of such a relationship depends on the statistical distribution of the slopes for a considered discretely sampled weld population (or, alternatively, the wavelength-spectrum of a weld population). This subject will be reverted to later in this section and in section 4.9.

The wheel-rail system under consideration can be characterised in the frequency-domain by means of the spectrum $\tilde{F}(\omega)$ of the contact force due to a unit pulse excitation $\tilde{\chi}(s)=1$ (according to Eq. (3.23); $s=\sigma+i\omega$ with $\sigma=0$). The absolute value of this spectrum is shown in Fig. 4.33. A narrow and sharp peak appears around 45 Hz, followed by a gradually increasing and declining frequency content in the region between 200 and 3000 Hz, with a maximum around 1000 Hz. The peak at the lower frequency is completely governed by the rail support stiffness, as is shown in Fig. 4.33 for two variations of this stiffness. In the integration process for the excitations that will be considered, this peak only has a minor contribution, indicating a limited impact of the support stiffness on the time-domain response. It is remarked that in the simulations for measured welds the spectrum of the integrand is declining must faster after 2000 Hz than is shown in Fig. 4.33 for a white-noise excitation spectrum, due to the finite frequency content of measured rail welds.

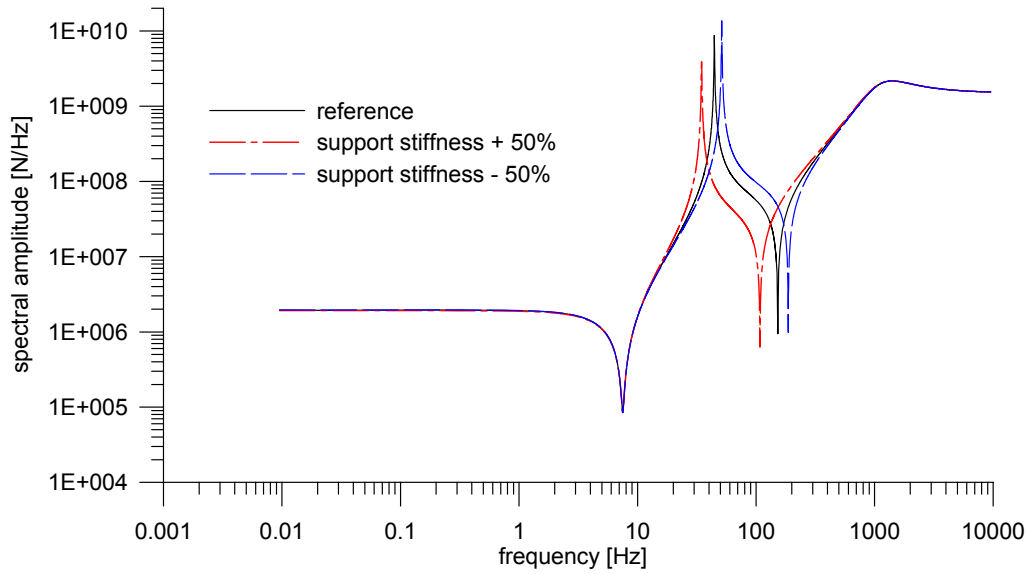


Fig. 4.33 Absolute value of the spectrum of the contact force in the wheel-rail system due to a unit pulse excitation for different values of the rail support stiffness

In the following, the influence of the velocity in Eq. (34) is considered, for the basic case of a single ramp. Because the linear influence of the slope θ is known, θ is kept constant at 2 mrad. The contact force is then calculated for the velocities 40, 80, 140, 200 and 300 km/h. For each case the maximum value of the contact force is determined, occurring in the time interval $2\Delta x/V$. The length of the basis is chosen as $\Delta x = 25$ mm. The time histories of these contact forces are shown in Fig. 4.34 for a duration $2\Delta x/V$. The contact force increases with the velocity, and a dip occurs after reaching the transition from the inclined part of the ramp to the horizontal part.

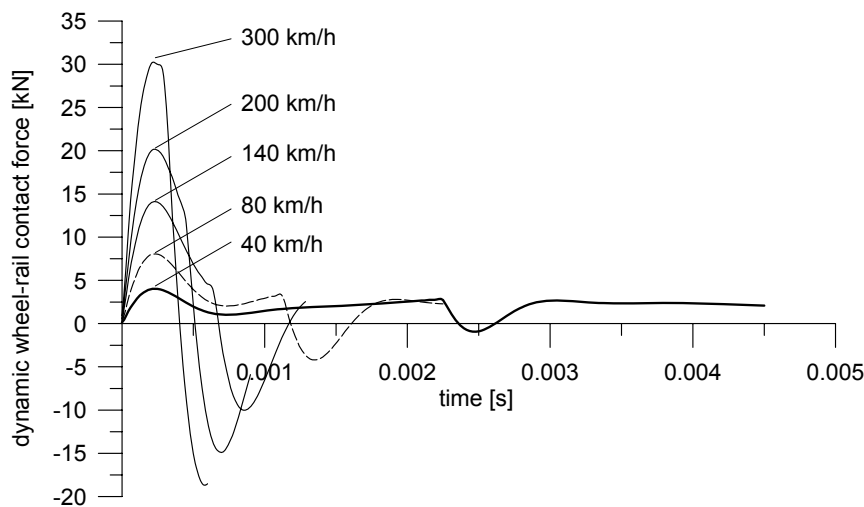


Fig. 4.34 Time histories of the wheel-rail contact force for a ramp with an inclination of 2 mrad, a basis of 25 mm and different train velocities

The extreme values of the force using a basis of 5 mm show no consistent trend, because for high velocities the time interval corresponding to this length is too short for the force to reach its first ‘natural’ peak, that would occur without the second transition in the ramp, from the inclined part of the ramp to the horizontal part (apart from the fact that a real contact patch is larger than 5 mm). For a basis of 25 mm this is no longer the case; the trend of the maximum dynamic contact force as a function of the velocity is shown in Fig. 4.35. It can be concluded that for a rail irregularity with a single constant slope, the first peak force (P_1) is perfectly and directly proportional to the speed.

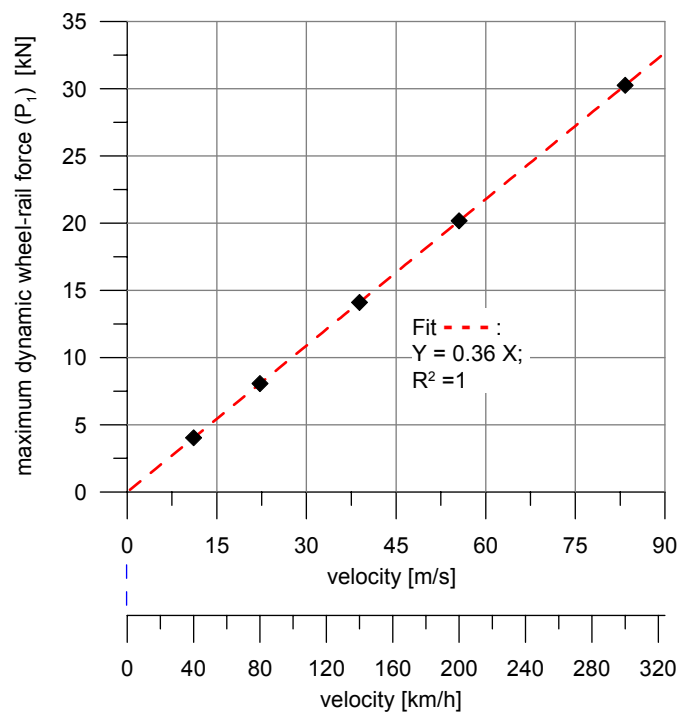


Fig. 4.35 Relationship between the maximum dynamic contact force (P_1) and the velocity, for a given ramp inclination (2 mrad) and basis length (25 mm)

In section 4.7, the following approximate relationship has been established between the maximum dynamic force occurring at rail welds, their maximum absolute inclination, and the train speed:

$$F_{\text{dyn, max}} = \beta \cdot V \cdot \theta \quad (\text{with } \theta = \tan \theta) \quad (F [\text{kN}]; V [\text{m/s}]; \theta [\text{mrad}]) \quad (4.17)$$

In the FEM-validation, the factor $\beta=0.22$ was found for this relationship. According to Fig. 4.35 the following relationship is found for a ramp with a single slope:

$$F_{\text{dyn, max}} = 0.36 \cdot V \text{ for } \theta = 2 \text{ mrad, or } \beta = 0.18 \quad (4.18)$$

The coefficients found in both cases are very similar. In section 4.7, the fact was remarked that the dimensions in Eqs. (4.12, 4.17, 4.18) are inconsistent for a dimensionless coefficient β , which is non-physical. Returning to the original expression (4.14) for the contact force shows that the dimensions in this equation are as follows:

$$F [\text{kN}] = \theta [\text{mrad}] \times V [\text{ms}^{-1}] \int_0^{\infty} \text{Re} \left(\frac{K}{s^2} \right) \left[\frac{\text{Nm}^{-1}}{\text{s}^{-2}} \right] d\omega [\text{s}^{-1}] \quad (4.19)$$

In Eq. (4.19) K represents some overall stiffness parameter. Comparing Eqs. (4.17-4.18) to Eq. (4.19) it is obvious that the integral expression has been replaced by the constant β , which therefore should have a dimension (namely $[\text{Ns/m}]$), accounting for the effect of a dynamic system stiffness (in which all masses and stiffnesses in the system are involved). Eqs. (4.17-4.18) can be used alternatively in a normalised form, with a dimensionless coefficient.

In Fig. 4.36 the effect of the length of the basis on the wheel-rail interaction is shown, for $\theta = 2 \text{ mrad}$, $V = 38.9 \text{ m/s}$ (140 km/h). The P_1 force needs, at this speed, a basis of about 10 mm (which is approximately the size of the contact patch) in order to reach its ‘natural’ peak value. If the basis is long enough, a narrow and a broad peak appear in the contact force (the P_1 and P_2 force, respectively). For the considered parameters (and without damping), the magnitude of P_2 exceeds P_1 . Very similar results to those obtained in Fig. 4.36 have been found by Frederick in 1979 with a simplified model for a ramp in the rail surface [4.44].

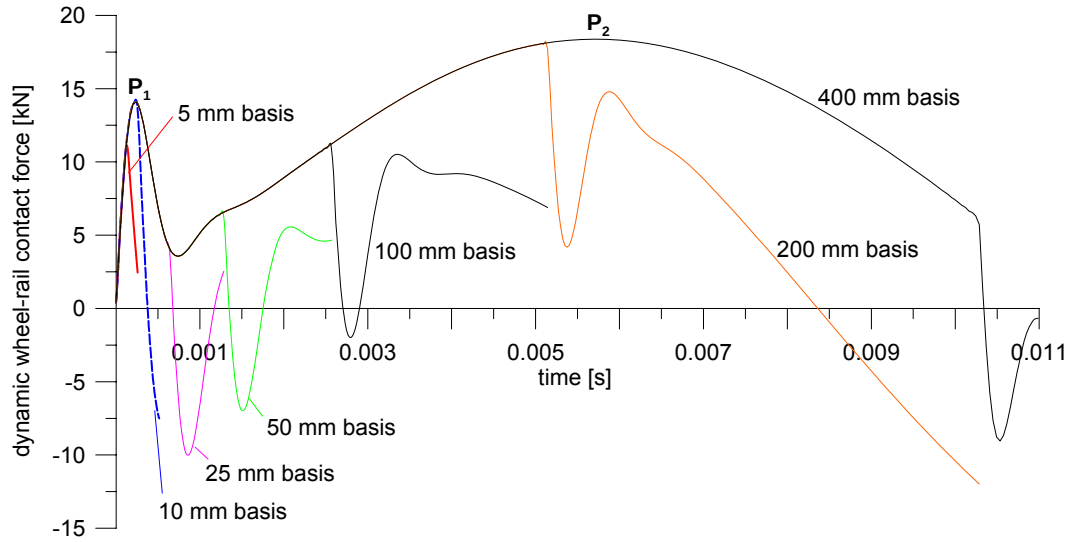


Fig. 4.36 Influence of the basis length of the ramp on the wheel-rail interaction force, for an inclination of 2 mrad and a speed of 140 km/h

The effect of different track parameters on the P_1 and P_2 forces is shown in Fig. 4.37. Simulations have been performed for a doubled and a halved foundation stiffness with respect to the reference situation, and a rail bending stiffness and a rail inertia both reduced with a factor 5. From this figure it can be concluded that the rail inertia, and to a lesser degree, the rail bending stiffness, govern the P_1 force, whereas the rail support stiffness, and to a small extent the rail bending stiffness, govern the P_2 force.

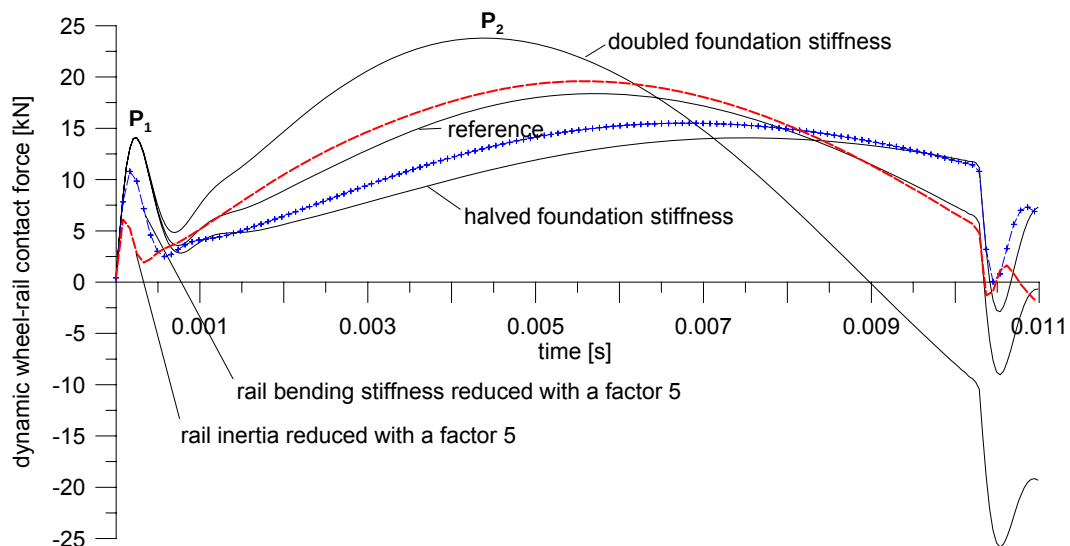


Fig. 4.37 Influence of the track parameters on the wheel-rail interaction force, for a ramp inclination of 2 mrad, a basis length of 400 mm and a train speed of 140 km/h

The diagrams in Fig. 4.38 show the dependence of the P_1 and P_2 forces on the rail properties more into detail. As the linear influence of the slope θ and the velocity V is known, their values are taken constant, at 2 mrad and 38.9 m/s (140 km/h) respectively. At the left, the influence of the rail bending stiffness (in comparison to the reference stiffness of the 54E1 profile) on the magnitude of P_1 and P_2 is shown, for a constant rail mass (54E1). In the region which is relevant for common rail profiles, both the trends for P_1 and P_2 show an approximating exponential behaviour as a function of the bending stiffness. At the right, the influence of the distributed rail mass (for a constant bending stiffness (54E1)) is shown. In the region of interest, the trends of both P_1 and P_2 can be approximated as linear. Both figures show that the influences of both the rail inertia and the rail bending stiffness on P_1 are most important, whereas the influence of the rail inertia on P_2 is negligible.

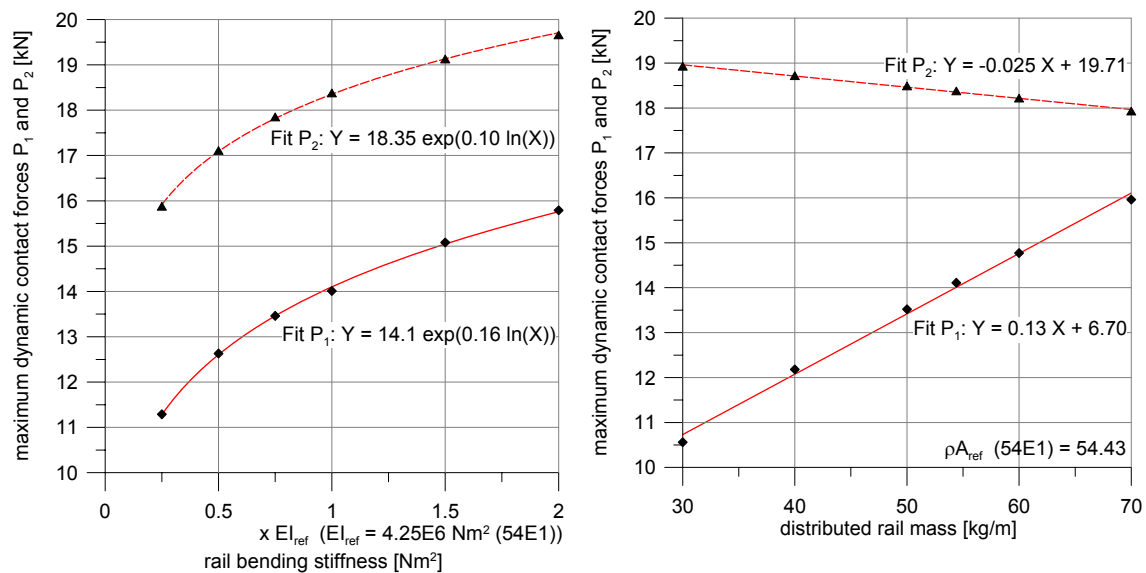


Fig. 4.38 Influence of the rail bending stiffness (left) and the rail mass (right) on the peak forces P_1 and P_2 , for a ramp inclination of 2 mrad, a basis length of 400 mm and a train speed of 140 km/h

It was shown in section 4.4 that during passage of a train wheel over a rail weld irregularity the track follows the excitation quasi-instantaneously, whereas the wheel displacement shows a delay in response relative to the track, owing to its larger inertia. This conclusion is further verified with the present model in Fig. 4.39, where the displacements of the rail and the wheel are shown for a ramp of 2 mrad, a train velocity of 38.9 m/s (140 km/h) and a basis length of 25 mm (to study the P_1 -behaviour; at the left) and 400 mm

(to study the P_2 -behaviour; at the right) respectively. In Fig. 4.39, also the differential displacement between wheel and rail is shown as a function of time. When this parameter is compared to the excitation as a function of time, it gives an indication of the contribution of the elastic component to the dynamic interaction force relative to the inertial component. When both functions coincide, the response is quasi-static.

From the left plot in Fig. 4.39, in which also the excitation in the time domain is plotted, it can be concluded that the rail indeed follows the irregularity to a large extent during the P_1 -interaction, whereas the wheel shows a negligible response within the considered time interval. The differential displacement, which almost coincides with the rail displacement, does not coincide with the excitation, revealing an important contribution of the inertial component to the P_1 -force.

From the right graph in Fig. 4.39 it can be concluded that both the wheel and rail displacements play an important role during the P_2 -interaction. The differential displacement coincides almost exactly with the excitation, indicating a negligible contribution of the inertial component to the P_2 -force.

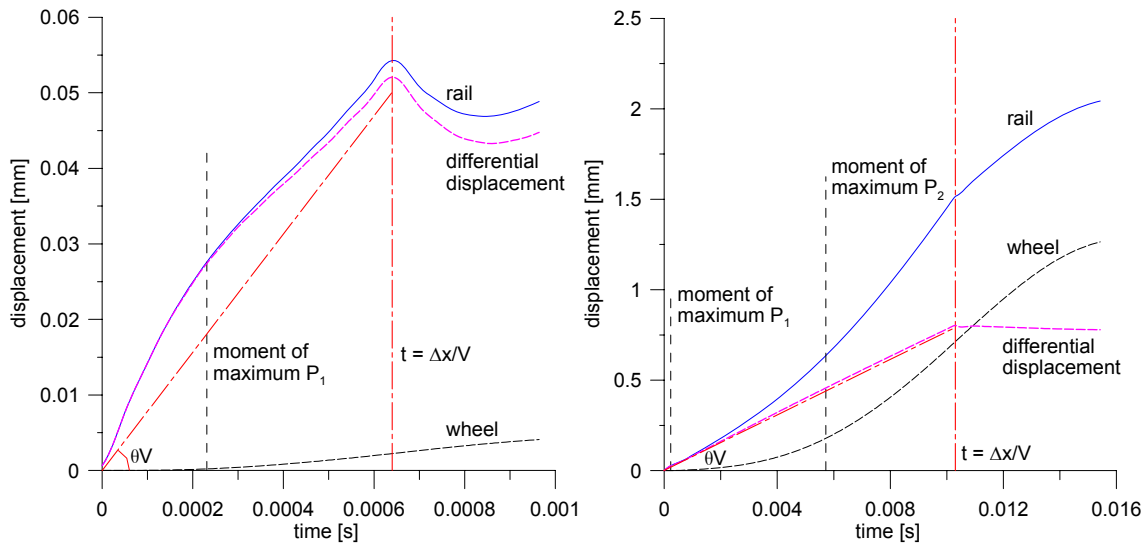


Fig. 4.39 Displacements of wheel and rail during the P_1 and P_2 interaction, for a ramp inclination of 2 mrad and a basis length of 25 (left) and 400 mm (right) respectively, at 140 km/h

In the previous, the characteristics of the dynamic wheel-rail interaction at short-length rail irregularities without dominating wavelength have been studied with the basic ramp model. In the following, real measured welds will be addressed. As has been discussed in section 4.3, a continuous 5-point-

smoothing (running average over 25 mm) is applied to the measurement signal before using it as an input in the calculations, however, omitting the filtering of each 5th point⁴. The latter filtering is applied for feasibility purposes in the assessment, but influences the computation accuracy negatively and unnecessarily; the discretisation basis in the input signal therefore remains 5 mm. Other filters have been proposed in the literature, such as the well known heuristic filter of Remington [4.45]. This filter however leads to negative values for short wavelengths just above the contact patch diameter. In Ref. [4.46] it is shown that the effect of contact filters becomes only relevant for wavelengths comparable to or smaller than the size of the contact patch along the rail.

The graphs in Fig. 4.40 show examples of the time histories of the contact force computed according to Eq. (4.15) for two measured welds, for a train velocity of 38.9 m/s (140 km/h). Also the measured geometry (sampling

⁴ In the present approach, it is assumed that the kinematical wheel centre trajectory, which is commonly used as an excitation function in dynamic wheel-rail interaction models, is just equal to the geometry of the irregularity along the rail. If the curvature of the rail irregularity is small relative to the circumferential wheel curvature, the error in taking the irregularity instead of the trajectory is negligible. Because the rail irregularity is measured, the direct use of this measurement is an important simplification. In order to validate the assumption, we consider a harmonic rail irregularity according to the following expression, along with its maximum curvature:

$$\zeta(x) = \hat{\zeta} \cos\left(\frac{2\pi x}{L}\right); \left|\frac{d^2 \zeta}{dx^2}\right|_{\max} = \frac{4\pi^2}{L^2} \hat{\zeta} \quad (4.a)$$

The length of a measurement is typically 1 m, yielding a maximum wavelength of 2 m. The amplitude will be taken in the same order of magnitude as the vertical tolerance, which is commonly 0.3 mm. In order to check the maximum curvatures, several amplitudes and wavelengths are compared to the circumferential wheel curvature in Table 4.1.

Table 4.1 Rail irregularity curvatures relative to the wheel circumferential curvature

wavelength [mm]	amplitude [mm]	max. curvature [1/m]	% of 1/R (R = 0.46 m)
2000	0.6	0.0059	0.27
500	0.6	0.095	4.4
300	0.5	0.22	10.1
100	0.3	1.18	54.4

From Table 4.1 it is obvious that the shorter the wave, the worse the approximation is, which also follows directly from expression (4.a): the maximum curvature is inversely proportional to the square of the wavelength. The last example in Table 4.1 implies that the height difference of 0.3 mm (which is the common vertical tolerance value) occurs over a length of 25 mm, which is an extreme situation (vertical step). For these extreme cases, the use of the rail geometry will lead to computational inaccuracies. However, an objective of this investigation is to establish a norm tool and limit values for geometrical weld assessment. It is obvious that especially in the last case (the shortest wave), the contact force will be the highest, and it is precisely the aim of an enhanced assessment method to avoid these situations. Therefore, the computations using the measured geometry will be correct for those welds satisfying the new norms.

interval 5 mm) and the geometry after continuous 5-point smoothing (i.e., the input for the calculation) are shown.

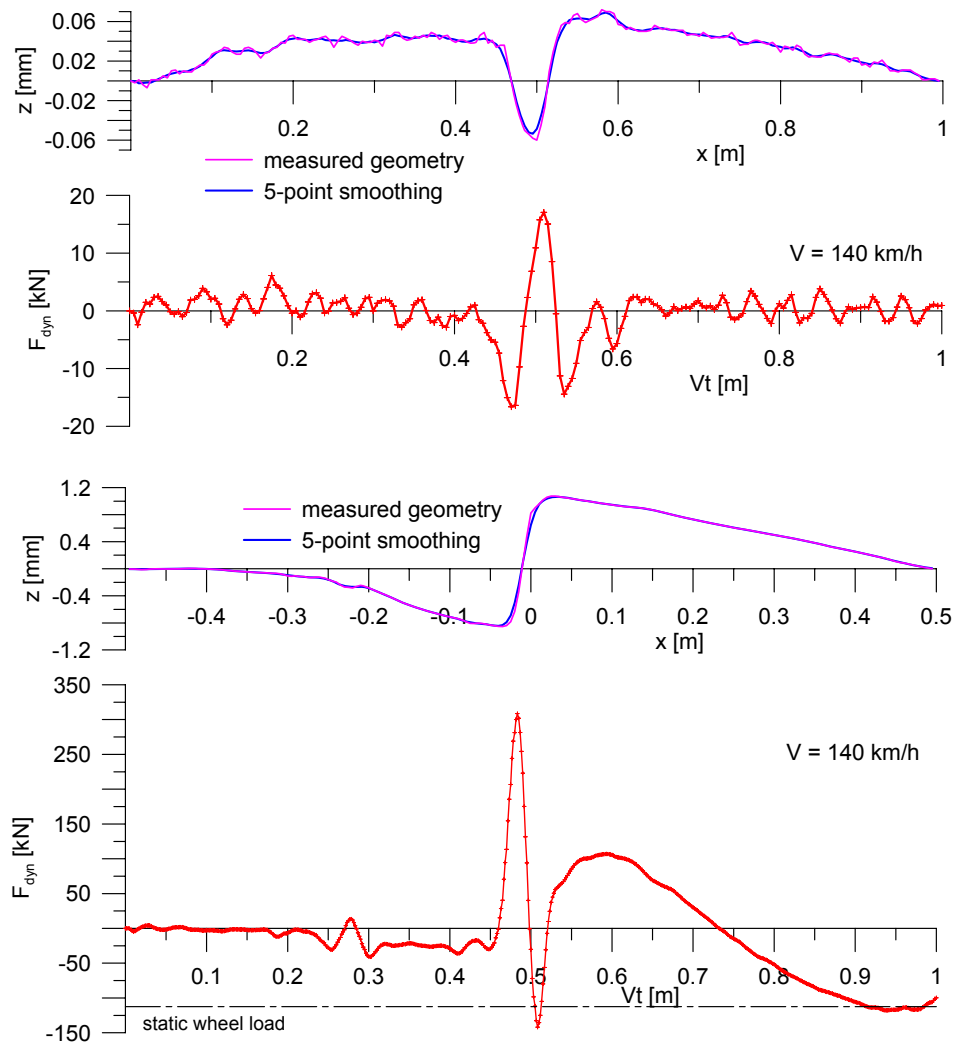


Fig. 4.40 Examples of measured welds, geometry after 5-point-smoothing and calculated time histories of the dynamic wheel-rail contact force at 140 km/h

The first weld is rather well aligned, but it shows a high degree of roughness on a small scale (order 1 mm) and, at the centre, the effect of non-homogeneous shrinkage of the weld and parent material after the grinding process, which has obviously been performed at a too high temperature. The second weld is smooth on a micro-scale, but shows a severe misalignment before the rail ends have been welded together, resulting in a pronounced step of about 2 mm in rail height over 25 mm. For the first weld the dynamic component of the contact force does not exceed 20 kN. For the second weld, the maximum dynamic force exceeds 300 kN, followed by a contact loss,

since the dynamic force component exceeds the static preload (112.5 kN). The real maximum contact force is expected to be somewhat higher due to the non-linear stiffening of the Hertzian contact spring during its almost instantaneous compression at $Vt = 0.47$ m. Furthermore, the linear calculation cannot deal with the contact loss. When the studied weld is overpassed in the track by trains in the positive x -direction, it will have a very short lifetime with a severe risk of brittle fracture. At 83.3 m/s (300 km/h) the maximum force increases to more than 800 kN followed by a dip to less than -500 kN in a linear calculation, indicating repeated loss of contact and impact loading.

As has been stated before, a perfectly linear relation between $F_{\max}$ and $\theta_{\max}$ does not exist for an arbitrary irregularity due to the mutual influence of the force histories for each discrete slope, especially with the sampling interval approaching zero in a continuous signal. Therefore, the accuracy of any relationship between the extreme value of the contact force and a geometrical parameter of the irregularity depends on the statistical distribution of this geometrical parameter over the irregularity. For example, if the first derivative of the irregularity has a narrow distribution, the slope variation along the irregularity is restricted and the mutual influence of the time histories of the force for each discrete slope is also restricted.

In the previous, the distribution of the discrete slopes was considered for a single given weld. Now, the distribution of the maximum absolute inclination for a large number of welds is considered. The higher this inclination, the more variation in slope is possible, implying that the mutual influence of the time histories of the contact force for the discrete slopes of a given irregularity will also increase. Therefore, when a large number of welds is considered, the scatter in the relationship between maximum inclinations and contact forces will increase with increasing maximum inclination, as will be confirmed by the results in the following.

In the following, a correlation analysis will be performed for the sample population of 239 measured rail welds, which was also used in the previous section. This analysis will be restricted to the dynamic contact force, which is the most important variable to be limited in the assessment of welds.

In Figs. 4.41 through 4.45 the maximum absolute dynamic wheel-rail contact forces (during the time interval of overpassing the weld) are shown as a function of the vertical absolute peak deviations (at the left) and the maximum absolute inclinations of the welds (at the right), for the train velocities 40, 80, 140, 200 and 300 km/h. It is observed, for each velocity, that

the scatter in the forces increases with increasing maximum absolute inclination, as has been explained. For all velocities, a linear fit through the origin is shown; its equation and coefficient of determination are displayed too. Apart from the linear fits through the origin, also the linear regression lines are shown in the cases that the coefficients of determination R^2 for both cases differ. The correlation between the forces and the inclinations is significantly better than that between the forces and the vertical deviations; the ratio between the coefficients of determination for both cases ranges from 2.2 (140 km/h) up to 4.8 (300 km/h). Similarly to the results obtained with the FE model [4.35], the correlation between forces and maximum inclinations increases significantly with the velocity. This can be explained by a more flat transfer of the system as the excitation frequency band shifts towards higher frequencies with increasing speed. For a perfectly flat transfer, there would be a perfect correlation between geometry and contact force. Furthermore, the scatter in the results may be partly explained by the fact that no difference is being made between P_1 - and P_2 -like forces: the maximum force is taken occurring over 1 m of welded rail.

For 140 km/h, the results are also shown on a logarithmic scale in Fig. 4.46; the results projected on both axes may then be represented as Gaussian distributed, with a given mean value. This result is valid for all velocities and will be addressed more extensively in section 9.

