

Dynamic behaviour of tunnel elements during the immersion process

A study to the influence of swell waves and wind waves on the immersion system

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Executive summary

Immersion of tunnels is an often used technique in rivers and canals. The transport of tunnel elements can be done under offshore conditions, but immersing the elements under these conditions is not common practise. The tunnel elements of the Busan-Geoje Fixed Link in South Korea were immersed in such a situation.

During this process, several loads act on the element and the equipment, such as current and wave loads. Wave loads consist of relatively long waves (swell waves) and short waves (wind waves). The wave induced motions of the tunnel element are restricted during immersion due to serviceability limit state conditions.

In this study, the influence of swell and wind waves on the immersion configuration is analysed. In order to solve this problem, equations of motion are composed and solved to analyse the dynamic behaviour.

In the first step, the natural frequencies of the configuration are determined, and in the next step the total response is analysed. The latter is given in response amplitude operators, that represent the ratio between motion of the tunnel element and height of the wave per frequency. Using this methodology, locating the resonance peaks in the graphs is straightforward. Rough estimations of added mass and damping values cause some uncertainty in the results.

One of the findings is that some natural frequencies of the configuration are close to the frequency of swell waves. Especially the frequency of the rotation of the tunnel element is close to the frequency of swell waves. The influence of wind waves on the forces in the immersion cables is negligible.

Large motions of the element and high forces in the cables may be expected during resonance. This should be avoided by adjusting the immersion configuration, for example by decreasing the cross section of the floaters. This will result in larger natural periods.

The main conclusion is that the influence of swell waves on the forces in the cables is significantly larger than the influence of wind waves.

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List of symbols

In Table I the most important parameters which are used in this report are summarized.

Symbol	Description	Value	SI-unit
A	Cross section	—	m^2
A	Added mass matrix	—	kg
C	Hydrodynamic damping matrix	—	Ns/m
d	Position of the TE measured from the water surface on the z-axis	1	m
dr_p	Draft of the pontoons	—	m
E	Young's Modulus	$105 \cdot 10^3$	N/mm^2
E	Eigenmatrix	—	N
F	Force	—	N
g	Gravitational acceleration	9.81	m/s^2
H	Wave height	—	m
h	Immersion depth	23	m
h_e	Height of the tunnel element	9.97	m
h_{pd}	Height of the pontoon deck	2.5	m
H_s	Significant wave height	—	m
J_e	Mass moment of inertia of the tunnel element	—	kgm^2
J_p	Mass moment of inertia of the pontoon	$3.74 \cdot 10^9$	kgm^2
K	Stiffness matrix	$2.75 \cdot 10^8$	N/m
k_c	Stiffness of the contraction cables	$2.29 \cdot 10^6$	N/m
k_m	Stiffness of the mooring cables	$1.41 \cdot 10^6$	N/m
k_s	Stiffness of the suspension cables	$5.63 \cdot 10^8$	N/m
k_w	Restoring coefficient of the water	$4.34 \cdot 10^6$	N/m
L_e	Length of the tunnel element	180	m
L_p	Length of the pontoon/ floaters	36	m
M	Mass matrix	—	kg
m_e	Mass of the tunnel element	$4.965 \cdot 10^7$	kg
m_p	Mass of the pontoon	$1.4 \cdot 10^6$	kg
p	Pressure	—	N/mm^2
t	Time	—	s
T_p	Wave period	—	s
T_0	Natural period	—	s
W_e	Width of the tunnel element	26.46	m
W_p	Width of the pontoon	42.5	m
x_i	Displacement of degree of freedom $i = 1,2,4,5$ (translation)	—	m
$\dot{x}_i$	Velocity of degree of freedom $i = 1,2,4,5$ (translation)	—	m/s
$\ddot{x}_i$	Acceleration of degree of freedom $i = 1,2,4,5$ (translation)	—	m/s^2
x_i	Displacement of degree of freedom $i = 3,6$ (rotation)	—	rad
$\dot{x}_i$	Angular velocity of degree of freedom $i = 3,6$ (rotation)	—	rad/s
$\ddot{x}_i$	Angular acceleration of degree of freedom $i = 3,6$ (rotation)	—	rad/s^2
Y_e	Distance from fixation of the suspension cables to the edge of the TE	0.665	m
Y_p	Distance from the edge of the pontoon to the point where the resulting restoring spring coefficient acts	3	m
α_c	Angle in contraction cables	$15 \cdot \pi/180$	rad
α_m	Angle in mooring cables	$10 \cdot \pi/180$	rad
ζ	Damping ratio	—	—
ζ	Wave amplitude	—	m
κ	Wave number	—	m^{-1}
ρ	Density of water	1025	kg/m^3
σ	Tension	—	N/mm^2
ω	Frequency	—	rad/s
ω_0	Natural frequency	—	rad/s

Table I: most important parameters.

Introduction and problem definition

Immersion of tunnel elements in offshore conditions is seldom applied. Nowadays there are some projects where tunnel elements are immersed in such conditions. An example is the immersed tunnel in the Busan-Geoje Fixed Link in South Korea, which is used as reference project in this study.

Offshore wave and weather conditions act on the tunnel element during transport and immersion. loads consist of swell waves with a period of approximately 8 seconds, wind waves with a period of approximately 4 seconds, currents and wind loads.

The wave loads result in motions of the tunnel element. Too high motions can result in high forces in the cables which connect the tunnel element and the pontoons. Damage to the tunnel elements will occur when the velocity of the tunnel element is too high at the moment the tunnel element reaches a previous installed element.

Before the immersion operation takes place it should be clear what the response is of the tunnel element and the pontoons to the offshore wave conditions.

The aim of this study is to predict the dynamic behaviour of a tunnel element during the immersion process. A contractor can use this information in the tender-phase of a future project. Conclusions can be drawn whether it is useful to have an extended analysis on basis of the results obtained in this report, before the immersion operation takes place.

The main problem can be divided into three sub problems:

- What are the natural frequencies of the system?
- What is the response of the system to different wave loads?
- What are the forces in the cables due to wave-induced loads?

In order to answer these questions a model is composed where the response of the immersion system is determined. As reference project, the characteristics of the tunnel in the Busan-Geoje Fixed Link are used.

For the analysis of the motions of the tunnel element during immersion boundary conditions are composed. The most important restrictions are:

- Three degrees of freedom are studied. Only the sway (horizontal translation), heave (vertical translation) and roll motions (rotation in the sway and heave plane) are in this thesis analysed. This means that only the motions, which are present in the z-y plane, are studied.
- The wave loads act perpendicular on the tunnel element as a result of the previous condition.
- The tunnel element is assumed to be a stiff box, so deformations of the tunnel element during immersion are ignored.
- Rotations are assumed to be small, so the following assumption is made:

$$\sin(\varphi) = \tan(\varphi) = \varphi$$

- A linear differential equation is used to calculate the dynamic behaviour of the tunnel element. Non-linear effects are linearised.
- The dynamic behaviour of the tunnel element will be calculated for the position of one meter below the water level, measured on the z-axis. This means that the distance between the deck of the tunnel element and the water surface is one meter.

Chapter 1 - 4 consists of a literature study.

A simple estimation of the natural frequencies is done in chapter 5. The motions were decoupled in this phase, which means that each time only one degree of freedom (DOF) is studied. Only mass contributions and restoring properties of the water are taken into account in the estimation. The stiffness of the cables is assumed to be infinite.

Hereafter an extended model with the complete stiffness of the system is taken into account. The motions are coupled in this phase and influences each other. The immersion system is described by a 2 mass-spring system which consists of six degrees of freedom. The natural frequencies are hard to determine by hand and are therefore calculated in a Maple-sheet.

The response of the system is calculated in the frequency domain. This is described in chapter 6.

In chapter 7 the forces in the cables between the tunnel element and the pontoons are analysed. This is performed for two specific wave loads, which are the swell waves and the wind waves.

Finally, a small sensitivity study is performed where some modifications of the immersion system are given to change the response of the system. This is described in chapter 8.

Conclusions are drawn in the last section of this thesis.

1 Immersed tunnels in a nutshell

In this chapter a general introduction is given about immersed tunnel topics.

Tunnels can be distinguished according to the applied construction method. Roughly speaking, these are: techniques for rock, shield-driven, cut-and-cover or immersed tunnel techniques.

In order to compare the different kind of crossings, selection criteria are set up. The most important criterion is the physical possibility of building certain structures in certain situations. Thereafter the construction, operating, maintenance, repair and user cost are of importance. The last criteria are the safety, the inconvenience and the environmental impact during both construction and use (Glerum 1988).

1.1 Immersed tunnels

An immersed tunnel consists of prefabricated elements that are transported floating to the site and installed one by one. An immersed tunnel is installed in a trench that has been dredged previously in the bottom of a waterway (Grantz and Saveur 1997).

The first immersed tunnel for road transport in the Netherlands was the Maastunnel, which was built in Rotterdam and completed in 1943. Nowadays immersed tunnel techniques have become more widely used. Two examples of a cross section of an immersed tunnel are given in Figure 1-1.

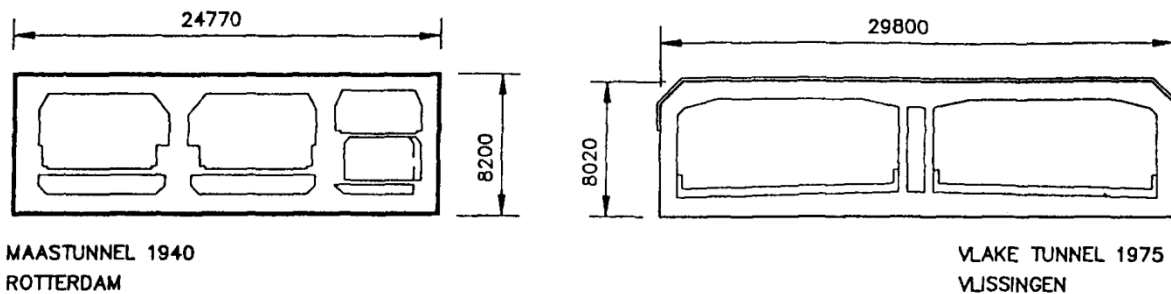


Figure 1-1: Cross section of the Maastunnel and the Vlaketunnel

An immersed tunnel is not always a road or railway tunnel, it can also be a service tunnel. The latter includes tunnels for conveyor belts, high- and low-voltage cables, utility pipelines, cooling water intakes and outfalls, sewer culverts and siphons (Rasmussen 1997). Two examples of a cross section of a service tunnel are shown in Figure 1-2.

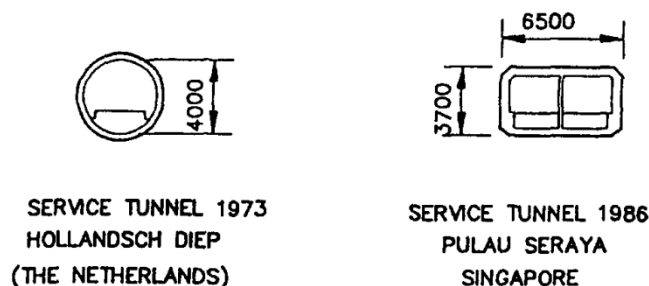


Figure 1-2: Examples of a cross section of immersed service tunnels

1.2 Tunnel elements

Tunnel elements are prefabricated in a casting basin or in a drydock. Construction of the elements can take place next to the immersion location or far away from it. The elements will be constructed in the dry. Thereafter, the casting basin will be flooded to allow the elements to float out and taken away (Ingerslev and Saveur 1997).

The tunnel elements are made buoyant by installing temporary bulkheads at the element ends.

In addition to providing proper structural strength and controlling the weight of the element, the main design and construction point of a tunnel is to provide a watertight structure (Rasmussen 1997).

Construction of concrete tunnel elements is relatively straightforward. However, great care is required in order to meet durability requirements. Also attention must be paid to avoid cracks in the concrete.

In the past, tunnel elements were made as a monolith structure with a length of about 100 meter. Problem with such a long tunnel element is that cracks in the concrete can occur due to shrinkage, temperature fluctuations and unequal settlements in longitudinal direction. Due to the cracks, problems can occur with the water tightness of the structure.

Therefore the monolith elements were replaced with elements divided in segments with expansion joints. This construction technique reduces the tensile stresses in the structure.

The elements are prestressed during construction stages. Once the elements are resting on their final foundation, the prestress is removed, so that the tunnel forms a flexible chain of segments (Glerum 1995).

As a remark on this it must be mentioned that crack width control is also possible by partially prestressing of the structure. One can use reinforcement bars on places in the structure where tensile stresses are expected.

A tunnel element normally consists of 4-8 segments. The traditional casting sequence of a segment is 'bottom-walls-roof'. Nowadays it is possible to cast a segment in one batch. The segments are approximately 15-25 m long.

The segments of the Øresund tunnel, which connects Denmark with Sweden, are poured in a fabrication plant. Once a segment was finished, the segment was pushed forwards to make space for a new segment. Once eight segments were casted, they were prestressed and pushed a further 100 m, so that the whole element was located within a bunded dock area. A sliding gate was closed between the element and the factory. Finally the dock was flooded which allows the elements to float out.

Whilst the completed element was being immersed, the factory was able to continue with construction of a new element. As a result of this process, there were no breaks in production (Marshall 1999).

1.3 Ancillary works and immersion equipment

Some ancillary work has to be done in order to proceed with the immersion process. Below, the basic equipment and additional works required for the immersion operation are given (Rasmussen 1997).

In order to float up or immerse a tunnel element, a water ballasting system is installed in the tunnel element.

After immersion, the temporary ballast tanks are replaced by permanent solid ballast. The permanent ballast can be placed inside and/or outside the tunnel element.

In order to allow precise and controlled immersion of a tunnel element one needs floating bodies with a water cutting cross section. The function of the floating body is to keep the element stable during immersion. Often pontoons are used to immerse tunnel elements. Two types of pontoon-configuration are more explained in detail.

The commonly-used configuration is the catamaran-type. A second possibility is deck-mounted pontoons. The advantage of the catamaran-type configuration is a larger correcting moment in comparison with the deck-mounted configuration. The latter has the advantage of a smaller width in comparison with the catamaran-type configuration. The two mentioned pontoon configurations are shown in Figure 1-3.

One can also use other floating equipment to immerse a tunnel element, such as floating cranes.

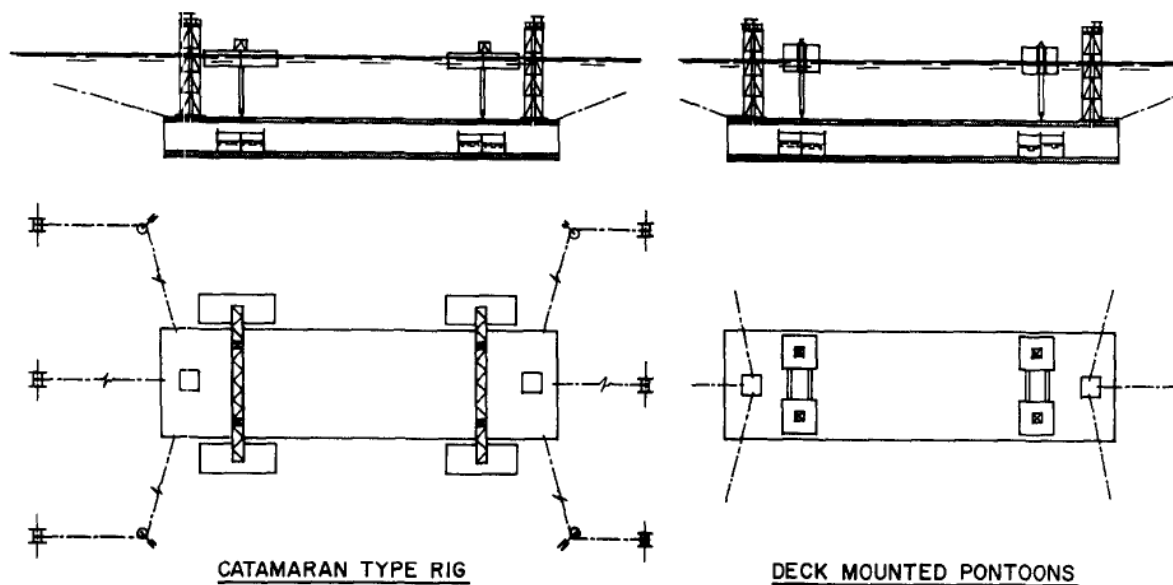


Figure 1-3: Pontoon configuration

Deck-layout, such as bollards, lifting lugs, pulling jacks and positioning system such as a beam and catch construction are installed in order to proceed with the immersion process.

Temporary alignment/ survey towers are mounted at each end of the tunnel element. One of the alignment/ survey towers can be used as an access shaft. This allows the entry of personnel or equipment to the interior of an immersed tunnel while submerged.

1.4 Immersion of tunnel elements

The four following main activities are related to the immersion process of a tunnel element (Molenaar 1988):

- Towing the element to the immersion location;
- Anchoring the element in order to allow precise manoeuvring;
- Ballasting the element to bring it down to the bottom;
- Connecting the element to the previous one.

The four activities are explained more in detail hereafter.

The elements must be transported from the casting yard or a mooring location to the immersion site. In some cases the casting yard has not enough space to cast all the elements in one batch. In order to proceed with the construction process of tunnel elements, the finished tunnel elements can be moored temporarily at a mooring location.

Nowadays the most tunnel elements are towed afloat, when the final ballast weight is not yet installed. A second possibility, which is not common anymore, is by hanging the elements under pontoons when the final ballast is already placed.

For offshore tunnels, the elements must be designed in such a way that the element and the immersion equipment can withstand the forces resulting from currents, wind waves and swell waves. The bulkheads, installed at the element ends, have to resist these forces as well.

When the tunnel element is towed to the immersion location, the next step is to anchor the element in order to allow precise manoeuvring. Anchors are installed on the bottom of the waterway. Cables are connected between the anchors and the tunnel element and immersion equipment. Relative heavy cables and anchors are required for offshore conditions involving high, strong currents.

For these tunnels, it is not difficult to imagine the complicated anchoring system that is required when the influences of wind waves and swell waves are added.

As an example, the mooring and immersion arrangement of the Elbe tunnel is shown in Figure 1-4.

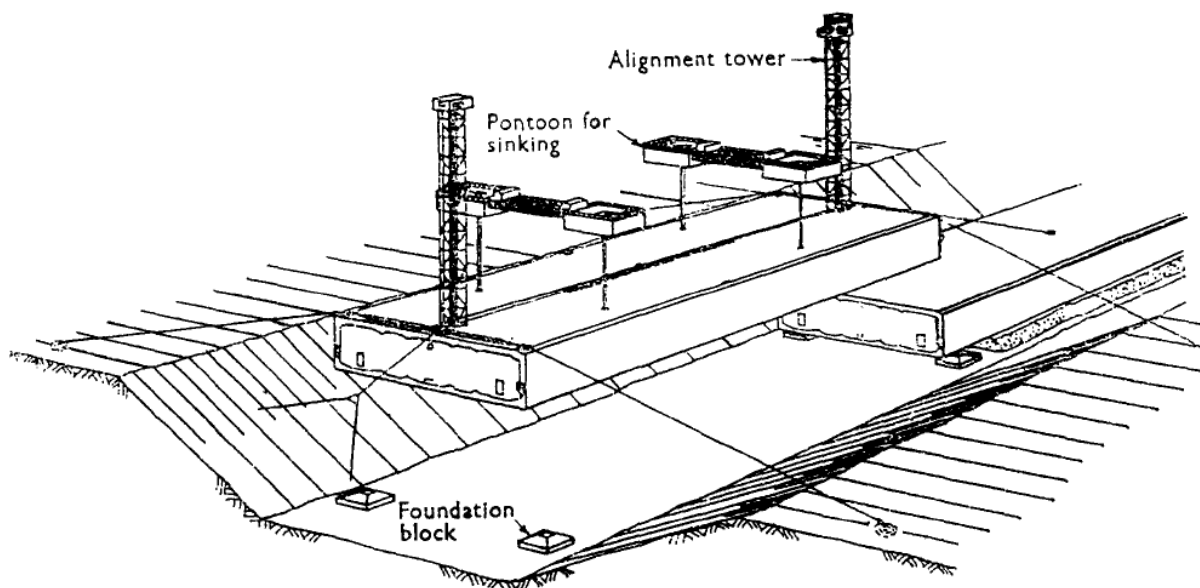


Figure 1-4: Mooring and sinking arrangement used for the Elbe tunnel

The next step in the immersion process is to fill the built-in ballast tanks with water in order to bring the tunnel element to the bottom. The ballast tanks are shown in Figure 1-5.

With the ballast tanks it is possible to change the buoyancy of the tunnel element. Positive buoyancy indicates that the element wants to float. This can be achieved by emptying the ballast tanks. Negative buoyancy means that the weight is greater than the buoyancy: the tunnel element wants to sink.

The element is lowered stepwise onto the foundation pads or the gravel bed. The position of the element is continuously monitored during the entire immersion operation. The foundation pads can be seen in Figure 1-4.

The disadvantage of a temporary ballast-system is that the ballast water has to be replaced by ballast concrete. The transport of ballast concrete to replace ballast water can cause logistical problems. Additionally to this issue, the casting of ballast concrete is time consuming and costly (Molenaar 1988).

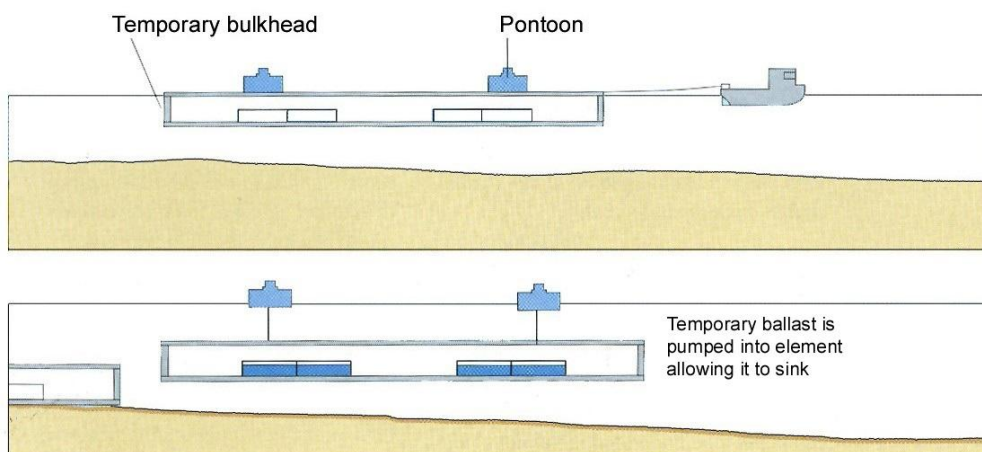


Figure 1-5: Ballast tanks

Once placed, the elements are joined. This is the last step in the immersion process. The elements are first joined together by bringing the rubber gasket into contact with the previously placed element. This rubber gasket, the Gina gasket: one of the most characteristic details of immersed tunnels, is explained more in detail hereafter.

The second phase in the connecting procedure is to empty the immersion joint. As a result of this the full hydrostatic water pressure on the tunnel cross-section is mobilized, which compresses the Gina gasket to get a watertight connection. This process can be seen in Figure 1-6.

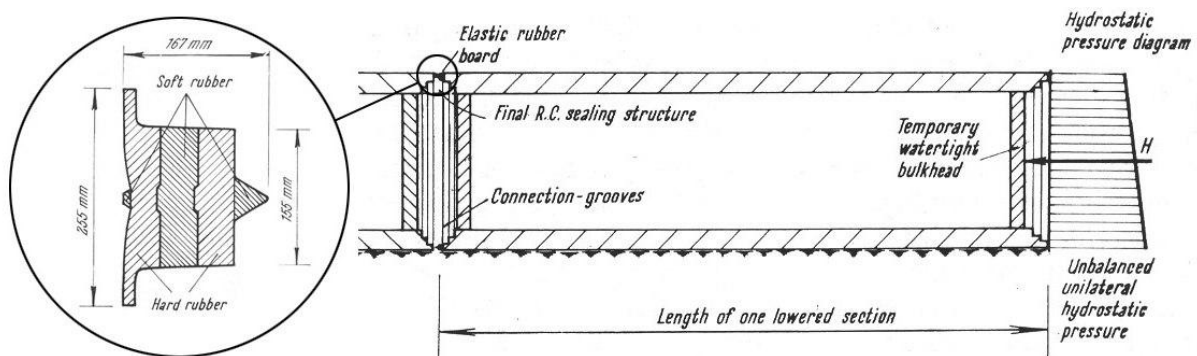


Figure 1-6: Joining procedure of the tunnel element

1.5 The Gina and Omega gasket

The Gina gasket consists of a rubber section able to transfer large compression forces, and a soft nose able to provide an initial seal under low compression forces. The gasket provides a temporary seal and compression contact face during immersion installation, and may provide a permanent seal at flexible joints.

Later on, a second gasket is installed: the Omega gasket. It may form a secondary permanent seal or it may become the primary seal. Because of its shape, it can sustain large longitudinal and transverse movements at the joint (Ingerslev and Saveur 1997).

The Gina and Omega gasket are shown in Figure 1-7. In the upper figure the uncompressed Gina gasket is shown. The lower figure shows the compressed Gina gasket and the rubber Omega Gasket.

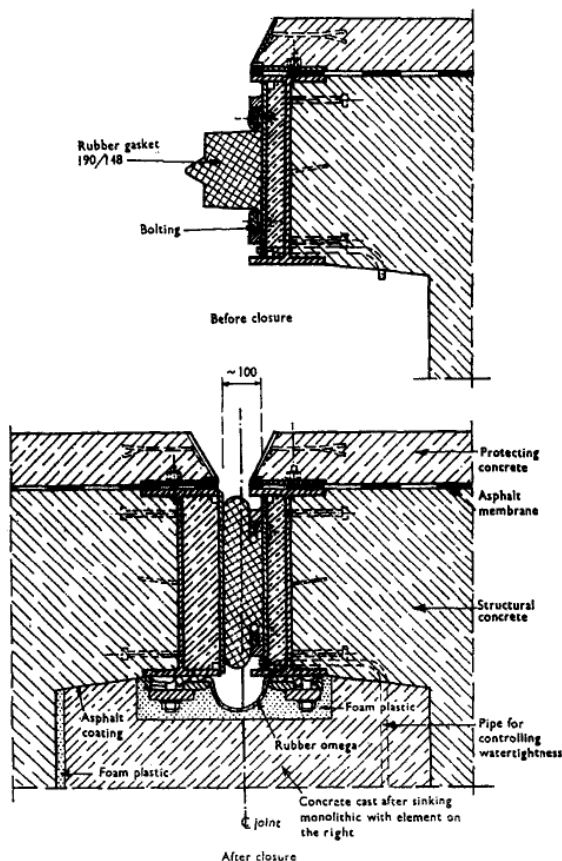


Figure 1-7: Gina gasket before and after closure

1.6 The trench and foundation of the tunnel element

The space between the trench bottom and the bottom of the tunnel element can be a previously prepared gravel bed or it can be sand bedding (Rasmussen 1997).

If the tunnel is to be founded on a jetted sand foundation, the element will be placed on temporary foundation blocks, close to the previously placed tunnel element. The foundation blocks are placed beforehand. Four heavy steel rams, activated by cylinders from inside the tunnel element is the original solution.

A solution which is common nowadays is that the front pair of cylinders is substituted by one or two brackets on the previously placed tunnel element.

When the tunnel element is immersed on its temporary supports, a sand-water mixture is pumped through openings in the floor slab into the gap between the underside of the element and the bottom of the trench. The water flows away and the grains of sand settle, forming a kind of circular 'pancake' around the injection point. As soon as a pancake is formed, the sand-water mixture is pumped through another opening. A pattern of overlapping 'pancakes' is formed which provides a good foundation. This process is monitored carefully by the hydraulic jacks or a survey system. As soon as the forces in the hydraulic jacks decrease, the sand package forms a good foundation. The principle of sand-jetting is shown in Figure 1-8 (Glerum 1995).

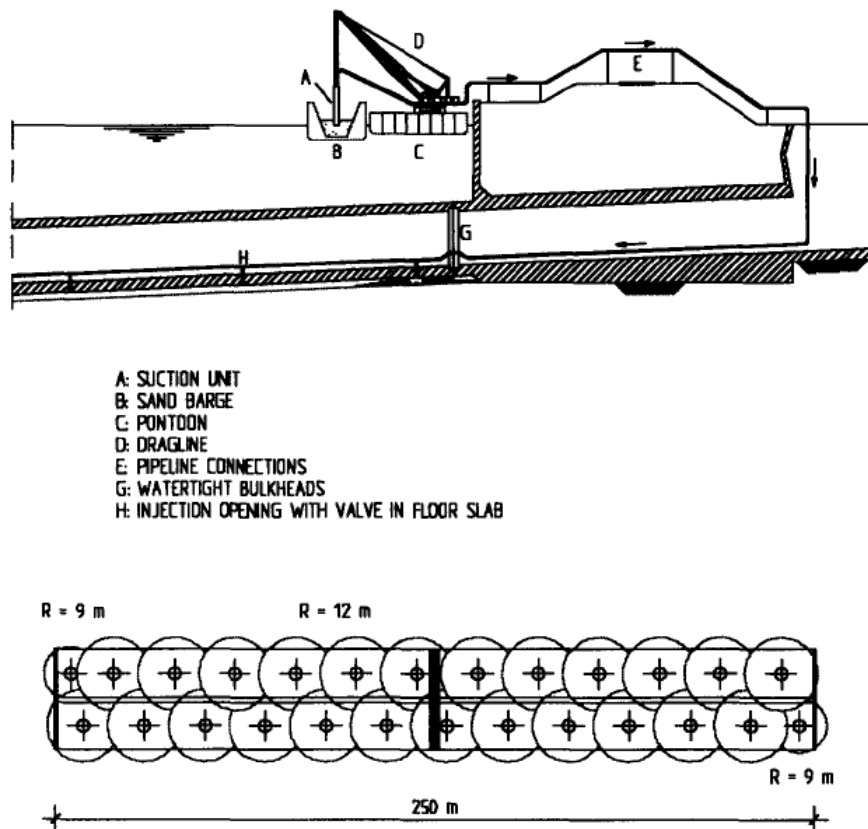


Figure 1-8: Sand-flow system (Vlaketunnel): equipment and pattern of sand cakes

A second possibility is to immerse the tunnel element on a previously prepared gravel bed. In this case there is no need for temporary supports or sand-jetting under the tunnel element.

The demands on the gravel bed are very high. Tolerances in the construction height are in order of a few centimetre. Therefore, the construction of the trench is a very difficult operation.

1.7 Complementary works

The trench is backfilled when the tunnel element rests on its permanent foundation. Protecting mattresses and armour rock are also placed. The latter has as purpose to protect the permanent foundation and the backfill from scour and protect the tunnel element from falling objects, such as a falling anchor. In Figure 1-9 one can see the different types of layers to protect the tunnel element.

The complementary works can be divided in two groups: civil works and mechanical and electrical works. The civil works consists of for example removal of the ballast tanks and bulkheads, casting of the permanent ballast concrete or asphaltting of the roadway.

The mechanical and electrical works consists of the installation of a permanent lightning en ventilation system, a drainage system, a fire-protection system and a traffic control system (Rasmussen 1997).

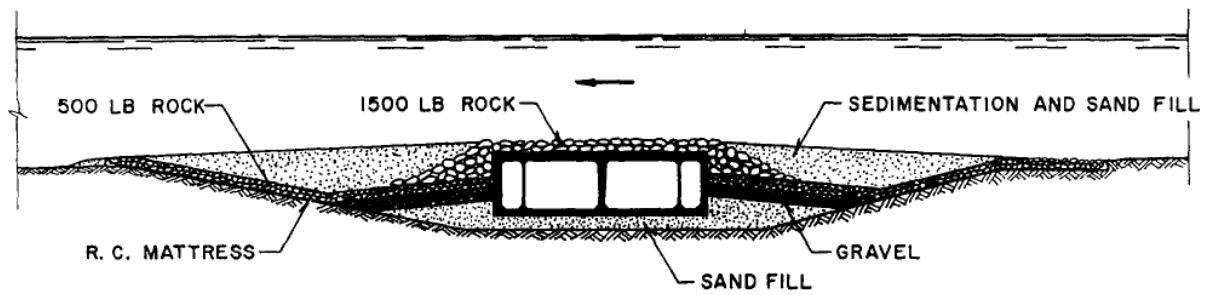


Figure 1-9: Backfilling of the tunnel trench

2 The immersed tunnel for the Busan- Geoje Fixed Link

In this chapter the tunnel project of the Busan- Geoje Fixed Link is described.

The Busan- Geoje Fixed Link, with a total length of 8.2 kilometre, is a major infrastructural project in South Korea. The project consists of two bridges with a length of 1650 meter and 1865 meter and also includes an immersed tunnel with a length of 3380 meter and a maximum foundation level of 48 meter below main sea level. The entire link has two lanes in each direction for car traffic (Heijmans and Meijnhardt 2008).

The immersion has to take place at deep depth, deeper than any immersion project before.

Dense ship traffic is predicted across the tunnel due to the completion of the Busan New Harbour which requires a water depth of at least 20 meter.

The area is prone to seismic activity. The tunnel will therefore be designed in accordance with Korean standards (Chang, et al. 2006), (Heo, et al. 2006).

An overview map of the project is shown in Figure 2-1.

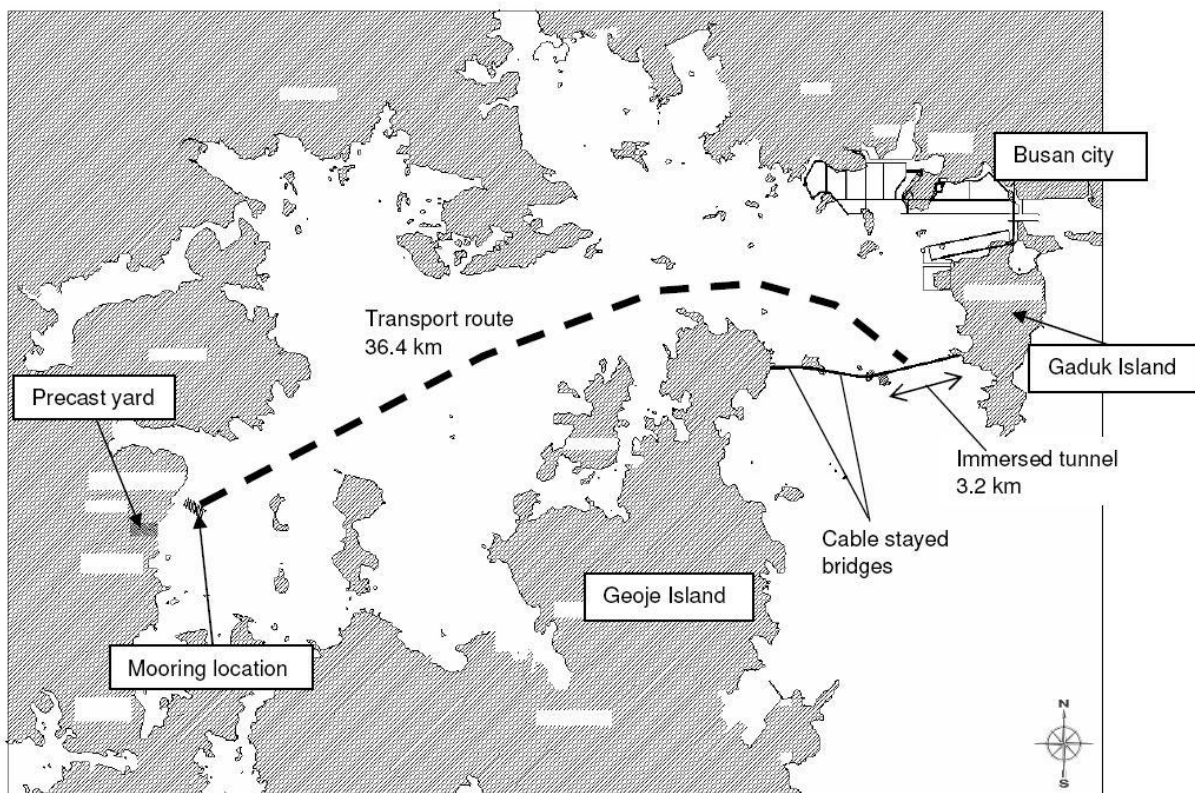


Figure 2-1: Overview map of the Busan - Geoje Fixed Link

The contractor is GK Fixed Link Construction Consortium with Daewoo Engineering and Construct as representing company. The Design Contract has been awarded to COWI from Denmark and Daewoo Engineering for design of the immersed tunnel works. The technical service for the immersion is provided by TEC from the Netherlands.

Commissioning authority is Busan Metropolitan City together with the Province of Gyeongnam. A 40-year contract has been concluded for the construction and operation of the link.

Strukton Afzinktechnieken has been awarded with a part of the engineering and the execution of the floatation, transport and immersion of the tunnel elements. This work will be carried out by a subsidiary of Strukton called Mergor Underwater Construction.

2.1 The tunnel elements

The immersed tunnel in the Busan – Geoje Fixed Link consists of 16 ordinary elements and 2 elements with a climbing lane. The elements with a climbing lane are element 17 and 18, which can be seen in the vertical alignment in Figure 2-2.

The total length of the immersed tunnel is 3380 meter. The tunnel elements are build in a pre-cast yard. The pre-cast yard has space to build 4 or 5 elements at a time. The elements are stored at a mooring location, close to the pre-cast yard and 36,4 kilometre away from the immersion location. The latter is shown in Figure 2-1.

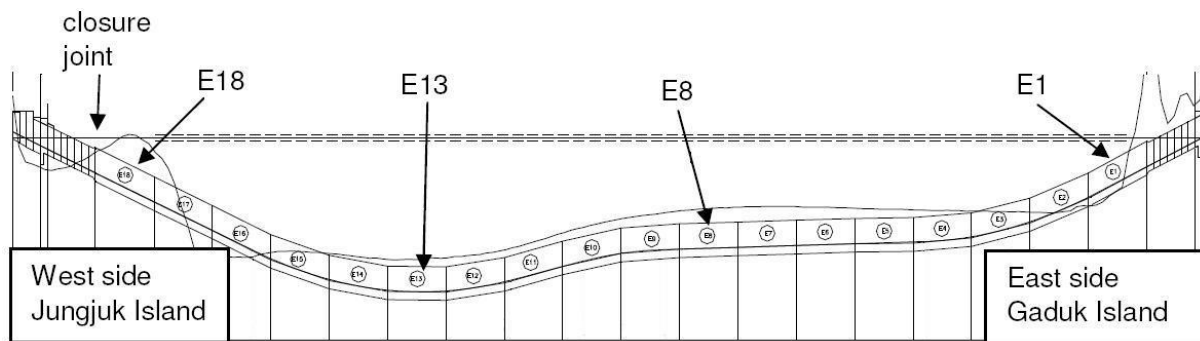


Figure 2-2: Vertical alignment

The width of one of the ordinary tunnel elements is 26.46 meter, the two elements with the climbing lane are 28.46 m. The height of all the elements is 9.97 m. The cross section of an ordinary tunnel element can be seen in Figure 2-3.

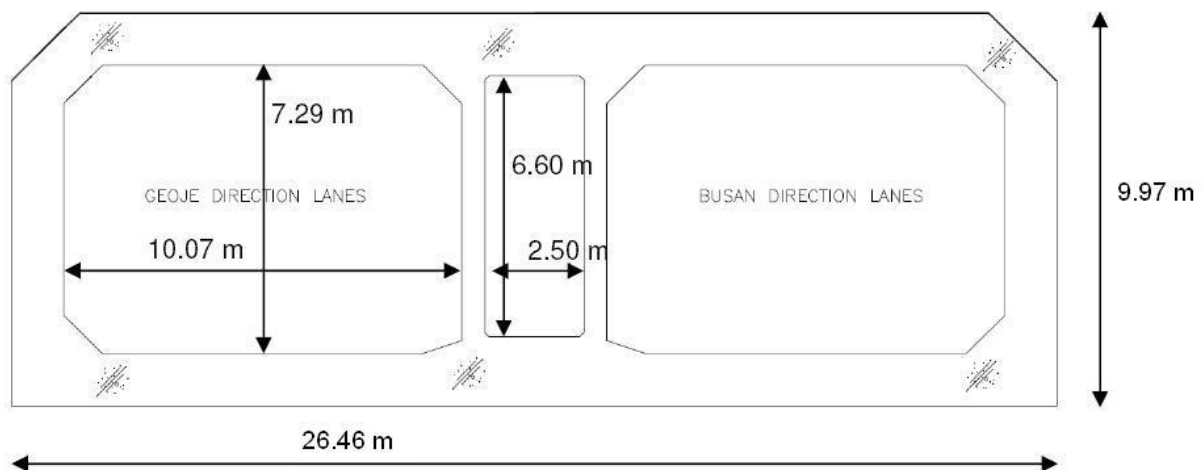


Figure 2-3: Cross section of an ordinary tunnel element

2.2 Offshore conditions for transport and immersion

The tunnel elements are moored, transported and immersed in offshore conditions. These offshore conditions consists of wind, waves and currents, which act on the tunnel and the immersion equipment.

Most waves are generated by winds in the area including tropical storms and typhoons. Waves generated by distant storms can also reach the tunnel alignment from southerly directions. The latter type of waves are called swell waves.

During transport from the mooring location to the immersion site and the start of the immersion operation, the short wind waves and the longer swell-waves affect the dynamic behaviour of the tunnel element. The tunnel element is also sensitive to swell waves on a greater depth.

Model research is done to determine the influence of the wave height and period on the entire system. These tests are done by Marin in Wageningen.

A numerical model is set up and calibrated by scale models which are performed in a wave simulation basin. The results of the numerical model and the predicted wave conditions determine whether the element can be immersed in a safe way.

The model tests are performed for two situations. The first situation is when the element is located at a distance of one meter below the sea level. The influence of the waves in this situation is significant for the forces in wires, tunnel motion and bending moments in the element. The main design criteria for this situation are the maximum forces in the wires.

The second situation is when the tunnel is located at a distance of 0.5 meter from the bottom of the immersion trench. The accepted motions of the element are restricted in this situation, because the element is in proximity of the previous immersed element. The forces in the wires are lower compared to the first situation because the influence of the waves at a greater depth decreases (Groot and Jille 2009).

2.2.1 Immersion of the tunnel

The tunnel elements are prepared for the immersion operation at the mooring location, which can be seen in Figure 2-1. Prior to the transport, two immersion pontoons are positioned over the tunnel element. The pontoons are of the catamaran type. This type of pontoons is shown in Figure 1-3. The pontoons consist of a main deck (size of 42,5 x 24 x 2.5 meter) and two floaters (36 x 6 x 6 meter each) (Vlaanderen Oldenzeel, Groot and Reijm 2010).

The deck-layout, consisting of bollards, lifting lugs and a landing tower are installed on the tunnel deck in the mooring location. Also the guide beams are installed on the primary side of the element and the catches on the secondary side. The guide beam and catches are used to guide the element during the last phase of the immersion operation.

The first tunnel elements are immersed with an access shaft mounted on the element. At a greater depth the use of an access shaft is not possible anymore. As a result of this, the equipment in the tunnel element is remote controlled. However, sometimes it might be necessary to enter the element during the immersion process. This can be done by using a self propelled diving bell (SPDP), which can be connected to the element on the landing tower (Groot and Jille 2009).

As soon as predicted weather and wave conditions are within the limits, the total system is transported to the immersion location.

2.2.2 Wires

Three types of wires can be distinguished. The mooring wires are connected between the anchors and the pontoons. These wires are used for positioning of the immersion pontoons above the alignment. The contraction wires run from the pontoon, through a pool on the deck, over the tunnel element to the anchors. These wires are used for positioning the tunnel element sideways and in longitudinal direction. Suspension wires are connected between the pontoons and the element. These wires carry the overweight of the tunnel element. The wires are shown in Figure 2-4.

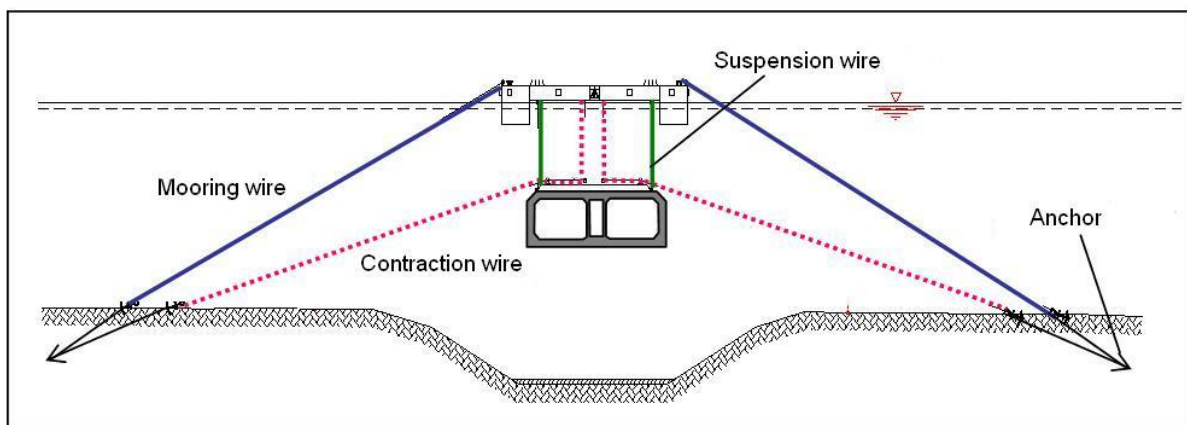


Figure 2-4: Immersion spread

2.2.3 External Positioning System

An innovative solution is used to connect the element on a safe way against the previous placed element, without the influence of the waves.

The External Positioning System (EPS) consists of two large portal constructions which are placed over the primary and secondary sides of the tunnel element. The tunnel element is immersed on the gravel bed with a distance of 0.5 meter from the previous placed element. Thereafter the tunnel element is lifted by the EPS and pulled towards the previous placed element.

Hydraulic jacks push the legs of the EPS out and it starts to lift the tunnel element just above the seabed for connection to the previous element. The EPS steps the complete element towards the previous element for connection, with no risk of damaging the previous element by wave induced motions. The EPS is shown in Figure 2-5 (Vlaanderen Oldenzeel, Groot and Reijm 2010).

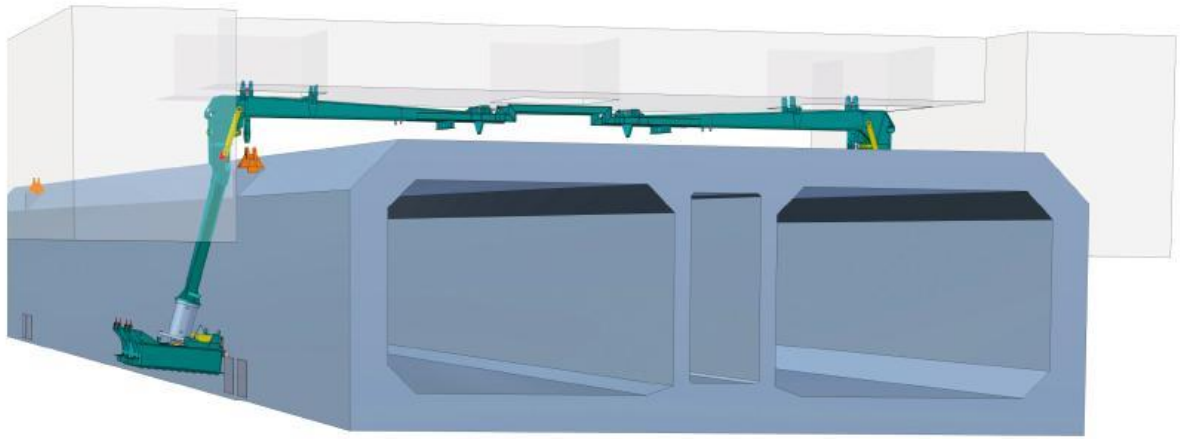


Figure 2-5: The External Positioning System

2.2.4 Summary

Traditionally, immersed tunnels have been installed as river crossings or in areas with protection against offshore conditions. The tunnel in the Busan – Geoje Fixed Link is constructed in an area which is prone for wind and wave conditions that in certain periods prevent immersion of tunnel elements.

Model tests and numerical simulations are carried out to determine what the effects are of the wave loads on the dynamic behaviour of the entire system and the loads in the wires and the bending moments in the tunnel element. The structural strength of the element is determined by the numerical models and the model tests.

The structural capacity of the element and the immersion equipment is in combination with a wave forecast system the basis to determine whether the immersion operation can take place or not.

3 Experiences of tunnel elements in offshore conditions

The immersed tunnel in the Busan – Geoje Fixed Link is the first tunnel which is immersed in offshore conditions. These harsh conditions for an immersed tunnel have never been seen before (Heo, et al. 2006). As a result of this there is not much experience gained about immersion of tunnel elements in off-shore conditions. In the past however, there are some examples in which tunnel elements are transported in off-shore conditions.

The experiences of tunnel elements in offshore conditions is the topic of this chapter.

For the design of an immersed tunnel, which is transported and/or immersed in offshore conditions, geological, hydraulic and meteorological surveys must be performed during all phases of the tunnel project. For offshore tunnels such surveys can be decisive for the design.

The required hydraulic survey should include information on current velocities and directions, tides, height and frequencies of wind and swell waves and the difference in specific weight of the water. The behaviour of the surf is also of importance in coastal zones (Molenaar 1988).

On the basis of the above survey, conclusions can be reached about the following aspects of construction:

- The procedure for and timing of immersion;
- The forces on the tunnel elements and the equipment;
- The type of equipment to be used;
- The ballasting of the tunnel elements.

A meteorological survey is required when the hydraulic aspects are influenced by the weather conditions.

For economic reasons, procedures and equipment are not designed to withstand extreme weather and wave conditions. Therefore, the weather and wave conditions, which influences the hydraulic aspects, must be studied continuously (Molenaar 1988).

3.1 Transport of tunnel elements in offshore conditions

In the past, some tunnels were transported in offshore conditions. For example, the tunnel elements of the Second Downtown Tunnel, which crosses the Elizabeth River in Norfolk and Portsmouth, Virginia, were fabricated in Corpus Christi, Texas. Eight elements were transported, two at a time, for the long 3000 km voyage through the Gulf of Mexico, across Florida and up to Norfolk (Hakkaart 1997).

Immersed tunnels are designed to be placed on prepared foundations. The only large, permanent, loads on the tunnel are the soil and water pressure and possible differential settlements in the soil.

During floating however, more loading cases can result from factors such as: the weight of the bulkheads, the equipment mounted for immersion and the off-shore wave height and period. These factors must be taken in account into the design of the element and provisions for the method of transport. The amount of prestressing in the elements is determined using these factors.

Tunnel elements under tow must withstand loading conditions comparable to ordinary ships. These include wave-induced transverse, longitudinal and torsion moments. Local loads, which act at the

tunnel elements as well, consist of wave forces which act on the bulkheads or collisions with floating objects, for example.

The magnitude of these forces depends largely on the expected sea conditions during transport and the length of time of exposure during transport.

Once these conditions have been determined, the next part of evaluation starts. The tunnel elements must be designed in such a way that overstresses do not occur. An envelope of moments for all load cases must be developed. Based on that, the configuration of the towing system can be evaluated (Hakkaart 1997).

In the Netherlands, the elements of the Wijker Tunnel have been transported over a distance of 70 km over the North Sea. The main challenge of maritime transport lies in the exposure of a tunnel element to waves (Zitman 2003).

3.2 Probabilistic design procedure for transportation of Wijker Tunnel

In the design procedure of the elements of the Wijker Tunnel was not focused on achieving a high degree of seaworthiness. Transportation of the elements could only take place during periods of moderate wave and weather conditions.

Scale models provided the design necessary to ensure that tunnel elements would be able to withstand certain wave conditions.

The critical design criteria was the amount of pressure in the joints between the segments. Leakage of the elements can occur when wave-induced moments become so large that the pressure in the joints is disappeared. The minimum pressure in the joints was set on $0,3 \text{ N/mm}^2$.

