Resonant Tunneling of Interacting Electrons in a One-Dimensional Wire

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We consider the conductance of a one-dimensional wire interrupted by a double-barrier structure allowing for a resonant level. Using the electron-electron interaction strength as a small parameter, we are able to build a nonperturbative analytical theory of the conductance valid in a broad region of temperatures and for a variety of the barrier parameters. We find that the conductance may have a nonmonotonic crossover dependence on temperature, specific for a resonant tunneling in an interacting electron system.

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The phenomenon of resonant tunneling is well known in the context of electron transport physics [1]. The hybridization of a discrete state localized in the barrier with the extended states outside the barrier may strongly enhance the transmission coefficient for electrons incident on the barrier with energy matching the energy of the localized state. For a single electron, the transmission coefficient at energies close to the resonance is given by the Breit-Wigner formula [1]. However, if the barrier carrying the resonant level separates conductors which in equilibrium have a finite density of mobile electrons, the problem of resonant tunneling becomes more complex due to the electron-electron interaction. Manifestation of resonant tunneling in the conductance of a solid-state device is inevitably sensitive to this interaction.

Some of the effects of electron-electron interaction do not depend on the dimensionality d of the conductors—leads separated by the barrier. For instance, the on-site repulsion together with the hybridization of the localized state with the states of continua lead to the Kondo effect in the transmission across the barrier [1] at any d. The Fermi-edge singularity also strongly affects the resonant tunneling [2] in any dimension. The electron-electron interaction within the leads, however, does not have a strong effect if $d > 1$, and if the leads are not disordered. In contrast, tunneling across a barrier interrupting a one-dimensional (1D) wire is modified drastically by the interaction within the wire [3–5]. Theory [6] predicts also a strong interaction-induced modification of the resonant tunneling in 1D wires. Results of the experiments with nanotubes containing a quantum dot [7] apparently deviate from the corresponding predictions [6] of the Luttinger liquid theory. These predictions were challenged recently in Ref. [8], where a somewhat different theoretical model of a wire was considered.

The electron-electron repulsion enhances the backscattering off the barrier. This enhancement is the strongest for the states with low energies. Even at weak "bare" backscattering, when the high-temperature conductance approaches the unitary limit, the zero-temperature conductance $G(T \to 0) \to 0$. Thus, because of interaction, the electron tunneling rate becomes energy dependent, even if the bare scattering amplitude is independent of energy. The Luttinger liquid theory yields the high- and low-temperature asymptotes for $G(T)$, and provides a qualitative description of the corresponding crossover regime [3]. However, if the bare scattering amplitude strongly depends on the electron energy, such as in the case of resonant tunneling, the general theory [3,6] does not capture the crossover regime. This may cast some doubt on the low-temperature asymptote for the conductance [3], as its derivation assumes that the tunneling rate is a smooth function of energy in the crossover regime.

In this Letter, we find the full crossover behavior of the resonant tunneling conductance for an arbitrary asymmetry of the barrier and arbitrary position of the resonant level with respect to the Fermi level, in the limit of weak interaction. In general, the temperature dependence of the conductance $G(T)$ is not monotonic. However, its low-temperature asymptote agrees with the one found in Ref. [3]. We perform an analytical calculation of $G(T)$ by a method similar to the one of Ref. [9]. Within this method, the complicated picture of many-electron transport is considered within the traditional Landauer-Büttiker elastic scattering formalism. The role of the interaction is to renormalize the elastic scattering amplitudes. The renormalization brings about an extra energy dependence of these amplitudes. In the limit of weak interaction, the most divergent terms in perturbation theory indeed correspond to the purely elastic processes [9], thus justifying the method.

To adapt the method of Ref. [9] to the conditions of resonant tunneling, we first generalize it onto the case of arbitrary energy dependence of scattering amplitudes. To start with, we derive the first-order interaction correction to scattering amplitudes. This can be readily done along the lines of Ref. [9]. The correction to transmission amplitude reads

$$\delta t(\epsilon) = \frac{t(\epsilon)}{2} \int_{-\infty}^{0} \frac{d\epsilon'}{\epsilon' - \epsilon} [\alpha_L r_L(\epsilon) r_L^*(\epsilon') + \alpha_R r_R^*(\epsilon') r_R(\epsilon)]. \quad (1)$$

Here the $r_{L(R)}$ are the reflection amplitudes for electrons incoming from the left (right), and the coefficients $\alpha_{L(R)}$ represent the interaction within the left (right) part of the 1D wire; energies ϵ and ϵ' are measured from the Fermi level. Transmission and reflection amplitudes $r_{L,R}$ satisfy the unitarity relation: $r_R t^* = -r_L^* t$. The coefficients α are related to the Fourier components $V(k)$ of the corresponding electron-electron interaction potential by $\alpha = [V(0) - V(2k_F)]/2\pi v_F$. In the limit of weak interaction, these coefficients determine the exponents in the edge density of states [3] for each part of the channel, $\nu(\epsilon) \propto \epsilon^{\alpha}$.

