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Convergence of Stochastic PDMM

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I. Abstract

In this work, we analyse a stochastic version of the primal-dual method of multipliers (PDMM), which is a promising algorithm in the field of distributed optimisation. So far, its convergence has been proven for synchronous implementations of the algorithm [1], [2]. Simulations have shown that PDMM also converges if it is implemented asynchronously, having the advantage that there is no need for clock synchronisation between the nodes in a distributed network. Furthermore, a broadcast implementation of asynchronous PDMM can be derived, instead of the usual unicast implementation. This broadcast implementation comes with a number of benefits. For example, it is a lot simpler to implement and requires less transmissions per iteration. Broadcast PDMM also lends itself to an efficient privacy preservation method that was introduced in [3].

In this paper, we analyse the convergence properties of different implementations of PDMM. In order to perform a rigorous analysis of a number of empirical findings, first a general stochastic version of PDMM is introduced. This general definition encompasses both asynchronous updating and transmission losses. Next, a formal proof is derived for the convergence of stochastic PDMM. This proof follows similar steps to the ones taken in [4] and builds upon a previous unfinished proof from [5]. The convergence proof makes use of the fact that the sequence of auxiliary errors of PDMM forms a non-negative supermartingale. By using Markov's inequality and Borel Cantelli's lemma, stochastic PDMM can be shown to converge almost surely to a bounded random variable that is supported by the set of fixed points of the standard PDMM operator. These points correspond to primal optimal points of the optimisation problem in question. The only assumption required for convergence is the fact that all edge variables must have a non-zero probability of updating.

In the case of unicast PDMM, asynchronous PDMM and PDMM with transmission losses can both be seen as specific instances of stochastic PDMM and thus also converge almost surely. Broadcast PDMM, however, requires each auxiliary variable to be stored at two nodes. In the case of transmission losses, a mismatch occurs between the two values stored for the same variable. This mismatch causes the algorithm to reach a fixed point that does not correspond to a primal optimal solution. As long as the two versions of the same variable are never mismatched, broadcast PDMM is equivalent to unicast PDMM. This why asynchronous broadcast PDMM does converge. With unicast PDMM, each auxiliary variable is only needed at one node, which makes unicast PDMM inherently robust against transmission loss and thus favourable when compared to broadcast PDMM. In Fig. 1 simulation results are given to show the difference in convergence behaviour between unicast and broadcast PDMM in the presence of transmission losses.

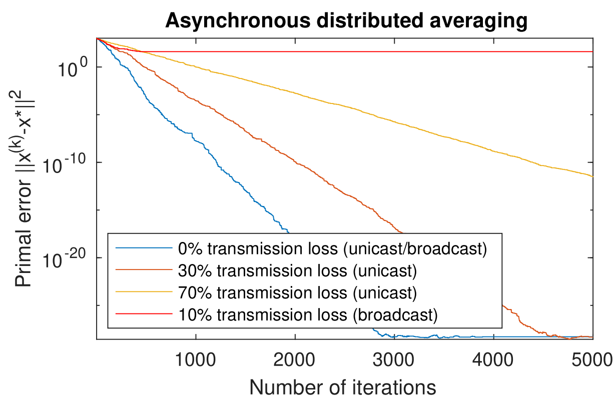

Fig. 1. Experimental convergence results for distributed averaging in the presence of transmission losses. Simulations are performed for a a random geometric network with 30 nodes and asynchronous PDMM is used as optimisation algorithm.

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