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Scenario tree airline fleet planning for demand uncertainty

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Abstract

This paper proposes an innovative multi-period modeling approach to solve the airline fleet planning problem under demand uncertainty. The problem is modeled using a scenario tree. The tree is composed of nodes, which represent points of decision in multiple time stages of the planning horizon, and branches, representing demand variation scenarios. The branches link the decision nodes in consequent time stages and compose scenario paths. Fleet decisions are modeled according to these scenario paths, resembling the real-life process in which fleet plans are not defined in a single moment but instead are adjusted according to the demand development. Given that some scenario paths share common decision nodes, decisions among scenarios are synchronized. A mixed-integer linear programming model is proposed to determine the ideal fleet composition for each scenario. Results for two real-world based case studies show that the proposed approach can provide flexible multi-period airline fleet plans.

Keywords: Fleet planning; airline strategic planning; scenario tree; demand uncertainty; multi-period planning.

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1. Introduction

The airline industry is one that faces many challenges. Whilst demand has grown steadily at an average of around 5% over the past 30 years, the industry still struggles to be profitable (Belobaba et al., 2009; Pearce, 2013). Profits have shown to be cyclical, with airlines generating major losses in some years and some profits in others. As all other operational decisions are dependent on the available fleet, it is of vital importance for an airline to optimize the fleet for their operations (Bazargan, 2010). The process of determining the ideal fleet to operate is called fleet planning. It entails deciding between many different aircraft families and types, their quantities and when to acquire them.

Much research has already been done in the field of vehicle fleet planning. However, most research has focused on other industries than the airline industry. Nonetheless, the general problem of determining the right vehicle and number of vehicles in a fleet can generally be applied. The simplest approach is to use a deterministic model that optimizes the fleet for a certain expected future. Research dates back to Gertsbach and Gurevich (1977) and Abara (1989), who developed methods to respectively minimize an uniform fleet of vehicles and aircraft to serve a particular schedule. The following research developments have focused on studying how to optimally change the schedule depending on the capacity and available demand. Bertsekas (1998) performed this optimization for a general fleet planning problem using a single vehicle type, minimizing the fleet size whilst serving all available demand. Similarly, Xinlian et al. (2000) optimized a fleet of ships to serve all demand whilst minimizing the cost of doing so. The authors already considered the case of fleets composed of vehicle with different characteristics. Revenue maximization has been the focus of most recent works. Instead of serving all demand, Sayarshad and Ghoseiri (2009) optimized a rail car fleet for profit. Demand could thus be spilled if this would improve the profitability. In a fairly recent study, Bazargan and Hartman (2012) compared the options of aircraft purchasing and leasing in the airline fleet replacing problem. The results showed that aircraft leasing is, up to a certain extend, preferable to the purchase of aircraft.

A downside of the aforementioned models is their deterministic basis. In real life, the future almost never turns out as expected. An airline that does not consider this might be left short-handed if the future turns out more favorable than expected, or on the other hand might be stuck with overcapacity in case of a downturn. In an industry that has low profit margins at best, a well-structured fleet can be the difference between thriving or perishing. Stochastic programming is the most common method to consider this uncertainty. List et al. (2003) constructed such a type of model, in which the decision process is split into two stages. In the first stage, the fleet sizing decision is made, after which these aircraft are assigned to routes in the second stage. These second stage decisions are based on demand forecast values that are modeled according to random variables. The generation of these random demand values can be done in various ways. A traditional approach to this is to assume that demand values follows a normal distribution. These distributions are used for their simplicity (Sherali and Zhu, 2008). Other approaches proposed in the literature to model the randomness of demand include, for instance, Gaussian white noise (Bojovi, 2002), upper partial moment to avoid too high losses (List et al., 2003), Monte Carlo simulations for probability prediction (Khoo and Teoh, 2013) and scenario based approaches to limit the required computing power (Listes and Dekker, 2005). Each approach has its benefits, with a decision for a model mainly being driven by the allowed complexity of the model and which risks an airline wants to protect itself from.

This paper focuses on the long-term airline fleet planning problem. This problem involves decisions that are not always taken at the same time. Airlines may decide to invest (or *disinvest*) in their fleets in multiple stages, adapting their plans to new information and spreading the investment over time. Thus, to model this long-term problem, it is necessary to include several years in the optimization. A scenario tree can be used to model this multi-stage decision process and to capture the development of uncertain parameters over time. In generic terms, there are two options in the way the scenario tree approach can be used to optimize a fleet. The first option comprises the optimization of a singular fleet development plan. In such a case, the optimizer finds one fleet that leads to the most optimal result considering the various scenarios. This was the model adopted by Hsu et al. (2011). The authors focused on the multi-period aircraft replacement problem. However, they have only dealt with uncertainty in two stages, after which the remaining stages were presumed to be equal for all scenarios. The second option is to find the optimal fleet for each individual scenario, whilst considering the fact that, due to the scenario tree structure, these scenarios overlap early on in the time horizon and it is hence required to have the same fleet at these overlapping nodes. This is a better representation of the actual fleet planning process in which the plans are adjusted according to how the future unfolds. The principle, first described by Ahmed et al. (2003), was used by Pantuso et al. (2015b) to optimize a fleet of ships considering many factors on the available ships. Their model was still limited though, as it only had two stages, of which the second stage spanned multiple years. Also, the outcome could only be used for here-and-now decisions, limiting the use of the multi-period model. In a consecutive paper, Pantuso et al. (2015a) improved their model by making it a true multi-stage model. Because of the computational problems inherent to these problems, they however focused their research on a novel solution technique and failed to explore the potential benefit and application of such a model.

The work presented in this paper aims to develop a modeling approach that best resembles the decision process of an airline and helps it to hedge against uncertainty. Therefore, it will make use of a scenario tree and provide multiple fleet development paths similar to Pantuso et al. (2015a). The modeling approach consists of aggregated level decisions, determining at each stage which fleet to operate in the next period, and detailed level decisions about how to assign these aircraft to different routes. Stochastic programming will be used to capture the corresponding uncertainty in demand and to model decisions over the multiple periods considered in the planning horizon. The optimal solution will be determined on the basis of the estimated airline operations profit maximization. The Pyomo based PySP software package (Watson et al., 2012) is used in combination with th IBM CPLEX linear optimizer. Our work will be the first to apply such modeling approach to the airline fleet planning problem. In addition, given that the work from Pantuso et al. mainly focused on the difficulties in solving these models, our study will be the first to focus on the benefit of adopting such type of modeling approach in the fleet planning problem under uncertainty and to show the feasibility of this approach to provide robust multi-period fleet plans for practical case studies.

