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## AN ANALYTICAL APPROACH FOR PREDICTING THE COLLAPSE PRESSURE OF THE FLEXIBLE RISERS WITH INITIAL OVALIZATION AND GAP

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### ABSTRACT

A flexible riser is a flexible pipe that transports materials between seafloor and topside structures. As oil and gas production heads to water depths greater than 3000 meters, huge hydrostatic pressure may cause the collapse failure of flexible risers. Generally, the collapse strength of a flexible riser is designed by considering the effects of initial imperfections, e.g., ovality of the carcass, and radial gap between the carcass/liner and pressure armor. These two imperfections may cause a significant reduction in the collapse strength of flexible risers under the flooded annulus condition. However, there are few analytical models available in the public literature that could take those factors into account. In this paper, an analytical approach is presented to predict the critical collapse pressure of the flexible risers with initial imperfections. The analytical results were compared with the numerical simulation, which showed reasonably good agreement.

Keywords: collapse strength, flexible risers, initial ovality, radial gap

### 1. INTRODUCTION

As the oil and gas production is heading towards subsea fields with water depth higher than 3000 meters, flexible risers are required to resist the huge hydrostatic pressure [1–2]. Normally, the external pressure is resisted by all the layers within the pipe since the flexible riser is a multi-layer structure, as shown in Figure 1 [3]. However, the outer sheath may be worn out due to frequent movements on the seafloor [4], resulting in a flooded annulus. In this scenario, the carcass and the pressure armor are the main layers for collapse resistance, as

they contribute the most to radial stiffness. Once the external pressure exceeds the collapse capacity of a flexible riser, the collapse failure happens, which is called “wet collapse” [5].



FIGURE 1: MAIN COMPONENT LAYERS OF A FLEXIBLE RISER [3]

Mostly, the wet collapse pressure is higher than the collapse limit of the carcass since the surrounded pressure armor provides a significant support effect [6]. However, this critical pressure may be weakened by two initial imperfections, which are ovality of the carcass, and the radial gap between carcass/liner and pressure armor, respectively. The ovality of the carcass is generated from the manufacture tolerance while the radial gap is caused by the volume change or extrusion of the liner, as shown in Figure 2 [7–8]. Although tensile armors can reduce the radial gaps by squeezing the pressure armor, it is hard to close these gaps completely if the pipe is under a longitudinal tension. According to some researchers' investigation, it indicates that the critical pressure of flexible risers is sensitive to those two imperfections [7–10].

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**FIGURE 2:** EXTRUSION OF THE LINER INTO THE ADJACENT INTERLOCKED CARCASS [11]

Up till now, most studies related to initial imperfections were conducted by numerical simulations, which are time-consuming for design purposes of flexible risers. In this regard, an analytical approach is proposed in this paper for the imperfection issues. When the carcass is encased in the pressure armor, two possible collapse modes may happen, ovalization mode (doubly symmetric) and heart mode (singly symmetric) [12]. The present study is focused on the ovalization mode. The analytical model could take two initial imperfections, ovality and radial gap, into account, and give a prediction of the wet collapse pressure for a flexible riser with ovalization mode collapse. The content of this paper is organized as follows: Section 2 gives an overview of analytical models for predicting the critical pressure of flexible pipes. After that, the analytical approach for assessing the collapse strength of flexible pipes with initial imperfections is presented in Section 3. In Section 4, a previously presented FE model (Gay Neto and Martins, 2014) is used to check the proposed analytical model. Some conclusions are drawn in the final section.

## 2. REVIEW OF ANALYTICAL MODELS

Although the collapse of flexible risers has been studied for years, the analytical models, for the most part, are developed from the ring buckling theories [13]

$$P_{cr} = \frac{3EI}{(1-\nu^2)R^3} \quad (1)$$

Where  $EI$  is the bending stiffness of the ring and  $R$  is the ring radius,  $\nu$  is Poisson's ratio.

