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Chapter 28
Experimental Parameter Identification of Nonlinear Mechanical Systems via Meta-heuristic Optimisation Methods

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Abstract Meta-heuristic optimisation algorithms are high-level procedures designed to discover near-optimal solutions to optimisation problems. These strategies can efficiently explore the design space of the problems; therefore, they perform well even when incomplete and scarce information is available. Such characteristics make them the ideal approach for solving nonlinear parameter identification problems from experimental data. Nonetheless, selecting the meta-heuristic optimisation algorithm remains a challenging task that can dramatically affect the required time, accuracy, and computational burden to solve such identification problems. To this end, we propose investigating how different meta-heuristic optimisation algorithms can influence the identification process of nonlinear parameters in mechanical systems. Two mature meta-heuristic optimisation methods, i.e. particle swarm optimisation (PSO) method and genetic algorithm (GA), are used to identify the nonlinear parameters of an experimental two-degrees-of-freedom system with cubic stiffness. These naturally inspired algorithms are based on the definition of an initial population: this advantageously increases the chances of identifying the global minimum of the optimisation problem as the design space is searched simultaneously in multiple locations. The results show that the PSO method drastically increases the accuracy and robustness of the solution, but it requires a quite expensive computational burden. On the contrary, the GA requires similar computational effort but does not provide accurate solutions.

Keywords Experimental nonlinear analysis · Nonlinear dynamics · Parameter identification · Meta-heuristic optimisation · Nonlinear frequency response

28.1 Introduction

The increasing demand for lightweight, high-performance, and flexible structures shows the ultimately nonlinear nature of mechanical systems, prompting the study of nonlinear dynamics in many fields of science [1]. In the literature, many techniques have been proposed for identifying the parameters of nonlinear systems [2, 3]; between them, there are linearisation methods, time-domain methods, frequency-domain methods, time-frequency methods, modal methods, black-box methods, and model updating methods. The latter methods try to extract information from the experimental data to update a numerical/analytical model [3]. Meta-heuristic optimisation algorithms lend themselves to similar tasks and can be adopted for model updating processes [4–7], minimising the difference between the experimental and numerical data. Such algorithms can efficiently search the design space of the optimisation problem [8] by seeking a near-optimal solution.

Many examples of similar optimisation processes are available in the scientific literature. Pelteret et al. [5] applied five different meta-heuristic algorithms for the parameter identification of nonlinear constitutive laws that describe coupled, magnetic-field responsive materials. The authors demonstrated that, for the class of problem and with the adopted setting, the genetic algorithm (GA) provided the best performance in terms of the accuracy of the predicted parameters. Yousri et al. [6]

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investigated the performance of several meta-heuristic optimisation algorithms for the parameter identification of fractional order chaotic systems, combining the optimisation procedures with different objective functions. The authors numerically demonstrated that, between the considered algorithms and objective functions, the flower pollination algorithm with integral of squared error represents the most effective combination for identifying the parameters. Recently, Safari et al. [7] presented a semi-parametric identification framework for nonlinear systems. The framework is based on optimisation strategies which allow identifying both the parameters and the type of nonlinear characteristics that better describe the system. The authors proved the efficacy of the proposed methodology by applying the framework to numerical and experimental examples. The significant number of works available in the literature [9–21] demonstrates the solid scientific interest in the parameter identification of nonlinear systems via meta-heuristic optimisation.

However, the definition of the objective function and the selection of the optimisation algorithm can drastically change the number of required iterations to reach the solution, hence the computational burden [6]. In some instances, the adoption of one method rather than another can prevent the attainment of a correct solution, as demonstrated in many works [5, 6]. Therefore, it is not completely clear which meta-heuristic algorithm provides the best performance for the identification process of nonlinear systems. In this paper, we propose the performance comparison of two mature meta-heuristic optimisation algorithms, i.e. the PSO and the GA, for parameter identification problems of nonlinear mechanical systems. These naturally inspired algorithms are based on the definition of a population, which initially colonises the whole design space of the problem, and they are chosen as they advantageously increase the chances of identifying the optimum solution to the identification problem. A clamped-clamped nonlinear two-degrees-of-freedom system is studied, and its parameters are identified from the experimental data. Firstly, the linear parameters are deduced with the Kennedy-Pancu method [22]. Then, the nonlinear parameters of the model are identified via an error minimisation process using the two meta-heuristic algorithms mentioned above. Finally, the performance of the two methods is compared in terms of computational burden, required time, and accuracy.