In Figs. 4.41 through 4.45, the maximum absolute inclination of the 5-point-smoothed signal (with a sampling interval of 5 mm) is plotted on the horizontal axes. This signal, artificially ‘filtered’ by the contact patch, also served as an input in the force calculations; the timestep for the evaluation of the integrals was $\Delta t = 5 \text{ mm} / V$, which lead to convergent results. In the determination of the Quality Index (QI) of rail welds (see section 4.3), the maximum absolute inclination of this signal is taken on a different, 25 mm basis (each 5th sampling point is used), resulting in a loss of information, but rendering the method more apt for practical purposes. Therefore, the results in Ref. [4.35] are not directly comparable to the results in Figs. 4.41 through 4.45. The relationship between both the maximum absolute inclinations of the continuously 5-point smoothed signal with the original sampling interval of 5 mm and the one with extended sampling interval of 25 mm (as used in the determination of the QI) is shown in Fig. 4.47. For a correct comparison, it should be noted that the extension of the sampling basis results in a maximum absolute inclination which is equal to or smaller than the original inclination.

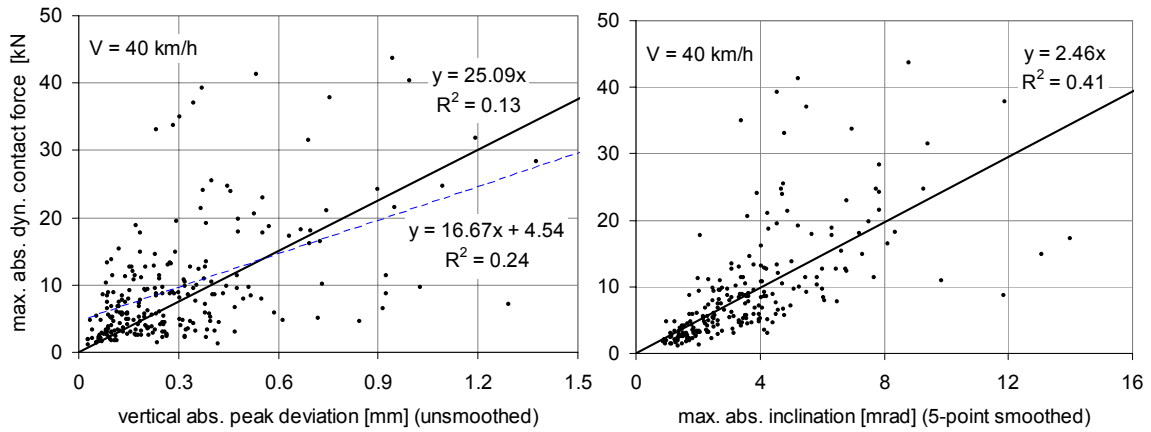


Fig. 4.41 Peak value of the dynamic wheel-rail force as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 40 km/h

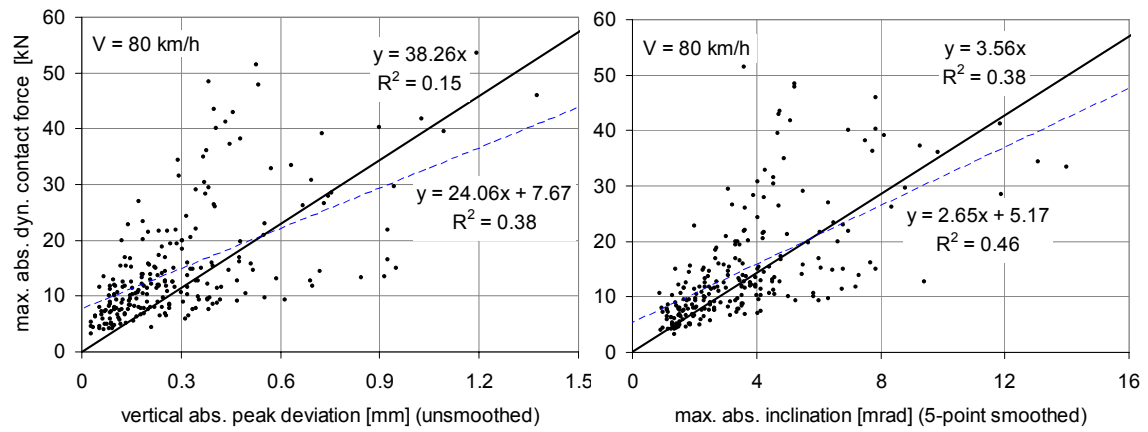


Fig. 4.42 Peak value of the dynamic wheel-rail force as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 80 km/h

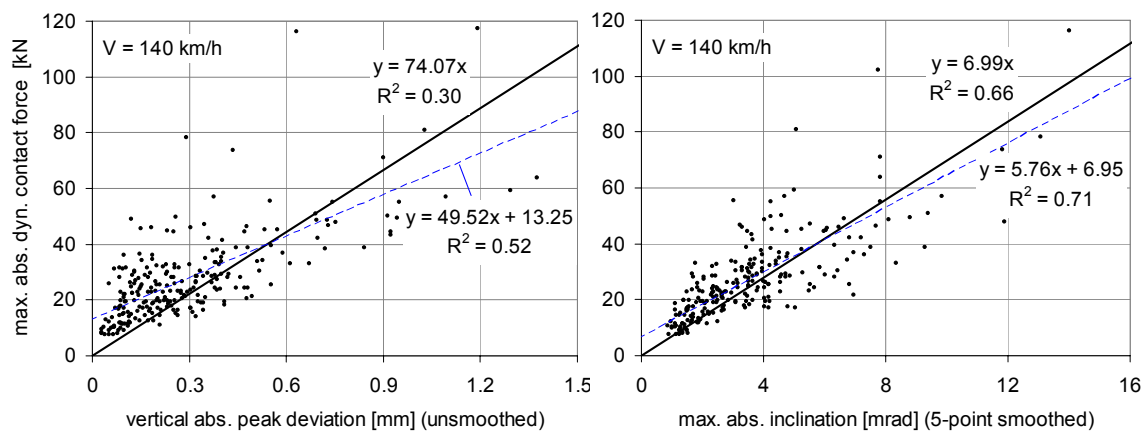


Fig. 4.43 Peak value of the dynamic wheel-rail force as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 140 km/h

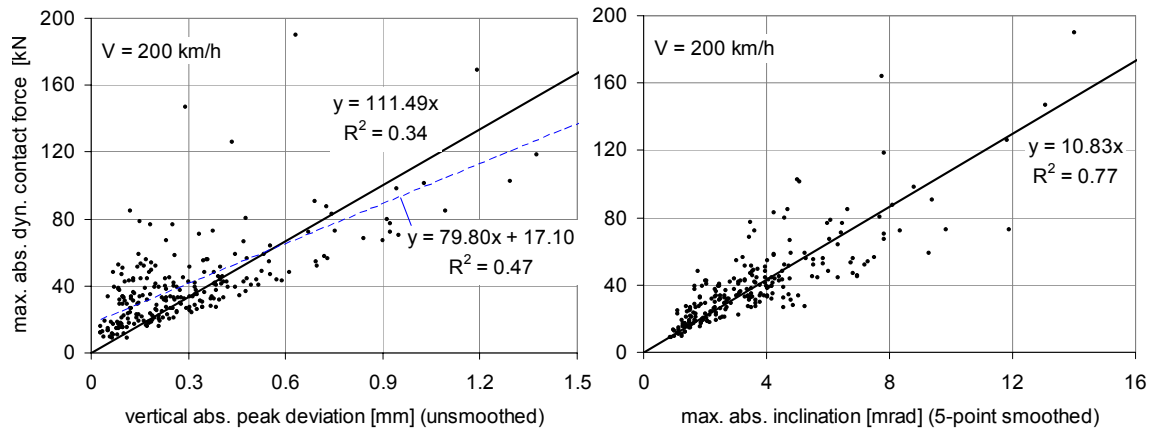


Fig. 4.44 Peak value of the dynamic wheel-rail force as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 200 km/h

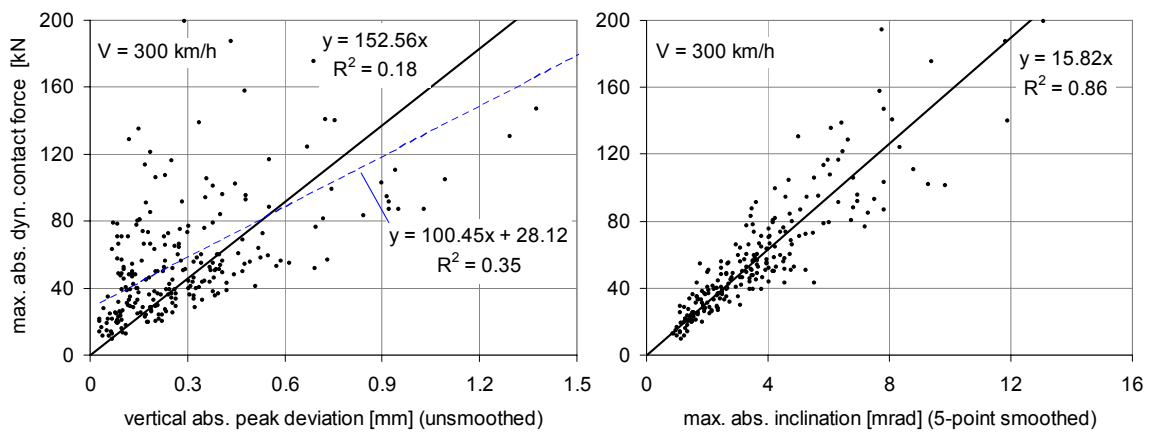


Fig. 4.45 Peak value of the dynamic wheel-rail force as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 300 km/h

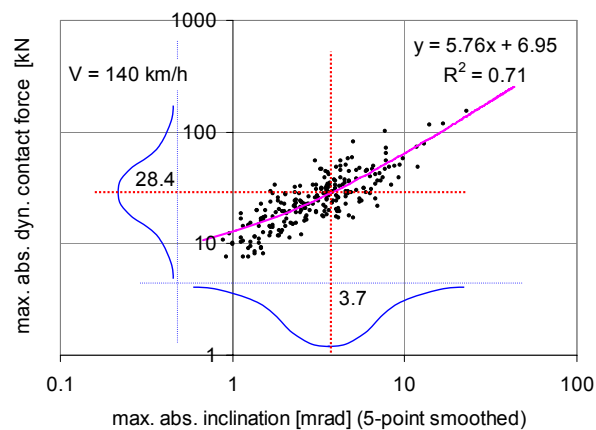


Fig. 4.46 Peak values of the dynamic wheel-rail force as a function of the maximum absolute inclinations and their normal distributions on a log-scale, at 140 km/h

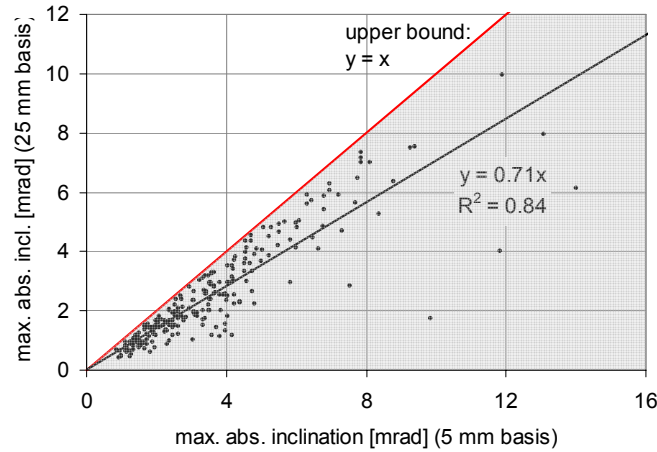


Fig. 4.47 Relationship between the maximum absolute inclinations of the 5-point smoothed signal with and without 25 mm filtering

The linear relationships between the dynamic force and the maximum absolute inclination from Figs. 4.41 through 4.45 are reproduced in Fig. 4.48 (left) for all train velocities. At the right, the dependence of the dynamic force on the train speed is shown for a given maximum inclination (5 mrad). It is obvious that the relationship between the dynamic force and the train speed can be described very well as directly proportional, which confirms the results of the analysis in section 4.7.

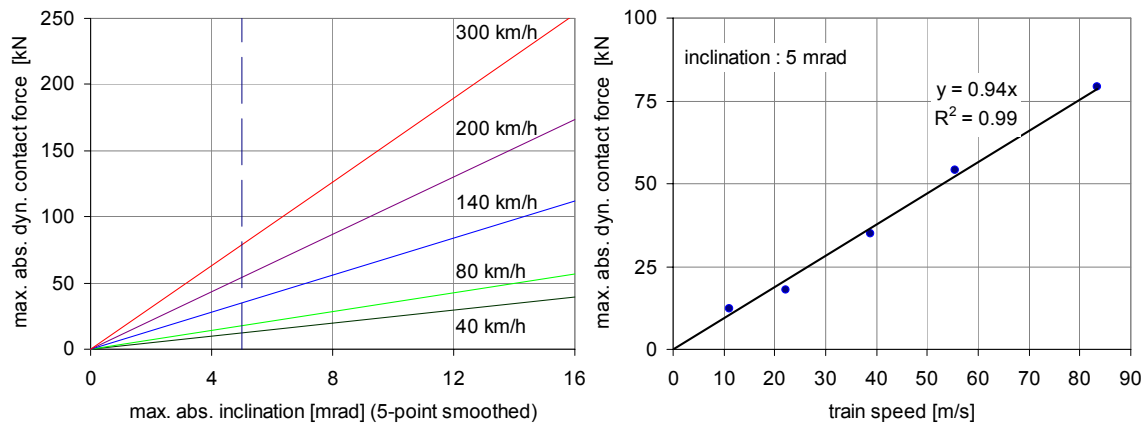


Fig. 4.48 Dependence of the dynamic wheel-rail contact force at welds on the maximum absolute inclination (for different train speeds) (left) and on the train speed (for 5 mrad) (right)

With the results shown in Fig. 4.48, the coefficient β in the general relationship (4.17) can be validated for rail welds, in the speed interval 0-300 km/h, resulting into the following normalised expression (4.20):

$$\frac{F_{\text{dyn,max}}}{F_{\text{ref}}} = 0.19 \cdot \theta \cdot \frac{V}{V_{\text{ref}}} \quad (F \text{ [kN]}; V \text{ [m/s]}; \theta \text{ [mrad]}; F_{\text{ref}} = 1 \text{ kN}; V_{\text{ref}} = 1 \text{ m/s})$$

From the FE analysis in section 4.7, $\beta = 0.22$ was found for rail welds. Previously in this section, $\beta = 0.18$ was found for the linear relationship for the theoretical case of a ramp with one single slope, and $\beta = 0.19$ is found for rail welds. The numbers are very close; the slightly higher value from the FE analysis may be explained from the fact that inclinations on a 25 mm basis have been used. As has been remarked before (Fig. 4.47), the inclination can only decrease when the sample interval is extended from 5 to 25 mm. Because the forces remain unaffected, the coefficient will then increase. It can be concluded that $\beta \approx 0.2$ in practice (for 54 E1 rail).

As an illustration, the examples of measured wheel-rail contact forces on the Swedish network (7 peaks in about 22 m) reported in Ref. [4.47], Figs. 8-9, can be used. Also the geometry of an irregularity corresponding to one of the peaks (without indication to the exact position of the peak) is reported. Application of formula (41) yields for this case ($\theta \approx 15 \text{ mrad}$; $V = 48.6 \text{ m/s}$): $F_{\text{dyn,max}} = 0.19 \cdot 15 \cdot 48.6 = 139 \text{ kN}$. The static load of 60 kN yields a total maximum load of approximately 200 kN. The order of magnitude (ranging from 150 to 200 kN for all force peaks; the applied low-pass filtering in the registration is unknown) is correct.

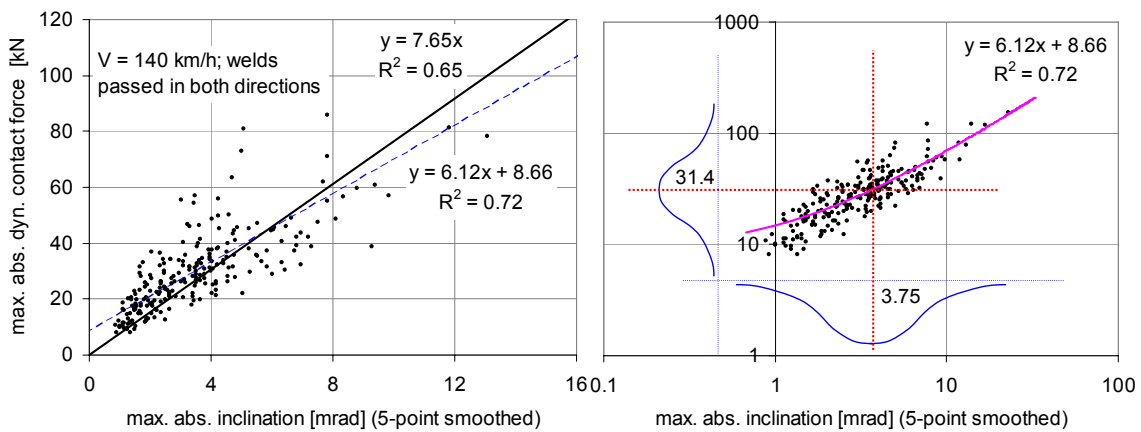


Fig. 4.49 Peak value of the dynamic wheel-rail force as a function of the maximum absolute weld inclination on linear (left) and log-scales (right); the welds are passed bi-directionally at 140 km/h

The results in Figs. 4.41 through 4.45 have been computed for a train, passing the welds in one single direction. This is realistic for a railway

network, on which trains, on a given piece of track, commonly run in only one direction. However, one would expect a better correlation between maximum absolute inclinations (which are direction-independent) and forces if the extreme value of the force is taken for a train passing the welds in both directions. The results are shown for 38.9 m/s (140 km/h) in Fig. 4.49. It is observed that the gradient of the linear fit increases (indicating a higher force level for a given maximum absolute inclination), whereas the correlation does not change significantly. This is confirmed by the results for the other velocities; the force levels when considering two directions are about 5-10% higher than in the case of only one running direction.

4.9 Statistical properties of dynamic wheel-rail interaction at rail welds

It has been demonstrated in section 4.6 that the discrete first derivatives of rail welds are Gaussian distributed with zero mean value, except for the tails. The maximum discrete first derivatives of a large population of welds was shown to be very well approximated by a lognormal distribution.

In Fig. 4.50, the maximum absolute inclination and its logarithm (for the 5-point smoothed signals) are shown as a function of the standard normal variable for the measured weld sample (239 welds) which was used in the simulations.

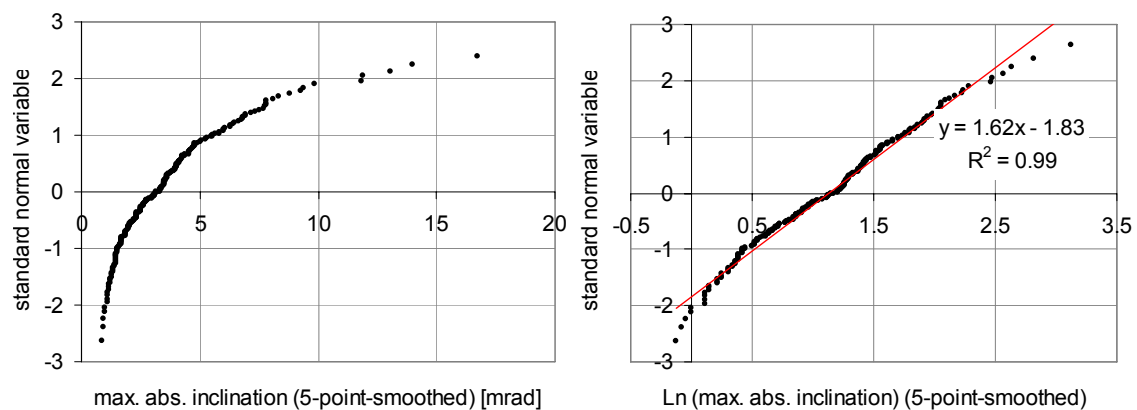


Fig. 4.50 Maximum absolute inclinations of the weld geometry; normal probability test (left) and lognormal probability test (right)

If the logarithm is taken, the result is approximately linear. This confirms that the maximum absolute inclination of a large number of rail welds may be represented by a lognormal distribution. Therefore, the projections of the data

points in Figs. 4.41 through 4.45 and 4.46 on the horizontal axes may be considered as lognormally distributed (as is shown in these figures for 140 km/h).

Now, the distribution of the maximum dynamic forces in Figs. 4.41 through 4.45, for the whole sample, is addressed. Thus, the projection of the data points on the vertical axes is considered. This distribution gives the distribution of the dynamic forces occurring over a railway network at welds, for a given distribution of the maximum absolute inclinations of the welds. It gives no information about the scatter in the deterministic relationship (4.20), for a given inclination; this aspect, which is linked to a vertical cross-section in Figs. 4.41 through 4.45, will be addressed later. In Fig. 4.51 at the left, the maximum dynamic contact forces and the maximum of their absolute values are plotted versus the standard normal variable (for 140 km/h). In Fig. 4.51 at the right, this is repeated for the logarithm of the forces. The resulting functions in the right graphs can be well approximated by a straight line (a base of 10 leads to similar results).

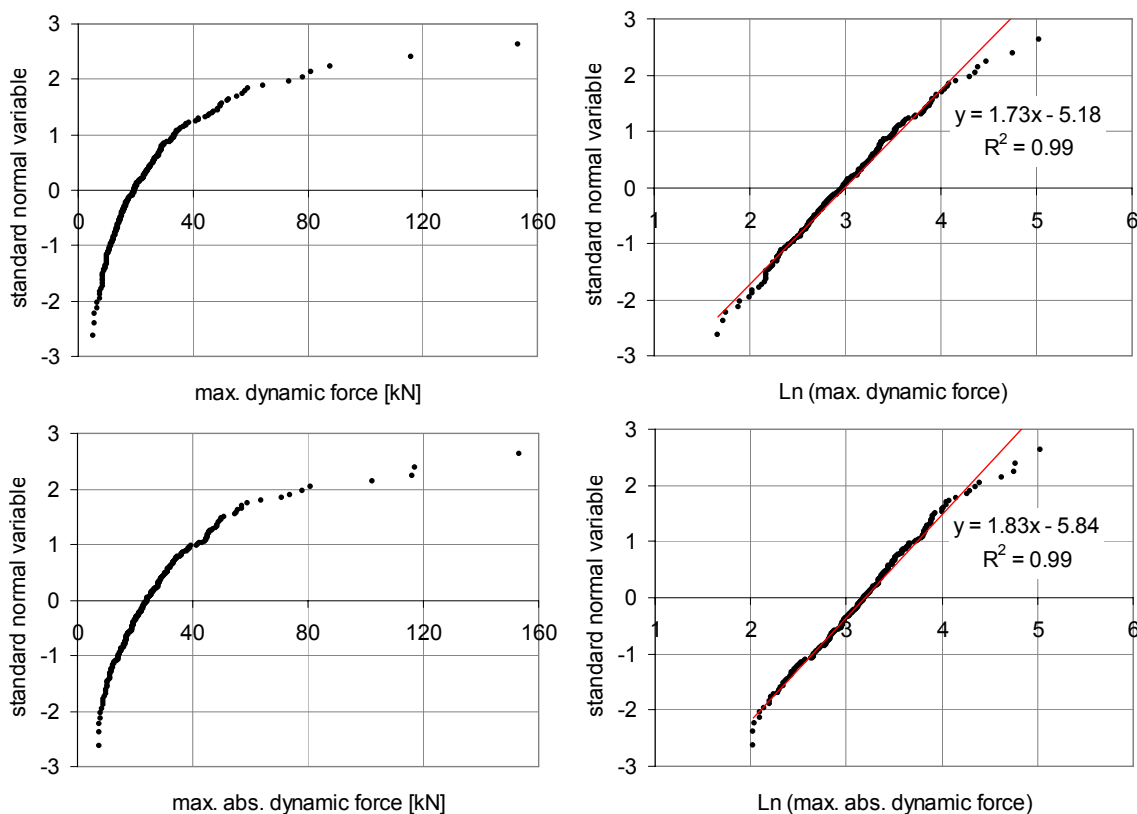


Fig. 4.51 Maximum dynamic contact forces (top) and their absolute values (bottom) (at 140 km/h); normal probability test (left) and lognormal probability test (right)

This result is valid for all velocities. Therefore, the maximum dynamic contact force occurring at rail welds may be represented by a lognormal distribution, with mean and standard deviation varying as a function of the train speed. In Fig. 4.46 this is shown for 140 km/h (vertical axis).

In Fig. 4.52 the probability density functions of the maximum absolute inclination and of the maximum dynamic contact force are shown for different velocities. The probability density function of the force shifts towards higher values with increasing speed, the top value drops and the function becomes more ‘stretched’.

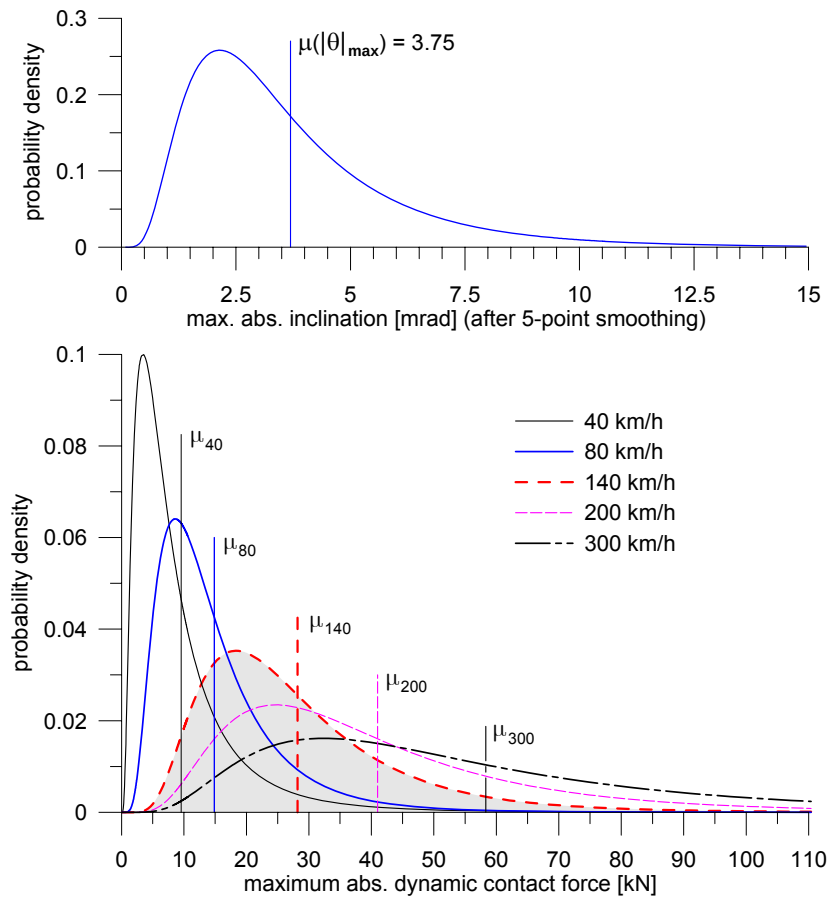


Fig. 4.52 Lognormal distribution of the maximum absolute inclination of rail welds and corresponding lognormal distributions of the maximum absolute values of the dynamic wheel-rail contact force at different train velocities

Fig. 4.53 shows the linear relationship between the mean value of the maximum dynamic wheel-rail contact force and the train velocity. For different velocities, the probability density corresponding to the force distributions is now quantified on a separate vertical axis. The values of the statistical parameters of the distributions are given in Table 4.2. With Fig. 4.53

the distribution of forces occurring at rail welds over a railway network can be easily estimated for different train velocities, once the mean value of the maximum absolute inclination of a population of measured rail welds is known. The standard deviation must be estimated as a percentage of the mean value. For this reason also the coefficient of variation is given in Table 4.2, as a measure of the statistical dispersion of the force. There is no clear link between the standard deviation and the velocity, which is related to the non-uniform frequency transfer of the wheel-rail system.

Table 4.2 Statistical properties of rail weld geometry (239 measurements) and maximum dynamic contact force

	Mean value	Standard deviation	Coefficient of variation
max. abs. inclination	3.75 mrad	2.7 mrad	0.72
force; 40 km/h	9.6 kN	9.2 kN	0.96
force; 80 km/h	15.0 kN	10.5 kN	0.70
force; 140 km/h	28.4 kN	18.3 kN	0.64
force; 200 km/h	41.5 kN	31.1 kN	0.75
force; 300 km/h	58.8 kN	45.8 kN	0.78

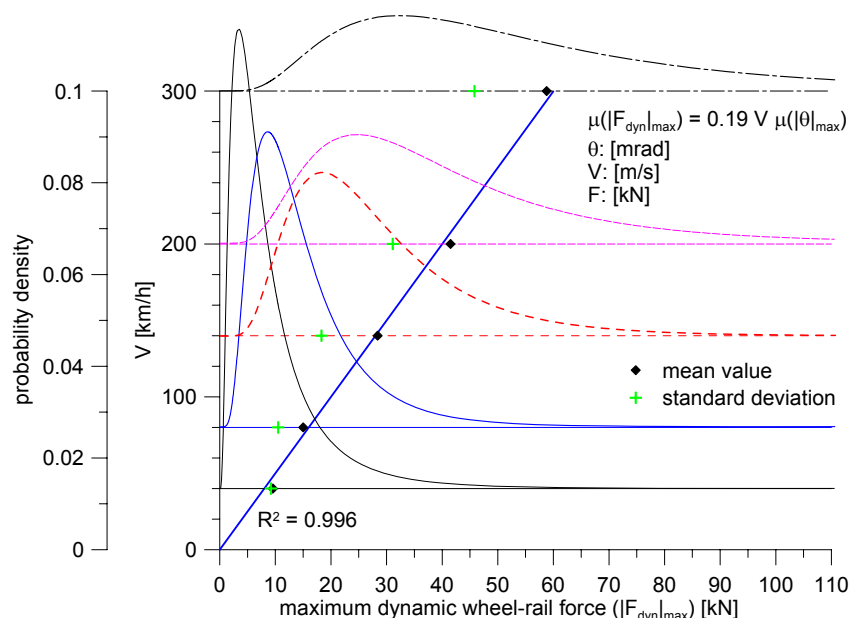


Fig. 4.53 Linear relationship between the mean value of the extreme dynamic wheel-rail force and the train speed for a weld population with a given mean value for the maximum inclination (lognormal contact force distributions are shown for different speeds)

The graphs in Figs. 4.41 - 4.45 and 4.49 show a considerable scatter in the results for a given inclination. Studying the results for a given inclination is equivalent to considering a vertical cross-section in the graphs. This scatter in the deterministic relationship (41) is due to the impossibility to reduce the dynamic wheel-rail interaction to a purely geometrical problem. The fact that the scatter in the results increases with increasing inclination was already explained in the previous section. However, the distribution of the forces for a given inclination and its relation to the value of the inclination cannot be further analysed, by lack of data: 4 values of the inclination with 5 data points (4 sets of 5 welds with the same maximum absolute inclination) and 1 inclination value with 6 data points are available; all other inclination values occur less frequently. Therefore, results of the normal or lognormal probability test do not give a clear answer.

4.10 The feasibility limit in standardization: high-speed norms

In section 4.5 the QI was introduced as a relative measure for the geometrical weld quality. The norm values of the first derivative in its definition must be determined for different train speeds. The norm value for high-speed lines, for which a norm speed of 300 km/h is chosen, is the most strict one in the speed range. Its value can be based on the geometrical quality of new straightened rail: there is no reason why rail welds should have a better geometrical quality than the rails themselves. This quality has been determined by measurements on new straightened longrails (120 m); a sample population of 100 segments of 1 m length have been measured (the rails were newly laid rails on the Dutch HSL-South [4.34]). The distribution function of the absolute maximum first derivatives (25 mm basis) for these segments is shown in Fig. 4.54. The 95 percentile value of the first derivative in this figure is 0.7 mrad. Therefore, 0.7 mrad could be taken as an appropriate value for the intervention level of first derivative of the weld geometry for high-speed lines. This accuracy is very close to the maximum obtainable accuracy in welding and grinding with grinding trains. Tests in this regard have been carried out during the construction of the Dutch HSL-South [4.26].