If no radial gaps exist, the sum of elastic collapse pressure from carcass and pressure armor is regarded as the critical pressure of the whole flexible pipes [14]

$$P_{cr} = \sum_{n=1}^{N_p} \frac{3(EI)_n}{(1-\nu^2)R_n^3} \quad (2)$$

Where  $(\cdot)_n$  represents the parameters of corresponding metallic layers,  $N_p$  is the total number of the layers.

However, such a formula is not applicable for the wet collapse situation. The wet collapse of flexible pipes can be described as the radial buckling of the confined carcass under external pressure [6]. In terms of this kind of buckling, a closed-

form analytical solution of an elastic cylinder confined in a rigid cavity was presented by Glock [15]

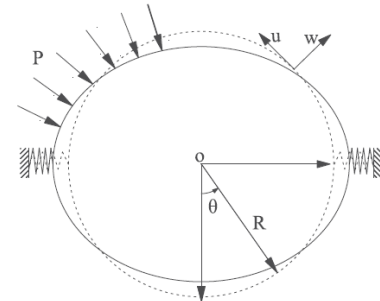
$$P_G = \frac{EI}{1-\nu^2} \left(\frac{t}{D}\right)^{2.2} \quad (3)$$

Where  $t$  and  $D$  are the thickness and mean diameter of the cylinder, respectively. Glock's formula was then extended by others [16-17] to consider a tightly (small gap) or loosely fitted (large gap) concentric rings. Although those formulae could consider the gap effect, they give an overestimated prediction of critical pressure for flexible pipes. This is because the pressure armor supports the carcass more like an elastic medium rather than a rigid cavity. With this regards, an elastic ring model with horizontal spring supports was proposed as [6]

$$P_{cr1} = \frac{3E_i I_i}{R_i^3} + \frac{2}{3} \frac{8E_o I_o}{(\pi^2 - 7)R_o^3} \quad (4)$$

Where the subscript  $(\cdot)_i$  and  $(\cdot)_o$  represent the inner and outer ring, separately.

The pressure armor is considered as springs in the above model which supports the carcass in the horizontal direction, as shown in Figure 3 [6]. However, this model is developed for the flexible risers with no gap in-between and only provides an elastic solution. Since the flexible risers are more likely to be collapsed in the plastic range in a deep water condition [18], efforts were made to predict the elastic-plastic collapse pressure of the flexible risers with initial imperfections in this paper.



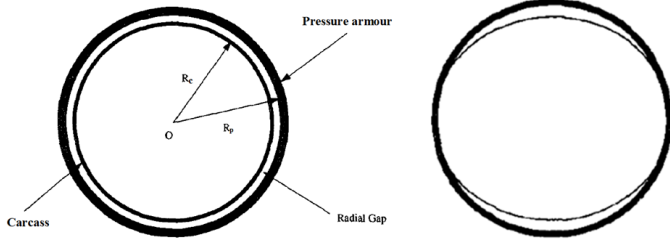
**FIGURE 3:** BUCKLING OF THE INNER CYLINDER WITH SPRING SUPPORTS [6]

## 3. ANALYTICAL METHOD

For the pressure armor which confines the carcass, its support effect is affected by the value of the radial gap between these two layers. Intuitively, a larger gap width leads to a less collapse resistance of the carcass due to a weaker constraint from the pressure armor. Figure 4 shows a typical progressive buckling progress of a flexible riser with an initial layer gap.

Two phases occur during this process: pre-contact and post-contact phases. During the pre-contact phase, the carcass is deformed as a buckled single layer ring. The pressure armor provides no constraints on the radial deformation of the carcass

until the contact happens. Once the contact occurs, the inner carcass starts to be divided into two portions: attached and detached portions. In this post-contact phase, the collapse pressure of the carcass is decided by the buckling strength of the detached portion [19].