The integration over ϵ' in the first-order correction Eq. (1), in general, yields a logarithmic divergence at $\epsilon \to 0$. This indicates that the perturbation series in the interaction potential can be resummed with the renormalization method. To account for the most divergent term in each order of the perturbation theory in α, we proceed with the renormalization in a usual way [10]. On each step of the renormalization, we concentrate on the electron states in a narrow energy strip around $-E$, with $E > 0$ being the running cutoff. We evaluate the interaction correction due to the electrons in these states to the scattering amplitudes at energies ϵ close to Fermi level, $|\epsilon| < E$. These amplitudes are thus functions of both ϵ and E. We correct those amplitudes according to Eq. (1), reduce the running cutoff by the width of the energy strip, and repeat the procedure. This yields the following renormalization equation:

$$\frac{\partial t(\epsilon, E)}{\partial \ln E} = \frac{t(\epsilon, E)}{2}[\alpha_L r_L(\epsilon, E) r_L^*(-E) + \alpha_R r_R^*(-E) r_R(\epsilon, E)], \qquad (2)$$

provided that $|\epsilon| < E$. We abbreviate here $r(\epsilon) \equiv r(\epsilon, |\epsilon|)$ (and similar for t) indicating that the renormalization of scattering amplitudes stops when the running cutoff approaches $|\epsilon|$. The initial conditions for this differential equation are set at upper cutoff energy Λ. If the ϵ dependence of the transmission amplitude in the absence of interaction, $t(\epsilon, \Lambda)$, can be disregarded, then all the energy dependence of renormalized amplitudes comes about as a result of the renormalization procedure. The corresponding simplification of Eq. (2) then reads

$$\frac{\partial |t(\epsilon)|^2}{\partial \ln \epsilon} = (\alpha_R + \alpha_L)|t(\epsilon)|^2(1 - |t(\epsilon)|^2), \qquad (3)$$

and contains the transmission probabilities only. This coincides with the results of Ref. [9]. However, the above simplification is not possible in the more general case we consider here. One cannot even deal with a single equation: Eq. (2) shall be supplemented with a similar equation for one of the reflection amplitudes,

$$\frac{\partial r_L(\epsilon, E)}{\partial \ln E} = \frac{1}{2}\{\alpha_L[-r_L(-E) + r_L^2(\epsilon, E) r_L^*(-E)] + \alpha_R r_R^*(-E) t^2(\epsilon, E)\}. \qquad (4)$$

To describe resonant tunneling, we consider a compound scatterer made of two tunnel barriers with tunnel amplitudes $t_{1,2} \ll 1$ separated by a distance $\pi v_F/\delta$. This gives rise to a system of equidistant transmission resonances separated by energy δ. We assume that one of the resonances is anomalously close to Fermi energy and concentrate on this one disregarding the others. The scattering amplitudes in the absence of interaction are then given by common Breit-Wigner relations:

$$t(\epsilon, \Lambda) = \frac{i\sqrt{\Gamma_L \Gamma_R}}{(\Gamma_L + \Gamma_R)/2 - i(\epsilon - \Delta)},$$
$$r_L(\epsilon, \Lambda) = \frac{(-\Gamma_L + \Gamma_R)/2 - i(\epsilon - \Delta)}{(\Gamma_L + \Gamma_R)/2 - i(\epsilon - \Delta)},$$

where $\Gamma_{L,R} = |t_{1,2}|^2 \delta/2\pi$ are the level widths with respect to the electron decay into the left (right) lead and Δ is the energy shift of the resonance with respect to the Fermi Level; we assume here $\Delta \ll \delta$. We disregard possible energy dependence of $t_{1,2}$ that could be relevant at higher energies, which allows us to take the upper cutoff Λ to be of the order of δ. The corresponding transmission probability before the renormalizations,

$$|t(\epsilon, \Lambda)|^2 = \frac{\Gamma_L \Gamma_R}{(\epsilon - \Delta)^2 + (\Gamma_L + \Gamma_R)^2/4},$$

is the usual Lorentzian function of energy. The interaction corrections to Δ and $\Gamma_{L,R}$ which come from bigger energy scales, $\delta < E < E_F$, are assumed to be included in the definitions of these quantities.