Two case studies are used in this paper to explore the possible applicability and potential benefit of adopting the model approach proposed. First, a proof of concept is performed with a simple real-word based case involving the definition of the fleet of an airline flying ten long-haul routes. This simple case study is used to highlight the potential benefits of the approach proposed. The second case study is performed on a real-life problem of a major intercontinental airline involving different fleet development strategies. For this case the model is more limited in its options, since the airline brings several operational requirements with it. Nevertheless, this case study allows to form a better understanding of the use of the model in a real-life problem. All simulations will be run using IBM ILOG CPLEX 12.6 on an Intel Core i5-2450M CPU with 4Gb RAM.

The remainder of this paper is structured as follows. First, the scenario tree structure will be introduced, followed by the mathematical model itself, explaining the assumptions it is based on, sets and parameters it includes and the constraints it is subject to. Next, the two case studies are explained, followed by an analysis of the outcome of the simulations. Finally, conclusions are drawn which will be provided together with recommendations for further research.

2. Scenario tree modeling

Finding a single solution for a fleeting decision years from now is not an accurate representation of the real-life decision process. Instead, each year the fleet plan will be reviewed and decisions will be made dependent on how the uncertain parameters have developed. This can be modeled with the introduction of a scenario tree. An example of a representation of such tree is given in Figure 1.

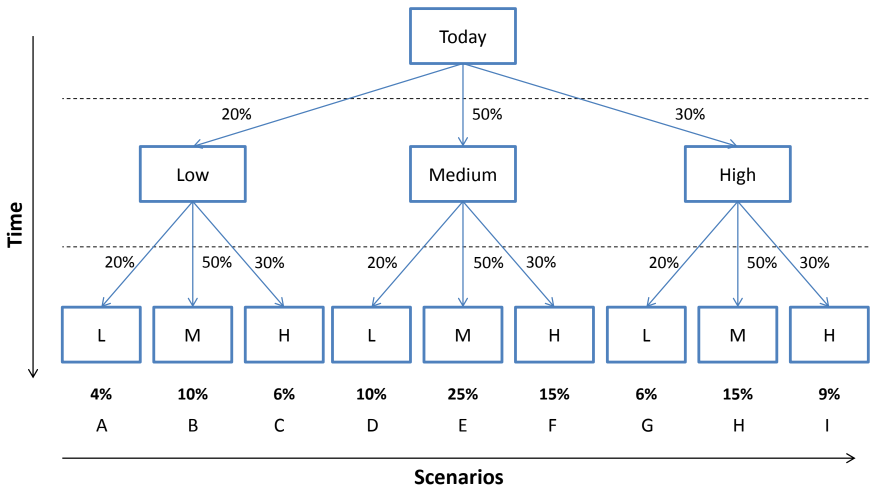

Figure 1: Example three-stage scenario tree

In this figure, there are multiple decision nodes (represented by rectangles) in which fleet decisions are possible. The horizontal dashed lines separate different time stages in the planning horizon. Three stages are considered, meaning that decisions can be taken at three points in time. Each node in the first and second stage has tree branches linking the decisions in the initial stages with the decisions of future stages. The tree branches are formed by their respective demand change (Low, Medium or High) and the probability of each demand change to happen (20%, 50% or 30%). Scenarios are represented in this figure by the paths linking the root node ('Today') with the end nodes at the bottom of the figure. There are in total nine scenarios in this figure – named A, B, . . . , I. This representation of a scenario tree allows solutions to vary across the scenarios. Nevertheless, each of the nine scenarios originates from the same node in the first stage. This means that a single decision has to be made in the first stage and this decision needs to be considered in the following stages of each scenario. In the second stage, each node is still relevant for multiple scenarios, but not to all. For example, the decisions in the second stage have to be the same for scenario A, B and C, but these can differ from scenario D, E and F, and G, H and I. Finally, each scenario has an individual node in stage three, and the decisions can hence be completely different. Each scenario thus consists of a demand growth path with the probability of occurring formed by the multiplication of the probabilities of each stage.

In some particular cases, we can assume that different nodes in the scenario tree represent the same future situation in terms of demand. That is the case, e.g., of the final nodes in scenarios B and D in Figure 1. Both scenarios follow a low and medium demand variation in the two stages represented. However, these variations take place in a different order for each scenario. That is, despite the fact that both scenarios may have the same demand value at the end of stage two, they describe a different demand development pattern. The scenarios should therefore be modeled differently and the resulting fleets may end up being different. This path dependency aspect is modeled in our approach by considering that each scenario has its own independent variables and that these variables are bounded by constraints. The constraints need to guarantee an equal value for the decision variables referring to decision nodes where scenarios overlap. This means that decisions may be different for each scenario but fleets need to be equal for nodes common to multiple scenarios. In this scenario tree approach proposed one is completely free to bound different nodes of the tree and the scenario tree can thus take on any form. Figure 2 illustrates how to structure these constraints for the case of the scenario tree presented in Figure 1.

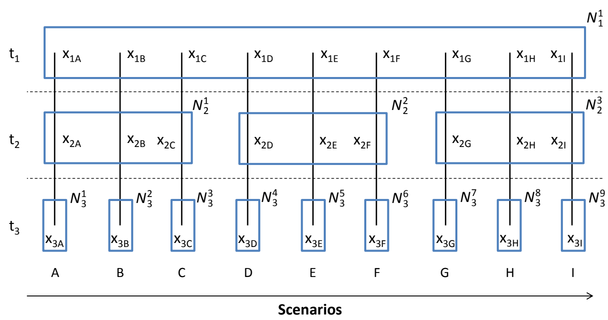

Figure 2: Scenario tree constraints example

In this example, the vertical lines represent individual scenarios, each with a variable

x in t_1, t_2, and t_3. The decision nodes are represented by the boxes. They are given a name N dependent on the time and the number of nodes at that specific time stage. Whenever the scenarios fall into the same node, their variables have to be equal. Hence, $x_{1A} = x_{1B} = ... = x_{1I}$ because they are all part of N_1^1. Next, $x_{2A} = x_{2B} = x_{2C}$ and $x_{2D} = x_{2E} = x_{2F}$, but, since the first are part of N_2^1 and the latter of N_2^2, the two sets do not have to be equal. In such a manner, all variables at overlapping nodes are bounded to each other.