28.2 Experimental Test Rig and Nonlinear Analysis

The nonlinear system is constituted of two adjustable masses and two supports. One support is fixed on the vibration isolation table and blocks the structure on the ground, while the second is attached to a shaker (LDS-V403) and excites the system. The masses and the supports are made in PLA (polylactic acid) and are obtained through FDM (filament deposition modelling), a common additive manufacturing process. Two parallel beams in stainless steel sustain the masses, allowing them to oscillate in the horizontal plane. Such a configuration provides a hardening nonlinear stiffness characteristic to the model: when the masses move from the resting position, the beams are subjected to both bending and axial deformation. The latter changes the bending stiffness resulting in a hardening cubic characteristic.

The main dimensions of the components of the system are reported in Table 28.1. The two masses are separated by a distance of 160 mm, while each mass is 90 mm apart from the adjacent constraint. Moreover, the support beams are separated by a distance of 35 mm. Figure 28.1 shows the experimental test rig and the configuration set-up to obtain the data: two accelerometers are connected to the masses (PCB Piezotronics Model: 352C22) while a laser vibrometer (Polytec PVD 100) is used to measure the velocity of the base.

The nonlinear analysis is performed by recording the acceleration time histories of the two masses when a voltage input signal is provided to the shaker. The sinusoidal input signal is produced by the National Instruments (NI) unit NI-9263, and it is designed to smoothly change frequency every 30 s. This allows the creation of forward/backwards sweep frequency signals from 11 Hz to 16 Hz with a variation of frequency of 0.1 Hz. The acquired raw time signal is recorded with the unit NI-9234 and it is post-processed with the following steps: the signal is divided into time portions corresponding to a well-defined frequency, each block is filtered and integrated to obtain the displacement and the velocity of the masses, and then the transient is removed from each portion of the signal. Finally, the amplitude of each steady-state signal block is computed from the averaged peaks. The resulting transfer functions[1] (TFs) are described by Fig. 28.2. The figure shows the

Table 28.1 Dimensions of the masses and the distances between the masses and the supports

Component	Length	Width	Height
Beams	500 mm	0.5 mm	20 mm
Mass	50 mm	55 mm	50 mm

[1] For transfer function of nonlinear system, we mean the frequency response curve of the system for a given input.

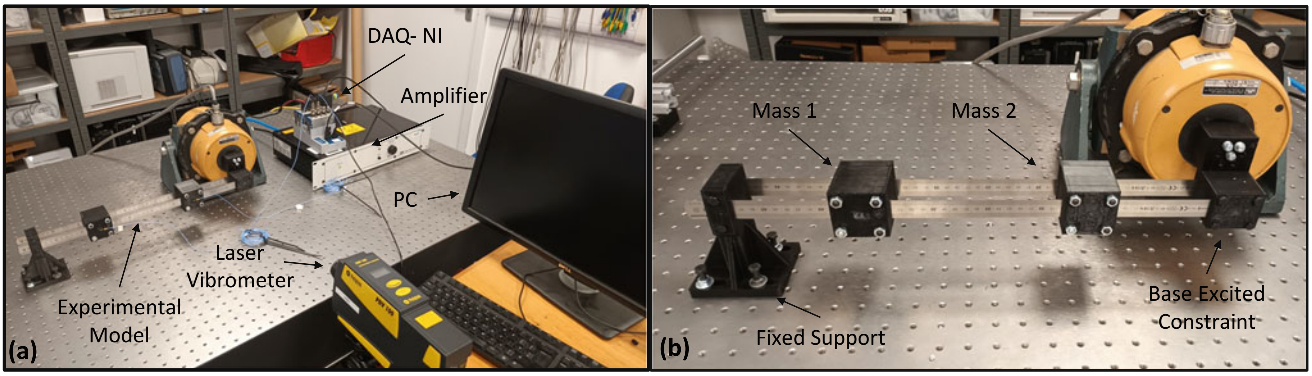

Fig. 28.1 Experimental set-up for the analysis of the two-mass system (**a**) and model description (**b**)