The next step is to solve the renormalization Eqs. (2) and (4). To stay within the accuracy of the method, in the solution we need to retain the terms $\propto \alpha^n[\ln(\Lambda/\epsilon)]^n$ while same-order terms with a lower exponent of the logarithmic factor should be disregarded. This allows for a substantial simplification. We proceed by solving Eqs. (2) and (4) at higher energy (far from the resonance), where the reflection from the compound scatterer is almost perfect. In this case, we approximate $|r_{L,R}(-E)| \approx 1$. It is possible to see that in this case the renormalization of the tunnel amplitudes $t_{1,2}$ of each constituent of our compound scatterer occurs *separate* from each other, $d\ln t_{1,2}/d\ln\epsilon = \alpha_{L,R}/2$. This renormalization can be incorporated into the energy dependence of the effective level widths, $\Gamma_{R,L}(\epsilon) = \Gamma_{R,L}(\epsilon/\Lambda)^{\alpha_{R,L}}$. The result for $|t(\epsilon)|^2$ thus reads

$$|t(\epsilon)|^2 = \frac{\Gamma_L(\epsilon)\Gamma_R(\epsilon)}{(\epsilon - \Delta)^2 + [\Gamma_L(\epsilon) + \Gamma_R(\epsilon)]^2/4}. \qquad (5)$$

The above approximation of the integrand in Eq. (2) becomes invalid at lower energies, where the transmission coefficient may become of the order of unity. The energy scale $\tilde{\epsilon}$ at which this occurs can be evaluated from Eq. (5), and is given by the solution of equation $2\tilde{\epsilon} = \Gamma_L(\tilde{\epsilon}) + \Gamma_R(\tilde{\epsilon})$. If $\alpha_L = \alpha_R \equiv \alpha \ll 1$, it is $2\tilde{\epsilon} = (\Gamma_L + \Gamma_R)[(\Gamma_L + \Gamma_R)/2\Lambda]^\alpha$. At energies below $\tilde{\epsilon}$, the reflection amplitudes in the integrand can be approximated as $r(\epsilon') \approx r(\epsilon)$, and we immediately recover Eq. (3). Its solution at $|\epsilon| < \tilde{\epsilon}$ yields

$$|t(\epsilon)|^2 = \frac{\tilde{\Gamma}_L(\epsilon)\tilde{\Gamma}_R(\epsilon)}{(\epsilon-\Delta)^2 + \tilde{\Gamma}_L(\epsilon)\tilde{\Gamma}_R(\epsilon) + [\Gamma_L(\tilde{\epsilon}) - \Gamma_R(\tilde{\epsilon})]^2/4}, \tag{6}$$

with $\tilde{\Gamma}_{L,R}(\epsilon) = \Gamma_{L,R}(\tilde{\epsilon})|\epsilon/\tilde{\epsilon}|^{(\alpha_R+\alpha_L)/2}$. Relation (6) determines the full crossover function for the resonant tunneling, if $\tilde{\epsilon} \gtrsim |\Delta|$. In the opposite case of a resonance distant from the Fermi level, $|\Delta| \gtrsim \tilde{\epsilon}$, we shall change the approximation at $\epsilon = |\Delta|$. The answer is thus given by Eq. (6) with $\tilde{\epsilon}$ being *replaced* by $|\Delta|$. The definition of the crossover energy $\tilde{\epsilon}$ and the condition $|\Delta| = \tilde{\epsilon}$ of the crossover between the low energy cutoffs could contain any other numerical factors of the order of 1. Fixing the numerical factors with a greater precision would exceed the accuracy of our renormalization method. In other words, the energy dependence of Γ in all above relations is assumed to be very slow, which is the case in the limit $\alpha \ll 1$.

It is important to notice that the tunneling rate in the interesting domain of energies, $|\epsilon| \lesssim \tilde{\Gamma}_L + \tilde{\Gamma}_R$, reflects the electron dynamics at long time scales. The transient charge associated with the tunneling process is spread over a distance $\sim v_F/|\epsilon|$, greatly exceeding the physical size of the double-barrier system (v_F/δ). Therefore the transient charge accumulation in the vicinity of the barriers, which occurs at shorter scales, does not affect Eq. (6). By the same token, the finite range of interaction in the model of Ref. [8] should not affect the results either [11]. As seen from Eq. (6) and from Ref. [3], the resonance in $|t(\epsilon \to 0, \Delta)|^2$ becomes infinitely sharp, if the barrier is symmetric, $\Gamma_L = \Gamma_R$. The result of Ref. [8] is at odds with our conclusion. This discrepancy, demonstrated here in the limit of $\alpha \ll 1$, may be an artifact of the approach of Ref. [8]. Apparently, it is this discrepancy that leads to the disagreement of the main prediction of [8] for $G(T)$ at resonance with the known result [3] of the Luttinger liquid theory.