The required minimum pressure in the joints was transformed into a requirement for a probabilistic approach.

A safety level has been defined for the offshore transport of elements of the Wijker Tunnel. The latter was focused on preventing leakage in the joints.

A tool that has been developed shows for any wave forecast whether the non-exceedance probability of the minimum allowed pressure in the joints is larger or less than the predefined criteria. If it is larger, transport of the element needs to be postponed (Zitman 2003).

The results are shown in Figure 3-1. The encircled numbers are the conditions in which the elements are transported. The swell heights are represented by the lines in the graph in Figure 3-1.

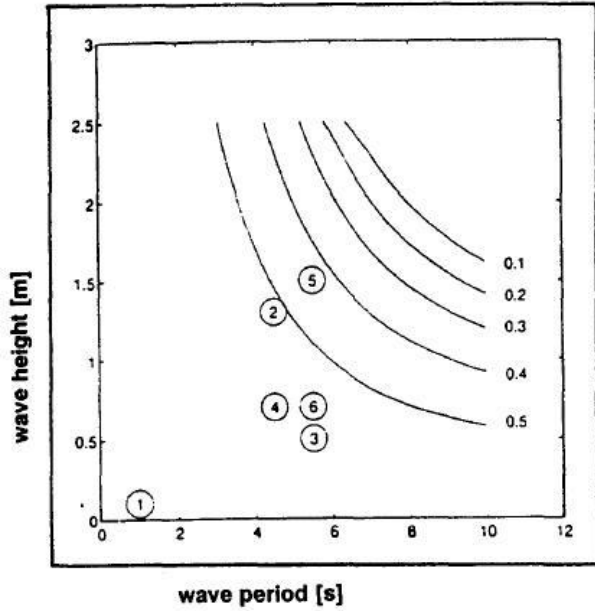


Figure 3-1: Curves reflecting the wave forecasts for which the safety level is only satisfied

With the graph in Figure 3-1 a manageable probabilistic tool has been obtained. With this tool, one can evaluate the forecast of wave conditions whether the safety level set for transporting of tunnel elements will be satisfied (Zitman 2003).

3.3 Immersion of tunnel elements in offshore conditions

The immersion of large-scale tunnel elements is one of the most important procedures of immersed tunnel construction.

Up to now, there is not so much literature related to the dynamic behaviour of a tunnel element which is exposed to offshore conditions during immersion. However, some research is done (Chen, Hou, et al. 2009), (Chen, Wang and Wang 2009).

3.3.1 Equation of motion

The dynamic behaviour of a tunnel element can be described by Newton's Second Law:

$$\sum_{j=1}^6 M_{kj} \ddot{\eta}_j = - \sum_{j=1}^6 A_{kj} \ddot{\eta}_j - \sum_{j=1}^6 C_{kj} \dot{\eta}_j - \sum_{j=1}^6 K_{kj} \eta_j + F_k$$

($k, j = 1, 2, \dots, 6$)

In which the complex displacement is given by:

$$\eta_j = \bar{\eta}_j \cdot e^{-i \cdot \omega \cdot t}$$

This system of equations describes the dynamic behaviour of the tunnel element in all 6 degrees of freedom. The mass matrix is given by M . The damping of the system is given in matrix C . The matrix K represents the stiffness of the entire system and the forces are given in vector F .

The matrix A is the added mass matrix.

The term $\bar{\eta}_j$ represents the motion amplitude of the tunnel element in the j^{th} mode.

Under only wave loading, the loads acting on the tunnel element are relatively large near the water surface and they decrease with the increase of immersing depth. The motion responses of the tunnel

element are also generally large near the water surface and decrease as the immersing depth increases.

3.3.2 Hydrodynamic coefficients

The calculation of added-mass and damping coefficients, which are called the 'hydrodynamic coefficients', are important for determining of the tunnel motions.

Several methods are developed to approximate the hydrodynamic coefficients. The coefficients are not straightforward to determine. Several researchers have already paid special attention to it (Guedes Soares and Ramos 1997).

The hydrodynamic coefficients are related to the shape of the cross-section. In the past, research is done to determine the hydrodynamic coefficients for ships (Oortmerssen 1976). Rectangular shapes have also been studied.

Results are used which are obtained by previous research (Vugts 1971), (Newman 1979).

4 Dynamic behaviour: theory

In the previous chapters literature is studied with the purpose to find a method wherein the dynamic behaviour of tunnel elements during the immersion operation can be determined. In this chapter the method is defined.

4.1 Equation of motion

Motions, such as translations or rotations, of the tunnel element and the pontoons can be described with a linear equation of motion, derived from Newton's Second Law. For translations (heave, sway or surge) the equation of motion holds (Journée and Massie 2000):

$$(M + A) \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = F$$

In which:

M = solid mass of the cylinder [kg]

A = hydrodynamic mass coefficient [kg]

C = hydrodynamic damping coefficient [kg/s]

K = restoring spring coefficient [kg/s²]

F = Force [N]

x = Displacement [m]

The terms $A \cdot \ddot{x}$ and $C \cdot \dot{x}$ are caused by the hydrodynamic reaction as a result of the movement of the cylinder with respect to the water.

4.1.1 Free decay test

The equation of motion which describes free decay of a cylinder in heaving is given by:

$$(M + A) \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = 0$$

When the cylinder is given an initial displacement at time $t = 0$ and thereafter is released, the cylinder oscillates in water and the motion can die out freely.

During the oscillation of the cylinder, waves are generated, which propagate from the cylinder. These waves transport energy, so they withdraw energy of the oscillated cylinder. As a result of this the motion of the cylinder will die out.

This so-called wave damping is proportional to the velocity $\dot{x}$ of the cylinder. The coefficient C is called wave or potential damping and has the dimension of mass per unit of time.

In an actual viscous fluid however, also other phenomena with respect to damping are present. Vortices and separation phenomena for example are mostly described in the equation of motion in non-linear contributions (Journée and Massie 2000).

The hydro mechanical part $A \cdot \ddot{x}$ is proportional to accelerations that are given to the water particles near the oscillated cylinder. This part of the hydro mechanical forces does not dissipate energy from the system.

The coefficient A is called hydrodynamic mass or added mass and has the dimension of mass.

When the system is linear the motions of the cylinder can be seen as a superposition of the motions of the cylinder oscillating in still water and the forces on the restrained cylinder in waves. This principle is shown in Figure 4-1.

The hydro mechanical parameters are determined by oscillating the cylinder in still water. The wave loads which result in forces on the cylinder are determined by restraining the cylinder in waves.

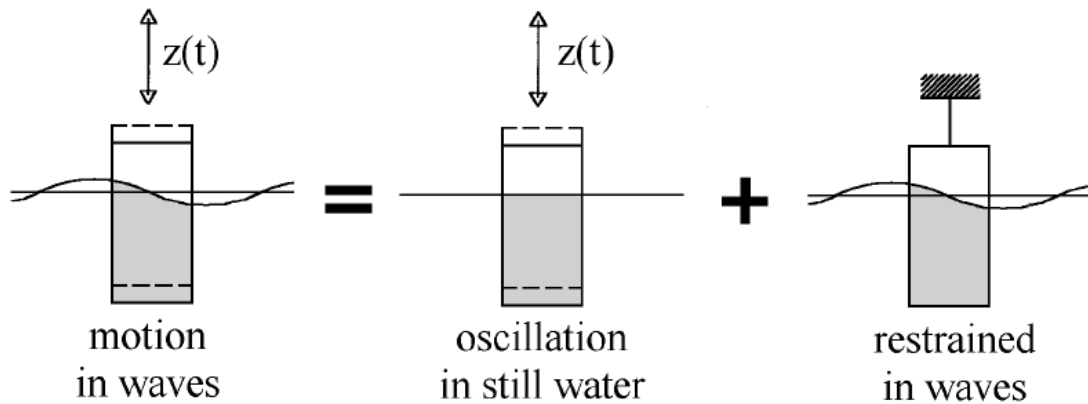


Figure 4-1: Superposition of hydro mechanical and wave loads

4.1.2 Frequency domain

Only linear contributions are taken into account in the equation of motion, which is given in paragraph 4.1. In reality however, also non-linear contributions are present. For example quadratic damping, forces and moments due to currents, wind, anchoring and second order wave loads have a non-linear contribution in the equation of motion.

Although the linear equation of motion does not describe reality very accurate, it has an advantage to use the linear equations of motion instead of an equation of motion with the added non-linear contributions. If the system is linear, then the behaviour of the system can be studied in the frequency domain. This means that, at each frequency, the different ratios between the motion amplitudes and the wave amplitudes are constant. Doubling the wave amplitude results in a doubled motion amplitude (Journée and Massie 2000).

4.1.3 Time domain

If the equations of motion are non-linear, then the superposition principle, which is used in the frequency domain, is no longer valid. However, it is possible to solve the non-linear equation of motion in the time-domain.

If the behaviour of a cylinder in irregular waves is analysed, all the possible wave combinations should be taken into account, which is time-consuming.

4.2 Waves

Ocean surface waves consist of wind waves and swell. Wind waves are irregular and are generated by the local wind field. High waves are followed unpredictably by low waves.

Swell waves are waves which are propagated out of the local wind field in which they are generated. Swell waves are no longer dependent on the wave field and can propagate for hundreds of

kilometres through areas where the wind is calm. Individual waves are more regular than wind waves. The wave height of swell waves is also more predictable.

The surface elevation can be described with a Normal Distribution. The wave amplitude (or the wave height) is given in a Rayleigh distribution.

The probability density function of the normal distribution is defined as (Journée and Massie 2000):

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\left(\frac{x}{\sigma \cdot \sqrt{2}}\right)^2\right)$$

The probability density function of the Rayleigh distribution is equal to:

$$f(x) = \frac{x}{\sigma^2} \cdot \exp\left(-\left(\frac{x}{\sigma \cdot \sqrt{2}}\right)^2\right)$$

In which x is the variable being studied and σ is its standard deviation.

4.2.1 Significant wave height

It is obvious to define the wave height H as the vertical distance between the highest and the lowest surface elevation in a wave. In a wave record with N waves, the mean wave height $\bar{H}$ is defined as (Holthuijsen 2007):

$$\bar{H} = \frac{1}{N} \cdot \sum_{i=1}^N H_i$$

Where i is the sequence number of the wave in the record.

The significant wave height is defined as the mean of the highest one-third of waves in the record:

$$H_{1/3} = \frac{1}{N/3} \cdot \sum_{j=1}^{N/3} H_j$$

Where j is the rank number of the wave, based on the wave height.

The significant wave height is close to the value of the visually estimated wave height. The significant wave height can also be estimated from the wave spectrum, which will be explained in paragraph 4.2.3.

4.2.2 Wave spectrum

The wave elevation can be seen as a superposition of many simple, regular harmonic wave components. Each single sine wave consists of an amplitude, length, period or frequency and direction of propagation, which is shown in Figure 4-2 (Holthuijsen 2007).

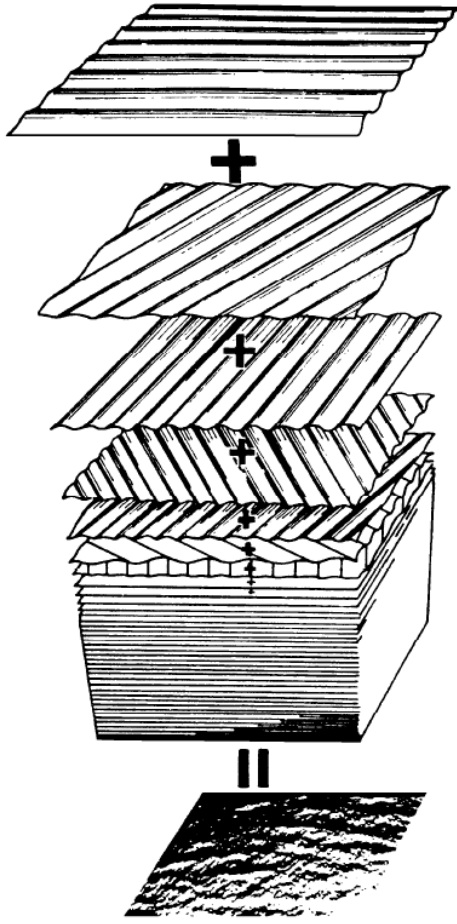


Figure 4-2: An irregular sea can be approximated by a sum of many simple sine waves

The wave elevation can be reproduced as the sum of a large number of harmonic wave components with a Fourier analysis. Therefore, one should have a time record segment, which is called N , which contains many waves (Journée and Massie 2000).

$$\zeta(t) = \sum_{n=1}^N \zeta_{an} \cdot \cos(k_n \cdot y - 2 \cdot \pi \cdot f_n \cdot t + \alpha_n)$$

ζ_{an} , α_n and k_n are respectively the amplitude, the phase and the wave number. f_n is the frequency of wave n . The record N is exactly reproduced by substituting these parameters in the equation above. If enough Fourier series terms are added the entire time record at that point can be reproduced using this set of values.

In practice, the exact water level at some time t is not of importance, because this is already history. More useful are the statistical properties in terms of frequency and amplitude.

The variance of the wave elevation is equal to:

$$\begin{aligned} \sigma_\zeta^2 &= \overline{\zeta^2} \\ &= \frac{1}{N} \cdot \sum_{n=1}^N \zeta_n^2 = \frac{1}{N \cdot \Delta t} \cdot \sum_{n=1}^N \zeta_n^2 \cdot \Delta t \\ &= \frac{1}{\tau} \cdot \int_0^\tau \zeta(t)^2 \cdot dt = \frac{1}{\tau} \cdot \int_0^\tau \left\{ \sum_{n=1}^N \zeta_{an} \cdot \cos(k_n \cdot y - 2 \cdot \pi \cdot f_n \cdot t + \alpha_n) \right\}^2 dt \end{aligned}$$

$$= \sum_{n=1}^N \frac{1}{2} \cdot \zeta_{an}^2$$

The wave amplitude can be expressed in a wave spectrum:

$$S_{\zeta}(\omega_n) \cdot \Delta\omega = \sum_{\omega_n}^{\omega_n + \Delta\omega} \frac{1}{2} \cdot \zeta_{an}(\omega)^2$$

The energy per unit of area of the waves is given when this expression is multiplied with $\rho \cdot g$. By letting $\Delta\omega \rightarrow 0$ the definition of the wave energy spectrum becomes:

$$S_{\zeta}(\omega_n) \cdot d\omega = \frac{1}{2} \cdot \zeta_{an}^2$$

The variance of the water surface elevation is equal to the area under the spectrum (Journée and Massie 2000).

$$\sigma_s^2 = \int_0^{\infty} S_{\zeta}(\omega_n) d\omega = m_0$$

4.2.3 Estimation significant wave height from a wave spectrum

The amplitude of waves are Rayleigh distributed. With $a = \frac{1}{2} \cdot H$ and $\sigma^2 = m_0$, the transferred Rayleigh distribution in terms of wave height is given by (Holthuijsen 2007):

$$p(H) = \frac{H}{4 \cdot m_0} \cdot \exp\left(-\frac{H^2}{8 \cdot m_0}\right)$$

The mean value of the highest one-third of the waves is defined as the significant wave height. This fraction can be determined from the Rayleigh distribution. The wave heights that are involved in this fraction are defined by:

$$\int_{H^*}^{\infty} p(H) dH = \int_{H^*}^{\infty} \frac{H}{4 \cdot m_0} \cdot \exp\left(-\frac{H^2}{8 \cdot m_0}\right) dH = \frac{1}{3}$$

Where H^* is equal to:

$$H^* = \sqrt{-8 \cdot \ln\left(\frac{1}{3}\right) \cdot m_0}$$

The mean values of this wave heights is by definition the significant wave height. The significant wave height can be determined as an expected value, with the zeroth- and the first order moments of the highest third of the Rayleigh distribution:

$$H_{m0} = \frac{\int_{H^*}^{\infty} H \cdot p(H) dH}{\int_{H^*}^{\infty} p(H) dH} = \frac{\int_{H^*}^{\infty} H \cdot p(H) dH}{\frac{1}{3}}$$

Substituting the expression for the Rayleigh distribution in the equation above, will result in:

$$H_{m0} = 4.004 \dots \cdot \sqrt{m_0}$$

Or, for all practical purposes (Holthuijsen 2007):

$$H_{m0} = 4 \cdot \sqrt{m_0}$$

So, the significant wave height can be estimated from a wave spectrum.

4.2.4 Waves in Korea

The waves in Korea can be described with a double peaked Jonswap spectrum, because two types of waves can be distinguished. These are swell waves and wind waves. The Jonswap (Joint North Sea Wave Project) spectrum describes the waves in the North Sea but also fits the situation in Korea.

The spectral density of the Jonswap spectrum at wave frequency ω is given in $S_\zeta(\omega)$.

$$S_\zeta(\omega) = \alpha \cdot g^2 \cdot \omega^{-5} \cdot \exp\left(-1.25 \cdot \left(\frac{\omega}{\omega_0}\right)^{-4}\right) \cdot \gamma^{\exp\left(\frac{(\omega-\omega_0)^2}{2 \cdot \sigma^2 \cdot \omega_0^2}\right)}$$

$$\sigma = \begin{cases} \sigma_a & \text{for } \omega \leq \omega_0 \\ \sigma_b & \text{for } \omega > \omega_0 \end{cases}$$

ω_0 is the peak frequency and the gravitational acceleration is given with g . γ , σ_a and σ_b are parameters of the Jonswap spectrum and are respectively 3.0, 0.07 and 0.09.

The factor α is chosen such, that the following relation is fulfilled:

$$H_{1/3} = 4 \cdot \int_0^\infty S_\zeta(\omega) d\omega = 4 \cdot \sqrt{m_0}$$

4.3 Wave loads

The wave force follows from the integration of the water pressure on the body in the undisturbed waves. This force is called the Froude-Krylov force.

The deepwater pressure, which is caused by waves is:

$$p = \rho \cdot g \cdot \zeta_a \cdot \exp(\kappa \cdot z) \cdot \cos(\omega \cdot t - \kappa \cdot y)$$

The Froude-Krylov force follows from the integration of the pressure over the body:

$$F_{FK} = \int_A p dA = \zeta_a \cdot \int_A \rho \cdot g \cdot \exp(\kappa \cdot z) \cdot \cos(\omega \cdot t - \kappa \cdot y) dA = \zeta_a \cdot \hat{F}_{FK}(\omega)$$

$$F_{FK} = \zeta_a \cdot \hat{F}_{FK}(\omega) \cdot \exp(i \cdot (\omega \cdot t - \kappa \cdot y))$$

Actually however, a part of the waves will be diffracted. This requires a correction of the Froude-Krylov-force. Using the relative motion principle, one finds additional force components. The diffraction can be described by:

$$F_D(\omega) = (-A(\omega) \cdot \omega^2 + C(\omega) \cdot i \cdot \omega) \cdot \zeta$$

The diffraction forces are corrections on the Froude-Krylov force due to diffraction of the waves by the presence of the body in the fluid. The water particle acceleration and velocity at an arbitrary depth is given by $\ddot{\zeta}$ and $\dot{\zeta}$.

The total wave force is equal to the Froude-Krylov force and the diffraction force:

$$F_W = F_{FK} + F_D$$

4.4 Response amplitude operator (RAO)

The equation of motion is given by:

$$(M + A) \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = F_{FK} + F_D$$

The wave elevation ζ is a sine function, so the response function x is also assumed to be a sine function. The complex response function is given by:

$$x = x_a \cdot \exp(i \cdot \omega \cdot t) \cdot \exp(-i \cdot \kappa \cdot y)$$

Also the wave elevation can be expressed in complex notation:

$$\zeta = \zeta_a \cdot \exp(\kappa \cdot z) \cdot \exp(i \cdot \omega \cdot t) \cdot \exp(-i \cdot \kappa \cdot y)$$

The motion of a cylinder in heave can be described with:

$$(M + A) \cdot \ddot{\zeta} + C \cdot \dot{\zeta} + K \cdot \zeta = F_{FK} + F_D = O \cdot \zeta^* + C \cdot \dot{\zeta}^* + A \cdot \ddot{\zeta}^*$$

Where O is the cross section of the cylinder.

When the diameter of the cylinder is assumed to be small, relative to the wave length ($k \cdot y \approx 0$).

Then, ζ^* is equal to:

$$\zeta^* = \zeta_a \cdot \exp(\kappa \cdot z) \cdot \exp(i \cdot \omega \cdot t)$$

The equation of motion can be rewritten in:

$$\begin{aligned} (K + i \cdot \omega \cdot C - \omega^2 \cdot (M + A)) \cdot x_a \cdot \exp(i \cdot \omega \cdot t) = \\ (O + i \cdot \omega \cdot C - \omega^2 \cdot A) \cdot \zeta_a \cdot \exp(\kappa \cdot z) \cdot \exp(i \cdot \omega \cdot t) \end{aligned}$$

$$RAO = \frac{x_a}{\zeta_a} = \exp(\kappa \cdot z) \cdot \frac{O + i \cdot \omega \cdot C - \omega^2 \cdot A}{K + i \cdot \omega \cdot C - \omega^2 \cdot (M + A)}$$

The response amplitude operator is defined as x_a/ζ_a . Doubling the wave amplitude results in a doubled motion amplitude.

4.5 Motion in irregular waves

The wave energy spectrum is derived in paragraph 4.2.2 and is equal to:

$$S_\zeta(\omega) \cdot d\omega = \frac{1}{2} \cdot \zeta_a^2(\omega)$$

Analogous to this, the energy spectrum of a response can be defined:

$$\begin{aligned} S_u(\omega) \cdot d\omega &= \frac{1}{2} \cdot x_a^2(\omega) \\ &= \left| \frac{x_a}{\zeta_a}(\omega) \right|^2 \cdot \frac{1}{2} \cdot \zeta_a^2(\omega) \\ &= \left| \frac{x_a}{\zeta_a}(\omega) \right|^2 \cdot S_\zeta(\omega) \cdot d\omega \end{aligned}$$

The response spectrum of a motion can be found by using the transfer function of the motion and the wave spectrum (Journée and Massie 2000).

$$S_u(\omega) = \left| \frac{x_a}{\zeta_a}(\omega) \right|^2 \cdot S_\zeta(\omega_n)$$

5 Natural frequencies

In this chapter the natural frequencies are determined. The aim of this analysis is to compare the frequencies of the waves with the natural frequencies of the system. Resonance can occur when one or more natural frequencies of the system are equal or close to the frequency of the load.

The response of the system is not determined in this chapter. Actually, the response of the system during resonance is not of great importance, because resonance need to be avoided anyway.

5.1 Approach

The theory to determine the natural frequencies and response of the tunnel elements during immersion is described in chapter 4. This theory is used in this chapter to determine the natural frequencies.

This has been done using the following steps:

Schematisation of the immersion system into a model. This is done in paragraph 5.2.

Determining the natural frequencies with a simple estimation (paragraph 5.3). Only one degree of freedom is studied per calculation, which means that the motions are decoupled. Only restoring properties of the water and structural mass contributions are used. Added mass is not taken into account in these simple hand calculations.

Determining the natural frequencies with a coupled model (paragraph 5.4). In this model the stiffness of the cables is taken into account. A Maple-sheet is used to analyse the natural frequencies. Added mass contributions are not taken into account in this step.

Including of added mass contributions at the coupled model (paragraph 5.5). The natural frequencies are determined with added mass contributions included.

5.2 Schematisations

In this thesis is focused on a water depth of 23 m. and the position of the tunnel element at 1 m. below water surface. The wave forces have a maximum influence when the tunnel element is 1 meter below water surface. This position of the tunnel element is shown in Figure 5-1.

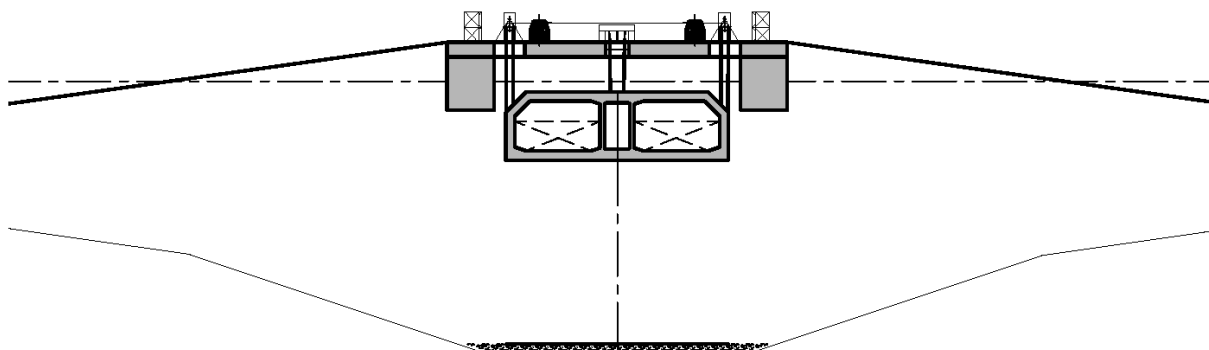


Figure 5-1: Tunnel element at 1 meter below water surface

Motions are restricted when the tunnel element is close to the bottom. This position of the tunnel element is shown in Figure 5-2.

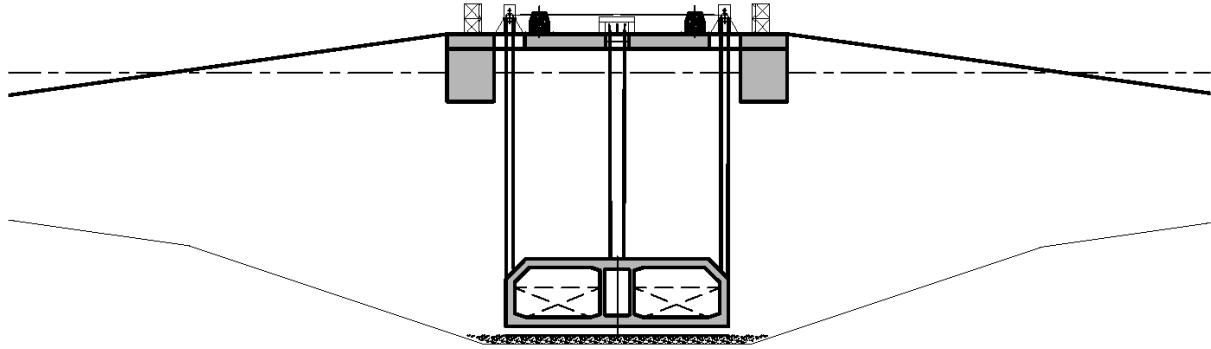


Figure 5-2: Tunnel element at 0.5 meter above the bottom

The symbols, which represents different lengths are shown in Figure 5-3.

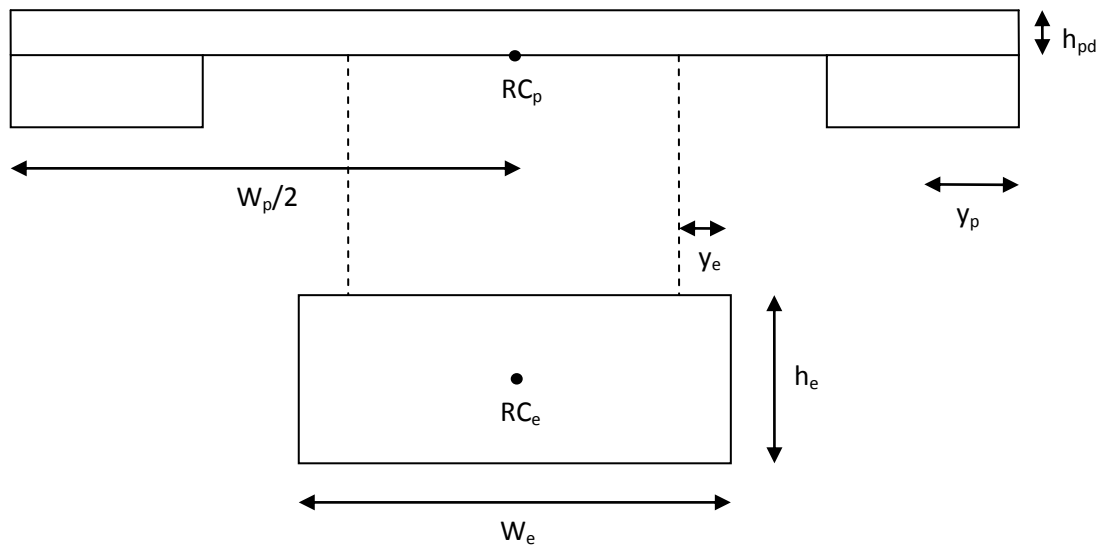


Figure 5-3: Symbols with respect to length

The width, W_e , and height, h_e , of the tunnel element are equal to respectively 26,46 m and 9,97 m. The width of the pontoon, W_p is equal to 42,5 m and the height of the pontoon deck, h_{pd} , is 2,5 m. The distance measured from the edge of the pontoon to the place where the resulting reaction force of the water is located is called y_p and is equal to 3 m. The distance measured from the edge of the tunnel element to the place where the suspension cables are mounted is called y_e and is equal to 0,665 m.

The degrees of freedom are shown in Figure 5-4. These degrees of freedom are taken into account in the model.

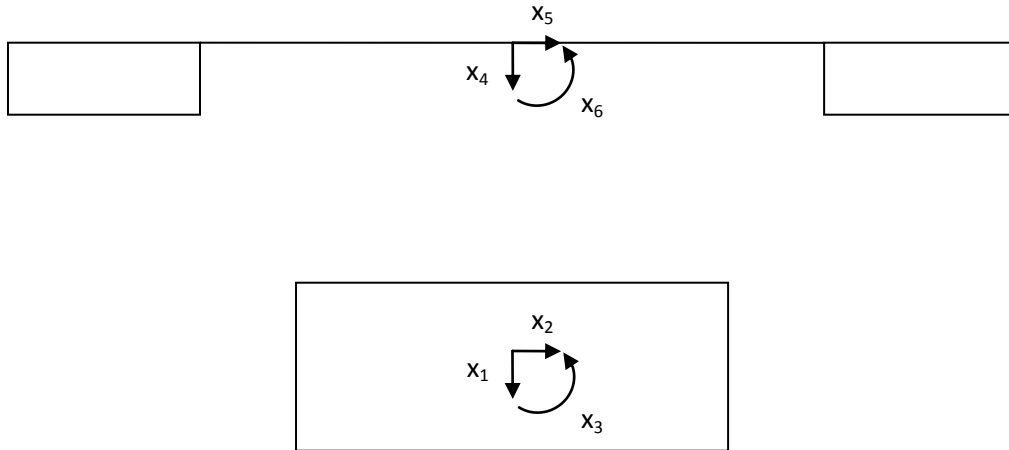


Figure 5-4: Degrees of Freedom

The names of the translations and rotations are defined in Figure 5-5.

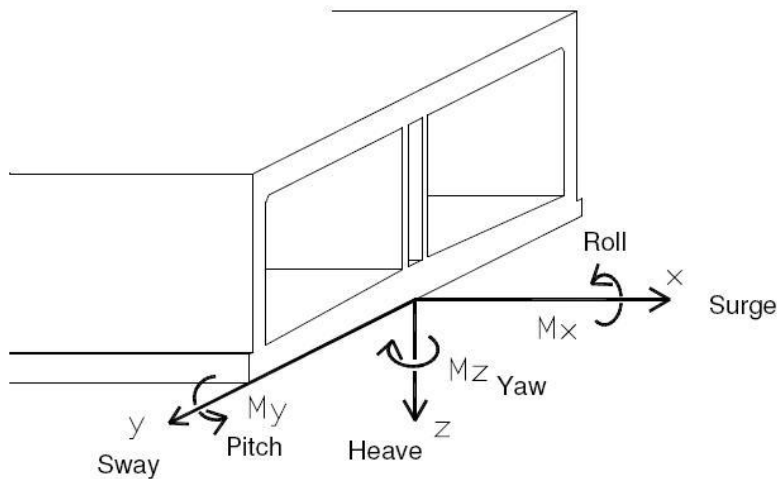


Figure 5-5: Six Degrees of Freedom

The parameters for the model are derived and given in Appendix A. These includes for example the derivation of the spring stiffness of the cables.

5.3 Estimation natural periods

A first step in the determination of the natural frequencies and periods of the motions is to estimate them with a very simple hand calculation. In a later stage of the calculation process the results of the more extensive analyse can be compared with the results which are obtained from this calculation.

In this simple calculation, only the mass of the tunnel element and the restoring coefficient of the water is taken into account. Forces, added mass and damping are in this analysis ignored.

The first natural period which is estimated is shown in Figure 5-6. This is the vertical motion of the tunnel element.

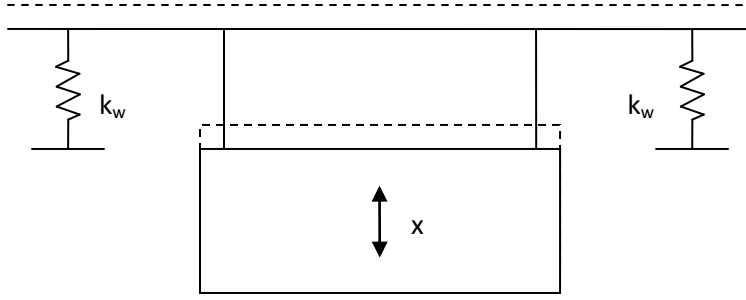


Figure 5-6: Vertical motion of the tunnel element

The spring stiffness in Figure 5-6 is equal to:

$$k = 2 \cdot k_w = 2 \cdot 4343868 = 8687736 \text{ N/m}$$

The natural period is equal to:

$$T_0 = \frac{2 \cdot \pi}{\omega_0} = \frac{2 \cdot \pi}{\sqrt{\frac{k}{m}}} = \frac{2 \cdot \pi}{\sqrt{\frac{8687736}{49650000}}} = 15 \text{ s}$$

The rotation of the tunnel element is shown in Figure 5-7.

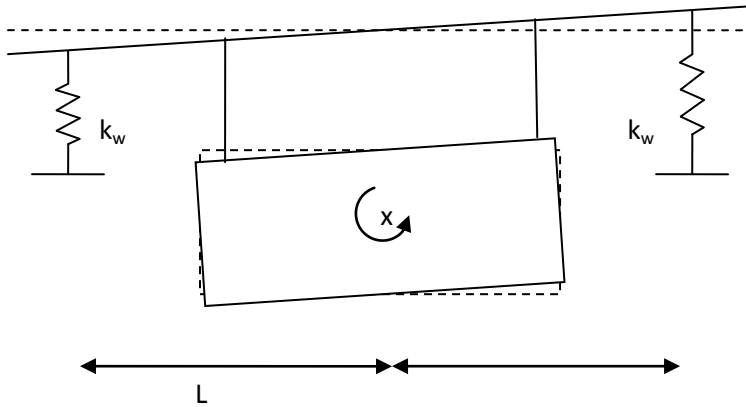


Figure 5-7: Rotation of the tunnel element

The rotation stiffness is equal to:

$$k_\varphi = 2 \cdot k_w \cdot L^2 = 2 \cdot k_w \cdot \left(\frac{W_p}{2} - y_e \right)^2 = 2 \cdot 4340000 \cdot \left(\frac{42.5}{2} - 3 \right)^2 \rightarrow k_\varphi = 2.89 \cdot 10^9 \text{ Nm}$$

The natural period of rotation of the tunnel element is equal to:

$$T_0 = \frac{2 \cdot \pi}{\omega} = \frac{2 \cdot \pi}{\sqrt{\frac{k_\varphi}{J_e}}} = \frac{2 \cdot \pi}{\sqrt{\frac{2.89 \cdot 10^9}{3.74 \cdot 10^9}}} = 7.1 \text{ s}$$

The vertical motion of the pontoon is shown in Figure 5-8.

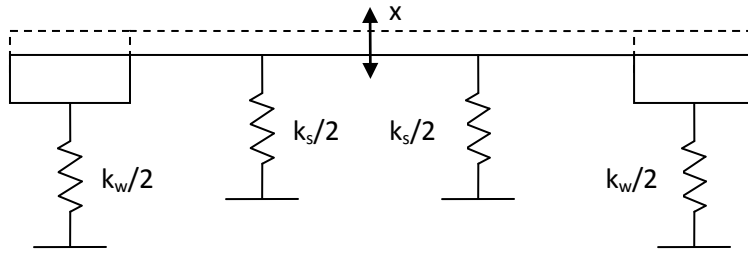


Figure 5-8: Vertical motion of the pontoon

The pontoon is connected to the tunnel element by the suspension cables.

The stiffness of the springs is equal to:

$$k = 2 \cdot \frac{k_s}{2} + 2 \cdot \frac{k_w}{2} = 563000000 + 4340000 = 5.67 \cdot 10^8 \text{ N/m}$$

The natural period of the vertical motion of the pontoon becomes:

$$T_0 = \frac{2 \cdot \pi}{\omega} = \frac{2 \cdot \pi}{\sqrt{\frac{5.67 \cdot 10^8}{1400000}}} = 0.32 \text{ s}$$

The rotation of the pontoon is shown in Figure 5-9.

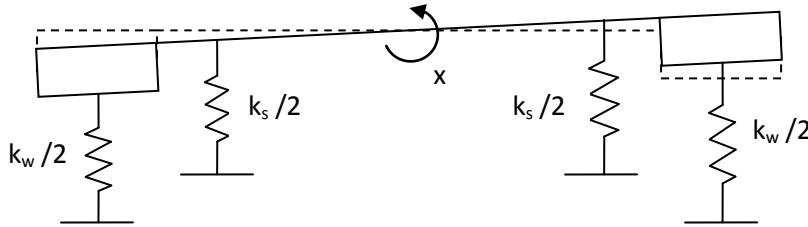


Figure 5-9: Rotation of the pontoon

The rotation stiffness of the pontoon is equal to:

$$k_\varphi = 2 \cdot \frac{k_w}{2} \cdot \left(\frac{W_p}{2} - y_p\right)^2 + 2 \cdot \frac{k_s}{2} \cdot \left(\frac{W_e}{2} - y_e\right)^2 =$$

$$4340000 \cdot \left(\frac{42.5}{2} - 3\right)^2 + 563000000 \cdot \left(\frac{26.46}{2} - 0.665\right)^2 = 9.03 \cdot 10^{10} \text{ Nm}$$

The natural period of the rotation of the pontoon is:

$$T_0 = \frac{2 \cdot \pi}{\omega} = \frac{2 \cdot \pi}{\sqrt{\frac{k_\varphi}{J_p}}} = \frac{2 \cdot \pi}{\sqrt{\frac{9.03 \cdot 10^{10}}{2.75 \cdot 10^8}}} = 0.35 \text{ sec}$$

5.4 Natural frequencies coupled model without added mass and damping

The next step in the analysis is to take the stiffness of the entire system into account. The latter consists of the stiffness of the water and the stiffness of the cables.

The equations of motion are derived with Newton's second Law. The forces on the system are derived with the displacement method and are given in Appendix B. The equations of motions are linear. This means that the non-linear effects are neglected.

A flexible structure in nodal coordinates is represented by the following second-order differential equation (Gawronski 2004):

$$M \cdot \ddot{\underline{x}} + C \cdot \dot{\underline{x}} + K \cdot \underline{x} = \underline{F}$$

In this equation $\underline{x}$ is the displacement vector, $\dot{\underline{x}}$ is the velocity vector and $\ddot{\underline{x}}$ is the acceleration vector. The mass matrix is given by M , C is the damping matrix and K is the stiffness matrix. The forces are given in vector $\underline{F}$.

5.4.1 Analysis of the natural periods

The theory to determine the natural frequencies of a system with linear equations of motions is described in Appendix C.1. The equations of motion which govern small vibrations are derived in Appendix B.

The mass matrix and the stiffness matrix are derived from the equations of motions (see Appendix B). The mass matrix is defined as:

$$M = \begin{bmatrix} m_e & 0 & 0 & 0 & 0 & 0 \\ 0 & m_e & 0 & 0 & 0 & 0 \\ 0 & 0 & J_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \cdot m_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \cdot m_p & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \cdot J_p \end{bmatrix}$$

When the parameters in the mass matrix and stiffness matrix are changed with numerical values, the following matrices are obtained (see Appendix B):

$$M = \begin{bmatrix} 4.965 \cdot 10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.965 \cdot 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.74 \cdot 10^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.8 \cdot 10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.8 \cdot 10^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.5 \cdot 10^8 \end{bmatrix}$$

$$K = \begin{bmatrix} 1.129 \cdot 10^9 & 0 & 0 & -1.13 \cdot 10^9 & 0 & 0 \\ 0 & 4.272 \cdot 10^6 & -3.644 \cdot 10^7 & 0 & 0 & 0 \\ 0 & -3.644 \cdot 10^7 & 1.782 \cdot 10^{11} & 0 & 0 & -1.779 \cdot 10^{11} \\ -1.13 \cdot 10^9 & 0 & 0 & 1.14 \cdot 10^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.741 \cdot 10^6 & -1.713 \cdot 10^7 \\ 0 & 0 & -1.779 \cdot 10^{11} & 0 & -1.713 \cdot 10^7 & 1.809 \cdot 10^{11} \end{bmatrix}$$

The natural frequencies can be obtained by calculating the determinant of $(K - \omega^2 \cdot M) = 0$, in which ω^2 is the Eigenvalue and ω is the natural frequency. This condition leads to a polynomial of degree n in ω^2 , which is called the characteristic polynomial.

For systems with more than two degrees of freedom, it is possible to formulate the characteristic equation by hand, but the determination of the roots is often not possible without numerical

methods. Therefore, a computer program is used for finding the solution of the Eigenvalue problem for systems with more than two degrees of freedom (Spijkers, Vrouwenfelder and Klaver 2006).

The matrix of natural frequencies is defined by:

$$\Omega = \begin{bmatrix} 20.728 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.414 & 0 & 0 & 0 & 0 \\ 0 & 0 & 19.383 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.277 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.030 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.831 \end{bmatrix}$$

The natural periods, which are equal to $(2 \cdot \pi)/\omega_i$ are given by:

$$T_0 = \begin{bmatrix} 0.303 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.167 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.324 & 0 & 0 & 0 \\ 0 & 0 & 0 & 22.707 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.101 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7.559 \end{bmatrix}$$

The mode shapes, which are given in the Eigenmatrix are:

$$E = \begin{bmatrix} 0.056 & 0.710 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.000 & 0.996 & -0.010 & 0.058 \\ 0 & 0 & 0.143 & 0.013 & 0.014 & -0.048 \\ -0.998 & 0.704 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.016 & 0.086 & -0.999 & -0.996 \\ 0 & 0 & -0.990 & 0.013 & 0.013 & 0.047 \end{bmatrix}$$

In this analysis, the entire stiffness of the system is taken into account. The motions are now coupled. Natural frequencies, which are obtained by this analysis, are almost equal to the estimated frequencies which are given in paragraph 5.3.

In the estimation of the heave and roll motion of the tunnel element, the assumption is made that the stiffness of the suspension cables is infinite stiff. In the extensive model the stiffness of the cables is approached by calculating them.

As mentioned before, the results of both computations are quite comparable. The influence of the suspension cables on the accuracy of the results is analysed in paragraph 5.4.3.

5.4.2 Validation equations of motion

The static equilibrium is determined to validate the equations of motion. The corresponding maple file is given in Appendix F.

The equation of motion of the completely submerged tunnel element and the pontoons in vertical direction (heave) is derived by adding all the forces which act on the tunnel element and the pontoons:

$$m_e \cdot \ddot{x} + 2 \cdot m_p \cdot \ddot{x} = m_e \cdot g + 2 \cdot m_p \cdot g - F_b - F_{HD} - 2 \cdot k_w \cdot x$$

Where m_e is the mass of the tunnel element, m_p is the mass of the pontoon, x is the displacement, $\ddot{x}$ is the acceleration, F_b is the buoyancy force, F_{HD} are the hydrodynamic forces and the restoring coefficient is given by k_w .

The connection between the tunnel element and the pontoons is assumed to be infinite stiff in this paragraph. Therefore, there is one equation of motion to describe the heave motion of the combined system.

The displacement is given by a static part and a dynamic component for each degree of freedom. This is for the heave motion of the element and the pontoons given by:

$$x(t) = z_0 + z_{dyn}(t)$$

The displacement is substituted in the equations of motion:

$$m_e \cdot \ddot{z}_{dyn} + 2 \cdot m_p \cdot \ddot{z}_{dyn} = m_e \cdot g + 2 \cdot m_p \cdot g - F_b - F_{HD} - 2 \cdot k_w \cdot z_0 - 2 \cdot k_w \cdot z_{dyn}$$

This can be rewritten into:

$$m_e \cdot \ddot{z}_{dyn} + 2 \cdot m_p \cdot \ddot{z}_{dyn} = -F_{HD} - 2 \cdot k_w \cdot z_{dyn}$$

Where:

$$2 \cdot k_w \cdot z_0 = m_e \cdot g + 2 \cdot m_p \cdot g - F_b$$

The static displacement of the combined system can be found by:

$$z_0 = \frac{m_e \cdot g + 2 \cdot m_p \cdot g - F_b}{2 \cdot k_w}$$

In reality, the static displacement of the tunnel element and the pontoons are not exactly the same, because the connection between tunnel element and pontoons is not infinite stiff.

The contributions $m_e \cdot g$, F_b are added by the equation of motion which describes heave of the tunnel element. $2 \cdot m_p \cdot g$ is added by the equation of motion which describes the heave motion of the pontoons. Thereafter, twelve initial conditions are defined. For each equation of motion the initial displacement and velocity is given at time $t = 0$.

The initial displacements are set at $x_i = 0.1$. The initial velocities are set at $\dot{x}_i = 0$.

When the initial displacements and velocities are equal to zero, some degrees of freedom are already in equilibrium. Therefore, all the initial displacements are set at $x_i = 0.1 \text{ m}$.

Damping is applied to determine the steady-state solution. The static displacement is given by the steady state solution and is a straight line, because the transient motions are died out.

The amount of damping is not of importance as long as c is greater than 0.

The equations of motion are validated with this method. One should expect that the static displacement of both heave motions is greater than 0. The static displacement of the sway motions and the rotations are expected to be exactly 0.

5.4.3 Parametric study influence stiffness suspension cables

The stiffness of the suspension cables is determined as 563.000 kN/m .

The influence of this model parameter can be validated by varying it. Therefore a second analysis is performed. In this calculation the natural frequencies are re-calculated with a lower stiffness of the suspension cables of 100.000 kN/m .

The results of this analysis are:

The matrix of natural frequencies is:

$$\Omega = \begin{bmatrix} 8.95 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.407 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.406 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.275 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.022 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.817 \end{bmatrix}$$

The natural periods are:

$$T_0 = \begin{bmatrix} 0.702 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.450 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.747 & 0 & 0 & 0 \\ 0 & 0 & 0 & 22.807 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.148 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7.689 \end{bmatrix}$$

The Eigenmatrix is:

$$E = \begin{bmatrix} 0.054 & 0.723 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.001 & 0.996 & 0.009 & -0.070 \\ 0 & 0 & 0.134 & 0.014 & -0.012 & 0.055 \\ -0.999 & 0.691 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.087 & 0.086 & 1 & 0.995 \\ 0 & 0 & -0.987 & 0.013 & -0.011 & 0.051 \end{bmatrix}$$

One can see that only two frequencies are significantly changed when the frequencies are compared with the ones which are obtained in paragraph 5.3. The changed frequencies are related to the heave and roll motion of the pontoons (respectively ω_{11} and ω_{33}).

The natural heave and roll frequencies of the pontoon are dominated by the stiffness of the suspension cables, which is shown in Figure 5-8 and Figure 5-9. Therefore it is expected that these frequencies decrease when the stiffness of the cables is reduced.

The natural periods of the heave and roll motion are not changed. This means that the stiffness of the suspension cables have a negligible influence on these frequencies.

This can be explained by calculating the stiffness of the element in heave motion. The springs are attached in series to the tunnel element, which can be seen in Figure 5-6.

The stiffness of the suspension cables is equal to 563.000 kN/m which is more stiff with respect to the restoring spring coefficient of the water (4340 kN/m). Therefore, the spring stiffness of the tunnel element in heave motion is equal to:

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_w} \approx \frac{1}{\infty} + \frac{1}{k_w} \rightarrow k = k_w$$

The natural frequencies of the heave and roll motion of the element can be determined accurate, because these motions are not affected by the stiffness of the cables. The only parameters which are relevant are the restoring spring stiffness of the water and inertia properties of the tunnel element.

The natural periods of the tunnel element, which are estimated in paragraph 5.3, are in fact accurate values.

The influence of the suspension cables on the motions of the tunnel element is small. The forces in the cables are a function of the stiffness and the elongation. Larger dynamic forces in the cables are introduced when the stiffness of the cables is enlarged.

5.4.4 Non-linear contributions of the cables to the equation of motion

The equations of motion which are used are linear, which means that all non-linear terms are neglected.

In reality the suspension cables affect the horizontal motions between the tunnel element and the pontoons.

The suspension cables are elongated when the tunnel element and the pontoon have a difference in horizontal shift between each other. The elongation will result in a force in the suspension cables, which can be divided in a horizontal and a vertical component.

In the linear equations of motion only the vertical forces in the suspension cables are taken into account. It is possible to take the horizontal reaction forces also into account by adding non-linear contributions in the equation of motion.

This equations are given in Appendix B.5 but are not solved.

5.5 Natural frequencies coupled model with added mass

The equation of motion, which describes the motions of a body oscillating in water, consist partially of hydrodynamic terms.

$$(M + A) \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = F$$

The term $A(\omega)$ is the added mass and $C(\omega)$ is the hydrodynamic damping. Both parameters are frequency dependent.

5.5.1 Added mass

The added mass coefficients should be determined for the heave, sway and roll motion of the tunnel element and the pontoons.

One can see in Figure 5-10 the added mass coefficients for a rectangular cylinder in heave motion, determined by Vugts. These coefficients are determined for floating bodies. Vugts calculated the hydrodynamic coefficients for rectangular cross-sections with a strip theory (Vugts 1971).

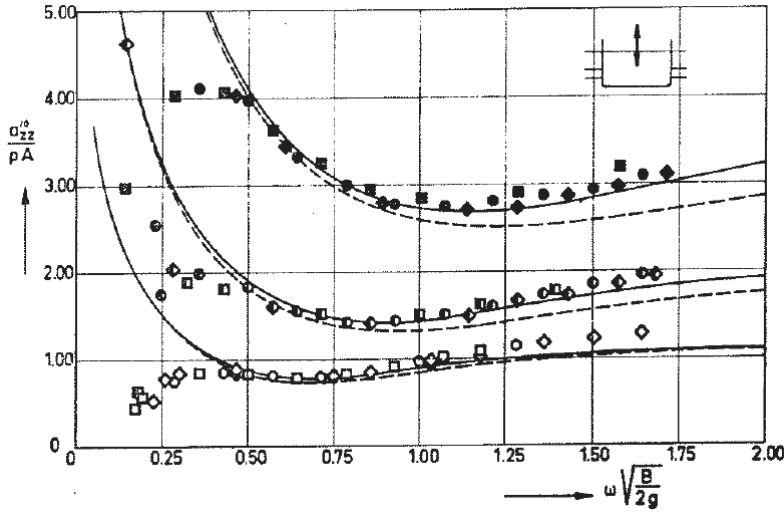


Figure 5-10: Added mass coefficients for heave motions

The added mass coefficient for a tunnel element in heaving is estimated with the graph of Vugts, despite the fact that the tunnel element is submerged instead of floating.

The graph gives an approximation of the coefficients. Better values of the added mass coefficients can be obtained by model tests or extensive computer simulations. Computer software should take a lot of factors into account such as the shape of the tunnel element, the influence of the bottom and the frequency of the oscillation.

The breadth over draught ratio (B/T) of the body of 8 is represented by the upper line in Figure 5-10. A B/T ratio of 4 is given in the central line and a B/T ratio of 2 is represented by the bottom line. The tunnel element has a B/T ratio of 2.6.

The added mass coefficients are given per unit length. The small figures in the graph represent results of model proofs which are performed by Vugts.