3. Mathematical model

A mixed-integer linear programming model is used to determine the ideal fleet composition in multiple periods. It does so considering several demand scenarios and the structure of constraints discussed in the previous section. We start this section by presenting the nomenclature used, followed by the model formulation. The explanation and justification of the model is presented in the final part of this section.

Sets:

T	set of time periods ($t = t_1, .., t_{\|T\|}$), where $\|T\|$ is the number of periods considered in the model;
A	set of aircraft type ($a = a_1, .., a_{\|A\|}$);
R	set of routes ($r = r_1, .., r_{\|R\|}$);
S	set of scenarios ($s = s_1, .., s_{\|S\|}$).

Decision variables:

x_{ta}^s	number of aircraft of type a owned in period t and scenario s;
y_{tar}^s	assigned weekly frequency of aircraft type a on route r in period t and scenario s;
z_{ta}^s	number of aircraft of type a that are acquired in period t and scenario s;
u_{ta}^s	number of aircraft of type a that are disposed in period t and scenario s;
q_{tr}^s	number of passengers transported on route r in period t and scenario s.

Model Formulation:

$$\text{maximize: } \sum_{s\in S} p_s \sum_{t\in T} \delta_t \times n_t \sum_{a\in A}\sum_{r\in R} [2 \times fare_r \times q_{tr}^s - (vc_{ar} \times y_{tar}^s + fc_{ta} \times x_{ta}^s + pen_{ta} \times u_{ta}^s)] \quad (1)$$

Subject to:

$$BT_a \times x_{ta}^s \geq 2 \times \sum_{r\in R} (OT_r + TAT_a) \times y_{tar}^s \quad , \quad \forall t \in T, a \in A, s \in S \quad (2)$$

$$q_{tr}^s \leq dem_{tr}^s \quad , \quad \forall t \in T, r \in R, s \in S \quad (3)$$

$$q_{tr}^s \leq \sum_{a\in A} cap_a \times LF_r \times y_{tar} \quad , \quad \forall t \in T, r \in R, s \in S \quad (4)$$

Parameters:

p_s	probability of scenario s;
δ_t	discount factor translating the profit of period t into its present day value;
fc_{ta}	cost of owning aircraft type a in period t;
vc_{ar}	cost of operating route r with aircraft type a;
pen_{ta}	penalty cost related to disposing of an aircraft of type a in period t;
$fare_r$	average fare paid by a passenger for one leg on route r;
BT_a	block time (i.e. the maximum number of operating hours per week) that is available for aircraft type a;
OT_r	time required to operate route r;
TAT_a	turnaround time of aircraft type a;
dem^s_{tr}	average weekly demand on both legs of route r in period t and scenario s;
n_t	number of weeks in period t;
cap_a	the seat capacity of aircraft type a;
LF_r	the maximum load factor on route r;
$MFrq_{tr}$	minimum frequency on route r in period t;
IF_a	aircraft of type a that are already owned or leased for the first period;
$W^{s,s'}_t$	auxiliary matrix with value of 1 whenever the decision node of scenario s is equal to the node of scenario s' at period t, and 0 otherwise.

$$\sum_{a \in A} y^s_{tar} \geq MFrq_{tr} \quad , \quad \forall t \in T, r \in R, s \in S \tag{5}$$

$$x^s_{1a} \geq IF_a \quad , \quad \forall a \in A, s \in S \tag{6}$$

And:

$$x^s_{ta} = x^s_{(t-1)a} + z^s_{(t-1)a} - u^s_{(t-1)a} \quad , \quad \forall t = (2, ..., T), a \in A, s \in S \tag{7}$$

$$W^{s,s'}_t (x^s_{ta} - x^{s'}_{ta}) = 0 \qquad \forall t \in T, a \in A, s, s' \in S \tag{8}$$

$$W^{s,s'}_t (y^s_{tar} - y^{s'}_{tar}) = 0 \qquad \forall t \in T, a \in A, r \in R, s, s' \in S \tag{9}$$

$$W^{s,s'}_t (z^s_{ta} - z^{s'}_{ta}) = 0 \qquad \forall t \in T, a \in A, s, s' \in S \tag{10}$$

$$W^{s,s'}_t (u^s_{ta} - u^{s'}_{ta}) = 0 \qquad \forall t \in T, a \in A, s, s' \in S \tag{11}$$

$$W^{s,s'}_t (q^s_{tr} - q^{s'}_{tr}) = 0 \qquad \forall t \in T, r \in R, s, s' \in S \tag{12}$$

With:

$$x^s_{ta} \in \mathbb{Z}^+, \; y^s_{tar} \in \mathbb{Z}^+, \; z^s_{ta} \in \mathbb{Z}^+, \; u^s_{ta} \in \mathbb{Z}^+, \; q^s_{tr} \in \mathbb{R}^+ \tag{13}$$

The objective function (1) concerns the maximization of the sum of the discounted operational profit of all the considered periods for multiple demand scenarios multiplied by their probability. The first term represents the probability that a scenario occurs (p_s), followed by the discount factor by which the result of each period is multiplied by (δ_t). The expected operational profit per time period is computed with the four terms included in the square brackets. The first term refers to the revenue obtained per week by transporting passengers and the second term represents the weekly operational costs, which are generated when aircraft are operated. These costs are calculated on a route and frequency basis and comprise airport and en-route taxes, fuel, crew, and maintenance cost. These two terms are computed assuming a standard week of operations during period t. To convert these values to the full period they are multiplied by the number of weeks in the period (n_t) and by 2 in order to include both legs of each route. The third term in the square brackets represents ownership costs. They reflect either the lease costs or the depreciation costs per period. For the sake of simplicity, it is assumed in this formulation that these costs are exactly the same but the formulation can be easily extended to include different ownership costs in the case of leased or purchased aircraft. The airline is subjected to this cost for each aircraft in the fleet, regardless of whether it is operated or not. The last term is a penalty cost related to disposing an aircraft. For leased aircraft, this consists of the charge that will be applied when an aircraft is returned before the end of the leasing contract. For aircraft owned by the airline, this penalty cost can reflect the difference between the selling value and its 'book' value. This cost for purchased aircraft can be caused by the imbalance between the accounting depreciation and the actual depreciation of the aircraft and can be either positive or negative. Figure 3 provides an example of this imbalance when linear depreciation is used for accounting purposes and the real value of the aircraft depreciates according to an exponential decay function.