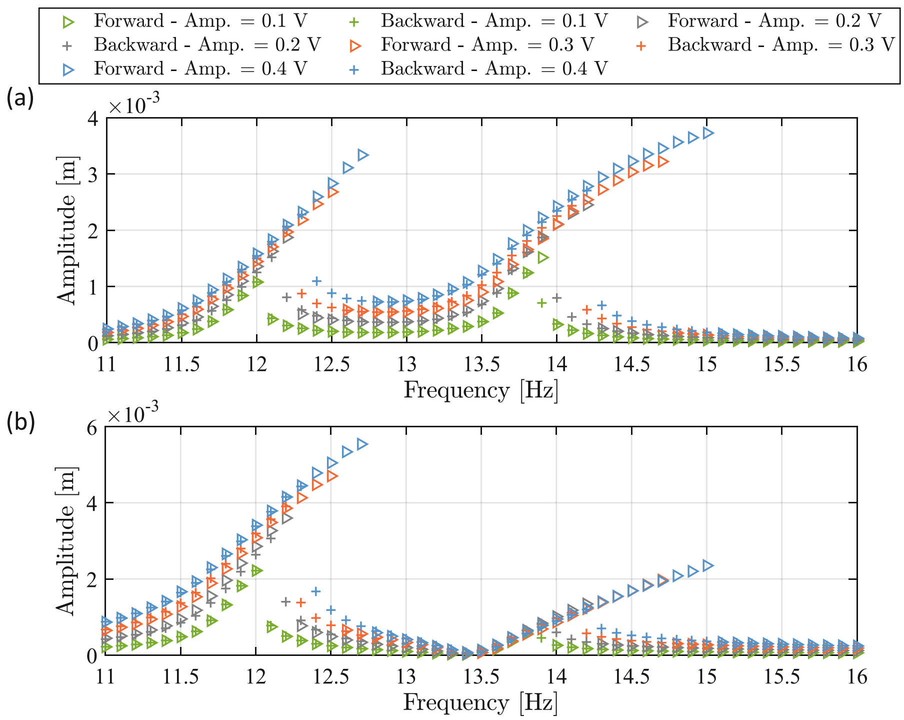

Fig. 28.2 Nonlinear transfer function between the voltage input and the first mass displacement (**a**) and the second mass displacement (**b**). The TFs are computed at different amplitudes of input voltages to show the effect of the external excitation

TFs from the input voltage signal to the output displacement of the masses; when the signal input amplitude is small, the system behaves almost linearly, while at high input amplitudes, the typical jump phenomena occur in the forward frequency sweeps after the system resonances. The backward frequency sweeps, instead, show the low-amplitude dynamics of the system. Finally, it is worth remembering that these TFs include the dynamics of the shaker; thus, the following numerical analyses need to take this aspect into account.

28.3 Model Updating of the Underlying Linear System

The experimental system can be mathematically described with a lumped parameter model with asymmetrical coefficients. The model is graphically represented by Fig. 28.3, where k_1, k_2, and k_3 denote the linear stiffness, c_1, c_2, and c_3 are the viscous damping coefficients, μ_1, μ_2, $\mu_{2,b}$, and μ_3 represent the cubic stiffness coefficients, m indicates the mass of the system, and y is the displacement of the moving constraint. Given the nature of the nonlinearities, the equations of motion contain unsymmetrical cubic stiffness coefficients, as described by Eq. 28.1.

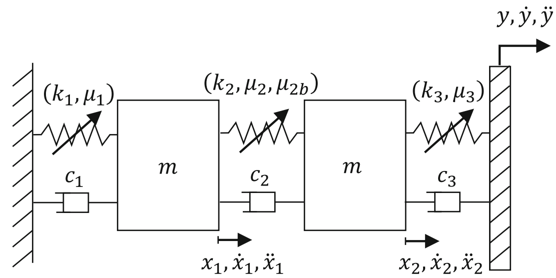

Fig. 28.3 Lumped parameter model representing the experimental test rig

$$m\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + \mu_1x_1^3 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + \mu_2x_1^3 - \mu_{2,b}x_1^2x_2 + \mu_{2,b}x_1x_2^2 - \mu_2x_2^3 = 0 \tag{28.1a}$$

$$m\ddot{x}_2 + c_3(\dot{x}_2 - \dot{y}) + k_3(x_2 - y) + \mu_3(x_2 - y)^3 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) - \mu_2x_1^3 + \mu_{2,b}x_1^2x_2 - \mu_{2,b}x_1x_2^2 + \mu_2x_2^3 = 0 \tag{28.1b}$$