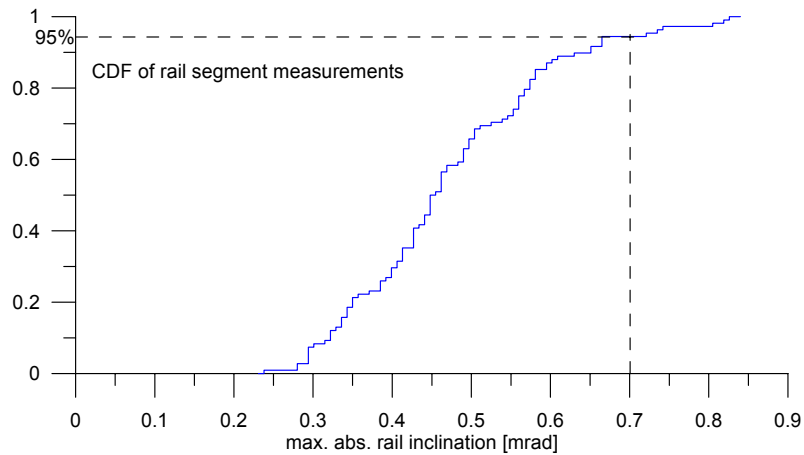


Fig. 4.54 Cumulative distribution function of the maximum absolute first derivative (25 mm basis) for new straightened rails (100 segments of 1 m)

In Fig. 4.55 the cumulative distribution of the maximum absolute first derivatives (25 mm basis) of 296 test welds (both flash butt and thermite) is shown before and after grinding with a GWM 550 grinding train. Before grinding with the GWM the 95 percentile value of the maximum first derivative is 5.3; this reduces to 1.1 afterwards.

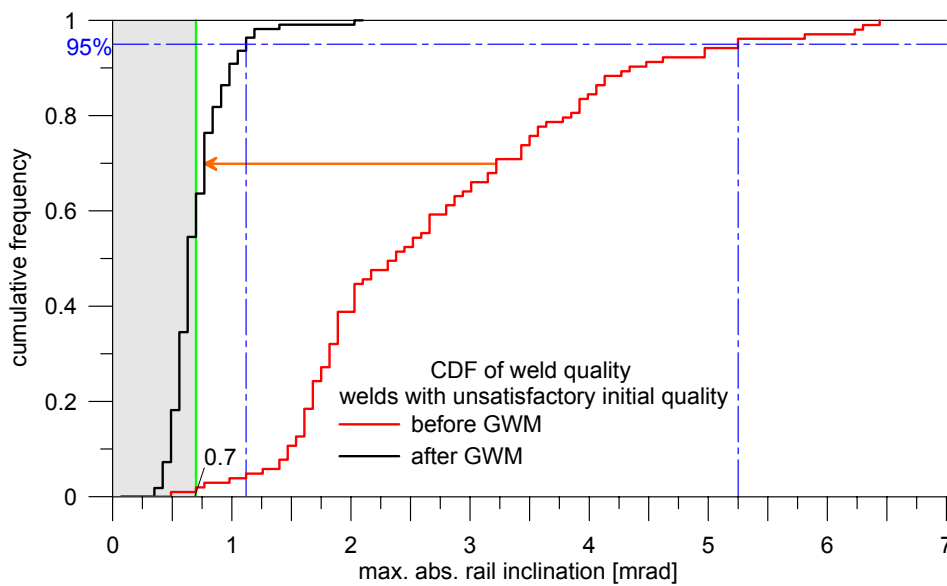


Fig. 4.55 Effect of GWM-grinding on the cumulative distribution of the maximum absolute first derivative of rail welds on the Dutch HSL-South

In Fig. 4.56, the cumulative distributions are shown of the maximum absolute first derivative of the rail welds on a double track test section of 6 km on the HSL-South. These welds were manually pre-ground. The distributions are

shown both before and after final grinding with a Speno grinding train (types RR 24 and 48), separately for the west and the east track. The 95 percentile of the maximum first derivative reduces from 2.5 and 2.2 respectively for the West and East track section, to 1.3 for both sections.

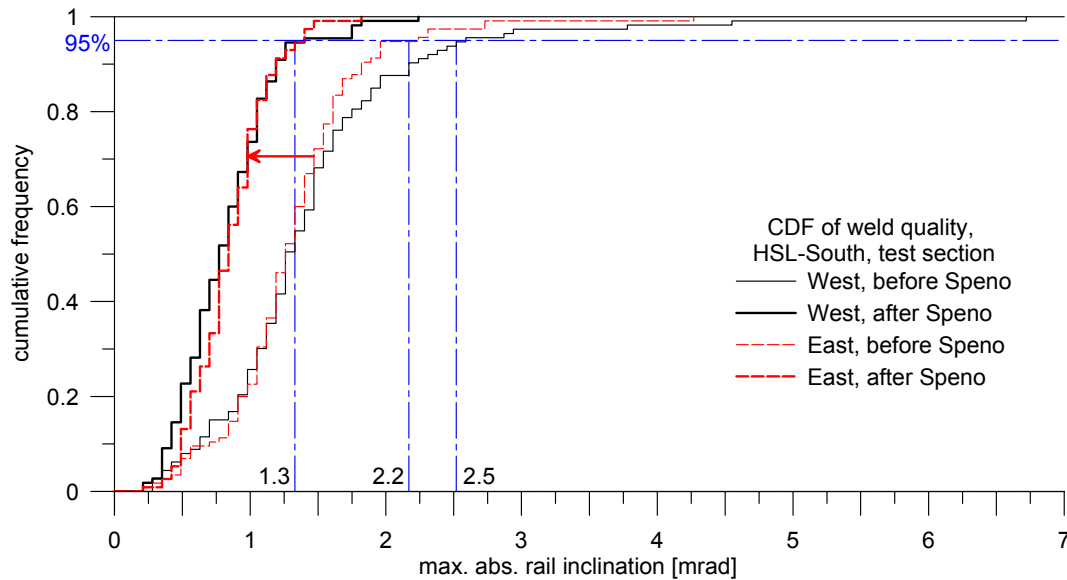


Figure 4.56 Effect of Speno grinding on the cumulative distribution of the maximum absolute first derivative of rail welds on a representative section of the Dutch HSL-South

The difference in the final quality obtained after grinding in Figs. 4.55 and 4.56 is due to the applied grinding principle: the GWM 550 works with a translatory iterative grinding principle in a stationary setup, whereas Speno uses a continuous rotatory grinding in a mobile setup. Therefore, the GWM can in principle grind to any required perfection, independent of the initial surface quality, whereas Speno reaches a rather constant improvement.

Using the 95 percentile values in Fig. 4.56 and Eq. (4.13), the total wheel-load reduces from 130 kN to 109 kN, which is a reduction with a factor 1.2. This reduction results in an extension of the fatigue lifetime of the welds with a factor 1.2^3 or 1.7. A similar calculation for the graph in Fig. 4.55 (GWM grinding) yields a total wheel-load reduction (from 181 to 106 kN) with a factor 1.7, or a significant lifetime extension with a factor 4.9.

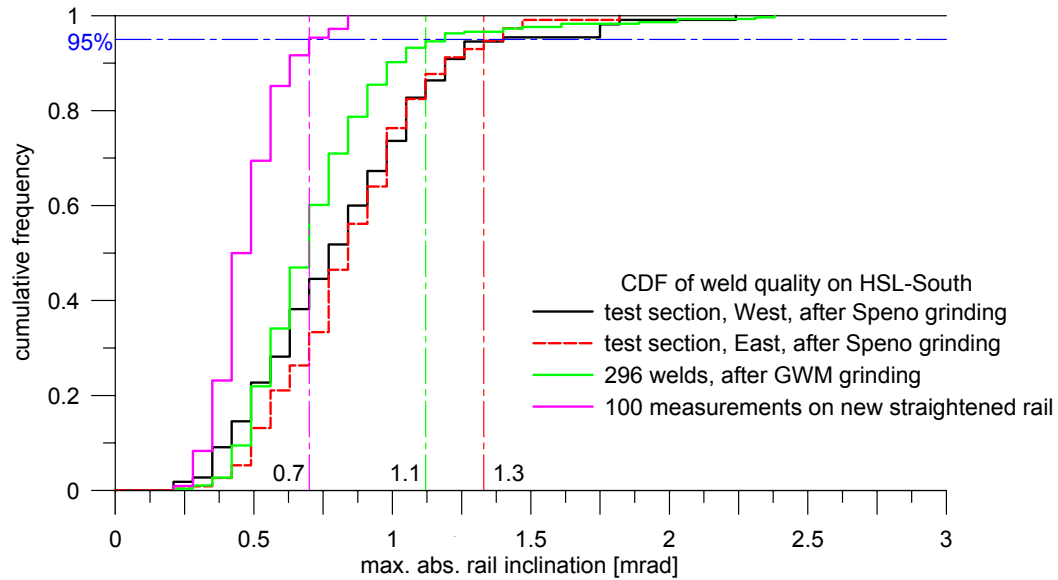


Fig. 4.57 Cumulative distributions of maximum absolute first derivatives on the Dutch HSL-South; comparison of measurements on new straightened rail segments and measurements of rail welds after grinding with grinding trains

In Fig. 4.57, the results obtained by Speno (from Fig. 4.56), by the GWM (from Fig. 4.55) and the measurements on new straightened rail (from Fig. 4.54) are compared. Based on the results in this figure, the 0.7 mrad limit seems a too strict value, and 1 mrad is proposed as a limit value in the QI determination for welds in new high-speed tracks (norm speed 300 km/h).

4.11 Application: the Dutch rail welding regulations (2005)

Eqs. (4.13, 4.20) establish the quantitative relationship between the maximum dynamic component of the wheel-rail contact force, the maximum inclination of the measured weld geometry and the train speed. In order to consistently determine speed-dependent norm values for the inclination, a predefined dynamic force level should be admitted. This force level should be related to the damage level, but since little is known of this relationship and furthermore this is strongly frequency-dependent, any choice of this level is rather arbitrary. Two facts may be taken as a point of departure:

(i) the empirical vertical tolerance of 0.3 mm has been used worldwide for several decades, not leading to systematic problems; the maximum peak deviation resulting from the standardized inclination should, for 140 km/h

(which is a conventional passenger train speed), not have a different order of magnitude;

(ii) the norm value for high-speed lines (300 km/h) has been fixed at the feasibility limit of 1 mrad in the previous section.

The norm value of 1 mrad at 300 km/h leads, according to Eq. (4.17) (with $\beta = 0.2$), to a dynamic force level of 17 kN. The value of 0.7 mrad, obtained for new straightened longrail, leads to a force level of 12 kN. The curves according to Eq. (4.17) corresponding with both force levels are depicted in Fig. 4.58. The following speed intervals have been used, according to the situation on the Dutch rail network: 0-40 km/h, 40-80 km/h, 80-140 km/h, 140-200 km/h (conventional lines) and 200-300 km/h (high-speed lines).

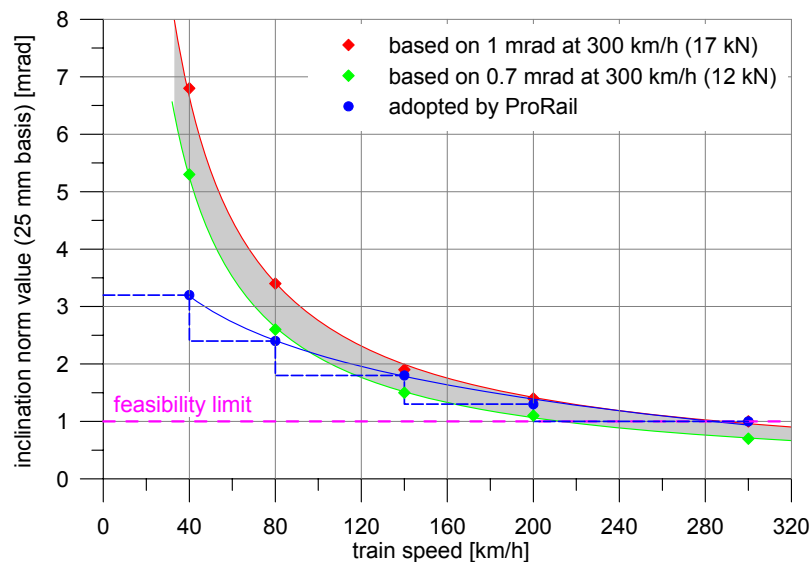


Figure 4.58 Determination of speed-dependent norm values in the QI definition

The norm values should be ideally situated in the hatched area between both curves, with a cut-off by the feasibility limit, in Fig. 4.58. However, due to the hyperbolic behaviour, the values for low speeds grow to infinity, whereas it is easy to grind, even manually, to a much better quality. Therefore, the smooth function depicted in Fig. 4.58, leaving the hatched area at 40 and 80 km/h, has been adopted by the Dutch Rail Infra Manager ProRail. In 2005, the geometrical standards as shown in Table 4.3 have been introduced for metallurgical rail welds in the Netherlands [4.48].

Table 4.3 Inclination norm values adopted by ProRail for the QI determination

Train speed [km/h]	max. abs. inclination (25 mm basis) [mrad]
0 – 40	3.2
40 – 80	2.4
80 – 140	1.8
140 – 200	1.3
200 – 300	1
<i>N.B.: 60, 100 (special tracks)</i>	2.8, 2.2

In Table 4.4 the amplitudes are given of a sinusoidally shaped weld with a half wavelength of 1 m. These amplitudes (equal to $|dz_i/dx|_{\max}/\pi$) give an impression of the maximum vertical peak deviations corresponding to the inclination values. Using the 1.8 mrad criterion, the maximum amplitude equals 0.57 mm at 140 km/h, which is larger than - but in the same order of magnitude as the 0.3 mm tolerance (i).

Table 4.4 Inclination norm values and the resulting 2 m - wave amplitude

V	based on Eq. (4.17) at 17 kN	resulting 2 m - wave amplitude	based on Eq. (4.17) at 12 kN	resulting 2 m - wave amplitude	ProRail values	resulting 2 m - wave amplitude
[km/h]	[mrad]	[mm]	[mrad]	[mm]	[mrad]	[mm]
40	6.8	2.16	5.3	1.69	3.2	1.02
80	3.4	1.08	2.6	0.83	2.4	0.76
140	1.9	0.60	1.5	0.48	1.8	0.57
200	1.4	0.45	1.1	0.35	1.3	0.41
300	1	0.32	0.7	0.22	1	0.32

In Fig. 4.59, a comparison is made of the acceptance level of welds, according to the conventional standards based on vertical tolerances (0 – 0.3 mm, independent of the speed), and the method based on first derivatives (the norm values adopted by ProRail have been used). The analysis has been performed for the sample of 239 welds which has been used for the investigation in sections 4.7 and 4.8. It is observed that the acceptance level increases drastically (except for 300 km/h). At the conventional 140 km/h the acceptance increases with 30 percent. This relaxation, which, according to the investigation in this chapter should not lead to an increasing rate of

deterioration, is due to the fact that peak deviations larger than 0.3 mm are accepted, provided that the contact geometry is smooth.

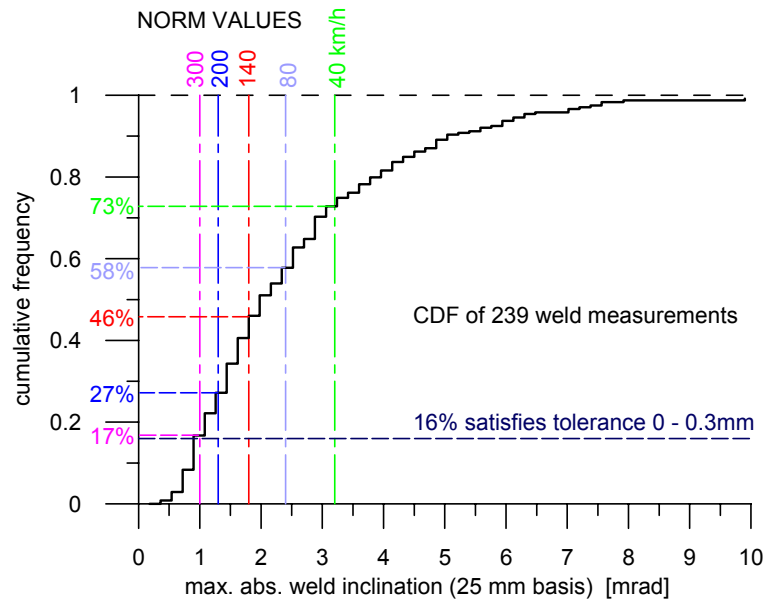


Fig. 4.59 Comparison of acceptance levels of rail welds according to the traditional method based on tolerances and the QI-method

4.12 Geometrical rail weld assessment in practice, according to the QI-method

The weld assessment method that was elaborated in this chapter can be simply implemented in practice. This is briefly illustrated in this section. After welding and cooling down of the rails, the geometrical quality can be measured using an electronic digital straightedge. In the processor the routines for the calculation of the quality index can be programmed. The device samples the rail geometry and plots the normalised first derivative (dependent on the train speed) and the QI on a screen. With the help of this output the geometry can be optimised such that the standards are met. An example of such a screen output is given in Fig. 4.60. At the location where the longitudinal rail geometry shown a relatively large inclination, the requirement is not met (the QI is 1.7), and the weld should be ground before acceptance.

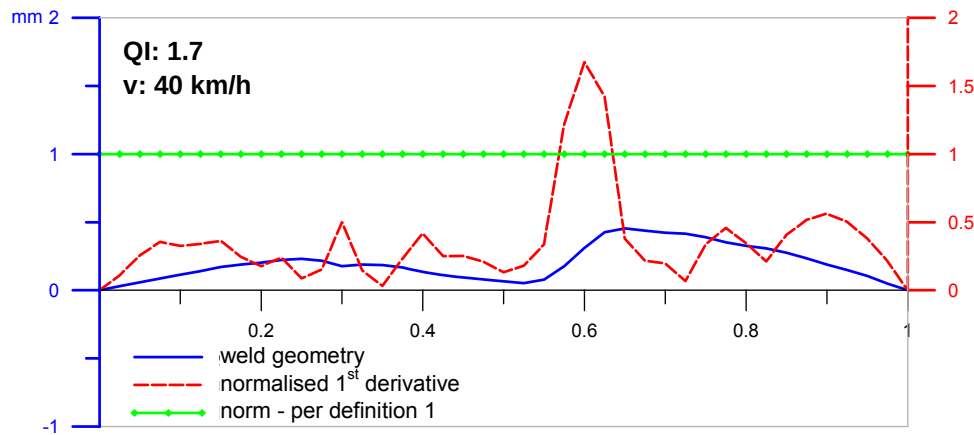


Fig. 4.60 Example of the screen output of a digital measuring device for the assessment of the geometry of rail welds

In Fig. 4.61, the steps for the assessment of rail welds are shown in practice.



Fig. 4.61 Semi-mechanical grinding of the rail weld surface in longitudinal direction, ground rail and electronic geometry measurement

4.13 Considerations on rail welds in heavy haul, conventional and high-speed lines

In paragraph 4.2, the relationship between the axle load, including the dynamic component, and the lifetime of the rail in relation to low-cycle bending fatigue was discussed. When considering welds on different track types, namely heavy haul tracks, conventional tracks and high-speed tracks, it must be realised that the total axle load on a rail weld depends, apart from the dynamic component, on the static axle load. This static axle load is essentially different on the three mentioned track types, and therefore it must be included in a lifetime assessment.

It has been established in paragraph 4.4 that the maximum dynamic wheel-rail contact force is generally generated by the rail reaction to the shortest irregularity, due to the relative and significant difference in inertia between the

wheels (the unsprung mass) and the rails (the equivalent track mass). The unsprung mass of heavy haul trains, conventional trains and high-speed trains is not essentially different. The high-speed axles in some cases (e.g. Shinkansen) even have the highest mass due to the presence of additional braking discs. Therefore, it is necessary to consider the static axle loads. According to Eq. (4.1), the following relationship is valid for the lifetime extension factor λ due to a weld improvement:

$$\lambda = \frac{\text{Lifetime}_{\text{new}}}{\text{Lifetime}_{\text{orig.}}} = \left(\frac{F_{\text{tot, orig.}}}{F_{\text{tot, new}}} \right)^3; \quad F_{\text{tot}} = F_{\text{stat}} + F_{\text{dyn}} \quad (4.21)$$

Assuming that the dynamic component of the load for a given weld geometry is equal for all cases, the total axle load is determined by the static component, which is chosen as follows:

$$\begin{aligned} F_{\text{stat, conv.}} &= 112.5 \text{ kN} \\ F_{\text{stat, hh}} &= 175 \text{ kN} \\ F_{\text{stat, hsl}} &= 85 \text{ kN} \end{aligned} \quad (4.22)$$

These values are based on nominal axle loads of 17.0 [4.49], 22.5 [4.27] and 35.0 tonnes [4.50] for high-speed trains, conventional trains and heavy haul trains. The lifetime extension factor λ is, according to Eq. (4.21), given by the third power of the total force – ratio before and after weld improvement. On heavy-haul tracks, the dynamic component is a smaller part of the total axle load than on conventional tracks. A reduction in dynamic load therefore has a smaller influence on the force ratio than the same reduction for a conventional track. The lifetime extension factor is therefore significantly smaller, due to the third power relationship. For high-speed tracks the opposite is valid: a given weld improvement leads to a significantly higher lifetime extension than on a conventional track.

To illustrate the above, in the following an example is elaborated. According to Eq. (4.20) or (4.30), $F_{\text{dyn}} \approx 0.2V\theta_{\text{max}}$. For heavy haul lines, the speed 80 km/h (22.2 m/s) is adopted; for conventional lines 140 km/h (38.9 m/s) and for high-speed lines 300 km/h (83.3 m/s). A weld improvement is considered, yielding a change in maximum inclination from 3 to 1.5 mrad. Table 4.5 shows the lifetime extension factors calculated for the three track types.

Table 4.5 Lifetime effect of a constant weld improvement on different track types

		Heavy Haul	Conventional	High-speed
	F_{stat} [kN]	175	112.5	85
$\theta_{\text{max}} = 3 \text{ mrad}$	F_{dyn} [kN]	13.3	23.3	50.0
	F_{tot} [kN]	188.3	135.8	135
$\theta_{\text{max}} = 1.5 \text{ mrad}$	F_{dyn} [kN]	6.7	11.7	25.0
	F_{tot} [kN]	181.7	124.2	110
λ		1.1	1.3	1.8

From the previous considerations and the results in Table 4.5, the following conclusions may be drawn: for a given weld geometry, the dynamic forces are the highest on high-speed track, and weld geometry improvement is most profitable on high-speed track.

In Table 4.6 a different improvement is considered; an original weld with a maximum inclination of 4 mrad is improved to the norm value for the speed corresponding to each track type (according to Table 4.3): 2.4 mrad for the heavy haul line, 1.8 mrad for the conventional line and 1 mrad for the high-speed line. The results for the lifetime extension factor show that the difference for the three track types becomes even more pronounced.

Table 4.6 Effect of weld improvement according to the norm for different track types

		Heavy Haul	Conventional	High-speed
	F_{stat} [kN]	175	112.5	85
$\theta_{\text{max}} = 3 \text{ mrad}$	F_{dyn} [kN]	13.3	23.3	50.0
	F_{tot} [kN]	188.3	135.8	135
$\theta_{\text{max, norm}}$ [mrad]		2.4 (80 km/h)	1.8 (140 km/h)	1.0 (300 km/h)
	F_{dyn} [kN]	10.7	14.0	16.7
	F_{tot} [kN]	185.7	126.5	101.7
λ		1.0	1.2	2.3

The previously obtained results may suggest that weld geometry improvement is not favourable on heavy haul tracks. It is stressed however that the considerations in this paragraph were limited to low-cycle bending fatigue. There is no reason to assume that heavy haul lines are less susceptible to high-cycle rail fatigue (RCF-damage types) than conventional or high-speed lines. Furthermore, the dynamic component of the force is, independent from the static axle load, responsible for track damage such as local ballast

decompaction and settlement, and broken clips and sleepers, as has been argued in section 4.4. From this viewpoint it remains necessary to limit the dynamic wheel-rail interaction force also on heavy haul lines.

4.14 *Weld geometry, energy input into the track and deterioration*

In the previous, a weld assessment based on a limitation of dynamic forces in the wheel-rail interface was elaborated. From the viewpoint of mechanical energy input into the track, its dissipation and the resulting damage, especially on the long term, as has been discussed in sections 1.3 and 3.4, it would be consistent to limit the energy or power input into the track. More strictly, the power spectrum should be limited in order to avoid damage, because different frequency regimes are dissipated in different components. The latter condition however seems not feasible in practice. Therefore, a correlation between the maximum first derivative (or another geometrical parameter describing the weld geometry) and the energy or maximum power input into the track can be looked for. This correlation is different from that between the maximum first derivatives and the maximum contact forces, due to the fact that the dynamic stiffness spectrum of the track (including both effects from stiffness/elasticity and inertia) is a non-uniform spectrum. Depending on the geometry of the excitation, different frequency regions of the dynamic stiffness spectrum may play a role.

In order to illustrate the computational procedure outlined in section 3.4, Fig. 4.62 shows the contact force, the vertical rail velocity and the power input along the track (or energy input into the track section) for both welds displayed in Fig. 4.40, when a train axle passes at 140 km/h. The first weld, which is rather smooth but suffers from some shrinkage of the weld material, yields a maximum power input only between 2 and 3 kW. The second weld, containing a severe step, yields a maximum power input between 700 and 800 kW, for each passing wheel (using a linear calculation). Keeping in mind that modern locomotives generally have a traction power between 2000 and 8000 kW (the latter for high-speed trains), a power loss of 800 kW per wheel into the track due to a bad weld is very severe.

Fig. 4.63 shows the correlation between the maximum absolute value of the power input into the rail and the vertical peak deviations and the maximum first derivatives respectively of the weld population used in section 4.8. The

same procedure is repeated in Fig. 4.64 for the total mechanical energy input into the track over the 1 m length over which the rail geometry is sampled. Both Figs. 4.63 and 4.64 show computational results only for a conventional train speed of 140 km/h. It is obvious that the linear correlation is rather bad in all graphs. A second-order correlation between maximum inclinations and maximum power input, and vertical deviations and energy input seems to make more sense. The first is confirmed by the relationships shown in Fig. 4.65. This figure shows computational results for the elementary case of a ramp in the rail surface, which was considered already in Figs. 4.34 and 4.35. At the left, maxima of the power input into the track due to P_1 are shown as a function of the ramp inclination, for a given train speed of 140 km/h. At the left this is repeated as a function of the velocity, for a given inclination of 2 mrad. The basis length of the ramp equals 25 mm in both cases. In both cases the relationship is exactly second-order polynomial.

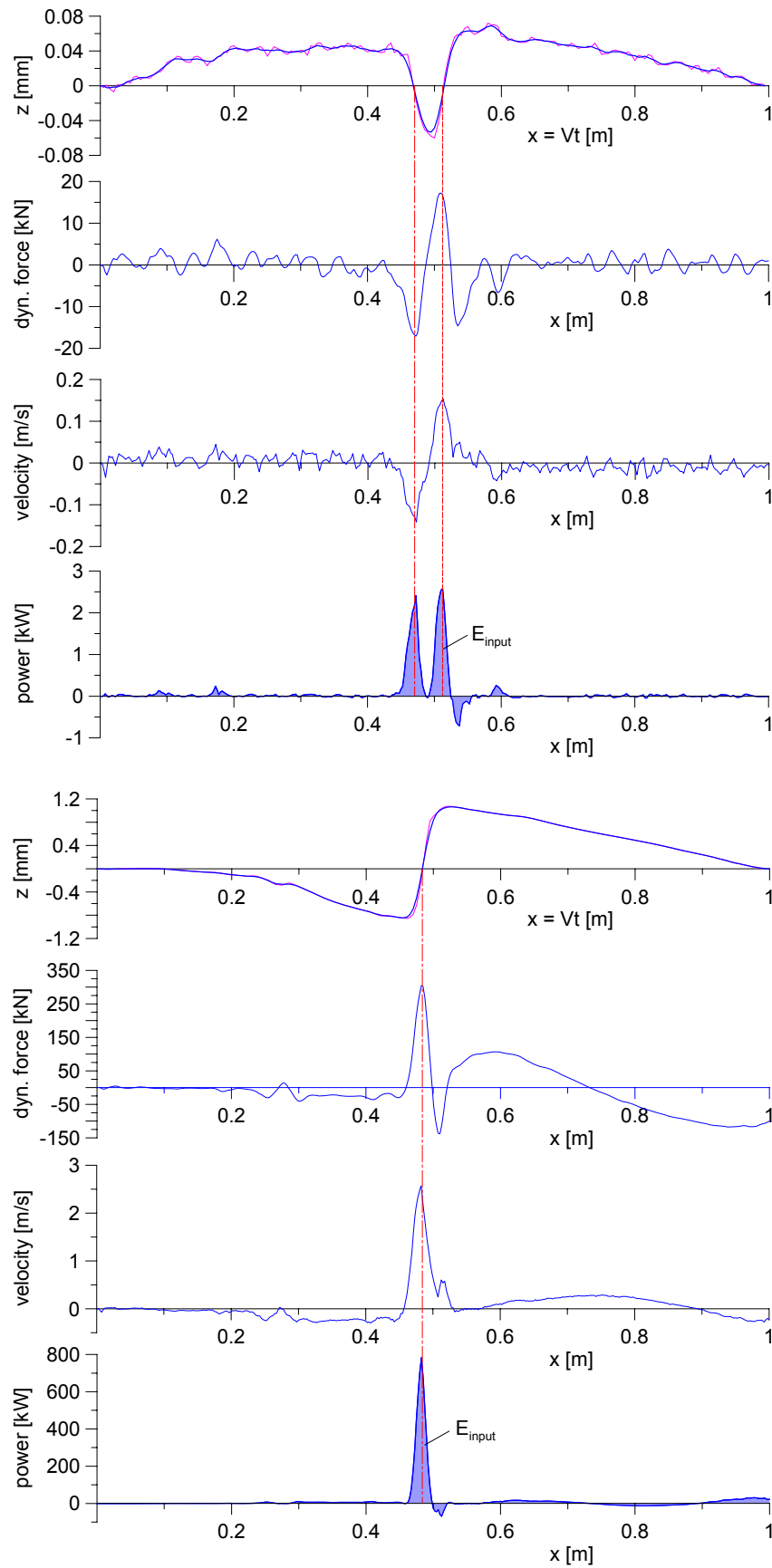


Fig. 4.62 Power input into the track for the welds depicted in Fig. 4.40 at 140 km/h

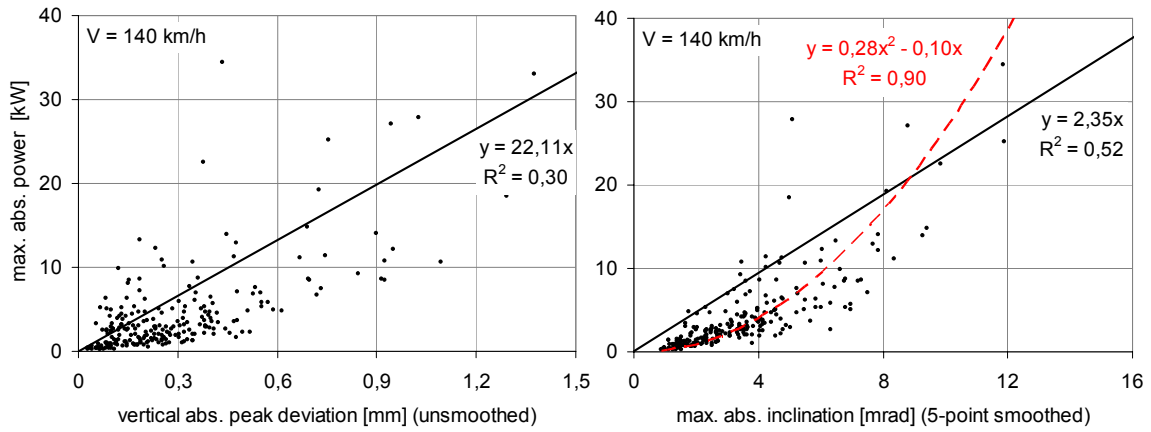


Fig. 4.63 Maximum absolute mechanical power input into the track as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 140 km/h

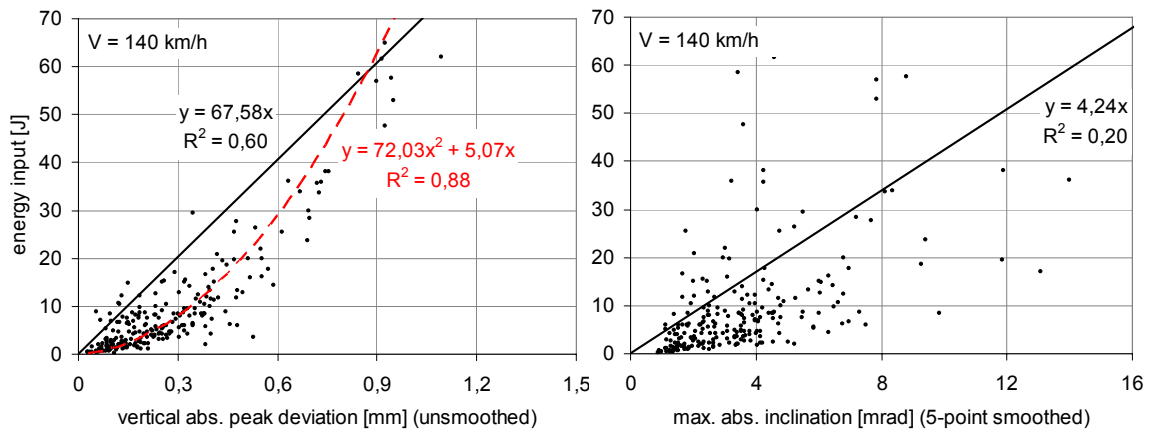


Fig. 4.64 Mechanical energy input into a 1 m track section as a function of the vertical peak deviation (left) and the maximum absolute inclination (right) respectively, at 140 km/h

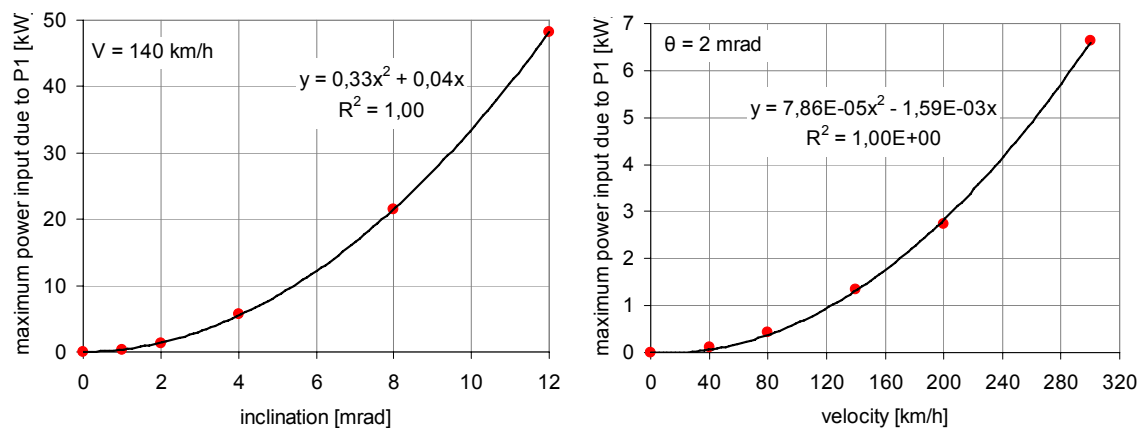


Fig. 4.65 Maxima of the power input into the track due to P_1 as a function of the ramp inclination, for 140 km/h (left) and as a function of the velocity, for an inclination of 2 mrad (right). The basis length equals 25 mm.