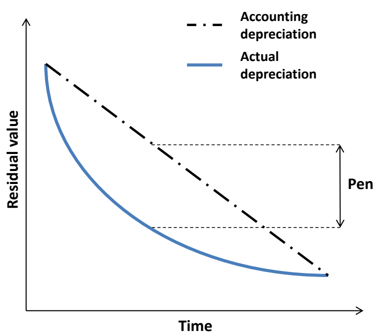

Figure 3: Penalty for sale of owned aircraft

We can divide the constraints from the model in two parts. The first part, including the five set of constraints following the objective function (2-6), refers to the basic functionality of the single period fleet planning model. Constraints 2 assure that the operated hours per aircraft type cannot exceed the available block time per week, given the frequency per route and the number of aircraft available in the fleet. Next, the sets of constraints 3 and 4 ensure that the number of transported passengers can neither exceed the available demand nor the capacity offered. A maximum load factor can be set to account for the fact that on average some routes never exceed a certain load factor. An important aspect of constraints 3 is that not all the demand needs to be served and some passenger may be spilled. Constraint 5 is added to allow the model to be forced to operate certain routes with a particular minimum frequency. For some strategic or political reasons, the airline may decide to operate routes even if they are not profitable. The initial fleet of the airline is considered in the last set of constraints 6.

The second part of the constraints (8 - 12) extend the single period fleet planning model to a multi-period fleet planning framework. Constraints 7 reflect the fact that decisions in one period $t-1$ only take effect in the following period t. They also guarantee that for each scenario there is consistency between the fleet in period $t-1$, the acquisition of new aircraft or disposal of existing aircraft and the fleet in period t. The other five sets (8-12) are used to enforce the scenario tree structure. The constraints are only active in the case $W_t^{s,s'}$ is equal to 1. This happens whenever two scenarios overlap at a certain decision node. In these cases, the decision variables value for scenarios s and s' should be the same for time period t in which the constraint apply.

Finally, equations 13 governs the fact that x, y, z and u can only take on non-negative integer values, whereas q can assume non-negative continuous values.

The model is meant to be a strategic planning model. Hence, average fares and a frequency schedule are used. To further simplify the model, in order to limit the required computing power, some additional assumptions were made. In particular, demand was assumed to be inelastic and the network to be point-to-point. That is, demand is assumed to be leg-based. This means that the outbound and inbound leg of a route can be combined into a single flight, halving the number of flights under consideration. This also means that no constraints were required to ensure that the outbound leg is flown by the same aircraft as the inbound leg. In addition, a single cabin type is assumed and the potential influence of cargo is not considered. Finally, all parameters are given on a weekly basis, with that week being representative for the entire period of a time stage.

4. Case studies

We make use of two cases studies to test the proposed methodology and to illustrate the type of results that can be obtained with it. Both case studies are based on real-world data from a reference intercontinental carrier and they provide the required flexibility to explore the performance of the scenario tree methodology. The first case concerns the definition of the fleet composition to operate in a set of long-haul routes of the carrier. The second case study focus on planning the transition between generations of aircraft, studying a set of fleet composition options with the scenario tree approach proposed.

4.1. Case 1: Fleet composition

To validate the premise of the research that the novel model would help airlines better hedge against uncertainty, a small case is used to to explore the relation between the degree of uncertainty and the benefit of the novel model. The network resembles part of the long-haul network of the reference carrier and the parameters values are estimate based on actual operational figures provided by the carrier.

A detailed description is given of the used parameters, followed by a discussion of the performance of the model and how its results can be used for long-term fleet planning.

4.1.1. Model parameters

The parameters can be separated into three categories: aircraft related, route related, and aircraft-route related. The applied values of the parameters of these categories are therefore also given in three separate tables. First, a selection is made of three aircraft types, with significantly different capacities and cost figures. We opted for these three aircraft because the reference airline was able to provide operational data for them. Table 1 gives the ownership cost, capacity, available block time, turnaround time and penalty cost for each aircraft type. The ownership costs and penalty costs are both assumed to be equal in all periods. Next, the operating time (or block time), average fare, average demand of each route and maximum load factor are given in Table 2. Average fares are calculated by dividing the total revenue on a route by the number of passengers, and are thus the combined average of both flight legs. These include taxes and fuel surcharge. Demand is taken as the average of the forecasts of both flight legs of each route. Finally, the only route-aircraft related parameters are the operational cost of each aircraft type per route. These figures are provided in Table 3 in thousands of dollars.

Table 1: Case 1 aircraft related parameters

Aircraft	fc [$/week]	cap [seats]	BT [h/week]	TAT [h/flight]	pen [$]
B772	140,000	322	96	1.5	10,000
B773	175,000	401	96	1.5	10,000
B788	95,000	234	96	1.0	10,000

Table 2: Case 1 route related parameters

Route	OT [h/flight]	$fare$ [$/pax]	dem_1 [-/week]	LF [%]
A	9.00	435	3365	90
B	6.10	267	3091	90
C	8.70	504	1724	90
D	5.25	226	2137	90
E	9.00	500	2569	90
F	3.50	298	768	90
G	3.00	260	729	90
H	4.25	325	2655	90
I	9.60	350	5000	90
J	10.50	477	1366	90

To provide more flexibility to the model and to eliminate the influence of contextual parameters on the outcome of the fleet optimization model, it was assumed that there is no minimum frequency required (i.e., $MFreq = 0$) and that the airline does not have any

Table 3: Case 1 variable costs per aircraft-route combination (*1000 USD)

Aircraft	A	B	C	D	E	F	G	H	I	J
B772	207	132	207	113	225	72	75	93	200	240
B773	249	158	249	136	269	87	90	112	240	288
B788	150	95	150	82	161	52	54	67	144	173

initial fleet (i.e., $IF = 0$). In addition, it was assumed that each period represents a year (i.e., $n_t = 52$ for all t). For the sake of simplicity, the discount factor δ_t was considered to be equal to 1 in each period, meaning that it was presumed that all future values are constant and equal to their present day value. Discount factors lower than one would mean that futures values (costs and revenues) would have a lower importance in the objective function, resulting in more fleet changes due to lower discounted penalty costs in later stages.