The added mass coefficient with a B/T ratio of 2 reaches asymptotic a value of approximately 1. The heave frequency of the tunnel element without added mass is equal to 0.414 rad/s:

$$\omega \cdot \sqrt{\frac{B}{2 \cdot g}} = 0.414 \cdot \sqrt{\frac{26.46}{2 \cdot 9.81}} = 0.48$$

The following relation holds, according to Figure 5-10:

$$\frac{a'_{zz}}{\rho \cdot A} \approx 1$$

The added mass of the tunnel element in heaving is approximately the submerged volume of the tunnel element multiplied with the density of the water.

$$a_{11} = 1 \cdot A \cdot L \cdot \rho = 1 \cdot 9.97 \cdot 26.46 \cdot 180 \cdot 1025 = 4.87 \cdot 10^7 \text{ kg}$$

A similar graph is available for sway motions, which is given in Figure 5-11.

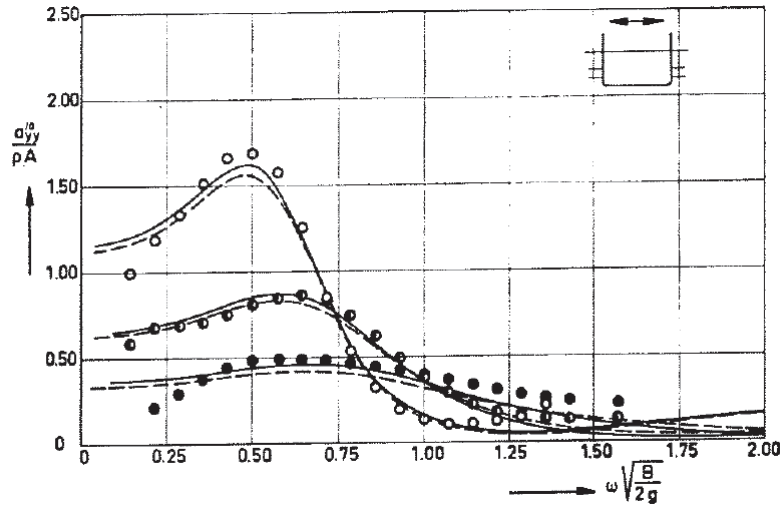


Figure 5-11: Added mass coefficients for sway motions

One can see that at high frequencies the added mass coefficient for sway reaches zero. In this graph, the B/T ratio of 8 is the bottom line. A B/T ratio of 2 is shown in the upper line and the central line represents a B/T ratio of 4.

The sway frequency of the tunnel element is equal to 0.227 rad/s, which results in:

$$0.227 \cdot \sqrt{\frac{26.46}{2 \cdot 9.81}} = 0.32$$

The sway added mass coefficient for the tunnel element is equal to:

$$\frac{a'_{yy}}{\rho \cdot A} \approx 1.25$$

The added mass for the sway motion of the tunnel element is approximately:

$$a_{22} = \rho \cdot A \cdot L = 1.25 \cdot 9.97 \cdot 26.46 \cdot 180 \cdot 1025 = 6.08 \cdot 10^7 \text{ kg}$$

Added mass coefficients for roll motions are also studied by Vugts. These results are shown in Figure 5-12.

The symbols represents different fixed roll angles. The little squares represent a roll angle of 0.20, the circles represent an angle of 0.10 and the triangles represent a fixed roll angle of 0.05.

The roll frequency is equal to 0.831 rad/s.

$$0.831 \cdot \sqrt{\frac{26.46}{2 \cdot 9.81}} = 0.97$$

The added inertia coefficient for the roll motion is given by:

$$0.025 = \frac{a'_{\varphi\varphi}}{\rho \cdot A \cdot B^2}$$

The roll added inertia is equal to:

$$a_{33} = 0.025 \cdot \rho \cdot A \cdot B^2 \cdot L = 0.025 \cdot 1025 \cdot 9.97 \cdot 26.46 \cdot 180 \cdot 26.46^2 = 8.52 \cdot 10^8 \text{ kgm}^2$$

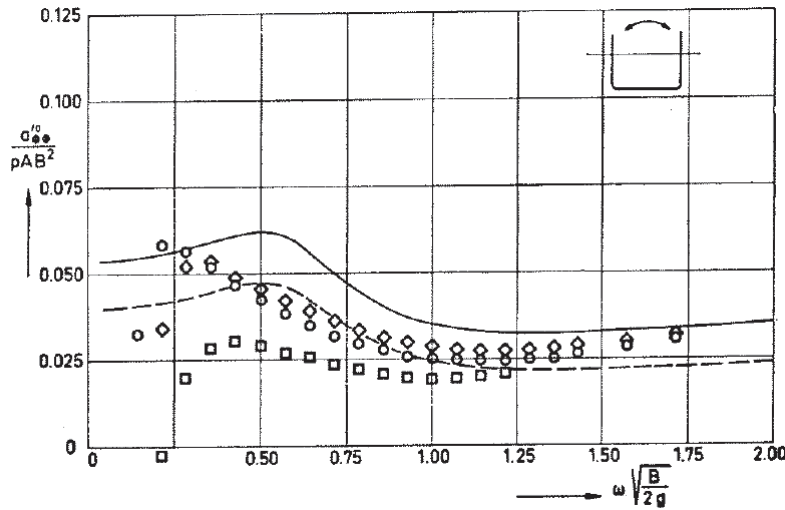


Figure 5-12: Added mass coefficients for roll motions

The motions of the pontoons are also affected by the added mass. The first step to determine the added mass coefficients for the pontoons is to calculate the B/T ratio.

The draft of the floaters can be determined by dividing the forces, which act downwards on the pontoon, by the restoring spring stiffness of the pontoon.

The draught of the floaters is:

$$dr_p = T = \frac{F}{k} = \frac{2 \cdot 1400 \cdot 1000 \cdot 9.81 + 0.02 \cdot 180 \cdot 26.46 \cdot 9.97 \cdot 1025 \cdot 9.81}{2 \cdot 4343868} = 4.26 \text{ m}$$

The width of one floater is equal to 6 m, which gives a B/T ratio of 1.3. The lines which represent a B/T ratio of 2 are used to determine the added mass coefficients.

Added mass coefficients for the pontoon motions are estimated with the graphs which are obtained by Vugts. The added mass coefficients for heave motions are shown in Figure 5-10.

The heave frequency of the tunnel element is 20.73 rad/s:

$$20.73 \cdot \sqrt{\frac{6}{2 \cdot 9.81}} = 11.46$$

This value is not shown in the graph. Since the line in Figure 5-10 is asymptotic and converges, a value of 1 is used:

$$\frac{a'_{zz}}{\rho \cdot A} \approx 1$$

The heave added mass for one pontoon consists of the added mass of two floaters and is equal to:

$$a_{44} = 2 \cdot 1 \cdot \rho \cdot T \cdot B \cdot L = 2 \cdot 1025 \cdot 4.26 \cdot 6 \cdot 36 = 1.89 \cdot 10^6 \text{ kg}$$

The sway added mass coefficients are shown in Figure 5-11. The corresponding frequency of the pontoon is equal to 1.03 rad/s:

$$1.03 \cdot \sqrt{\frac{6}{2 \cdot 9.81}} = 0.57$$

In the graph in Figure 5-11, one can see that at 0.57 the added mass coefficient is approximately 1.25.

The added mass for the pontoons in sway motion is:

$$a_{55} = 2 \cdot 1.25 \cdot \rho \cdot T \cdot B \cdot L = 2 \cdot 1.25 \cdot 1025 \cdot 4.28 \cdot 6 \cdot 36 = 2.37 \cdot 10^6 \text{ kg}$$

The roll motion of the pontoon can be described as two cylinders in heave motion, which oscillates opposite to each other. The floaters itself do not roll.

The added inertia coefficients of the roll motion can be determined by multiplying the added mass coefficients of the heave motion with the distance measured from the rotation centre of the pontoon to the point where the reaction force of the water acts, which is shown in Figure 5-3.

The roll frequency of the pontoons is equal to 19.4 rad/s:

$$19.4 \cdot \sqrt{\frac{6}{2 \cdot 9.81}} = 10.73$$

The corresponding added mass coefficient is approximately 1, according to Figure 5-10. The added mass for one floater becomes:

$$\frac{a}{\rho \cdot A} \approx 1$$

$$a = 1 \cdot \rho \cdot d r_p \cdot B \cdot L = 1 \cdot 1025 \cdot 4.28 \cdot 6 \cdot 36 = 9.45 \cdot 10^5 \text{ kg}$$

$$a_{66} = 2 \cdot \left(\frac{W_p}{2} - y_p \right)^2 \cdot a = 2 \cdot \left(\frac{42.5}{2} - 3 \right)^2 \cdot 9.45 \cdot 10^5 = 6.32 \cdot 10^8 \text{ kg m}^2$$

The added mass matrix A is assembled by substituting the previous determined values in matrix A. One can see in the added mass matrix that only the diagonal terms are non-zero. In reality the cross-coupling coefficients such as a_{12} and a_{23} are non-zero, implying that the hydrodynamic force differs in direction from the acceleration (Newman 1977).

Vugts calculated also coupling coefficients of sway into roll and roll into sway (Vugts 1971). These coefficients are small in comparison with the diagonal coefficients and are therefore neglected.

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \cdot a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \cdot a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \cdot a_{66} \end{bmatrix}$$

$$A = \begin{bmatrix} 4.9 \cdot 10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6.1 \cdot 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.52 \cdot 10^8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.79 \cdot 10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.74 \cdot 10^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.26 \cdot 10^9 \end{bmatrix}$$

5.5.2 Natural periods with added mass

The method which is given in Appendix C.1 is used to determine the natural frequencies of the system.

$$\det(K - \omega^2 \cdot (M + A)) = 0$$

The matrix of natural frequencies is:

$$\Omega = \begin{bmatrix} 13.576 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.293 & 0 & 0 & 0 & 0 \\ 0 & 0 & 11.748 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.186 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.741 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.576 \end{bmatrix}$$

The natural frequencies are changed, because the added mass is taken into account. Therefore a second iteration is done to determine new added mass values.

The heave frequency of the tunnel element is 0.293 rad/s

$$0.293 \cdot \sqrt{\frac{26.46}{2 \cdot 9.81}} = 0.34$$

The graph in Figure 5-10 gives still a value of 1, so this added mass coefficient for the heave motion of the tunnel element is unchanged.

The frequency of the sway motion is changed to 0.186 rad/s.

$$0.186 \cdot \sqrt{\frac{26.46}{2 \cdot 9.81}} = 0.22$$

Figure 5-11 gives a value of 1.25 which is the same as the value which is determined earlier.

The frequency of the roll motion of the tunnel element is equal to 0.567 rad/s.

$$0.576 \cdot \sqrt{\frac{26.46}{2 \cdot 9.81}} = 0.97$$

The graph in Figure 5-12 gives a value of 0.035, which is different than the value of 0.025, which is found earlier.

The added mass for the roll motion is changed in:

$$a_{33} = 0.035 \cdot \rho \cdot A \cdot B^2 \cdot L = 0.035 \cdot 1025 \cdot 9.97 \cdot 26.46 \cdot 180 \cdot 26.46^2 = 1.19 \cdot 10^9 \text{ kgm}^2$$

The heave frequency of the pontoons is equal to 13.57 rad/s.

$$13.57 \cdot \sqrt{\frac{6}{2 \cdot 9.81}} = 7.50$$

Figure 5-10 gives a value of 1, which is equal to the value which was found earlier.

The frequency of the sway motion of the pontoon is equal to 0.74 rad/s.

$$0.74 \cdot \sqrt{\frac{6}{2 \cdot 9.81}} = 0.41$$

The graph in Figure 5-11 gives a value of 1.5, which is more than the value which is determined earlier.

$$a_{55} = 2 \cdot 1.5 \cdot \rho \cdot T \cdot B \cdot L = 2 \cdot 1.5 \cdot 1025 \cdot 4.28 \cdot 6 \cdot 36 = 2.84 \cdot 10^6 \text{ kg}$$

The frequency of the roll motion of the pontoon is equal to 6.47 rad/s.

$$11.7 \cdot \sqrt{\frac{6}{2 \cdot 9.81}} = 6.47$$

The graph in Figure 5-10 gives a value of 1. This means that the added mass for this motion is unchanged.

The new added mass matrix is equal to:

$$A = \begin{bmatrix} 4.9 \cdot 10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6.1 \cdot 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.2 \cdot 10^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.8 \cdot 10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7 \cdot 10^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.26 \cdot 10^9 \end{bmatrix}$$

The matrix of natural frequencies is:

$$\Omega = \begin{bmatrix} 13.576 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.293 & 0 & 0 & 0 & 0 \\ 0 & 0 & 11.635 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.186 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.719 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.545 \end{bmatrix}$$

The natural periods are:

$$T_0 = \begin{bmatrix} 0.462 & 0 & 0 & 0 & 0 & 0 \\ 0 & 21.450 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.540 & 0 & 0 & 0 \\ 0 & 0 & 0 & 33.807 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.470 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11.52 \end{bmatrix}$$

The Eigenmatrix is:

$$E = \begin{bmatrix} 0.066 & 0.710 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.001 & 0.996 & 0.066 & -0.016 \\ 0 & 0 & 0.341 & 0.012 & -0.096 & 0.013 \\ -0.998 & 0.704 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.014 & 0.086 & 0.989 & 1 \\ 0 & 0 & -0.940 & 0.012 & -0.095 & 0.013 \end{bmatrix}$$

A third iteration is also performed, but the added mass values are during this process unchanged.

5.6 Conclusions natural frequencies

The natural frequencies and periods of the roll and heave motions without added mass, can be determined relative accurate. These frequencies are dependent of the restoring coefficient of the water and inertia properties of the system. These parameters can be determined precisely, which results in natural frequencies which has to be accurate as well.

When added mass contributions are included, the accuracy of the results decreases. The method which is used to determine the added mass values is rough, which results in uncertainty in the results.

The influence of the stiffness of the suspension cables on the roll and heave frequencies is negligible small. The natural frequencies obtained by the estimation and the natural frequencies determined by the coupled model are almost equal to each other. From the stiffness contributions only the restoring coefficient of the water is of importance for the heave and roll motions.

6 Response amplitude operators

In this chapter the response of the system to wave loads is determined. Results are presented in response amplitude operators (RAO). In dynamics, these functions are better known as frequency response functions (FRF), where the ratio between the dynamic deflection over the static deflection is determined.

6.1 Approach

In chapter 5, the structural mass matrix M , added mass matrix A and stiffness matrix K are derived. To determine the response of the system damping parameters (C) and force contributions (F) have to be added to the model.

The response of the system is determined using the following steps:

Determining of the damping values. Damping values are only of importance for motions which have a frequency close or equal to one of the natural frequencies of the system. This is described in paragraph 6.2.

Determining of the force contributions. For the force contribution only the Froude-Krylov is taken into account. The Froude-Krylov Force is the integration of the undisturbed wave pressure over the tunnel surface. Diffraction forces are not taken into account. This is described in paragraph 6.3.

Determining of the response amplitude operators. All the necessary contributions are determined. The structural mass matrix M , added mass matrix A and stiffness matrix K are already derived in chapter 5.

6.2 Damping

The tunnel element and the pontoons are during the motion affected by damping. Damping can be defined as the dissipation of vibration energy from the system.

In principle, there are two ways in a system where energy may disappear from a vibration. The first one is conversion into heat and the second way is emission to the surroundings.

The energy dissipation is a complex phenomenon in which a large number of dissipation mechanisms are involved. Usually the damping which is applied in a model is a simple approximation of a much more complex reality.

Constant damping implies that the damping is proportional to the velocity:

$$F_d = c \cdot \dot{u}$$

This damping force is added by the equation of motion and approximates the energy dissipation in the system.

The reason that the damping force is often described as $F_d = c \cdot \dot{u}$, is that it leads to a linear differential equation, which can be solved easy.

In case of quadratic damping the damping force is dependent of the square of the velocity:

$$F_d = \frac{1}{2} \cdot c \cdot \dot{u} \cdot |\dot{u}|$$

Often, quadratic damping is expressed to an equivalent damping factor, because then a linear mathematical model is obtained, which has mathematical advantages.

When for example a 1-DOF mass-spring-damper system is loaded with a harmonic load, than the approximation of the damping constant needs to be accurate for frequencies in the vicinity of the natural frequency. In this cases damping becomes important, because mass terms and spring terms neutralize each other. This is shown in Figure 6-1.

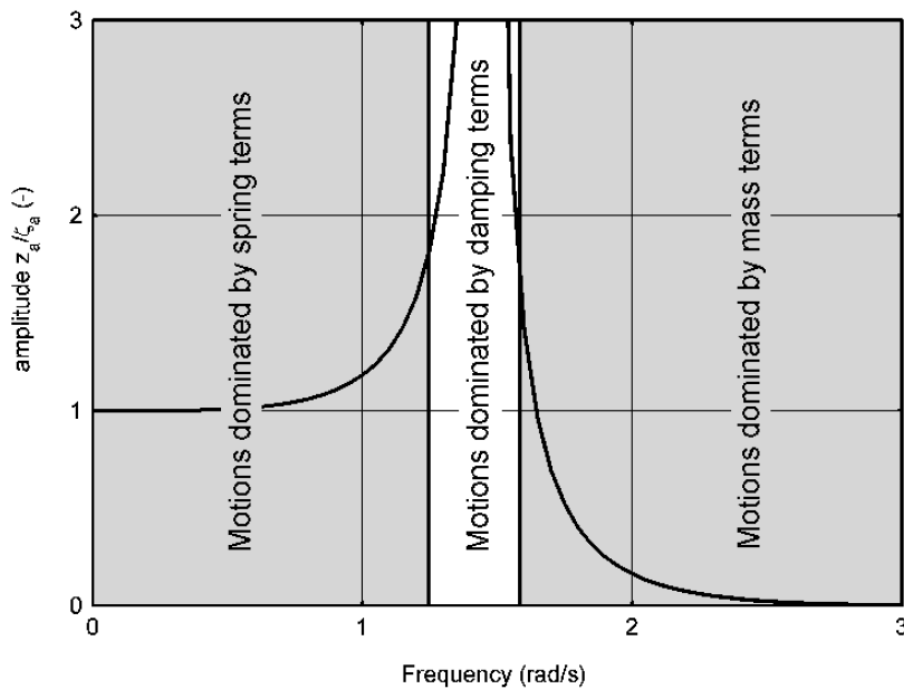


Figure 6-1: Frequency areas with respect to motional behaviour

6.2.1 Damping in the system

In this thesis several damping mechanisms are involved. The two most important damping mechanisms are potential (or wave) damping and viscous damping.

Potential damping is caused by the generated waves which dissipate energy from the moving system. This damping mechanism can be described with a linear contribution in the equation of motion.

Viscous damping consists of viscous effects, such as skin friction or vortices. In the equation of motion these contributions are often described as quadratic terms and need to be linearised when solved in the frequency domain.

In most cases viscous effects are neglected in motion calculations of offshore structures. The major part of the damping is caused by potential damping. Viscous damping contribution are in these cases of minor importance (Journée and Massie 2000).

However, there are examples where the viscous damping components can be significant for a certain motion. For example viscous damping can be significant for rolling ships. This is because a circular cylinder rotating about its centre does not generate waves. So, the potential damping in this case is relative small. The viscous damping contributions can be significant due to the presence of for example bilge keels. In this example the viscous damping contributions are relative large compared with the potential damping contributions (Journée and Massie 2000).

In this thesis it is questionable whether potential damping is of major importance compared with the viscous damping.

It is assumed that the tunnel element itself, while oscillating does not generate waves at the water surface. So the potential damping in this case is relative small. The viscous damping contributions are of major importance for the tunnel element.

The pontoons generate waves when they oscillate. Here, the potential damping components are of major importance.

The damping in this thesis consists of many mechanisms that dissipate energy from the system, that it is almost impossible to calculate or simulate it with a computer model sufficiently accurate. An extra factor which makes it hard to determine the parameters is that the motions are coupled. This is one of the reasons why often model tests are performed in a water basin.

The damping parameters are not analysed in this project. It is too time consuming to determine these coefficients sufficiently accurate.

The damping in a system is described as a percentage of the critical damping. This percentage is called the damping ratio:

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2 \cdot \sqrt{k \cdot m}}$$

In which the critical damping is defined as (Spijkers, Vrouwenvelder and Klaver 2006):

$$c_{cr} = 2 \cdot \sqrt{k \cdot m}$$

Four damping ratios are analysed, $\zeta = 0$, $\zeta = 0.01$, $\zeta = 0.03$ and $\zeta = 0.05$. The first damping ratio is the undamped situation. The colours of the lines which represents the different damping ratios in the graphs are respectively black, red, blue and green.

It is assumed that the damping is governed by this values. Nevertheless, it is strongly recommended that this damping ratios are analysed in another project. One way to determine these parameters is to perform model tests in a water basin.

6.2.2 Damping in an N-DOF system

If the response of an N-DOF system is analysed, damping has to be taken into account. In almost all cases the orthogonal relation is not valid for the damping matrix. This means that the equations of motion are coupled which leads to time-integration.

In practice the response is determined with the assumption that the system is uncoupled. For each mode a modal damping factor is defined. The estimation of this values is problematic when the system has various damping mechanisms. Often, the modal damping forces commonly used for the

single degree of freedom system are assumed and estimated conservatively. Exact estimation of the damping, when relatively small, is not of great importance, because resonances needed to be avoided anyway (Spijkers, Vrouwenvelder and Klaver 2006).

6.3 Froude Krylov force

The Froude Krylov force follows from an integration of the pressures on the body in the undisturbed wave (Journée and Massie 2000).

6.3.1 Trajectories

Water particles carries out an oscillation in the y - and z - directions due to passing waves. The trajectories of water particles are ellipses in general case. For long waves in shallow water the trajectories are shown in Figure 6-2 (Journée and Massie 2000).

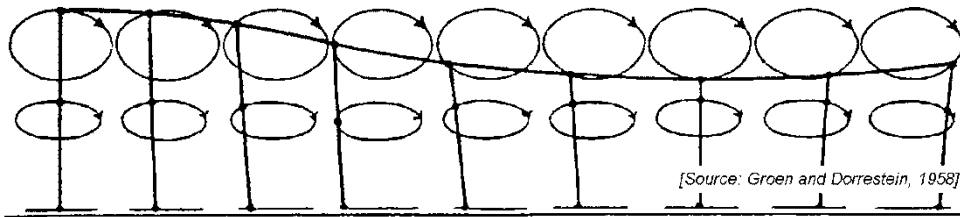


Figure 6-2: Trajectories of water particles in long or shallow water waves

The trajectories for short waves in deep water are shown in Figure 6-3.

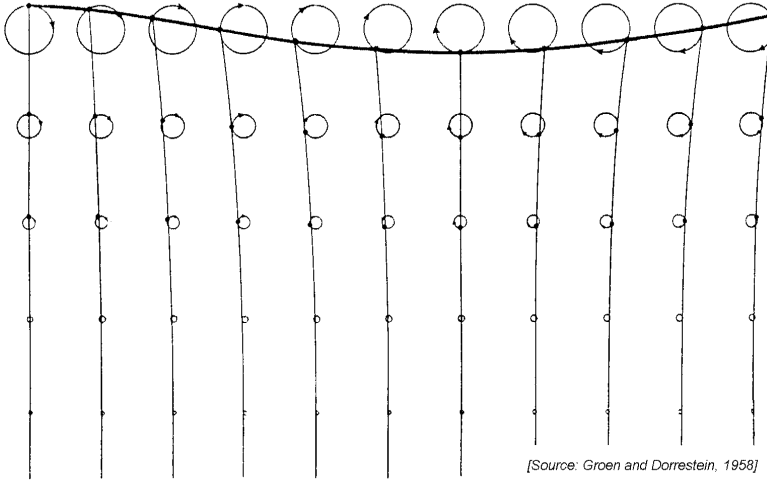


Figure 6-3: Trajectories of water particles in short or deep water waves

6.3.2 Forces

The wave-induced pressures are used to determine the force on the tunnel element and the pontoons.

The wave pressure at an arbitrary place in y and z direction and at time t is equal to:

$$p = \frac{H}{2} \cdot \rho \cdot g \cdot \frac{\cosh(\kappa \cdot (h + z))}{\cosh(\kappa \cdot h)} \cdot \cos(\omega \cdot t - \kappa \cdot y)$$

Or, in complex notation:

$$p = \frac{H}{2} \cdot \rho \cdot g \cdot \frac{\cosh(\kappa \cdot (h + z))}{\cosh(\kappa \cdot h)} \cdot \exp(i \cdot \omega \cdot t) \cdot \exp(-i \cdot \kappa \cdot y)$$

The wave pressures which act on the tunnel element are shown in Figure 6-4.

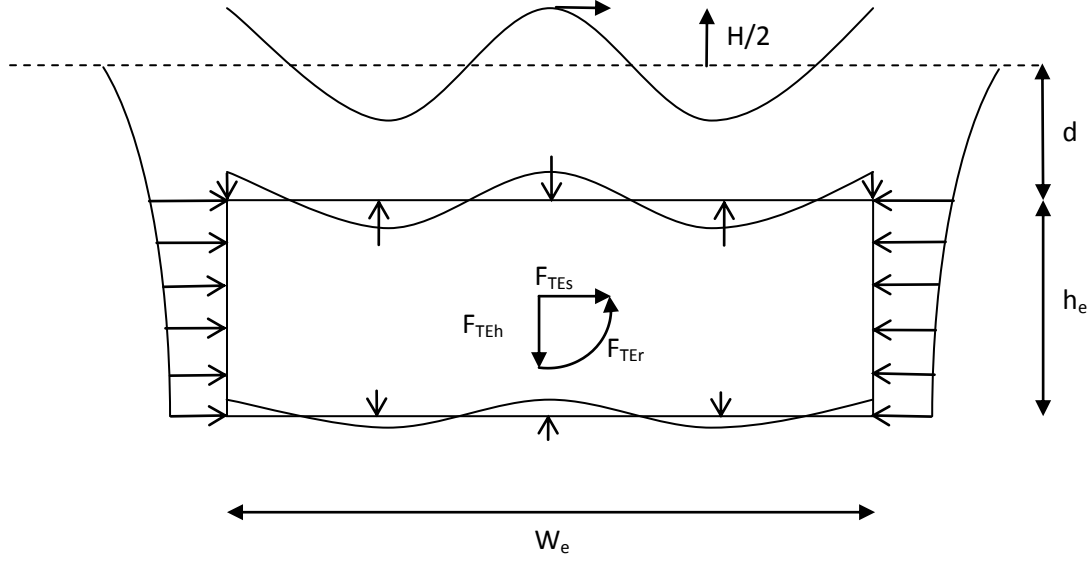


Figure 6-4: Wave pressures on tunnel element

The wave forces and moments follow from an integration of the pressure p over the submerged surface S of the body:

$$\vec{F} = - \iint_{dS} p \cdot \vec{n} dS$$

$$\vec{M} = - \iint_{dS} p \cdot (\vec{r} \cdot \vec{n}) dS$$

In which $\vec{n}$ is the outward normal vector on surface dS and $\vec{r}$ is the position vector of surface dS in the coordinate system (Journée and Massie 2000).

6.3.3 Tunnel element heave

The heave force on the tunnel element can be determined by:

$$F_{TEh} = \rho \cdot g \cdot \frac{H}{2} \cdot L_e \cdot \left(-\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} + \frac{\cosh(\kappa \cdot (h - (d + h_e)))}{\cosh(\kappa \cdot h)} \right) \cdot \int_{-\frac{W_e}{2}}^{\frac{W_e}{2}} \cos(\omega \cdot t - \kappa \cdot y) dy$$

Or, in complex notation:

$$\begin{aligned} F_{TEh} = & \rho \cdot g \cdot \frac{H}{2} \cdot L_e \cdot \left(-\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} + \frac{\cosh(\kappa \cdot (h - (d + h_e)))}{\cosh(\kappa \cdot h)} \right) \cdot \\ & \int_{-\frac{W_e}{2}}^{\frac{W_e}{2}} \exp(i \cdot \omega \cdot t) \cdot \exp(-i \cdot \kappa \cdot y) dy \end{aligned}$$

The vertical axis in the coordinate system is positive in downwards direction, see Figure 5-4. Therefore, a positive amplitude or translation is downwards directed.

When these equations are integrated, one can see that the wave force is dependent of the wave height H , the wave number κ , the time t and the frequency ω . All other parameters in the equations are constants.

The equations are solved in the frequency domain, so the time related parts can be removed from the equations. The wave height is given in the wave spectrum. This means for the equations above that the part $H/2$ can be removed.

The wave number κ is related to the frequency by the dispersion relationship:

$$\omega^2 = \kappa \cdot g \cdot \tanh(\kappa \cdot h)$$

This means that the wave force is only dependent of the wave frequency ω .

6.3.4 Tunnel element sway

The sway force which act on the tunnel element can be determined on a similar way.

The wave induced pressure is equal to:

$$p = \rho \cdot g \cdot \frac{H}{2} \cdot \frac{\cosh\left(\kappa \cdot \left(h - d - \frac{h_e}{2} + z\right)\right)}{\cosh(\kappa \cdot h)} \cdot \exp(i \cdot \omega \cdot t) \cdot \exp(-i \cdot \kappa \cdot y)$$

The sway, or horizontal, force follows from the integration of the pressure over the tunnel height.

$$F_{TEs} = \rho \cdot g \cdot \frac{H}{2} \cdot L_e \cdot \exp(i \cdot \omega \cdot t) \cdot \left(\exp\left(i \cdot \kappa \cdot \frac{W_e}{2}\right) - \exp\left(-i \cdot \kappa \cdot \frac{W_e}{2}\right) \right) \cdot \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \frac{\cosh\left(\kappa \cdot \left(h - d - \frac{h_e}{2} + z\right)\right)}{\cosh(\kappa \cdot h)} dz$$

6.3.5 Tunnel element roll

The roll force follows from the integration of the pressure multiplied with the distance vector.

$$F_{TEr} = \rho \cdot g \cdot \frac{H}{2} \cdot L_e \cdot \left(\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} - \frac{\cosh(\kappa \cdot (h - (d + h_e)))}{\cosh(\kappa \cdot h)} \right) \cdot \int_{-\frac{W_e}{2}}^{\frac{W_e}{2}} y \cdot \exp(i \cdot \omega \cdot t) \cdot \exp(-i \cdot \kappa \cdot y) dy - \rho \cdot g \cdot \frac{H}{2} \cdot L_e \cdot \exp(i \cdot \omega \cdot t) \cdot \left(\exp\left(i \cdot \kappa \cdot \frac{W_e}{2}\right) - \exp\left(-i \cdot \kappa \cdot \frac{W_e}{2}\right) \right) \cdot \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} z \cdot \frac{\cosh\left(\kappa \cdot \left(h - d - \frac{h_e}{2} + z\right)\right)}{\cosh(\kappa \cdot h)} dz$$

6.3.6 Froude Krylov forces on tunnel element

The Froude Krylov forces which act on the tunnel element are shown in Figure 6-5. The red line represents the force in vertical direction (heave), the green line represents the horizontal force (sway) and the moment (roll) is given with the blue line.

The x-axis of the graph represents the frequency ω of the wave. The y-axis of the graphs gives the maximum force per wave amplitude ζ in Newton (N).

The graph in Figure 6-5 gives the maximum forces for a wave amplitude which is 1 m high ($H = 2$ m). When for example the wave height doubles, the forces will double too.

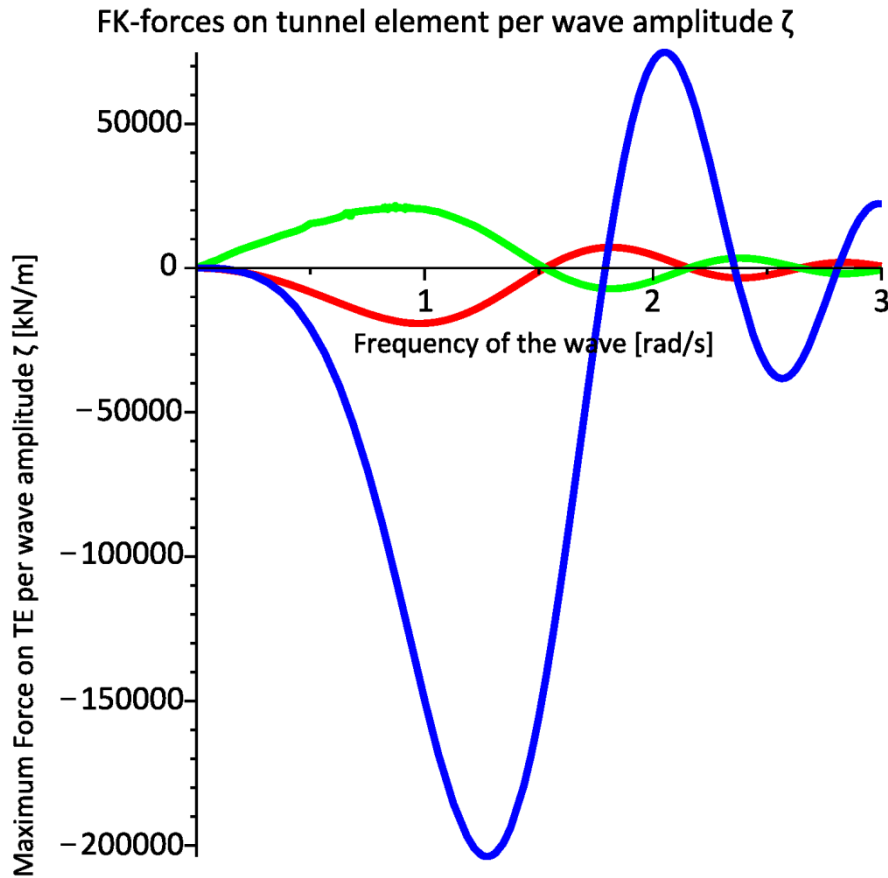


Figure 6-5: Froude Krylov Forces on Tunnel Element. Red=horizontal force, green=vertical force, blue=moment

6.3.7 Pontoon heave

The Froude Krylov force which act on the pontoon is determined by adding the forces which act on the floaters of the pontoon.

The first step is to determine the draft of the pontoon. The floaters of the pontoons carry the dead load of the pontoons itself and 2% overweight of the tunnel element.

$$dr_p = \frac{F}{k} = \frac{(2 \cdot m_p + 0.02 \cdot m_e) \cdot g}{2 \cdot k_w}$$

The pressures which act on the pontoons are given in Figure 6-6.

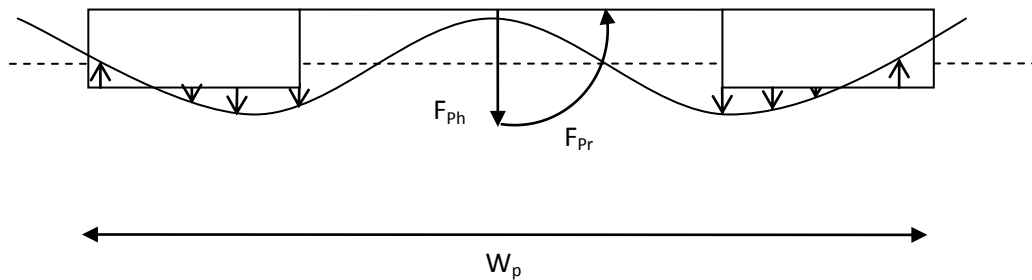


Figure 6-6: Wave pressures on pontoon

The total heave force which act on the pontoons is equal to:

$$F_{ph} = \rho \cdot g \cdot \frac{H}{2} \cdot 2 \cdot L_p \cdot \left(\frac{\cosh(\kappa \cdot (h - dr_p))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(i \cdot \omega \cdot t) \cdot \left(\int_{-\frac{W_p}{2}}^{-\frac{W_p}{2} + 2 \cdot y_p} \exp(-i \cdot \kappa \cdot y) dy + \int_{\frac{W_p}{2} - 2 \cdot y_p}^{\frac{W_p}{2}} \exp(-i \cdot \kappa \cdot y) dy \right)$$

6.3.8 Pontoon sway

The horizontal force which act on the pontoon does not influence the motions of the tunnel element. This can be seen in the Eigenmatrices which are given in paragraph 5.4.1 and paragraph 5.5.2.

Also, a good approximation of the force is hard to determine. The approach which is used to determine the sway force for the tunnel element is not valid for this situation, because it is not valid for points which are above the still water line ($z > 0$).

6.3.9 Pontoon roll

Only the heave-contribution is taken into account by the roll moment. The sway force is not determined for the pontoon and gives therefore also no contribution to the wave induced moment which act on the pontoons.

$$F_{pr} = -\rho \cdot g \cdot \frac{H}{2} \cdot 2 \cdot L_p \cdot \left(\frac{\cosh(\kappa \cdot (h - dr_p))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(i \cdot \omega \cdot t) \cdot \left(\int_{-\frac{W_p}{2}}^{-\frac{W_p}{2} + 2 \cdot y_p} y \cdot \exp(-i \cdot \kappa \cdot y) dy + \int_{\frac{W_p}{2} - 2 \cdot y_p}^{\frac{W_p}{2}} y \cdot \exp(-i \cdot \kappa \cdot y) dy \right)$$

6.3.10 Froude Krylov forces on pontoons

The Froude Krylov which acts on the pontoons are shown in Figure 6-7. The x-axis represents the wave frequency and the y-axis gives the forces in Newton per wave amplitude ζ . This is explained in paragraph 6.3.6.

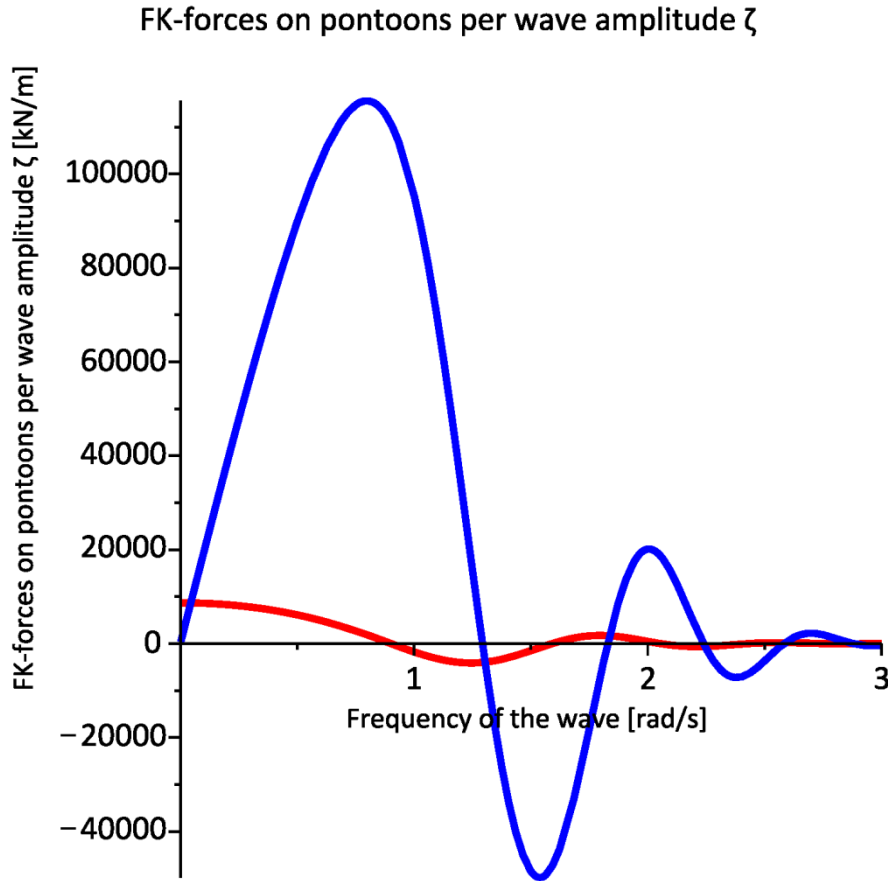


Figure 6-7: Froude Krylov Forces on pontoons. Red=horizontal force, blue=moment

6.3.11 Diffraction force

A part of the waves will be diffracted, requiring a correction of the Froude-Krylov force. One finds the additional components by using the relative motion principle. One component is proportional to the acceleration of the water particles and one is proportional to the velocity of the water particles (Journée and Massie 2000). The total force on the tunnel element is the Froude-Krylov Force and the diffraction force, which is given by:

$$F = F_{FK} + A(\omega) \cdot \ddot{\zeta} + C(\omega) \cdot \dot{\zeta}$$

The diffraction force consists of added mass coefficients, damping coefficients multiplied with respectively accelerations and velocities of water particles.

The acceleration of a water particle is equal to (Journée and Massie 2000):

$$\dot{u} = +\zeta_a \cdot \omega^2 \cdot \frac{\cosh(\kappa \cdot (h + z))}{\sinh(\kappa \cdot h)} \cdot \sin(\omega \cdot t - \kappa \cdot y)$$

$$\dot{w} = -\zeta_a \cdot \omega^2 \cdot \frac{\sinh(\kappa \cdot (h + z))}{\sinh(\kappa \cdot h)} \cdot \cos(\omega \cdot t - \kappa \cdot y)$$

Generally it can be said that the diffraction part of the total force is small for waves with low frequencies (long waves). At higher frequencies there is an influence of diffraction on the wave force. In other words, diffraction forces can be significant when the sectional dimensions of the tunnel element are a substantial fraction of the wavelength.

The diffraction force is not taken into account in this report.

6.4 Definition response amplitude operators

The response of the system can be calculated by determining the response amplitude operators (RAO).

The equation of motion is given by:

$$(M + A) \cdot \ddot{\underline{x}} + C \cdot \dot{\underline{x}} + K \cdot \underline{x} = \underline{F}(t)$$

The load function and the response function are given by respectively:

$$\underline{F}(t) = \underline{\hat{F}} \cdot \exp(i \cdot \omega \cdot t)$$

$$\underline{x}(t) = \underline{\hat{x}} \cdot \exp(i \cdot \omega \cdot t)$$

Substituting of the load function and the response function in the equation of motion gives:

$$(-\omega^2 \cdot (M + A) \cdot \underline{\hat{x}} + i \cdot \omega \cdot C \cdot \underline{\hat{x}} + K \cdot \underline{\hat{x}}) \cdot \exp(i \cdot \omega \cdot t) = \underline{\hat{F}} \cdot \exp(i \cdot \omega \cdot t)$$

This can simplified into:

$$(-\omega^2 \cdot (M + A) + i \cdot \omega \cdot C + K) \cdot \underline{\hat{x}} = \underline{\hat{F}}$$

The response amplitude operator is defined as:

$$\underline{\hat{x}} = (-\omega^2 \cdot (M + A) + i \cdot \omega \cdot C + K)^{-1} \cdot \underline{\hat{F}}$$

This analysis is very simple to understand and to perform. The disadvantage of this method is that the inverse operation in the previous equation is hard to calculate for systems with many degrees of freedom.

When this strategy is used some strange phenomena occurs in the output of the analysis. The computer software which is used (Maple) is apparently unable to determine the inverse in a good way. This is an essential step in the solving strategy and it lead to problems in the output of the results (see Appendix E). It is not analysed why Maple cannot handle with this command. It is assumed that another software package, for example Matlab, is able to solve the problem.

Therefore the Modal Analysis is used to determine the amplitudes. This method is explained in Appendix C.3.2.

6.5 Results

The response amplitude operators give information about the influence of waves per frequency. These graphs are given in Figure 6-8 to Figure 6-13. Different damping ratios are represented by the lines in different colours. The line in black is the solution without damping ($\zeta = 0$). The lines in red, blue and green represent a modal damping ratio of respectively $\zeta = 0.01$, $\zeta = 0.03$ and $\zeta = 0.05$.

In Figure 6-8 and Figure 6-9 it can be seen that the response amplitude operators of both heave motions are almost equal to each other. This is caused by the relative stiff connection between the tunnel element and the pontoons by the suspension cables.

The response amplitude operators of the roll motions of the tunnel element and the pontoons are also almost identical to each other, see Figure 6-12 and Figure 6-13. This is also caused by the stiff connection between the tunnel element and the pontoons.

Different response amplitude operators are obtained at the sway motions. This is caused by the fact that there is no direct (relative stiff) connection in horizontal direction between the tunnel element and the pontoons. The response operators of the sway motions are given in Figure 6-10 and Figure 6-11.

The system can be divided in two separate systems which can be seen in the Eigenmatrix (see paragraph 5.5.2). The heave motions are a separate system and the sway and roll motions are a separate system.

These systems have respectively two degrees of freedom and four degrees of freedom. A result for the response amplitude operators is that the graphs will have respectively a maximum amount of two and four resonance peaks.

The graphs which represents the response amplitude operator of the heave motion of the tunnel element and the pontoons will have a maximum amount of resonance peaks of two. The graphs which represent the roll and sway motions of the tunnel element and the pontoon will have a maximum amount of resonance peaks of four.

6.5.1 RAO of heave motions

The response amplitude operators which show the heave motions of the tunnel element and the pontoons are given in respectively Figure 6-8 and Figure 6-9.

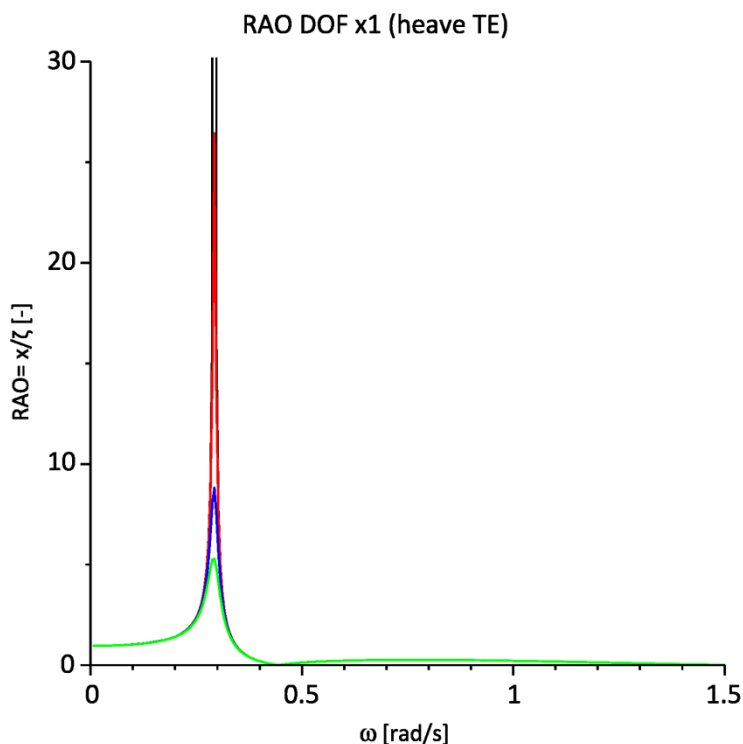


Figure 6-8: Response amplitude operator of degree of freedom x1 (heave tunnel element)

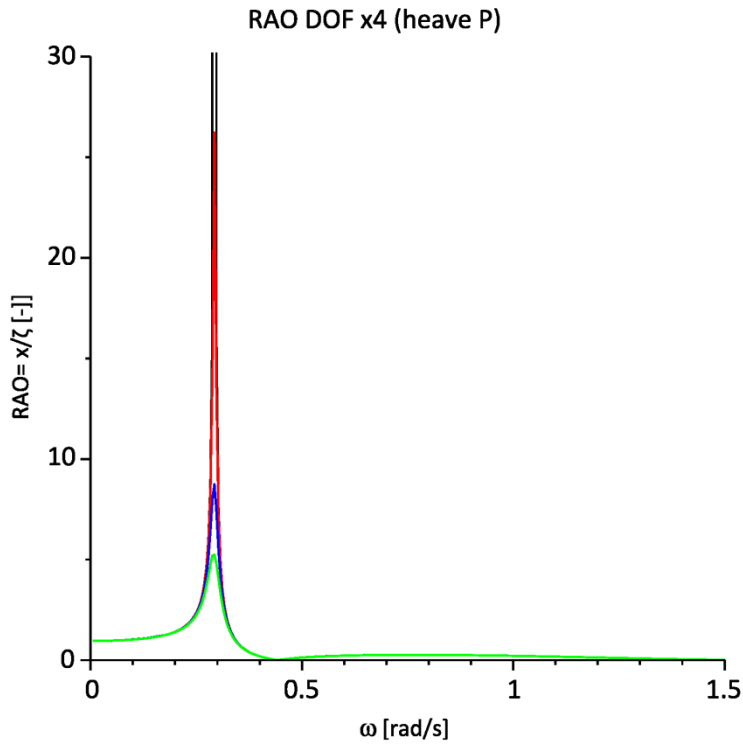


Figure 6-9: Response amplitude operator of degree of freedom x4 (heave pontoons)

6.5.2 RAO of sway motions

The response amplitude operators which show the sway motions of the tunnel element and the pontoons are given in respectively Figure 6-10 and Figure 6-11.

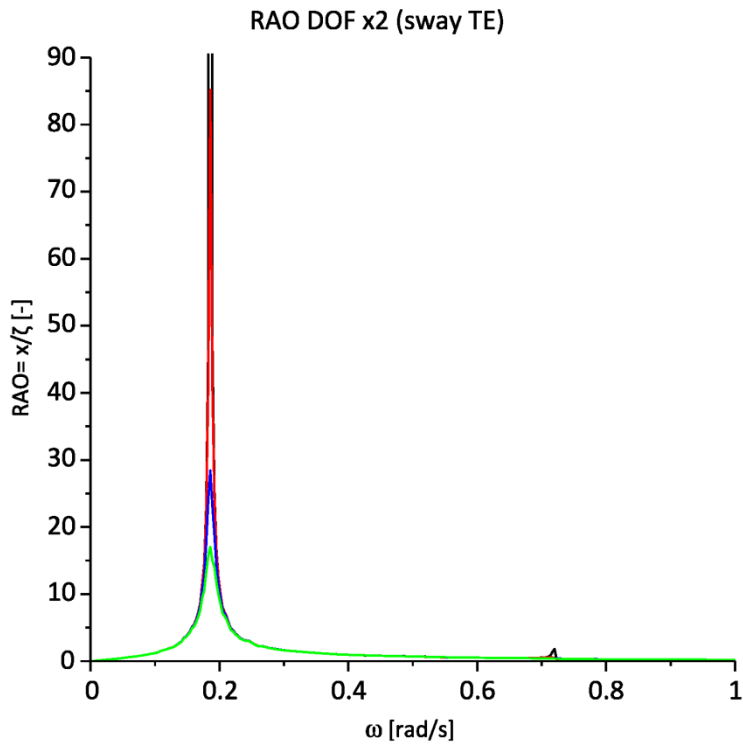


Figure 6-10: Response amplitude operator of degree of freedom x2 (sway tunnel element)

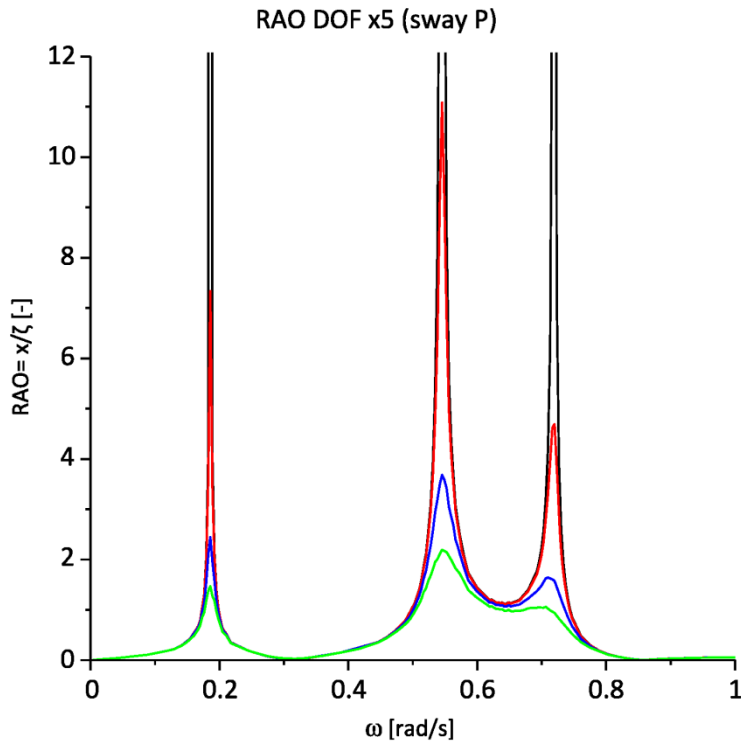


Figure 6-11: Response amplitude operator of degree of freedom x5 (sway pontoons)

6.5.3 RAO of roll motions

The response amplitude operators which show the roll motions of the tunnel element and the pontoons are given in respectively Figure 6-12 and Figure 6-13.