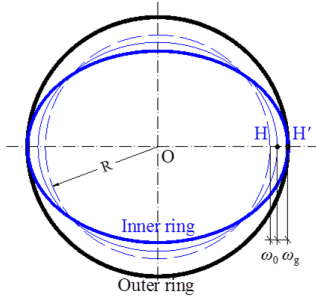


**FIGURE 4: PROGRESSIVE BUCKLING PROCESS FOR A SYMMETRICAL COLLAPSE MODEL**

The presented analytical scheme adopts different analytical models for these two phases. For the pre-contact phase, formulae of the ring buckling are used to solve the external pressure  $P_{con}$  at contact moment. For the post-contact phase, formulae of the arch buckling are employed to determine the buckling pressure  $P_{arch}$  of the detached portion. The shape of the detached portion of the carcass in the post-contact phase is decided by the initial ovality and gap width together. The pressure armor which restrains the deformation of the detached portion at its two ends is regarded as springs. The critical collapse pressure of a flexible pipe is the sum of  $P_{con}$  and  $P_{arch}$ .

### 3.1 PRE-CONTACT PHASE

For a carcass encased in the pressure armor, it might have two different buckling situations, depending on the value of radial gap between the carcass/liner and pressure armor. The inner carcass would be collapsed as a single ring if the gap width is large enough; otherwise, layer contact happens followed by the post-contact phase, as shown in Figure 5.



**FIGURE 5: CONTACT MOMENT OF A CONCENTRIC RING STRUCTURE**

The formula given by Timoshenko and Gere for the plastic collapse of a single ring with an initial deflection  $\omega_0$  can be used for the first situation [13]

$$P_y^2 - \left[ \frac{\sigma_y t_c}{R_c} + (1 + 6 \frac{\omega_0}{t_c}) P_{cr} \right] P_y + \frac{\sigma_y t_c}{R_c} P_{cr} = 0 \quad (5)$$

The maximum horizontal displacement  $\omega_{max}$  of the point H at the collapse pressure of the single inner ring, as shown in Figure 5, is given as

$$\omega_{max} = \frac{\omega_0 P_y}{P_{cr} - P_y} \quad (6)$$

Where  $\omega_0$  is the initial radial deflection of the ring, depending on its initial ovalization;  $\sigma_y$  is the material yielding stress;  $R_c$  and  $t_c$  are mean radius and equivalent thickness of the carcass;  $P_{cr}$  is the elastic critical pressure obtained from Eq. (1).

If the gap width  $g_w$  is smaller than  $\omega_{max}$ , then the radial deformation of the inner carcass is restrained after it contacts with the pressure armor. Therefore, the external pressure at the contact moment can be determined by equating the horizontal displacement  $\omega_1$  of point H (shown in Figure 5) to the gap width  $g_w$ , which is given by

$$P_{con} = \frac{g_w P_{cr}}{(g_w + \omega_0)} \quad (7)$$

However, it should be noted that the external pressure could cause a reduction on the wall thickness of the liner since it has a very low stiffness. This reduction postpones the contact moment between carcass and pressure armor and thus, Eq.(7) should be improved as

$$\begin{cases} P_{con} = \frac{(g_w + t_{br}) P_{cr}}{(g_w + t_{br} + \omega_0)} \\ t_{br} = t_{br}(P_{con}) \end{cases} \quad (8)$$

Where  $t_{br}$  is the reduction of wall thickness of the liner. Its value is determined by external pressure and the material constitutive equation of the liner.

The bending moment and hoop force at the crown point of the inner carcass can be calculated by

$$M_{con} = P_{con} R_c \frac{\omega_0}{1 - P_{con} / P_{cr}} \quad (9)$$

$$N_{con} = P_{con} R_c \quad (10)$$

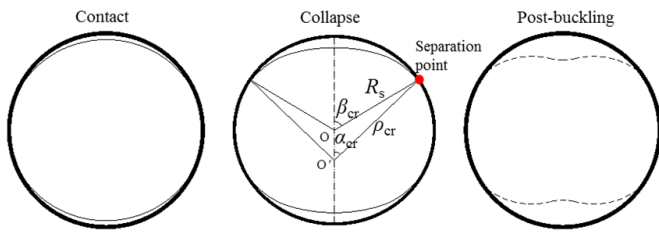
And thus, the maximum compressive stress  $\sigma_{con}$  at the crown point for the contact moment is

$$\sigma_{con} = \frac{P_{con} R_c}{t_c} + \frac{6 P_{con} R_c}{t_c^2} \frac{\omega_0}{1 - P_{con} / P_{cr}} \quad (11)$$