For this case study, we assumed a tree with three branches at each node and a three-stage model like the one presented in Figure 1 and discussed in Section 2. The L/M/H branches are considered to be related to demand changes of -10%/+5%/+15% and probabilities 20%/50%/30%, respectively. The most likely demand growth reflects the average demand growth in the air transport industry over the last 30 years (Pearce, 2013) and the other values were obtained by consulting some of the carrier strategic planning experts. They reflect the extreme annual growth scenarios assumed by these experts. It is assumed that on all routes the demand changes in a similar way and there are thus no differences between routes.

4.1.2. Results

The analysis of the results for this case study is divided in two parts. First we discuss the results from the scenario tree methodology presented in this paper. In the second part of this sub-section we compare these results with the results obtained following a deterministic single scenario approach.

Scenario tree results

The best fleet for each decision node in the scenario tree is presented in Table 4. The first column in the table indicates the stage of the decision node, while the second column describes each node according to the L/M/H branches followed by the scenario path. The next two columns indicate the demand variation and the probability associated with each decision node. The demand variation reflects the ratio between the demand considered in the decision node and the initial demand. The last four columns present the number of aircraft per aircraft type considered and in total in the fleet.

The ideal fleet for the first node of the tree comprise ten aircraft, 7 B773s and 3 B788s. This is also the ideal fleet for the three decision nodes in Stage 2. Meaning that given the uncertainty for Stage 2 the best option is not to change the fleet when making decisions in Stage 1. However, depending on the branch in the second stage, the fleets vary for Stage 3. For the scenarios with high demand variation in the second stage, the solution for the third stage involves the acquisition of two more aircraft, one B773 and one B788. For the

Table 4: Ideal fleet for each scenario path for Case 1

Stage	Node	Demand variation	Probability	B772	B773	B788	Total
1	-	1.00	1.00	0	7	3	10
2	H	1.15	0.30	0	7	3	10
2	M	1.05	0.50	0	7	3	10
2	L	0.95	0.20	0	7	3	10
3	HH	1.32	0.09	0	8	4	12
3	HM	1.21	0.15	0	8	4	12
3	HL	1.09	0.06	0	8	4	12
3	MH	1.21	0.15	0	7	3	10
3	MM	1.10	0.25	0	7	3	10
3	ML	1.00	0.10	0	7	3	10
3	LH	1.09	0.06	0	6	3	9
3	LM	1.00	0.10	0	6	3	9
3	LL	0.90	0.04	0	6	3	9

medium demand variation scenarios (in the second stage) there is no variation on the fleet while for the low demand variation scenarios the solution for the third stage is to dispose of one of the B773s. The B772 aircraft is never chosen to be part of any of the ideal fleets.

It is important to notice the path dependency effect in these results. For instance, the scenario ending at the node 'HM' is equivalent to the scenario ending at the node 'MH' - demand variation of 1.21 and probability of 15%. However, the fleets for both scenarios are different. This reflects the fact that it is less risky to investing in a larger fleet when being at Stage 2, in a node with a high demand scenario (15% more than in the initial node). That is, when deciding the Stage 3 fleet for nodes branching from the high demand scenario in Stage 2, there is 80% probability that the demand variation in Stage 3 will be higher or equal to 1.21. For branches deriving from the medium demand scenario in Stage 2 the probability of having a demand variation of 1.21 is only equal to 30%. The inverse effect can be observed when comparing the medium demand scenario with the low demand scenario.

The analysis of the previous table, especially in the case that more stages and branches per node are considered, can be a hard task for the decision-maker. Thus, another way to present the results is to do it in terms of a fleet probability table, such as the one presented in Table 5. For each period, aircraft type and number of aircraft, the probabilities of the scenarios in which this instance occurs are summed to give an individual probability distribution for each aircraft type. A similar process is used for the total fleet size, but without making a distinction between aircraft type. The percentages resemble the probability that a particular number of aircraft is advised. This is an important feature of the presented methodology. While other methods focus on providing a single fleet solution, the modeling methodology proposed takes into consideration future uncertainty to provide the probability that a certain number of aircraft is necessary for future operations.

Table 5 presents this fleet probability for the last stage of our Case 1. It can be concluded from this table that, according to our analysis, with 100% certainty no B772 is advised. The

Table 5: Fleet probability for Stage 3 for Case 1

	Stage 3			
# aircraft	B772	B773	B788	Total
0	100%			
1				
2				
3			70%	
4			30%	
5				
6		20%		
7		50%		
8		30%		
9				20%
10				50%
11				
12				30%

fleet for Stage 3 should have at least 9 aircraft. But most likely it should have 10 aircraft. The other option is to consider 12 aircraft in the fleet, but that solution would only results in profit for 30% of the scenarios. This can help the decision-maker to decide to plan the fleet for 10 aircraft and consider, for instance, the purchase option or the short-term leasing of 2 more aircraft in the case of high-demand outcome. Curious is the fact that no scenario resulted in a fleet of 11 aircraft. In terms of fleet composition, it seems that the fleet should have, at least, 3 B788s and 6 B773s. In the case of going for a 10 aircraft fleet, the 10th aircraft should be a B773 (with 80% chances of being needed). The 12 aircraft option should consist of one more B773 and another B788.

Scenario tree vs single scenario solutions

To compare the proposed scenario tree approach with the traditional approach of using a single scenario to plan the future fleet, we computed the optimal fleet for each scenario independently. These best fleets per scenario are of course always better or equal to the solution obtained from the scenario tree approach for each scenario. The computation of the best fleets does not consider the demand uncertainty or the interdependencies between stages, it just involves a three-stages model with a single scenario. The objective function values for the best solution for each scenario, the worst solution from the ones tested, the solution for the most likely scenario (the 'MM' scenario) and the scenario tree solution are provided in Table 6 for every scenario in the scenario tree. The second column in the 'most likely solution' and the 'scenario tree solution' refers to the decay in the objective function value when compared with the best solution. The last line of the table presents the results for the entire set of scenarios, considering the performance of each solution and the probability per scenario.