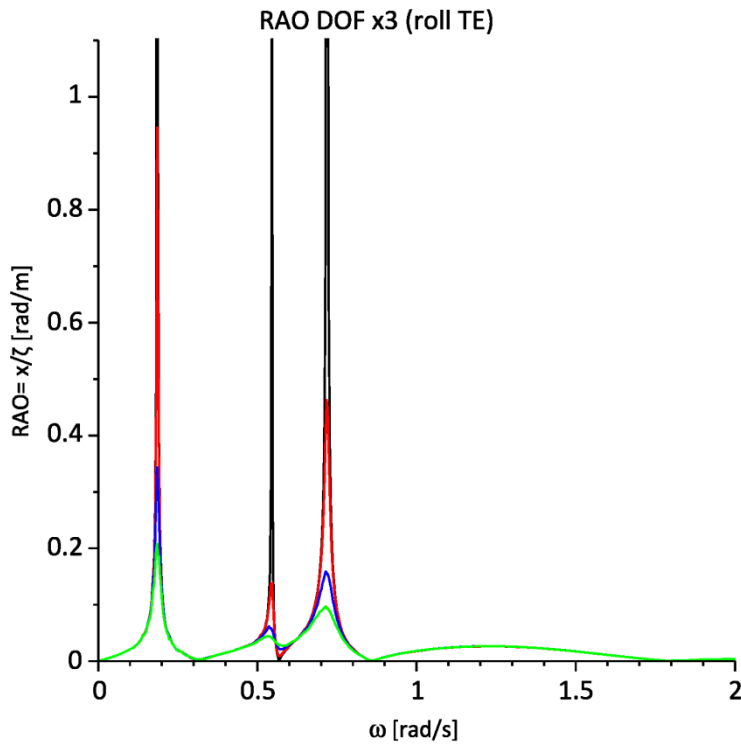


Figure 6-12: Response amplitude operator of degree of freedom x3 (roll tunnel element)

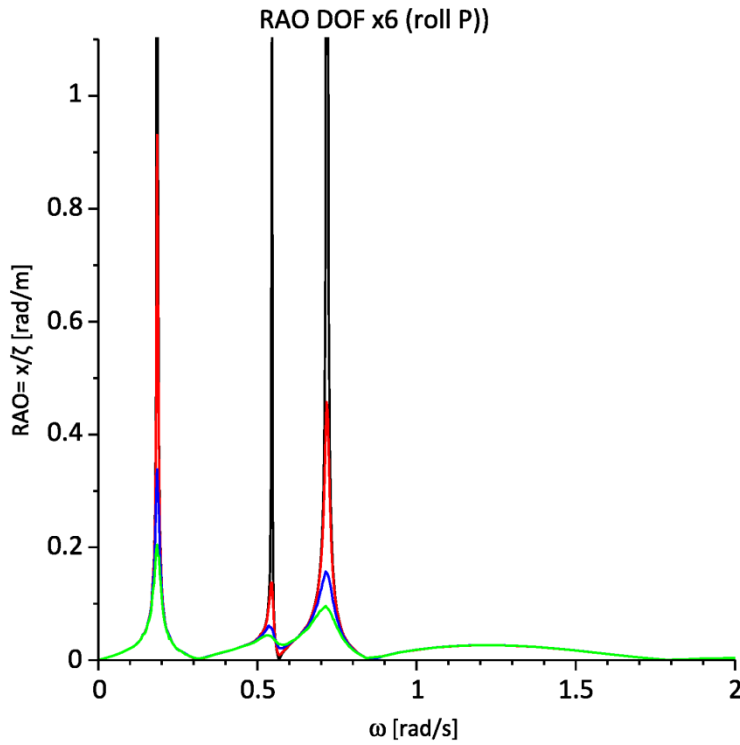


Figure 6-13: Response amplitude operator of degree of freedom x6 (roll pontoons)

6.5.4 Analysis of the results

It can be concluded that the heave motions of the tunnel element and the pontoons are relative small, when the frequency of the waves is higher than 0.45 rad/s . This is equal to a natural period T_0 which is smaller than 14 s .

In reality there is a second peak in the graph at $\omega \approx 13.6 \text{ rad/s}$ (which is equal to $T_0 \approx 0.46 \text{ s}$). This peak is only theory, because in reality damping is present. A relative small amount of damping causes the complete vanishing of the peak which belongs to the high frequency.

This high frequency corresponds to the mode where the heave motion of the pontoons dominates.

Three different peaks can be distinguished in the response amplitude operators of the sway and roll motions (see Figure 6-10 to Figure 6-13). The fourth peak is a high frequency, which is not plotted. The high frequency corresponds to the mode where the roll motion of the pontoons dominates. This frequency is observed at $\omega \approx 11.6 \text{ rad/s}$, which is approximately equal to $T_0 \approx 0.54 \text{ s}$.

As said before, it is almost impossible to obtain resonance in this high frequencies, because the peak in the graphs are vanished when damping is added. A second reason that it is hard to get resonance in these frequencies is that a wave field with waves with a period of $T < 0.6 \text{ s}$ do not exist at the immersion site. The two high frequencies have no effect on the motions of the system. Therefore these frequencies are neglected.

6.5.5 Frequency range for resonance

Resonance can occur in the frequency range between approximately 0.2 rad/s and 0.8 rad/s . Four of the in total six frequencies are located in this range. The two other frequencies are higher than 11.5 rad/s and are not of importance, which is explained in paragraph 6.5.4.

The corresponding periods of this frequency range is approximately $8.5 \text{ s} < T_0 < 22 \text{ s}$.

Wave fields with a large amount of relative high waves should be avoided in this frequency range, when this is possible. This can be concluded on the basis of two results: the response amplitude operators and the matrix of natural frequencies which is given in paragraph 5.5.2.

6.5.6 Motion in irregular waves

The response spectrum of a motion can be found by using the transfer function of the motion and the wave spectrum. This is described in paragraph 4.5.

The response spectrum can be calculated by the wave spectrum multiplied with the square of the response amplitude operator.

The variance of the motion spectrum is given by the integration of the motion spectrum. The significant motion amplitudes are defined as the mean value of the highest one-third part of the amplitudes (Journée and Massie 2000):

$$x_{1/3} = 2 \cdot \sqrt{m_{0x}}$$

The significant motion amplitudes are not determined.

The variances of the six motion spectrums are:

$$\begin{bmatrix} m_{o;x1} \\ m_{o;x2} \\ m_{o;x3} \\ m_{o;x4} \\ m_{o;x5} \\ m_{o;x6} \end{bmatrix} = \begin{bmatrix} 0.000574 \\ 0.000862 \\ 0.0000343 \\ 0.000571 \\ 0.00411 \\ 0.0000335 \end{bmatrix}$$

6.6 Conclusions response amplitude operators

The response of the system is dependent of mass, damping, stiffness and force contributions.

Damping, added mass and force contributions contain a certain amount of uncertainty. For the added mass contributions this is already described in paragraph 5.6.

Damping parameters are the most difficult parameters to determine accurate for hydro mechanical problems. Model tests or computer simulations can provide good estimations for one mass-spring systems. However, in this thesis the motions of the pontoons and tunnel element are coupled, which is a complicating factor to determine damping parameters accurate.

Three critical damping ratios are assumed for determining the response of the system. It is recommended that accurate damping parameters need to be determined in another thesis or project.

It must be mentioned that damping parameters are only relevant during resonance. When the motions are determined for frequencies which are not in the vicinity of the natural frequencies, damping is not of importance.

Force contributions consists in this model only of the Froude-Krylov force. The influence of the diffraction force is assumed to be negligible small.

7 Forces in suspension cables

When the tunnel element is lowered and is located close to the bottom, motions of the tunnel element are restricted. Damage can occur when a tunnel element collides with a previous installed tunnel element. Therefore, the motions are limited when the tunnel element is close to the bottom.

In the beginning of the immersion operation, the tunnel element is close to the water surfaces. Here, motions (displacements, velocities or accelerations) of the tunnel element itself are not problematic, because the tunnel element can oscillate without any risk of colliding to a previous installed tunnel element. However, the forces in the cables need to transfer the forces which act on the immersion system. The forces in the cables are restricted to a minimum and a maximum allowed value.

In this chapter the focus is on the forces in the suspension cables which are caused by wave-induced loads.

In paragraph 7.1 the method to determine the forces in the cables is described. This theory is used to determine the forces in the suspension cables. The wave-induced forces in the contraction and mooring cables are not analysed in this report. Also currents and other loads are ignored.

7.1 Method to determine the forces in the cables

The forces in the cables are derived by:

$$F_{cable} = k \cdot x$$

The force in the cable is equal to the stiffness [N/m] multiplied with the relative displacement [m].

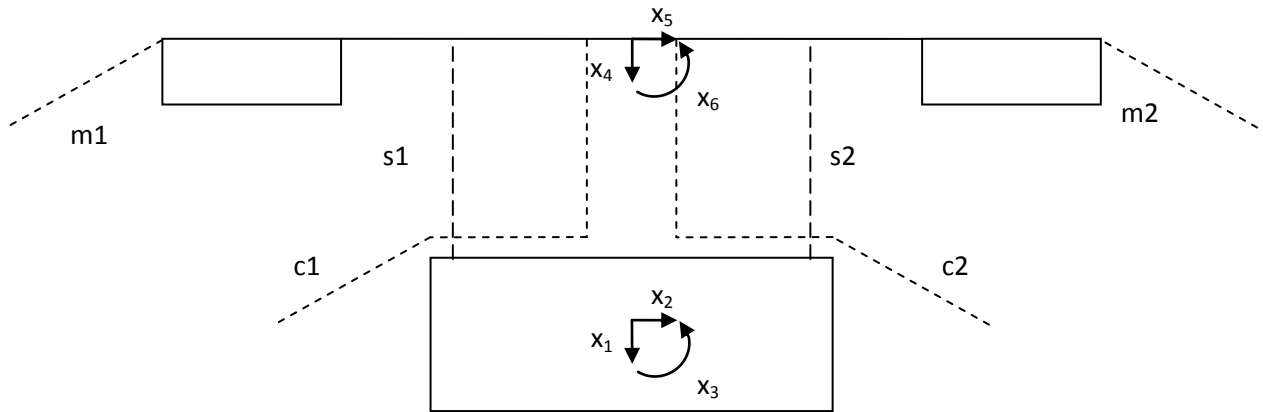


Figure 7-1: Forces in cables

The forces in the cables $s1$, $s2$, $c1$, $c2$, $m1$ and $m2$ are respectively (see Figure 7-1):

$$F_{s1} = k_s \cdot \left(x_1 - x_4 + (x_3 - x_6) \cdot \left(\frac{W_e}{2} - y_e \right) \right)$$

$$F_{s2} = k_s \cdot \left(x_1 - x_4 + (-x_3 + x_6) \cdot \left(\frac{W_e}{2} - y_e \right) \right)$$

$$F_{c1} = k_c \cdot \left(x_1 \cdot (1 - \sin(\alpha_c)) + x_2 \cdot \cos(\alpha_c) - x_3 \cdot \left(\frac{W_e}{2} \cdot \tan(\alpha_c) + \frac{h_e}{2} \right) \cdot \cos(\alpha_c) - x_4 \right)$$

$$F_{c2} = k_c \cdot \left(x_1 \cdot (1 - \sin(\alpha_c)) - x_2 \cdot \cos(\alpha_c) + x_3 \cdot \left(\frac{W_e}{2} \cdot \tan(\alpha_c) + \frac{h_e}{2} \right) \cdot \cos(\alpha_c) - x_4 \right)$$

$$F_{m1} = k_m \cdot \left(-x_4 \cdot \sin(\alpha_m) + x_5 \cdot \cos(\alpha_m) - x_6 \cdot \left(\frac{W_p}{2} \cdot \tan(\alpha_m) + h_{pd} \right) \cdot \cos(\alpha_m) \right)$$

$$F_{m2} = k_m \cdot \left(-x_4 \cdot \sin(\alpha_m) - x_5 \cdot \cos(\alpha_m) + x_6 \cdot \left(\frac{W_p}{2} \cdot \tan(\alpha_m) + h_{pd} \right) \cdot \cos(\alpha_m) \right)$$

The displacements are determined from the displacements which are determined in the frequency domain in paragraph 6.5. The displacements for a specific frequency can be determined by:

$$x = \text{Re}(\hat{x} \cdot \exp(i \cdot \omega \cdot t))$$

The force in the suspension cables consists of a constant tension force related to the overweight of the tunnel element and a force which is related to the wave-load.

The overweight of the tunnel element is transferred to the pontoons by 4 suspension cables. The tension force in one suspension cable is equal to:

$$F_s = \frac{0.02 \cdot g \cdot \rho \cdot V}{4} = \frac{0.02 \cdot 9.81 \cdot 1025 \cdot (180 \cdot 26.46 \cdot 9.97)}{4} = 2387 \text{ kN}$$

The minimum force in a suspension cable should be 500 kN in the serviceability limit state. The maximum force should be 5000 kN. The restrictions to the minimum and maximum forces in the cables are given in Appendix A.3.

Two different kind of waves are analysed. The first wave is a wind wave with a period of $T = 4 \text{ s}$ and a wave height of $H = 0.8 \text{ m}$. The second wave is a swell wave with a height of $H = 0.4 \text{ m}$ and a period of $T = 8 \text{ s}$. Both wave loads are also summarized.

Damping is assumed to be 3 percent of the critical damping.

7.2 Results

In Figure 7-2 the force in the suspension cable s1 (see Figure 7-1) is plotted which is caused by waves with a height of $H = 0.8 \text{ m}$ and a period of $T = 4 \text{ s}$.

The force in suspension cable s1 is also determined for a wave load with a period $T = 8 \text{ s}$ and a wave height of $H = 0.4 \text{ m}$. These forces are shown in Figure 7-3. A summation of both forces is shown in Figure 7-4.

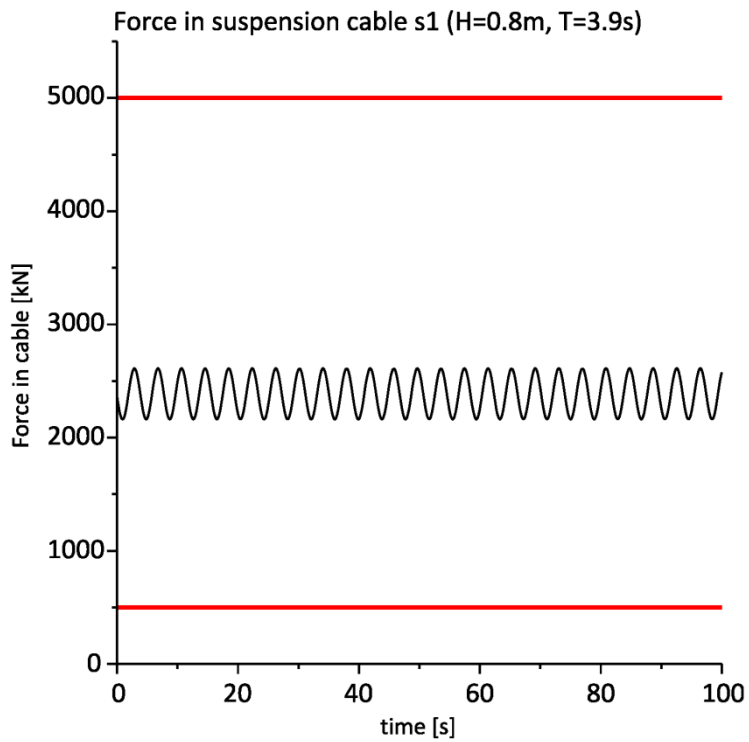


Figure 7-2: Force in suspension cables by a wave load ($H=0.8\text{m}$, $T=3.9\text{s}$)

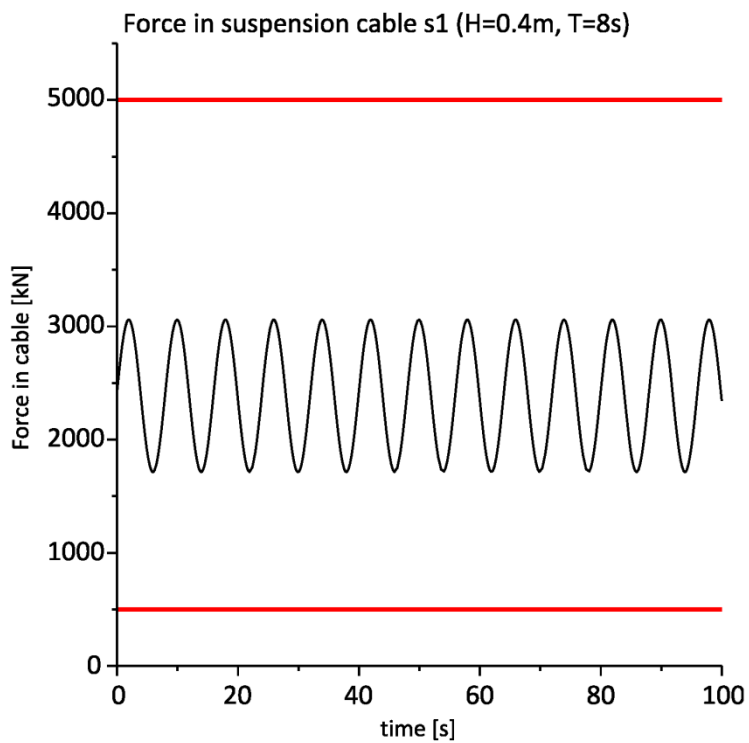


Figure 7-3: Force in suspension cables by a wave load ($H=0.4\text{m}$, $T=8\text{s}$)

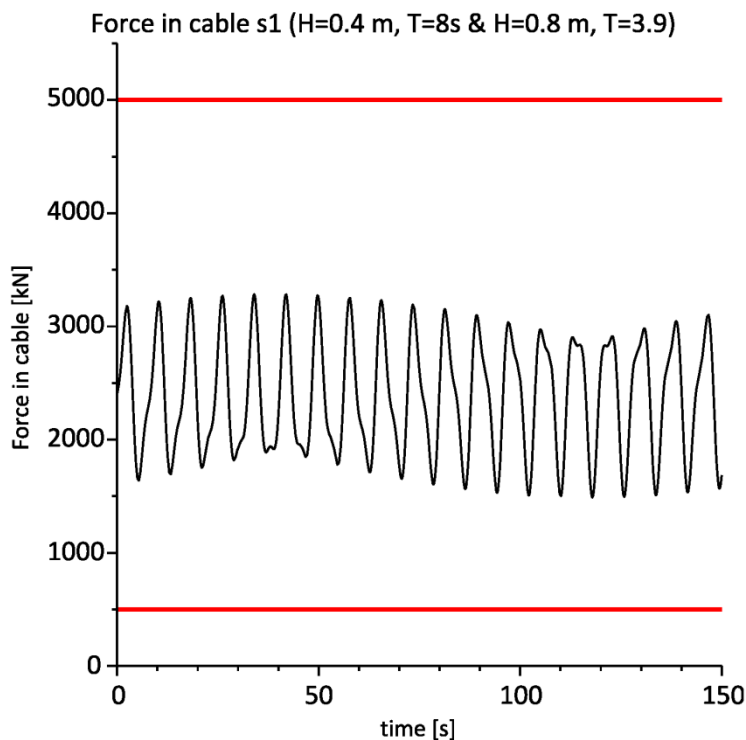


Figure 7-4: Force in suspension cables by a wave load ($H=0.4\text{ m}$, $T=8\text{ s}$ and $H=0.8\text{ m}$, $T=3.9\text{ s}$)

The minimum force in the cables by a wave with a period of $T=3.9\text{ s}$ and a wave height $H=0.8\text{ m}$ is 2150 kN. The maximum force is equal to 2600 kN.

The minimum and maximum force in the cable for the wave with a period of $T=8\text{ s}$ and a wave height of 0.4 m are respectively 1750 kN and 3150 kN. For the summation of the cable forces, the minimum and maximum forces are respectively 1500 kN and 3250 kN.

When the forces in Figure 7-2 and Figure 7-3 are compared it can be seen that short wind waves have a small influence on the total force in the cables. Longer swell waves cause higher forces in the cables, even when the wave height of the swell waves is half the wave height of the wind waves.

It can be seen in the matrix of natural frequencies, which is given in paragraph 5.5, that one of the natural frequencies of the system is almost equal to 8 s. During resonance, relative high forces in the cables are expected. Very low frequencies and very high frequencies of waves have a negligible influence on the forces in the cables, because they are not in the vicinity of one of the natural frequencies.

7.3 Design criteria

In the graphs which gives the forces in the suspension cables no safety philosophy is applied. This is not a main topic of this report, so the theory is briefly described.

For design of the immersion equipment, wave combinations should be studied which are representative for design criteria. In this study only the influence of the periods of wind waves and swell waves is analysed. Wave heights are in the analysis not of importance, because the system is linear. Doubling the wave amplitude result in a doubled amplitude of the response of the motion.

For design of the immersion equipment a maximum wave height should be determined. Also a safety factor for the strength should be applied.

Maximum wave heights follow a Rayleigh distribution. A possibility to determine the wave height for design in SLS conditions is given in paragraph 7.3.1.

7.3.1 Maximum wave height

It is often desirable to make a statistically-based guess of the highest wave that can be expected in a storm. One reasoning is to assume that the chance that this wave will be exceeded is zero. The wave height which belongs by a probability of 0 is $H = \infty$ m. This is not a practical result for engineers. A pragmatic method is to choose the maximum wave height that will be exceeded once in every 1000 waves. It will take at least 3 hours for 1000 waves to pass by in a storm. By that time the worst peak of a storm will probably be past.

The corresponding rule of thumb for H_{max} is derived hereafter (Journée and Massie 2000).

The wave amplitudes will follow a Rayleigh distribution (see paragraph 4.2). The probability density function of the Rayleigh distribution is given by:

$$f(x) = \frac{x}{\sigma^2} \cdot \exp\left(-\left(\frac{x}{\sigma \cdot \sqrt{2}}\right)^2\right)$$

The probability that a wave amplitude ζ_a exceeds a certain threshold value a can be calculated by:

$$\begin{aligned} P(\zeta_a > a) &= \int_a^{\infty} f(x) dx \\ &= \frac{1}{\sigma^2} \cdot \int_a^{\infty} x \cdot \exp\left(-\left(\frac{x}{\sigma \cdot \sqrt{2}}\right)^2\right) dx \\ &= \exp\left(-\frac{a^2}{2 \cdot \sigma^2}\right) \end{aligned}$$

In paragraph 4.2.3 it is described that the significant wave height can be derived from the standard deviation:

$$H_{1/3} = 4 \cdot \sigma$$

It follows that (Journée and Massie 2000):

$$P(H_w > H) = \exp\left(-2\left(\frac{H}{H_{1/3}}\right)^2\right)$$

With $P(H_w > H) = 1/1000$, H becomes:

$$\exp\left(-2\left(\frac{H}{H_{1/3}}\right)^2\right) = \frac{1}{1000}$$

Which can be rewritten in:

$$H = H_{1/3} \cdot \sqrt{\frac{\ln\left(\frac{1}{1000}\right)}{-2}} = 1.86 \cdot H_{1/3}$$

The wave height which can be expected with an exceeding probability of 1/1000 is equal to $1.86 \cdot H_{1/3}$.

7.3.2 Maximums according to Longuet Higgins

The most probable maximum can be determined according to the following formula of Longuet Higgins, which is equal to the maximum wave height which is determined in paragraph:

$$x_{max} = x_{mean} + \sigma \cdot \sqrt{2 \cdot \ln(N)}$$

Where x_{mean} is the average value, σ is the standard deviation and N is the number of oscillations of a considered period.

The expected maximum in a span of time is subject to a statistical distribution. The Rayleigh distribution, where the most probable maximums are determined, is such that there is still a chance that the largest outcome will exceed this value.

There is a method to determine extreme values including a so-called risk parameter α to reduce the probability of exceedance. With this method not the most probable maximum is considered but a higher design value. For small α and N this design value can be calculated by:

$$x_{max} = x_{mean} + \sigma \cdot \sqrt{2 \cdot \ln\left(\frac{N}{\alpha}\right)}$$

A common value in the Serviceability Limit State condition for the risk parameter α is 1%. In the Ultimate Limit State condition a lower risk parameter of 0.01% can be used.

7.4 Safety factor for strength

A safety philosophy for the strength of the materials should be applied. Different strategies can be used to determine the safety factor of the strength. A simple way is to apply an overall safety factor of for example 1,5. It is also possible to split up the safety factor in different components. With this method a lower safety factor can be achieved.

7.5 Comparison with the Marin model

In the beginning of this thesis it was intended to compare the results with the outcome of the Marin-model. Unfortunately, the outcome of the Marin model differs from the outcome in this report. Natural frequencies of the system and response amplitude operators were desired in order to compare. The results of Marin are determined in a time-domain simulation with an irregular wave field. It is not possible to compare the time-domain results with the frequency response functions which are obtained in this thesis.

8 Sensitivity study

The response of the tunnel element and the pontoons to certain wave conditions can be modified by changing some model characteristics of the immersion system. This can be useful when waves with a certain frequency, which is close or equal to the natural frequency, often occur at the immersion location. The natural frequencies can be changed to avoid resonance. In this chapter some possible modifications of the immersion system are given to change the natural frequencies and the response of the system.

8.1 Floaters

The restoring coefficient of a pontoon, which consist of two floaters, is given by:

$$k_w = 2 \cdot \rho \cdot g \cdot A$$

Where A is the cross section of the floaters in the x- y- plane.

When the cross section of the floaters changes the stiffness k_w also changes. Enlarging the cross section A will result in a higher restoring coefficient.

The natural periods of the motions will decrease as a result of a higher restoring coefficient.

$$T_0 = \frac{2 \cdot \pi}{\sqrt{\frac{k}{m}}}$$

The natural period decrease with a factor $\sqrt{2}$ when the cross-section A of the floaters is doubled.

8.2 Mass of the tunnel element

The tunnel element has an overweight of 2 percent during immersion. This overweight is carried by four suspension cables. The dynamic load causes a change in the force in the cables.

It is required that there is always a tension force in the suspension cables. The tunnel element is uncontrollable, when the prestress force in the suspension cables is completely disappeared.

The force in the cable consists of a static part and a dynamic part:

$$F_{total} = F_{static} + F_{dynamic}(t)$$

The static force is always a tension force (positive) and is not dependent of time. The dynamic force can be negative too. A negative dynamic force means that the total force in the cable will decrease.

The total force in the suspension cables should not exceed a maximum value in the Serviceability Limit State. Both criteria can be summarized in the following requirement, which must be fulfilled during immersion.

The force in the suspension cables should not exceed the Ultimate Limit State (ULS) conditions:

$$0 < F_{s;cable}(t) \leq F_{s,ULS}$$

In fact always a safety margin is present, which leads to the following condition in the Serviceability Limit State (SLS):

$$F_{s,min} \leq F_{s,cable}(t) \leq F_{s,SLS}$$

This means that the total force in the cable is limited by a lower bound and an upper bound.

The force in the cable caused by the wave load can be so large that the prestress force in the cables is disappeared. The prestress force in the cable can be enlarged by increasing the overweight of the tunnel. This can be done by filling the ballast tanks with more water.

The force caused by the dynamic load in the cable is allowed to be larger as a result of a higher prestress force in the cables. It must be mentioned that this is only true for the lower bound. The extra prestress force can cause problems to the maximum allowed force in the cable.

8.3 Cables

The function of the suspension cables is to transfer the overweight of the tunnel element to the pontoons. The forces consist of a force caused by a static load and a force caused by dynamic loads. The total tension force in the cables should not exceed a minimum value and a maximum value. This is explained in the previous paragraph.

The stiffness of the cables is derived with Hooke's Law:

$$x = \frac{F \cdot L}{E \cdot A} \rightarrow F = \frac{E \cdot A}{L} \cdot x \rightarrow k = \frac{E \cdot A}{L}$$

One can see that the stiffness increases when the Young's Modulus or the cross section of the cable increases. The stiffness decrease when the length of the cable is increased.

More overweight leads to an increase of the forces in the cable. Therefore, a cable is necessary which can handle the required maximum force. However, enlarging the diameter of the cable will increase the force in the cable caused by the dynamic load.

The maximum allowed force in the cable is proportional to the cross-section A of the cable.

$$F_{max} = \sigma_{max} \cdot A_{cable}$$

Enlarging the diameter of the cable is not by definition sufficient when the maximum capacity of the cable is not large enough, because due to the higher stiffness larger forces are introduced.

Concluding remarks

In the previous sections the dynamic behaviour of tunnel elements during immersion is studied. In the introduction the research questions are stated. In this section the main conclusions of this study are summarized.

During the research it is found that the natural frequencies are reliable when added mass values are determined accurate. The stiffness of the cables, restoring properties of the water and inertia properties can be determined accurate. Added mass values contain some uncertainty, resulting in natural frequencies which are less accurate.

It is also found that the influence of the suspension cables on the natural frequencies of the system is small. When only the natural frequencies of the immersion system are analyzed, a simple hand calculation can be performed to estimate the natural frequencies.

Frequencies of swell waves are in the vicinity of one of the natural frequencies of the system, which means that swell waves have more influence on the motions of the system compared to the influence of wind waves.

For motions which are described during resonance, damping parameters need to be accurate. Damping is hard to describe in this problem, because several damping mechanisms can be distinguished which extract energy of the system.

The influence of the swell waves on the forces in the suspension cables is also larger compared to the influence of the wind waves, which is in accordance with the swell and wind wave induced motions.

To avoid resonance the system can be adjusted. This must be done in advance.

The response of the immersion system can partly be determined in a tender-phase of an immersed tunnel project with the theory and model which is described in this thesis. The analysis in this report provides a method to estimate the dynamic behaviour.

For a complete view of the response of the system, professional research should be done. Complex phenomena such as the response to oblique waves are for example not studied in this thesis.

The hydrodynamic coefficients in this study contain a lot uncertainty. It is recommended that these values are analyzed in following research in order to obtain reliable results of the response of the system during resonance. Especially the damping parameters need extra research. Added mass values contain also uncertainty and needs to be analyzed as well.

In the beginning of the research it was intended to compare the results with the model composed by Marin. Unfortunately, it was not possible to compare, because the results of the model of Marin did not contain the required information, such as the natural frequencies of the system and the response amplitude operators.

In order to give the accuracy of the results it is recommended that the results of this model are compared to the Marin data or data which is measured during immersion.

It is recommended to use Matlab instead of Maple, when the dynamic behaviour of a tunnel element is studied in a following project.

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Appendices

Appendix A Constraints and schematisations

In this Appendix the constraints are given and the schematisations for the model are derived. The first part of this appendix is related to the constraints.

A.1 Position

During the immersion of a tunnel element two positions of the tunnel element are considered governing.

The first position is when the tunnel element is approximately 1 m. below surface. Here, the tunnel element has a maximum influence from wave and current forces. The second position is when the tunnel element is close (0,5 m) to the gravel bed. In this position the constraints consists of minimum movements which are acceptable.

A.1.1 Immersion spread

Two different immersion spreads are studied in the pre-phase of the project in Korea. The first model tests, which are done by Marin, are carried out with the immersion spread as shown in Figure I.

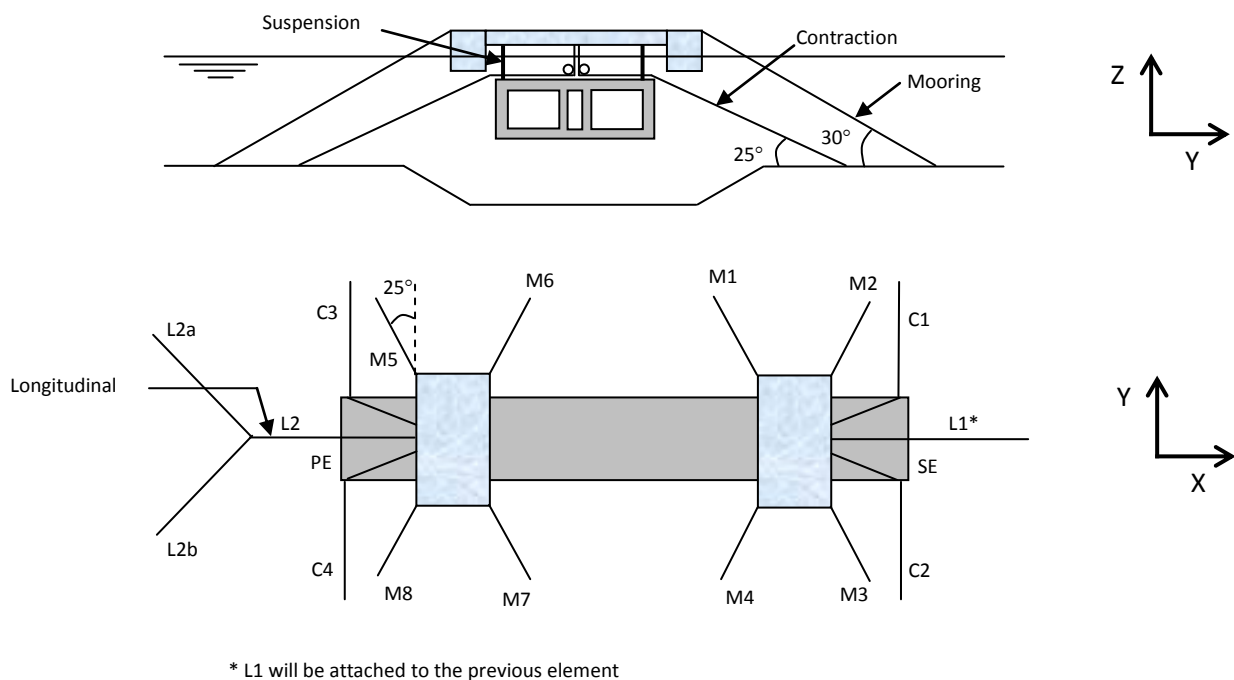


Figure I: The initial immersion spread

In the results of these model tests was concluded that the mooring and contraction lines should be installed less steep to decrease the vertical component of the force in these lines. Therefore a second immersion spread is set up. The tunnel elements in Korea are immersed according to the new immersion spread. Model tests and numerical simulations are also performed for the new immersion spread.

As a result of the larger length, the stiffness of the cables will be lower. Marin concluded that the latter will result in lower peak forces, but increases the movements of the pontoons and the tunnel element.

The second immersion spread, which is given in Figure II is also used to determine the motions of the tunnel elements in this study.

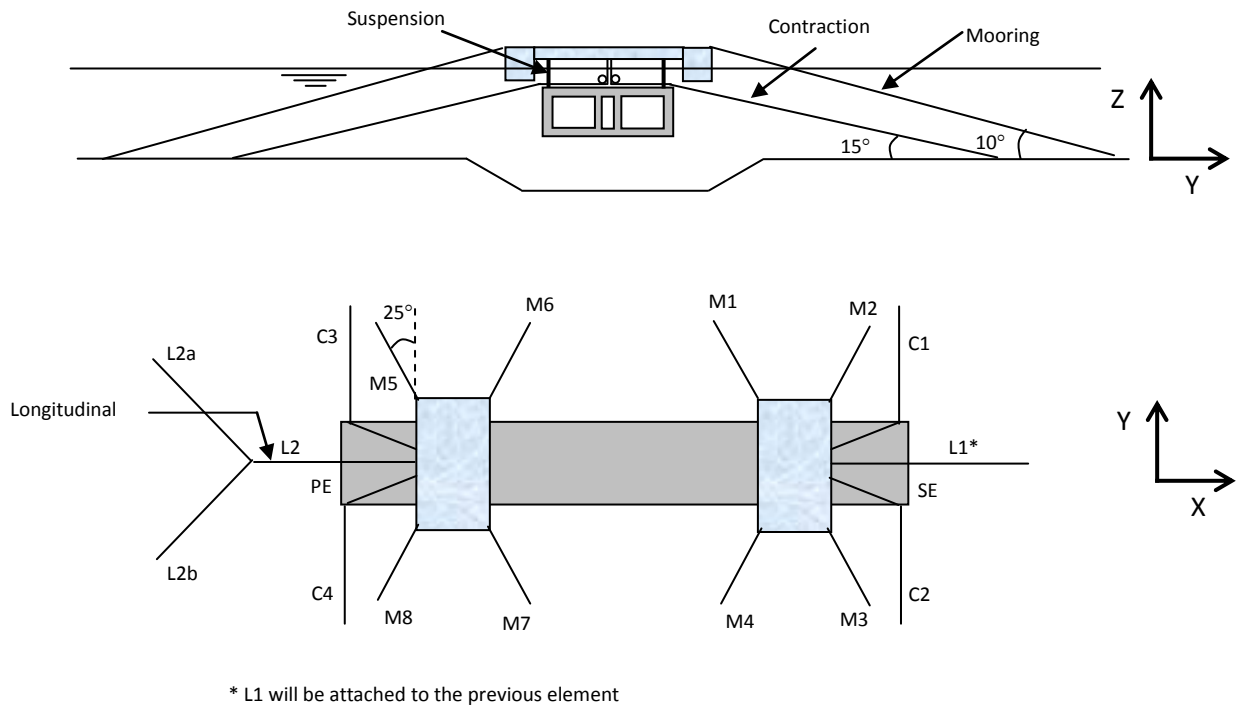


Figure II: Immersion spread which is used for immersing of the elements in Korea

A.1.2 Water depth

The water depth is the height between the bottom of the waterway and the water level. The depth of the trench is not included in this value.

Three average water depths can be distinguished. These are 12 meter, 23 meter and 32 meter. Approximately four tunnel elements must be immersed in the shallow part of 12 meter in the trench. These are the elements which are located near the land abutments (element 1, 2, 17, 18).

Half of the remained elements (element 3-9) must be immersed in the average water depth of 23 meter and the other (element 10-16) in an average depth of 32 meter. An overview of the positions of the tunnel elements is shown in Figure 2-2.

Marin assumed that a depth of 23 meter will result in governing forces as result of shorter lines with respect to a depth of 32 m. Therefore two critical depths can be distinguished, namely 12 and 23 meter.

With the two different positions and the two different average water depths, four combinations can be made which can be used for the numerical input of the model:

- Water depth of 12 m., tunnel element at 1 m. below surface;
- Water depth of 12 m., tunnel element at 0.5 m. above gravel bed;
- Water depth of 23 m., tunnel element at 1 m. below surface;
- Water depth of 23 m., tunnel element at 0.5 m. above gravel bed.

In this study is focused on the third combination, a water depth of 23 m. and the position of the tunnel element at 1 m. below surface, which is shown in Figure II.

A.2 Schematisations

The stiffness of the cables is given by Hooke's Law. This relation between force and displacement is only valid when the cables are prestressed.

$$F = \frac{EA}{L} \cdot x$$
$$k = \frac{EA}{L}$$

In reality however, the spring stiffness of the cables is non-linear, because the elasticity module of cables is non-linear (Feyrer 2007).

The cables consists of strands which are twisted around a core. The strands consists of wires. For the immersion operation 6x36 Warrington Seale cables with a steel core are used. This means that the cables consist of 6 strands and that the strands consist of 36 wires. The cross-section of a 6x36 Warrington Seale cable can be seen in Figure III.

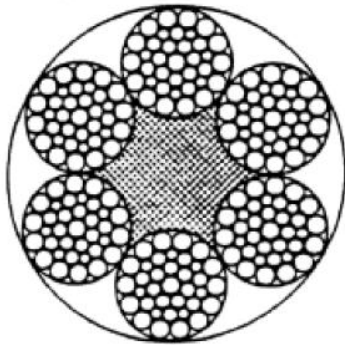


Figure III: Cross section of a 6x36 Warrington Seale cable

A.2.1 Cross-section of the cables

The ratio between the sum of the nominal metallic cross-section areas A of all wires in the rope and the circumscribed area A_u of the rope, based on its nominal diameter d , is called the fill factor f (Feyrer 2007).

$$f = \frac{A}{A_u}$$

The nominal cable diameter is the diameter of the circle circumscribing the rope cross-section.

The nominal metallic cross-sectional area A (sum of the wire cross-sections) is given by:

$$A = C \cdot d^2$$

With nominal metallic cross-sectional area factor C :

$$C = f \cdot \frac{\pi}{4}$$

The nominal metallic factor of the 6x36 Warrington Seale cables is 0.46.

The diameter of the contraction cables is 54 mm. The mooring cables have a diameter of 40 mm and the suspension cables are 58 mm.

A.2.2 Young's modulus of the cables

The elongation effect of materials under the effect of mechanical stresses is described by the elasticity module. The elongation of a wire rope depends on the elasticity module of wire materials. However, the wire rope elasticity module describing wire rope elongation differs from the wire elasticity module. The rope stress-extension curve is not linear. Therefore, the wire rope elasticity module is not constant but depends on the tensile stresses (Feyrer 2007).

As an example, the graph in Figure IV shows the increase of tensile stress as the rope extends of a Warrington 8x19-IWRC-sZ cable. For rope wires with a core (for example steel) progressive increase and hysteresis loops for loading and unloading occurs.

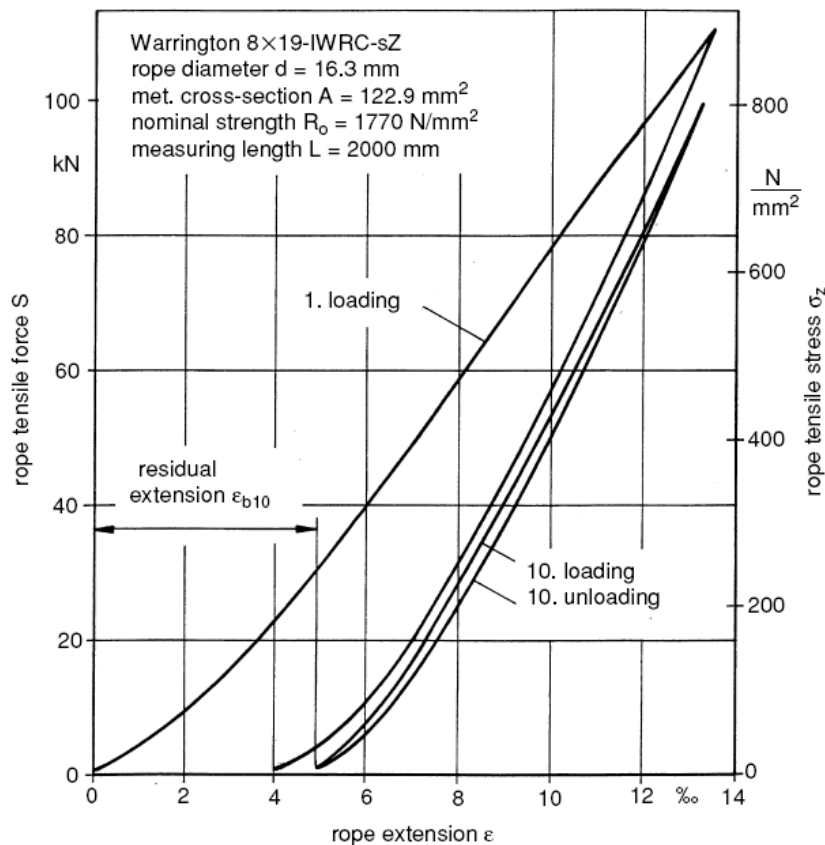


Figure IV: Stress-extension curves for a stranded wire rope with steel core

The most important conclusion about the graph in Figure IV is that the Young's modulus is not a constant value. One can see that the elasticity module differs with respect to the amount of loadings. During immersion it is unknown which line is governing. Therefore it is hard to describe the elasticity module correct, because the elasticity module is dependent on the current and previous elongation of the wire ropes.

The design value of the elasticity module is according to the manual of the wire rope supplier¹ equal to:

$$E = 105 \cdot 10^3 \text{ N/mm}^2$$

¹ Source: Mennens (1995), technisch vademecum, staalkabel-, hijs- en heftechniek, 3rd edition, page 62

This constant value is used in the analysis. The effect of the hysteresis loops is not taken into account.

A.2.3 Length of the cables

The length of the cables has been determined for a tunnel element 1 m. below surface and with a water depth of 23 m.

In Figure II one can see that the contraction cables run from the anchor to the tunnel element, thereafter over the tunnel deck to the pontoons. One can divide this cable in three parts.

The angle in the z-y plane is equal to 15 degrees. The length of the first part of the cable is equal to:

$$L = \frac{z}{\sin(15)} \approx 3.86 \cdot z$$

Thereafter, the cable runs over the tunnel deck. The length of this part is (given by Mergor):

$$L = \sqrt{8.138^2 + 29.52^2} = 30.62 \text{ m}$$

The last part of the cable runs from the tunnel deck to the pontoon. This length is equal to the distance between the pulley on the pontoon deck and the water surface and the distance between the water surface and the tunnel deck:

$$L = 6.5 + 1 = 7.5 \text{ m}$$

The total length of one contraction line is equal to:

$$L_c = 3.86 \cdot (23 - 1) + 30.62 + 7.5 = 123.04 \text{ m}$$

One can distinguish two different angles in the mooring cables. These can be seen in Figure II and Figure V. The angle in the z-y plane is 10 degrees. The angle in the x-y plane is equal to 25 degrees.

The length of the mooring cables is according to Figure V equal to:

$$L = \sqrt{z^2 + \left(\frac{z}{\tan(10) \cdot \cos(25)} \right)^2} \approx 6.34 \cdot z$$

The height z is dependent on the draft of the pontoons. The draft of the pontoons during the immersion operation is 3.162 m. The height of the floaters is 6 m and the thickness of the deck of the pontoons is 2.5 m. This results in a length of:

$$L_m = 6.34 \cdot (23 + 6 + 2.5 - 3.162) = 179.66 \text{ m}$$

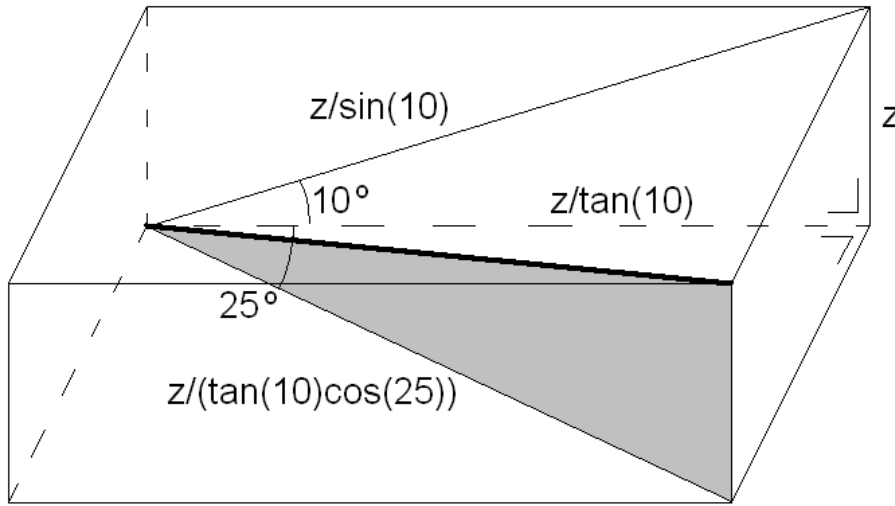


Figure V: Length mooring line

The suspension wires are on a pulley. This means that the suspension wires consists of 13 parallel lines on sheaves.

The length of the suspension cables measured from the pontoon to the pulley on the deck of the pontoon is equal to:

$$L_s = 7.5 \text{ m}$$

The lengths of the cables, which are given above, are not the entire length of the cables. The remaining part of the cables is rolled on the drums. In order to move the element or the pontoons, one can slack or haul the cables by rolling the wire rope drums.

The remaining length of the cables which is rolled on the drums is not included in the length which is used to determine the spring stiffness of the cables.

A.2.4 Spring stiffness of the cables

The wires consist of an axial stiffness and a length. The ratio between these two factors is the spring stiffness of the cables.

When the motions of the tunnel element are analysed in the z-y plane, the spring stiffness of the contraction cables is equal to the spring stiffness of two single contraction lines. The cables are placed one after another in the z-y plane. For example the spring stiffness of the contraction cable is equal to the stiffness of the cables C1 and C3. This can be seen in Figure II.

$$k_c = 2 \cdot \frac{0.46 \cdot d^2 \cdot E}{L} = 2 \cdot \frac{0.46 \cdot 0.054^2 \cdot 105 \cdot 10^6}{123.04} = 2289.4 \text{ kN/m}$$

The spring stiffness of the mooring cables consists of the equivalent stiffness of 4 single mooring lines. This is shown in Figure II. The cables have an angle of 25 degrees in the x-y plane, so the stiffness is reduced by a factor $\cos(25)^2$, which can be seen in Figure VI.

$$k_m = 4 \cdot \cos(\alpha)^2 \cdot \frac{0.46 \cdot d^2 \cdot E}{L} = 4 \cdot \cos(25)^2 \cdot \frac{0.46 \cdot 0.04^2 \cdot 105 \cdot 10^6}{179.66} = 1413.3 \text{ kN/m}$$

The stiffness of the suspension wires is equal to:

$$k_s = 2 \cdot 13 \cdot \frac{0.46 \cdot d^2 \cdot E}{L_s} = \frac{2 \cdot 13 \cdot 0.46 \cdot 0.058^2 \cdot 105 \cdot 10^6}{7.5} = 563000 \text{ kN/m}$$

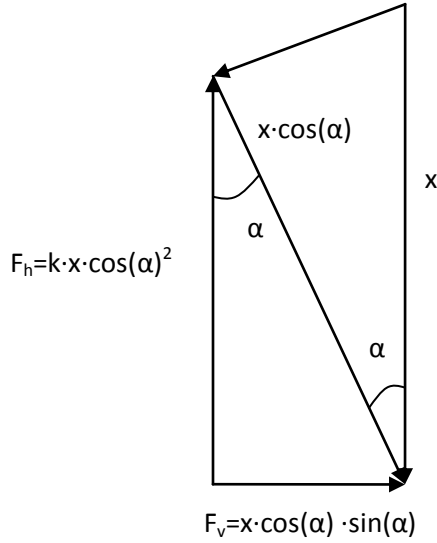


Figure VI: Displacement 'x' results in a force in the spring

A.2.5 Restoring coefficient of the water

The restoring coefficient of the pontoons is equal to:

$$k_w = 2 \cdot \rho \cdot g \cdot A = 2 \cdot 1025 \cdot 9.81 \cdot 36 \cdot 6 = 4343868 \text{ N/m}$$

A.2.6 Inertia properties of the pontoons

The function of the pontoon is to guide the tunnel element during the immersion operation. The pontoons can fulfil this demand only if the pontoons are afloat, so the pontoons must always have freeboard left for stability.

The pontoons consists of a top deck and two floaters. The immersion equipment is installed on the deck of the pontoons. The top deck transverse the overweight of the tunnel element to the floaters. A buoyancy force is generated by the floaters.

The mass of the pontoons consists of the mass of the floaters, the mass of the top deck and the mass of the immersion equipment which is installed at the top deck.

The mass of one floater is $176 \cdot 10^3 \text{ kg}$ and the mass of the top deck and the immersion equipment is equal to $1048 \cdot 10^3 \text{ kg}$.

The total weight of one pontoon is equal to:

$$m_p = 1400 \cdot 10^3 \text{ kg}$$

One can also determine the mass of the pontoon by measuring the draft of the pontoons. The amount of displaced water multiplied with the density of water is equal to the mass of the pontoon.