With this maximum compressive stress  $\sigma_{con}$ , the following task is to work out how much additional external pressure is needed in the post-contact phase for reaching the material yield stress.

### 3.2 POST-CONTACT PHASE

After the carcass contacts with the surrounded pressure armor, the contact point will keep moving upwards until the critical pressure is reached [20], as shown in Figure 6. An assumption from Timoshenko and Gere is used to define the elastic-plastic collapse of the flexible pipes: the collapse occurs at which yielding in the extreme fibers of the carcass begins [13], i.e. the carcass collapses when its maximum compressive stress equals to the material yield stress.



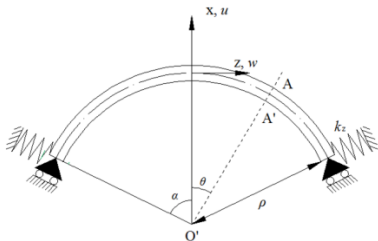
**FIGURE 6:** PROGRESSIVE BUCKLING PROCESS DURING THE POST-CONTACT PHASE

The detached portion can be regarded as a circular arch with a new center  $O'$ . The geometry of this arch is determined by gap width together with the initial ovalization of the carcass. When the position of the contact point at the collapse moment is determined, the geometry of this circular arch can be calculated by

$$\begin{cases} 2\pi R_c - 2R_s(\pi - 2\beta) = 4a\rho \\ \rho \sin a = R_s \sin \beta \end{cases} \quad (12)$$

Where  $R_s$  is the distance from the separation point to the ring center  $O$  at the collapse moment,  $\rho$  is the arch radius referred to the new center  $O'$ ;  $a$  and  $\beta$  are the angular quantities defined in Figure 6 (b);  $(\cdot)_{cr}$  denotes the parameters at the collapse moment.

Since the surrounded pressure armor is treated as springs, then the concentric ring structure could be simplified as a spring-supported arch model, as shown in Figure 7.

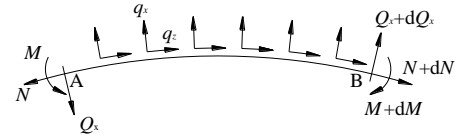


**FIGURE 7:** SPRING-SUPPORTED ARCH MODEL

The general linear equilibrium equation set for the differential element of a circular arch is expressed as [21]

$$\begin{cases} Q'_x - N + q_x \rho = 0 \\ N' + Q_x + q_z \rho = 0 \\ M' + Q_x \rho = 0 \end{cases} \quad (13)$$

Where  $M$ ,  $N$ , and  $Q_x$  are the bending moment, hoop force and radial shear force on the differential element;  $q_x$  and  $q_z$  are the uniform loads along the radial and circumferential directions, as shown in Figure 8.



**FIGURE 8:** EQUILIBRIUM OF A DIFFERENTIAL ELEMENT OF THE ARCH [21]

In the collapse analysis of flexible risers, values of those loads are given as

$$\begin{cases} q_x = -q = -(P - P_{con}) \\ q_z = 0 \end{cases} \quad (14)$$

Where  $q$  is the differential pressure between the external pressure  $P$  and the pressure at the contact moment  $P_{con}$ .