The linear behaviour of the system can be easily described by the linear part of Eq. 28.1; by considering a sinusoidal input signal y with constant acceleration amplitude and through mathematical manipulation, the linear receptance[2] H can be represented as follows:

$$H_1 = \frac{X_1}{\ddot{Y}} = \frac{X_2k_2 + i\Omega X_2c_2}{\ddot{Y}(k_1 + k_2 - \Omega^2m + \Omega i(c_2 + c_1))} \tag{28.2a}$$

$$H_2 = \frac{X_2}{\ddot{Y}} = \frac{X_1k_2 - \frac{\ddot{Y}k_3}{\Omega^2} + i\Omega X_1c_2 - \frac{i\ddot{Y}c_3}{\Omega}}{\ddot{Y}(k_2 + k_3 - \Omega^2m + \Omega i(c_2 + c_3))} \tag{28.2b}$$

where X_1 and X_2 are the complex amplitudes of the mass displacement, $\ddot{Y}$ represents the complex amplitude of the acceleration input, and Ω denotes the forcing frequency of the excitation.

The experimental linear behaviour of the system is evaluated by measuring the TF of the underlying linear system. This can be achieved by exciting the system with random vibrations; indeed, such an excitation attenuates the effect of the system nonlinearities and allows obtaining an averaged linear TF [22, 23]. The underlying linear TF is acquired with the unit *DataPhysics Abacus 901*(DP-901) and with the aid of the commercial software *SignalCalc 900 Series*. The linear TF is determined by considering the motion of the excited constraint as the input signal and the mass displacement as the output signal, and then an averaging process is adopted to reduce the effect of the noise. Such a linear TF represents the experimental counterpart of the receptance mathematically described in Eq. 28.2.

In order to identify the linear coefficients of the system, the experimental linear transfer functions are exploited; the Kennedy-Pancu method [22] represents a straightforward, effective tool to extract the natural frequencies ω_n, modal damping ratios ζ, and the modal matrix Ψ of MDOF mechanical systems. The modal damping ratios and the natural frequencies can be organised in matrix form as follows:

$$\mathbf{Z} = \begin{bmatrix} 2\zeta_1\omega_{n,1} & 0 \\ 0 & 2\zeta_2\omega_{n,2} \end{bmatrix} \tag{28.3a}$$

$$\mathbf{W}_n = \begin{bmatrix} \omega_{n,1}^2 & 0 \\ 0 & \omega_{n,2}^2 \end{bmatrix} \tag{28.3b}$$

The matrices of Eq. 28.3 can be used to obtain the stiffness and damping linear coefficients of the system, as follows:

$$\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \tag{28.4a}$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix} = (\mathbf{\Psi}^T)^{-1}(\mathbf{\Psi}^T\mathbf{M}\mathbf{\Psi}\mathbf{Z})(\mathbf{\Psi})^{-1} \tag{28.4b}$$

[2] For receptance, the authors mean the transfer function between input acceleration and the output displacement.

Table 28.2 Identified linear coefficients of the underlying linear system: initial set of coefficients (obtained from Kennedy-Pancu method) and optimised set of coefficients (obtained from the optimisation process)

Component	c_1 [Ns/m]	c_2 [Ns/m]	c_3 [Ns/m]	k_1 [N/m]	k_2 [N/m]	k_3 [N/m]
Initial coefficients	0.0454	0.0069	0.0370	726.3	85.5	621.5
Optimised coefficients	0.0454	0.0075	0.0366	714.9	89.6	624.9