The mass moment of inertia of a body with an axis passing through an arbitrary rotation centre is given by:

$$J = \int y^2 dm + \int r^2 dm + 2 \cdot r \cdot \int y dm$$

The first term is the mass moment of inertia of an object passing through its own centre of mass. The second term becomes $m \cdot r^2$ and the third part is by definition zero, because the origin is at the centre of mass. Therefore the mass moment of inertia passing through an arbitrary rotation centre becomes:

$$J = \int y^2 dm + m \cdot r^2$$

The mass moment of inertia of the pontoons is not straightforward to determine. One must know exactly where the mass on the pontoon is divided.

The rotation centre of the pontoon is assumed at the centre of the horizontal axis and at the bottom of the top deck.

The floaters are schematized as point masses. The mass moment of inertia of the floaters about the axis passing through its own centre of mass is negligible in comparison to the moment of inertia of the floaters through the centre of mass of the pontoon. Therefore the mass moment of inertia of one floater becomes:

$$J_{fl} = m_{fl} \cdot r^2 = 176 \cdot 10^3 \cdot \left(\frac{42.5}{2} - 3 \right)^2 = 0.5862 \cdot 10^8 \text{ kgm}^2$$

The top deck schematized as a beam. The dead load of the top deck and the load of the immersion equipment is assumed to be equally distributed.

The mass moment of the top deck is equal to:

$$J_{td} = \frac{1}{12} \cdot m_{td} \cdot W_p^2 = \frac{1}{12} \cdot 1048 \cdot 10^3 \cdot 42.5^2 = 1.577 \cdot 10^8 \text{ kgm}^2$$

The mass moment of inertia of one pontoon is equal to the sum of the moment of inertia of the floaters and the top deck:

$$J_p = J_{tp} + 2 \cdot J_{fl} = (1.577 + 2 \cdot 0.5862) \cdot 10^8 = 2.75 \cdot 10^8 \text{ kgm}^2$$

A.2.7 Inertia properties of the tunnel element

The tunnel element is schematized as a stiff box.

The cross section of the concrete is equal to (see Figure 2-3):

$$A_c = 26.46 \cdot 9.97 - 2 \cdot 10.06 \cdot 7.29 - 6.6 \cdot 2.5 = 100.49 \text{ m}^2$$

The specific weight of reinforced concrete is 2400 kg/m^3 .

The weight of the tunnel element is (without the weight of the bulk heads and the ballast water) equal to:

$$100.49 \cdot 180 \cdot 2400 = 43.41 \cdot 10^6 \text{ kg}$$

During immersion the mass of the tunnel element is equal to 1.02 times the buoyancy force:

$$m_e = 1.02 \cdot 26.46 \cdot 9.97 \cdot 180 \cdot 1025 = 49.65 \cdot 10^6 \text{ kg}$$

The mass moment of inertia of the tunnel element is determined by calculating the mass moment of inertia of a rectangle cross section. Thereafter three rectangles with the size of the direction lanes and the escape gallery are subtracted.

The centre of mass is assumed in the centre of the tunnel element (at 26.46/2 and 9.97/2).

The mass moment of inertia of a rectangle is equal to:

$$J = \frac{1}{12} \cdot m \cdot (y^2 + z^2)$$

$$J = \frac{1}{12} \cdot (26.46 \cdot 9.97 \cdot 180 \cdot 2400) \cdot (26.46^2 + 9.97^2) = 7.593 \cdot 10^9 \text{ kgm}^2$$

The mass moment of inertia of the direction lanes is equal to:

$$J = \frac{1}{2} \cdot \left(\frac{1}{12} \cdot (10.07 \cdot 7.29 \cdot 180 \cdot 2400) \cdot (10.07^2 + 7.29^2) + (10.07 \cdot 7.29 \cdot 180 \cdot 2400) \cdot \left(\frac{10.07}{2} + 0.6 + \frac{2.5}{2} \right)^2 \right) = 3.823 \cdot 10^9 \text{ kgm}^2$$

The mass moment of inertia of the escape gallery is equal to:

$$J = \frac{1}{12} \cdot (2.5 \cdot 6.6 \cdot 180 \cdot 2400) \cdot (2.5^2 + 6.6^2) = 0.030 \cdot 10^9 \text{ kgm}^2$$

The mass moment of inertia of the tunnel element is (exclusive ballast tanks and bulkheads):

$$J_e = (7.593 - 3.823 - 0.03) \cdot 10^9 = 3.74 \cdot 10^9 \text{ kgm}^2$$

A.3 Constraints

The constraints during immersion consist of maximum allowed forces in the wires and winches and maximum allowed velocities. The latter consists of rotational and translational velocities.

The constraints, which are given below, are equal to the ones which are used for the model tests performed by Marin.

The maximum forces in the wires are limited. These limitations are given in Table II.

Line	Normal conditions [kN] (SLS)	Extreme conditions [kN] (ULS)
Contraction + Longitudinal	600	NA
Mooring	350	NA
Suspension	max. 5000, min. 500	NA

Table II: Maximum forces in the serviceability limit state and the ultimate limit state

The suspension wires may not slacken ($F = 0 \text{ kN}$).

The vertical angle of the suspension wires should not be more than 5% in transversal direction. This is visualized in Figure VII.

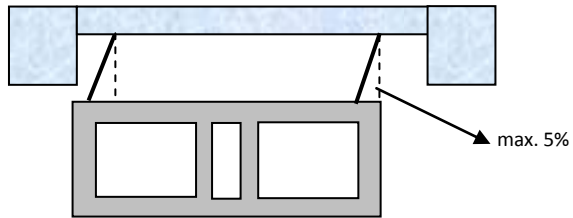


Figure VII: Maximum angle in transverse direction

The constraints to maximum movements, velocities and accelerations with respect to translations and rotations are given in Table III.

		TE lowering	TE close to touchdown
Movements to X, Y and Z	Speed	50 cm/s	0.17 cm/s
	Total	Suspension wires max 5%	Amplitude +/- 18.5 cm
Rotation mx, my and mz	Speed	1.5°/s	1.0°/s
	Total	2.0°	1.5°

Table III: Constraints to maximum movements and rotations

Appendix B Stiffness matrix

The stiffness matrix is derived with the displacement method. This method is used, because it directly provides the stiffness matrix.

The first step of determining of the stiffness of the system is to analyse the forces in the cables. The force in the cables is calculated by multiplying the stiffness of the cable with a displacement x . The latter is done for each degree of freedom.

Thereafter the reaction forces of the cables which act on the tunnel element and the pontoons are determined.

B.1 Forces in cables

The forces in the cables by a displacement x_1 are given in Figure VIII.

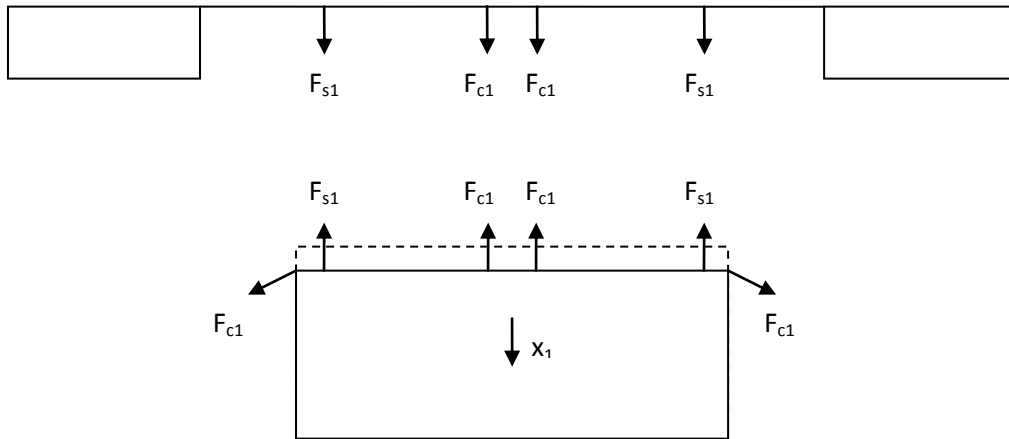


Figure VIII: Force in the cables by a displacement x_1

The forces become:

$$F_{c1} = k_c \cdot x_1 \cdot (1 - \sin(\alpha_c))$$

$$F_{s1} = k_s \cdot x_1$$

The forces in the cables by a displacement x_2 are shown in Figure IX.

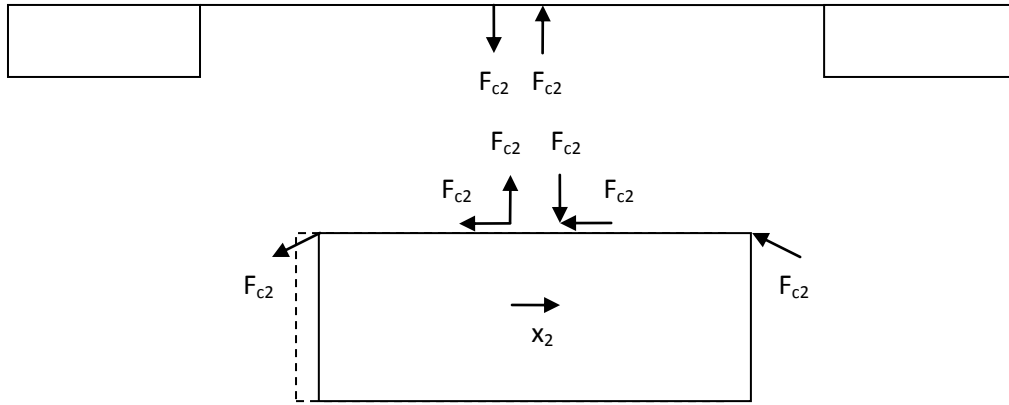


Figure IX: Force in the cables by a displacement x_2

The force in the contraction cables becomes:

$$F_{c2} = k_c \cdot x_2 \cdot \cos(\alpha_c)$$

The forces in the cables by a displacement x_3 are given in Figure X.

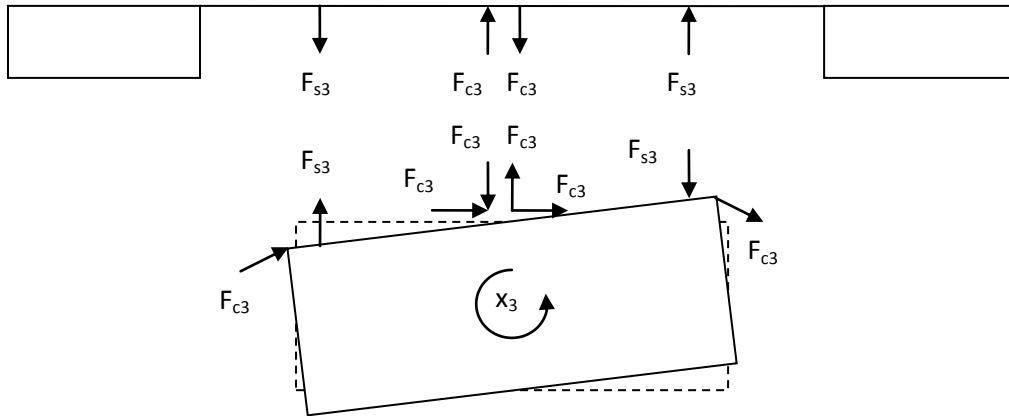


Figure X: Force in the cables by a displacement x_3

The elongation of the contraction cable by a rotation x_3 is shown in Figure XI and is given by:

$$arm \cdot x_3 = \left(\frac{W_e}{2} \cdot \tan(\alpha_c) + \frac{h_e}{2} \right) \cdot \cos(\alpha_c) \cdot x_3$$

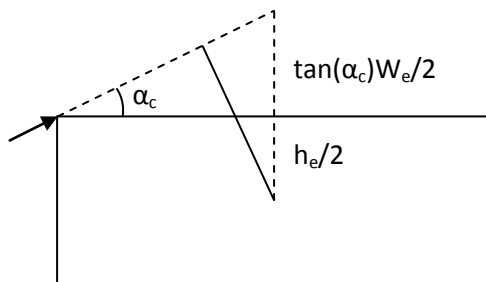


Figure XI: Elongation of the contraction cables by a rotation x_3

The forces in the cables become:

$$F_{c3} = \left(\frac{W_e}{2} \cdot \tan(\alpha_c) + \frac{h_e}{2} \right) \cdot \cos(\alpha_c) \cdot k_c \cdot x_3$$

$$F_{s3} = \left(\frac{W_e}{2} - y_e \right) \cdot k_s \cdot x_3$$

The forces in the cables by a displacement x_4 are shown in Figure XII.

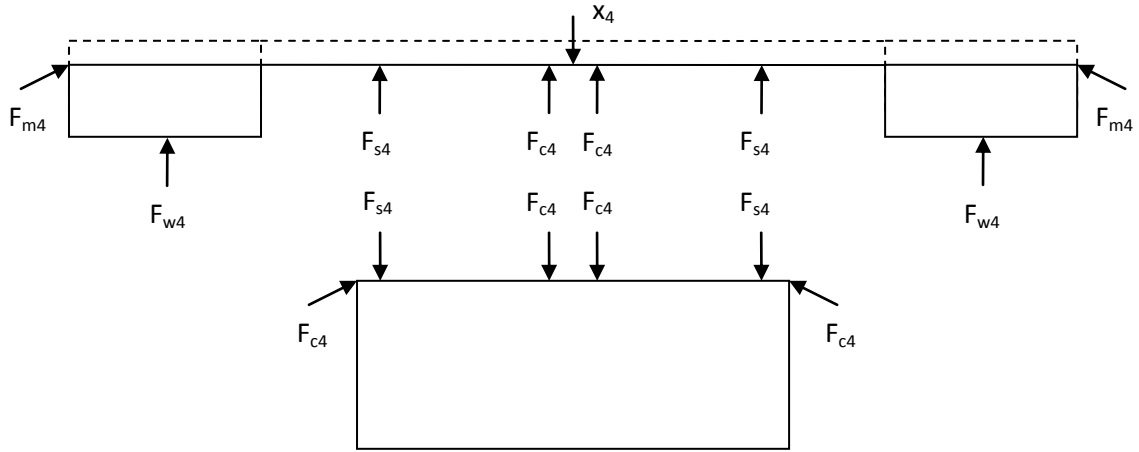


Figure XII: Forces in the cables by a displacement x_4

The forces become:

$$F_{c4} = k_c \cdot x_4$$

$$F_{m4} = k_m \cdot x_4 \cdot \sin(\alpha_m)$$

$$F_{s4} = k_s \cdot x_4$$

$$F_{w4} = k_w \cdot x_4$$

The forces in the cables by a displacement x_5 are given in Figure XIII.

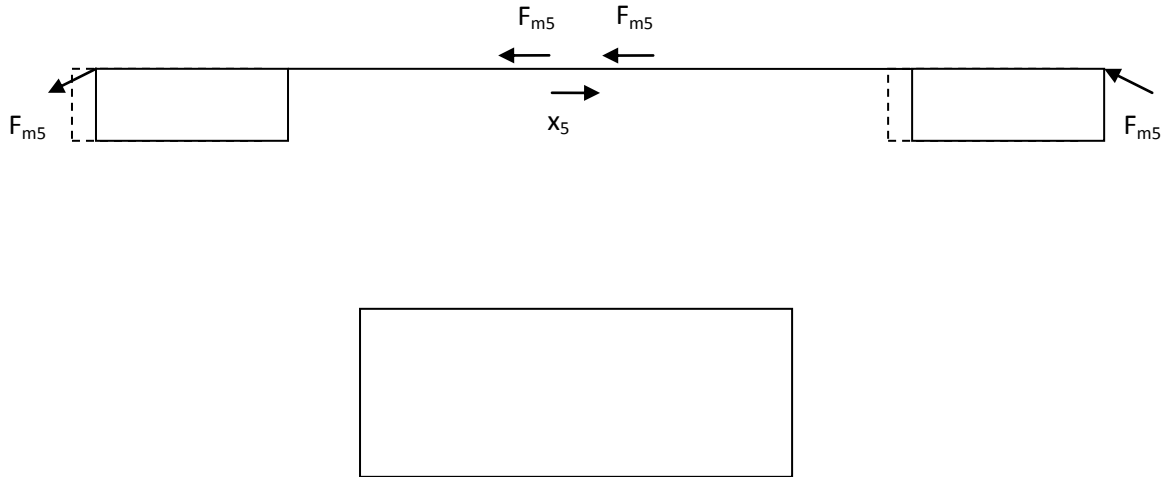


Figure XIII: Forces in the cables by a displacement x_5

The force in the mooring cable becomes:

$$F_{m5} = k_m \cdot x_5 \cdot \cos(\alpha_m)$$

The forces in the cables by a displacement x_6 are shown in Figure XIV.

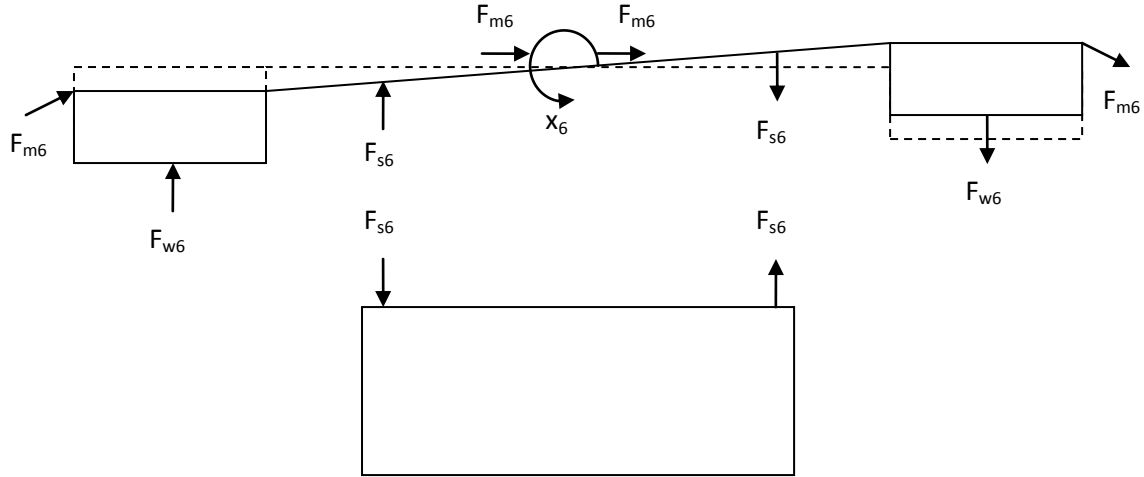


Figure XIV: Forces in the cables by a displacement x_6

The forces in the cables are equal to:

$$F_{m6} = \left(\frac{W_p}{2} \cdot \tan(\alpha_m) + h_{pd} \right) \cdot \cos(\alpha_m) \cdot k_m \cdot x_6$$

$$F_{s6} = \left(\frac{W_e}{2} - y_e \right) \cdot k_s \cdot x_6$$

$$F_{w6} = \left(\frac{W_p}{2} - y_p \right) \cdot k_w \cdot x_6$$

B.2 Reaction forces of the cables

The reaction force of the contraction and mooring cables on the pontoon and the tunnel element is shown in Figure XV.

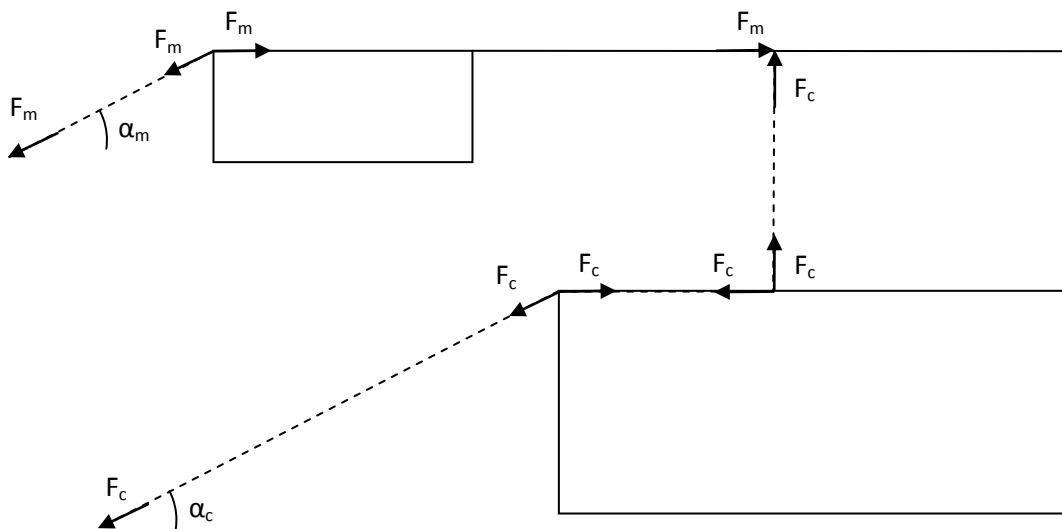
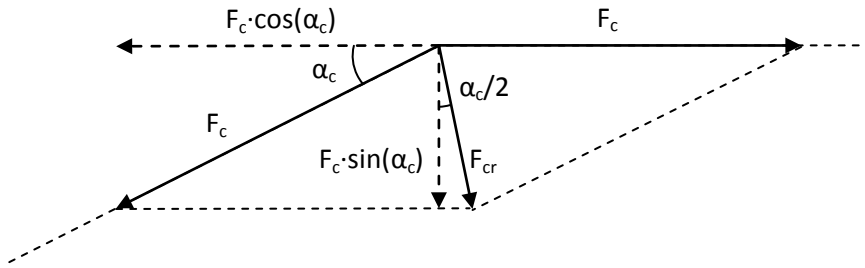


Figure XV: Reaction Force on Tunnel Element

The reaction force at the edge of the tunnel element of the contraction cables is shown in Figure XVI.



The reaction force which acts at the tunnel element is:

Figure XVI: Reaction force on the Tunnel Element

$$F_{cr} = F_c \cdot \frac{\sin(\alpha_c)}{\cos\left(\frac{\alpha_c}{2}\right)} = 2 \cdot F_c \cdot \sin\left(\frac{\alpha_c}{2}\right)$$

The reaction force of the contraction cables in the middle of the tunnel element is given in Figure XVII.

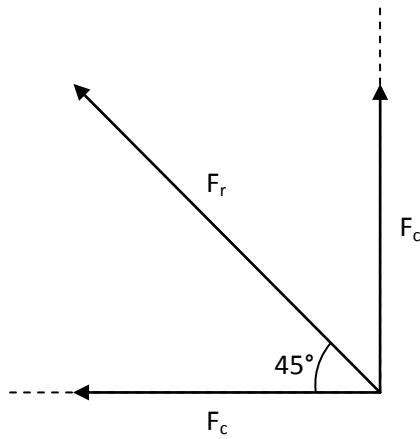


Figure XVII: Reaction force on the Tunnel Element

The reaction force which is given in Figure XVII is equal to:

$$F_r = \frac{F_c}{\cos(45)} = F_c \cdot \sqrt{2}$$

The reaction force of the mooring cables which act on the pontoon can be derived on a similar way as done for the contraction cables.

The reaction force of the mooring cables on the pontoon is:

$$F_{mr} = 2 \cdot F_m \cdot \sin\left(\frac{\alpha_m}{2}\right)$$

B.3 Forces on the tunnel element and the pontoons

The forces which act on the tunnel and element and the pontoons are shown in this paragraph.

The forces on the tunnel element are shown in Figure XVIII when:

$$x_1 > 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0$$

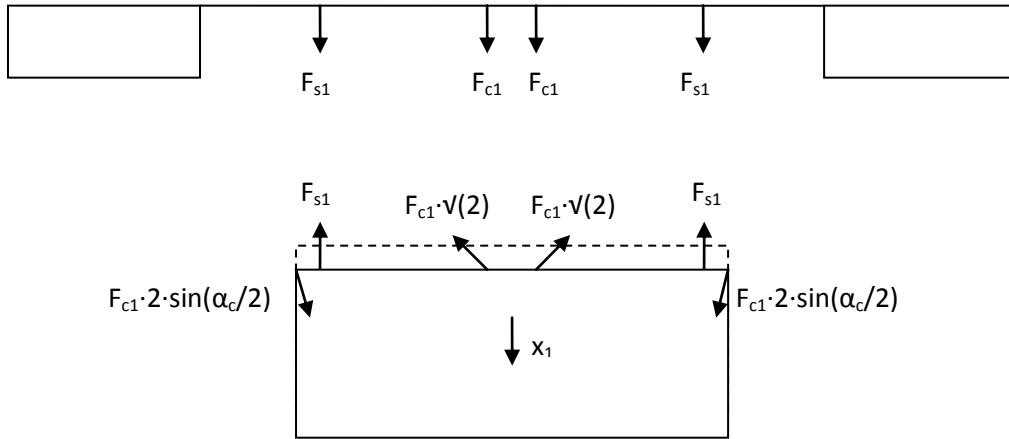


Figure XVIII: Forces when $x_1 > 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0$

The forces on the tunnel element are shown in Figure XVIII when:

$$x_1 = 0, x_2 > 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0$$

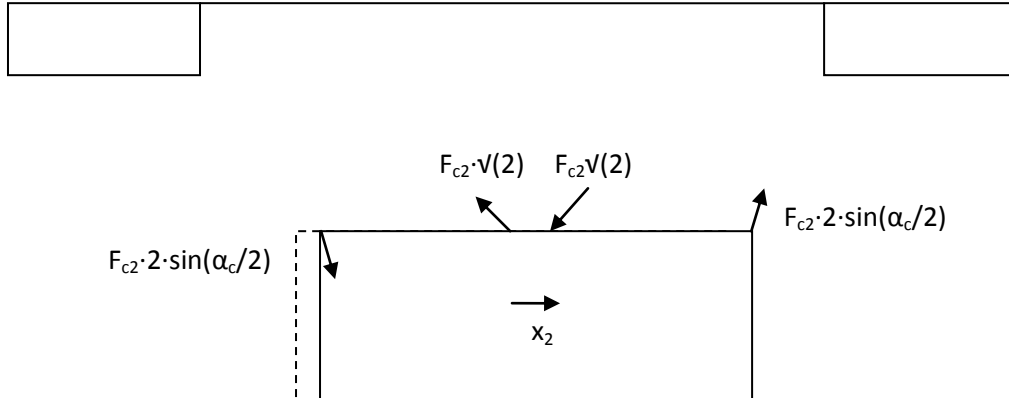


Figure XIX: Forces when $x_1 = 0, x_2 > 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0$

The forces on the tunnel element are shown in Figure XX when:

$$x_1 = 0, x_2 = 0, x_3 > 0, x_4 = 0, x_5 = 0, x_6 = 0$$

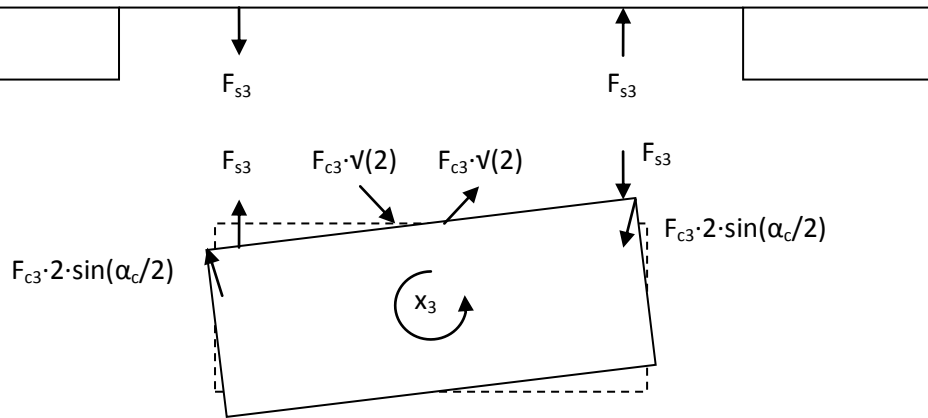


Figure XX: Forces when $x_1 > 0, x_2 = 0, x_3 > 0, x_4 = 0, x_5 = 0, x_6 = 0$

The forces on the tunnel element are shown in Figure XXI when:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 > 0, x_5 = 0, x_6 = 0$$

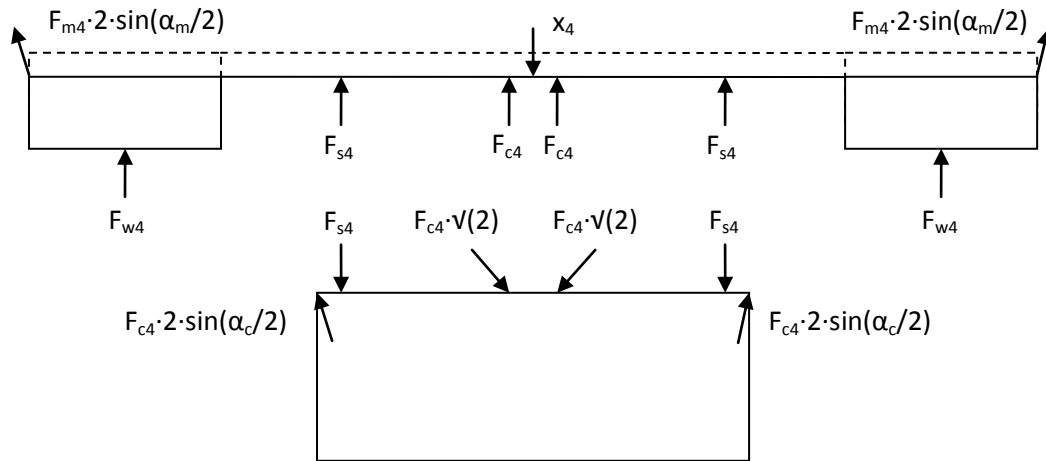


Figure XXI: Forces when $x_1 = 0, x_2 = 0, x_3 = 0, x_4 > 0, x_5 = 0, x_6 = 0$

The forces on the tunnel element are shown in Figure XXII when:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 > 0, x_6 = 0$$

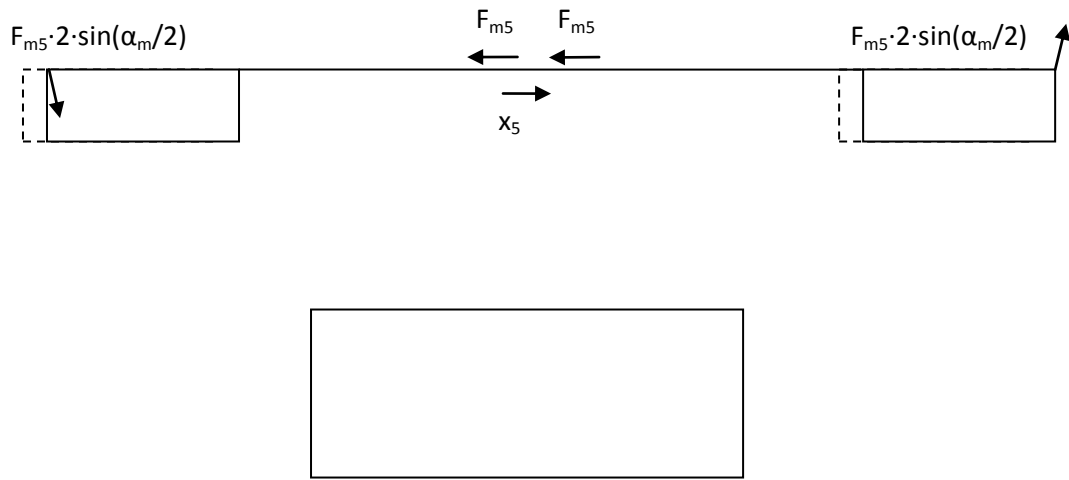


Figure XXII: Forces when $x_1=0, x_2=0, x_3=0, x_4=0, x_5>0, x_6=0$

The forces on the tunnel element are shown in Figure XXIII when:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 > 0$$

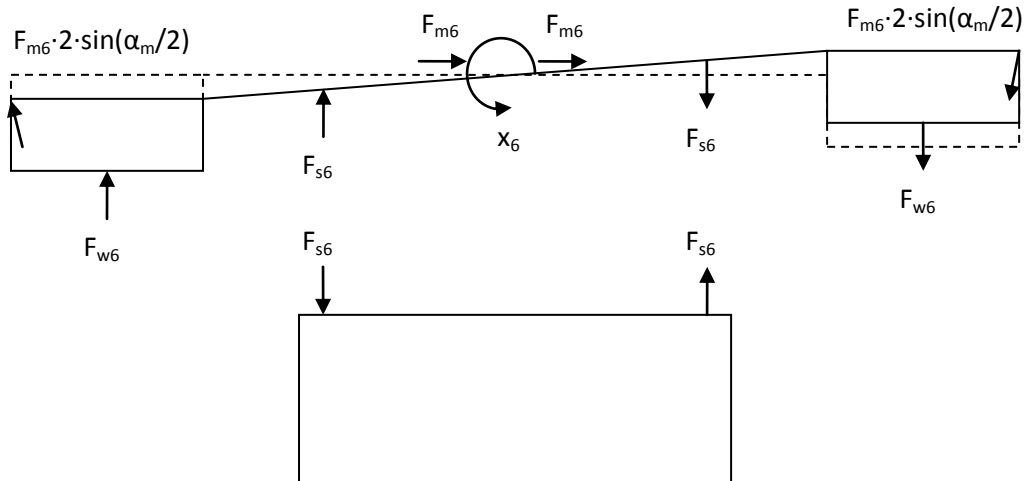


Figure XXIII: Forces when $x_1=0, x_2=0, x_3=0, x_4=0, x_5=0, x_6>0$

B.4 Equations of motion

The equations of motion without added mass, damping and forces becomes:

$$m_e \cdot \ddot{x}_1 = -2 \cdot F_{s1} - 2 \cdot F_{c1} + 2 \cdot F_{c1} \cdot 2 \cdot \sin\left(\frac{\alpha_c}{2}\right) \cdot \cos\left(\frac{\alpha_c}{2}\right) + 2 \cdot F_{s4} + 2 \cdot F_{c4} - 2 \cdot F_{c4} \cdot 2 \cdot \sin\left(\frac{\alpha_c}{2}\right) \cdot \cos\left(\frac{\alpha_c}{2}\right)$$

$$m_e \cdot \ddot{x}_2 = -2 \cdot F_{c2} + 2 \cdot F_{c2} \cdot 2 \cdot \sin\left(\frac{\alpha_c}{2}\right)^2 + 2 \cdot F_{c3} - 2 \cdot F_{c3} \cdot 2 \cdot \sin\left(\frac{\alpha_c}{2}\right)^2$$

$$\begin{aligned}
J_e \cdot \ddot{x}_3 = & 2 \cdot F_{c2} \cdot 2 \cdot \sin\left(\frac{\alpha_c}{2}\right) \cdot \left(\frac{W_e}{2} - \tan\left(\frac{\alpha_c}{2}\right) \cdot \frac{h_e}{2}\right) \cdot \cos\left(\frac{\alpha_c}{2}\right) + 2 \cdot F_{c2} \cdot \frac{h_e}{2} - 2 \cdot F_{c3} \cdot 2 \cdot \sin\left(\frac{\alpha_c}{2}\right) \cdot \\
& \left(\frac{W_e}{2} - \tan\left(\frac{\alpha_c}{2}\right) \cdot \frac{h_e}{2}\right) \cdot \cos\left(\frac{\alpha_c}{2}\right) - 2 \cdot F_{c3} \cdot \frac{h_e}{2} - 2 \cdot F_{s3} \cdot \left(\frac{W_e}{2} - y_e\right) + 2 \cdot F_{s6} \cdot \left(\frac{W_e}{2} - y_e\right) \\
2 \cdot m_p \cdot \ddot{x}_4 = & 2 \cdot F_{s1} + 2 \cdot F_{c1} - 2 \cdot F_{w4} - 2 \cdot F_{s4} - 2 \cdot F_{c4} - 2 \cdot F_{m4} \cdot 2 \cdot \sin\left(\frac{\alpha_m}{2}\right) \cdot \cos\left(\frac{\alpha_m}{2}\right) \\
2 \cdot m_p \cdot \ddot{x}_5 = & -2 \cdot F_{m5} + 2 \cdot F_{m5} \cdot 2 \cdot \sin\left(\frac{\alpha_m}{2}\right)^2 + 2 \cdot F_{m6} - 2 \cdot F_{m6} \cdot 2 \cdot \sin\left(\frac{\alpha_m}{2}\right)^2
\end{aligned}$$

$$\begin{aligned}
2 \cdot J_p \cdot \ddot{x}_6 = & 2 \cdot F_{s3} \cdot \left(\frac{W_e}{2} - y_e\right) + 2 \cdot F_{m5} \cdot h_{pd} + 2 \cdot F_{m5} \cdot 2 \cdot \sin\left(\frac{\alpha_m}{2}\right) \cdot \left(\frac{W_p}{2} - h_{pd} \cdot \tan\left(\frac{\alpha_m}{2}\right)\right) \cdot \\
& \cos\left(\frac{\alpha_m}{2}\right) - 2 \cdot F_{s6} \cdot \left(\frac{W_e}{2} - y_e\right) - 2 \cdot F_{m6} \cdot 2 \cdot \sin\left(\frac{\alpha_m}{2}\right) \cdot \left(\frac{W_p}{2} - h_{pd} \cdot \tan\left(\frac{\alpha_m}{2}\right)\right) \cdot \cos\left(\frac{\alpha_m}{2}\right) - 2 \cdot \\
& F_{m6} \cdot h_{pd} - 2 \cdot F_{w6} \cdot \left(\frac{W_p}{2} - y_p\right)
\end{aligned}$$

The equations of motion can be rewritten into:

$$\begin{aligned}
m_e \cdot \ddot{x}_1 = & -2 \cdot k_s \cdot x_1 + 2 \cdot k_c \cdot x_1 \cdot (2 \cdot \sin(\alpha_c) - 1 - \sin(\alpha_c)^2) + 2 \cdot k_s \cdot x_4 + 2 \cdot k_c \cdot x_4 \cdot (1 - \sin(\alpha_c)) \\
m_e \cdot \ddot{x}_2 = & -2 \cdot k_c \cdot x_2 \cdot \cos(\alpha_c)^2 + k_c \cdot x_3 \cdot (W_e \cdot \sin(\alpha_c) \cdot \cos(\alpha_c) + h_e \cdot \cos(\alpha_c)^2) \\
J_e \cdot \ddot{x}_3 = & k_c \cdot x_2 \cdot (W_e \cdot \sin(\alpha_c) \cdot \cos(\alpha_c) + h_e \cdot \cos(\alpha_c)^2) - k_c \cdot x_3 \cdot \\
& \left(\frac{W_e^2}{2} \cdot \sin(\alpha_c)^2 + W_e \cdot h_e \cdot \cos(\alpha_c) \cdot \sin(\alpha_c) + \frac{h_e^2}{2} \cdot \cos(\alpha_c)^2\right) - 2 \cdot k_s \cdot x_3 \cdot \left(\frac{W_e}{2} - y_e\right)^2 + 2 \cdot k_s \cdot \\
& x_6 \cdot \left(\frac{W_e}{2} - y_e\right)^2 \\
2 \cdot m_p \cdot \ddot{x}_4 = & 2 \cdot k_s \cdot x_1 + 2 \cdot k_c \cdot x_1 - 2 \cdot k_c \cdot x_1 \cdot \sin(\alpha_c) - 2 \cdot k_s \cdot x_4 - 2 \cdot k_c \cdot x_4 - 2 \cdot k_w \cdot x_4 - 2 \cdot k_m \cdot \\
& \sin(\alpha_m)^2 \cdot x_4 \\
2 \cdot m_p \cdot \ddot{x}_5 = & -2 \cdot k_m \cdot \cos(\alpha_m)^2 \cdot x_5 + k_m \cdot x_6 \cdot (W_p \cdot \sin(\alpha_m) \cdot \cos(\alpha_m) + 2 \cdot h_{pd} \cdot \cos(\alpha_m)^2) \\
2 \cdot J_p \cdot \ddot{x}_6 = & 2 \cdot k_s \cdot x_3 \cdot \left(\frac{W_e}{2} - y_e\right)^2 + k_m \cdot x_5 \cdot (W_p \cdot \sin(\alpha_m) \cdot \cos(\alpha_m) + 2 \cdot h_{pd} \cdot \cos(\alpha_m)^2) - 2 \cdot k_s \cdot \\
& \left(\frac{W_e}{2} - y_e\right)^2 \cdot x_6 - 2 \cdot k_w \cdot \left(\frac{W_p}{2} - y_p\right)^2 \cdot x_6 - k_m \cdot x_6 \cdot \\
& \left(\frac{W_p^2}{2} \cdot \sin(\alpha_m)^2 + 2 \cdot W_p \cdot h_{pd} \cdot \sin(\alpha_m) \cdot \cos(\alpha_m) + 2 \cdot h_{pd}^2 \cdot \cos(\alpha_m)^2\right)
\end{aligned}$$

The stiffness of the system can be numerically approximated by:

$$K = \begin{bmatrix} 1.129 \cdot 10^9 & 0 & 0 & -1.13 \cdot 10^9 & 0 & 0 \\ 0 & 4.272 \cdot 10^6 & -3.644 \cdot 10^7 & 0 & 0 & 0 \\ 0 & -3.644 \cdot 10^7 & 1.782 \cdot 10^{11} & 0 & 0 & -1.779 \cdot 10^{11} \\ -1.13 \cdot 10^9 & 0 & 0 & 1.14 \cdot 10^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.741 \cdot 10^6 & -1.713 \cdot 10^7 \\ 0 & 0 & -1.779 \cdot 10^{11} & 0 & -1.713 \cdot 10^7 & 1.809 \cdot 10^{11} \end{bmatrix}$$

One can see that the stiffness matrix is symmetric.

B.5 Non-linear equations of motion, without added mass, damping and forces

If one wants to determine the natural frequencies and the periods with a relation between the horizontal motions of the tunnel element and the pontoons the non-linear effect of the suspension cables must be taken into account.

This effect is shown in Figure XXIV.

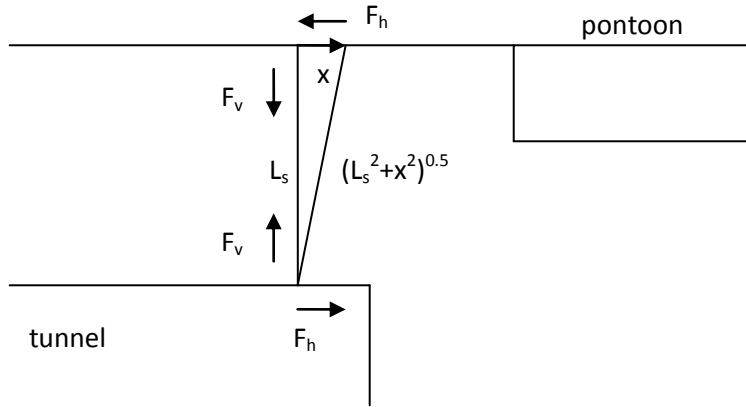


Figure XXIV: Non linear effect of the suspension cables on the horizontal translations

The force in the suspension cable is equal to the spring stiffness multiplied by the elongation of the cable:

$$F_{cable} = k_s \cdot \left(\sqrt{L_s^2 + x^2} - L_s \right)$$

The angle of the cable is equal to:

$$\alpha = \arctan\left(\frac{x}{L_s}\right)$$

The force in the cable can be divided in a horizontal and a vertical component:

$$F_h = \sin\left(\arctan\left(\frac{x}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x^2} - L_s \right)$$

$$F_v = \cos\left(\arctan\left(\frac{x}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x^2} - L_s \right)$$

The coupled equations of motion which are used to analyse the natural periods are linearly dependent of the displacement $x_1, x_2, \dots, x_6$. When the above mentioned effect of the suspension cables is taken into account, the equations of motion are no longer linearly dependent of the

displacement x . Therefore the theory, which is given in paragraph C.1, cannot be used to determine the natural frequencies and periods.

However, it is possible to solve the equations of motions in the time-domain.

Numerical methods should be used to integrate the six equations in order to approximate the natural periods of the system. The results of the numerical analysis should be plotted and the natural periods can be distinguished from the graphs.

One needs 12 initial conditions for the six coupled second order differential equations. The initial conditions consists of a deflection x and a velocity $\dot{x}$ at time $t = 0$ for each degree of freedom.

At $t = 0$ all initial conditions should be 0 except the deflection of degree of freedom i . The displacement of degree of freedom i should be $x_i = 1$. Thereafter, the natural period T_i of motion x_i can be determined by integrating the equations of motion. The output of the analysis can be plotted and the natural period can be read from the graph.

The integration process of the equations of motions is done with help of a computer program.

The effect of the suspension cables on the horizontal translations of the tunnel element and the pontoons is neglected. In this paragraph the non-linear effect of the suspension cables is derived.

The equations of motion in Appendix B.4 are added with the following components, which makes the equations non-linear:

The forces are shown in Figure XXV when:

$$x_1 = 0, x_2 > 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0$$

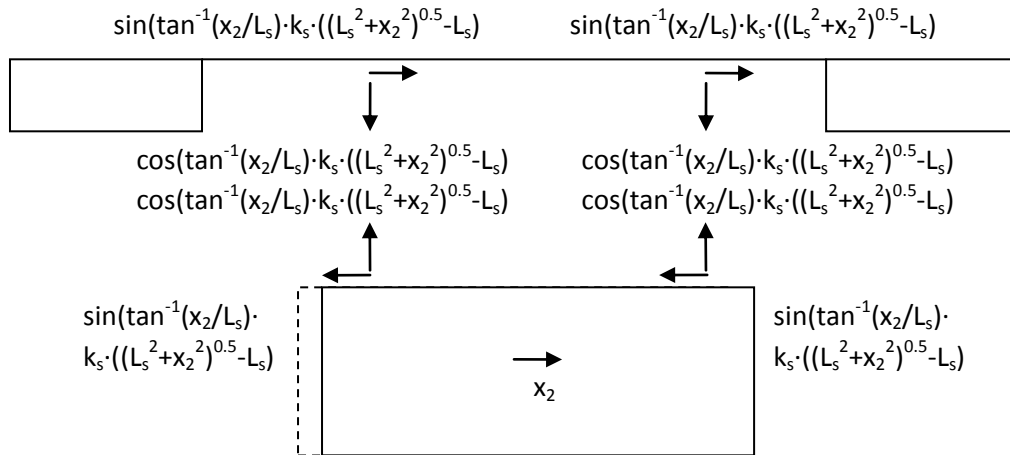


Figure XXV: Forces when $x_1=0, x_2>0, x_3=0, x_4=0, x_5=0, x_6=0$

The reaction forces are shown in Figure XXVI when:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 > 0, x_6 = 0$$

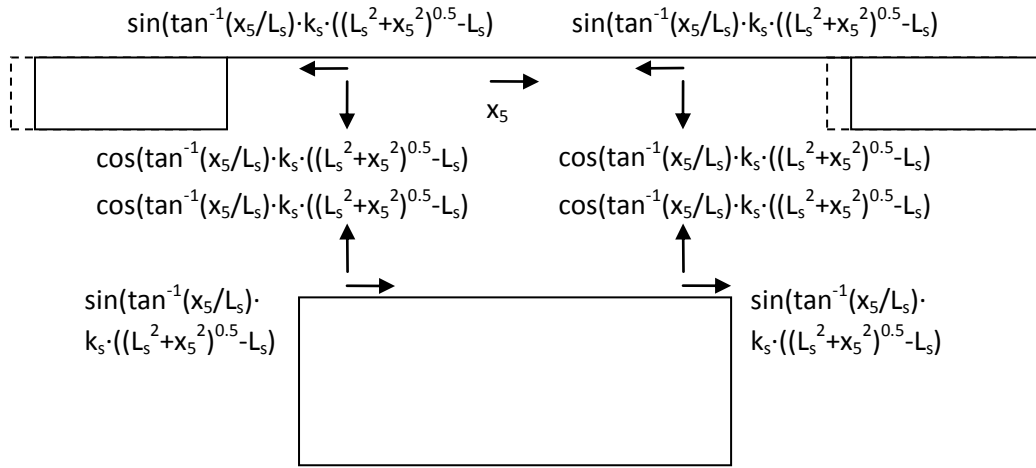


Figure XXVI: Forces when $x_1=0$, $x_2=0$, $x_3=0$, $x_4=0$, $x_5>0$, $x_6=0$

The following non-linear terms are added by the linear equations of motion:

$$m_e \cdot \ddot{x}_1 =$$

$$-2 \cdot \cos\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) - 2 \cdot \cos\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)$$

$$m_e \cdot \ddot{x}_2 =$$

$$-2 \cdot \sin\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) + 2 \cdot \sin\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)$$

$$J_e \cdot \ddot{x}_3 =$$

$$2 \cdot \sin\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) \cdot \frac{h_e}{2} - 2 \cdot \sin\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right) \cdot \frac{h_e}{2}$$

$$2 \cdot m_p \cdot \ddot{x}_4 =$$

$$2 \cdot \cos\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) + 2 \cdot \cos\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)$$

$$2 \cdot m_p \cdot \ddot{x}_5 =$$

$$2 \cdot \sin\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) - 2 \cdot \sin\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)$$

$$2 \cdot J_p \cdot \ddot{x}_6 = 0$$

The equations of motions with the non-linear effect of the suspension cables are given by:

$$m_e \cdot \ddot{x}_1 =$$

$$-2 \cdot k_s \cdot x_1 + 2 \cdot k_c \cdot x_1 \cdot (2 \cdot \sin(\alpha_c) - 1 - \sin(\alpha_c)^2) + 2 \cdot k_s \cdot x_4 + 2 \cdot k_c \cdot x_4 \cdot (1 - \sin(\alpha_c)) - 2 \cdot \cos\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) - 2 \cdot \cos\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)$$

$$\begin{aligned}
m_e \cdot \ddot{x}_2 = & -2 \cdot k_c \cdot x_2 \cdot \cos(\alpha_c)^2 + k_c \cdot x_3 \cdot (W_e \cdot \sin(\alpha_c) \cdot \cos(\alpha_c) + h_e \cdot \cos(\alpha_c)^2) - 2 \cdot \sin\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot \\
& k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) + 2 \cdot \sin\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)
\end{aligned}$$

$$\begin{aligned}
J_e \cdot \ddot{x}_3 = & k_c \cdot x_2 \cdot (W_e \cdot \sin(\alpha_c) \cdot \cos(\alpha_c) + h_e \cdot \cos(\alpha_c)^2) - k_c \cdot x_3 \cdot \\
& \left(\frac{W_e^2}{2} \cdot \sin(\alpha_c)^2 + W_e \cdot h_e \cdot \cos(\alpha_c) \cdot \sin(\alpha_c) + \frac{h_e^2}{2} \cdot \cos(\alpha_c)^2\right) - 2 \cdot k_s \cdot x_3 \cdot \left(\frac{W_e}{2} - y_e\right)^2 + 2 \cdot k_s \cdot \\
& x_6 \cdot \left(\frac{W_e}{2} - y_e\right)^2 + 2 \cdot \sin\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) \cdot \frac{h_e}{2} - 2 \cdot \sin\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \\
& \left(\sqrt{L_s^2 + x_5^2} - L_s\right) \cdot \frac{h_e}{2}
\end{aligned}$$

$$\begin{aligned}
2 \cdot m_p \cdot \ddot{x}_4 = & 2 \cdot k_s \cdot x_1 + 2 \cdot k_c \cdot x_1 - 2 \cdot k_c \cdot x_1 \cdot \sin(\alpha_c) - 2 \cdot k_s \cdot x_4 - 2 \cdot k_c \cdot x_4 - 2 \cdot k_w \cdot x_4 - 2 \cdot k_m \cdot \\
& \sin(\alpha_m)^2 \cdot x_4 + 2 \cdot \cos\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) + 2 \cdot \cos\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \\
& \left(\sqrt{L_s^2 + x_5^2} - L_s\right)
\end{aligned}$$

$$\begin{aligned}
2 \cdot m_p \cdot \ddot{x}_5 = & -2 \cdot k_m \cdot \cos(\alpha_m)^2 \cdot x_5 + k_m \cdot x_6 \cdot (W_p \cdot \sin(\alpha_m) \cdot \cos(\alpha_m) + 2 \cdot h_{pd} \cdot \cos(\alpha_m)^2) + 2 \cdot \\
& \sin\left(\arctan\left(\frac{x_2}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_2^2} - L_s\right) - 2 \cdot \sin\left(\arctan\left(\frac{x_5}{L_s}\right)\right) \cdot k_s \cdot \left(\sqrt{L_s^2 + x_5^2} - L_s\right)
\end{aligned}$$

$$\begin{aligned}
2 \cdot J_p \cdot \ddot{x}_6 = & 2 \cdot k_s \cdot x_3 \cdot \left(\frac{W_e}{2} - y_e\right)^2 + k_m \cdot x_5 \cdot (W_p \cdot \sin(\alpha_m) \cdot \cos(\alpha_m) + 2 \cdot h_{pd} \cdot \cos(\alpha_m)^2) - 2 \cdot k_s \cdot \\
& \left(\frac{W_e}{2} - y_e\right)^2 \cdot x_6 - 2 \cdot k_w \cdot \left(\frac{W_p}{2} - y_p\right)^2 \cdot x_6 - k_m \cdot x_6 \cdot \\
& \left(\frac{W_p^2}{2} \cdot \sin(\alpha_m)^2 + 2 \cdot W_p \cdot h_{pd} \cdot \sin(\alpha_m) \cdot \cos(\alpha_m) + 2 \cdot h_{pd}^2 \cdot \cos(\alpha_m)^2\right)
\end{aligned}$$

The Non-linear equations are only derived and are not used to determine the dynamic behaviour.