As the linear relationship between strains and displacements is given as

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \\ \varepsilon_z = \frac{\partial w}{\rho \partial \theta} + \frac{u}{\rho} \\ \varepsilon_{xz} = \frac{\partial u}{\rho \partial \theta} + \frac{\partial w}{\partial x} - \frac{w}{\rho} \end{cases} \quad (15)$$

then the equilibrium equation set Eq.(13) can be redefined with Eqs.(14) and (15)

$$\begin{cases} -E_c A_c \frac{w'' + u'}{\rho} = 0 \\ E_c A_c \frac{w' + u}{\rho} + \frac{E_c I_c}{\rho^3} (u^{IV} + 2u'' + u) = -Rq \end{cases} \quad (16)$$

Where  $u$  and  $w$  are the displacements of the differential element along the radial and circumferential directions, separately;  $\theta$  is the angle for an arbitrary cross-section of the

arch; the items with subscript (c) refers to the parameters of the carcass; (l) represents  $\frac{\partial}{\partial \theta}$ . If taking  $K$  as

$$K = -\frac{\rho^3}{E_c I_c} (\rho q + E_c A_c \frac{w' + u}{\rho}) \quad (17)$$

then the general solution of Eq.(16) can be written as

$$\begin{cases} u = KC_1 \cos \theta + KC_2 \theta \sin \theta + K \\ w = -KC_1 \sin \theta - KC_2 (\sin \theta - \theta \cos \theta) + KC_3 \theta \end{cases} \quad (18)$$

Where  $C_1 \sim C_3$  are constants that determined by the boundary conditions, and therefore, the forces for any arbitrary cross section are given as

$$\begin{cases} N = \frac{E_c A_c K}{\rho} (1 + C_3) + \frac{E_c I_c K}{\rho^3} (1 + 2C_2 \cos \theta) \\ M = \frac{E_c I_c K}{\rho^2} (2C_2 \cos \theta + 1) \\ Q_x = \frac{E_c I_c K}{\rho^3} (2C_2 \sin \theta) \end{cases} \quad (19)$$

For a spring-supported arch model as given in Figure 7, it has boundary conditions at its arch ends

$$\begin{cases} w|_{\theta=\alpha} = 0 & \text{No hoop displacement} \\ N|_{\theta=\alpha} = N_{thrust} & \text{Force equilibrium in hoop direction} \\ Q_x + k_t u = 0 & \text{Force equilibrium in radial direction} \end{cases} \quad (20)$$

Where  $N_{thrust}$  is the hoop thrust force at the arch end [22];  $k_t$  is the elastic stiffness of the pressure armor together with the liner.

The liner between the carcass and pressure armor may reduce the support effect of the pressure armor greatly due to its low stiffness. According to Ref. [18], the liner and pressure armor should be considered as two series of springs to provide support to the carcass against buckling. Therefore, the value of  $k_t$  takes form as

$$k_t = \frac{k_p k_b}{k_p + k_b} \quad (21)$$

Where  $k_p$  and  $k_b$  are the elastic stiffness of the pressure armor and the liner, separately, which can be obtained by

$$k = \frac{9\pi EI}{4R^3} \quad (22)$$

With the above boundary conditions Eqs.(20), the formula of  $C_1 \sim C_3$  can be derived

$$\begin{cases} C_1 = \frac{D_5}{D_6} - \frac{D_4}{D_3 D_6} k_t \\ C_2 = (1 + C_1 \cos \alpha) \frac{k_t}{D_3} \\ C_3 = \frac{C_1 \sin \alpha + C_2 (\sin \alpha - \alpha \cos \alpha)}{\alpha} \end{cases} \quad (23)$$