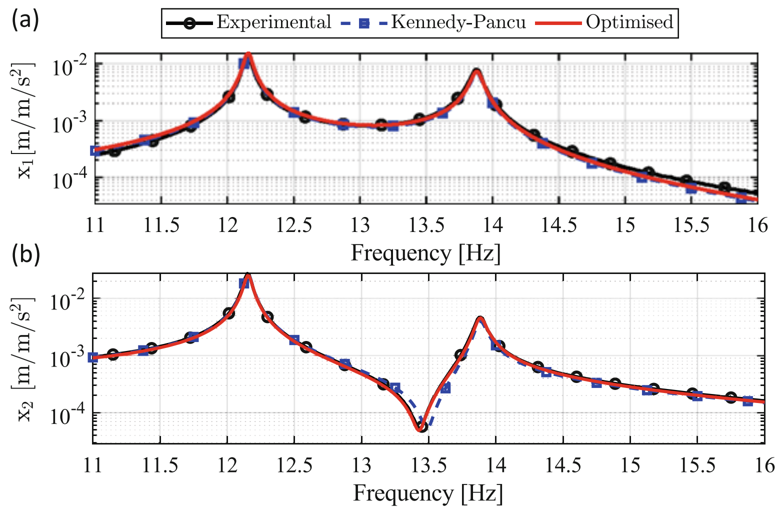

Fig. 28.4 Comparison between the experimental and analytical receptances of the first (**a**) and the second (**b**) degree of freedom of the system

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} = (\boldsymbol{\Psi}^T)^{-1}(\boldsymbol{\Psi}^T \mathbf{M} \boldsymbol{\Psi} \mathbf{W}_n)(\boldsymbol{\Psi})^{-1} \tag{28.4c}$$

where M represents the diagonal mass matrix, C is the linear damping matrix, and K denotes the linear stiffness matrix. Now, considering that both the masses of the system are equal to 0.113 kg, the unknown coefficients of the underlying linear system, i.e. c_1, c_2, c_3 , k_1, k_2, and k_3, can be evaluated with the Kennedy-Pancu method and by exploiting Eqs. 28.3 and 28.4. The obtained set of coefficients represents a rough but effective approximation of the linear part of the system; thus, it is used as a starting point for an optimisation procedure. The optimisation aims to minimise the difference between the experimental and analytical receptances in terms of amplitude and phase, allowing a more accurate definition of the linear coefficients of the system. Equations 28.2a and 28.2b can be solved to obtain the analytical expression of the amplitude and phase of the system receptances and their differences with respect to the experimental data can be minimised with the *lsqnonlin* MATLAB function, which is used in the optimisation process. The results of the optimisation process are reported in Table 28.2.

The results of Table 28.2 show that the identified initial coefficients are already a very good approximation of the linear system; however, the additional optimisation process allows for reaching a more accurate definition of the linear parameters which is beneficial for the following nonlinear parameter identification. Figure 28.4 shows the experimental and analytical receptances for the two sets of coefficients. The figure demonstrates that the Kennedy-Pancu prediction is very near to the optimal configuration, but a small difference in the anti-resonance response is still present. The optimised TFs, instead, can better fit the experimental data, proving to be a good approximation, even at low amplitude responses.

28.4 Nonlinear Model Updating

Once the linear coefficients are known, the nonlinear system can be identified with the aid of the meta-heuristic optimisation algorithms, namely, GA and PSO. Both the algorithms are based on the definition of a population; the GA [24–26] initialises the domain with a random initial population, whose size is defined by the user. Then, the algorithm creates the next generation of individuals starting from the current population, which is assessed on the base of the objective function value of each individual. Some individuals, generally with low objective function values, are selected as parents according to the selection criteria and used to create new individuals. The children are created in three different ways: elite individuals survive the generational change as they have the lowest values of the objective function, crossover children are obtained by combining the genes of the parents according to the selected crossover function, and mutated children are generated by introducing random mutation to a single parent. The algorithm stops when the change in the objective function is less than the prescribed tolerance. The PSO [27, 28], instead, is based on swarm intelligence. Similarly to the GA, the algorithm begins uniformly populating the whole domain. The algorithm randomly assigns the initial position and velocity to the particles and computes the objective function of each particle. At this point, the algorithm can identify the best position d associated with the best function value b, and all the particle positions are stored in the matrix p. Then the iteration process starts: for each particle i, a random subset S of N particles is chosen. This set does not include the particle i-th. The best position g and the best function value f are identified in the subset S whose objective function values are already known. With this information, the algorithm can compute the new velocity and position with the following expressions:

$$v(i) = Wv(i) + y_1 u_1(p(i) - x(i)) + y_2 u_2(g - x(i)) \tag{28.5a}$$

$$x(i) = x(i) + v(i) \tag{28.5b}$$

where $x(i)$ and $v(i)$ represent the position and the velocity of the i-th particle, y_1, y_2, u_1, and u_2 are PSO tuning coefficients, and W represents the inertia coefficient. The updated position $x(i)$ is, now, used to evaluate again the objective function $F(x(i))$ and if its value is lower than $F(p(i))$ the optimal position $p(i)$ is updated with new identified position $x(i)$. Finally, if $F(x(i)) < b$ then the optimal value b is updated and the neighbourhood N is modified accordingly. At each iteration, the bounds are enforced, and the process is repeated until the change in the objective function is less than the prescribed tolerance.

For both methods, the optimisation procedure tries to minimise the following objective function:

$$F(x) = \sum_{j=1}^{P} \sum_{i=1}^{R} (|T_{num}(x) - T_{exp}|) \tag{28.6}$$

where x is the design variable vector, constituted of the nonlinear unknown coefficients μ_1, μ_2, $\mu_{2,b}$, and μ_3, R is the number of discrete frequencies at which the function is evaluated, P is the number of degrees of freedom of the problem, and T_{exp} and T_{num} represent, respectively, the experimental and numerical transfer functions between the shaker input voltage and the mass displacement. The numerical transfer function T_{num} is computed via numerical integration of Eq. 28.1 with the aid of the MATLAB built-in function *ode45*; such procedure must be performed by utilising the experimental displacement and velocity of the moving constraint. This is necessary to account for the dynamics of the shaker, which could not be considered otherwise. The optimisation procedure is performed by using the MATLAB built-in functions *ga* and *particleswarm* with the default options. The optimisation processes are repeated three times to generate more robust results, and a multi-core computer (32 cores—Intel(R) Xeon(R) Silver 4214 CPU @ 2.10 GHz, RAM 129 Gb) is used to perform the analyses with parallel computing. The largest amplitude responses of the experimental TFs are used to compute the considered objective function: this guarantees to better represent the nonlinear characteristic of the experimental data. Therefore, the optimisation is carried out considering the experimental data generated by a voltage input amplitude of 0.4 V for the forward sweep of discrete frequencies in the ranges from 11.6 Hz to 12.7 Hz and from 13.6 Hz to 15.0 Hz, i.e. where the system shows the resonances. Finally, the lower and upper boundaries of the optimisation are set, respectively, at $\mu = 10^3$ N/m^3 and $\mu = 10^8$ N/m^3, for all the design variables.

The results of the optimisations with the two meta-heuristic algorithms are reported in Tables 28.3 and 28.4. The tables show that the PSO accurately converges to the same optimal condition for three different cases with a similar amount of time and iterations. The minimum objective function value is 2.458e-3 which represents the best-optimal identified condition. On the contrary, the GA requires less time to perform the optimisation, but it identifies three very different optimal conditions. Moreover, these optimal conditions are associated with higher-objective function values. Figure 28.5 provides a graphical interpretation of the optimisation results of Tables 28.3 and 28.4, showing the comparison between the optimised numerical TFs and the experimental ones. Figure 28.5a and b show the numerical TFs for the first and second degree of freedom of the system for the best solution of the PSO algorithm. The figures demonstrate that the PSO numerical results can fit the experimental TFs with high fidelity. Conversely, the GA optimal results generate poorly converged numerical TFs, as shown in Fig. 28.5c and d. The results of Tables 28.3, 28.4, and Fig. 28.5 demonstrate the robustness of the PSO and its ability to solve identification problems of nonlinear systems, confirming its superiority with respect to the GA when common default options are used in the optimisation procedure. Moreover, these results agree with previous studies [4, 12, 14, 21] which, differently from this case, adopted time data to identify the system parameters.