Appendix C Modal analysis

In this Appendix the Modal Analysis is explained which is used to determine the natural frequencies and the response amplitude operators.

C.1 Method to determine the natural frequencies

Consider free vibrations of a structure without damping and without external excitation. The equation of motion in this case turns into the following equation (Gawronski 2004):

$$M \cdot \ddot{\underline{x}} + K \cdot \underline{x} = \underline{0}$$

The solution of the above equation is:

$$\underline{x} = \underline{\hat{x}} \cdot e^{i \cdot \omega \cdot t}$$

Hence, the second derivative of the solution is:

$$\ddot{\underline{x}} = -\omega^2 \cdot \underline{\hat{x}} \cdot e^{i \cdot \omega \cdot t}$$

Introducing the latter u and $\ddot{u}$ into $M \cdot \ddot{u} + K \cdot u = 0$ gives:

$$(K - \omega^2 \cdot M) \cdot \underline{\hat{x}} \cdot e^{i \cdot \omega \cdot t} = \underline{0}$$

This is a set of homogeneous equations for which a non-trivial solution exists if the determinant $K - \omega^2 \cdot M$ is equal to zero:

$$\det(K - \omega^2 \cdot M) = 0$$

The above determinant equation is satisfied for a set of n values of frequency ω . These frequencies are denoted $\omega_1, \omega_2, \dots, \omega_n$, and their number n does not exceed the number of degrees of freedom, i.e. $n \leq n_d$. The frequency ω_i is called the i^{th} natural frequency (Gawronski 2004).

Substituting ω_i in $(K - \omega^2 \cdot M) \cdot \underline{\hat{x}} \cdot e^{i \cdot \omega \cdot t} = \underline{0}$, yields the corresponding set of vectors $\{\underline{\hat{x}}_1, \underline{\hat{x}}_2, \dots, \underline{\hat{x}}_n\}$ that satisfy this equation. The i^{th} vector $\underline{\hat{x}}_i$ corresponding to the i^{th} natural frequency is called the i^{th} natural mode or mode shape. The natural modes are not unique, since they can be arbitrarily scaled.

The matrix of natural frequencies is defined by:

$$\Omega = \begin{bmatrix} \omega_1 & 0 & \dots & 0 \\ 0 & \omega_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \omega_n \end{bmatrix}$$

The Eigenmatrix or matrix of mode shapes or modal matrix E , of dimensions $n_d \times n$, which consists of n natural modes of a structure is given by (Gawronski 2004):

$$E = [\underline{\hat{x}}_1 \quad \underline{\hat{x}}_2 \quad \dots \quad \underline{\hat{x}}_n] = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{21} & \dots & \hat{x}_{n1} \\ \hat{x}_{12} & \hat{x}_{22} & \dots & \hat{x}_{n2} \\ \dots & \dots & \dots & \dots \\ \hat{x}_{1n_d} & \hat{x}_{2n_d} & \dots & \hat{x}_{nn_d} \end{bmatrix}$$

where $\hat{x}_{ij}$ is the j^{th} displacement of the i^{th} mode.

The free vibration of the system consists of the summation of the n principal modes of vibrations (Spijkers, Vrouwenvelder and Klaver 2006):

$$\underline{x}(t) = \sum_{i=1}^n \hat{x}_i \cdot u_i(t) = E \cdot \underline{u}(t)$$

C.2 Modal analysis

The particular solutions are also assumed to be a summation of eigenvectors.

$$M \cdot E \cdot \underline{\ddot{u}} + K \cdot E \cdot \underline{u} = \underline{F}(t)$$

Pre-multiplication of the left and right expressions by the transposed Eigenmatrix gives:

$$E^T \cdot M \cdot E \cdot \underline{\ddot{u}} + E^T \cdot K \cdot E \cdot \underline{u} = E^T \cdot \underline{F}(t)$$

The Matrices M^* and K^* are defined as the Modal Mass Matrix and the Modal Stiffness Matrix:

$$M^* = E^T \cdot M \cdot E$$

$$K^* = E^T \cdot K \cdot E$$

The Matrices M^* and K^* are diagonal due to orthogonality conditions (Spijkers, Vrouwenvelder and Klaver 2006).

The equations of motion can be rewritten into:

$$M^* \cdot \underline{\ddot{u}} + K^* \cdot \underline{u} = E^T \cdot \underline{F}$$

The following relation holds between the diagonal matrices M^* and K^* :

$$\omega^2 \cdot M^* = K^*$$

Which can be written as (Spijkers, Vrouwenvelder and Klaver 2006):

$$\begin{bmatrix} \omega_1^2 & 0 & \dots & 0 \\ 0 & \omega_2^2 & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \omega_n^2 \end{bmatrix} \cdot \begin{bmatrix} m_{11}^* & 0 & \dots & 0 \\ 0 & m_{22}^* & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & m_{33}^* \end{bmatrix} = \begin{bmatrix} k_{11}^* & 0 & \dots & 0 \\ 0 & k_{22}^* & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & k_{33}^* \end{bmatrix}$$

Because it concerns a fully uncoupled system, the equations above can also be written as:

$$m_{ii}^* \cdot \ddot{u}_i + m_{ii}^* \cdot \omega_i^2 \cdot u_i = \hat{x}_i^T \cdot \underline{F}(t) \quad (i = 1, 2, \dots, n)$$

$$\ddot{u}_i + \omega_i^2 \cdot u_i = \frac{\hat{x}_i^T \cdot \underline{F}(t)}{\hat{x}_i^T \cdot M \cdot \hat{x}_i} = \frac{F_i^*(t)}{m_{ii}^*} \quad (i = 1, 2, \dots, n)$$

C.3 Frequency Response Functions

In this paragraph the response of a harmonic load function on the system is analysed. The harmonic load function is given by:

$$F(t) = \hat{F} \cdot \exp(i \cdot \omega \cdot t)$$

The particular solution is assumed to be also a harmonic time function, having the same frequency as the load function. The amplitude is unknown yet:

$$u_i(t) = \hat{u}_i \cdot \exp(i \cdot \omega \cdot t)$$

C.3.1 Response on harmonic load (undamped)

The amplitude can be determined by substituting the load and response function in the decoupled system. The equation of motion without damping is equal to:

$$M^* \cdot \ddot{\underline{u}} + K^* \cdot \underline{u} = E^T \cdot \underline{F}$$

Substituting the load function and amplitude function into the equations of motion gives:

$$(-\omega^2 \cdot \hat{u}_i + \omega_i^2 \cdot \hat{u}_i) \cdot \exp(i \cdot \omega \cdot t) = \frac{\hat{\underline{x}}_i^T \cdot \hat{\underline{F}} \cdot \exp(i \cdot \omega \cdot t)}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

For the amplitude follows (Spijkers, Vrouwenvelder and Klaver 2006):

$$\hat{u}_i = \frac{1}{\omega_i^2 - \omega^2} \cdot \frac{\hat{\underline{x}}_i^T \cdot \hat{\underline{F}}}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

If the load vector consists of only one harmonic load and is active at degree of freedom x_p , the load vector reads:

$$\hat{\underline{F}} = [0 \quad 0 \quad \dots \quad 0 \quad \hat{F}_p \quad 0 \quad \dots \quad 0]^T$$

The product $\hat{\underline{x}}_i^T \cdot \hat{\underline{F}}$ simplifies to one term:

$$\hat{\underline{x}}_i^T \cdot \hat{\underline{F}} = [\hat{x}_{1i} \quad \hat{x}_{2i} \quad \dots \quad \hat{x}_{pi} \quad \dots \quad \hat{x}_{ni}] \cdot \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ \hat{F}_p \\ 0 \\ \dots \\ 0 \end{bmatrix} = \hat{x}_{pi} \cdot \hat{F}_p$$

The amplitude $\hat{u}_i$ is equal to:

$$\hat{u}_i = \frac{1}{1 - \left(\frac{\omega}{\omega_i}\right)^2} \cdot \frac{1}{\omega_i^2} \cdot \frac{\hat{x}_{pi} \cdot \hat{F}_p}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

The frequency response function is defined as (Spijkers, Vrouwenvelder and Klaver 2006):

$$H_{u_i F_p}(\omega) = \frac{u_i(t)}{F_p(t)} = \frac{\hat{u}_i}{\hat{F}_p} = \frac{1}{1 - \left(\frac{\omega}{\omega_i}\right)^2} \cdot \frac{1}{\omega_i^2} \cdot \frac{\hat{x}_{pi}}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n), (p = 1, 2, \dots, n)$$

i and p can assume values between 1 and n . As a result, n^2 different response frequencies functions are defined:

$$H_{u_i F_p} = \begin{bmatrix} H_{u_1 F_1} & H_{u_1 F_2} & \dots & H_{u_1 F_n} \\ H_{u_2 F_1} & H_{u_2 F_2} & \dots & H_{u_2 F_n} \\ \dots & \dots & \dots & \dots \\ H_{u_n F_1} & H_{u_n F_2} & \dots & H_{u_n F_n} \end{bmatrix}$$

The relation between the harmonic functions $\hat{\underline{u}}$ and $\hat{\underline{F}}$ is

$$\hat{\underline{u}} = H_{u_i F_p} \cdot \hat{\underline{F}}$$

The response formulated in the physical degrees of freedom $\underline{x}(t)$ is equal to:

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_n(t) \end{bmatrix} = \sum_{i=1}^n \hat{x}_i \cdot \frac{1}{1 - \left(\frac{\omega}{\omega_i}\right)^2} \cdot \frac{1}{\omega_i^2} \cdot \frac{\hat{x}_{pi} \cdot \hat{F}_p}{\hat{x}_i^T \cdot M \cdot \hat{x}_i} \cdot \exp(i \cdot \omega \cdot t)$$

The expression given above is the steady-state response. The steady-state response is the response after the transient motion has died out.

Again, a frequency response function can be made which concerns a harmonic load at the position of degree of freedom x_q (Spijkers, Vrouwenvelder and Klaver 2006):

$$H_{x_q F_p}(\omega) = \frac{x_q(t)}{F_p(t)} = \frac{\hat{x}_q}{\hat{F}_p} = \sum_{i=1}^n \frac{1}{1 - \left(\frac{\omega}{\omega_i}\right)^2} \cdot \frac{1}{\omega_i^2} \cdot \frac{\hat{x}_{qi} \cdot \hat{x}_{pi}}{\hat{x}_i^T \cdot M \cdot \hat{x}_i} \quad (p = 1, 2, \dots, n), (q = 1, 2, \dots, n)$$

This is the frequency response function (FRF) of degree of freedom i to the force applied to degree of freedom p .

The frequency response function gives vertical asymptotes at the position of the natural frequencies ω_i ($i = 1, 2, \dots, n$).

The frequency response matrix $H_{x_q F_p}$ contains n^2 possible combinations:

$$H_{x_q F_p} = \begin{bmatrix} H_{x_1 F_1} & H_{x_1 F_2} & \dots & H_{x_1 F_n} \\ H_{x_2 F_1} & H_{x_2 F_2} & \dots & H_{x_2 F_n} \\ \dots & \dots & \dots & \dots \\ H_{x_n F_1} & H_{x_n F_2} & \dots & H_{x_n F_n} \end{bmatrix}$$

The frequency response matrix $H_{x_q F_p}$ is symmetrical, because of Maxwell's Law (Spijkers, Vrouwenvelder and Klaver 2006).

The relation between the amplitude vectors of the response and load is given by:

$$\hat{\underline{x}} = H_{x_q F_p} \cdot \hat{\underline{F}}$$

C.3.2 Response on harmonic load (damped)

The ratio between the amplitudes of the load and the amplitude of the response is infinity at the position of the natural frequency for the undamped frequency response functions.

In reality however, this is impossible, because damping is present.

The equation of motion with linear damping is equal to:

$$M \cdot \ddot{\underline{x}} + C \cdot \dot{\underline{x}} + K \cdot \underline{x} = \underline{F}(t)$$

The forced vibration is assumed to be expanded in eigenvectors, which are known from the undamped system.

$$\underline{x}(t) = \sum_{i=1}^n \hat{x}_i \cdot u_i(t) = E \cdot \underline{u}(t)$$

Multiplication with the Eigenmatrix and the transposed Eigematrix gives:

$$E^T \cdot M \cdot E \cdot \ddot{\underline{u}} + E^T \cdot C \cdot E \cdot \dot{\underline{u}} + E^T \cdot K \cdot E \cdot \underline{u} = E^T \cdot \underline{F}(t)$$

The modal mass and modal stiffness matrix are defined in paragraph C.2. The modal damping matrix is defined as:

$$C^* = E^T \cdot C \cdot E$$

Until now, no special requirements have been laid upon the damping matrix C . In general, the modal damping matrix is not diagonal, because the eigenvectors are not orthogonal with respect to the damping matrix (Spijkers, Vrouwenvelder and Klaver 2006).

The modal analysis for damped systems does not result in a fully uncoupled system, and is therefore in its most simple form not applicable for damped systems.

In this report, the damping matrix is assumed to be diagonal. The advantage of this assumption is that the set of differential equations is fully decoupled:

$$M^* \cdot \ddot{\underline{u}} + C^* \cdot \dot{\underline{u}} + K^* \cdot \underline{u} = E^T \cdot \underline{F}(t)$$

The differential equation can be rewritten into:

$$\ddot{u}_i + \frac{\hat{\underline{x}}_i^T \cdot C \cdot \hat{\underline{x}}_i}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \cdot \dot{u}_i + \omega_i^2 \cdot u_i = \frac{\hat{\underline{x}}_i^T \cdot \underline{F}(t)}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

The modal damping ratio ξ_i is defined as the relative damping ratio per uncoupled degree of freedom.

$$2 \cdot \xi_i \cdot \omega_i = \frac{\hat{\underline{x}}_i^T \cdot C \cdot \hat{\underline{x}}_i}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

The differential equation gets the following form:

$$\ddot{u}_i + 2 \cdot \xi_i \cdot \omega_i \cdot \dot{u}_i + \omega_i^2 \cdot u_i = \frac{\hat{\underline{x}}_i^T \cdot \underline{F}(t)}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

The response of a harmonic load function is analysed. $\hat{\underline{F}}$ is a vector which can be complex.

$$\underline{F}(t) = \hat{\underline{F}} \cdot \exp(i \cdot \omega \cdot t)$$

The response function is assumed to be harmonic too, having the same frequency as the load function, but with a phase shift φ_i .

$$u_i(t) = \hat{u}_i \cdot \exp(i \cdot \omega \cdot t)$$

The amplitude is derived by substituting the load function and response function in the equation of motion.

$$\hat{u}_i = \frac{1}{-\omega^2 + 2 \cdot i \cdot \xi_i \cdot \omega \cdot \omega_i + \omega_i^2} \cdot \frac{\hat{\underline{x}}_i^T \cdot \hat{\underline{F}}}{\hat{\underline{x}}_i^T \cdot M \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

The phase shift is already included in the previous expression, because the sine function is written in the complex notation. The phase shift can also be determined separately:

$$\tan(\varphi_i) = \frac{2 \cdot \xi_i \cdot \frac{\omega}{\omega_i}}{1 - \left(\frac{\omega}{\omega_i}\right)^2} \quad (i = 1, 2, \dots, n)$$

The amplitude is also equal to:

$$\hat{u}_i = \frac{\hat{\underline{x}}_i^T \cdot \hat{\underline{F}}}{-\omega^2 \cdot \hat{\underline{x}}_i^T \cdot \underline{M} \cdot \hat{\underline{x}}_i + i \cdot \omega \cdot \hat{\underline{x}}_i^T \cdot \underline{C} \cdot \hat{\underline{x}}_i + \hat{\underline{x}}_i^T \cdot \underline{K} \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n)$$

If the load vector consists of only one harmonic load and is active at degree of freedom x_p , the load vector reads:

$$\hat{\underline{F}} = [0 \quad 0 \quad \dots \quad 0 \quad \hat{F}_p \quad 0 \quad \dots \quad 0]^T$$

The product $\hat{\underline{x}}_i^T \cdot \hat{\underline{F}}$ simplifies to one term:

$$\hat{\underline{x}}_i^T \cdot \hat{\underline{F}} = [\hat{x}_{1i} \quad \hat{x}_{2i} \quad \dots \quad \hat{x}_{pi} \quad \dots \quad \hat{x}_{ni}] \cdot \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ \hat{F}_p \\ 0 \\ \dots \\ 0 \end{bmatrix} = \hat{x}_{pi} \cdot \hat{F}_p$$

The amplitude in this case is equal to:

$$\hat{u}_i = \frac{1}{-\omega^2 + 2 \cdot i \cdot \xi_i \cdot \omega \cdot \omega_i + \omega_i^2} \cdot \frac{\hat{x}_{pi} \cdot \hat{F}_p}{\hat{\underline{x}}_i^T \cdot \underline{M} \cdot \hat{\underline{x}}_i}$$

The frequency response function $H_{u_i F_p}(\omega)$ is defined as (Spijkers, Vrouwenvelder and Klaver 2006):

$$H_{u_i F_p}(\omega) = \frac{\hat{u}_i}{\hat{F}_p} \neq \frac{u_i(t)}{F_p(t)}$$

$$H_{u_i F_p}(\omega) = \frac{\hat{u}_i}{\hat{F}_p} = \frac{1}{-\omega^2 + 2 \cdot i \cdot \xi_i \cdot \omega \cdot \omega_i + \omega_i^2} \cdot \frac{\hat{x}_{pi}}{\hat{\underline{x}}_i^T \cdot \underline{M} \cdot \hat{\underline{x}}_i} \quad (i = 1, 2, \dots, n), (p = 1, 2, \dots, n)$$

The presence of damping causes a phase shift. The phase shift occurs in the response $u_i(t)$ with respect to the load function $F_p(t)$. Here the advantage of writing the sine and cosine function in complex notation appears, because the phase shift is already included in the analysis.

The frequency response functions are summarized in a matrix:

$$H_{u_i F_p} = \begin{bmatrix} H_{u_1 F_1} & H_{u_1 F_2} & \dots & H_{u_1 F_n} \\ H_{u_2 F_1} & H_{u_2 F_2} & \dots & H_{u_2 F_n} \\ \dots & \dots & \dots & \dots \\ H_{u_n F_1} & H_{u_n F_2} & \dots & H_{u_n F_n} \end{bmatrix}$$

The response in the physical degree of freedom x is:

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_n(t) \end{bmatrix} = \sum_{i=1}^n \underline{\hat{x}}_i \cdot \frac{1}{-\omega^2 + 2 \cdot i \cdot \xi_i \cdot \omega \cdot \omega_i + \omega_i^2} \cdot \frac{\hat{x}_{pi} \cdot \hat{F}_p}{\underline{\hat{x}}_i^T \cdot M \cdot \underline{\hat{x}}_i} \cdot \exp(i \cdot \omega \cdot t)$$

Again, a frequency response function can be made which concerns a harmonic load at the position of degree of freedom x_q (Spijkers, Vrouwenvelder and Klaver 2006):

$$H_{x_q F_p}(\omega) = \frac{\hat{x}_q}{\hat{F}_p} = \sum_{i=1}^n \frac{1}{-\omega^2 + 2 \cdot i \cdot \xi_i \cdot \omega \cdot \omega_i + \omega_i^2} \cdot \frac{\hat{x}_{qi} \cdot \hat{x}_{pi}}{\underline{\hat{x}}_i^T \cdot M \cdot \underline{\hat{x}}_i} \quad (p = 1, 2, \dots, n), (q = 1, 2, \dots, n)$$

Note that when the damping factor ξ_i is equal to 0, the response function is equal to the undamped function which is given in paragraph .

The frequency response matrix $H_{x_q F_p}$ contains n^2 possible combinations:

$$H_{x_q F_p} = \begin{bmatrix} H_{x_1 F_1} & H_{x_1 F_2} & \dots & H_{x_1 F_n} \\ H_{x_2 F_1} & H_{x_2 F_2} & \dots & H_{x_2 F_n} \\ \dots & \dots & \dots & \dots \\ H_{x_n F_1} & H_{x_n F_2} & \dots & H_{x_n F_n} \end{bmatrix}$$

The total response is equal to:

$$\underline{\hat{x}} = H_{x_q F_p} \cdot \underline{\hat{F}}$$

Appendix D Maple file

The motions of the tunnel element and the pontoons are determined with Maple. The entire Maple sheet is given in this Appendix.

restart : with(LinearAlgebra) :

$M := \text{Matrix}([[me, 0, 0, 0, 0, 0], [0, me, 0, 0, 0, 0], [0, 0, Je, 0, 0, 0], [0, 0, 0, 2 \cdot mp, 0, 0], [0, 0, 0, 0, 2 \cdot mp, 0], [0, 0, 0, 0, 0, 2 \cdot Jp]])$: $A := \text{Matrix}([[a11 \cdot \rho \cdot We \cdot he \cdot Le, 0, 0, 0, 0, 0], [0, a22 \cdot \rho \cdot We \cdot he \cdot Le, 0, 0, 0, 0], [0, 0, a33 \cdot \rho \cdot We^3 \cdot he \cdot Le, 0, 0, 0], [0, 0, 0, 2 \cdot 2 \cdot a44 \cdot \rho \cdot Wf \cdot drp \cdot Lp, 0, 0], [0, 0, 0, 0, 2 \cdot 2 \cdot a55 \cdot \rho \cdot Wf \cdot drp \cdot Lp, 0], [0, 0, 0, 0, 0, 2 \cdot 2 \cdot \left(\frac{Wp}{2} - yp \right)^2 \cdot \rho \cdot drp \cdot Wf \cdot Lp \cdot a66]])$: $drp := \frac{(2 \cdot mp + 0.02 \cdot me) \cdot g}{2 \cdot kw}$:

$Kx := \text{Matrix}([[[2 \cdot ks - 2 \cdot kc \cdot (2 \cdot \sin(\alpha c) - 1 - \sin(\alpha c))^2], 0, 0, -2 \cdot ks - 2 \cdot kc \cdot (1 - \sin(\alpha c))], [0, 2 \cdot kc \cdot \cos(\alpha c)^2, -kc \cdot (We \cdot \sin(\alpha c) \cdot \cos(\alpha c) + he \cdot \cos(\alpha c)^2), 0, 0, 0], [0, -kc \cdot (We \cdot \cos(\alpha c) \cdot \sin(\alpha c) + he \cdot \cos(\alpha c)^2), kc \cdot \left(\frac{We^2}{2} \cdot \sin(\alpha c)^2 + We \cdot he \cdot \cos(\alpha c) \cdot \sin(\alpha c) + \frac{he^2}{2} \cdot \cos(\alpha c)^2 \right) + 2 \cdot \left(\frac{We}{2} - ye \right)^2 \cdot ks, 0, 0, -2 \cdot ks \cdot \left(\frac{We}{2} - ye \right)^2], [-2 \cdot ks - 2 \cdot kc + 2 \cdot kc \cdot \sin(\alpha c), 0, 0, 2 \cdot ks + 2 \cdot kw + 2 \cdot kc + 2 \cdot km \cdot \sin(\alpha m)^2], [0, 0, 0, 2 \cdot km \cdot \cos(\alpha m)^2, -km \cdot (Wp \cdot \sin(\alpha m) \cdot \cos(\alpha m) + 2 \cdot hpd \cdot \cos(\alpha m)^2)], [0, 0, -2 \cdot ks \cdot \left(\frac{We}{2} - ye \right)^2, 0, -km \cdot (Wp \cdot \cos(\alpha m) \cdot \sin(\alpha m) + 2 \cdot hpd \cdot \cos(\alpha m)^2), 2 \cdot ks \cdot \left(\frac{We}{2} - ye \right)^2 + 2 \cdot kw \cdot \left(\frac{Wp}{2} - yp \right)^2 + km \cdot \left(\frac{Wp^2}{2} \cdot \sin(\alpha m)^2 + 2 \cdot Wp \cdot \sin(\alpha m) \cdot \cos(\alpha m) \cdot hpd + 2 \cdot hpd^2 \cdot \cos(\alpha m)^2 \right)]]])$:

$We := 26.46 : he := 9.97 : Wp := 42.5 : Wf := 6 : \alpha c := \frac{15}{180} \cdot \text{Pi} : \alpha m := \frac{10}{180} \cdot \text{Pi} : hpd := 2.5 : yp := 3 : km := 1413.3 \cdot 1000 : ks := 563268 \cdot 1000 : kc := 2289.4 \cdot 1000 : kw := 4343868 : ye := 0.665 : h := 23 : g := 9.81 : d := 1 : \rho := 1025 : Le := 180 : Lp := 36 : K := \text{Matrix}(6, 6) :$

for i from 1 to 6 do for j from 1 to 6 do $K[i, j] := \text{evalf}(Kx[i, j])$ end end ; K;

$$\begin{bmatrix} 1.12905136010^9 & 0. & 0. & -1.12992971910^9 & 0. & 0. \\ 0. & 4.27207855910^6 & -3.64406926210^7 & 0. & 0. & 0. \\ 0. & -3.64406926210^7 & 1.78167468510^{11} & 0. & 0. & -1.77856630610^{11} \\ -1.12992971910^9 & 0. & 0. & 1.13988776810^9 & 0. & 0. \\ 0. & 0. & 0. & 0. & 2.74136758110^6 & -1.71251816610^7 \\ 0. & 0. & -1.77856630610^{11} & 0. & -1.71251816610^7 & 1.80857169810^{11} \end{bmatrix}$$

$me := 1.02 \cdot \rho \cdot he \cdot We \cdot Le : Je := 3.74 \cdot 10^9 : mp := 1400 \cdot 10^3 : Jp := 2.75 \cdot 10^8 : a11 := 1 : a22 := 1.25 : a33 := 0.035 : a44 := 1 : a55 := 1.5 : a66 := 1 : In := \text{Matrix}(6, 6) :$

for i from 1 to 6 do for j from 1 to 6 do $In[i, j] := \text{evalf}(M[i, j] + A[i, j])$ end end ; M; A; In;

$$\begin{bmatrix}
4.96456887810^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 4.96456887810^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 3.74000000010^9 & 0 & 0 & 0 \\
0 & 0 & 0 & 2800000 & 0 & 0 \\
0 & 0 & 0 & 0 & 2800000 & 0 \\
0 & 0 & 0 & 0 & 0 & 5.50000000010^8
\end{bmatrix}$$

$$\begin{bmatrix}
4.86722439010^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.08403048810^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.19269416010^9 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.79291377610^6 & 0 & 0 \\
0 & 0 & 0 & 0 & 5.68937066410^6 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.26327734410^9
\end{bmatrix}$$

$$\begin{bmatrix}
9.83179326810^7 & 0. & 0. & 0. & 0. & 0. \\
0. & 1.10485993710^8 & 0. & 0. & 0. & 0. \\
0. & 0. & 4.93269416010^9 & 0. & 0. & 0. \\
0. & 0. & 0. & 6.59291377610^6 & 0. & 0. \\
0. & 0. & 0. & 0. & 8.48937066410^6 & 0. \\
0. & 0. & 0. & 0. & 0. & 1.81327734410^9
\end{bmatrix}$$

$NFsquared, Ex := Eigenvectors(K, In) :$

$T := \text{Vector}(6) : w := \text{Vector}(6) :$

for i **from** 1 **to** 6

do $w[i] := evalf(\text{sqrt}(\text{Re}(NFsquared[i]))) :$

end; $w, E := \text{Matrix}(6, 6) :$

for i **from** 1 **to** 6

do $T[i] := evalf\left(\frac{2 \cdot \text{Pi}}{\text{sqrt}(\text{Re}(NFsquared[i]))}\right) :$

end; $T;$

$$\begin{bmatrix}
13.57548412 \\
0.2929311951 \\
11.63499590 \\
0.1858562629 \\
0.7189311863 \\
0.5452028587
\end{bmatrix}$$

$$\begin{bmatrix}
0.4628332405 \\
21.44935539 \\
0.5400247117 \\
33.80669130 \\
8.739619908 \\
11.52449076
\end{bmatrix}$$

for i **from** 1 **to** 6 **do** **for** j **from** 1 **to** 6 **do** $E[i,j] := \text{Re}(\text{evalf}(Ex[i,j]))$: **end**: **end**;

E ;

$$\begin{bmatrix} 0.06635769708 & 0.7100273792 & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.0008320524674 & 0.9961620685 & 0.06638096247 & -0.01632914772 \\ 0. & 0. & 0.3414127388 & 0.01245481117 & -0.09624316801 & 0.01280203185 \\ -0.9977958990 & 0.7041740700 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.01403794973 & 0.08576531970 & 0.9885834315 & 0.9997037622 \\ 0. & 0. & -0.9398082706 & 0.01226054483 & -0.09504535424 & 0.01272221335 \end{bmatrix}$$

$Kstar1 := \text{Transpose}(E).$ $K.E : Kstar := \text{Matrix}(6, 6)$:

for i **from** 1 **to** 6 **do** $Kstar[i,i] := \text{Re}(\text{evalf}(Kstar1[i,i]))$: **end**:

$Kstar$;

$$\begin{bmatrix} 1.28946859510^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.53370719710^6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.94643410410^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.82523290110^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.66218701010^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.85823535910^6 \end{bmatrix}$$

$Instar1 := \text{Transpose}(E).$ $\text{In}.E : Instar := \text{Matrix}(6, 6)$:

for i **from** 1 **to** 6 **do** $Instar[i,i] := \text{Re}(\text{evalf}(Instar1[i,i]))$: **end**:

$Instar$;

$$\begin{bmatrix} 6.99681058410^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.28350620010^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.17652802310^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.10739735510^8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.08542445010^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.61571845810^6 \end{bmatrix}$$

$Cstar := \text{Matrix}(6, 6)$:

for i **from** 1 **to** 6 **do** $Cstar[i,i] := 2 \cdot \text{sqrt}(Kstar[i,i] \cdot Instar[i,i])$; **end**:

$Cstar$;

$$\begin{bmatrix} 1.89970182010^8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.09540757210^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.06477892810^{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.11633467810^7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.01878652110^8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.04850343810^7 \end{bmatrix}$$

Froude Krylov Forces

$$FTEh := \rho \cdot g \cdot Le \cdot \left(-\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} + \frac{\cosh(\kappa \cdot (h - (d + he)))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \int \left(\exp(-I \cdot \kappa \cdot y), y = -\frac{We}{2} .. \frac{We}{2} \right) :$$

$$FTEs := \rho \cdot g \cdot Le \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\exp\left(\frac{I \cdot \kappa \cdot We}{2}\right) - \exp\left(-\frac{I \cdot \kappa \cdot We}{2}\right) \right) \cdot \int \left(\frac{\cosh(\kappa \cdot (h - 0.5 \cdot he - d + z))}{\cosh(\kappa \cdot h)}, z = -\frac{he}{2} .. \frac{he}{2} \right) :$$

$$FTEr := \rho \cdot g \cdot Le \cdot \left(\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} - \frac{\cosh(\kappa \cdot (h - (d + he)))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \int \left(y \cdot \exp(-I \cdot \kappa \cdot y), y = -\frac{We}{2} .. \frac{We}{2} \right) - \rho \cdot g \cdot Le \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\exp\left(\frac{I \cdot \kappa \cdot We}{2}\right) - \exp\left(-\frac{I \cdot \kappa \cdot We}{2}\right) \right) \cdot \int \left(z \cdot \frac{\cosh(\kappa \cdot (h - 0.5 \cdot he - d + z))}{\cosh(\kappa \cdot h)}, z = -\frac{he}{2} .. \frac{he}{2} \right) :$$

$$FPh := \rho \cdot g \cdot 2 \cdot Lp \cdot \left(\frac{\cosh(\kappa \cdot (h - drp))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\int \left(\exp(-I \cdot \kappa \cdot y), y = -\frac{Wp}{2} .. -\frac{Wp}{2} + 2 \cdot yp \right) + \int \left(\exp(-I \cdot \kappa \cdot y), y = \frac{Wp}{2} - 2 \cdot yp .. \frac{Wp}{2} \right) \right) :$$

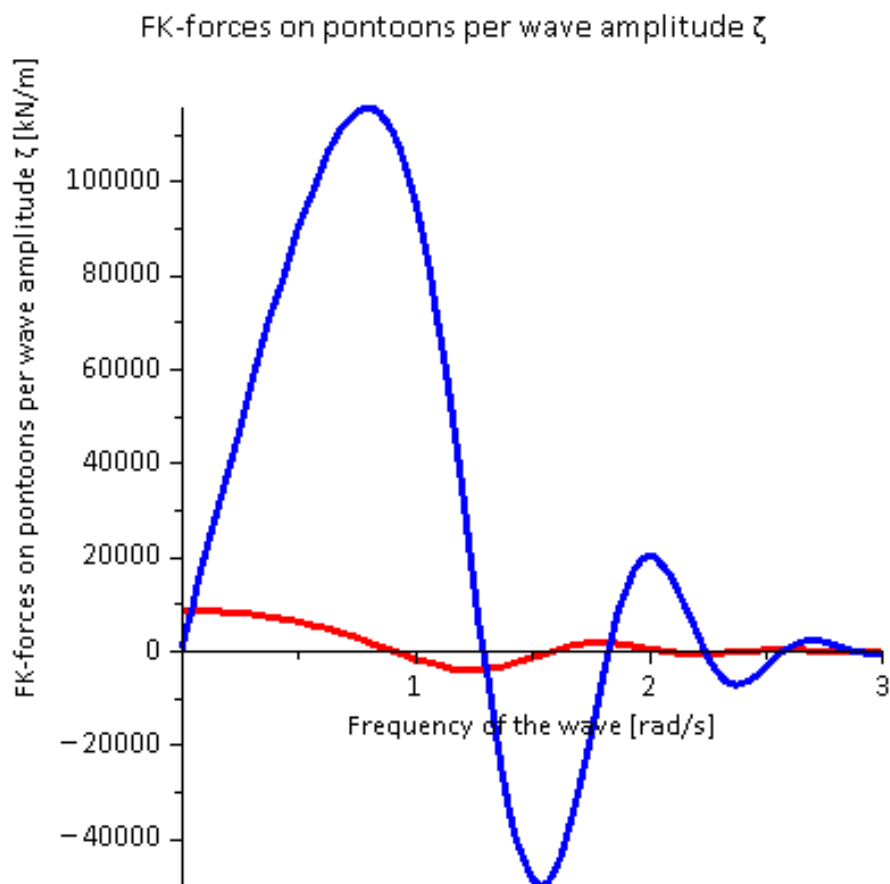
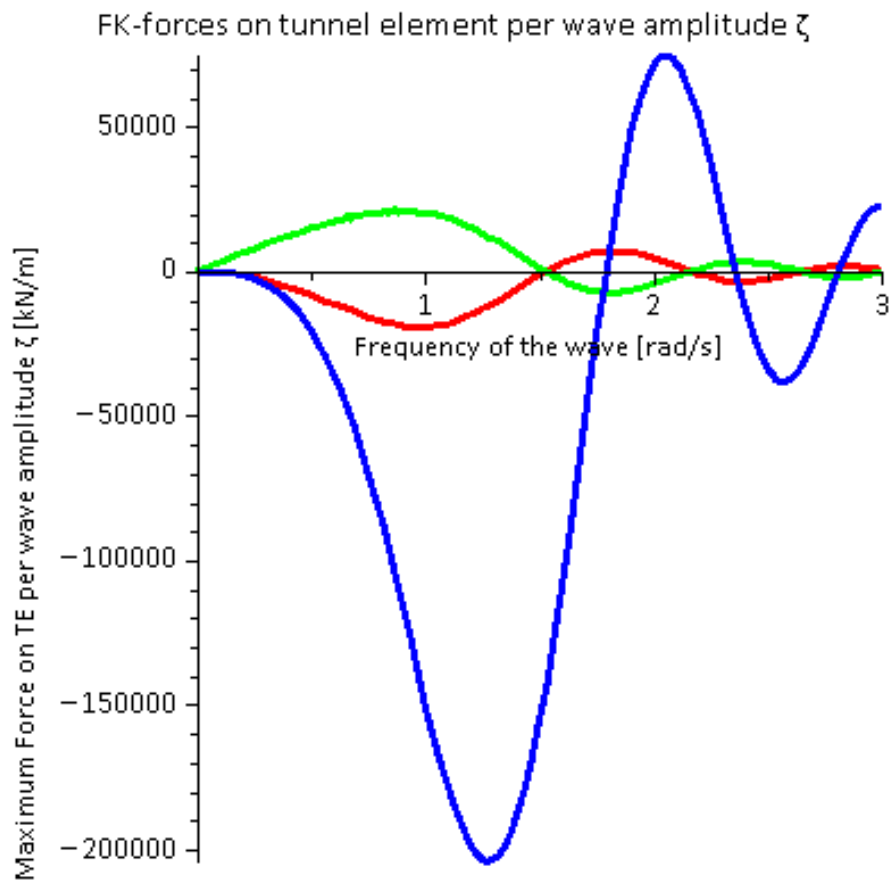
$$FPs := 0 :$$

$$FPr := -\rho \cdot g \cdot 2 \cdot Lp \cdot \left(\frac{\cosh(\kappa \cdot (h - drp))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\int \left(y \cdot \exp(-I \cdot \kappa \cdot y), y = -\frac{Wp}{2} .. -\frac{Wp}{2} + 2 \cdot yp \right) + \int \left(y \cdot \exp(-I \cdot \kappa \cdot y), y = \frac{Wp}{2} - 2 \cdot yp .. \frac{Wp}{2} \right) \right) :$$

$$F := Vector(6) : F[1] := FTEh : F[2] := FTEs : F[3] := FTEr : F[4] := FPh : F[5] := FPs : F[6] := FPr : \kappa := \text{abs} \left(\frac{RootOf(_Z g (e^{-Z})^2 - _Z g - \omega^2 h (e^{-Z})^2 - \omega^2 h)}{h} \right) :$$

$$t := 0 : \text{plot} \left(\left[\frac{\text{Re}(FTEh)}{1000}, \frac{\text{Im}(FTEs)}{1000}, \frac{\text{Im}(FTEr)}{1000} \right], \omega = 0 .. 3, \text{thickness} = 3, \text{color} = [\text{red}, \text{green}, \text{blue}], \text{title} = \text{"FK-forces on tunnel element per wave amplitude"} \xi, \text{labels} = [\text{"Frequency of the wave [rad/s]"}, \text{"Maximum Force on TE per wave amplitude"} \xi, \text{[kN/m]}], \text{labeldirections} = [\text{horizontal}, \text{vertical}], \text{font} = [\text{Calibri}, 1, 12], \text{labelfont} = [\text{Calibri}, 1, 10]) :$$

$$\text{plot} \left(\left[\frac{\text{Re}(FPh)}{1000}, \frac{\text{Im}(FPr)}{1000} \right], \omega = 0 .. 3, \text{color} = [\text{red}, \text{blue}], \text{thickness} = 3, \text{title} = \text{"FK— forces on pontoons per wave amplitude"} \xi, \text{labels} = [\text{"Frequency of the wave [rad/s]"}, \text{"FK— forces on pontoons per wave amplitude"} \xi, \text{[kN/m]}], \text{labeldirections} = [\text{horizontal}, \text{vertical}], \text{font} = [\text{Calibri}, 1, 12], \text{labelfont} = [\text{Calibri}, 1, 10]) :$$



Response amplitude operator (RAO)/ Frequency response function (FRF)

```

HuFaux := Transpose(E) : HuF := Matrix(6) :
for i from 1 to 6 do
for p from 1 to 6 do
HuF[i,p] := HuFaux[i,p]/(-Instar[i,i]·ω2 + Kstar[i,i]); end do
end do;
evalf(HuF) :
HxF := (E.HuF) :
xhead := evalf(HxF.F) :

#plot({Re(xhead[1]), Im(xhead[1]), abs(xhead[1])}, ω = 0..2, y = -2..4, numpoints = 10, color = [red, blue,
black], title = "Response Amplitude Operator of DOF x1 (Undamped)", labels = ["ω [rad/s]", "RAO = x/ζ [-]"],
labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);

#plot({Re(xhead[2]), Im(xhead[2]), abs(xhead[2])}, ω = 0..3, y = -2..4, numpoints = 10, color = [red, blue,
black], title = "Response Amplitude Operator of DOF x2 (Undamped)", labels = ["ω [rad/s]", "RAO = x/ζ [-]"],
labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);

#plot({Re(xhead[3]), Im(xhead[3]), abs(xhead[3])}, ω = 0..2, y = -0.2..0.5, numpoints = 10, color = [red, blue,
black], title = "Response Amplitude Operator of DOF x3 (Undamped)", labels = ["ω [rad/s]",
"RAO = x/ζ [rad/m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1,
10]);

#plot({Re(xhead[4]), Im(xhead[4]), abs(xhead[4])}, ω = 0..2, y = -2..8, numpoints = 10, color = [red, blue,
black], title = "Response Amplitude Operator of DOF x4 (Undamped)", labels = ["ω [rad/s]", "RAO = x/ζ [-]"],
labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);

#plot({Re(xhead[5]), Im(xhead[5]), abs(xhead[5])}, ω = 0..2.5, y = -2..4, numpoints = 10, color = [red, blue,
black], title = "Response Amplitude Operator of DOF x5 (Undamped)", labels = ["ω [rad/s]", "RAO = x/ζ [-]"],
labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);

#plot({Re(xhead[6]), Im(xhead[6]), abs(xhead[6])}, ω = 0..1.5, y = 0..0.4, numpoints = 10, color = [red, blue,
black], title = "Response Amplitude Operator of DOF x6 (Undamped)", labels = ["ω [rad/s]",
"RAO = x/ζ [rad/m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1,
10]);

```

```

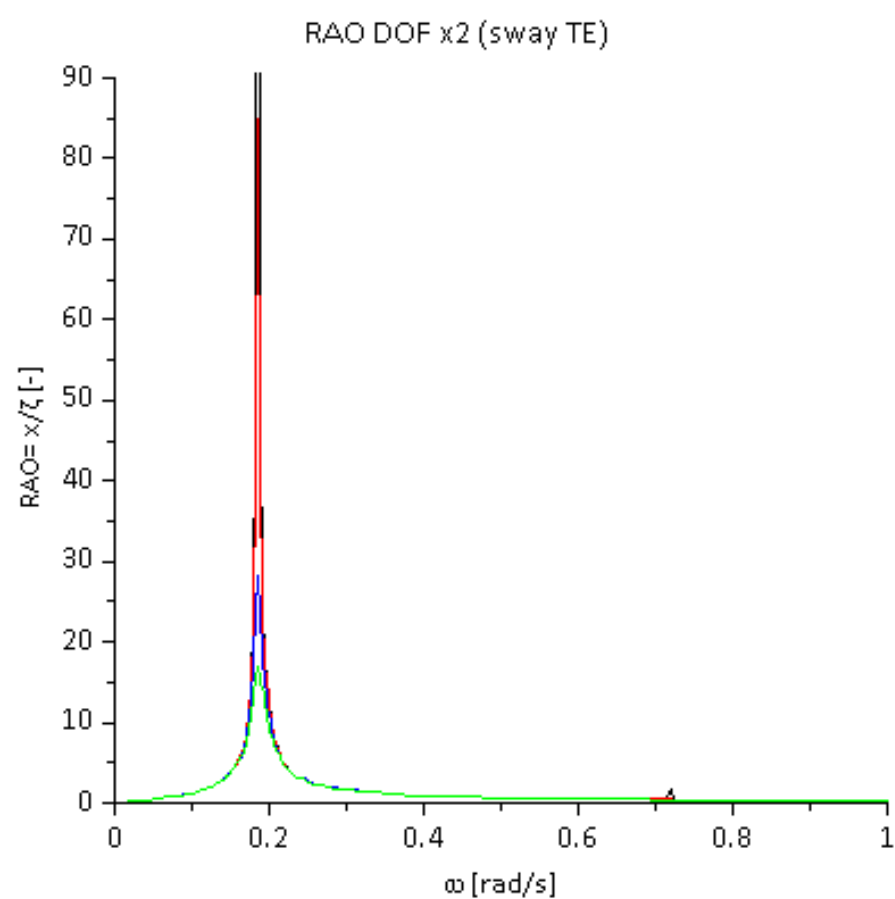
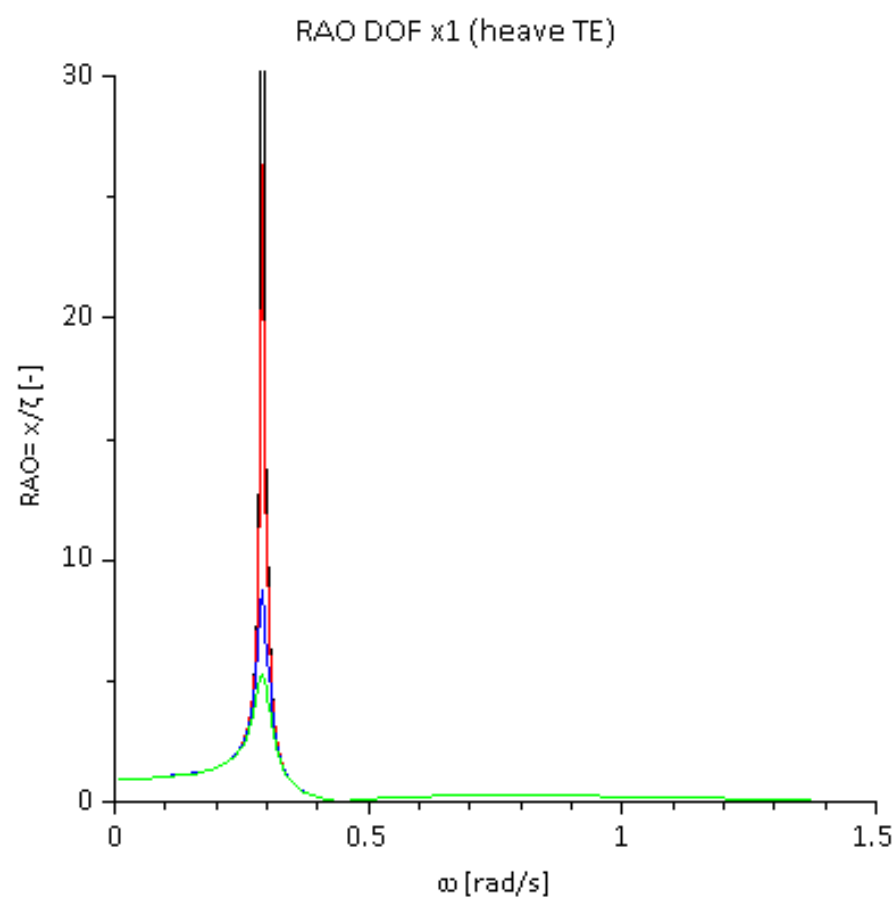
HuFauxc1 := Transpose(E) : HuFc1 := Matrix(6) :
for i from 1 to 6 do
for p from 1 to 6 do
HuFc1[i,p] := HuFauxc1[i,p]/(-Instar[i,i]·ω2 + I·ω·0.01· Cstar[i,i] + Kstar[i,i]); end do
end do;
evalf(HuFc1) :
HxFc1 := (E.HuFc1) :
xheadc1 := evalf(HxFc1.F) :
HuFauxc2 := Transpose(E) : HuFc2 := Matrix(6) :
for i from 1 to 6 do
for p from 1 to 6 do
HuFc2[i,p] := HuFauxc2[i,p]/(-Instar[i,i]·ω2 + I·ω·0.03· Cstar[i,i] + Kstar[i,i]); end do
end do;
evalf(HuFc2) :
HxFc2 := (E.HuFc2) :
xheadc2 := evalf(HxFc2.F) :
HuFauxc3 := Transpose(E) : HuFc3 := Matrix(6) :
for i from 1 to 6 do
for p from 1 to 6 do
HuFc3[i,p] := HuFauxc3[i,p]/(-Instar[i,i]·ω2 + I·ω·0.05· Cstar[i,i] + Kstar[i,i]); end do
end do;
evalf(HuFc3) :
HxFc3 := (E.HuFc3) :
xheadc3 := evalf(HxFc3.F) :

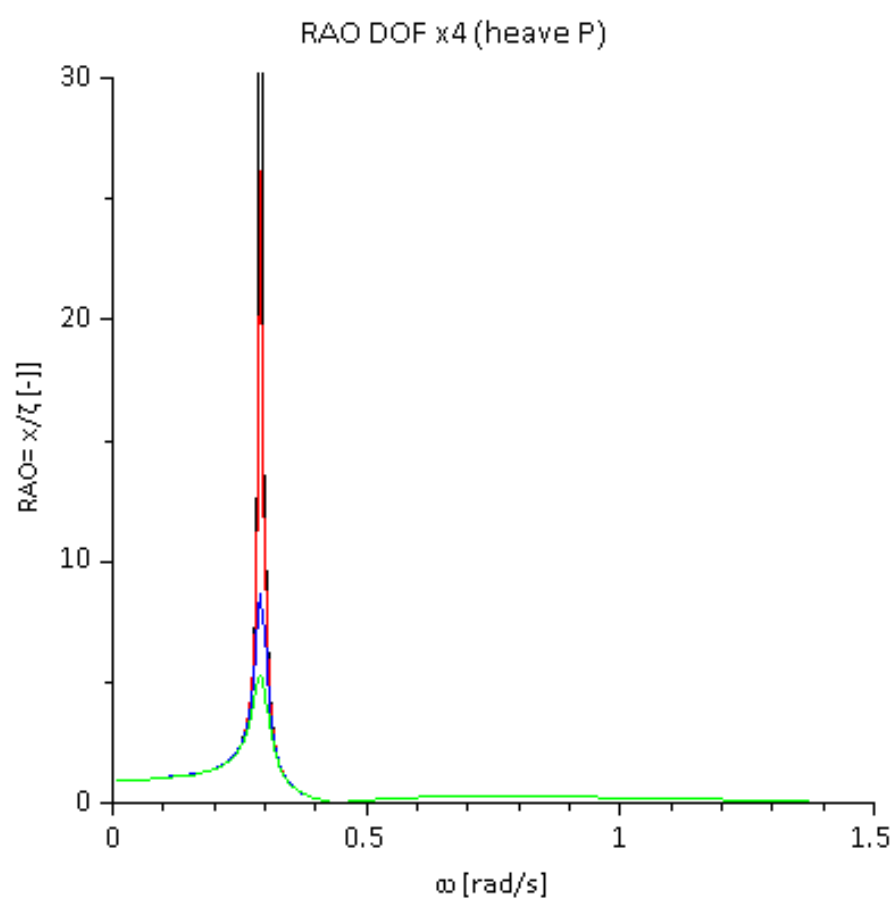
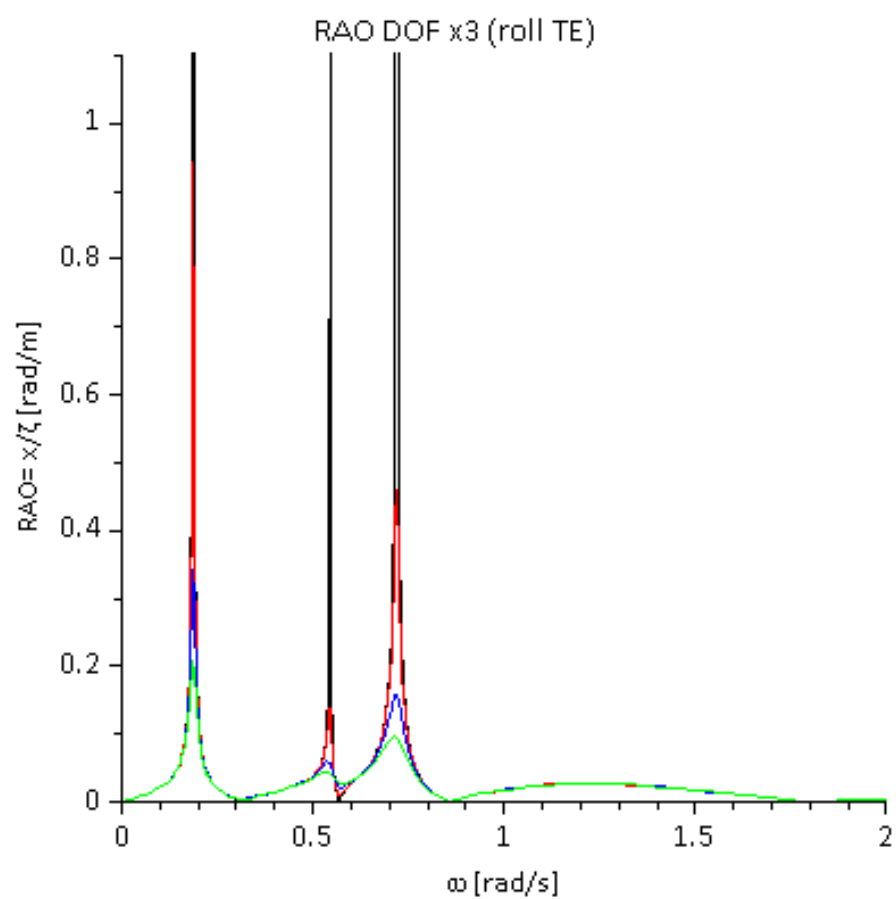
```