To present quantitative conclusions, we discuss the linear conductance $G(T)$ at $\alpha_R = \alpha_L \equiv \alpha$. Within the Landauer formalism, the conductance is given by

$$G(T) = G_Q \int_{-\infty}^{\infty} \frac{d\epsilon}{4T\cosh^2(\epsilon/2T)} |t(\epsilon)|^2, \tag{7}$$

where the conductance quantum unit for one fermion mode is $G_Q = e^2/2\pi\hbar$. The results strongly depend on the ratio of Γ_R and Γ_L. We will characterize this ratio by the asymmetry parameter $\beta \equiv |\Gamma_L - \Gamma_R|/(\Gamma_R + \Gamma_L)$ which ranges from 0 to 1 and does not depend on energy, provided that $\alpha_R = \alpha_L$. To emphasize the effect of interaction, let us recall that in the case of free electrons one finds $G(T) \propto 1/T$ at temperatures $T \gg \Gamma, \Delta$; in the limit $T \to 0$, the conductance saturates at a finite value, which reaches $(1-\beta^2)G_Q$ if the Fermi level is tuned to the resonance ($\Delta = 0$). Interaction changes this picture noticeably. Let us start the discussion with the case $\Delta = 0$. At high temperatures, $T \gtrsim \tilde{\epsilon}$, the conductance can be estimated as $G_{\Delta=0}(T)/G_Q = [\pi(1-\beta^2)/4](T/\tilde{\epsilon})^{\alpha-1} \simeq \Gamma(T)/T$. The unusual temperature dependence thus can be ascribed to the interaction-induced renormalization of Γ. The low-temperature behavior differs strikingly for symmetric ($\beta = 0$) and asymmetric ($\beta \neq 0$) resonances. For symmetric resonance, the conductance saturates at the ideal value of G_Q. For $\beta \neq 0$, the conductance reaches at $T \approx \tilde{\epsilon}$ its maximum value, which is smaller than $(1-\beta^2)G_Q$, and drops to zero with the further decrease of temperature,

$$G_{\Delta=0}(T)/G_Q = (1/\beta^2 - 1)(T/\tilde{\epsilon})^{2\alpha}, \qquad T \lesssim \tilde{\epsilon}. \tag{8}$$

The temperature exponents at $T \lesssim \tilde{\epsilon}$ agree with those obtained in Refs. [3,6] at any β. The exponent at $\beta \neq 0$ is the same as for a single tunnel barrier interrupting the 1D channel. It indicates that at low energies the electrons get over the compound scatterer in a single quantum transition.

The increase of Δ leads to a decrease of the conductance. For noninteracting electrons, the conductance stays at a level of the order of its maximal value, $G_{\Delta=0}$ for Δ less than $\Gamma_L + \Gamma_R$, which determines the width of the resonance in $G(\Delta)$ at $T \lesssim \Gamma_L + \Gamma_R$. At higher temperatures, the effective resonance width is $w \simeq T$. Let us discuss now the temperature dependence $w(T)$ and the shape of the resonance $G(\Delta)$ at fixed temperature in the presence of interaction. For $T \gg \tilde{\epsilon}$, the width $w \simeq T$ does not reveal any anomalous exponent. The shape of the resonance in this regime is mainly determined by the thermal-activated exponential contribution $G(\Delta) \simeq \exp(-|\Delta|/T)\Gamma(T)/T$ in Eq. (7). However, at large $\Delta \gg w$, the power-law "cotunneling" tail $G_{\rm tail}(\Delta) = G_Q(1-\beta^2)(T/\tilde{\epsilon})^{2\alpha}\tilde{\epsilon}^2/\Delta^2$, replaces that exponential dependence [12]. The crossover occurs at $\Delta \simeq T\ln(G_Q/G_{\Delta=0})$ and corresponds to the conductance $G_{\rm cross} \simeq G_{\Delta=0}^2/G_Q$, this being much smaller than $G_{\Delta=0}$.