Table 28.3 Optimisation results for the identification process with the PSO algorithm. The optimisations are carried out with a swarm size of 40 particles

PSO	Time [min]	Iterations	Func. counts	μ_1 [N/m^3]	μ_2 [N/m^3]	$\mu_{2,b}$ [N/m^3]	μ_3 [N/m^3]	Func. value
Test 1	566.1	77	3120	2.653e6	1.314e6	8.889e6	7.071e6	2.461e-3
Test 2	524.7	74	3000	2.732e6	1.324e6	8.850e6	7.069e6	2.458e-3
Test 3	591.8	87	3520	2.651e6	1.312e6	8.891e6	7.074e6	2.461e-3

Table 28.4 Optimisation results for the identification process with the GA. The optimisations are carried out with a population of 50 individuals

GA	Time [min]	Iterations	Func. Counts	μ_1 [N/m^3]	μ_2 [N/m^3]	$\mu_{2,b}$ [N/m^3]	μ_3 [N/m^3]	Func. Value
Test 1	471.8	55	2638	1.259e7	1.685e7	9.162e6	3.104e6	40.705e-3
Test 2	489.3	54	2591	8.093e6	1.785e7	1.446e6	3.363e6	38.705e-3
Test 3	497.5	57	2732	1.002e3	8.612e6	3.537e5	1.296e7	22.668e-3

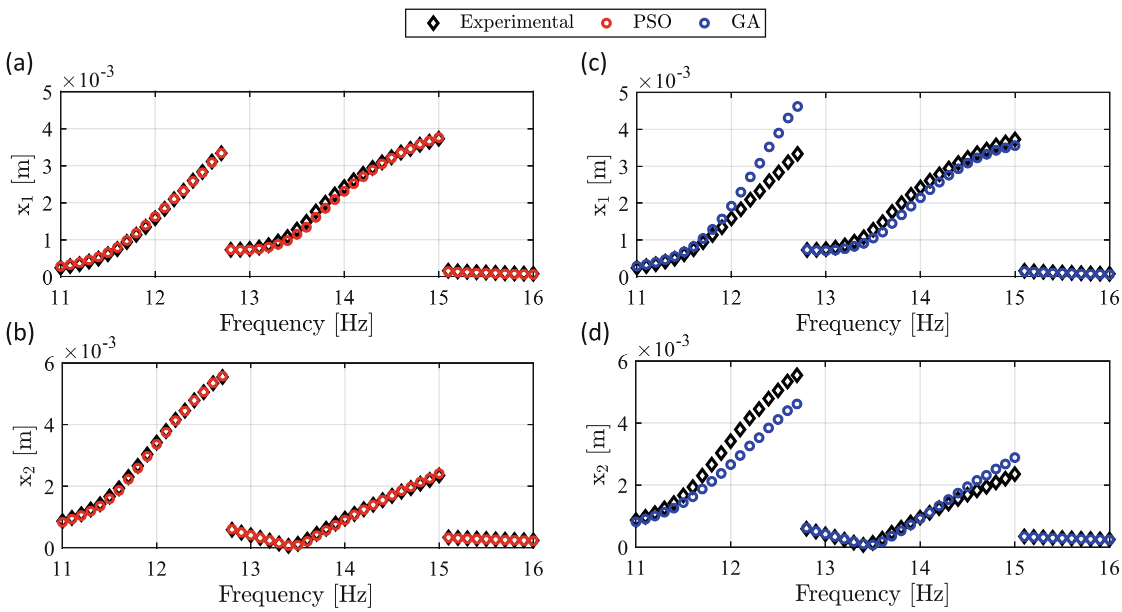

Fig. 28.5 Comparison between the experimental transfer function and the numerical optimised transfer functions with PSO (**a–b**) and with GA (**c–d**). The results are obtained by using the experimental transfer function with a voltage input amplitude of 0.4 V

Finally, the optimal results are experimentally validated: a different set of experimental data, i.e. the TFs associated with an input voltage amplitude of 0.3 V and 0.4 V, is used to validate the identified nonlinear coefficients with the PSO method. Figure 28.6 shows the comparison between such experimental results and the numerical transfer functions with the optimal PSO coefficients identified in the $Test$ 2 of Table 28.3. The figure demonstrates that the experimental and numerical TFs are in extreme agreement even for different sets of experimental data, validating the identified numerical model.