The most important conclusion that can be derived from these results is that the scenario

Table 6: Comparison of results per scenario between the scenario tree solution and single-scenario solutions - operational profits in 10^3 USD/week

Scenario	Probability	Best solution	Worst solution	Most likely solution		Scenario tree solution	
		OF	OF	OF	Dif.Best	OF	Dif.Best
HH	0.09	2567.9	2448.5	2533.1	-1.36%	2549.4	-0.72%
HM	0.15	2482.2	2394.6	2464.0	-0.73%	2471.9	-0.42%
HL	0.06	2350.4	2254.2	2332.1	-0.78%	2293.0	-2.41%
MH	0.15	2408.7	2333.7	2400.1	-0.36%	2392.7	-0.67%
MM	0.25	2327.5	2227.1	2327.5	**0.00%**	2324.7	-0.12%
ML	0.10	2203.7	2049.1	2162.1	-1.89%	2192.8	-0.49%
LH	0.06	2156.7	1978.3	2114.8	-1.94%	2108.2	-2.25%
LM	0.10	2085.4	1874.5	2008.7	-3.68%	2047.8	-1.80%
LL	0.04	1979.7	1708.4	1864.0	-5.80%	1925.9	-2.72%
Total	-	2305.5	2221.9	2297.9	-0.33%	2305.5	**0.00%**

tree solution is more robust than the most likely solution, or than any other single optimal solutions computed. The scenario tree solution is never as good as the optimal solution for each scenario, but it always performs reasonably well in all the scenarios. The maximum difference to the best value of the objective function is -2.72% while the most likely solution can result in profit losses of 5.80%. This is even more evident if we compare it with the potential lost of adopting some of the other optimal solutions for single scenarios. For instance, the profit lost can be of almost 14% if we adopt the best solution obtained for the 'HH' scenario and scenario 'LL' turns out to be the one better describing the demand evolution. This may represent a lost of 271.3 thousand USD per average week, even for this simple case study. The scenario tree solution performs really well in scenarios with high probability of happening and less well for scenarios with low probability of happening. It is, however, the best solution when we computed the objective function for the entire scenario tree.

The drawback of the proposed methodology is the computational effort needed to determine the best solution for the scenario tree. Given the complexity of the problem, the scenario tree approach is very demanding with regard to computing power, as discussed by Pantuso et al. (2015b). After one hour of computation, the scenario tree provided a solution that is 0.33% better than the most likely scenario solution. The single scenario solution was however computed in less than 8 minutes. The optimality gap in the case of the scenario tree solution was equal to 0.54%, meaning that the scenario tree approach can potentially lead to even better results if the optimizer runs for longer times. Despite this long computing times, it is important to notice that after 5 minutes of computation it could already provide solutions with a gap lower than 1.0%. This last solution was already better than the solution for the most likely scenario and it could have been already used as a 'good enough' solution to support the robust long-term fleet planning decisions. Nonetheless, it needs to be recognized that the complexity of the scenario tree approach raises some challenges in the case we want to study a problem with more stages, more branches and more aircraft options.

4.2. Case 2: NextGen Aircraft

The second case study concerns a real case fleet planning problem faced by the reference airline. The airline currently operates a fleet of ten B737s that are usually dedicated to 22 medium-haul routes of its network. With the next generation B737 on the market, the airline is exploring options to transition to this NextGen aircraft. In addition the airline is set to expand this network with two additional routes and to increase frequency on some other routes. The purpose of this case study is therefore twofold. The primary goal is to asses the benefit of the model when more limiting considerations are introduced, and secondary to prove the value of the proposed methodology in advising an airline on how the transition between aircraft technologies and different fleet composition options.

We start by setting-up the case study, discussing the options available and how the optimization model needs to be adapted in order to capture all the feature of this particular problem. Next we present the model parameters, followed by the discussion of the results.

4.2.1. Study set-up

The airline currently has four options under consideration to structure their fleet (Table 7). Each of which entails different ways of transition (or not) to the NextGen aircraft. The horizon is limited to the year 2027 because that would be the end term of any lease on current generation B737s. The fleet options considered are defined according to possible combinations of the following four lease alternatives:

- CGC – Current Generation aircraft Currently detained by the airline (referring to five B737s operational until 2019 via a long-term lease that can be extended until 2027);
- CGL – newly Current Generation aircraft acquired via Long-term lease;
- CGS – newly Current Generation aircraft acquired via Short-term lease; or
- NG – NextGen aircraft acquired via long-term leased aircraft and that is only available to the airline from 2020.

Table 7: Different fleet options under consideration by the airline

Option	2016-2019	2020-2027
No NextGen	CGC & CGL	CGC & CGL
Mixed	CGC & CGS	CGC & NG
Reverse Mixed	CGC & CGL	CGL & NG
Full NextGen	CGC & CGS	NG

In the 'No NextGen' option, the airline will continue to operate the CGCs until 2027 and for any additional aircraft it acquires CGLs. Similarly, for the 'Mixed' option it operates the CGCs until 2027, but the airline can acquire CGSs for the first four years, which will be then replaced by NGs in 2020. For the 'Reverse Mixed', the airline gets CGLs and replaces the CGCs with NGs in 2020. Finally, the 'Full NextGen' option consists of CGCs and CGSs, which will be then all replaced by NGs in 2020. Additionally, for the sake of comparison, a fifth option is considered where no restriction is imposed in terms of the combination of leasing options and the optimal fleet can be obtained. This fifth option we called the 'Free' option.

The transition problem will be modeled using a five-stage scenario tree. Starting in 2016, the first four stages represent two years of operations each (i.e., $n_t = 2 \times 52$), whereas the final stage will span four years (i.e., $n_t = 4 \times 52$). These four stages between 2016 and 2023 and the single stage between 2024-2027 represent the periods in which the airline expects to be able to make changes to the fleet. Figure 4 gives a graphical representation of the set-up and the notation corresponding to each period.