4.15 *Final considerations and conclusions*

The FE-model used in the simulations in section 4.7 modelled the rail as a Timoshenko beam, including shear deformation. The analytical simulations in section 4.8 were based on the Euler-Bernoulli beam theory for the rail. This theory does not account for shear deformation of the rail. Studies [4.51] have shown that Euler beam models are correct if the excitation frequency of the rail does not exceed 500 Hz; for higher frequencies the Timoshenko model gives more accurate results and gives good results up to 2000 Hz. It should be noted furthermore that the beam model cannot account for local railhead deformations. Very short defects do not lead only to rail bending, but to dynamic stresses (stress waves) which may only penetrate the railhead and the wheel tire, and in this case the rail should be modelled as a 3D dispersive medium to calculate the dynamic wheel-rail interaction in more detail, as was also discussed in chapter 1 (see e.g. Refs. [4.24, 4.25]). Such a more detailed FE-model of the rail could be used in combination with a more global track model (see e.g. Refs. [4.52-4.54]). Especially when investigating rail damage such as crack propagation close to or in a weld, it is important to account for the correct dynamic stress distribution along the height of the rail. This level of detail however goes beyond the scope of the presented research.

In the input for the simulations, the vertical geometry of the weld surface irregularities had a sampling interval of 5 mm, like in the measurements. In the definition of the QI, the interval of the averaged signal was extended to 25 mm, for practical reasons. The length of the wheel-rail contact patch is generally in the order 5-12 mm [4.29, 4.30]. The sensitivity of the level of the contact force to the sampling basis is automatically accounted for in a more detailed investigation as discussed above, a pure FEM approach. Again, this level of detail is interesting to study single cases of welds, but does not enter in the context of this chapter: a unique ‘threshold’ value for the length of the sampling basis, below which the contact force does not change, cannot be established in general. It depends on the length of the longitudinal axis of the wheel-rail contact patch, which does not only depend on the geometrical properties of wheel and rail, but also on the actual axle load.

From the study in this chapter, which considered the dynamic wheel-rail response to rail weld surface irregularities on ballasted track, the following main general conclusions can be drawn:

- 1) During train wheel passage, the rail follows a vertical rail weld surface irregularity quasi-instantaneously, whereas the wheel displacement shows a delay in response relative to the track. This phenomenon causes P_1 and P_2 wheel-rail forces. For the theoretical case of a ramp in the longitudinal rail surface, the maximum contact force P_1 is directly proportional to both the inclination of the ramp and the train speed. P_1 and P_2 increase with increasing rail bending stiffness; P_1 increases strongly and approximately linearly with increasing distributed rail mass, and P_2 decreases slightly with increasing rail mass. During P_1 - interaction, the rail follows a surface irregularity to a large extent, whereas the wheel displacement is negligible. Both inertial and elastic contributions to the dynamic force are of major importance within this time-interval. During P_2 - interaction both wheel and rail displacements have comparable magnitudes. The inertial component of the dynamic force is negligible in comparison to the elastic one.
- 2) The maximum dynamic contact force is generated by the irregularity with the shortest length-scale. However, the corresponding force peaks are rather narrow, and the corresponding amount of energy is rather small; these high-frequency peak forces damage mostly the rail and railhead itself. The delayed reaction of the wheel mass results in a carrier frequency in the contact force. The highest energy of the dynamic contact force is contained in this relatively low frequency. The relatively large energy contained in this carrier frequency is mainly responsible for ballast bed deterioration, as it is not efficiently dissipated in the rail and the railpads.
- 3) There is no correlation between maximum dynamic wheel-rail forces and the peak deviation of the vertical weld geometry. Therefore, an assessment based on the principle of peak deviations satisfying tolerances is inadequate to assess the geometry of rail welds from the viewpoint of dynamic wheel-rail forces and the resulting track deterioration. The maximum absolute inclination or first derivative of the vertical weld geometry on the contrary gives a good indication of the maximum dynamic wheel-rail forces occurring for a train wheel passing the weld. Furthermore, the level of the dynamic wheel-rail force is found to increase linearly with the train speed up to 300 km/h. Significant tailing-off due to non-linearities occurs for undamaged welds only at higher speeds. Therefore, the QI is a suitable tool to assess rail weld geometry from a viewpoint of the dynamic wheel-rail response. Fig. 4.58 specifies suitable norm values for a maximum weld inclination as a function of the train line speed.

- 4) The change from bolted rail joints to welded rail joints reduced the dynamic loading at the connections with globally a factor 3.

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5 Wheel flats⁵

5.1 General

Wheel flats generally arise when a train wheel is locked during braking, as a result of poorly adjusted or defective brake systems or a braking force which temporarily exceeds the available wheel-rail friction force. A second reason for the wheel flat generation is a local wheel-rail adhesion force reduction of an operating train wheel, leading to slip. The latter reduction can occur due to snow on the rails, lubrication or the presence of leaves in autumn (see, for example, [5.1]). Fig. 5.1 shows an example of a wheel flat, formed by abrasion.



Fig. 5.1 Illustration of a wheel flat formed by wheel-rail slip in the contact

⁵ The content of this chapter is reflected in the following international journal publications:

- M.J.M.M. Steenbergen. *The role of the contact geometry in wheel-rail impact due to wheel flats*. Vehicle System Dynamics, 2007, 45 (12), 1097-1116.
- M.J.M.M. Steenbergen. *The role of the contact geometry in wheel-rail impact due to wheel flats, Part II*. Vehicle System Dynamics, 2007, in press.

Wheel flats may cause significant damage to both railway vehicles and track due to the resulting high impact forces. Damage types include hot or broken axles and axle boxes, damaged bearings, cracks penetrating into the wheels [5.2] and rail fracture with the risk of derailment and cracking of concrete sleepers. Apart from damage, wheel flats produce excessive noise and vibration levels.

The wheel flat problem, which is an old problem, remains of current interest because trains are operating on the limit of the possible wheel-rail tractions in accelerating and decelerating in many countries, as a consequence of the optimised and tight time schedules as well as the increased train speeds. This favours the formation of circumferential wheel tread defects like wheel flats.

Wheel flats and the corresponding wheel-rail interaction have been subject to investigation from about 1950. A brief overview of some important steps is presented in the following. Popp (1952) [5.3] published a first and analytical investigation into the effects of wheel flats. Jenkins *et al.* (1974) [5.4] performed model simulations for harmonically shaped wheel flats and presented trends of rail displacements, bending moments and contact stresses as a function of the train speed. This work was further extended by Frederick (1979) [5.5], who presented both model simulations and experimental results. Vér *et al.* (1976) [5.6] studied noise generation due to wheel and rail irregularities like rail joints and wheel flats. They were the first to derive a criterion for loss of contact dependent on the shape of the wheel-rail interface. Newton and Clark (1979) [5.7] presented results for wheel-rail interaction due to wheel flats from both measurements and numerical simulations; they studied a harmonically worn flat, the shape of which is ground into the rail. Kumagai *et al.* (1991) [5.8] studied the causes of wheel flat occurrence and presented statistics on wheel flats. Dukkupati and Dong (1999) [5.9] developed and validated a numerical model for dynamic wheel-rail interaction due to wheel flats and investigated the effect of some parameter changes. Jergéus *et al.* (1999) [5.2] presented extensive full-scale experiments for the formation of wheel flats under controlled parameter conditions. They investigated the shape of the flats, but mainly focussed on the metallurgical phase transformations and cracks which arise due to the heat generation during the flat formation. Nielsen and Johansson (2000) [5.10] presented an overview of earlier experiments on dynamic wheel-rail interaction due to wheel flats by Ahlbeck *et al.*, Kalay *et al.* and Stone *et al.* Wu and Thompson

(2001) [5.11] developed and validated a model to simulate the dynamic wheel interaction due to wheel flats. They performed parameter studies and used the model for the prediction of noise generation. Johansson and Nielsen (2003) [5.12] presented results of extensive field tests to determine the track response and the contact forces caused by wheel flats, for different velocities and axle loads. Baeza *et al.* (2006) [5.13, 5.14] studied dynamic wheel-rail interaction due to wheel flats with a non-Hertzian contact model and compared results to the ones obtained with Hertzian models.

Other contributions in the literature, which are not mentioned explicitly, deal with wheel out-of-roundness (OOR), which leads to high dynamic contact force amplifications, but not to impact. The length-scale of an OOR-defect is typically the wheel circumference (which is approximately 3 m).

Several contributions in the literature (e.g. refs. [5.7, 5.12]) deal with the registration of dynamic wheel-rail contact forces in the presence of wheel flats. These measurements have in common that they predict a local maximum in impact contact forces at a train speed between 30 and 60 km/h, whereas in many detailed models (see e.g. refs. [5.11, 5.13]) such an effect cannot be established. It is remarkable that these discrepancies and scatter in the results are reflected in the very different actual criteria for wheel flats over many countries. There exist two criteria types: force-based criteria, which are based on the principle of impact load detectors in the track (like the WILD/Salient system [5.15] or the Dutch Gotcha system [5.16]) and criteria which specify an allowable length or depth of the flat spot, or a combination of both. For example, the British Network Rail prescribes a maximum flat length between 40 and 80 mm, depending on the axle load [5.17]. On the Japanese Shinkansen network, a 50 mm length limit is adopted (30 mm is used in practice), and on the conventional network the limits are 75 mm for a single flat and 50 mm if two flats occur on a wheel [5.17]. Italian railways prescribe a maximum flat length of 40 mm [5.18]. Spanish railways use a 50 mm limit [5.11]. UIC (leaflet 510-2) prescribes a maximum length of 60 mm [5.19]. Apart from the difference in the standardization values, also the force-based and the geometry-based criteria do not match. It will be shown in this chapter that this is due to the fact that no unique relationship exists between the length of a flat and the maximum dynamic contact force.

In the following, a brief overview is given of the structure and the content of the present chapter. Section 5.2 starts with a classification of wheel

flats according to their growth stage. In sections 5.3 and 5.4 a space- and time-domain analysis is performed of a wheel with a flat, rolling on a rigid rail. In section 5.5, the consequences of the finite contact stiffness, wear and plasticity are taken into consideration. In section 5.6 the analysis is extended to situations of contact loss, by introduction of a transcritical speed domain. The established behaviour of the wheel-rail contact force as a function of its constitutive variables is considered in more detail in section 5.7. In section 5.8, frequency-domain expressions are derived for excitation functions due to wheel flats in the different growth stages. They serve as an input for dynamic wheel-rail interaction models, which include a non-rigid railway track and are formulated in the frequency-domain. This is the subject of section 5.9, in which also simulation results are presented and discussed. In section 5.10 the obtained results are confronted with results from the literature. Sections 5.11 and 5.12 deal with the experimental registration of wheel-rail contact forces in the presence of wheel flats; the equivalent rail indentation and the rail bending strain measurement method are discussed respectively.

5.2 *The stages of wheel flat development and classification*

In the development of a fresh wheel flat, formed by wheel blocking and successive abrasion of the wheel material, three growth stages can be distinguished, which are shown in Fig. 2. It should be remarked that if the wheel is not blocked completely, allowing partial slip in the contact, the type of wheel flat which is initially formed is stage III. Further, eventual thermal damage, such as crumbling away of wheel material due the formation of a martensite plug in the wheel surface caused by heat generation and steel phase transformation during wheel slide (see refs. [5.2, 5.20, 5.21]), is not taken into consideration, since it leads in any case to impact. Wheels with such types of damage should therefore always be taken out of service immediately, as confirmed in Ref. [5.21].

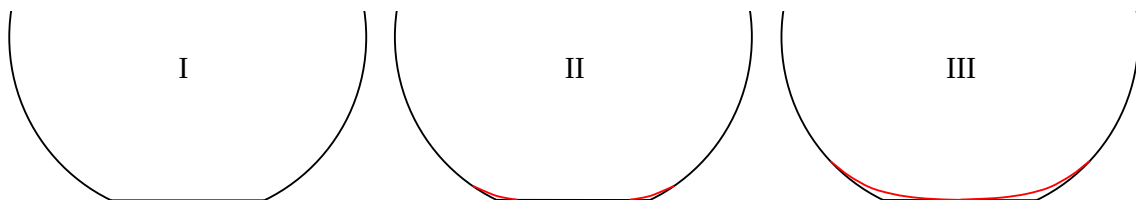


Fig. 5.2 Definition of the three different stages of abrasive wheel flat development

The three stages or categories and their features are discussed in the following.

- I. The flat is newly formed by abrasion of wheel material. It can be simply described by a chord of the circle which describes the wheel circumference through the contact point of the undisturbed wheel. The circumferential curvature of the wheel tread is zero along the whole flat.
- II. The flat is worn at both edges. The start of the wear and/or plastic flow from the edges is geometrically determined, since it is impossible for the material between the edges and the centre of the flat to come into single-point contact with the rail. The circumferential curvature of the wheel tread is non-zero along the worn parts of the flat, but the minimum curvature of the wheel tread occurs in the remaining flat part of the flat spot, and is still zero.
- III. The flat spot is completely worn up to its centre. The minimum circumferential curvature of the wheel tread becomes non-zero and positive.

An example of a measurement of the development of a fresh flat into a worn flat is given in Ref. [5.21]. In this example, the length in stage III is almost four times the original chord length, whereas the depth remains constant.

The stage I flat has no large practical importance because it can only exist immediately after its generation if the wheel blocks completely, but it will prove to simplify a theoretical analysis significantly.

5.3 *Space and time-domain analysis of a wheel with a flat, rolling on a rigid foundation*

In this section a train wheel having a flat and rolling at the train speed V on an infinitely rigid foundation is considered. As has been discussed in chapter 2, in the absence of a flat the wheel-rail contact force would have no contribution originating from the vertical inertia of the wheel. Fig. 5.3 shows the rolling train wheel with a fresh or stage I flat, described by a circle chord. The chord connects both endpoints of the flat, with length l and depth d . Position A corresponds to the position where the contact radius vector, perpendicular to the foundation, is R ; position B corresponds to the wheel position where this contact radius vector is R' . A detail of the rotating wheel with the flat, between the positions A and B, is shown in Fig. 5.4.

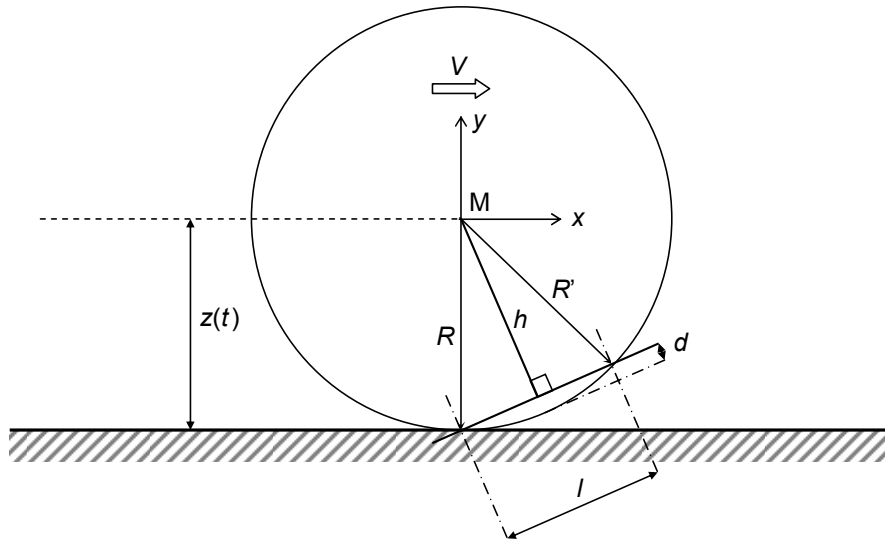


Fig. 5.3 Rolling train wheel with a stage I flat on a rigid foundation

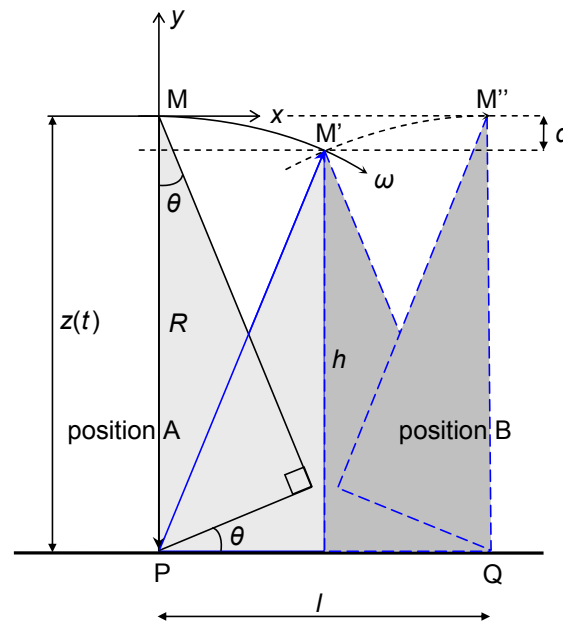


Fig. 5.4 Detail of the contact geometry in Fig. 5.3

In the following, it is assumed that the angular velocity ω of the wheel is constant. The same holds for the train speed V , which is further assumed to be relatively small. The effects of an increasing speed will be investigated in section 5.6.

In position A, the centre of rotation changes from the wheel centre M to the contact point P . The wheel gravity centre M then describes a circular curve until it reaches M' . In this position the distance between the wheel centre and the foundation reaches a minimum and is given by the difference

of the wheel radius R and the flat depth d . The instantaneous centre of rotation changes abruptly from P to Q, and the wheel gravity centre describes a second circular curve with radius R , until it reaches M". In this position (B), the rotation centre of the wheel changes again to its 'conventional' position M".

In Fig. 5.5, the distance $z(t) (= y(t) + R)$ of the wheel gravity centre to the foundation is shown as a function of time in the upper graph. In position A this centre experiences a smooth transition between a straight line and a circle segment (the tangent to the circle segment is horizontal in A). $z(t)$ reaches the minimum $R - d$ at $(t_A + t_B)/2$. After this time moment, $z(t)$ is symmetric with respect to the line $t = (t_A + t_B)/2$.

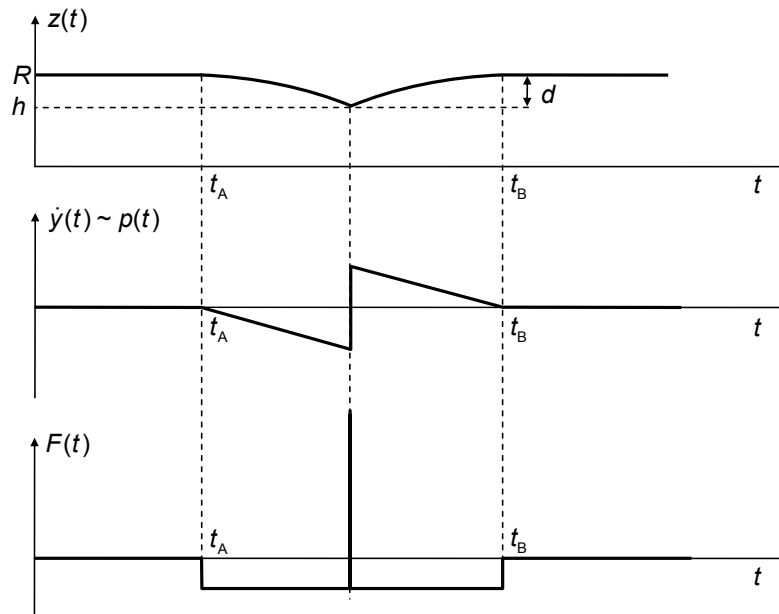


Fig. 5.5 Trajectory of the gravity centre of the wheel, vertical velocity of the wheel mass and contact force corresponding to Figs. 5.3 and 5.4, for a stage I flat

The time-derivative of $z(t)$ ($\dot{z}(t) = \dot{y}(t)$) represents the vertical velocity of the wheel mass. When the circle segments in $z(t)$ are approximated by a second-order polynomials with their respective axes of symmetry in t_A and t_B (which is a very good approximation, as will be shown in the following), their derivatives are given by linear functions. A jump (discontinuity) between both linear functions occurs at $(t_A + t_B)/2$. The time-derivative of the trajectory is shown in the second graph of Fig. 5.5.

The vertical linear momentum $p(t)$ of the wheel mass is proportional to this velocity, as it is given by the product of the wheel mass m and its vertical velocity.

The dynamic component of the vertical contact force between the wheel and the foundation can be simply obtained by differentiation of the vertical momentum over time, according to $F(t) = \dot{p}(t) = m\ddot{y}(t)$. This force is shown in the graph at the bottom of Fig. 5.5. It is given by a constant negative value between t_A and t_B , and the Dirac delta-function at $t = (t_A + t_B)/2$, which means that it grows to infinity in an infinitely short time-period at that moment. This is the cause of the impact force occurring in the contact for rolling wheels with a flat.

5.4 Mathematical problem formulation for a rigid foundation

In the following, expressions are derived for the trajectory of the wheel centre, its vertical velocity, its vertical momentum and the contact force. The Cartesian coordinate system (x, y) with its origin in the wheel mass centre in position A is used (Fig. 5.3), and still a rigid foundation is being considered. For $x \geq 0$ the trajectory can be written as (H represents the Heaviside function):

$$y(x) = \left(\sqrt{R^2 - x^2} - R \right) H(-x + l/2) + \left(\sqrt{R^2 - (x - l)^2} - R \right) (H(x - l/2) - H(x - l)) \quad (5.1)$$

A numerical example of this function is shown in Fig. 5.6.

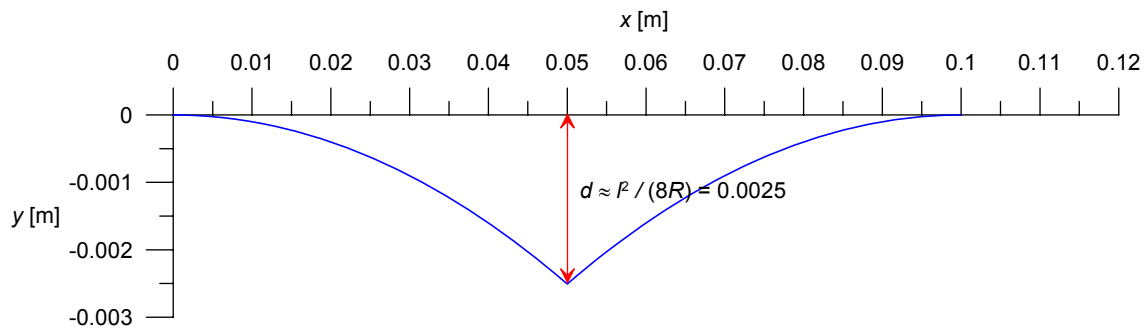


Fig. 5.6 Graphical example of $y(x)$ for the values $R = 0.5\text{m}$ and $l = 0.1\text{m}$, according to expression (5.1)

The spatial derivative of Eq. (5.1) is given by:

$$\begin{aligned} \frac{dy(x)}{dx} = & \left(\frac{-x}{\sqrt{R^2 - x^2}} \right) H(-x + l/2) \\ & + \left(\frac{-x + l}{\sqrt{R^2 - (x - l)^2}} \right) (H(x - l/2) - H(x - l)) \end{aligned} \quad (5.2)$$

A numerical example of this function is shown in Fig. 5.7 (which is related to Fig. 5.6). It is observed that the resulting function is almost bi-linear in the considered time-interval.

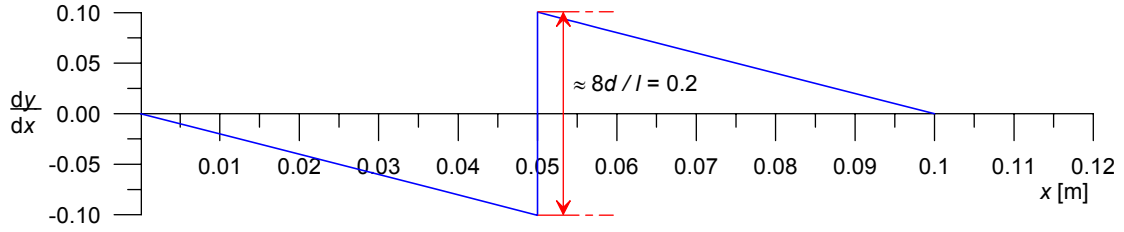


Fig. 5.7 Exact derivative $dy(x)/dx$ for the trajectory in Fig. 5.6

With $x = Vt$ the following is obtained in the time domain:

$$\begin{aligned} y(t) = & \left(\sqrt{R^2 - V^2 t^2} - R \right) H(-t + 0.5l/V) \\ & + \left(\sqrt{R^2 - (Vt - l)^2} - R \right) (H(t - 0.5l/V) - H(t - l/V)) \end{aligned} \quad (5.3)$$

and

$$\begin{aligned} \frac{dy(t)}{dt} = & \frac{-V^2 t}{\sqrt{R^2 - V^2 t^2}} H(-t + 0.5l/V) \\ & - \frac{V(Vt - l)}{\sqrt{R^2 - (Vt - l)^2}} (H(t - 0.5l/V) - H(t - l/V)) \end{aligned} \quad (5.4)$$

Consequently, the vertical momentum of the wheel mass is given by:

$$\begin{aligned}
p(t) = m\dot{y}(t) = & \frac{-mV^2t}{\sqrt{R^2 - V^2t^2}} H(-t + 0.5l/V) \\
& - \frac{mV(Vt - l)}{\sqrt{R^2 - (Vt - l)^2}} (H(t - 0.5l/V) - H(t - l/V))
\end{aligned} \tag{5.5}$$

The jump in this function, representing the discrete change in vertical momentum of the wheel mass at $t = (t_A + t_B)/2$ equals:

$$\Delta p|_{t=l/(2V)} = \frac{mVl}{\sqrt{R^2 - l^2/4}} \tag{5.6}$$

This exact expression for a perfect wheel flat can be expressed in terms of l and d . From Fig. 5.3 follows:

$$R^2 = (l/2)^2 + (R - d)^2 \tag{5.7}$$

With $d \ll l, R$ (see Fig. 5.6) d^2 is second-order and may be neglected. Then it follows:

$$R = \frac{l^2}{8d} \tag{5.8}$$

This yields for the denominator of Eq. (5.6):

$$\sqrt{R^2 - l^2/4} = \frac{l}{2} \sqrt{\left(\frac{l}{4d}\right)^2 - 1} \tag{5.9}$$

so that Eq. (5.6) can be written as:

$$\Delta p|_{t=l/(2V)} = \frac{2mV}{\sqrt{(l/(4d))^2 - 1}} \tag{5.10}$$

A simple approximation of this expression can be derived. For $l \gg d$, $l/(4d) \ll 1$, or $(l/(4d))^2 \ll 1$. Then, $\sqrt{(l/(4d))^2 - 1} \approx l/(4d)$. This results in the following approximation for expression (5.10):

$$\Delta p|_{t=l/(2V)} \approx 8mVd/l \quad (5.11)$$

The contact force between the wheel and the foundation is obtained by differentiation of the momentum (5.5) over time, yielding:

$$F(t) = \frac{-mV^2R^2}{\sqrt{(R^2 - V^2t^2)^3}} H(-t + 0.5l/V) + \delta(t - 0.5l/V) - \frac{mV^2R^2}{\sqrt{(R^2 - (Vt - l)^2)^3}} (H(t - 0.5l/V) - H(t - l/V)) \quad (5.12)$$

At $t = 0.5l/V$ the force is described by the Dirac delta-function. The force is symmetric around the line $t = 0.5l/V$. A numerical example of Eq. (5.12) is shown in Fig. 5.8; this figure shows the second spatial derivative of the function in Fig. 5.6. It is remarked that the spatial and temporal derivatives differ only by a constant in this case.