At $T \ll \tilde{\epsilon}$, the apparent width of the nonsymmetric resonance saturates at $w \simeq \tilde{\epsilon}$. The conductance thus drops uniformly at any Δ following the power law (8). The symmetric resonance presents an exception. In this case, the width shrinks with the decreasing temperature, $w(T) \simeq (T/\tilde{\epsilon})^\alpha\tilde{\epsilon}$, and $G(T,\Delta)$ acquires the scaling form, $G(T,\Delta) = G_Q/\{1 + [\Delta/w(T)]^2\}$, in agreement with Ref. [6].

We further illustrate our results by a numerical evaluation of Eq. (7) (see Figs. 1 and 2). For this calculation, we choose $\alpha = 0.2$. By virtue of our approach, the relative accuracy of the results is expected to be of the order of α. The dependence $G(T)$ is not monotonic, and in the limit $T \to 0$ the conductance drops to zero at any $\beta \neq 0$,

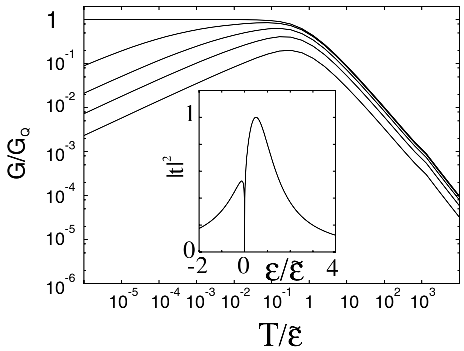

FIG. 1. Temperature dependence of resonant ($\Delta = 0$) tunneling conductance. The asymmetry parameter $\beta = 0$ (top curve), 0.2, 0.4, 0.6, and 0.8 (bottom curve). For symmetric resonance ($\beta = 0$), the conductance saturates at $T = 0$. Inset: The typical energy dependence of transmission coefficient consists of a Lorentz-like contour with a sharp dip at the Fermi level.

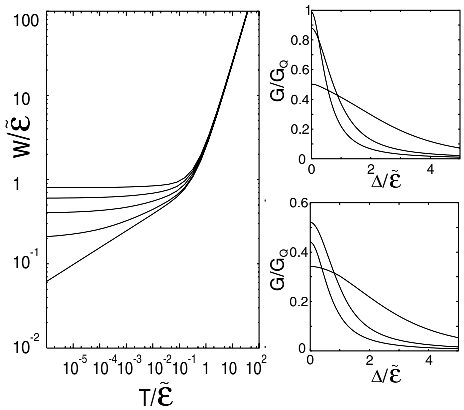

FIG. 2. Left: Half width at half maximum w vs temperature T for the values of asymmetry parameter $\beta = 0$, 0.2, 0.4, 0.6, and 0.8 (bottom to top curve). With the decreasing temperature, the half width saturates for a nonsymmetric resonance, and continuously decreases for the symmetric one. Right: The conductance dependence on the position of the resonant level with respect to the Fermi level, $G(\Delta)$, for symmetric (top) and nonsymmetric with $\beta = 0.5$ (bottom) resonances at three temperatures $T/\tilde{\epsilon} = 0.04$, 0.2, and 1.

although for small β this is noticeable only at very low temperatures (Fig. 1). The temperature dependence $w(T)$ of the width of the resonance $G(\Delta)$ is shown in the left panel of Fig. 2. If $\beta \neq 0$, this dependence saturates at some value $w(0) \neq 0$.

The differences and similarities of symmetric and nonsymmetric resonances are further illustrated in the right panels of Fig. 2. The three pairs of line shapes there correspond to "high," "medium," and "low" temperatures, respectively. The two high-temperature curves (the smallest values of $G_{\Delta=0}$) are hardly distinguishable from each other, and correspond to the resonance width $w \simeq T$. Both medium-temperature curves show a more narrow resonant peak with increased conductivity $G_{\Delta=0}$, and are still similar to each other, apart from the scale. The real difference becomes visible for the low-temperature curves. In the case of nonsymmetric resonance, the low-temperature curve is just reduced in height with no noticeable change of the shape. This is in contrast to the symmetric resonance, where the resonance peak gets taller and thinner.

In conclusion, we have investigated the transmission resonances of interacting electrons in 1D wires. For a weak electron-electron interaction, the transmission can be considered as an elastic process, which allowed us to build a comprehensive theory of the resonances, valid in a broad range of temperature and parameters of the resonant level. The temperature dependence of the maximum conductance in general is not monotonic, and reveals important differences between symmetric and nonsymmetric resonances. The obtained quantitative results present a comprehensive and consistent picture of the effect. It assures us in the qualitative validity of the picture at an arbitrary interaction strength. Although we are not able to come up with an explicit expression for the crossover function $G(T)$ in this case, such a function, with known high- and low-temperature asymptotes, does exist by virtue of the renormalizability.

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