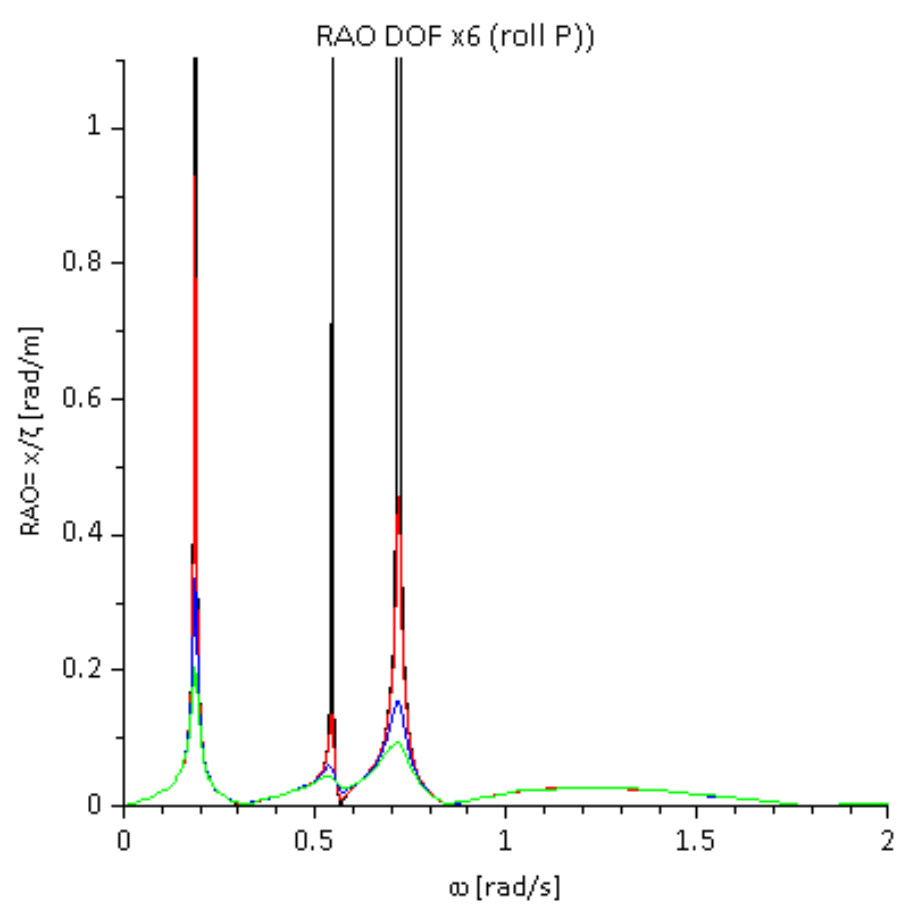
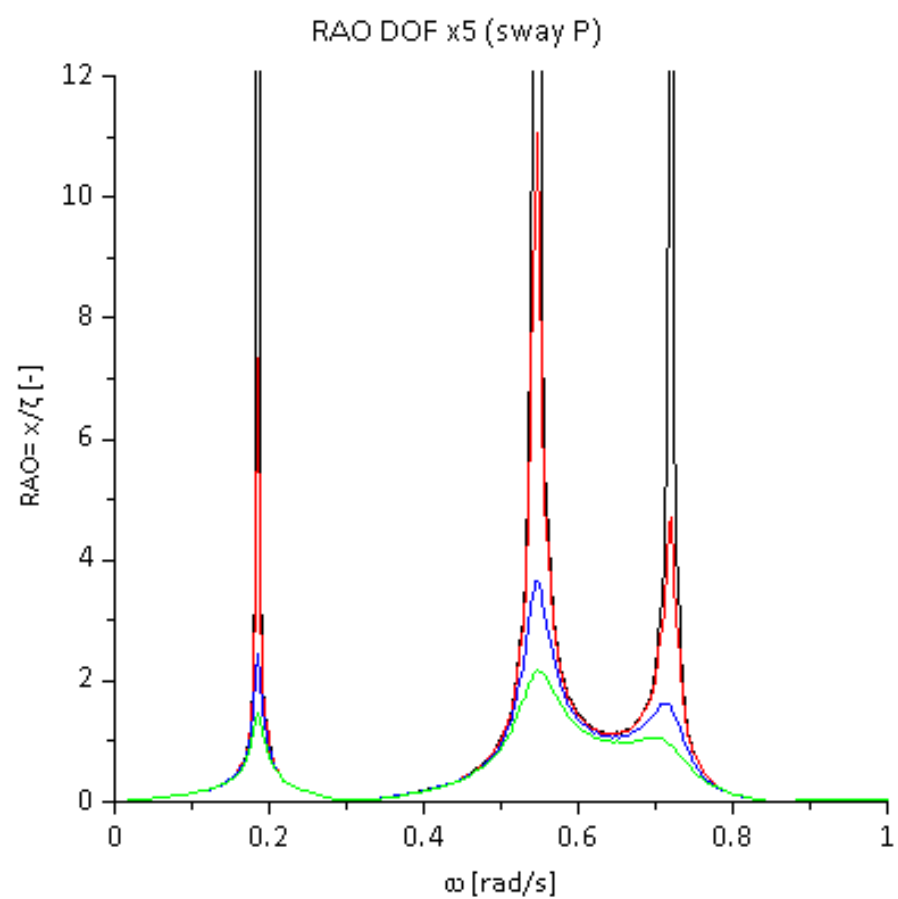
```

plot([abs(xhead[1]),abs(xheadc1[1]),abs(xheadc2[1]),abs(xheadc3[1])], ω = 0..1.5, y = 0..30, color
= black, numpoints = 10, color = [black, red, blue, green], title = "RAO DOF x1 (heave TE)", labels
= ["ω [rad/s]", "RAO= x/ζ [-]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
plot([abs(xhead[2]),abs(xheadc1[2]),abs(xheadc2[2]),abs(xheadc3[2])], ω = 0..1, y = 0..90, color = black,
numpoints = 10, color = [black, red, blue, green], title = "RAO DOF x2 (sway TE)", labels = ["ω [rad/s]",
"RAO= x/ζ [-]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(xhead[3]),abs(xheadc1[3]),abs(xheadc2[3]),abs(xheadc3[3])], ω = 0..2, y = 0..1.1, color = black,
numpoints = 20, color = [black, red, blue, green], title = "RAO DOF x3 (roll TE)", labels = ["ω [rad/s]",
"RAO= x/ζ [rad/m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1,
10]);
plot([abs(xhead[4]),abs(xheadc1[4]),abs(xheadc2[4]),abs(xheadc3[4])], ω = 0..1.5, y = 0..30, color
= black, numpoints = 10, color = [black, red, blue, green], title = "RAO DOF x4 (heave P)", labels
= ["ω [rad/s]", "RAO= x/ζ [-]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
plot([abs(xhead[5]),abs(xheadc1[5]),abs(xheadc2[5]),abs(xheadc3[5])], ω = 0..1, y = 0..12, color = black,
numpoints = 10, color = [black, red, blue, green], title = "RAO DOF x5 (sway P)", labels = ["ω [rad/s]",
"RAO= x/ζ [-]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(xhead[6]),abs(xheadc1[6]),abs(xheadc2[6]),abs(xheadc3[6])], ω = 0..2, y = 0..1.1, color = black,
numpoints = 20, color = [black, red, blue, green], title = "RAO DOF x6 (roll P)", labels = ["ω [rad/s]",
"RAO= x/ζ [rad/m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1,
10]);

```





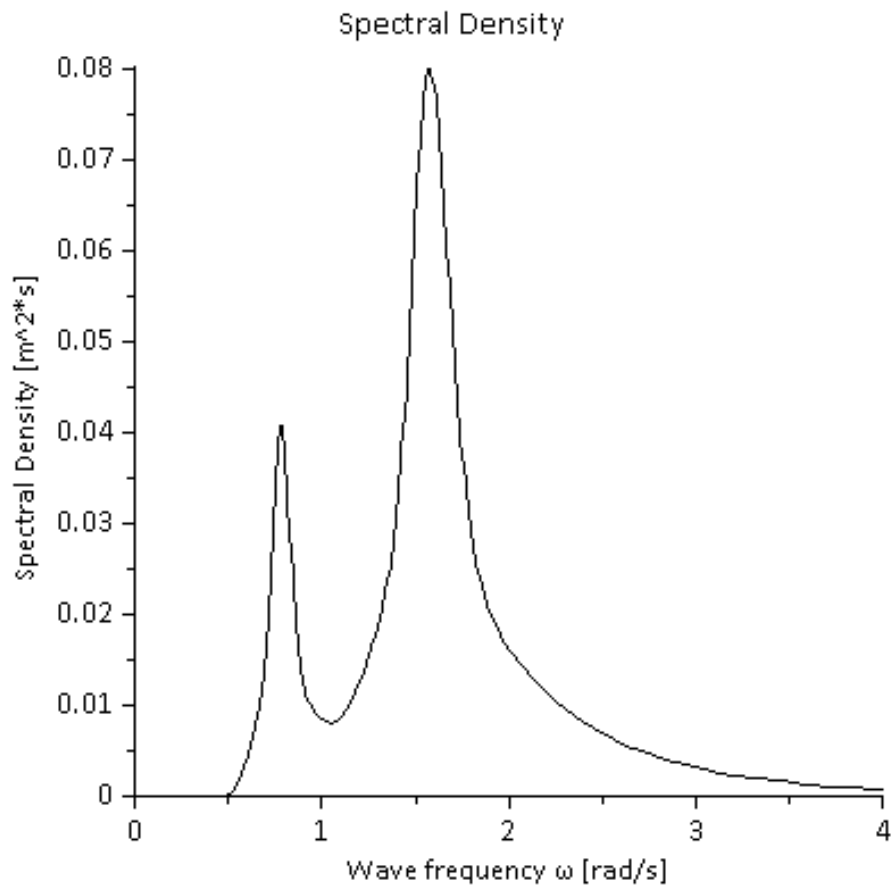


$$S1 := \alpha1 \cdot g^2 \cdot \omega^{-5} \cdot \exp\left(-1.25 \cdot \left(\frac{\omega}{\omega01}\right)^{-4}\right) \cdot \gamma1 \left(\exp\left(-\frac{(\omega - \omega01)^2}{2 \cdot \sigma^2 \cdot \omega01^2}\right)\right) : S2 := \alpha2 \cdot g^2 \cdot \omega^{-5} \cdot \exp\left(-1.25 \cdot \left(\frac{\omega}{\omega02}\right)^{-4}\right) \cdot \gamma2 \left(\exp\left(-\frac{(\omega - \omega02)^2}{2 \cdot \sigma^2 \cdot \omega02^2}\right)\right) :$$

$$\sigma := \text{piecewise}(\omega \leq \omega01, \sigma a, \omega > \omega01, \sigma b) : \sigma a := 0.07 : \sigma b := 0.09 : g := 9.81 : \gamma1 := 3.3 : \gamma2 := 3.3 : \omega01 := \frac{2 \cdot \text{Pi}}{T1} : \omega02 := \frac{2 \cdot \text{Pi}}{T2} : \omega := '\omega':$$

$$T1 := 4 : T2 := 8 : \alpha1 := 0.00830104 : \alpha2 := 0.00013512 : Hs1 := 4 \cdot \text{sqrt}(\text{int}(S1, \omega = 0..10, \text{numeric})) : S3 := S1 + S2 : Hs2 := 4 \cdot \text{sqrt}(\text{int}(S2, \omega = 0..10, \text{numeric})) : Hstot := 4 \cdot \text{sqrt}(\text{int}(S3, \omega = 0..10, \text{numeric})) :$$

$$\text{plot}(S3, \omega = 0..4, \text{numpoints} = 10, \text{color} = \text{black}, \text{title} = \text{"Spectral Density"}, \text{labels} = [\text{"Wave frequency } \omega \text{ [rad/s]"}, \text{"Spectral Density [m}^2\text{s}]), \text{labelfont} = [\text{Calibri}, 1, 12], \text{labeldirections} = [\text{horizontal}, \text{vertical}], \text{font} = [\text{Calibri}, 1, 12], \text{labelfont} = [\text{Calibri}, 1, 10]) :$$

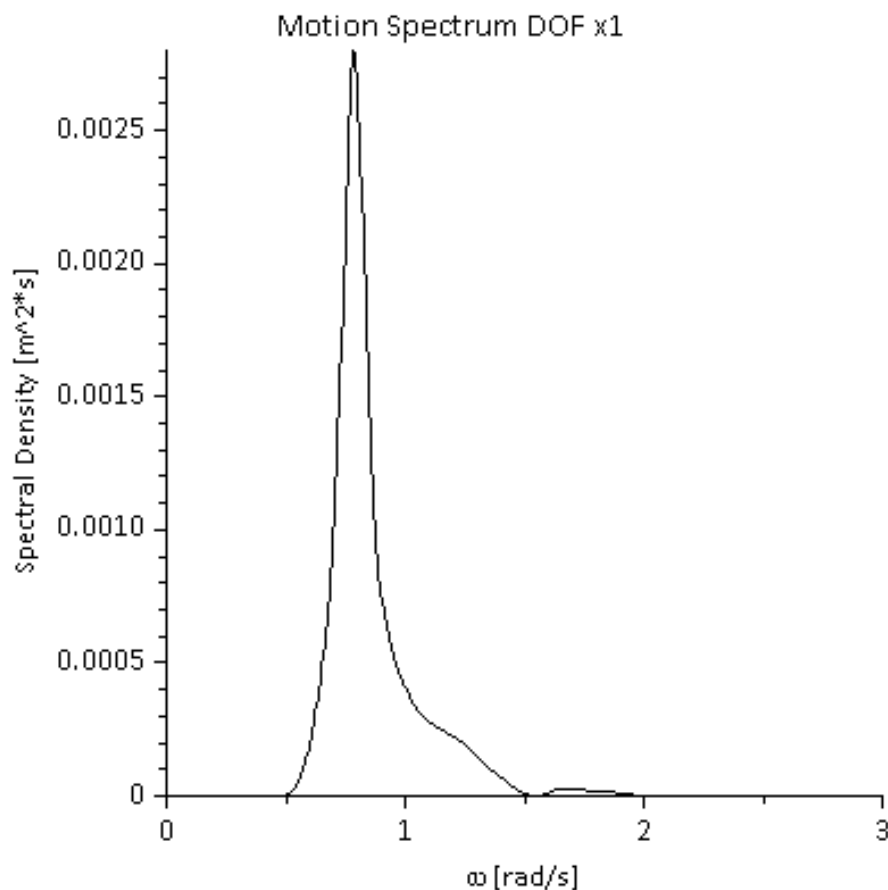


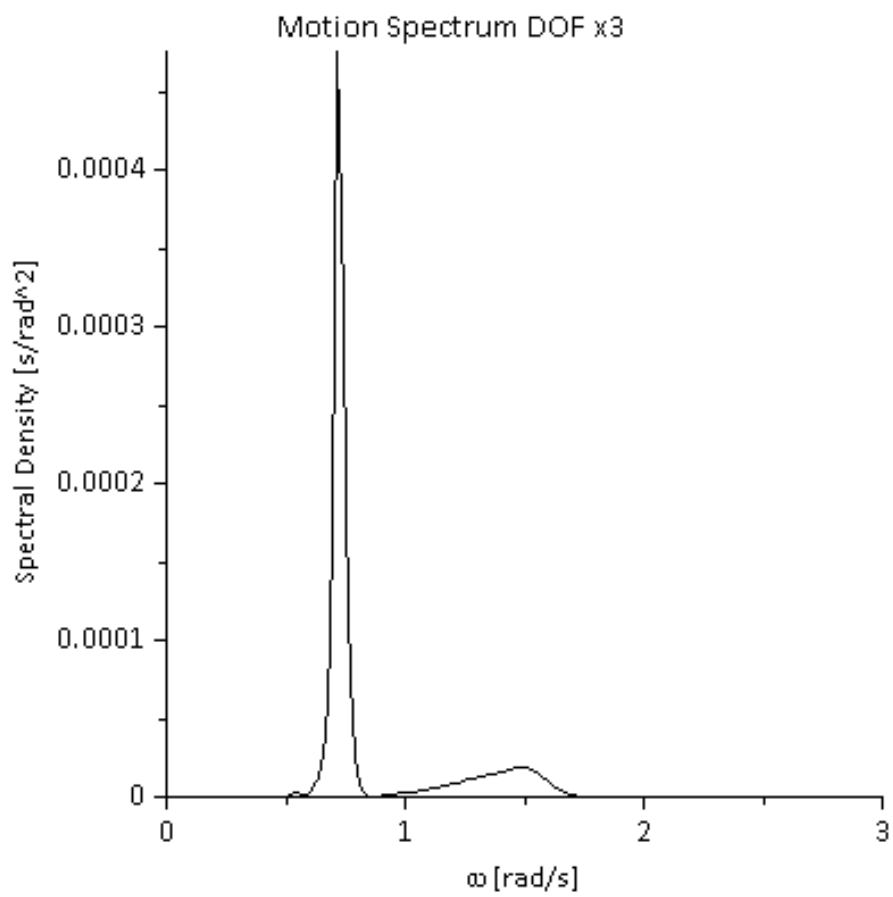
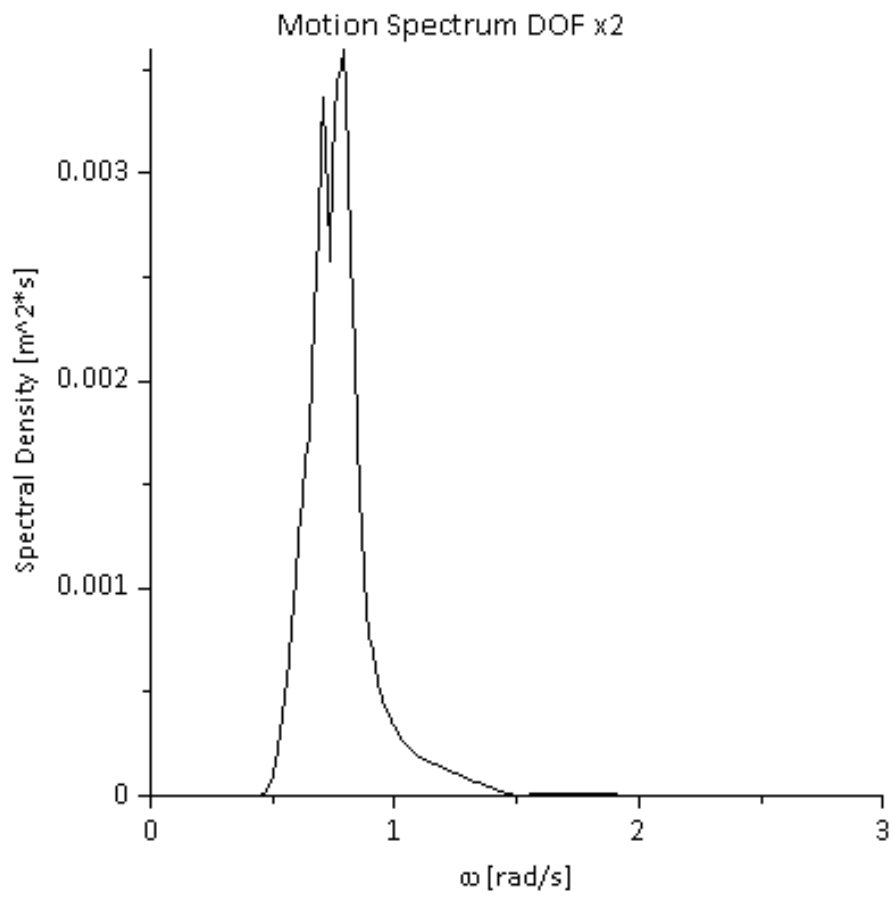
$$\text{Motspec} := \text{Vector}(6) : \text{for } i \text{ from } 1 \text{ to } 6 \text{ do } \text{MotSpec}[i] := \text{abs}(xheadc2[i]^2 \cdot S3) \text{ end: } \omega := '\omega':$$

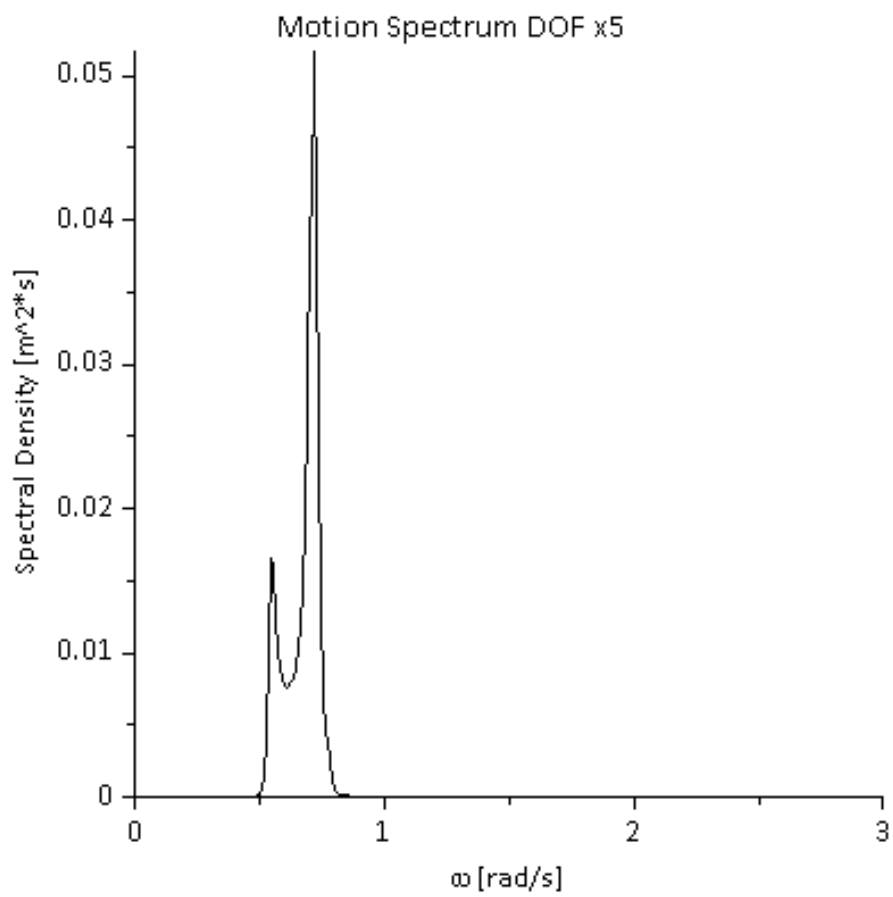
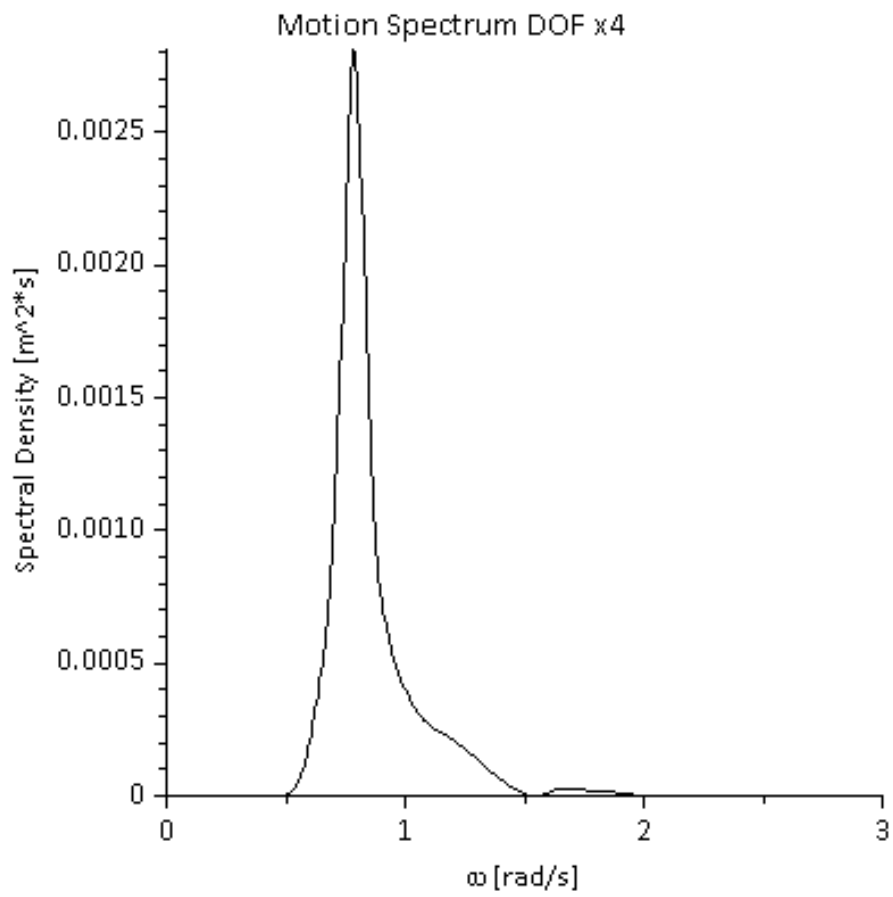
```

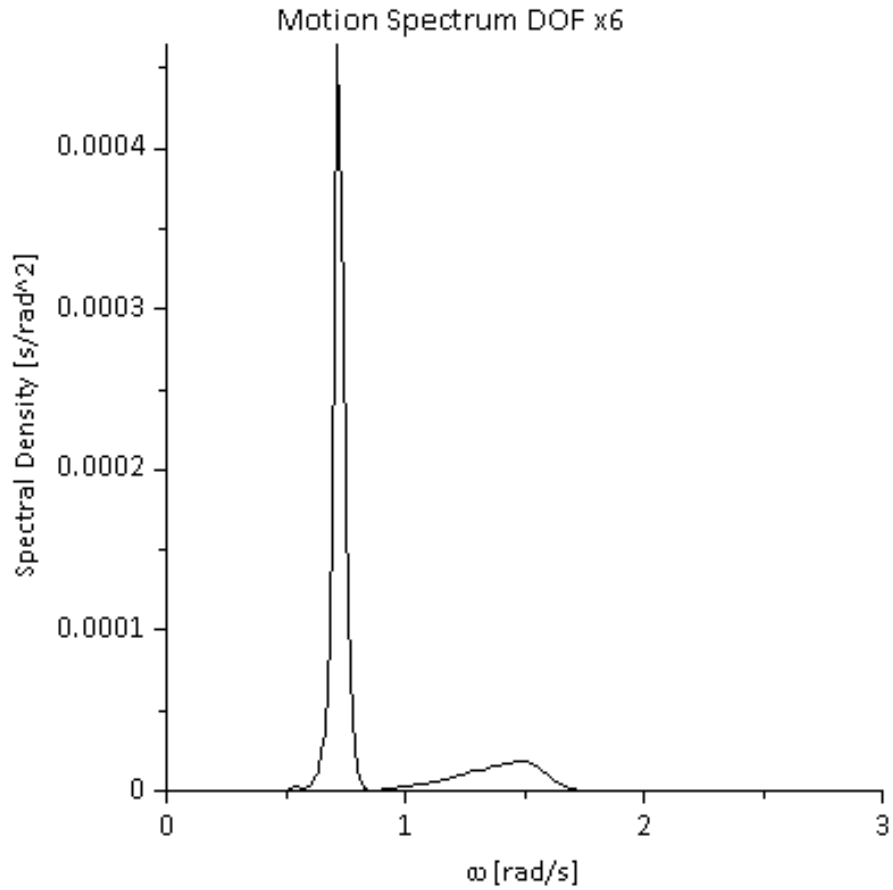
plot(MotSpec [1],  $\omega$  = 0 ..3, numpoints = 10, color = black, color = black, title = "Motion Spectrum DOF x1", labels
= [" $\omega$  [rad/s]", "Spectral Density [m^2s]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12],
labelfont = [Calibri, 1, 10]);
plot(MotSpec [2],  $\omega$  = 0 ..3, numpoints = 10, color = black, title = "Motion Spectrum DOF x2", labels = [" $\omega$  [rad/s]",
"Spectral Density [m^2s]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
plot(MotSpec [3],  $\omega$  = 0 ..3, numpoints = 10, color = black, title = "Motion Spectrum DOF x3", labels = [" $\omega$  [rad/s]",
"Spectral Density [s/rad^2]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
plot(MotSpec [4],  $\omega$  = 0 ..3, numpoints = 10, color = black, title = "Motion Spectrum DOF x4", labels = [" $\omega$  [rad/s]",
"Spectral Density [m^2s]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
plot(MotSpec [5],  $\omega$  = 0 ..3, numpoints = 10, color = black, title = "Motion Spectrum DOF x5", labels = [" $\omega$  [rad/s]",
"Spectral Density [m^2s]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
plot(MotSpec [6],  $\omega$  = 0 ..3, numpoints = 10, color = black, title = "Motion Spectrum DOF x6", labels = [" $\omega$  [rad/s]",
"Spectral Density [s/rad^2]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);

```









```

sigmasq := Vector(6) :
for ii from 1 to 6 by 1 do:
sigmasq[ii] := 0.01· sum(subs(omega = 0.01·kk, MotSpec [ii]), kk = 1 ..300) :
end do:
sigmasq;

```

```

[ 0.0005741437557
  0.0008622235166
  0.00003430482011
  0.0005705085212
  0.004109889286
  0.00003351226718]

```

```

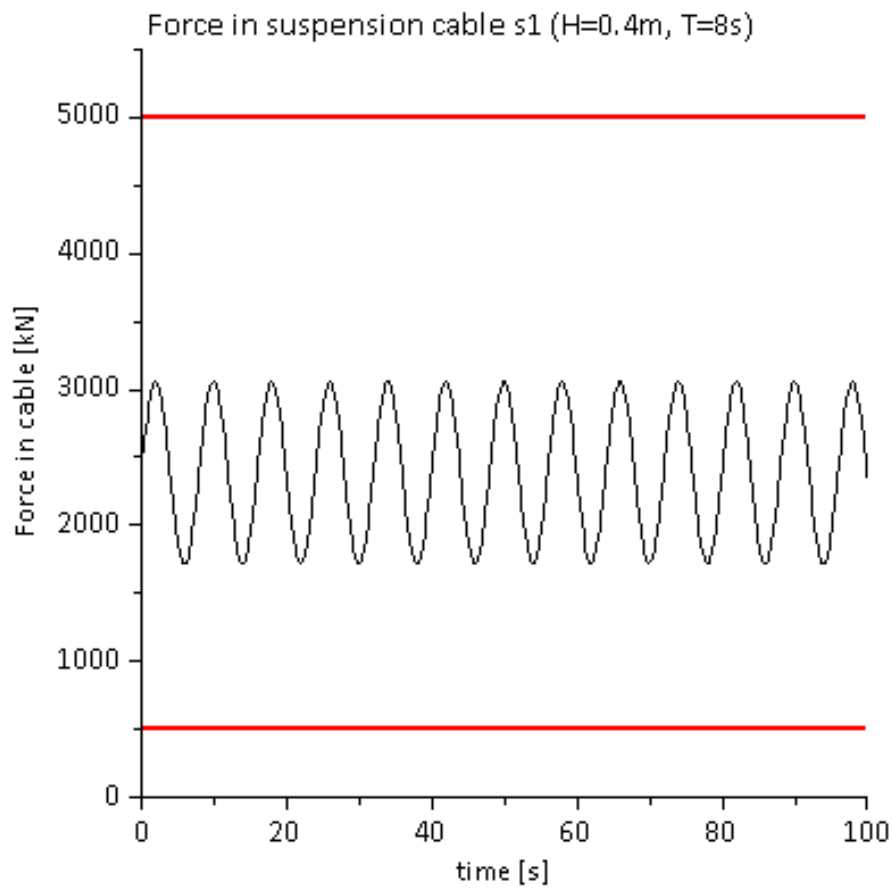
t := 't': H := 0.4 : T := 8 : omega := evalf( (2·Pi) / T ) :
Fs1a := evalf( Re( ( ks / 2 · H / 2 · exp(I·omega·t) · ( xheadc2 [1] - xheadc2 [4] + (xheadc2 [3] - xheadc2 [6]) · ( We / 2 - ye ) ) ) ) ) :
Fs2a := evalf( Re( ( ks / 2 · H / 2 · exp(I·omega·t) · ( xheadc2 [1] - xheadc2 [4] - (xheadc2 [3] - xheadc2 [6]) · ( We / 2 - ye ) ) ) ) :

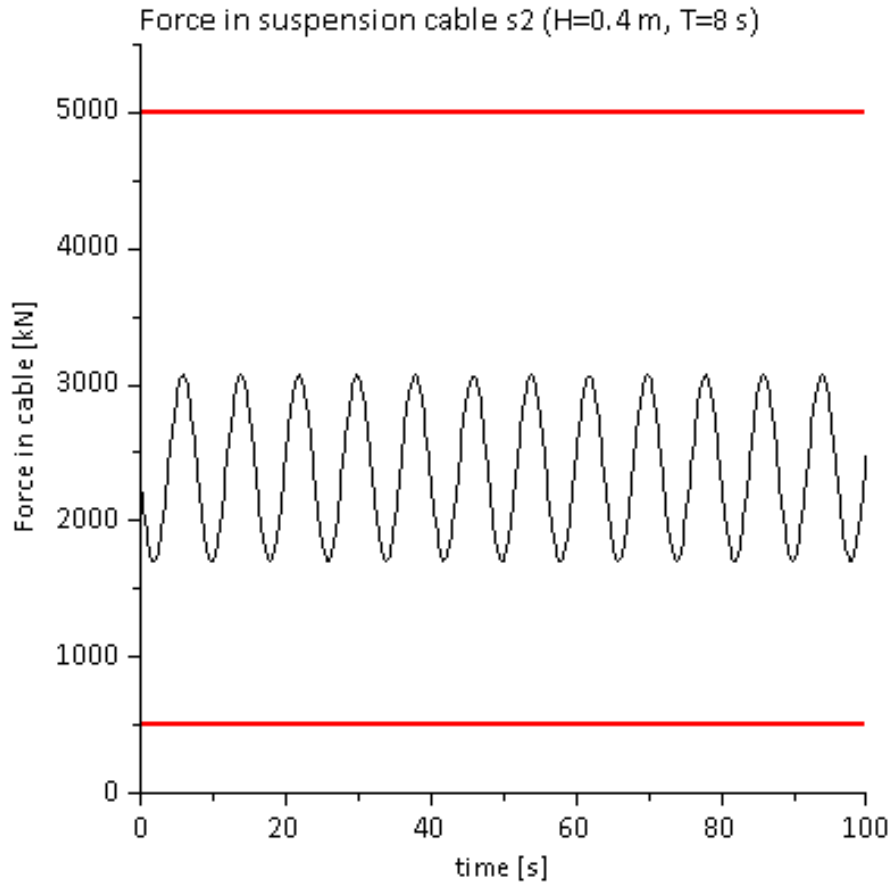
```

```

plot( $\left[ \frac{2387000 + Fs1a}{1000}, 500, 5000 \right]$ , t = 0 ..100, y = 0 ..5500, color = [black, red, red], thickness = [1, 2, 2], title
= "Force in suspension cable s1 (H=0.4m, T=8s)" labels = ["time [s]", "Force in cable [kN]"], labeldirections
= [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot( $\left[ \frac{2387000 + Fs2a}{1000}, 500, 5000 \right]$ , t = 0 ..100, y = 0 ..5500, color = [black, red, red], thickness = [1, 2, 2], title
= "Force in suspension cable s2 (H=0.4 m, T=8 s)" labels = ["time [s]", "Force in cable [kN]"], labeldirections
= [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);

```





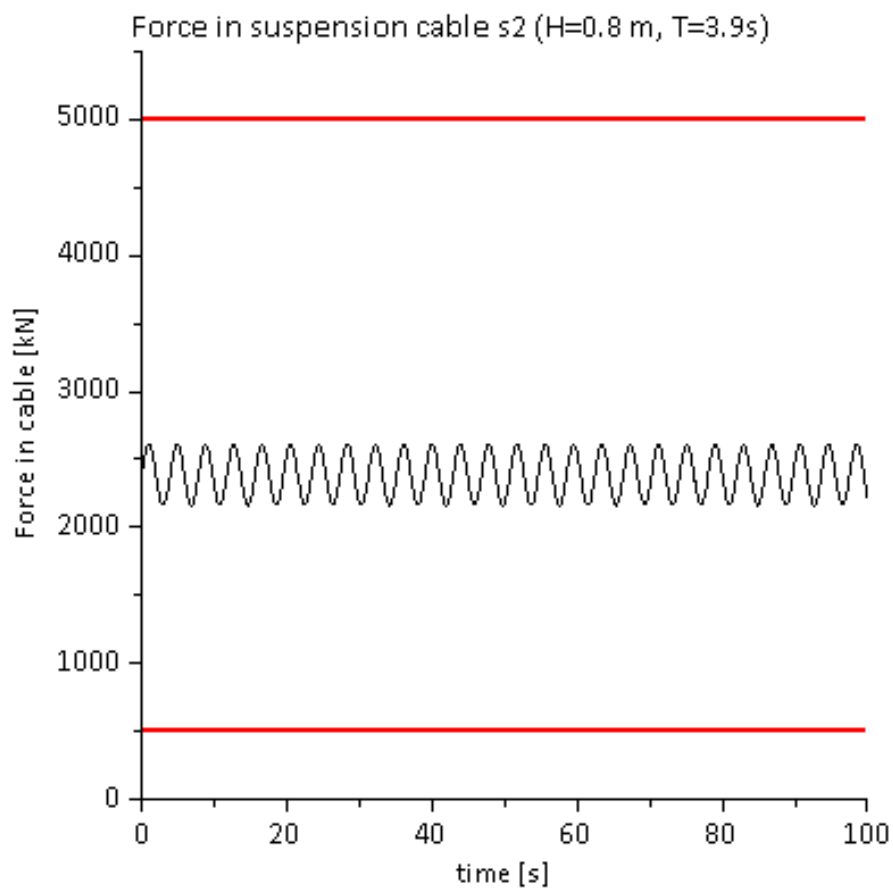
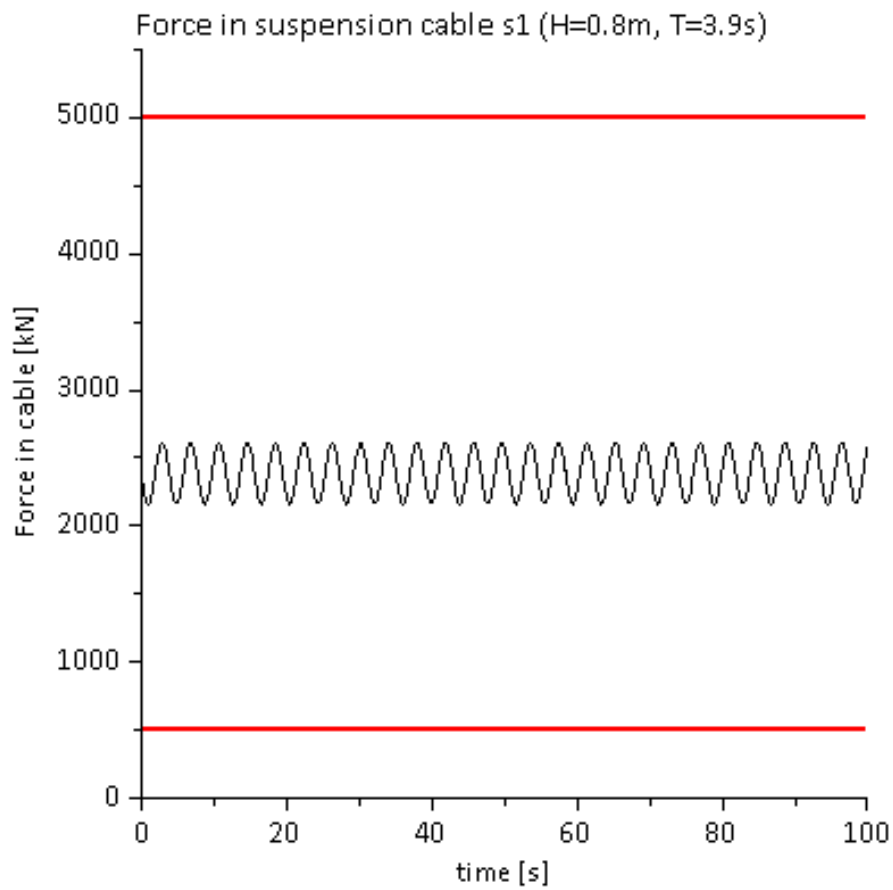
```

t := 't'; H := 0.8; T := 3.9; ω := evalf( (2·Pi) / T );
Fs1b := evalf( Re( (ks/2) · (H/2) · exp(I·ω·t) · (xheadc2[1] - xheadc2[4] + (xheadc2[3] - xheadc2[6]) · (We/2 - ye) ) ) );
Fs2b := evalf( Re( (ks/2) · (H/2) · exp(I·ω·t) · (xheadc2[1] - xheadc2[4] - (xheadc2[3] - xheadc2[6]) · (We/2 - ye) ) ) );

plot( [ (2387000 + Fs1b) / 1000, 500, 5000 ], t = 0..100, y = 0..5500, color = [black, red, red], thickness = [1, 2, 2], title
= "Force in suspension cable s1 (H=0.8m, T=3.9s)", labels = ["time [s]", "Force in cable [kN]"], labeldirections
= [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10] );

plot( [ (2387000 + Fs2b) / 1000, 500, 5000 ], t = 0..100, y = 0..5500, color = [black, red, red], thickness = [1, 2, 2], title
= "Force in suspension cable s2 (H=0.8 m, T=3.9s)", labels = ["time [s]", "Force in cable [kN]"], labeldirections
= [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10] );

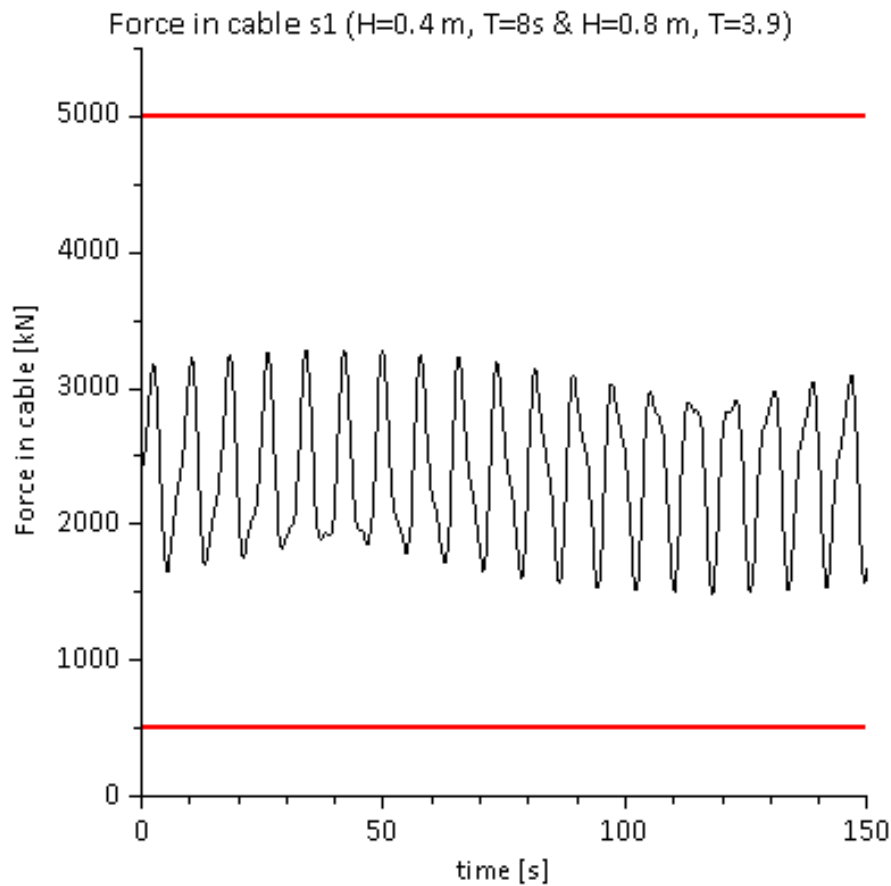
```



```

plot([ [  $\frac{2387000 + Fs1a + Fs1b}{1000}$ , 500, 5000 ], t = 0 .. 150, y = 0 .. 5500, color = [black, red, red], thickness = [1, 2, 2], title = "Force in cable s1 (H=0.4 m, T=8s & H=0.8 m, T=3.9)", labels = ["time [s]", "Force in cable [kN]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10] );

```



```

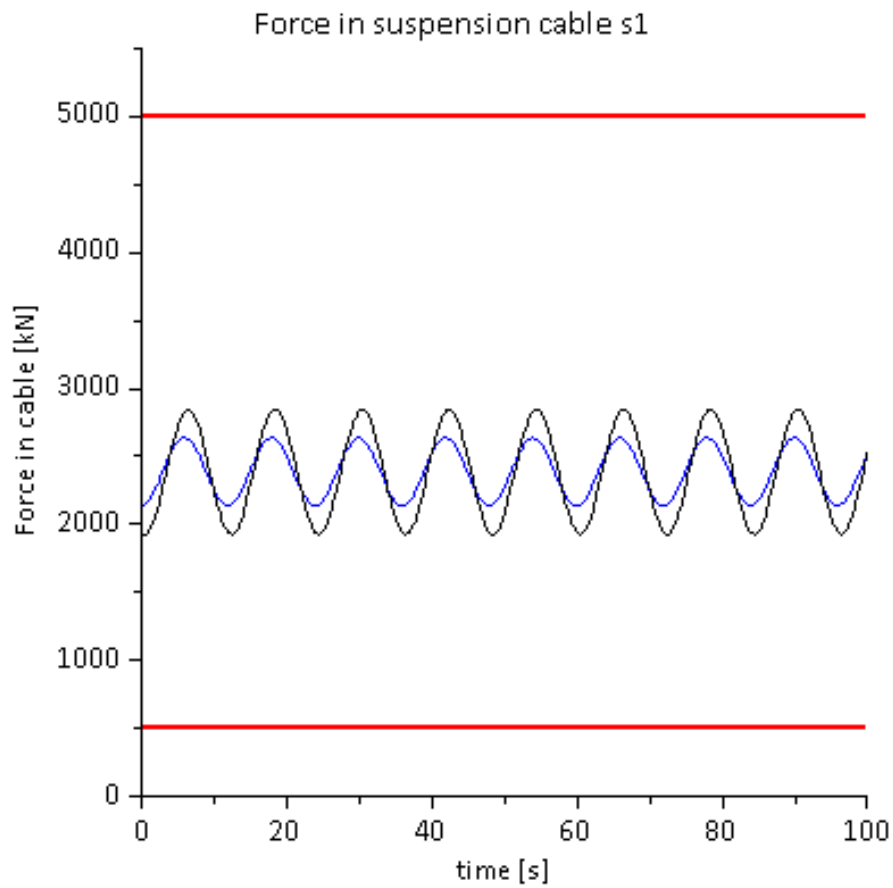
t := 't': H := 0.8 : T := 12 :  $\omega := \text{evalf}\left(\frac{2 \cdot \text{Pi}}{T}\right)$  :
Fs1c := evalf( Re(  $\frac{ks}{2} \cdot \frac{H}{2} \cdot \exp(I \cdot \omega \cdot t) \cdot (xheadc2[1] - xheadc2[4] + (xheadc2[3] - xheadc2[6]) \cdot \left(\frac{We}{2} - ye\right))$  ) ) :
Fs2c := evalf( Re(  $\frac{ks}{2} \cdot \frac{H}{2} \cdot \exp(I \cdot \omega \cdot t) \cdot (xheadc2[1] - xheadc2[4] - (xheadc2[3] - xheadc2[6]) \cdot \left(\frac{We}{2} - ye\right))$  ) ) :
Fs1d := evalf( Re(  $\frac{ks}{2} \cdot \frac{H}{2} \cdot \exp(I \cdot \omega \cdot t) \cdot (xhead[1] - xhead[4] + (xhead[3] - xhead[6]) \cdot \left(\frac{We}{2} - ye\right))$  ) ) :
Fs2d := evalf( Re(  $\frac{ks}{2} \cdot \frac{H}{2} \cdot \exp(I \cdot \omega \cdot t) \cdot (xhead[1] - xhead[4] - (xhead[3] - xhead[6]) \cdot \left(\frac{We}{2} - ye\right))$  ) ) :

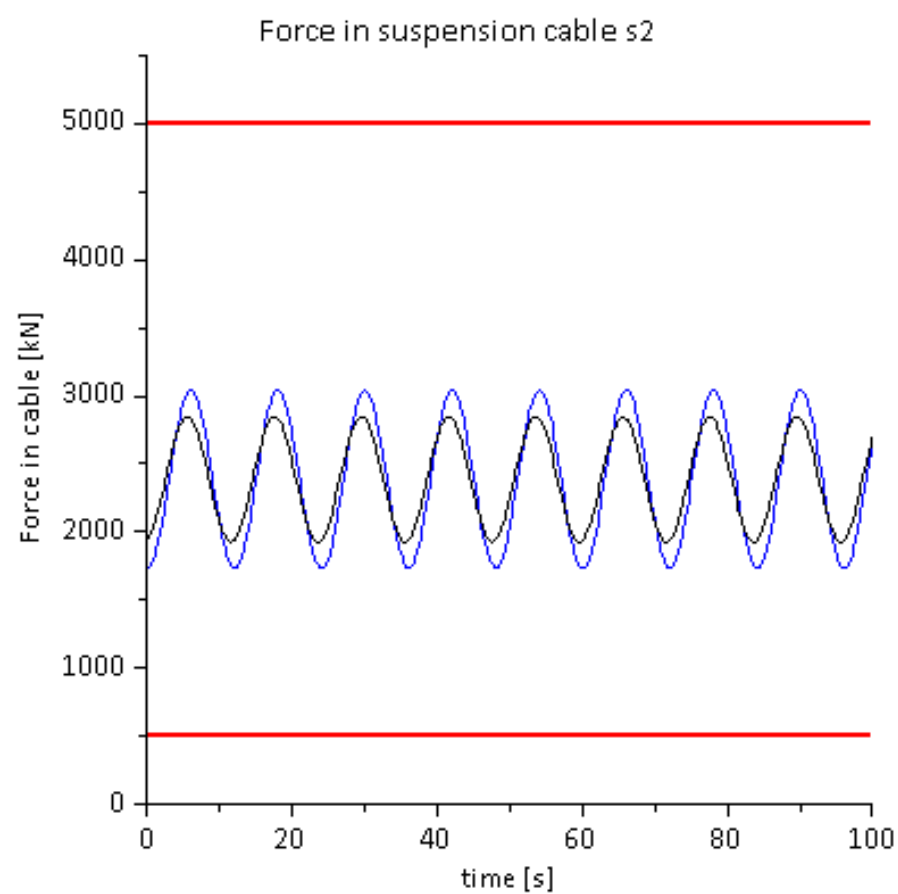
```

```

plot( $\left[ \frac{2387000 + Fs1c}{1000}, \frac{2387000 + Fs1d}{1000} \right]$ , 500, 5000, t = 0 ..100, y = 0 ..5500, color = [blue, black, red, red],
    thickness = [1, 1, 2, 2], title = "Force in suspension cable s1", labels = ["time [s]", "Force in cable [kN]",
    labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10] );
plot( $\left[ \frac{2387000 + Fs2c}{1000}, \frac{2387000 + Fs2d}{1000} \right]$ , 500, 5000, t = 0 ..100, y = 0 ..5500, color = [blue, black, red, red],
    thickness = [1, 1, 2, 2], title = "Force in suspension cable s2", labels = ["time [s]", "Force in cable [kN]",
    labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10] );

```





Appendix E Inverse

In this Maple file the motions are determined by:

$$\hat{\underline{x}} = (-\omega^2 \cdot M + i \cdot \omega \cdot C + K)^{-1} \cdot \hat{\underline{F}}$$

Maple was unable to perform this analysis, which is described in paragraph 6.