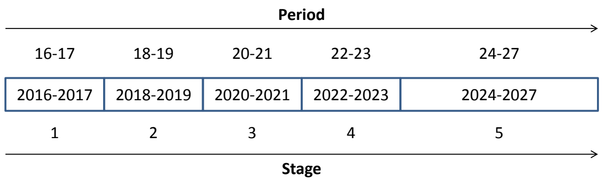

Figure 4: NextGen case study model set-up

The case study comprises some contextual requirements that need to be considered in the optimization model. For instance, the number of CGCs may not increase beyond the five the airline has in its initial fleet. We can easily incorporate this requirement by adding an additional set of constraints to enforce this. This can be done by using the following formulation:

$$x^s_{t,CGC} \leq 5 \qquad \forall t \in T, s \in S \tag{14}$$

Several other context requirements are considered. That is the case of newly acquired current generation aircraft. They can only either be CGL or CGS. The NextGen aircraft will only be available in period three and CGCs could only be handed back in the year 2020, coinciding with the assumed earliest delivery date of the NextGen aircraft. Once a new long-term lease starts with a current generation aircraft or a NextGen aircraft is acquired, the contracts can not be canceled before 2027. To ensure all these requirements, instead of adding additional constraints to the model, we made use of very large ownership or penalty costs coefficients in the objective function. These large costs, expressed by a constant M, are associated to some of the lease alternatives in stages that these these alternatives can not be started or broken. This way, we ensure that the associated variable will be equal to zero in the obtained solution. For example, a large ownership cost will be assumed for the NextGen aircraft in the years 2016 to 2019 in all options in consideration, in order to not select this leasing option in stages that these new aircraft are not yet available. Table 8 illustrates which parameters are set to M. In this table, the dashes represent periods where the cost values do not change.

Table 8: Costs associated to the starting and ending of leasing option

Aircraft	Parameter	16-17	18-19	20-21	22-23	24-27
CGS	*pen*	M	-	-	-	-
CGC	*pen*	M	-	M	M	M
CGL	*pen*	M	M	M	M	M
NG	fc	M	M	-	-	-
	pen	M	M	M	M	M

4.2.2. Model parameters

Once again, we divided the main parameters of the model in three tables. All aircraft parameters are provided in Table 9; the route related parameters for period 16-17 are provided in Table 10; while the variable cost are provided in Table 11. The values from these tables were obtained from current operational airline's records, from a five-year plan recently produced by the airline and via some interviews with airline's operators.

Table 9: Case 2 aircraft related parameters for the 4 leasing options

Lease option	fc [$/week]	*cap* [seats]	BT [h/week]	TAT [h/flight]	*pen* [$]	*IF* [-]
CGS	106,350	145	110	0.75	0	0
CGC	90,000	145	110	0.75	23,000	5
CGL	90,000	145	110	0.75	-	0
NG	110,000	162	110	0.75	-	0

Aircraft of different leasing alternatives may have differences in cost. These lease costs were obtained from current market prices. The weekly leasing costs of CGS aircraft is higher than the costs of the other CG leasing options given that they involve shorter leasing periods (of two or four years). The NextGen leasing was assumed to be 22% more expensive to reflect the additional costs of acquiring a new, more efficient and higher capacity aircraft. The penalty cost of early lease termination of these CGCs is estimated using the knowledge of previous leasing deals. It was assumed that short leasing contracts can be stopped in any of the five periods of analyses with no penalty costs and that long-term leasing options, namely CGL and NG can not be canceled. The block times per aircraft per week are also presented in Table 9. These values were considered to be equal to all aircraft, regardless of the lease alternative, and were set as such that the optimization leads to the same number of aircraft in the first stage as the airline has in its five-year plan. A discount factor δ of 1 is also considered in this case study.

The demand values and the minimum frequency values per routes for the initial stages were obtained from the five-year plan of the airline. The plan presents frequency schedules, expected fares and demand forecast values on the routes considered in our case study. It was assumed that from 2020 onward the minimum frequency schedule and the fares remain the same. Uncertainty is introduced by multiplying the base demand forecast values for

Table 10: Case 2 route related parameters for the period 16-17

Route	OT [h/flight]	$MFrq_{16-17}$ [flights/week]	$fare$ [$/pax]	dem_{16-17} [pax/week]	LF [%]
A	9.05	3	418	340	85
B	8.25	4	389	501	85
C	7.63	3	386	345	85
D	8.71	3	387	309	85
E	5.96	7	282	808	85
F	4.09	3	326	311	85
G	5.55	0	330	(316)	85
H	1.38	7	178	803	85
I	4.45	3	314	311	85
J	5.43	4	236	429	85
K	2.96	3	282	375	85
L	1.83	3	301	252	85
M	3.64	5	270	608	85
N	3.38	2	270	202	85
O	4.13	19	330	2000	85
P	4.17	2	284	218	85
Q	5.00	7	353	700	85
R	5.57	3	364	336	85
S	3.38	2	277	198	85
T	1.00	6	100	761	85
U	4.33	2	354	203	85
V	3.00	3	216	315	85
W	4.38	4	327	462	85
X	3.33	0	354	(343)	85

Table 11: Case 2 variable costs per aircraft-route combination (*1000 USD)

Aircraft	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
CG	97	109	102	113	55	44	50	17	43	50	37	36	34	38	42	45	53	65	38	13	43	37	43	34
NG	92	103	96	107	52	42	48	16	41	48	35	34	32	36	40	42	50	62	36	12	41	35	41	32

16-17 with a yearly change of -2%/+2%/+6%. Because the time between the start of each period is two years, the model uses -4.0%/+4.0%/+12.4%. The probabilities to each growth branch are chosen to be 25%/50%/25%, respectively. This results in a scenario tree with 81 scenarios, with demand in the final period ranging between +59.4% and -14.9% of the base demand, with a most likely variation of +17.2%. Routes G and X are two new routes in which the airline plans to start operations in the period 18-19. It was assumed that, although the airline is not being flown in 16-17, their potential demand in 16-17 is equal to 316 and 343 passengers per week, respectively for route G and X. After this period the scenario tree demand uncertainty branches were applied to estimate the future demand on these two routes. The maximum load factor for all the routes in the network was assumed to be 85%. This value is in line with the estimations described in the five-year plan, but it is higher than the current average load-factor experienced by the airline in its long-haul network. Still, we assumed it to be realistic for this case study.

The variable cost for the current generation aircraft are taken from the reference airline's records. Around 65% of these cost are deemed to aircraft dependent cost (i.e., fuel and maintenance cost). Manufacturer information suggests that the NextGen aircraft has an operational efficiency gain of 8%. Thus, the cost of the NextGen are taken as 94.8% of the current generation aircraft.

4.2.3. Results

The fleet compositions an operational profit for the four fleet option under consideration by the airliner and for the free option are given in Table 12.