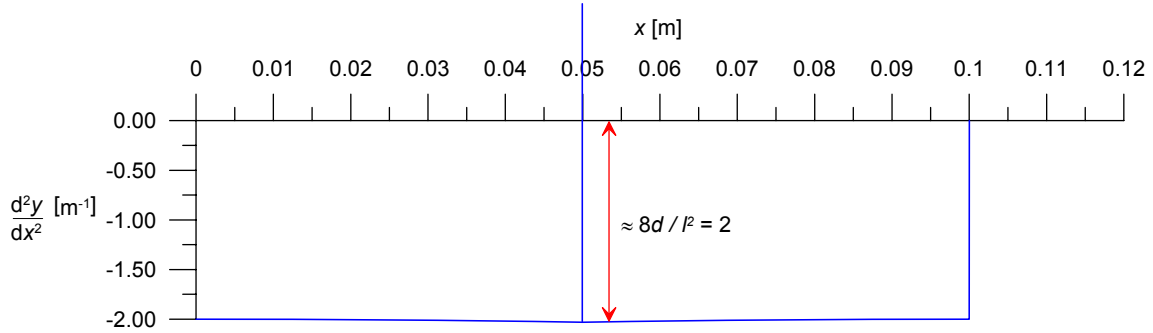


Fig. 5.8 Exact derivative $d^2y(x)/dx^2$ for the trajectory in Fig. 5.6

As can also be observed from Fig. 5.8, both factors preceding the Heaviside-functions in Eq. (5.12) are nearly constants for the considered parameter domains. Therefore, a good approximation to expression (5.12) can be obtained through use of simplification (5.11):

$$F(t) = \frac{-8mV^2d}{l^2} H(-t + l/V) + \delta(t - 0.5l/V) \quad (5.13)$$

5.5 *The consequences of the finite contact elasticity and starting plasticity and wear*

The considerations in this section will still be limited to a wheel rolling on a rigidly supported rail. The expressions (5.12) and (5.13) predict an infinite contact force at a discrete time moment for a rigid wheel with a stage I flat rolling on a rigid foundation. The fact that the flat is perfect causes the singularity in the contact force. In reality, the finite stiffness in the contact attenuates the singularity in the contact force to some finite value. Further, after a fresh flat has been generated, the high contact force immediately initiates plastic flow and wear, starting from the flat edges, as has been discussed in section 5.2. The process of interaction between the geometry and the contact force continues until a balance situation is reached. Such a balance situation may be stable or unstable, but the geometry of the flat will remain according to stage III.

The results of the non-zero and finite elasticity in the contact and starting wear and/or plastic flow are shown qualitatively in the graphs in Fig. 5.9, for the trajectory of the wheel gravity centre, its vertical momentum and the contact force.

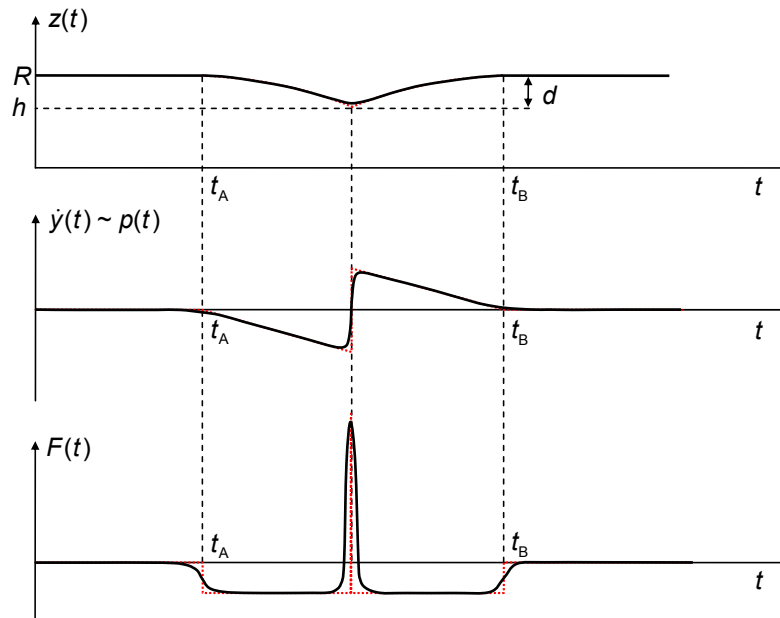


Fig. 5.9 Trajectory of the gravity centre of the wheel, vertical velocity of the wheel mass and contact force corresponding to Figs. 1 and 2, after occurrence of plastic flow and wear and accounting for the finite contact stiffness (qualitative graphs)

In the following, again the three stages of wheel flat growth which were introduced in section 5.2 are considered, with a focus on the trajectory of the wheel gravity centre.

As has been established, in stage I the circumferential curvature of the wheel tread is zero along the whole flat spot, yielding a theoretically infinite curvature of the wheel centre trajectory.

In stage II the circumferential curvature of the wheel tread is non-zero (between 0 and $1/R$) along the worn parts of the flat, but the minimum curvature of the wheel tread occurs in the remaining flat part of the flat, and is still zero. The edge wear increases the time interval $t_A - t_B$ and ‘stretches’ both circular segments of the trajectory. The zero minimum wheel tread curvature yields a theoretically infinite maximum curvature of the wheel centre trajectory, along with the theoretical singularity in the contact force. Therefore, in the transition from stage I to stage II the peak value of the contact force remains unaffected, but the behaviour as a function of time changes.

In stage III, the flat part has worn out and becomes convex, resulting into a positive minimum circumferential curvature of the wheel tread. As a result the maximum curvature of the wheel centre trajectory becomes finite; the singularity in the contact force disappears and the impact force is attenuated. Therefore, the transition from stage II to stage III yields, apart from a further change in the time-dependency of the contact force, an attenuation of its peak value, the impact. The quantitative relation between the degree of attenuation and the curvature of the wheel tread will be discussed further in this section.

Fig. 5.10 shows, in the graph at the top, the qualitative kinematical trajectory of the wheel gravity centre in the three growth stages. The graph in the middle shows the wheel tread curvature along the wheel circumference. For an undisturbed wheel this is $1/R$. In stage I the value jumps instantaneously to infinity at the first flat edge and immediately afterwards to zero, jumps to infinity at the second flat edge and then back to $1/R$. In stage II the transitions are smoothed, and in stage III the value gradually decreases and increases without reaching zero. In the graph at the bottom the wheel tread curvature is shown again, but now for a rolling wheel at uniform speed in the subcritical speed regime, experienced in the contact point. In stage I, the value jumps to infinity at the first flat edge, remains at infinity during pivoting, drops to zero at the moment in which the flat comes into contact

with the rail, immediately jumps to infinity again, and after pivoting drops to its initial value $1/R$. In Fig. 5.10, the change in the length of the total wheel circumference due to the presence of a flat is neglected.

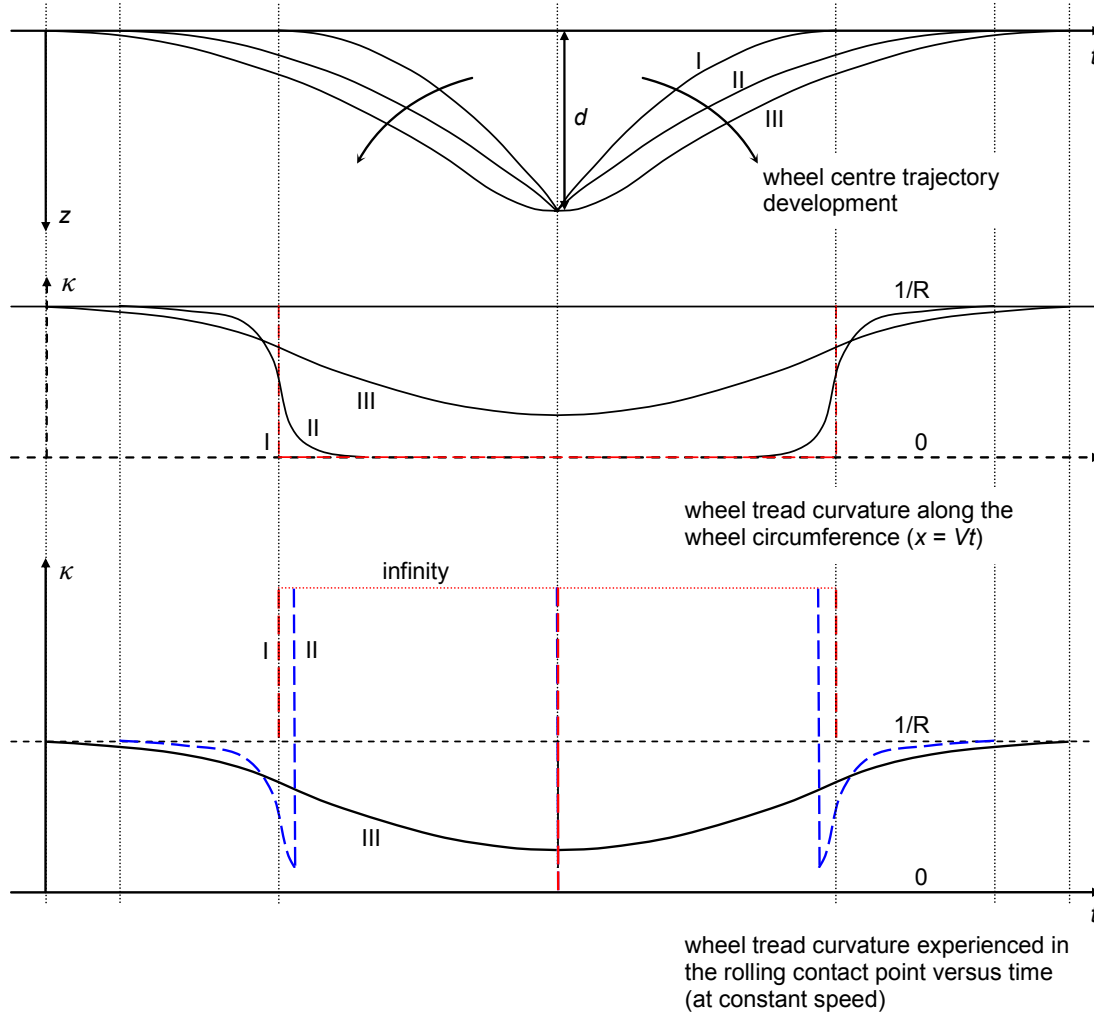


Fig. 5.10 Development of the kinematical wheel centre trajectory according to the 3 stages of wheel flat development depicted in Fig. 5.2 (top); wheel tread curvature along the wheel circumference (centre) and experienced by the steadily rolling wheel in the contact point (bottom).

We now return to Fig. 5.9, which shows qualitatively the influence of a finite contact elasticity and the effect of starting wear resulting into a non-zero minimum wheel tread curvature on the vertical wheel momentum and the contact force. In the following the function $p(t)$ will be considered only in the interval $[(t_A + t_B)/2, t_B]$ to simplify a quantitative description of the attenuated impact force. The function $p(t)$ is pointwise symmetric with respect to $((t_A + t_B)/2, 0)$ and $F(t)$ is symmetric with respect to the line

$t = (t_A + t_B)/2$. Therefore it is sufficient to consider this time-interval to obtain the value of the maximum contact force. In this interval, the following general form of $p(t)$ may be assumed for a worn flat:

$$p(t) = p_0 (1 - e^{-t/\tau}) (1 - at); \quad t \in [(t_A + t_B)/2, t_B] \quad (5.14)$$

The first function between brackets describes the sharp increase of $p(t)$ from zero to its maximum in a short time-interval; τ [s] is a time-scale for this process. The second function between brackets describes the theoretically linear decrease of $p(t)$ for the whole time-interval. The maximum contact force can be found as the maximum of the derivative of Eq. (5.14). This derivative is given by:

$$\dot{p}(t) = \frac{p_0}{\tau} (e^{-t/\tau} (1 - at + a\tau) - a\tau); \quad t \in [(t_A + t_B)/2, t_B] \quad (5.15)$$

In Fig. 5.11 two numerical examples are shown of the functions (5.14) and (5.15).

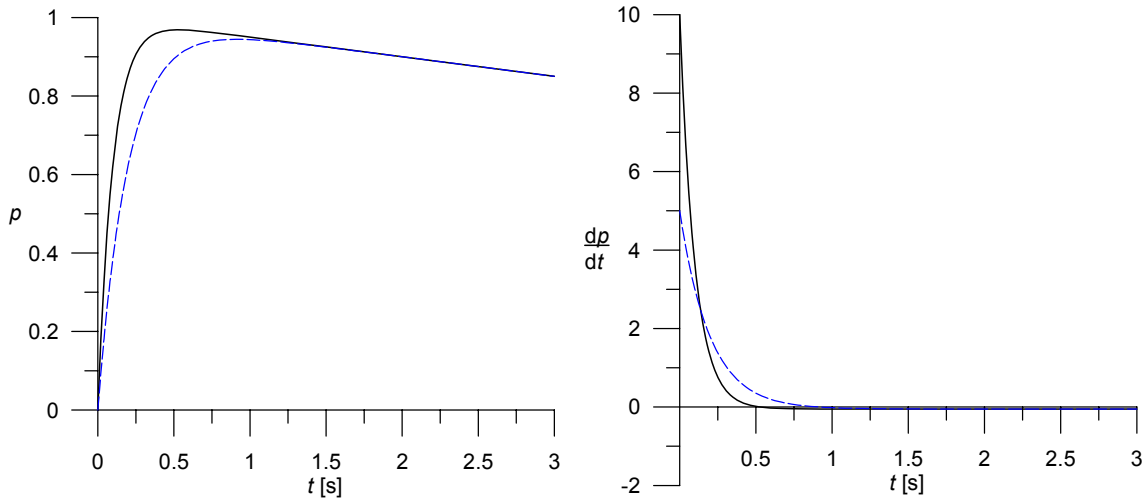


Fig. 5.11 Example of expression (5.14) and its time-derivative for the parameter values $P_0 = 1$; $\tau = 0.1$ (solid line) and $\tau = 0.2$ (dashed line) respectively; $a = 0.05$.

The maximum of Eq. (5.15) occurs at $t = 0$ and equals:

$$F(0) = \dot{p}(0) = p_0 / \tau \quad (5.16)$$

To determine the magnitude of p_0 , which represents basically half of the jump Δp in Fig. 5.9, it is convenient to use the simplified expression (5.11). This yields the following expression for the maximum contact force:

$$F_{\max} = \frac{4mVd}{l\tau} \quad (5.17)$$

The decrease in the contact force before and after impact is unaltered; it follows from Eq. (5.13) and is given by:

$$\Delta F = \frac{8mV^2d}{l^2} \quad (5.18)$$

In the previous, the wheel was assumed to roll on an infinitely rigidly supported rail. In reality, a railway track is built up from components with a finite inertia and elasticity in vertical direction. However, the behaviour of a wheel with a fresh flat rolling on a rigid foundation is not essentially different from the behaviour of such a wheel rolling on a flexible track. With an increasing time-scale τ (τ increases with a decreasing contact stiffness), the inertia of the rail in vertical direction will become increasingly important. The resulting rail displacement will, apart from the effect of wear, further attenuate the impact force. Due to the higher wheel impedance relative to the track impedance, V  r *et al.* [5.6] assume that the relative displacement of wheel and track will be accommodated by the rail, which could be most easily implemented by replacing the wheel mass m in Eq. (5.18) by an equivalent track mass. Although the assumption is generally correct for short-wave track irregularities, it is however not necessarily correct in the case of singularities like fresh wheel flats, which have theoretically an unbounded uniform wavelength spectrum. Simulations presented by Wu and Thompson [5.11] for a wheel with a fresh flat running on a flexible railway track show that at 30 km/h the wheel accommodates approximately 80 per cent of the flat depth (2 mm) and the same 80 per cent of the effective flat depth (0.67 mm; this concept will be discussed in the next section) at 80 km/h; the dynamic rail displacement shows - during impact - much smaller percentages.

In a simple approach, the effect of the track displacement or the effective track mass can therefore be simply and efficiently dealt with by introducing a dimensionless validation constant in expression (5.17). In principle, this is

equivalent to substituting some effective or equivalent mass, either of the wheel or the track. In order to relate the time-scale τ exclusively to the degree of wear along the flat, it is chosen to extend this expression with a separate constant factor η accounting for the consequences of the track elasticity. In this factor, the constant 4 from Eq. (5.17) is included for simplicity. The resulting expression for a wheel with a flat running on a flexible railway track becomes:

$$F_{\max} = \eta \frac{mVd}{l\tau} \quad (5.19)$$

It is observed that the scale parameter τ is of paramount importance, with the same proportionality as, for example, the train speed. For $\tau \rightarrow 0$ the contact force grows to infinity, which is the theoretical case shown in Fig. 5.5. For reasons explained above, this parameter and its magnitude will be analysed more into detail. It has been discussed already that in theory the singularity in the contact force remains as long as the flat has a part with zero curvature, yielding an infinite curvature of the trajectory (see Fig. 5.8). The geometrical relationship between the maximum curvature of the trajectory (which controls the parameter τ) and the minimum curvature of the wheel tread in circumferential direction can be described as (c_1 is a constant):

$$\kappa_{\max, \text{trajectory}} = c_1 \left(1/\kappa_{\min, \text{tread}} - R \right) / R^2; \quad 0 \leq \kappa_{\min, \text{tread}} \leq 1/R \quad (5.20)$$

This function is shown at the left-hand side in Fig. 5.12. Time-scale τ may be written as a function of the maximum curvature of the trajectory in the following form (c_2 is a constant):

$$\tau = c_2 / \left(\kappa_{\max, \text{trajectory}} V \right) \quad (5.21)$$

Combination of Eqs. (5.20) and (5.21) yields the following expression for τ (c is a constant):

$$\tau = \frac{cR^2}{V \left(1/\kappa_{\min, \text{tread}} - R \right)}; \quad 0 \leq \kappa_{\min, \text{tread}} \leq 1/R \quad (5.22)$$

This function is shown at the right-hand side in Fig. 5.12. Substitution of Eq. (5.22) into Eq. (5.19) yields for the impact force (μ is a track-dependent constant):

$$F_{\max} = \mu \frac{mV^2 d \left(\frac{1}{\kappa_{\min, \text{tread}}} - R \right)}{R^2}; \quad 0 \leq \kappa_{\min, \text{tread}} \leq 1/R \quad (5.23)$$

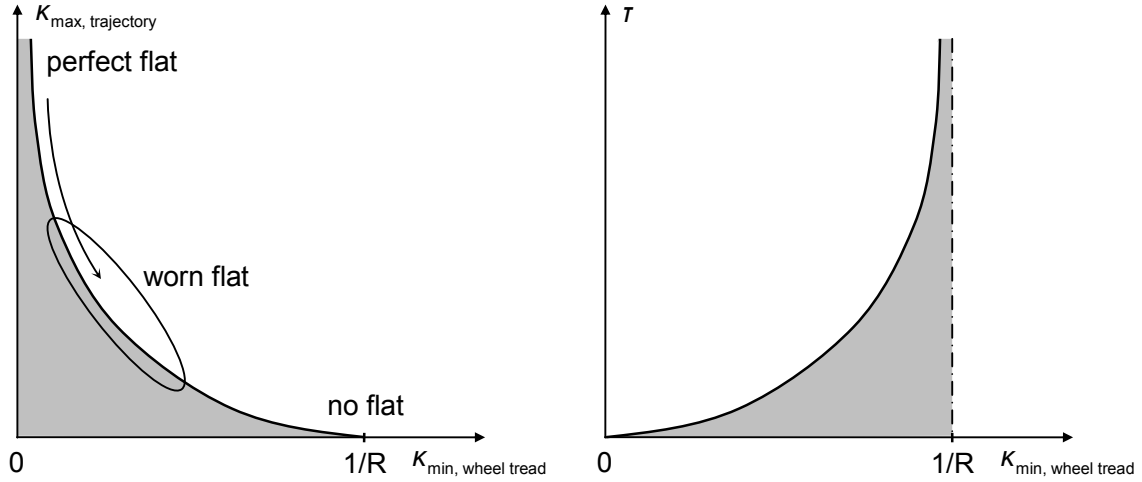


Fig. 5.12 (Left) behaviour of the maximum curvature of the trajectory of the wheel gravity centre as a function of the minimum curvature of the wheel tread and (right) impact time-scale τ as a function of the minimum curvature of the wheel tread (for a given velocity)

In general, experimental results are classified as a function of the flat depth or length, or their ratio. It follows from the above that this is only consistent in the case of equally worn out flats, in the sense that their minimum curvatures are equal (Fig. 5.13). If this is ignored, large scatter in experimental results can be expected. This may be one of the reasons for the significant scatter which is observed in some experiments in the literature (see e.g. [5.12]). Different track receptances at different positions along the track can only to a small extent contribute to this scatter; as has been discussed the rail mass is the main track parameter which is involved, and the rail parameters are almost deterministic.

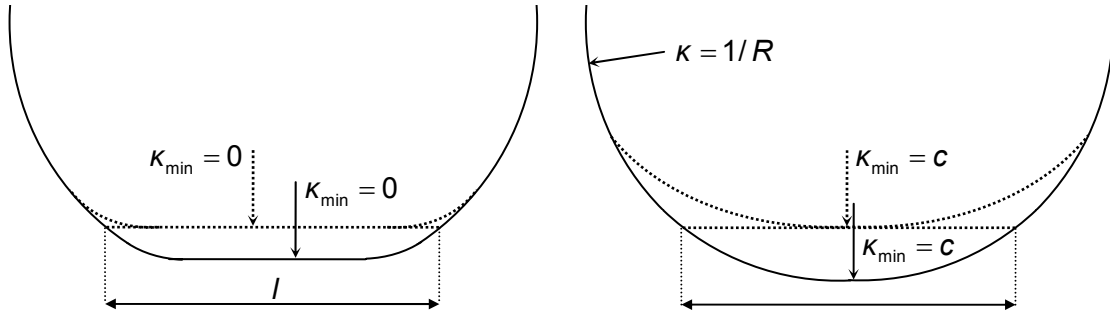


Fig. 5.13 Equally worn wheel flats (not to scale); (left) stage II, with zero minimum curvature and (right) stage III, with small positive minimum curvature

According to Fig. 5.12, the parameter τ grows with increasing wear. However, once a flat has developed into a severely worn stage III flat, the approximate relationships (5.19, 5.23) will no longer be valid. With increasing wear, the geometrical deviation yields, for a given speed, an excitation frequency spectrum of the wheel-rail system with an increasing contribution in the low-frequency regime (as will be further established in section 5.8). This excitation frequency spectrum will interact with the non-uniform frequency response of the track in the low-frequency regime, thus introducing important non-linearity in the time domain. The subject will be dealt with more extensively in section 5.9. A sharp transition between the domain of validity and non-validity does not exist, but it can be stated that the exactness of formula (5.23) increases with decreasing minimum circumferential curvature of the wheel thread.

5.6 The influence of the horizontal velocity of the wheel mass and the static axle load

In position A in Fig. 5.4 the mass centre M of the wheel, rolling with horizontal velocity V on a rigid foundation, starts describing a downward circular curve with radius R . However, if the downward curvature of $y(x)$ (which is $1/R$), converted to the time domain, exceeds a certain value, M will no longer follow this curve, but a loss of contact will occur and M will describe a parabolic curve corresponding to the fall of the wheel mass with horizontal velocity V in the gravity field and under the influence of the vertical static axle load (which, by means of the compressed suspension, acts as a pre-stress). In this section, a criterion is derived for this phenomenon to occur.

The situation is shown graphically in Fig. 5.14. In this figure, apart from the original bi-circular trajectory from Fig. 5.4, two more parabolic trajectories are shown, one with a curvature which is larger than the original trajectory (solid line), and one with a smaller curvature (dotted line). If the hatched area in Fig. 5.14 is larger than or equal to zero (or if the parabolic trajectory lies below the circular trajectory), V has no effect, and the mass centre will follow the bi-circular trajectory as has been assumed and described in the previous sections. Then, there is permanent contact between the wheel and the foundation. If this is not the case, the mass centre will follow the dotted parabolic line segment in Fig. 5.14, until it crosses the second circular part of the original trajectory, where the wheel mass is forced to follow this line again. For the parabolic part of the trajectory (MM') there will be no contact between the wheel and the foundation, whereas there will be contact again for the circular part of the trajectory (M'M'').

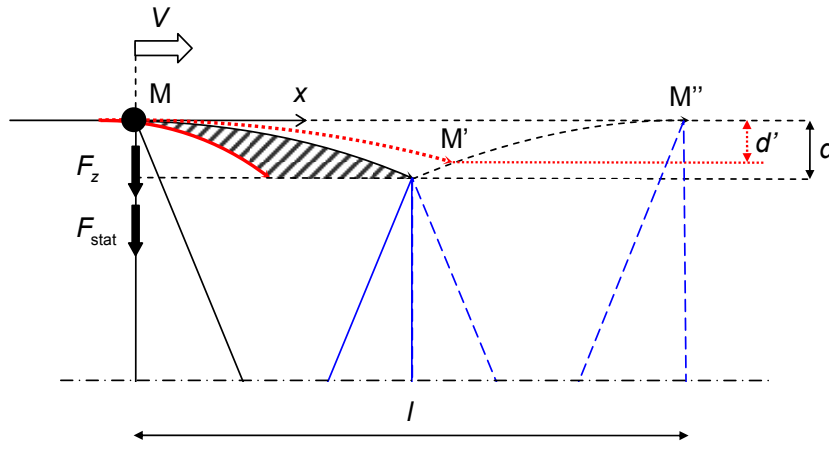


Fig. 5.14 The influence of the horizontal speed and the vertical static preload of the wheel mass in the gravity field on the trajectory of the mass centre

In the following, only the time interval $[t_A, (t_A + t_B)/2]$ is considered. The y -coordinate of the parabolic trajectory of M corresponding to a fall of the wheel mass in the gravity field and under the influence of the static preload of the wheel is described in this interval by:

$$y(t) = -\frac{1}{2} \left(g + \frac{F_{\text{stat}}}{m} \right) t^2 \quad (5.24)$$

With the help of Eq. (5.1) the criterion for excluding the influence of the horizontal speed V can be derived as:

$$\Delta y = \left(\sqrt{R^2 - V^2 t^2} - R \right) - \left(-\frac{1}{2} \left(g + \frac{F_{\text{stat}}}{m} \right) t^2 \right) \geq 0 \quad (5.25)$$

This corresponds to the situation in which the hatched area in Fig. 5.14 is larger than or equal to zero. Elaboration of Eq. (5.25) yields an expression for V which depends on t , because both terms in (5.25) have a different polynomial order. Therefore, it is convenient to use the parabolic approximation of Eq. (5.1), $y(x) = -x^2/(2R)$, which is quite accurate. Then criterion (5.25) is given in terms of the difference between two quadratic functions and can be written as:

$$\Delta y = -\frac{V^2 t^2}{2R} - \left(-\frac{1}{2} \left(g + \frac{F_{\text{stat}}}{m} \right) t^2 \right) \geq 0 \quad (5.26)$$

Elaboration yields, for arbitrary t , the following simple expression:

$$V \leq \sqrt{\left(g + \frac{F_{\text{stat}}}{m} \right) R} \quad (5.27)$$

Without static preload, this limit (designated by V_{crit}) is expressed only in terms of the gravitational acceleration and the wheel radius, as is expected. For $g = 9.81 \text{ m/s}^2$ and $R = 0.46 \text{ m}$ it is found that $V \leq 2.12 \text{ m/s}$ (7.64 km/h). This means that in general without static preload the wheel mass will experience a free fall with the corresponding loss of contact. Assuming a wheel mass (half unsprung mass) of 900 kg and a static wheel-load of $112 \cdot 10^3 \text{ N}$ results in $V \leq 7.86 \text{ m/s}$ (28.29 km/h). Common ratios of the static wheel-load to the wheel mass yield a critical velocity ranging between 6 and 10 m/s (22-36 km/h). Therefore, in general a loss of contact will occur in the case of wheels running with fresh flats.

An expression similar to (5.27) has been presented by V  r et al. [5.6] for the case of a longitudinal disturbance of the railhead.

In the case of a non-rigid track, Eq. (5.27) is still expected to give a correct indication of the critical velocity for initial contact loss. As soon as the wheel-rail contact force drops to zero, the non-zero inertia of the rail mass

prevents an immediate restoring of this force, and the wheel gravity centre will follow the parabolic trajectory. When the rail mass subsequently starts moving upward, the wheel mass will follow the parabolic path almost undisturbed due to its much larger inertia. Also this effect can be observed in the results (at 80 km/h) presented by Wu and Thompson [5.11], which have been discussed in the previous section.

The trajectory in Fig. 5.14 shows that in the most common case where the criterion (5.27) is not satisfied, the interaction between the wheel having a stage I flat and the foundation basically is not affected. The functions shown in Figs. 5.5 and 5.9 will become asymmetric as a function of time; before the impact peak in the force graph, the contact force drop to zero, whereas after the peak only a reduction of the static contact force will occur due to the negative dynamic component. Further, the magnitude of the jump in vertical momentum of the wheel mass (according to Eq. (5.11)) decreases to $\Delta p|_{l=l/(2V)} \approx 8mVd'/l$, but the impact itself remains and is not attenuated. The resulting function of the contact force is shown qualitatively in Fig. 5.15.

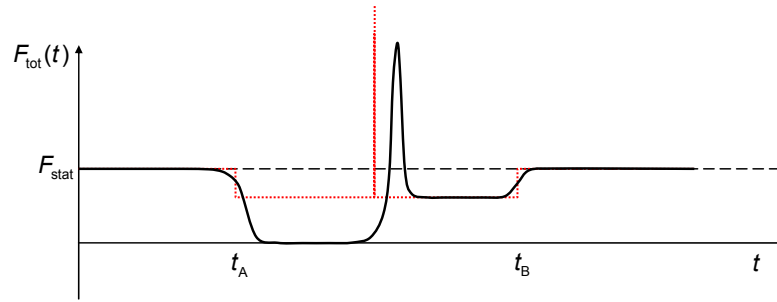


Fig. 5.15 Qualitative asymmetric behaviour of the net contact force for a wheel flat in the transcritical speed regime

Expressions for the magnitude of d' , which is introduced as the effective flat depth, and the corresponding x -coordinate can be easily derived by equating Eq. (5.24) for the parabolic trajectory MM' (substituting $t = x/V$) and the equation of the second circular part of the trajectory (M'M''). For this second part, the quadratic approximation is used for simplicity, which is given by:

$$y(x) = -\frac{4d}{l^2}x^2 + \frac{8d}{l}x - 4d \quad \text{or} \quad y(x) = -\frac{1}{2R}x^2 + \frac{l}{R}x - \frac{l^2}{2R} \quad (5.28)$$

The coordinate of impact is given by the positive root of the resulting quadratic equation:

$$x = \frac{\left(\sqrt{(g + F_{\text{stat}}/m)R - V^2}\right)Vl}{(g + F_{\text{stat}}/m)R - V^2} \quad (5.29)$$

It is important to realize that this coordinate represents a *virtual* impact position. It is the vertical projection of the position of the discontinuity in the derivative of the trajectory on the foundation. The real or physical impact position for a stage I flat in the transcritical speed regime is at the second flat edge ($x = l$).

Leaving out the minus-sign ($y = -d'$), the final expression for the effective flat depth $d_{\text{eff}} = d'$ is given as:

$$d_{\text{eff}} = \frac{(g + F_{\text{stat}}/m)l^2 \left(\sqrt{(g + F_{\text{stat}}/m)R - V^2}\right)^2}{2 \left((g + F_{\text{stat}}/m)R - V^2\right)^2} \quad (5.30)$$

Substitution of this value into expression (5.19) or (5.23) yields the corresponding expression for the contact force during impact. In Fig. 5.16, an example is shown of the behaviour of d_{eff} as a function of the train speed for common parameter values. Its value decreases with $1/V^2$ and asymptotically reaches zero for infinite speed. In Fig. 5.17, the corresponding x -coordinate of the position of impact is shown. For infinite speed, its value asymptotically reaches the flat length l ; the position of impact moves towards the end of the flat. However, the order of the increase, V^0 , is much smaller than the order of decrease for the effective depth.