28.5 Conclusion

This paper presents the experimental parameter identification of a nonlinear two-degrees-of-freedom system via meta-heuristic optimisation procedures. Firstly, the experimental test rig and the adopted methodology for acquiring the experimental data are presented. Then, the identification of the linear parameters is described, and, finally, two mature meta-heuristic optimisation methods, i.e. PSO and GA, are introduced and used in the identification process of the nonlinear parameters. The optimisation analyses show that the PSO is able to identify the nonlinear parameters from experimental data with great accuracy and repeatability. Contrarily, the GA requires less computational effort but provides sparse and poor accurate solutions which result in the identification of the wrong nonlinear parameters. Finally, the PSO optimal parameters are experimentally validated. The comparison demonstrates that the obtained coefficients are valid not only for the first set of experimental data but also for other sets of experimental data which are obtained at different excitation conditions. This

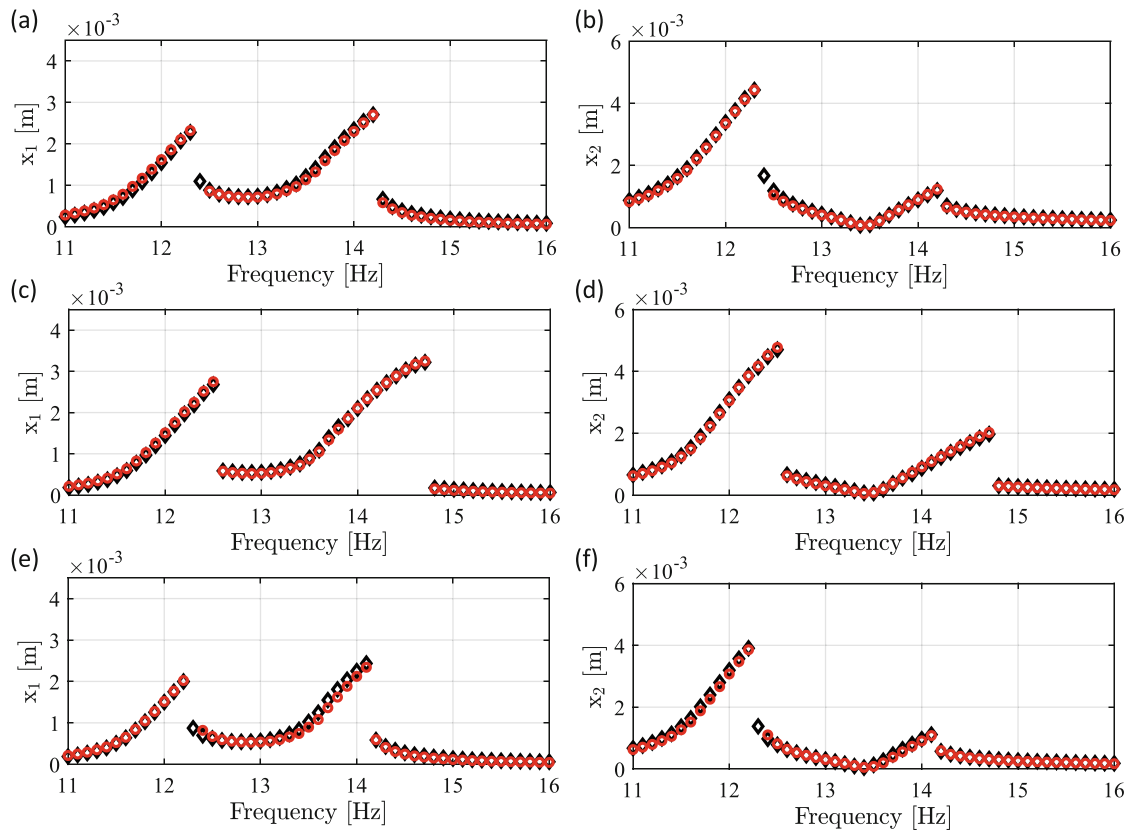

Fig. 28.6 Validation of the optimal nonlinear coefficients with the following experimental transfer functions: backward frequency sweep with an input voltage amplitude of 0.4 V (**a**–**b**), forward frequency sweep with an input voltage amplitude of 0.3 V (**c**–**d**), and backward frequency sweep with an input voltage amplitude of 0.3 V (**e**–**f**). The experimental results are denoted by the black diamond, while the numerical results with optimal PSO coefficients are identified by a red circle

validates the experimental identification procedure and demonstrates that the PSO can be successfully adopted for nonlinear parameter identification of mechanical systems.

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Data Availability: The experimental data are available at the following DOI [29]

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