and the coefficients in Eq.(23) can be calculated as

$$\begin{cases} D_1 = \frac{E_c A_c}{\rho}, \quad D_2 = \frac{E_c I_c}{\rho^3} \\ D_3 = (2D_2 - k_t \alpha) \sin \alpha \\ D_4 = D_1 \frac{\sin \alpha - \alpha \cos \alpha}{\alpha} + 2D_2 \cos \alpha \\ D_5 = -(\frac{N_{thrust}}{K} + D_1 + D_2) \\ D_6 = D_1 \frac{\sin \alpha}{\alpha} + \frac{D_4}{D_3} k_t \cos \alpha \\ D_7 = \frac{\sin \alpha}{\alpha D_6} + k_t \frac{\cos \alpha}{D_3 D_6} \frac{\sin \alpha - \alpha \cos \alpha}{\alpha} \\ D_8 = -\frac{k_t}{D_3} [\frac{D_4}{D_6} \frac{\sin \alpha}{\alpha} - (1 - \frac{D_4 k_t \cos \alpha}{D_3 D_6}) (\frac{\sin \alpha}{\alpha} - \cos \alpha)] \\ K = \frac{\rho q - N_{thrust} D_1 D_7}{(D_1 + D_2)(D_1 D_7 - 1) - D_1 D_8} \end{cases} \quad (24)$$

By substituting those coefficients into Eqs.(19), the external pressure can be written as a function of the maximum compressive stress at the crown point of the arch. Since the plastic collapse is defined by material yielding, therefore, the buckling pressure  $P_{arch}$  of this arch can be worked out as

$$\frac{N}{t_c} + \frac{6M}{t_c^2} = \sigma_y - \sigma_{con} \quad (25)$$

Where  $\sigma_y$  is the material yield stress of the carcass. By substituting Eqs.(11) and (19) into Eq.(25), the buckling pressure  $P_{arch}$  of this spring-supported arch can be obtained. Finally, the critical pressure  $P_{f-cr}$  of the flexible risers with initial ovality and gap is obtained as

$$P_{f-cr} = P_{con} + P_{arch} \quad (26)$$

### 3.3 SEPARATION POINT AT COLLAPSE MOMENT

If the position of the separation point, i.e.  $R_s$  and  $\beta_{cr}$ , at the collapse moment is determined, the arch geometry can be

obtained with Eq.(12), followed by the calculation of the critical pressure with the above-mentioned methods. However, this position is not easy to be determined since it is affected by multiple factors, such as gap width, initial ovaization and bending stiffness ratio between the outer and the inner layer. In order to tackle this problem, a formula needs to be proposed to estimate the value of  $R_s$  at the collapse moment. This formula for  $R_s$  should meet such rules:

I.  $R_s$  approximates to the sum of carcass radius, gap width, wall thickness reduction of the liner, and initial deflection when the outer layer becomes infinite rigid;

II.  $R_s$  approximates to the sum of carcass radius, initial deflection, and maximum horizontal displacement  $\omega_{max}$  when the outer layer's stiffness approaches zero;

III.  $R_s$  is not affected by the stiffness of the outer layer when  $g_w + t_{br} \geq \omega_{max}$ .

Therefore, a formula is proposed as

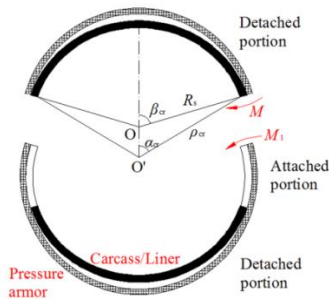
$$R_s = R_c + g_w + \omega_0 + t_{br} + (\omega_{max} - g_w - t_{br}) \left( \frac{g_w + \omega_0 + t_{br}}{\omega_{max} + \omega_0} \right) \Phi_k \quad (27)$$

$$0 \leq g_w + t_{br} \leq \omega_{max}$$

Where  $\Phi_k$  is the bending stiffness ratio between the carcass and liner/pressure armor, which is given as [6]

$$\Phi_k = \frac{k_t}{k_c} \quad (28)$$

With the value of  $R_{con}$  that estimated from Eq.(28), the buckling pressure  $P_{arch}$  of the arch can be determined by decreasing the value of  $\beta_{cr}$  (from  $\pi/2$  to 0) continually until the bending moments of the attached and detached portions at the contact position equal to each other, as shown in Figure 9.