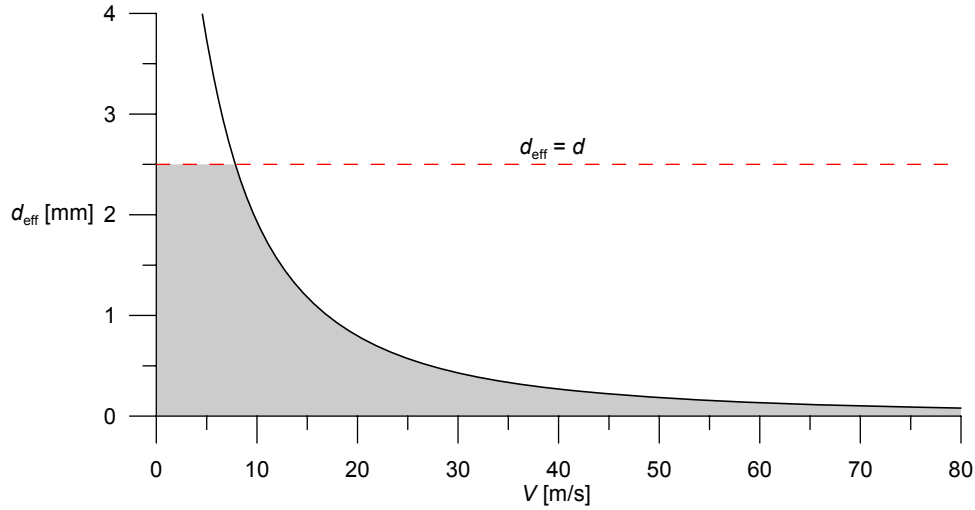


Fig. 5.16 Decrease of the effective flat depth ($O(V^{-2})$) as a function of the speed for the parameter values $F_{\text{stat}} = 112 \text{ kN}$; $m = 900 \text{ kg}$; $g = 9.81 \text{ m/s}^2$; $R = 0.46 \text{ m}$; $l = 0.1 \text{ m}$ ($V_{\text{crit}} = 7.86 \text{ m/s}$)

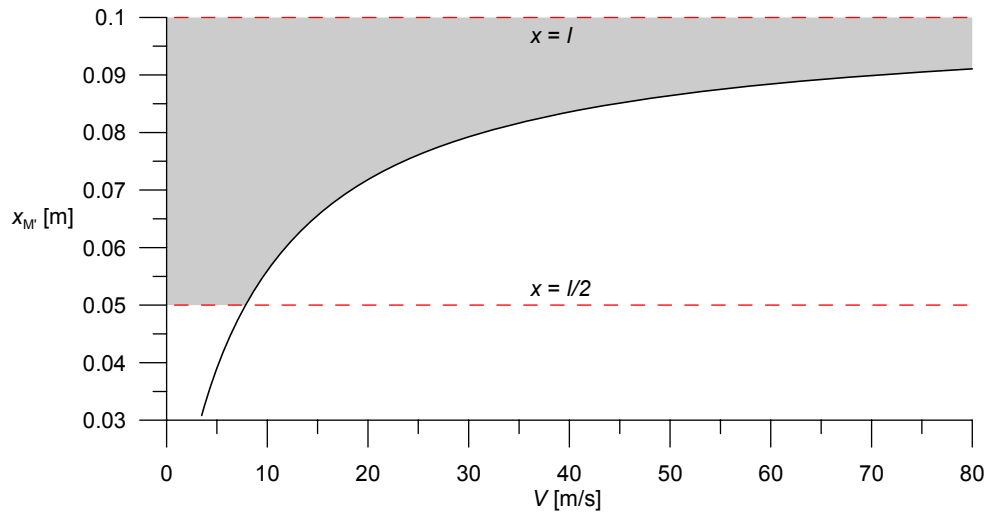


Fig. 5.17 Increase of the x -coordinate of M' (the *virtual* impact position) ($O(V^0)$) as a function of the speed for the parameter values used in Fig. 5.16

In the previous, the stage I flat was considered. We extend the analysis now to worn wheel flats (stage III), which are of more practical relevance. Fig. 5.10 showed the kinematical trajectories of the wheel centre in the presence of wheel flats in all three stages. The figure was valid for the subcritical speed regime. Fig. 5.18 extends this procedure to the transcritical speed regime; the figure is valid for one given value of the train speed.

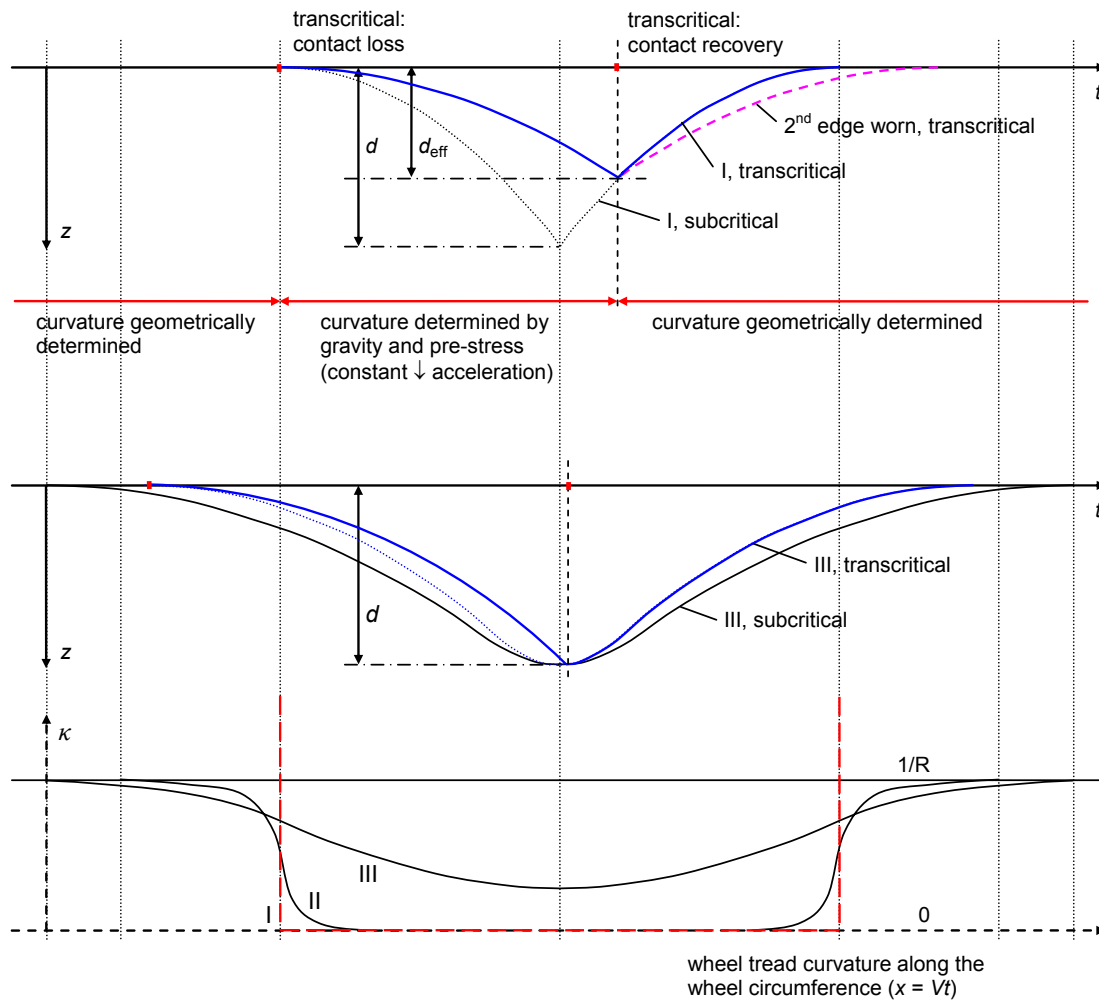


Fig. 5.18 Wheel centre trajectories in the transcritical speed regime; for a stage I flat, with and without starting wear at the 2nd flat edge (top); for a moderately worn and a severely worn stage III flat (middle). The wheel tread curvature along the wheel circumference is shown at the bottom.

At the top, the graph shows the change in the trajectory for a stage I flat from the subcritical to the transcritical speed regime. As has been discussed for Fig. 5.14, the flat depth ‘felt’ by the wheel-rail system is reduced to an effective flat depth. It is remarked that, unlike in Fig. 5.10, the trajectory is no longer strictly kinematical, because its first part is dictated by the wheel inertia. The top graph further shows the effect of wear of the second flat edge on the trajectory. In the transcritical regime the wear starts from this second flat edge, because it is the physical impact position. Further, additional slip in the wheel-contact is expected during contact recovery, wearing out the second flat edge, but also causing periodical rail damage. This effect was also observed in the measurements by Newton and Clark [5.7], who painted the rail during

their experiments. The basic shape of the trajectory in all these cases however remains unchanged, indicating that the dynamic wheel-rail interaction will remain qualitatively the same. This is not true for quantitative results.

The graph also shows the time interval during which the curvature of the trajectory is determined by the gravity force and the static preload acting on the wheel mass (parabolic trajectory segment, with constant second derivative or downward acceleration) and the intervals during which the curvature is geometrically determined.

The graph at the middle in Fig. 5.18 shows the trajectory for a moderately worn stage III flat, in the transcritical speed regime. The train speed is the same as in the graph at the top. The trajectory is also shown for a more severely worn stage III flat. For this flat, having a minimum tread curvature close to $1/R$, the speed is subcritical and no contact loss occurs. The figure clearly shows that the reduction from the flat depth to the effective flat depth is in general much more moderate for stage III worn flats than for fresh flats. The figure also demonstrates the principle of the speed-dependency of the effective flat depth, and the fact that the critical velocity increases with the degree of wear of a flat. This is consistent with the fact that for a zero curvature of the trajectory or a curvature $1/R$ of the wheel tread (no flat), the critical speed is infinity.

Finally, the figure shows the occurrence of impact for any flat at transcritical speeds, due to the fact that the trajectory in this speed regime always has a discontinuous time-derivative at the moment of contact recovery.

In the following, the condition for contact loss for worn wheel flats (stage II or III) will be established. The criterion for contact loss to occur in this case can be formulated as follows: the maximum downward curvature of the trajectory, induced by the wheel tread geometry, should exceed the constant curvature of the parabolic path resulting from gravity and static preload (in the time domain, downward accelerations must be compared, which are equivalent to curvatures in the space domain). Contact loss occurs at any time when the downward curvature of the trajectory (which is not necessarily the maximum value) exceeds the constant value from gravity and preloading. From the graph in the centre of Fig. 5.18 it can be observed that the subcritical trajectory for stage III flats starts with a downward or negative curvature, then has a central part with an upward or positive curvature, and finally again has a downward or negative curvature. It is important to note that in the formulation of the criterion for contact loss, the maximum

downward curvature should be taken, which is not equal to the maximum curvature as such, which is upward and occurs in the centre. Using Eq. (5.24) for the parabolic path and the conversion $x = Vt$, the condition can be elaborated to the following simple result:

$$V > \sqrt{\left(g + \frac{F_{\text{stat}}}{m_w}\right) \frac{1}{\kappa_{\text{max } \downarrow, \text{trajectory}}}} \quad (5.31)$$

Gravity and preload are not the only forces acting on the wheel mass; also the rail exerts an upward force on the wheel mass due to the quasi-static deflection field. However, similarly to the analysis of stage I flats, the above condition is based on the assumption that the rail has a given inertia, and that this inertia prevents the rail, which has a static deflection due to the static load, from immediately restoring the contact after occurrence of contact loss.

For a stage I – flat, $\kappa_{\text{max } \downarrow, \text{trajectory}} = 1/R$. Substituting this value into Eq. (5.31) yields expression (5.27) for the critical speed for stage I – flats.

According to Eq. (5.31), the following relationship is valid for the critical velocity:

$$V_{\text{crit}} \sim 1/\sqrt{\kappa_{\text{max } \downarrow, \text{trajectory}}} \quad (5.32)$$

The maximum downward curvature of the trajectory is directly related to the circumferential curvature of the wheel tread, experienced in the contact, during the considered time interval of downward acceleration. The downward curvature of the trajectory decreases with increasing curvature of the wheel tread (it approaches the value $1/R$). Therefore, it can be written:

$$V_{\text{crit}} \sim \sqrt{\kappa_{\text{experienced in the tread}}} \quad (5.33)$$

The found relationship is obvious when it is kept in mind that the curvature (or acceleration) that is experienced by the wheel mass is the one of the trajectory and not the one of the wheel tread. Fig. 5.19 shows the relationship (5.32) graphically.

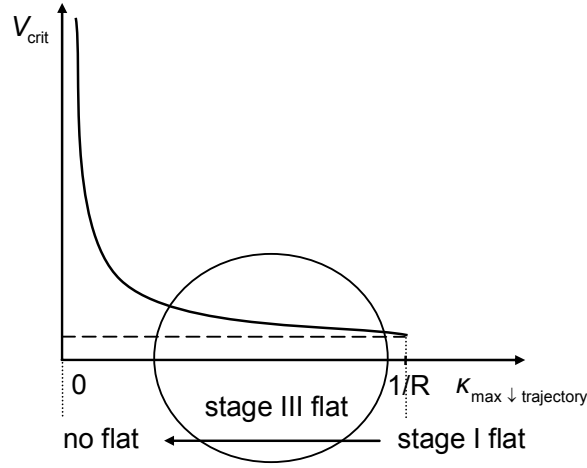


Fig. 5.19 Dependence of the critical velocity on the level of wear of a wheel flat according to Eq. (5.32)

5.7 Parametric analysis of the behaviour of the wheel-rail impact force

The impact force has been described by Eqs. (5.19) and (5.23), in terms of the time-scale for impact τ and the minimum curvature of the wheel tread $\kappa_{\min, \text{tread}}$ respectively. Herein d is defined as d_{eff} according to Eq. (5.30) in the transcritical speed regime, with V_{crit} according to expression (5.27). For a detailed examination of the behaviour of this force as a function of its constitutive variables, all expressions are summarised in the following. It is remarked that up to here, a rail on a rigid foundation has been considered. Eliminating the flat length l by substituting Eq. (5.8) into Eq. (5.19), the impact force in terms of τ is given by:

$$F_{\max} = \eta \frac{mV\sqrt{d}}{\tau\sqrt{8R}} \quad (5.34)$$

Since τ implicitly depends on the speed V (see Eq. (5.22)), Eq. (5.23) is more useful, as it describes the impact force only in terms of the independent geometrical parameters of the flat. Using again Eq. (5.8), it follows:

$$F_{\max} = \mu \frac{mV^2\sqrt{d}\left(1/\kappa_{\min, \text{tread}} - R\right)}{\sqrt{8R^5}}; \quad 0 \leq \kappa_{\min, \text{tread}} \leq 1/R \quad (5.35)$$

in which, for stage I:

$$d = d_{\text{eff}} = \frac{4\beta dR(\sqrt{\beta R} - V)^2}{(\beta R - V^2)^2}; \quad \beta = g + F_{\text{stat}} / m \quad (5.36)$$

for

$$V \geq V_{\text{crit}}; \quad V_{\text{crit}} = \sqrt{\left(g + \frac{F_{\text{stat}}}{m}\right)R} \quad (\text{stage I}) \quad (5.37)$$

$$V \geq V_{\text{crit}}; \quad V_{\text{crit}} = \sqrt{\left(g + \frac{F_{\text{stat}}}{m_w}\right) \frac{1}{\kappa_{\text{max} \downarrow, \text{trajectory}}}} \quad (\text{stages II, III}) \quad (5.38)$$

It is remarked that Eq. (5.35) has been derived for wheel flats which are already worn out to some extent. Expression (5.36) has been derived strictly for stage I flats. However, in stages II and III the effective flat depth is dependent upon the exact shape of the trajectory and therefore on the exact geometry of the flat, and a general expression cannot be given. As has been observed, although a reduction from the flat depth to the effective depth occurs, it is commonly much less for flats in these stages due to the convexity of the trajectory (Fig. 5.18). The more severely the flat is worn, the less the reduction will be. Therefore, in the transcritical speed domain expression (5.35) can only be used to predict a general trend. In Eq. (5.35), the minimum curvature should be taken as the *effective* minimum curvature (experienced at the impact position and the moment of contact recovery) in transcritical speed domain and for worn flats. This effective minimum curvature is larger (closer to $1/R$) than the minimum curvature itself. Summarizing, in the transcritical speed domain the effective depth decreases with respect to the flat depth, and the effective minimum curvature increases with respect to the minimum tread curvature. Considering the fact that the force according to Eq. (5.35) is proportional to the square root of the depth and inversely proportional to the minimum tread curvature, both changes decrease the impact force. Therefore, the increase of the effective curvature may compensate for the fact that the depth reduction in the transcritical regime is not as strong as calculated on the basis of Eq. (5.36) for worn flats.

When comparing Eqs. (5.37) and (5.38), it is remarked that wear along the flat edges increases the critical velocity, as this wear ‘stretches’ the trajectory; this became obvious already from Fig. 5.18.

Eq. (5.34) may be used to determine the constant in Eq. (5.35), when measurements are available at a given velocity; often the time-scale τ can be estimated (and is in the order 1-3 ms), whereas this is difficult for μ .

The behaviour of the impact force (5.35) as a function of the train speed is shown in Fig. 5.20, for a given wheel flat geometry. The same parameters have been used as in Fig. 5.16. The minimum tread curvature $\kappa_{\min, \text{tread}}$ is 0.2 m^{-1} , which is less than 10 percent of the undisturbed curvature $1/R$ (2.17 m^{-1}), indicating that the considered flat is only very slightly worn. In the subcritical speed regime, the magnitude of the impact force increases quadratically with the speed. In the transcritical speed domain the force further increases with the speed, but with a power which is much smaller than 2. In the transcritical speed regime the effective depth according to Eq. (5.36) decreases with the order V^{-2} ; because the force according to Eq. (5.35) increases with the square of the speed and only with the square root of the depth, the net variation (increase) of the force is of the order V .

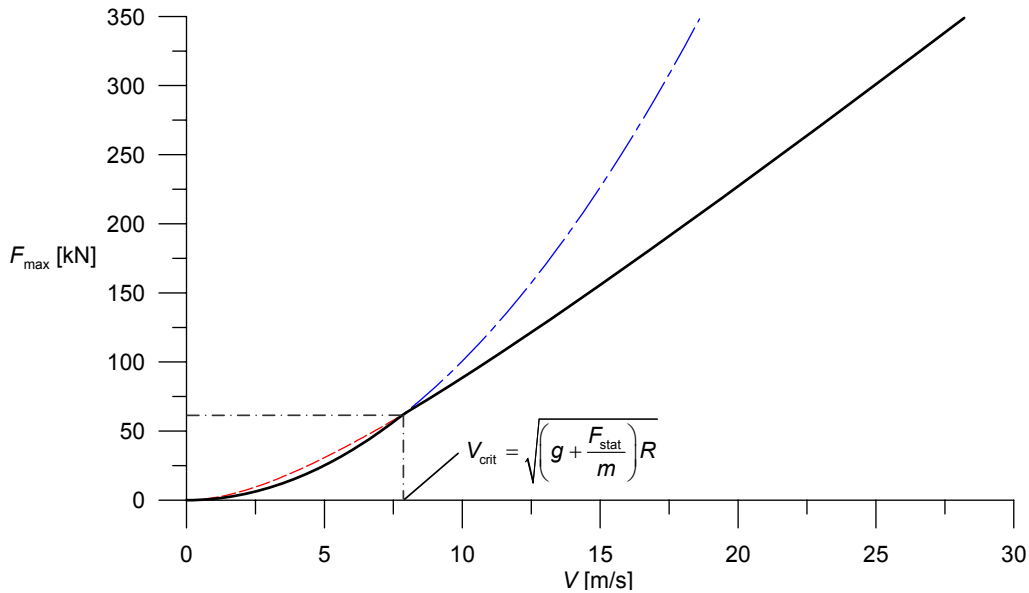


Fig. 5.20 Behaviour of the impact force according to Eq. (5.35) versus speed (solid line) for the parameter values used in Fig. 5.16; $d = 2.5 \text{ mm}$, $\kappa_{\min, \text{tread}} = 0.2 \text{ m}^{-1}$, $\mu = 2$

In Fig. 5.21, at the left the impact force according to Eq. (5.35) is shown as a function of the minimum curvature of the wheel tread for a

constant flat depth. At the right it is shown as a function of the flat depth for a constant minimum curvature (which is taken again as 0.2 m^{-1}). Both graphs are for a subcritical velocity. The impact magnitude decreases with the inverse of the minimum tread curvature for given flat depth and increases with the square root of the flat depth for a given curvature. Therefore, the variation with the tread curvature is stronger than with the flat depth.

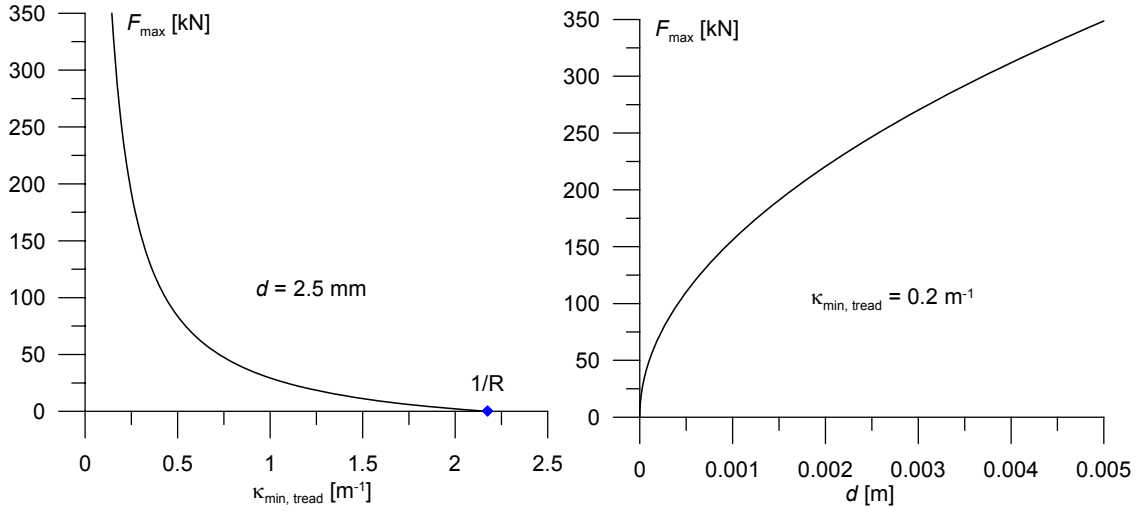


Fig. 5.21 Impact force versus minimum curvature of the wheel tread (left) and flat depth (right), for the parameter values $m = 900 \text{ kg}$; $R = 0.46 \text{ m}$; $V = 7 \text{ m/s}$; $\mu = 10$

5.8 The stages of wheel flat development: mathematical descriptions of the trajectories and frequency domain analysis

In section 5.2 the three stages of wheel flat growth were defined. The corresponding wheel centre trajectories were addressed already in a qualitative manner in sections 5.4, 5.5 and 5.6. In this chapter, the kinematical trajectories for the three stages will be described mathematically in the time and the frequency domain. The kinematical trajectory of the wheel mass can serve as an input for the excitation of dynamic wheel-rail interaction models. Therefore, frequency spectra of excitation functions form an important tool in the analysis of dynamic wheel-rail interaction due to wheel flats. The mathematical expressions will be used in the next section, in which a non-rigid railway track will be introduced and in which computations will be made for wheels with flats in the different categories, rolling on such a track. A severe drawback of the frequency-domain approach is that it does not allow for

contact loss and is therefore necessarily limited to the subcritical speed regime. On the other hand this approach will prove to provide much more insight in the fundamental features of the dynamic wheel-rail interaction than a time-domain modelling.

In the following, mathematical expressions are given for the wheel centre trajectory in stages I, II and III of wheel flat generation and growth, along with their maximum spatial curvatures. Also the Laplace-transforms of these expressions are given, which will be used in simulations. The Laplace-transform is chosen because a wheel flat excites a transient train-track vibration.

I) The trajectory is given by Eq. (5.1). In a very good polynomial approximation, this expression is represented in the time domain by:

$$\begin{aligned} \tilde{z}(t) = & -\frac{1}{2R}V^2t^2H(-Vt + l/2) \\ & + \left(-\frac{1}{2R}V^2t^2 + \frac{l}{R}Vt - \frac{l^2}{2R} \right) H(Vt - l/2) \cdot H(l - Vt) \end{aligned} \quad (5.39)$$

Its Laplace-image is given by:

$$\tilde{z}_I(s) = \frac{V}{Rs^3} \left(sle^{-sl/(2V)} + Ve^{-sl/V} - V \right) \quad (5.40)$$

The maximum curvature of $z(x)$ is given by:

$$\kappa_{\max} \approx \left. \frac{d^2\tilde{z}}{dx^2} \right|_{\max} \rightarrow \left. \frac{d^2\tilde{z}}{dt^2} \right|_{t=l/(2V)} = \infty \quad (5.41)$$

II) The length of a fresh flat has been defined as l . The extended length of a flat which is worn along the edges (stage II) is defined as l_{II} . The trajectory is given by:

$$\begin{aligned} \mathfrak{z}(t) = & \left(\frac{l}{l_{\text{II}}} \right)^2 \left(-\frac{1}{2R} V^2 t^2 H(-Vt + l_{\text{II}}/2) + \right. \\ & \left. \left(-\frac{1}{2R} V^2 t^2 + \frac{l_{\text{II}}}{R} Vt - \frac{l_{\text{II}}^2}{2R} \right) H(Vt - l_{\text{II}}/2) \cdot H(l_{\text{II}} - Vt) \right) \end{aligned} \quad (5.42)$$

The Laplace-image is given by:

$$\tilde{\mathfrak{z}}_{\text{II}}(s) = \left(\frac{l}{l_{\text{II}}} \right)^2 \frac{V}{R s^3} \left(s l_{\text{II}} e^{-s l_{\text{II}}/(2V)} + V e^{-s l_{\text{II}}/V} - V \right) \quad (5.43)$$

and the maximum curvature of $\mathfrak{z}(x)$ follows from:

$$\kappa_{\text{max}} \approx \left. \frac{d^2 \mathfrak{z}}{dx^2} \right|_{\text{max}} \rightarrow \left. \frac{d^2 \mathfrak{z}}{dt^2} \right|_{t=l_{\text{II}}/(2V)} = \infty \quad (5.44)$$

III) Now, l_{III} is defined as the further extended length of a flat which is worn along its whole length. The trajectory can be described by different mathematical expressions. A superposition of two harmonic shapes is chosen for convenience:

$$\mathfrak{z}(t) = \frac{1}{2} \left(\frac{l^2}{8R} \cos \left(\frac{2\pi V t}{l_{\text{III}}} \right) + \frac{1}{2} c \left(1 - \cos \left(\frac{4\pi V t}{l_{\text{III}}} \right) \right) - \frac{l^2}{8R} \right) H(-Vt + l_{\text{III}}) \quad (5.45)$$

Its Laplace-image is given by:

$$\begin{aligned} \tilde{\mathfrak{z}}_{\text{III}}(s) = & \frac{\pi^2 V^2 \left(-s^2 l^2 l_{\text{III}}^2 - 16 l^2 \pi^2 V^2 + 16 s^2 c R l_{\text{III}}^2 + 64 c R \pi^2 V^2 \right) \left(1 - e^{-l_{\text{III}} s / V} \right)}{4 R s \left(s^4 l_{\text{III}}^4 + 20 s^2 l_{\text{III}}^2 \pi^2 V^2 + 64 \pi^4 V^4 \right)} \end{aligned} \quad (5.46)$$

and the maximum curvature of $\mathfrak{z}(x)$ follows from:

$$\kappa_{\text{max}} \approx \left. \frac{d^2 \mathfrak{z}}{dx^2} \right|_{\text{max: } x=l_{\text{III}}/2} = \frac{\pi^2}{4 R l_{\text{III}}^2} \left(l^2 + 16 c R \right) \quad (5.47)$$

This maximum curvature decreases with the square of the flat length in stage III, given an initial flat depth and length. For a given flat length, the maximum curvature is proportional to the parameter c (the amplitude of the second harmonic), which can therefore be used as a kind of ‘control parameter’. For c equal to zero a purely harmonic trajectory results.

The expression (5.45) can only represent a limited maximum curvature; for higher values positive values of $\varkappa(x)$ occur due to the fact that it is a superposition. A more complex alternative, but able to account for higher curvatures, is given by the following expression for the trajectory:

$$\begin{aligned} \varkappa(t) = & \\ -\frac{l^2}{16R} & \left(1 - \cos \left(\frac{2\pi}{l_{\text{III}}} \left(Vt - a \sin \left(\frac{\pi l^2}{4R l_{\text{III}}} \sin \left(\frac{2\pi Vt}{l_{\text{III}}} \right) \right) \right) \right) \right) H(-Vt + l_{\text{III}}) \end{aligned} \quad (5.48)$$

The maximum curvature of $\varkappa(x)$ is given as:

$$\varkappa_{\max} \approx \left. \frac{d^2 \varkappa}{dx^2} \right|_{\max: x=l_{\text{III}}/2} = \frac{\pi^2 l^2}{16R^3 l_{\text{III}}^6} (2R l_{\text{III}}^2 + a \pi^2 l^2)^2 \quad (5.49)$$

A disadvantage of the present formulation is that a is not proportional to the maximum curvature. Eq. (5.48) will not be used in simulations in this study because its analytical Laplace-transform is not straightforward.

Examples of the trajectories $\varkappa(t)$ according to expressions (5.39), (5.42), (5.45) and (5.48) are given in Fig. 5.22, for an initial flat length $l = 60$ mm in stage I, $l_{\text{II}} = 80$ mm in stage II and $l_{\text{III}} = 100$ mm with $c = 0.5$ mm and $c = 0$ (yielding a purely harmonic shape) in stage III. The train speed is 20 m/s. In principle, the trajectory will move outward within the hatched area during wheel flat growth.

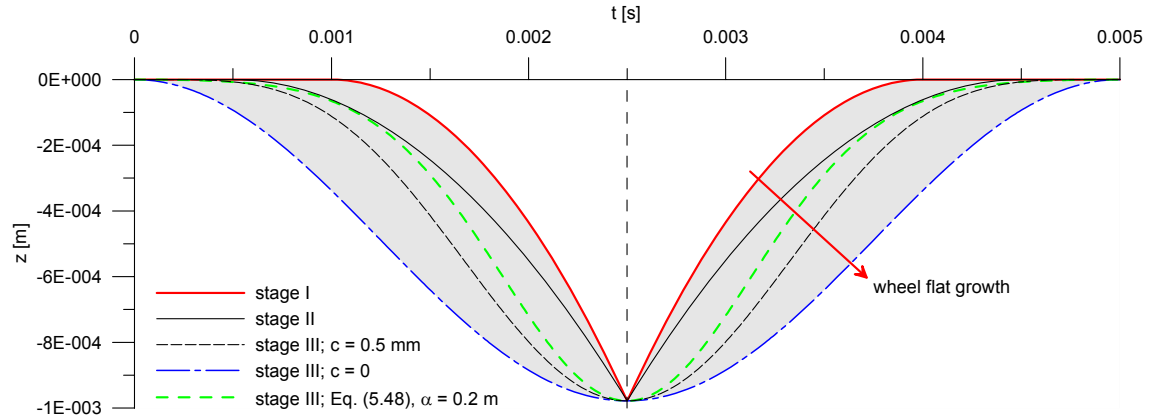


Fig. 5.22 Wheel gravity centre trajectories according to expressions (5.39, 5.42, 5.45, 5.48), in stages I, II and III of wheel flat growth ($v = 20$ m/s, $l = 0.06$ m, $l_{II} = 0.08$ m and $l_{III} = 0.1$ m)

In Fig. 5.23, the frequency spectra of the Laplace-transformed trajectories in stages I, II and III are shown ($s = \sigma + i\omega$; $\sigma = 0$; parameter values according to Fig. 5.22). At the left, the real part is shown, and at the right the modulus (which gives a clearer impression of the decrease of the spectrum as a function of the frequency). Fig. 5.23 clearly shows the redistribution of the spectral amplitude over the frequencies during the growth of the initial flat. The contribution in the high-frequency regime vanishes, whereas the contribution in the low-frequency regime increases. From this observation follows immediately that the relevance of the track stiffness (which reduces the impact) in the dynamic wheel-rail interaction will increase during the growth of a flat.