In this Appendix the Maple file which is used for this analysis is shown. The graphs in this Appendix show strange behaviour.

restart : with(LinearAlgebra) :

$M := \text{Matrix}([[me, 0, 0, 0, 0, 0], [0, me, 0, 0, 0, 0], [0, 0, Je, 0, 0, 0], [0, 0, 0, 2 \cdot mp, 0, 0], [0, 0, 0, 0, 2 \cdot mp, 0], [0, 0, 0, 0, 0, 2 \cdot Jp]])$: $A := \text{Matrix}([[a11 \cdot \rho \cdot We \cdot he \cdot Le, 0, 0, 0, 0, 0], [0, a22 \cdot \rho \cdot We \cdot he \cdot Le, 0, 0, 0, 0], [0, 0, a33 \cdot \rho \cdot We^3 \cdot he \cdot Le, 0, 0, 0], [0, 0, 0, 2 \cdot 2 \cdot a44 \cdot \rho \cdot Wf \cdot drp \cdot Lp, 0, 0], [0, 0, 0, 0, 2 \cdot 2 \cdot a55 \cdot \rho \cdot Wf \cdot drp \cdot Lp, 0], [0, 0, 0, 0, 0, 2 \cdot 2 \cdot \left(\frac{Wp}{2} - yp \right)^2 \cdot \rho \cdot drp \cdot Wf \cdot Lp \cdot a66]])$: $drp := \frac{(2 \cdot mp + 0.02 \cdot me) \cdot g}{2 \cdot kw}$:

$Kx := \text{Matrix}([[[2 \cdot ks - 2 \cdot kc \cdot (2 \cdot \sin(\alpha c) - 1 - \sin(\alpha c)^2), 0, 0, -2 \cdot ks - 2 \cdot kc \cdot (1 - \sin(\alpha c)), 0, 0], [0, 2 \cdot kc \cdot \cos(\alpha c)^2, -kc \cdot (We \cdot \sin(\alpha c) \cdot \cos(\alpha c) + he \cdot \cos(\alpha c)^2), 0, 0, 0], [0, -kc \cdot (We \cdot \cos(\alpha c) \cdot \sin(\alpha c) + he \cdot \cos(\alpha c)^2), kc \cdot \left(\frac{We^2}{2} \cdot \sin(\alpha c)^2 + We \cdot he \cdot \cos(\alpha c) \cdot \sin(\alpha c) + \frac{he^2}{2} \cdot \cos(\alpha c)^2 \right) + 2 \cdot \left(\frac{We}{2} - ye \right)^2 \cdot ks, 0, 0, -2 \cdot ks \cdot \left(\frac{We}{2} - ye \right)^2], [-2 \cdot ks - 2 \cdot kc + 2 \cdot kc \cdot \sin(\alpha c), 0, 0, 2 \cdot ks + 2 \cdot kw + 2 \cdot kc + 2 \cdot km \cdot \sin(\alpha m)^2, 0, 0], [0, 0, 0, 2 \cdot km \cdot \cos(\alpha m)^2, -km \cdot (Wp \cdot \sin(\alpha m) \cdot \cos(\alpha m) + 2 \cdot hpd \cdot \cos(\alpha m)^2), [0, 0, -2 \cdot ks \cdot \left(\frac{We}{2} - ye \right)^2, 0, -km \cdot (Wp \cdot \cos(\alpha m) \cdot \sin(\alpha m) + 2 \cdot hpd \cdot \cos(\alpha m)^2), 2 \cdot ks \cdot \left(\frac{We}{2} - ye \right)^2 + 2 \cdot kw \cdot \left(\frac{Wp}{2} - yp \right)^2 + km \cdot \left(\frac{Wp^2}{2} \cdot \sin(\alpha m)^2 + 2 \cdot Wp \cdot \sin(\alpha m) \cdot \cos(\alpha m) \cdot hpd + 2 \cdot hpd^2 \cdot \cos(\alpha m)^2]]]])$:

$We := 26.46 : he := 9.97 : Wp := 42.5 : Wf := 6 : \alpha c := \frac{15}{180} \cdot \text{Pi} : \alpha m := \frac{10}{180} \cdot \text{Pi} : hpd := 2.5 : yp := 3 : km := 1413.3 \cdot 1000 : ks := 563268 \cdot 1000 : kc := 2289.4 \cdot 1000 : kw := 4343868 : ye := 0.665 : h := 23 : g := 9.81 : d := 1 : \rho := 1025 : Le := 180 : Lp := 36 : K := \text{Matrix}(6, 6) :$

for i from 1 to 6 do for j from 1 to 6 do $K[i, j] := \text{evalf}(Kx[i, j])$ end end ; K;

$$\begin{bmatrix} 1.12905136010^9 & 0. & 0. & -1.12992971910^9 & 0. & 0. \\ 0. & 4.27207855910^6 & -3.64406926210^7 & 0. & 0. & 0. \\ 0. & -3.64406926210^7 & 1.78167468510^{11} & 0. & 0. & -1.77856630610^{11} \\ -1.12992971910^9 & 0. & 0. & 1.13988776810^9 & 0. & 0. \\ 0. & 0. & 0. & 0. & 2.74136758110^6 & -1.71251816610^7 \\ 0. & 0. & -1.77856630610^{11} & 0. & -1.71251816610^7 & 1.80857169810^{11} \end{bmatrix}$$

$me := 49.65 \cdot 10^6 : Je := 3.74 \cdot 10^9 : mp := 1400 \cdot 10^3 : Jp := 2.75 \cdot 10^8 : a11 := 1 : a22 := 1.25 : a33 := 0.035 : a44 := 1 : a55 := 1.5 : a66 := 1 : In := \text{Matrix}(6, 6) :$

for i from 1 to 6 do for j from 1 to 6 do $In[i, j] := \text{evalf}(M[i, j] + A[i, j])$ end end ; M; A; In;

$$\begin{bmatrix}
4.96500000010^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 4.96500000010^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 3.74000000010^9 & 0 & 0 & 0 \\
0 & 0 & 0 & 2800000 & 0 & 0 \\
0 & 0 & 0 & 0 & 2800000 & 0 \\
0 & 0 & 0 & 0 & 0 & 5.50000000010^8
\end{bmatrix}$$

$$\begin{bmatrix}
4.86722439010^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.08403048810^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.19269416010^9 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.79300000010^6 & 0 & 0 \\
0 & 0 & 0 & 0 & 5.68950000010^6 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.26330606210^9
\end{bmatrix}$$

$$\begin{bmatrix}
9.83222439010^7 & 0. & 0. & 0. & 0. & 0. \\
0. & 1.10490304910^8 & 0. & 0. & 0. & 0. \\
0. & 0. & 4.93269416010^9 & 0. & 0. & 0. \\
0. & 0. & 0. & 6.59300000010^6 & 0. & 0. \\
0. & 0. & 0. & 0. & 8.48950000010^6 & 0. \\
0. & 0. & 0. & 0. & 0. & 1.81330606210^9
\end{bmatrix}$$

$NFsquared, Ex := Eigenvectors(K, In) :$

$T := \text{Vector}(6) : w := \text{Vector}(6) :$

for i **from** 1 **to** 6

do $w[i] := evalf(\text{sqrt}(\text{Re}(NFsquared[i]))) :$

end: $w; E := \text{Matrix}(6, 6) :$

for i **from** 1 **to** 6

do $T[i] := evalf\left(\frac{2 \cdot \text{Pi}}{\text{sqrt}(\text{Re}(NFsquared[i]))}\right) :$

end: $T;$

$$\begin{bmatrix}
13.57538243 \\
0.2929250517 \\
11.63492811 \\
0.1858526685 \\
0.7189291325 \\
0.5451990299
\end{bmatrix}$$

$$\begin{bmatrix}
0.4628367076 \\
21.44980523 \\
0.5400278583 \\
33.80734513 \\
8.739644872 \\
11.52457169
\end{bmatrix}$$

for i **from** 1 **to** 6 **do** **for** j **from** 1 **to** 6 **do** $E[i,j] := \text{Re}(\text{evalf}(Ex[i,j]))$: **end**: **end**;

E ;

$$\begin{bmatrix} 0.06635566329 & 0.7100273842 & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.0008320413753 & 0.9961621109 & 0.06638022440 & -0.01632845251 \\ 0. & 0. & 0.3414175332 & 0.01245477622 & -0.09624556262 & 0.01280185433 \\ -0.9977960342 & 0.7041740649 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.01403787334 & 0.08576483734 & 0.9885830199 & 0.9997037781 \\ 0. & 0. & -0.9398065300 & 0.01226051031 & -0.09504772643 & 0.01272203832 \end{bmatrix}$$

$Kstar1 := \text{Transpose}(E).$ $K.E : Kstar := \text{Matrix}(6, 6)$:

for i **from** 1 **to** 6 **do** $Kstar[i,i] := \text{Re}(\text{evalf}(Kstar1[i,i]))$: **end**:

$Kstar$;

$$\begin{bmatrix} 1.28946403210^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.53370725310^6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.94644793310^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.82523299810^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.66234587310^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.85822562910^6 \end{bmatrix}$$

$\text{Instar1} := \text{Transpose}(E).$ $\text{In.E} : \text{Instar} := \text{Matrix}(6, 6)$:

for i **from** 1 **to** 6 **do** $\text{Instar}[i,i] := \text{Re}(\text{evalf}(\text{Instar1}[i,i]))$: **end**:

Instar ;

$$\begin{bmatrix} 6.99689065310^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.28372788610^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.17656360310^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.10744021810^8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.08577229310^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.61582078010^6 \end{bmatrix}$$

$Cstar := \text{Matrix}(6, 6)$:

for i **from** 1 **to** 6 **do** $Cstar[i,i] := 0.03 \cdot 2 \cdot \text{sqrt}(Kstar[i,i] \cdot \text{Instar}[i,i])$; **end**:

$Cstar$;

$$\begin{bmatrix} 5.69912798610^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9.28641758410^5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.51944966310^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.23492431810^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.05650087610^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.14552169710^5 \end{bmatrix}$$

Froude Krylov Forces

$$FTEh := \rho \cdot g \cdot Le \cdot \left(-\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} + \frac{\cosh(\kappa \cdot (h - (d + he)))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \int \left(\exp(-I \cdot \kappa \cdot y), y = -\frac{We}{2} \dots \frac{We}{2} \right) :$$

$$FTEs := \rho \cdot g \cdot Le \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\exp\left(\frac{I \cdot \kappa \cdot We}{2}\right) - \exp\left(-\frac{I \cdot \kappa \cdot We}{2}\right) \right) \cdot \int \left(\frac{\cosh(\kappa \cdot (h - 0.5 \cdot he - d + z))}{\cosh(\kappa \cdot h)}, z = -\frac{he}{2} \dots \frac{he}{2} \right) :$$

$$FTEr := \rho \cdot g \cdot Le \cdot \left(\frac{\cosh(\kappa \cdot (h - d))}{\cosh(\kappa \cdot h)} - \frac{\cosh(\kappa \cdot (h - (d + he)))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \int \left(y \cdot \exp(-I \cdot \kappa \cdot y), y = -\frac{We}{2} \dots \frac{We}{2} \right) - \rho \cdot g \cdot Le \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\exp\left(\frac{I \cdot \kappa \cdot We}{2}\right) - \exp\left(-\frac{I \cdot \kappa \cdot We}{2}\right) \right) \cdot \int \left(z \cdot \frac{\cosh(\kappa \cdot (h - 0.5 \cdot he - d + z))}{\cosh(\kappa \cdot h)}, z = -\frac{he}{2} \dots \frac{he}{2} \right) :$$

$$FPh := \rho \cdot g \cdot 2 \cdot Lp \cdot \left(\frac{\cosh(\kappa \cdot (h - drp))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\int \left(\exp(-I \cdot \kappa \cdot y), y = -\frac{Wp}{2} \dots -\frac{Wp}{2} + 2 \cdot yp \right) + \int \left(\exp(-I \cdot \kappa \cdot y), y = \frac{Wp}{2} - 2 \cdot yp \dots \frac{Wp}{2} \right) \right) :$$

$$FPs := 0 :$$

$$FPr := -\rho \cdot g \cdot 2 \cdot Lp \cdot \left(\frac{\cosh(\kappa \cdot (h - drp))}{\cosh(\kappa \cdot h)} \right) \cdot \exp(I \cdot \omega \cdot t) \cdot \left(\int \left(y \cdot \exp(-I \cdot \kappa \cdot y), y = -\frac{Wp}{2} \dots -\frac{Wp}{2} + 2 \cdot yp \right) + \int \left(y \cdot \exp(-I \cdot \kappa \cdot y), y = \frac{Wp}{2} - 2 \cdot yp \dots \frac{Wp}{2} \right) \right) :$$

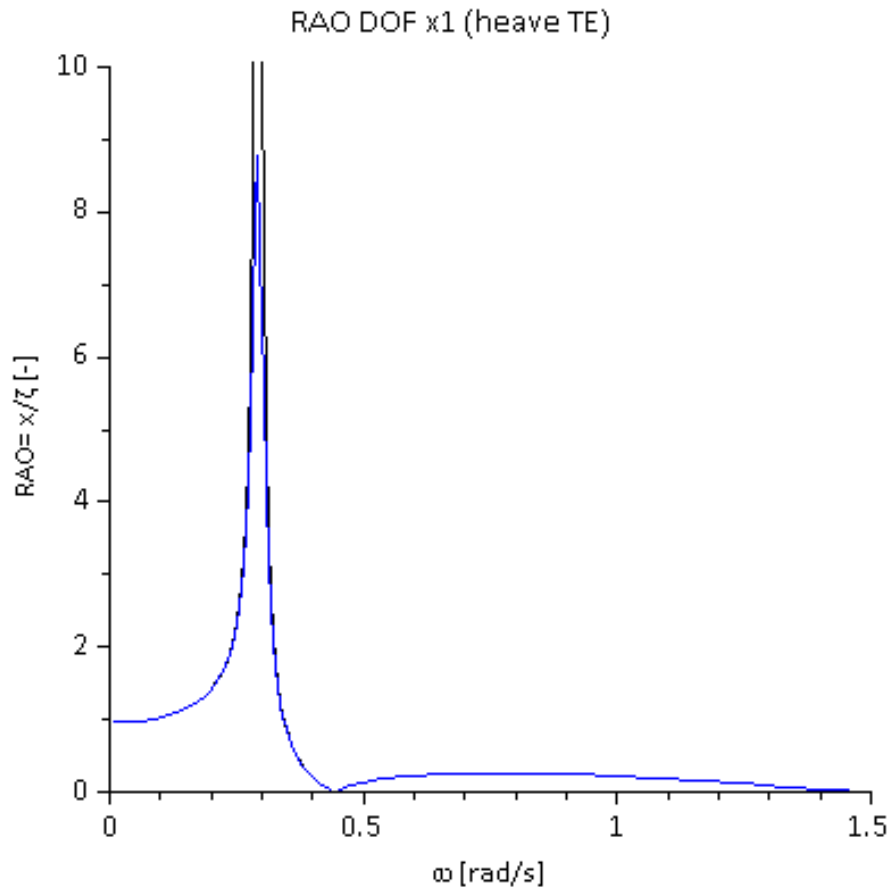
$$F := Vector(6) : F[1] := FTEh : F[2] := FTEs : F[3] := FTEr : F[4] := FPh : F[5] := FPs : F[6] := FPr : \kappa := \text{abs} \left(\frac{RootOf \left(\underline{Z} g \left(e^{-Z} \right)^2 - \underline{Z} g - \omega^2 h \left(e^{-Z} \right)^2 - \omega^2 h \right)}{h} \right) : t := 0 :$$

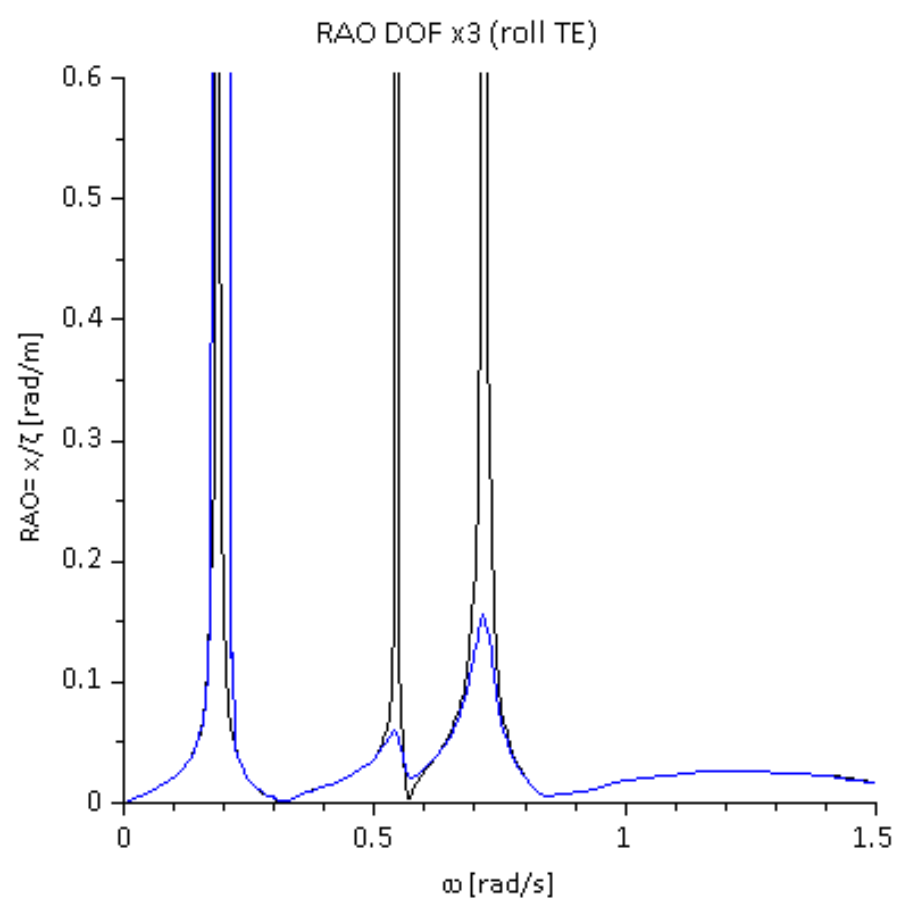
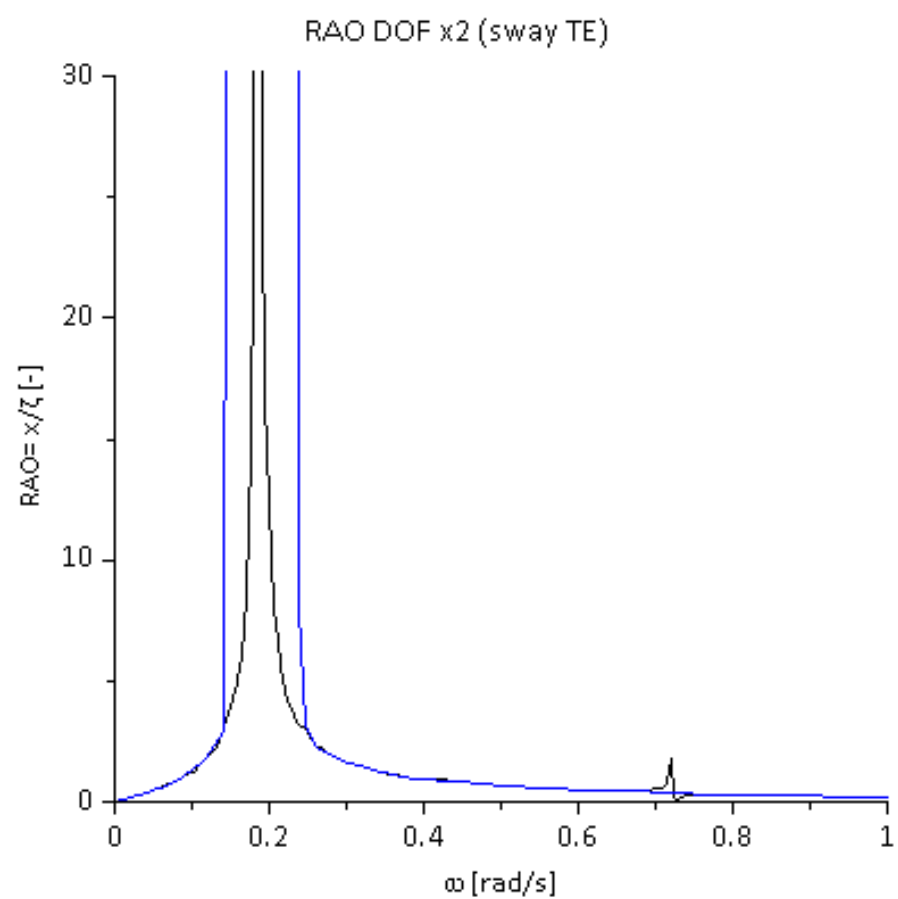
$$Cmod := MatrixInverse(Transpose(E)).Cstar.MatrixInverse(E) : evalf(%) : Cstar2 := Transpose(E).Cmod.E : x := MatrixInverse(-In \cdot \omega^2 + K).F : xc := MatrixInverse(-In \cdot \omega^2 + I \cdot \omega \cdot Cmod + K).F : t := 't' :$$

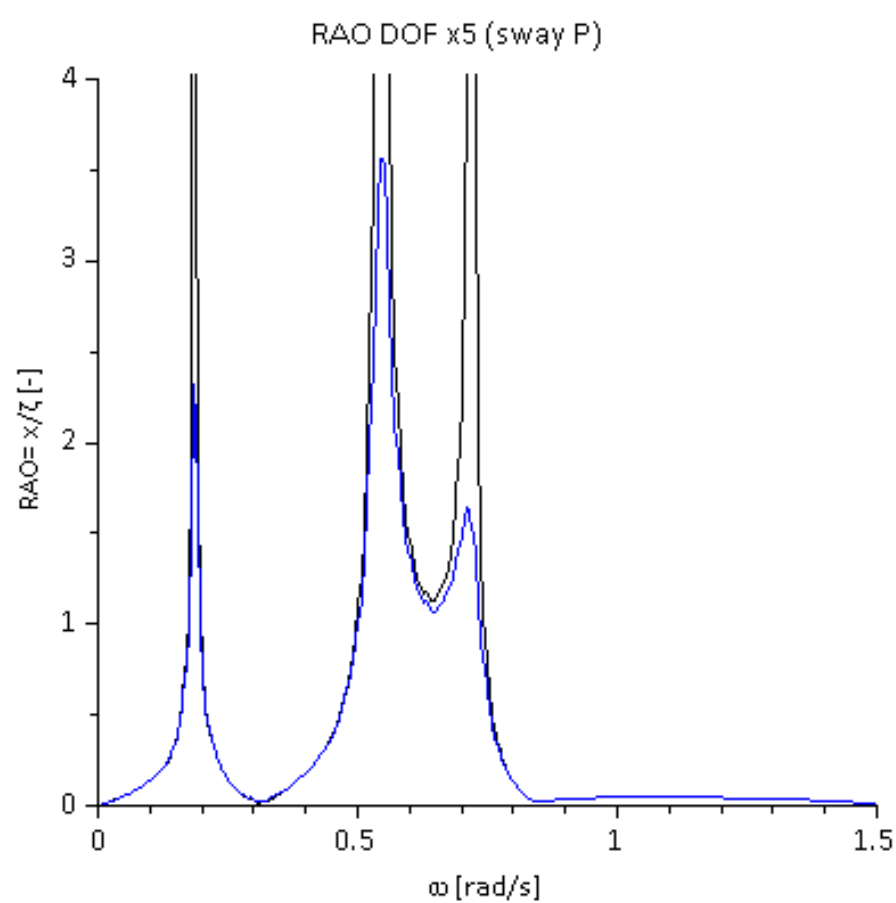
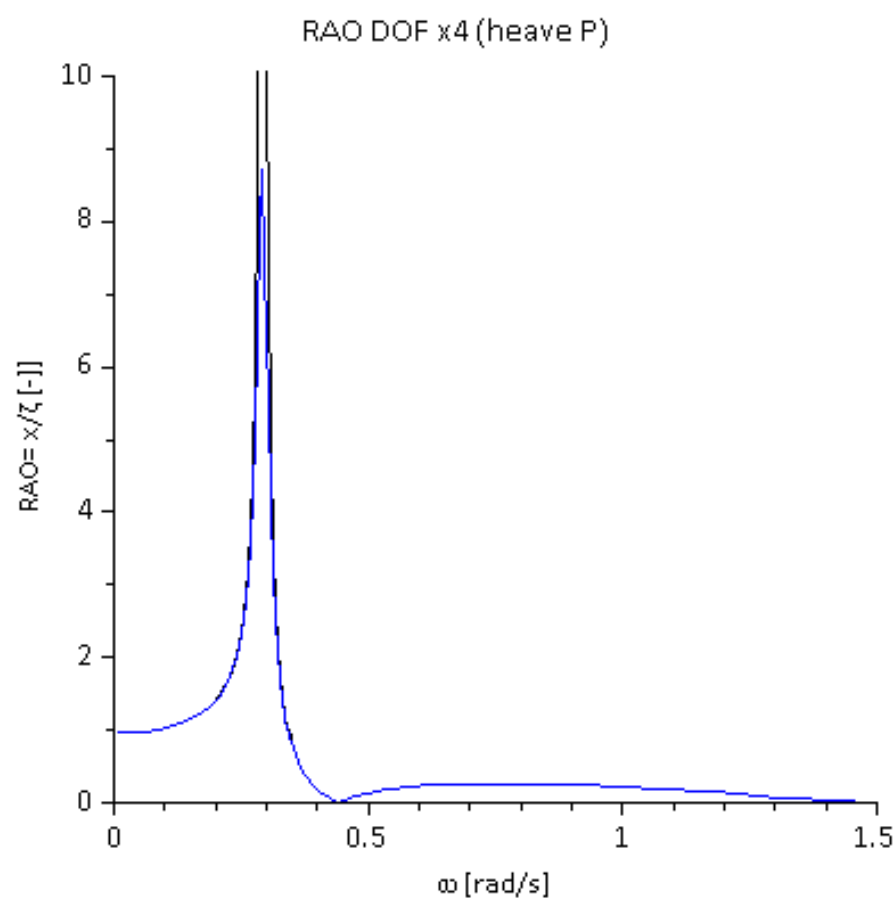
```

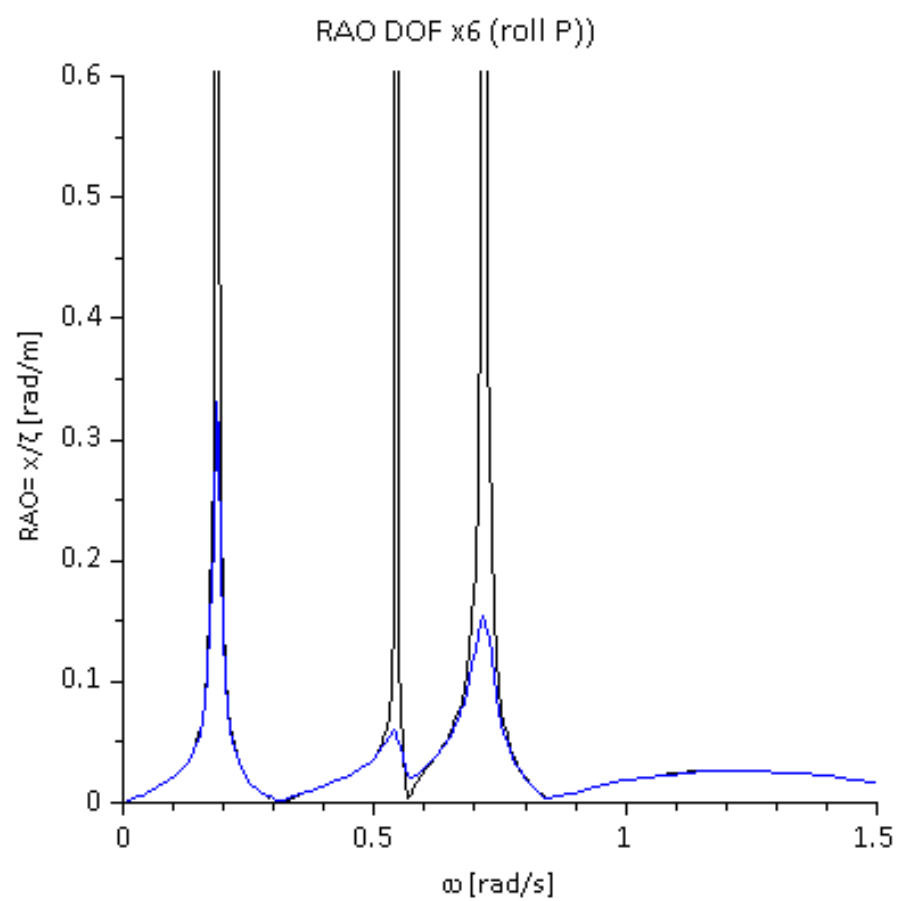
plot([abs(x[1]),abs(xc[1])], $\omega$  = 0 ..1.5, y = 0 ..10, numpoints = 10, color = [black, blue], title
    = "RAO DOF x1 (heave TE)", labels = [" $\omega$  [rad/s]", "RAO=  $x/\zeta$  [-]"], labeldirections = [horizontal, vertical],
    font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(x[2]),abs(xc[2])], $\omega$  = 0 ..1, y = 0 ..30, numpoints = 10, color = [black, blue], title
    = "RAO DOF x2 (sway TE)", labels = [" $\omega$  [rad/s]", "RAO=  $x/\zeta$  [-]"], labeldirections = [horizontal, vertical],
    font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(x[3]),abs(xc[3])], $\omega$  = 0 ..1.5, y = 0 ..0.6, numpoints = 10, color = [black, blue], title
    = "RAO DOF x3 (roll TE)", labels = [" $\omega$  [rad/s]", "RAO=  $x/\zeta$  [rad/m]"], labeldirections = [horizontal, vertical],
    font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(x[4]),abs(xc[4])], $\omega$  = 0 ..1.5, y = 0 ..10, numpoints = 10, color = [black, blue], title
    = "RAO DOF x4 (heave P)", labels = [" $\omega$  [rad/s]", "RAO=  $x/\zeta$  [-]"], labeldirections = [horizontal, vertical],
    font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(x[5]),abs(xc[5])], $\omega$  = 0 ..1.5, y = 0 ..4, numpoints = 10, color = [black, blue], title
    = "RAO DOF x5 (sway P)", labels = [" $\omega$  [rad/s]", "RAO=  $x/\zeta$  [-]"], labeldirections = [horizontal, vertical], font
    = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);
plot([abs(x[6]),abs(xc[6])], $\omega$  = 0 ..1.5, y = 0 ..0.6, numpoints = 10, color = [black, blue], title
    = "RAO DOF x6 (roll P)", labels = [" $\omega$  [rad/s]", "RAO=  $x/\zeta$  [rad/m]"], labeldirections = [horizontal, vertical],
    font = [Calibri, 1, 12], labelfont = [Calibri, 1, 10]);

```









Appendix F Static equilibrium

In this maple file the equations of motions are validated whether they are consistent.

restart : with(plots) :

$We := 26.46 : he := 9.97 : Wp := 42.5 : Wf := 6 : g := 9.81 : \rho := 1025 : Le := 180 : Lp := 36 : \alpha c := \frac{15 \cdot \text{Pi}}{180} :$
 $\alpha m := \frac{10 \cdot \text{Pi}}{180} : hpd := 2.5 : yp := 3 : km := 1413.3 \cdot 1000 : ks := 563000 \cdot 1000 : kc := 2289.4 \cdot 1000 : kw$
 $:= 4343868 : ye := 0.665 : me := 1.02 \cdot We \cdot he \cdot Le \cdot \rho : Je := 3.74 \cdot 10^9 : mp := 1400 \cdot 10^3 : Jp := 2.75 \cdot 10^8 : L$
 $:= 7.5 : h := 23 : d := 1 :$

$eq1 := me \cdot diff(x1(t), t\$2) = -2 \cdot ks \cdot x1(t) + 2 \cdot kc \cdot x1(t) \cdot (2 \cdot \sin(\alpha c) - 1 - \sin(\alpha c)^2) + 2 \cdot ks \cdot x4(t) + 2 \cdot kc$
 $\cdot x4(t) \cdot (1 - \sin(\alpha c)) - 8000000 \cdot diff(x1(t), t) + me \cdot g - We \cdot he \cdot Le \cdot \rho \cdot g : evalf(\%) ;$

$4.96456887810^7 \left(\frac{d^2}{dt^2} x1(t) \right) = -1.12851536010^9 x1(t) + 1.12939371910^9 x4(t) - 8.00000010^6 \left(\frac{d}{dt} x1(t) \right)$
 $+ 9.549494210^6$

$eq2 := me \cdot diff(x2(t), t\$2) = -2 \cdot kc \cdot x2(t) \cdot \cos(\alpha c)^2 + kc \cdot x3(t) \cdot (We \cdot \sin(\alpha c) \cdot \cos(\alpha c) + he \cdot \cos(\alpha c)^2)$
 $- 8000000 \cdot diff(x2(t), t) : evalf(\%) ;$

$4.96456887810^7 \left(\frac{d^2}{dt^2} x2(t) \right) = -4.27207855910^6 x2(t) + 3.64406926210^7 x3(t) - 8.00000010^6 \left(\frac{d}{dt} x2(t) \right)$

$eq3 := Je \cdot diff(x3(t), t\$2) = kc \cdot x2(t) \cdot (We \cdot \cos(\alpha c) \cdot \sin(\alpha c) + he \cdot \cos(\alpha c)^2) - kc \cdot x3(t) \cdot \left(\frac{We^2}{2} \cdot \sin(\alpha c)^2 \right.$
 $+ We \cdot he \cdot \cos(\alpha c) \cdot \sin(\alpha c) + \frac{he^2}{2} \cdot \cos(\alpha c)^2) - 2 \cdot \left(\frac{We}{2} - ye \right)^2 \cdot ks \cdot x3(t) + 2 \cdot \left(\frac{We}{2} - ye \right)^2 \cdot ks \cdot x6(t)$
 $- 5000000000 \cdot diff(x3(t), t) : evalf(\%) ;$

$3.74000000010^9 \left(\frac{d^2}{dt^2} x3(t) \right) = 3.64406926210^7 x2(t) - 1.78082845310^{11} x3(t) + 1.77772007410^{11} x6(t)$
 $- 5.00000000010^8 \left(\frac{d}{dt} x3(t) \right)$

$eq4 := 2 \cdot mp \cdot diff(x4(t), t\$2) = 2 \cdot ks \cdot x1(t) + 2 \cdot kc \cdot x1(t) - 2 \cdot kc \cdot x1(t) \cdot \sin(\alpha c) - 2 \cdot ks \cdot x4(t) - 2 \cdot kw \cdot x4(t)$
 $- 2 \cdot kc \cdot x4(t) - 2 \cdot km \cdot x4(t) \cdot \sin(\alpha m)^2 - 800000 \cdot diff(x4(t), t) + 2 \cdot mp \cdot g : evalf(\%) ;$

$2.80000010^6 \left(\frac{d^2}{dt^2} x4(t) \right) = 1.12939371910^9 x1(t) - 1.13935176810^9 x4(t) - 8.0000010^5 \left(\frac{d}{dt} x4(t) \right)$
 $+ 2.74680000010^7$

$eq5 := 2 \cdot mp \cdot diff(x5(t), t\$2) = -2 \cdot km \cdot x5(t) \cdot \cos(\alpha m)^2 + km \cdot x6(t) \cdot (Wp \cdot \sin(\alpha m) \cdot \cos(\alpha m) + 2 \cdot hpd$
 $\cdot \cos(\alpha m)^2) - 500000 \cdot diff(x5(t), t) : evalf(\%) ;$

$2.80000010^6 \left(\frac{d^2}{dt^2} x5(t) \right) = -2.74136758110^6 x5(t) + 1.71251816610^7 x6(t) - 5.0000010^5 \left(\frac{d}{dt} x5(t) \right)$

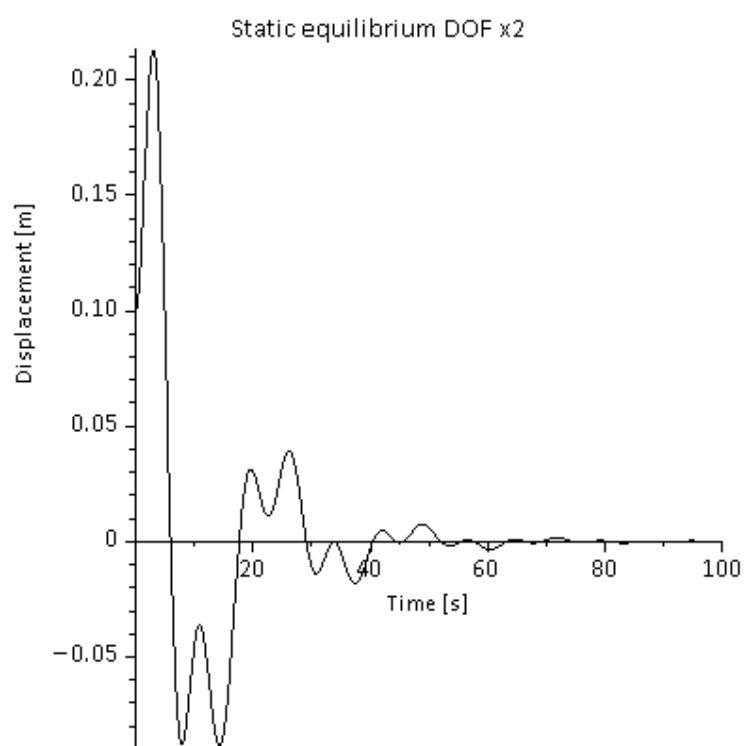
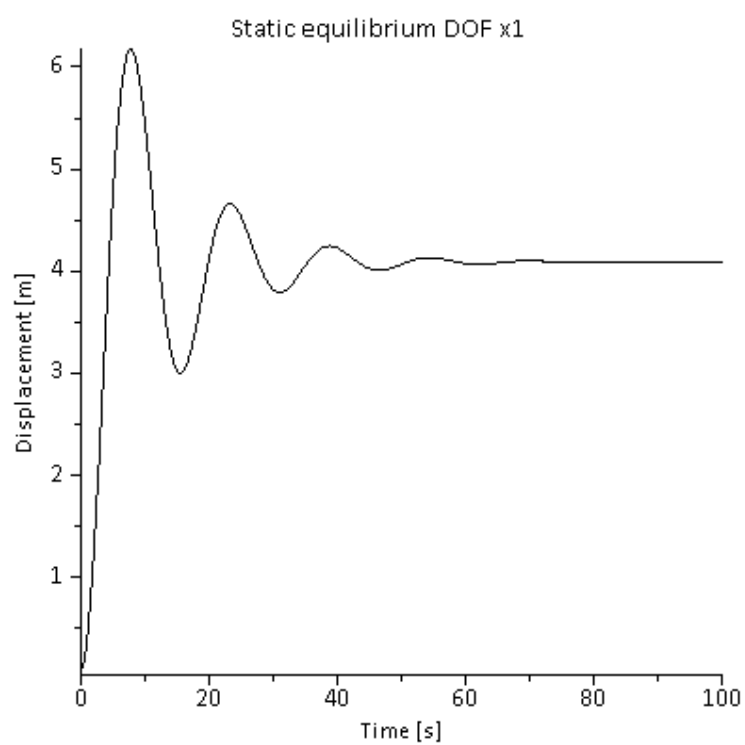
$$eq6 := 2 \cdot Jp \cdot diff(x6(t), t^2) = 2 \cdot ks \cdot x3(t) \cdot \left(\frac{We}{2} - ye \right)^2 + km \cdot x5(t) \cdot (Wp \cdot \cos(\alpha m) \cdot \sin(\alpha m) + 2 \cdot hpd \cdot \cos(\alpha m)^2) - 2 \cdot ks \cdot x6(t) \cdot \left(\frac{We}{2} - ye \right)^2 - 2 \cdot kw \cdot x6(t) \cdot \left(\frac{Wp}{2} - yp \right)^2 - km \cdot x6(t) \cdot \left(\frac{Wp^2}{2} \cdot \sin(\alpha m)^2 + 2 \cdot Wp \cdot \sin(\alpha m) \cdot \cos(\alpha m) \cdot hpd + 2 \cdot hpd^2 \cdot \cos(\alpha m)^2 \right) - 5000000 \cdot diff(x6(t), t) : evalf(\%);$$

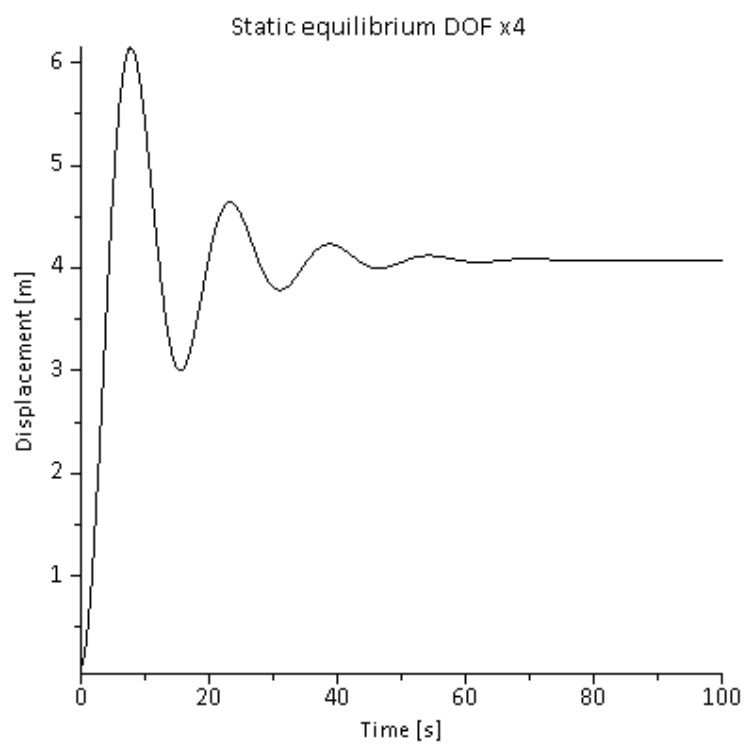
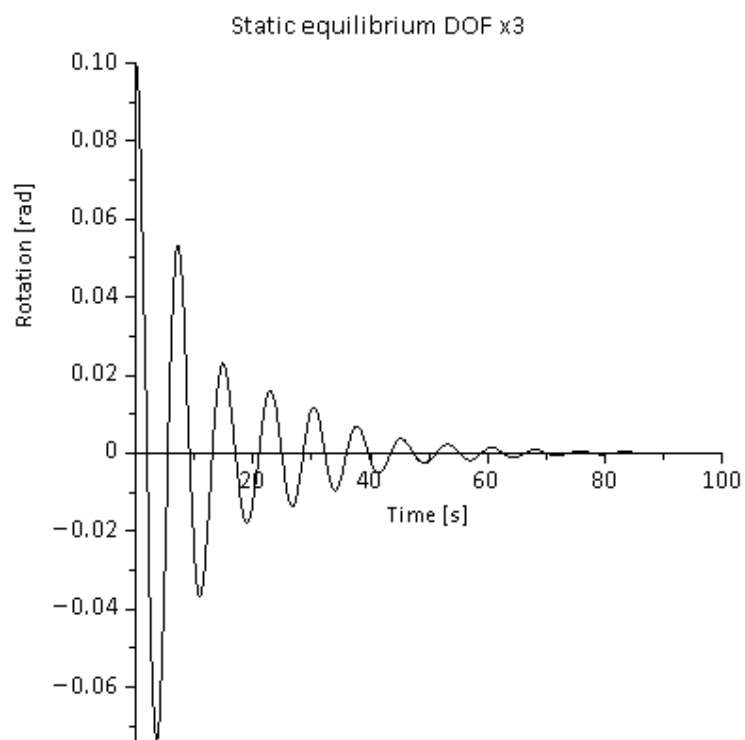
$$5.50000000010^8 \left(\frac{d^2}{dt^2} x6(t) \right) = 1.77772007410^{11} x3(t) + 1.71251816610^7 x5(t) - 1.80772546610^{11} x6(t) - 5.00000010^6 \left(\frac{d}{dt} x6(t) \right)$$

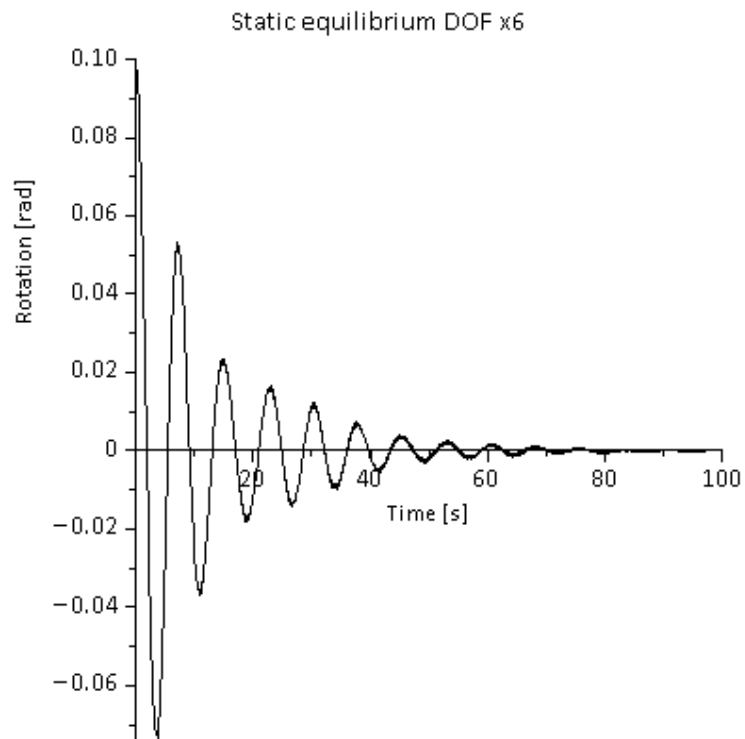
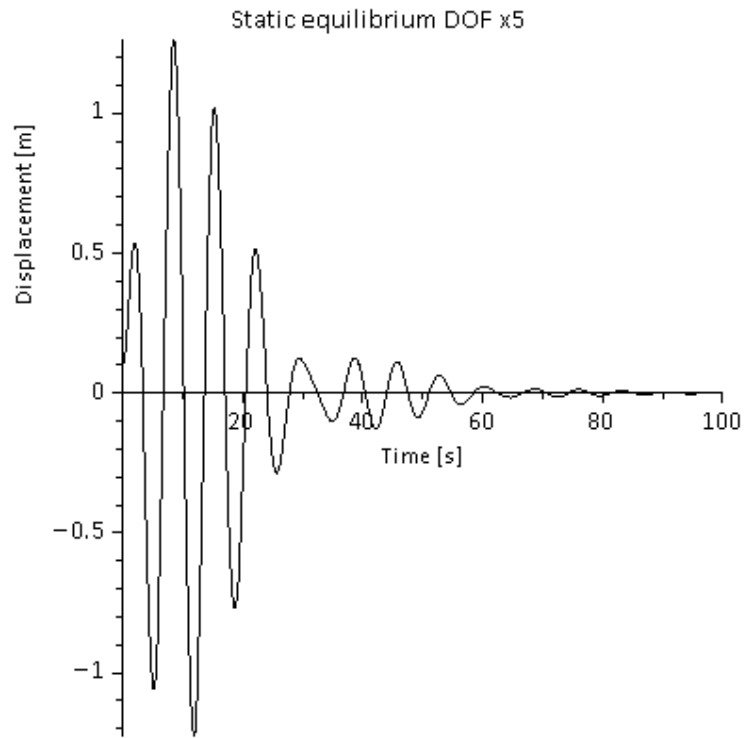
```

sol1 := dsolve([eq1, eq2, eq3, eq4, eq5, eq6, x1(0) = 0.1, D(x1)(0) = 0, x2(0) = 0.1, D(x2)(0) = 0, x3(0) = 0.1,
D(x3)(0) = 0, x4(0) = 0.1, D(x4)(0) = 0, x5(0) = 0.1, D(x5)(0) = 0, x6(0) = 0.1, D(x6)(0) = 0, ], numeric,
maxfun = 100000) :
odeplot(sol1, [t, x1(t)], 0..100, numpoints = 1000, color = black, title = "Static equilibrium DOF x1", labels
= ["Time [s]", "Displacement[m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
odeplot(sol1, [t, x2(t)], 0..100, numpoints = 1000, color = black, title = "Static equilibrium DOF x2", labels
= ["Time [s]", "Displacement[m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
odeplot(sol1, [t, x3(t)], 0..100, numpoints = 1000, color = black, title = "Static equilibrium DOF x3", labels
= ["Time [s]", "Rotation [rad]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
odeplot(sol1, [t, x4(t)], 0..100, numpoints = 1000, color = black, title = "Static equilibrium DOF x4", labels
= ["Time [s]", "Displacement[m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
odeplot(sol1, [t, x5(t)], 0..100, numpoints = 1000, color = black, title = "Static equilibrium DOF x5", labels
= ["Time [s]", "Displacement[m]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);
odeplot(sol1, [t, x6(t)], 0..100, numpoints = 1000, color = black, title = "Static equilibrium DOF x6", labels
= ["Time [s]", "Rotation [rad]"], labeldirections = [horizontal, vertical], font = [Calibri, 1, 12], labelfont
= [Calibri, 1, 10]);

```







```

x1(t) := x1 : x2(t) := x2 : x3(t) := x3 : x4(t) := x4 : x5(t) := x5 : x6(t) := x6 :
solve({eq1, eq2, eq3, eq4, eq5, eq6}, {x1, x2, x3, x4, x5, x6});
{x1 = 4.089717557, x2 = 0., x3 = 0., x4 = 4.078081459, x5 = 0., x6 = 0.}

```