Table 12: Fleet compositions and operational profit for the different options

Option	Leasing alternatives	Period 16-17	18-19	20-21	22-23	24-27	Operational profit [10^3 USD/week]
No NextGen	CGC	5	5	5	5	5	1 306.5
	CGL	5	6	7	7-8	7-9	
Mixed	CGC	5	5	5	5	5	1 569.9
	CGS	5	6	0	0	0	
	NG	0	0	7	7-8	7-9	
Reverse mixed	CGC	5	5	0	0	0	1 565.6
	CGL	5	6	6	6	6	
	NG	0	0	6	6-7	6-8	
Full NextGen	CGC	5	5	0	0	0	1 619.0
	CGS	5	6	0	0	0	
	NG	0	0	12	12	12-13	
Free	CGC	5	5	0	0	0	1 631.8
	CGS	4	5	0	0	0	
	CGL	1	1	1	1	1	
	NG	0	0	11	11	11-12	

The results suggest that, among the four options considered by the airline, the full renovation of the fleet in 2020 with new generation aircraft (NG) is the one that results in a higher operational profit. Nevertheless, a solution involving the CGS and CGL leasing alternatives together with the NG aircraft, such as the one suggested by the 'Free' option, seems to be even better than the 'Full NextGen' option. The difference between these two options is estimated to be of almost 13 thousand USD for an average week of operations. The difference is larger when we compare with the 'No NextGen' option, which turned out to be the least profitable option. The 'Full NextGen' option results in a weekly profit 312.5 thousand USD higher than the 'No NextGen' option. The two mixed options give intermediate results, when compared with these two last options.

In terms of fleet size, all solutions suggest an increase in fleet from ten aircraft in period 16-17 to twelve aircraft in period 20-21, regardless of the scenario in the scenario tree. For the periods 22-23 and 24-77, the 'No NextGen', the 'Mixed' and the 'Reversed Mixed' options involve the acquisition of one more aircraft per period in the case of scenarios with higher demand. This fleet enlargement is done with additional NG aircraft or, in the case of the 'No NextGen' option, with a long-term leasing of a CG aircraft. In the case of the two other options, the extra capacity of the NG aircraft acquired in period 20-21 is enough to cope with the demand variation until period 24-27. Only in this last period it suggested the acquisition of an additional NG aircraft for the extreme high demand scenarios. This can be observed in Table 13 where the fleet probabilities for the 'No NextGen' and the 'Free' options are compared. It is clear that with a high probability the most profitable fleet is composed of one CGL and eleven NG aircraft. The thirteenth aircraft is only profitable with 2.0% of probability. For the 'No NextGen' option, with almost 80% probability the most profitable fleet will be composed by the current five CGC and seven extra CGL. One additional CGL aircraft should be acquired in the case of high demand scenarios. There is a 18.8% probability of these scenarios to happen. The second additional CGL aircraft is only profitable with a probability of 1.6%.

5. Conclusions

In this paper an innovative multi-period scenario tree methodology aimed to help airlines better hedge against uncertainty is proposed. The methodology adopts the concept of scenario paths to develop fleet plans according to potential demand evolution scenarios. Given that some scenario paths share common decision points, the scenarios are solved dependently of each other. A mixed-integer linear programming model is used to determine the ideal fleet composition for each scenario in the tree and to capture these interdependencies. With two case studies inspired in real-world problems, it was demonstrated that the proposed methodology can be used for long-term fleet planning. The multi-period scenario approach leads to flexible and robust fleet plans. The plans are flexible because they comprise fleet composition solutions for all decision nodes in the scenario tree, reflecting in one hand the evolution of the fleet from previous time stages and on the other hand the potential demand evolution scenarios for future stages. This results in smooth transitions between fleet solutions along the multiple time stages of the scenario tree, facilitating the development of coherent and adaptable multi-period fleet investment plans. The fleet plans obtained with this approach are also more robust than the ones that can be determined by using the

Table 13: Fleet probability for Period 24-27 according to the 'No NextGen' and 'Free' options

	No NextGen						Free				
# aircraft	CGC	CGS	CGL	NG	Total	# aircraft	CGC	CGS	CGL	NG	Total
0		100%		100%		0	100%	100%			
1						1			100%		
2						2					
3						3					
4						4					
5	100%					5					
6						6					
7			79.7%			7					
8			18.8%			8					
9			1.6%			9					
10						10					
11						11				98.0%	
12					79.7%	12				2.0%	98.0%
13					18.8%	13					2.0%
14					1.6%	14					

traditional deterministic single-scenario approach usually followed in the literature. This happens because in the proposed approach multiple scenarios are taken into consideration when defining the best fleet for each decision node in the scenario tree.

The multi-scenario approach leads to as many advised fleets per stage as the number of decision nodes considered in each stage. When these fleets differ from each other, it is not possible to provide a single advise on the best fleet per stage. Although at first this can be seen as a limitation of the proposed method, this is in fact one strong point of the scenario tree approach. It generates additional information that can better help the fleet planner to hedge against uncertainty. By summing the probabilities of the scenarios resulting in the same fleet outcome, it is possible to construct fleet probability tables for each time stage in the scenario tree. These tables show the probabilities of having various fleet compositions as the best fleet for future stages. The tables provide, for example, advise on what is the most probable profitable fleet composition or on what is the minimum number of aircraft in the fleet necessary to have profit in a given percentage of the future scenarios considered. Depending on the airline's risk aversion and on the investment capabilities, the fleet planner can use this information to better decide on how many aircraft to order and how many of these orders should be via short-term or longer-term contracts. This is a very useful outcome for any fleet planner, given additional flexibility and robustness to the fleet planning process.

The case studies used in this paper demonstrated that the approach can be already used to solve real long-term fleet planning problems. Nevertheless, this paper opens the opportunity for further research. There are several topics that can still be further explored. For instance, it will be interesting to consider differing demand variations for different routes in the network. The impact of uneven growth of demand and the correlation between these growths may provide more realism to the case studies. The single period fleet planning mode in future studies can also be extended to consider the demand over origin-destination markets may involving connections at the airline hub. Other sources of uncertainty, not included in this paper, such as fuel prices, discount factor fluctuations or technology development may be object of analysis in future studies. When implementing these proposed extensions to the proposed approach, one has to bare in mind that the scenario tree methodology is very demanding in terms of computing power. The current version of the modeling approach, connecting Pyomo based PySp software package with the commercial IBM CPLEX linear optimizer, limits the application of the approach to large scenario trees or to handle very detailed fleet planning problems. Thus, further research can also focus on developing faster and more efficient solution techniques to handle the scenario tree approach proposed.

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