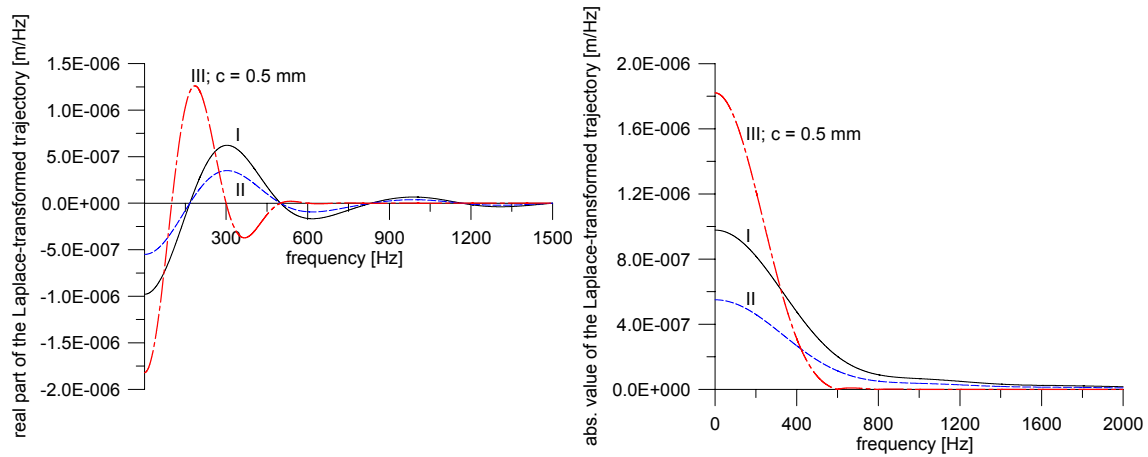


Fig. 5.23 Spectra of wheel centre trajectories according to expressions (5.3, 5.6, 5.9), in different stages of wheel flat development (parameter values according to Fig. 5.22)

Fig. 5.24 shows the spectra of the trajectory for three stage III – flats, for different values of the parameter ϵ , the control parameter of the maximum curvature. The graph shows, similarly to Fig. 5.23, a redistribution of the spectral values as a function of the maximum curvatures. The high-frequency contribution decreases with decreasing curvature.

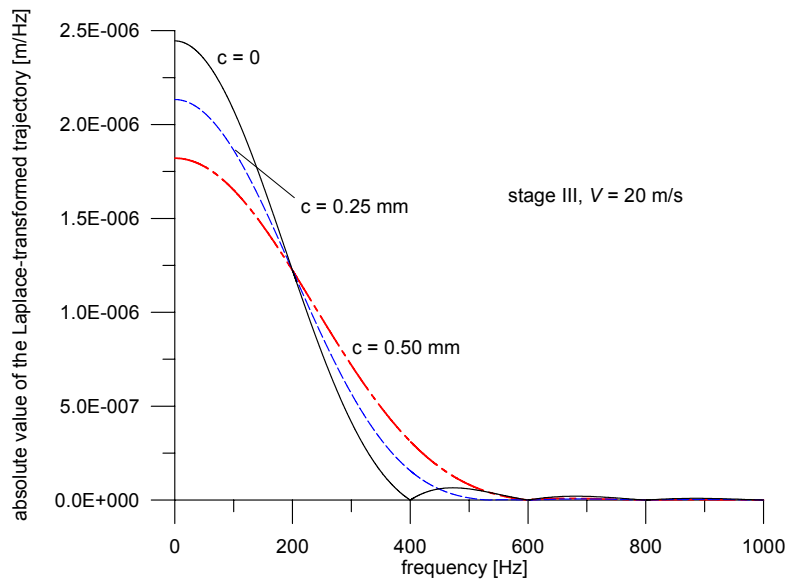


Fig. 5.24 Spectra of wheel centre trajectories according to expression (9), for different values of the maximum curvature (parameter values according to Fig. 5.22)

Finally, Fig. 5.25 shows the spectra for a stage III – flat for different train speeds. The higher the train speed, the larger the high-frequency contribution

in the spectrum. This indicates a higher relevance of the track stiffness (attenuating the impact) for lower train velocities.

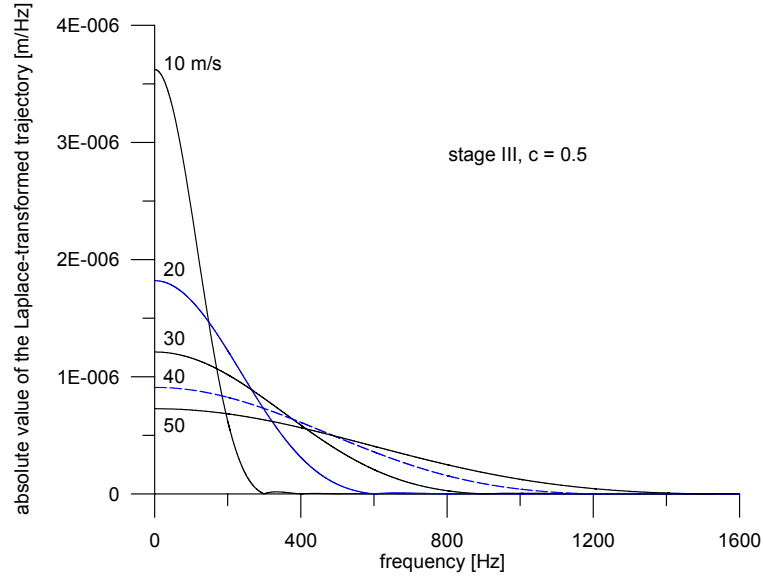


Fig. 5.25 Spectra of wheel centre trajectories according to expression (9), for a constant maximum curvature and varying train speed (parameter values according to Fig. 4)

5.9 Dynamic wheel-rail interaction for different wheel flats; modelling and simulation results for a non-rigid track

In chapter 3, a general model for dynamic wheel-rail interaction due to short interfacial irregularities was presented, and general expressions, in integral form, for the main involved dynamic variables were derived. The model did not include the individual sleepers. In the analysis of wheel flats this is an acceptable simplification. The wheel-rail impact is at first and mainly reacted by the rail mass, and only with a certain delay by the sleeper mass, especially on a relatively soft track. E.g. the simulations by Wu and Thompson [5.11] (Figs. 10 and 11 in this reference), as well as those by Baeza et al. [5.13] (Figs. 8 and 11 in this reference) and [5.14] (Fig. 13 in this reference) clearly show this effect: the dynamic contact force shows a double peak, but the first, due to the rail reaction, dominates by far. Because this study is mainly interested in the maximum value of the dynamic force, it seems not necessary to model the sleepers separately.

Fig. 5.26 shows the absolute value of the spectrum $\tilde{F}(\omega)$ of the contact force in the wheel-rail system to a unit pulse excitation $\tilde{\chi}(s)=1$ (thus,

according to equation (3.23): $\tilde{F}(s) = \left(k_H^{-1} + (m_w s^2 + k_1)^{-1} + (8EI k_0^3)^{-1} \right)^{-1}$; $s = \sigma + i\omega$ with $\sigma = 0$).

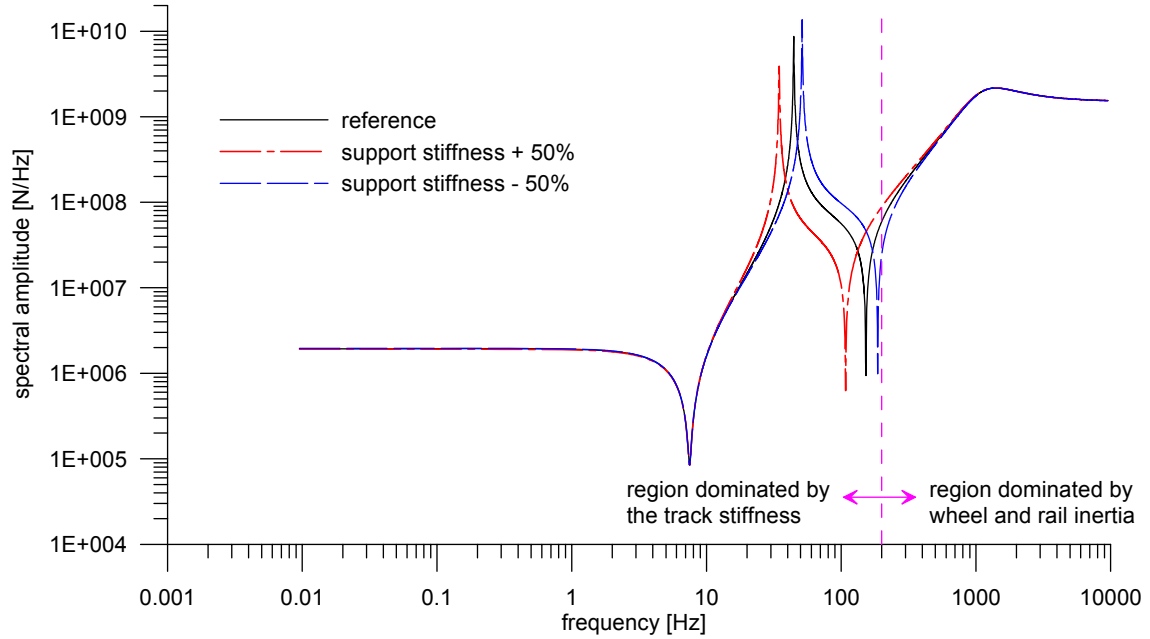


Fig. 5.26 Absolute value of the spectrum of the contact force in the wheel-rail system due to a unit pulse excitation

A narrow and sharp peak appears around 45 Hz, followed by a gradually increasing and declining frequency content in the region between 200 and 3000 Hz, with a maximum around 1000 Hz. The resonance (and anti-resonance) at the lower frequency is, for a given mass, completely governed by the rail support stiffness, as is shown for two variations of this stiffness. The high-frequency content (> 200 Hz) is completely governed by the wheel and rail inertia and their contact stiffness.

When determining the system response to wheel flat excitation, it is important to keep the model and its characteristics in mind. As has been discussed, a prerequisite for the applicability of the model was that the inertia should govern the response (impact or attenuated impact). For common wheel flats, with limited lengths, conventional train speeds and not too high wear levels this is the case. However, for longer flats which are severely worn out (which are basically already out-of-roundnesses), in combination with low speeds the response is no longer dominated by the inertial properties, but by the stiffness properties and the response will be extremely sensitive to the presence of particular resonances, and therefore strongly track-dependent. When for example a long, perfectly harmonic stage III flat excites at a given

train speed precisely a track resonance (e.g. the so-called pinned-pinned rail mode), it is essential for good predictions to account for this resonance in the model. From Fig. 5.26 it is relatively easy to estimate the boundary of the length-speed combination for which the model can be expected to give good results: $f = V / l_{\text{III}} > 200$ Hz.

Table 5.1 shows the admissible intervals for c for different stage III – flats, along with their lengths in the three stages. Also the train speed minimum for a good model performance is given. Fig. 5.27 shows the linear relationship, according to Eq. (5.47), between the maximum trajectorial curvature and the parameter c for these flats. As a reference to estimate the order of magnitude of the curvature, the circumferential curvature $1/R$ of the undamaged wheel is shown also.

Table 5.1 Different wheel flats and their lengths along with the admissible interval for c

Flat length stage I [mm]	Flat length stage II [mm]	Flat length stage III [mm]	Speed min. [m/s]	Interval c [mm]
60	80	100	20	[0 – 0.5]
50	60	70	14	[0 – 0.35]
40	50	60	12	[0 – 0.2]
30	40	50	10	[0 – 0.125]

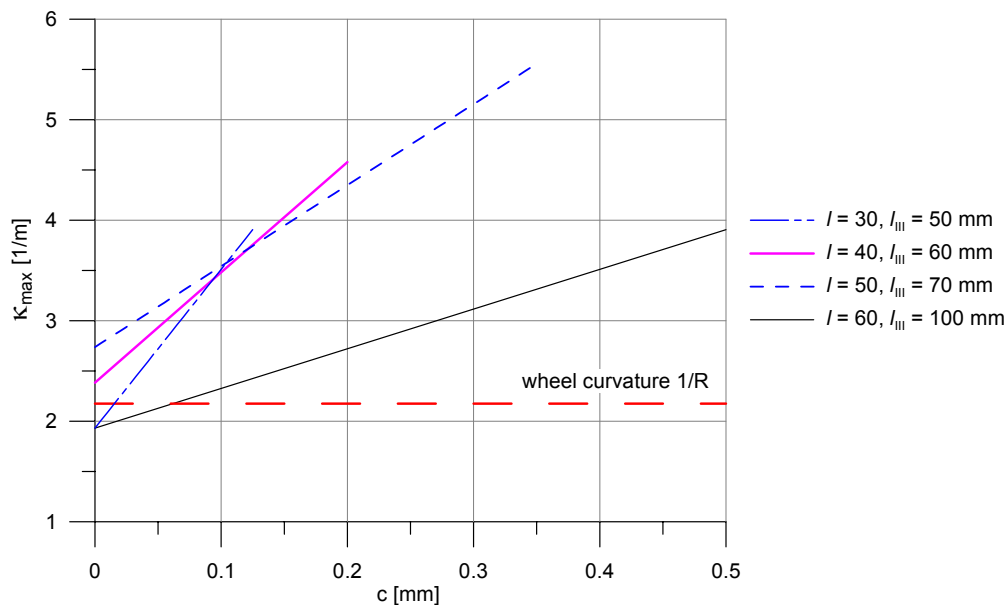


Fig. 5.27 Relationship between the parameter c and the maximum trajectorial curvature within the admissible interval for c (according to Table 2)

Fig. 5.28 shows the time histories of the dynamic wheel-rail contact force for a wheel flat in all three growth stages, calculated according to expression (3.30). It can be observed that the effect of wear along the flat (which is expressed in the minimum tread curvature) on the impact maximum is very significant. The initial dynamic unloading is less dependent on the flat geometry.

The impact magnitude of about 200 kN at 50 km/h corresponds to the value found by Baeza et al. [5.13], using a non-linear Hertz contact model, for a fresh wheel flat with $l = 50$ mm on a track with concrete sleepers.

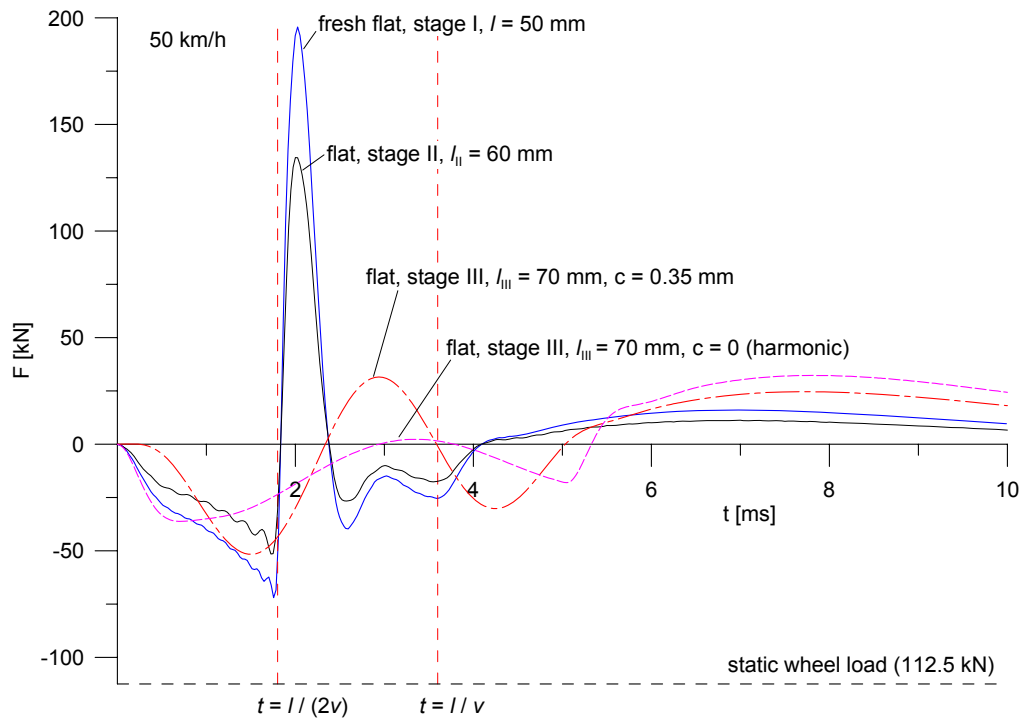


Fig. 5.28 Time histories of the dynamic wheel-rail contact force for a wheel flat in the three growth stages

In Fig. 5.29 the dynamic wheel-rail contact force according to Eq. (3.30) is shown for a stage III – flat ($l = 30$; $l_{III} = 50$ mm) with increasing maximum curvature of the trajectory, within the limits as shown in Fig. 5.27 for this particular case.

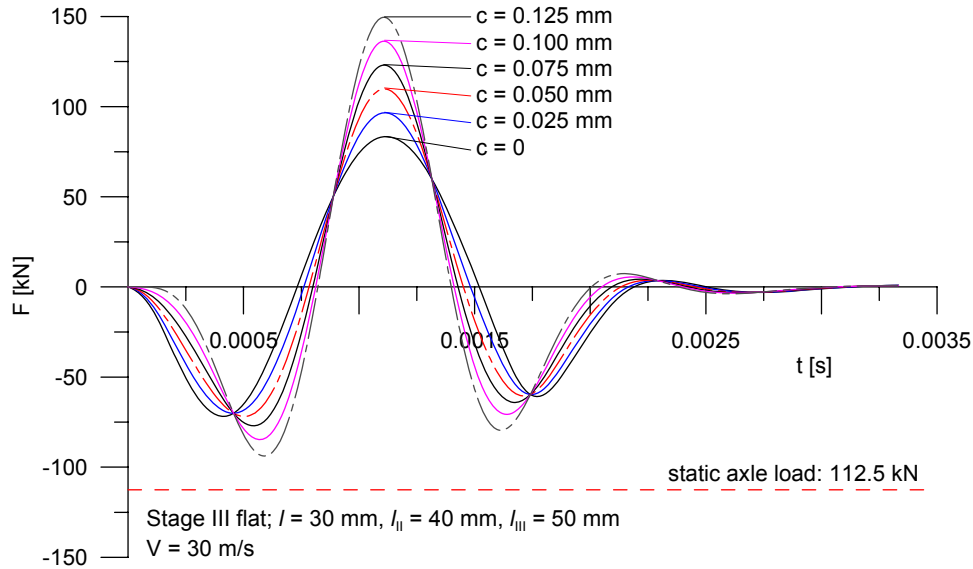


Fig. 5.29 Dynamic wheel-rail contact force for a stage III flat with increasing maximum curvature of the trajectory

The train velocity for the wheel flat under consideration can increase up to 40 m/s (± 140 km/h) before contact loss occurs. The minimum velocity for this particular flat to be in the inertia-dominated area is 10 m/s. The results for simulations at different train speeds have been summarized in Figs. 5.30 and 5.31. In Fig. 5.30, the impact magnitude is shown as a function of c for different values of the train speed. In Fig. 5.31, the impact magnitude is shown as a function of the train speed for different values of c . It can be observed that the impact force has a linear relationship with c for all speeds. Therefore, the relationship between the maximum trajectorial curvature and the impact force may be considered as linear. Further, the impact force in the inertia-dominated area ($V > 10$ m/s) shows an approximately linear behaviour as a function of the speed within the considered range for the trajectorial curvatures. The behaviour of the almost fully elastic contact force in the area governed by the track elasticity ($V < 10$ m/s) shows a local peak, which will be strongly dependent on the track properties.

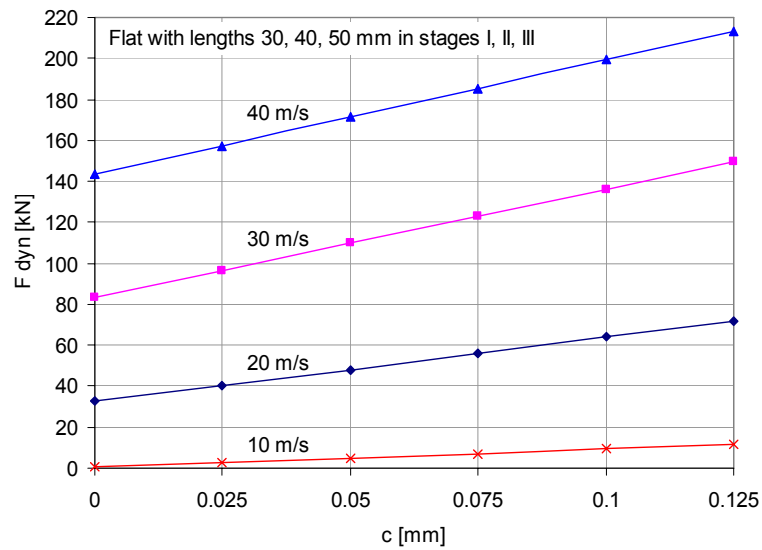


Fig. 5.30 Impact magnitude versus c for different velocities for a stage III wheel flat (no contact loss up to 140 km/h).

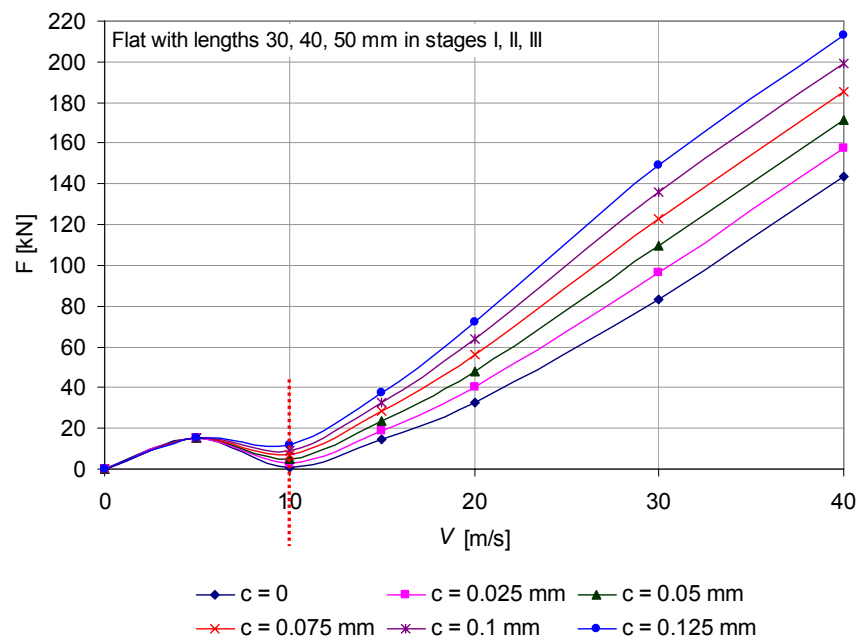


Fig. 5.31 Impact magnitude versus velocity for different values of c (bottom) for a stage III wheel flat (no contact loss up to 140 km/h).

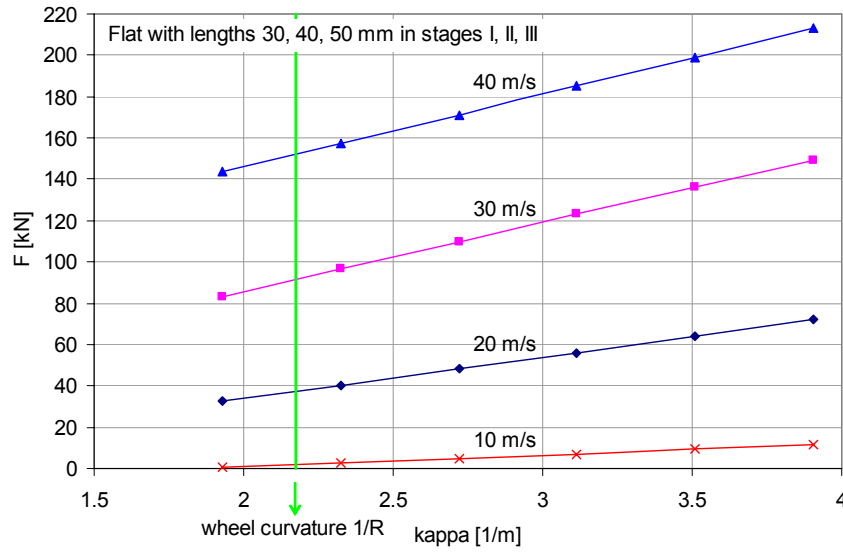


Fig. 5.32 Impact magnitude versus maximum trajectorial curvature for different velocities for a stage III wheel flat (no contact loss up to 140 km/h).

In Fig. 5.32 the impact magnitude is shown directly as a function of the trajectorial curvature. The translation has been made using the linear relationship between ϵ and the curvature from Fig. 5.27. As a reference, the wheel circumferential curvature is given again. The resulting relationship between impact magnitude and maximum trajectorial curvature, for a worn wheel flat without contact loss, is linear.

In section 5.7, Eq. (5.35) has been derived to describe the impact magnitude occurring for worn wheel flats (stage III):

$$F_{\max} = \mu \frac{m_w V^2 \sqrt{d} (1/\kappa_{\min, \text{tread}} - R)}{\sqrt{8R^5}}; \quad 0 \leq \kappa_{\min, \text{tread}} \leq 1/R \quad (5.50)$$

This formula has been derived without including a track model. Combination with Eq. (5.20), which describes the relationship between the maximum trajectorial curvature and the minimum circumferential curvature of the wheel tread, yields the following expression:

$$F_{\max} = \mu \frac{m_w V^2 \kappa_{\max, \text{trajectory}} R^2 \sqrt{d}}{c_1 \sqrt{8R^5}} \quad (5.51)$$

Also this relationship predicts a linear growth of the impact magnitude as a function of the maximum trajectorial curvature. Substituting the parameter

values from Table 3.1 and the flat depth ($d = l^2 / (8R) = 0.24 \text{ mm}$), for Eq. (5.51) is found (μ and c_1 are validation constants):

$$F_{\max} = \xi \frac{\mu}{c_1} V^2 \kappa_{\max, \text{trajectory}}; \quad \xi = 7.34 \text{ kg} \quad (5.52)$$

Eq. (5.52) predicts a proportional relationship between the impact force and the trajectorial curvature. In Fig. 5.33 such a directly proportional relation, a linear fit through the origin for each velocity, is shown. It can be concluded that, although the relationship is linear within the considered domain of the curvature, it is not proportional, especially for higher velocities. The smallest used curvature in Fig. 5.33 occurs for a perfectly harmonic shape of the trajectory (see Fig. 5.27); it is expected that curvatures below this value are not very relevant for practice, as they occur only for very ‘smooth’ flats, which lead to relatively low force levels. The linear relationship between the impact magnitude and the maximum trajectorial curvature also confirms the hyperbolic (first order) relationship between the impact magnitude and the minimum circumferential tread curvature in Eq. (5.50) (see Fig. 5.21).

Further, Eq. (5.52) predicts a quadratic increase of the force with the speed in the subcritical speed regime. This variation appears to be too strong; according to Fig. 5.31 the variation is almost linear in the inertia-dominated area for which the expression has been derived. However, the dependence for a given train-track configuration of the impact magnitude on the trajectorial or wheel tread circumferential curvature and the train speed as governing parameters appears to be correct.

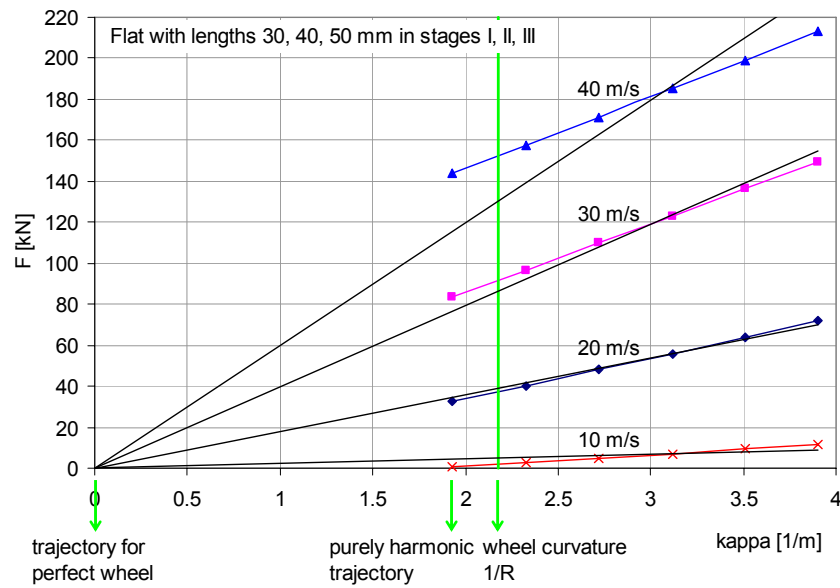


Fig. 5.33 Impact magnitude versus maximum trajectorial curvature for different velocities for a stage III wheel flat and linear fits through the origin.

5.10 Discussion; confrontation with reported results from the literature

The qualitative behaviour of the contact force derived in section 5.6 (Figs. 5.9 and 5.15) shows very good correspondence to measurement results, especially when it is kept in mind that the length of the horizontal non-zero parts of the force function depends on the length of the wheel flat. Examples of such measured time histories of the contact force are shown in Figs. 5.34 and 5.35, from Newton and Clark [5.7] and Johansson and Nielsen [5.12] respectively. It is remarked that Newton and Clark measured baseplate forces, and not wheel-rail contact forces. This will be dealt with more extensively with in section 5.12, but for a qualitative comparison this difference is not relevant. All graphs show an initial decrease of the contact force when a flat edge comes into contact with the rail or foundation, followed by a sudden sharp peak, and a second decrease before the contact force again comes to its static value. The general behaviour is only disturbed by the influence of the contact stiffness, which, apart from attenuating the impact, causes some minor vibrations afterwards. Further, this stiffness is of minor importance in the wheel-rail interaction during the rotation of a wheel with a flat, which is generally dominated by the variation of momentum of the interacting masses. Also the asymmetry in the contact force magnitude induced by the effects of gravity, the horizontal velocity of the wheel mass and the static preload are clearly

visible in the measurements at 50 and 117 km/h, although the effect is partly diluted by the wear of the flats in the measurements (all flats are stage III; the d/l^2 ratio is 0.096 m^{-1} in Fig. 5.34 and 0.11 m^{-1} in Fig. 5.35; according to $d/l^2 \approx 1/(8R)$ this should be 0.27 m^{-1} for a fresh flat). The different numerical simulations presented in references [5.7, 5.9, 5.11] and [5.22] show the asymmetry in the transcritical speed domain very clearly.

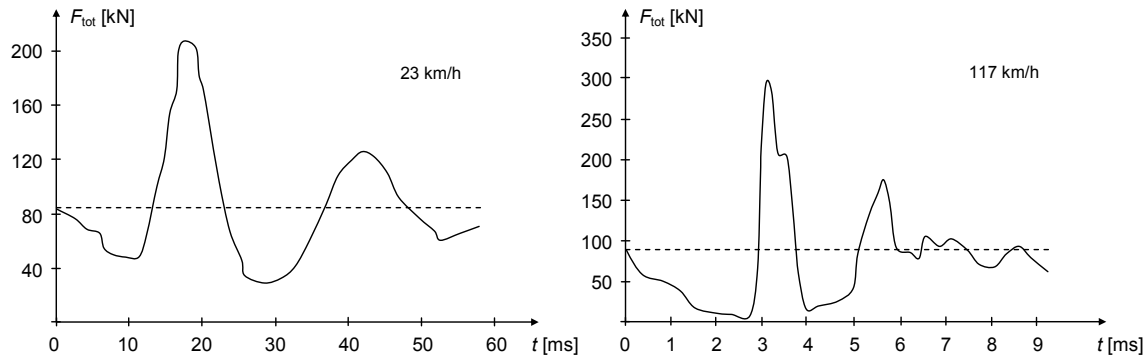


Fig. 5.34 Measured time histories of baseplate forces for a stage III wheel flat of 150 mm length and 2.15 mm depth (ground into the rail) (reproduced from [5.7])

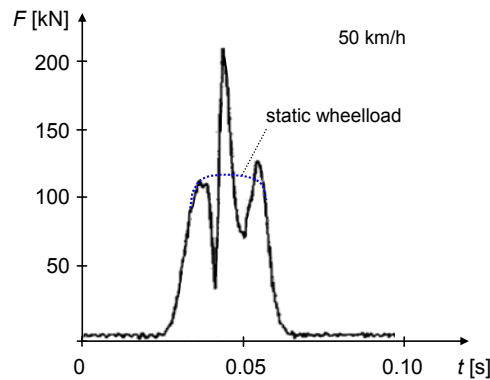


Fig. 5.35 Measured time history of the wheel-rail contact force for a stage III wheel flat with of 100 mm length and 1.1 mm depth (from [5.12])

In this chapter, it has become clear that the following aspects are necessary to be considered in a consistent analysis (either via experiments or via simulations) of dynamic train-track interaction in the presence of wheel flats:

- in which stage are the flats under consideration; what is their geometry and their critical circumferential curvature;
- is the flat length – train speed combination in the area governed by the track elasticity or by the rail and wheel inertia: trends of the impact

magnitude as a function of the train speed behave completely different in both regions;

- is the speed in the subcritical or transcritical regime;
- does a ‘geometrically induced’ and initial contact loss occur due to the fact that the combination flat geometry – speed is in the transcritical speed regime;
- does a ‘dynamically induced’ contact loss occur due to dynamic unloading after wheel-rail impact.

It is important to realize that the latter two types of contact loss have a different origin and can occur independently.

In Table 5.2, the most important results on the investigation of wheel flats reported in the literature have been summarized. The table mentions the author, the type of investigation – either experimentally or via simulation, the flat stage, the considered speed range, the general trend of the impact magnitude as a function of the train speed and the order of magnitude of the impact that has been found. Many of the investigations in Table 5.2 are not sufficiently documented in view of the relevant aspects mentioned at the beginning of this section. Generally, the exact wheel profiles are unknown, so that the degree of wear is unknown as well as the curvature along the flat, and in many cases the critical velocity (per flat) is unknown. Further, no distinction is made between the two types of contact loss that can occur. However, on the basis of Table 5.2 and the present investigation some general expectations can be formulated:

- a local force maximum occurs between 20-50 km/h, but only for longer flats, where the length-speed combination is in the frequency area dominated by the track elasticity (the forces are P_2 – like).
- this local peak disappears for shorter flats, since the whole relationship ‘shifts’ into the inertia-dominated frequency range (all loads become P_1 – like forces)
- it is hard to quantify the impact magnitude; qualitatively models results do correspond, but quantitatively large differences are found.
- when contact loss occurs at the critical speed, the impact magnitude becomes more or less speed-independent for higher speeds.

Table 5.2 Different investigations into train-track dynamics for wheel flats

Author	investigation type	flat stage	speed range	general trend	F, order of magnitude
Jenkins et al. [5.4]	simulation	III	0 - 180 km/h	increasing with local peak around 25 km/h and dip around 40 km/h; local extremes become more pronounced for deeper flats	DAF up to 2.5
Frederick [5.5]	simulation + experiment	III	0 - 120 km/h	increasing with local peak around 25 km/h and dip around 55 km/h; experiments tail off around 90 km/h	DAF up to 3.5 or 220 kN (experiments); DAF up to 6 in theory
Newton & Clark [5.7]	simulation + experiment	III	0 - 120 km/h	increasing with local peak between 20 – 30 km/h	DAF up to 4
Dukkipati & Dong [5.9]	simulation (+ validation)	III	0 - 140 km/h	increasing with local peak between 20 – 30 km/h	DAF up to 5
Wu & Thompson [5.11]	simulation	I, II, III	0 - 150 km/h	monotonic increase for stage III; increase until peak between 30 – 50 km/h and slight decrease afterwards up to 150 km/h for stages I, II	DAF up to 4 (III) 250 – 430 kN (I, II)
Johansson & Nielsen [5.12]	experiment + simulation	III	5 - 100 km/h	peak between 25 – 50 km/h; afterwards constant/slight decrease	50 – 220 kN
Baeza et al. [5.13]	simulation	I, II, III	10 - 200 km/h	almost monotonic increase	up to 350 kN
ORE D161 [5.19]	simulation	III	0 - 200 km/h	almost monotonic increase	up to 900 kN

It has been found in section 5.9 that the approximately quadratic – linear trend of the impact force as a function of the train speed, derived in section 5.7 on the basis of a rigid track model, overestimates the force in the subcritical domain, where the relationship appears instead to be linear. If the same overestimation could also be extended to the transcritical speed regime (which is only a hypothesis) the trend would be approximately linear – constant, which is indeed found experimentally (e.g. by Frederick [5.5], who finds experimental results tailing off from theoretical predictions around 70 km/h and becoming constant around 90 km/h for a 2.15 mm flat of 150 mm length, as also used by Newton and Clark [5.7]). Apart from this, the fact that the impact magnitude does not grow substantially after reaching the critical speed can be very well explained by the fact that the effective flat depth reduces with the second order of the speed in the transcritical speed regime (see Fig. 5.16), which will have anyhow a mitigating effect.

In view of the speed limits adopted in Table 5.3 and the rapid extension of high-speed rail connections it is important to extend the investigation to high-speed trains. In the high-speed regime (200 – 300 km/h) the probability of contact loss is much higher than for conventional train speeds. According to Eq. (5.37) the critical velocity is independent of the flat geometry for stage I flats; this is no longer true for stage III flats (Eq. 5.38), for which the wear increases the critical velocity, but also here the train speed may become easily transcritical, especially since the axle loads of high-speed trains are limited. Further, it is expected that for all common flat lengths the dynamic wheel-rail interaction will be in the inertia-dominated frequency area, especially since high-speed tracks (which are often slab tracks) are rather stiff.

5.11 *Experiments with wheel flats: the equivalent rail indentation and its applicability*

A wheel flat, like any circumferential wheel defect, may be represented by an equivalent indentation along the railhead. Before addressing the compatibility of both shapes, a general feature of such an equivalent rail indentation will be discussed. It has been concluded in section 5.5 (Fig. 5.12) that the minimum circumferential curvature of a wheel flat has a lower boundary 0 (stage I flat) and an upper boundary $1/R$ (undamaged wheel). This has its consequences for the maximum curvature of an equivalent indentation in the rail, which may reach, but not exceed, the value $1/R$. If it either reaches or exceeds this value,

transient double-point contact occurs, which results into a shift of the contact point along the wheel tread yielding impact. This phenomenon is illustrated in Fig. 5.36 (not to scale), which shows a discontinuous time-derivative (vertical wheel velocity) of the trajectory.

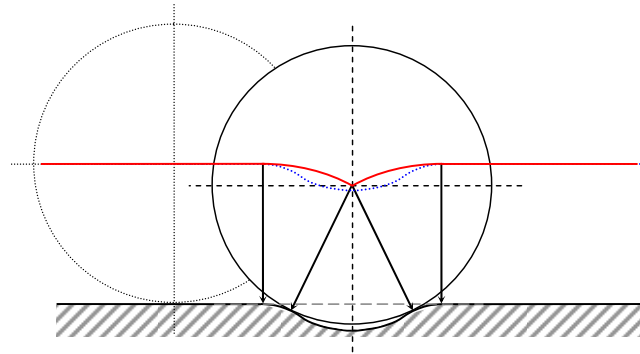


Fig. 5.36 Trajectory of the wheel gravity centre for a cosinusoidal indentation in the rail with a maximum curvature larger than $1/R$ (not to scale).

The compatibility requirement of the shapes of the wheel flat and the equivalent rail indentation depends on the contact type, and therefore, on the flat type which is studied. In the case of continuous single-point contact between wheel and rail, the radial deficiency $R - r$ of the surface of the flat (or the flat depth), as a function of the rotation angle φ , should be equal to the vertical depth z of the indentation, as a function of the longitudinal coordinate x along the rail.

In the case of transient double-point contact along the wheel circumference, the requirement is that the trajectory of the wheel centre is equal for both cases. This is equivalent to the simple requirement that the shape of the rail indentation is equal to the part of the wheel which virtually (in the absence of the flat) would overlap the rail when the wheel is positioned with the centre of the flat on the rail.

For a stage I flat, the second requirement is valid; for a stage III flat the first requirement. For a stage II flat, the second requirement is valid for the central part and the first requirement for both edges, leading to a superposition of two shapes.

The requirements and the resulting equivalent rail indentations are illustrated in Fig. 5.37 for a flat in stages I, II and III. It is important to note that for a fully worn wheel flat the length and depth of the flat are exactly equal to the length and depth of the indentation.

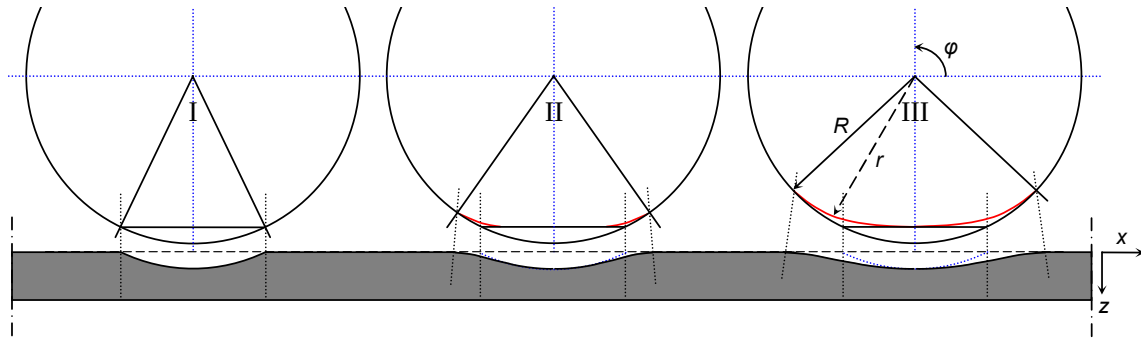


Fig. 5.37 Equivalent rail indentations for a wheel flat in stages I, II and III.

When using a dynamic wheel-rail interaction model with the Hertzian spring, such as the one introduced in chapter 3, commonly the kinematical trajectory of the wheel centre is used as an input for the excitation function. This is done because the mass of the wheel, modelled as a discrete rigid mass concentrated in its gravity centre, is not excited directly, but via the elastic Hertzian contact. In cases in which the curvature of the wheel-rail interfacial irregularity is negligible in comparison to the wheel tread circumferential curvature, this trajectory may be just taken as the geometry of the irregularity, because the error is negligible. In all other cases the trajectory must be derived (see Fig. 5.38). In cases of transient two-point-contact however, it is important to realise that the contact jumps between two positions, whereas the trajectory is a continuous function, be it with a discontinuous spatial and temporal derivative. The contact force calculated with a lumped wheel model on the basis of this trajectory therefore is a net force acting on the wheel and the rail. The distribution of this force over the contact points does not follow from the calculation. However, keeping in mind the derivation of the trajectory (see e.g. Figs. 5.10 and 5.18), the location of the impact can be specified.

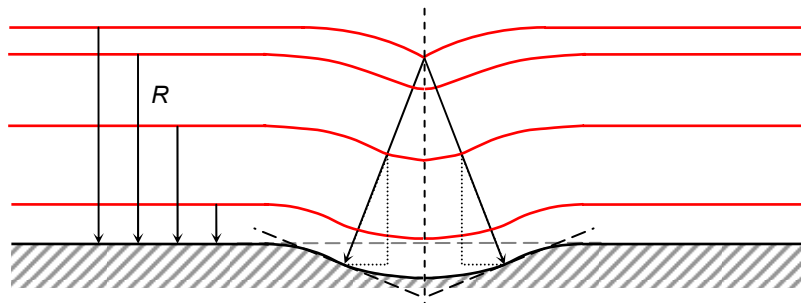


Fig. 5.38 Trajectories of the wheel centre for a given equivalent rail indentation and different wheel radii.

It is remarked that in FE modelling of the wheel and the rail the problem discussed in the above paragraph does not rise.

It is finally remarked that the stage III wheel flat studied experimentally by Newton and Clark [5.7] satisfies the criterion for the maximum curvature, derived in this section. They used an equivalent rail indentation with a simple cosinusoidal shape; the maximum curvature occurs at the centre of the indentation and can be calculated as 1.89 m^{-1} . For a conventional wheel radius R of 0.46 m , the curvature is given by $1/R = 2.17 \text{ m}^{-1}$ (for the given wheel radius of 0.5 m this limit equals 2 m^{-1}). The used value is about 6 per cent below the limit.

5.12 Experiments with wheel flats: the registration of wheel-rail contact forces

The dynamic wheel-rail contact force is a quantity that cannot be measured directly. Therefore, commonly indirect methods are applied, such as the registration of dynamic bending or shear strains. The bending strain can be said to be proportional to the elastic contribution to the dynamic contact force, whereas the inertial contribution is not accounted for. This causes a strongly frequency-dependent ‘filtering’ of the contact force. When this effect is not accounted for, significant errors can result. This will be considered in more detail in this section.

With the Kirchhoff assumptions for bending, the linear bending strain in the rail at the foot of the rail ($y = ab$) can be written as (the sign is a matter of convention):

$$\varepsilon(x, ab, t) = -\frac{\partial^2 w(x, t)}{\partial x^2} ab \quad (5.53)$$

Using the simplification of a stationary wheel model with moving irregularity ($w(x, t) \rightarrow w(0, t)$) this becomes:

$$\varepsilon(0, ab, t) = -\frac{\partial^2 w(x, t)}{\partial x^2} \bigg|_{x=0} ab \quad (5.54)$$

The bending strain in the fibre at the bottom of the rail foot in the Laplace- (or frequency-) domain follows simply from:

$$\varepsilon(0, ah, s) = - \frac{\partial^2 \tilde{w}(x, s)}{\partial x^2} \Big|_{x=0} ab \quad (5.55)$$

In chapter 3, expression (3.20) has been derived for the Laplace-image of the rail deflection along the rail. From this expression follows, after some elaboration:

$$\frac{\partial^2}{\partial x^2} \tilde{w}(x, s) = - \frac{\tilde{F}(s)}{8EI k_0} \left((-1-i) e^{(-1+i)k_0|x|} + (-1+i) e^{(-1-i)k_0|x|} \right) \quad (5.56)$$

or

$$\tilde{w}_{,xx}(0, s) = \frac{\tilde{F}(s)}{4EI k_0} \quad (5.57)$$

The bending strain in the ultimate fibre is now given as:

$$\tilde{\varepsilon}(0, ah, s) = - \frac{\tilde{F}(s)}{4EI k_0} ab \quad (5.58)$$

It is clear from Eq. (5.58) that the rail displacement and the contact force are directly related in the frequency domain. This is not the case in the time domain, which is obvious. The same holds for the bending strain and the force; the force-strain ratio is given by:

$$\frac{\tilde{F}(s)}{\tilde{\varepsilon}(0, ah, s)} = - \frac{4EI k_0}{ab} = - \frac{2\sqrt{2}EI^{3/4} (\varrho A s^2 + k_f)^{1/4}}{ab} \quad (5.59)$$

This ratio is exclusively determined by the track properties, and it is observed that the distributed rail mass plays an increasing role with increasing frequency (it is multiplied with the square of the frequency). The assumption of zero rail mass in Eq. (5.59) yields the quasistatic relationship:

$$\frac{\tilde{F}(s)}{\tilde{\varepsilon}(0, ah, s)} = - \frac{4EI k_0}{ab} = - \frac{2\sqrt{2}EI^{3/4} k_f^{1/4}}{ab} \quad (5.60)$$

which is of course frequency-independent. Normalisation of Eq. (5.59), including rail inertia, with Eq. (5.60), not accounting for rail inertia, yields the following complex transfer function, which can be considered as the transfer function for the quasistatic force-strain ratio to obtain the real force-strain ratio:

$$\tilde{G}(s) = \sqrt[4]{1 + \frac{\rho A}{k_f} s^2}; \quad s = \sigma + i\omega \quad (5.61)$$

The real or physical transfer function for a force spectrum, derived from dynamic strain measurements, becomes (substituting $s = i\omega$ with $\sigma = 0$):

$$G(\omega) = \text{Re} \left(\sqrt[4]{1 - \frac{\rho A}{k_f} \omega^2} \right) \quad (5.62)$$

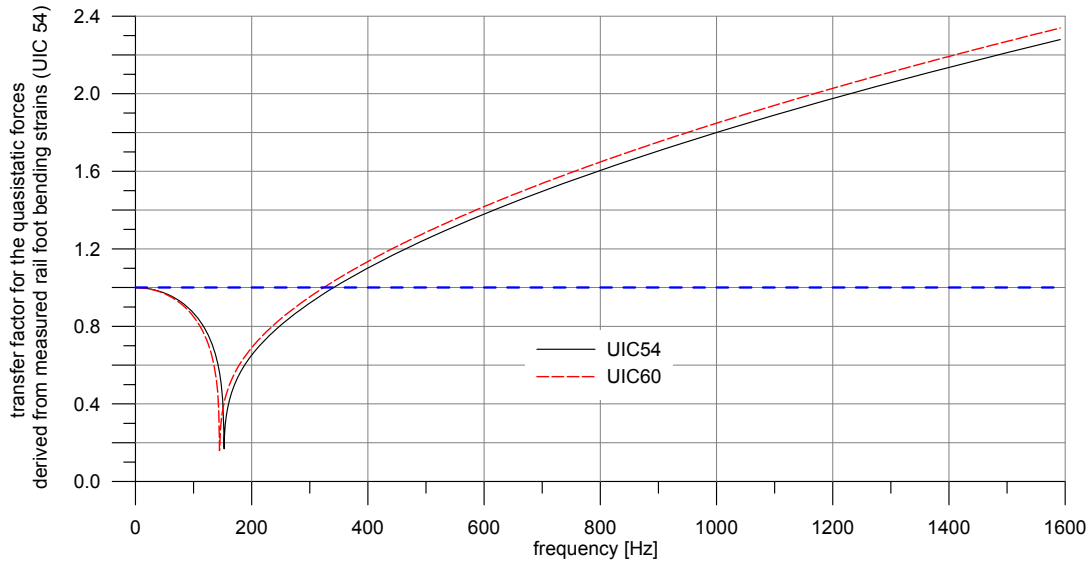


Fig. 5.39 Frequency-dependent error in relating dynamic rail bending strains and dynamic wheel-rail contact forces quasi-statically in the frequency domain (according to Eq. (5.62)).

It is remarked that this dimensionless function is governed by the rail mass; for low frequencies the transfer is close to 1, with increasing frequency the mass effect plays an increasing role. $G(\omega)$ is shown graphically in Fig. 5.39. From Fig. 5.39 it is evident that in the case that the frequency content of the excitation for a given wheel flat in combination with a given speed is mainly situated between 0-400 Hz, the force will be severely underestimated when

the rail mass is not accounted for. Fig. 5.25 shows frequency spectra of excitation functions for a worn wheel flat with a length of 100 mm and a depth of 0.24 mm for different speeds. Comparing these spectra with Fig. 5.39 is it clear that the influence of $G(\omega)$ reaches a local maximum around 10 m/s (36 km/h). For this train speed, the main body of the spectrum is located around the dip in $G(\omega)$ at 153 Hz.

The above could provide an explanation for the discrepancy between experiment and simulation found by Wu and Thompson ([5.11], Fig. 9). In this study, the flat shape used by Newton and Clark [5.7] was adopted in a simulation ($l_{\text{III}} = 150$ mm and $d = 2.15$ mm). Experimental results from Newton and Clark predict a local maximum between 30 and 40 km/h followed by a local dip around 60 km/h. Computational results from Wu and Thompson do not predict these local extremes, but a steadily increasing trend. However, Newton and Clark measured baseplate forces (and thus disregarded the effect of the rail inertia) and Wu and Thompson predict wheel-rail contact forces (and thus account for the rail mass). This is further illustrated by the following example. The expression for the rail bending strain at the rail in the time domain follows from Eq. (5.58) as:

$$\varepsilon(0, ab, t) = \frac{1}{\pi} \int_0^{\infty} \text{Re}(\tilde{\varepsilon}(0, ab, s) e^{st}) d\omega = -ab \frac{1}{\pi} \int_0^{\infty} \text{Re} \left(\frac{1}{4EI k_0} \cdot \tilde{F}(s) e^{st} \right) d\omega; \quad (5.63)$$

with $\tilde{F}(s)$ according to Eq. (3.23). Fig. 5.40 shows the extreme values of the registered bending strains in the rail foot during passage of a wheel with a stage III – worn flat (parameters equal to those used in Fig. 5.30) as a function of the train speed. Also the impact magnitude is shown. It is obvious that the local peak (in the area governed by the track stiffness) is much more pronounced for the strains (during impact) than for the impact magnitude.

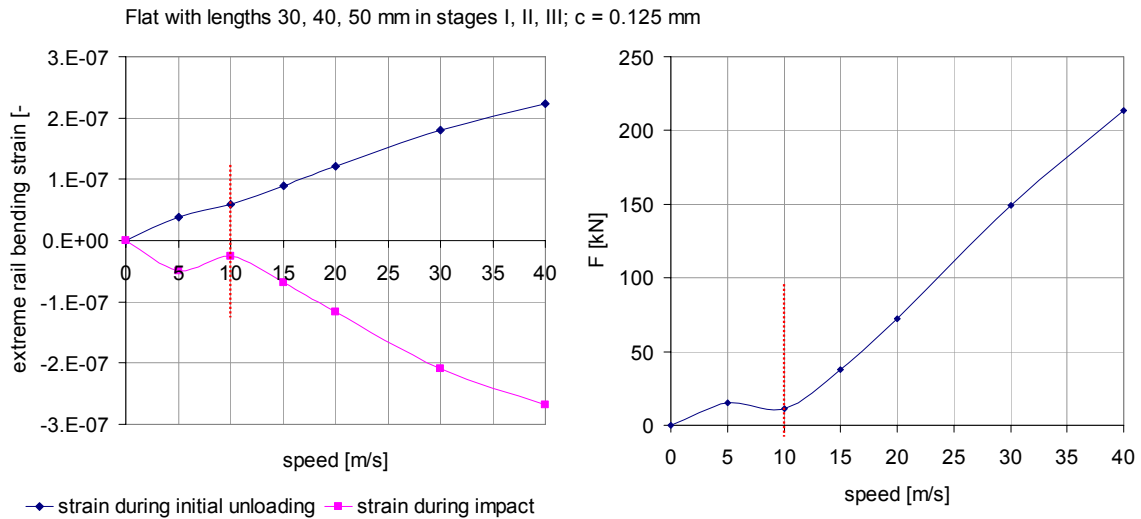


Fig. 5.40 Rail foot bending strains for a stage III – flat as a function of the train speed (left) and impact magnitude as a function of the train speed (right) (no contact loss).

5.13 Final remarks and conclusions

The main conclusion of the investigation in this chapter is that the minimum circumferential wheel tread curvature - which can also be conceived as the maximum deviation of this curvature from the constant circumferential curvature of a perfect wheel - is the critical parameter which governs the dynamic wheel-rail interaction in the presence of worn wheel flats (stage III):

- it determines whether the combination of the flat geometry and the train speed is located in the sub- or the transcritical speed regime, and thus whether contact loss occurs due to initial unloading, subsequently resulting in wheel-rail impact;
- it determines the magnitude of the contact force in the subcritical regime.

The dynamic wheel-rail interaction has been shown to be essentially different for the subcritical and the transcritical speed regime. The dynamic wheel-rail interaction is governed by the track stiffness for low train speeds or long flat lengths, whereas for high speeds and/or short flat lengths the interaction is governed by the inertial properties of the wheel and the rail. For a given flat geometry, non-linearities in the behaviour of the impact magnitude as a function of the train speed occur in the first case, whereas this relationship is approximately linear in the second case.

The model simulations in the present study were limited to the subcritical speed regime, due to the application of a frequency-domain model.

Therefore, several conclusions are only valid in the subcritical speed regime. The use of a time-domain model instead of a frequency-domain model is necessary to extend these conclusions to the transcritical speed regime. The same holds for those cases in which contact loss occurs after wheel-rail impact.

On the basis of a comparison of the results on wheel flats in the literature and the present investigation the following general expectations were formulated:

- a local force maximum occurs between 20-50 km/h, but only for longer flats, where the length-speed combination is in the frequency area dominated by the track elasticity (the forces are P_2 - like).
- this local peak disappears for shorter flats, since the whole relationship 'shifts' into the inertia-dominated frequency range (all loads become P_1 - like forces)
- it is hard to quantify the impact magnitude; qualitatively models results do correspond, but quantitatively large differences are found.
- when contact loss occurs at the critical speed, the impact magnitude becomes more or less speed-independent for higher speeds.

According to Fig. 5.12 the minimum curvature of the wheel flat increases during the wear process; the same holds for impact duration. However, it cannot increase to the limit value $1/R$ for an undamaged wheel. The question whether a stable balance situation is reached during the wear process and its position in the graph is a subject for further research. Work in this direction has been performed by Meywerk [5.23], Morys [5.24] and recently by Johansson [5.25], all mainly focusing on periodical out-of-roundness. It can be expected however that stabilizing or destabilizing mechanisms will be similar for worn wheel flats, and also strongly track-dependent.

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Summary

Short-wave irregularities in the wheel-rail interface are at the basis of track and vehicle damage and deterioration. On the short term, they result into high dynamic train-track interaction forces and a high energy input into the system that must be dissipated in the different system components or 'levels', leading on its turn to progressive deterioration on the long term. Furthermore, the short-wave defects grow into longer defects in the track geometry, due to the fact that the train is a travelling multi-body mass-spring system. The lifetime of the track and its components can be extended by adjusting the 'path' of the dissipated power spectrum through the system and adjusting component and system properties with respect to their hysteretic behaviour. Instead of life-time extension, such measures may also aim at an extension of maintenance intervals, which is important to optimise the availability of e.g. high-speed lines. The present study investigates two particular types of short defects in detail: rail welds and wheel flats.

In longitudinal direction and on a global scale, the contact between a rolling wheel and a rail can be distinguished into continuous single-point contact and transient double-point contact. The contact type that occurs depends on the actual geometry of the wheel-rail interface in the running direction. The first contact type leads to a dynamic amplification of the static axle load, whereas the second leads to wheel-rail impact. Especially the latter contact type is detrimental to the rail system and should be prevented as much as possible or detected at an early stage.

The introduction of rail welds instead of the traditionally bolted connections reduced the dynamic forces at rail joints globally with a factor three. However, welds remain potential damage initiators due to the local geometrical and metallurgical discontinuity. Investigations show an

approximately linear relationship between the extreme value of the dynamic wheel-rail contact force at a weld, the maximum absolute rail inclination and the train speed. The geometry of rail welds is traditionally assessed with the principle of vertical tolerances. A new assessment method for rail welds is proposed, with norm values for the allowable inclination depending on the line section train speed. This method is based on a relatively strong correlation between discretised maximum rail geometry inclinations (first derivatives) and extreme dynamic wheel-rail contact forces, relative to the poor correlation between tolerances and extreme forces. The method aims at a reduction and uniformisation of dynamic contact forces at rail welds, in order to avoid deterioration.

Wheel flats are commonly assessed on the basis of their length and/or depth, or automatically detected by wheel impact load detectors in the track. This study has shown that the minimum circumferential wheel tread curvature is the critical parameter that governs the dynamic wheel-rail interaction in the presence of wheel flats. It determines which contact type occurs for a given flat geometry: continuous single-point contact, in the subcritical speed regime, or transient double-point contact, in the transcritical speed regime. It furthermore determines the magnitude of the contact force in the subcritical regime. Both speed regimes are shown to exhibit essentially different features with respect to the dynamic wheel-rail interaction: the track stiffness governs the interaction for low train speeds and long flats, whereas for high speeds and/or short flats the inertial properties of the wheel and the rail govern the interaction. The force-speed relationship is non-linear in the first regime, whereas linearity is a good approximation in the second regime.

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Samenvatting

Korte oneffenheden langs het wiel-rail grensvlak liggen aan de basis van schade aan en aftakeling van het spoor en de voertuigen. Op de korte termijn veroorzaken zij hoge dynamische trein-spoor interactiekrachten en een significante energie-input in het systeem, die gedissipeerd moet worden in de verschillende systeem-componenten of 'lagen', hetgeen leidt tot voortschrijdende aftakeling op de lange termijn. Verder groeien de korte defecten uit tot langere defecten in de spoor-geometrie, als gevolg van het feit dat de trein een bewegend meervoudig massa-veer-systeem is. De levensduur van het spoor en haar componenten kan worden verhoogd door het afstemmen van het 'pad' van het spectrum van het gedissipeerde vermogen door het systeem en het afstemmen van component- en systeemeigenschappen met betrekking tot hun hysteretisch gedrag. In plaats van op levensduurverlenging kunnen dergelijke maatregelen ook gericht zijn op een verlenging van de onderhoudsintervallen, hetgeen belangrijk is om de beschikbaarheid van b.v. hogesnelheidslijnen te optimaliseren. De voorliggende studie verricht meer diepgaand onderzoek naar een tweetal specifieke typen korte defecten: metallurgische lassen in spoorstaven en vlakke plaatsen op wielen of wielbanden.

In langsrichting en op een macroscopisch schaalniveau kan rollend wiel-rail contact worden onderscheiden in continu enkel-punts contact en transient (of discontinu) twee-punts contact. Het type contact dat optreedt hangt af van de actuele geometrie van het wiel-rail grensvlak in de rijrichting. Het eerste type contact leidt tot een dynamische vergroting van de statische aslast, terwijl het tweede leidt tot wiel-rail impact. Het tweede type contact is buitengewoon schadelijk voor het spoorsysteem en moet ofwel zoveel mogelijk worden vermeden ofwel worden gedetecteerd in een vroeg stadium.

De introductie van lasverbindingen in spoorstaven in plaats van de traditionele geboute verbindingen heeft de dynamische krachten ter plaatse van de verbindingen met globaal een factor drie gereduceerd. Desondanks blijven lassen potentiële schade-initiatoren als gevolg van de lokale geometrische en metallurgische discontinuïteit. Onderzoek laat een bij benadering lineair verband zien tussen de extreme waarde van de dynamische wiel-rail contactkracht ter plaatse van een las, de maximale absolute helling van de rail en de treinsnelheid. De geometrie van lassen in spoorstaven wordt traditioneel beoordeeld op basis van het principe van verticale toleranties. Een nieuwe beoordelingsmethode voor lassen in spoorstaven wordt voorgesteld, met normwaarden voor de toelaatbare helling die afhankelijk zijn van de baanvak-snelheid. Deze methode is gebaseerd op een relatief sterke correlatie tussen gediscretiseerde maximale railhellingen (eerste afgeleiden) en extreme dynamische wiel-rail contactkrachten, ten opzichte van de zwakke correlatie tussen toleranties en krachten. De methode is gericht op een reductie en uniformisering van dynamische contactkrachten ter plaatse van lassen, om aftakeling tegen te gaan.

Het is gebruikelijk vlakke plaatsen op wielen te beoordelen op basis van hun lengte en/of diepte; ook worden ze wel automatisch gedetecteerd met wiel-impact detectoren in het spoor. Deze studie toont aan dat de minimale kromming van het loopvlak in omtreksrichting de kritieke parameter is die de dynamische wiel-rail interactie bepaalt wanneer vlakke plaatsen aanwezig zijn. Men kan ook spreken over het maximale krommingstekort ten opzichte van de constante kromming van een perfect wiel. Dit krommingstekort bepaalt welk type contact optreedt voor een gegeven geometrie van een vlakke plaats: continu enkel-punts contact, in het subkritische snelheidsregime, of transient twee-punts contact, in het transkritische snelheidsregime. Verder bepaalt dit tekort ook de grootte van de contactkracht in het subkritische regime. Beide snelheidsregimes blijken verschillende karakteristieke kenmerken te vertonen ten aanzien van de dynamische wiel-rail interactie: de spoorstijfheid domineert de interactie voor lage treinsnelheden en/of lange vlakke plaatsen, terwijl voor hogere snelheden en/of korte vlakke plaatsen deze rol wordt overgenomen door de traagheidseigenschappen van wielen en rails. Het verband tussen kracht en snelheid is niet-lineair in het eerste gebied, terwijl lineariteit een correcte benadering is in het tweede gebied.

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- (3) C. Esvelde, M.J.M.M. Steenbergen. *Force-based Assessment of Rail Welds*. 7th World Congr. on Railway Research, Montreal, Canada, June 4-8, 2006
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Note: chapter 4 of this thesis, on rail welds, will make part of a general chapter on rail welds in a Handbook of the Wheel-Rail Interface, in co-authorship with Peter Mutton, Monash Rail Research Centre, Australia (Woodhead Publishing, 2008).