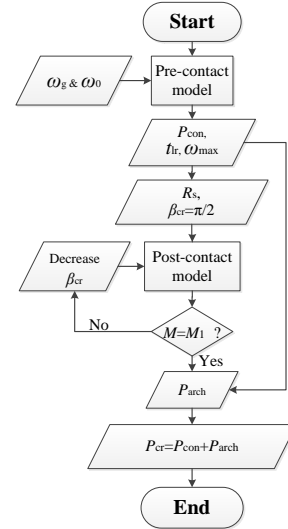


**FIGURE 9: BENDING MOMENTS AT THE CONTACT POSITION**

The bending moment for the detached portion can be calculated with Eq.(19). For the attached portion, the bending moment at the contact position is obtained by [13]

$$M_1 = E_c I_c \left( \frac{1}{\rho_{cr}} - \frac{1}{R_{con}} \right) \quad (29)$$

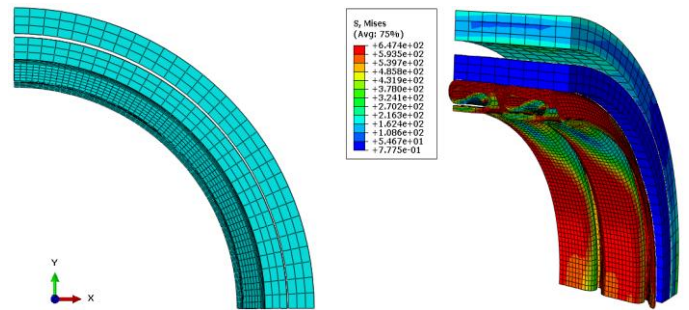
Once  $M$  and  $M_1$  are matched, the angle  $\beta_{cr}$  can be determined as well as the buckling pressure  $P_{arch}$  of the arch. A flowchart that shows the whole procedure is given as Figure 10.



**FIGURE 10: FLOWCHART OF THE WHOLE ANALYTICAL SCHEME**

#### 4. VERIFICATION

For verification purposes, a 4” flexible riser model presented by Gay Neto and Martins is used [5], which was also adopted by Edmans in his research [23]. This FE model given in the above-mentioned literature possesses three layers, the carcass, the inner liner and the pressure armor. The carcass was built with a detailed profile while the liner and pressure armor were represented as two homogeneous equivalent layers. Abaqus 6.13 is employed to recreate this FE model, as shown in Figure 11. The critical pressure given by this recreated FE model is 23.60 MPa, which is quite close to results of Gay Neto (25.56 MPa) and Edmans (24.1 MPa), with differences of 7.67% and 2.07%, respectively.



**FIGURE 11: FINITE ELEMENT MODEL OF A 4” ID FLEXIBLE RISER WITH INITIAL IMPERFECTIONS**

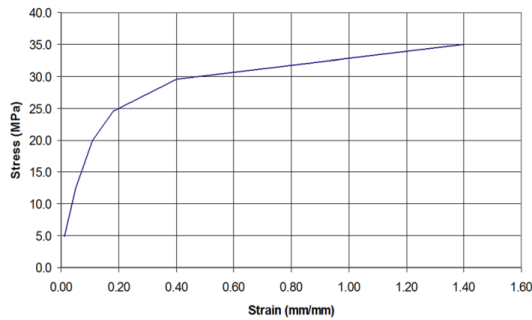
Table 1 lists the geometric and material data of this FE model. Geometric details of the carcass profile are given in the source reference. The material stress-strain curves of the liner



are shown in Figure 12 [5]. For the initial gap issue, a set of radial gap were considered in the FE models with a fixed initial ovalization of 0.5%. According to the numerical simulation, the maximum gap width for the carcass contacts with the pressure armor before its collapse is 0.36 mm, approximately. Therefore, the gap width for the following case study varies from 0 to 0.3 mm. For the initial ovalization issue, the simulation data for three kind of ovalization, i.e. 0.5%, 1.0%, 2.0%, from the work of Gay Neto, et al. and Edmans are used directly [5,23]. According to their work, the ovalization is defined as maximum radial deviation of the carcass inner surface from circular, divided by carcass undeformed inner radius [5].

**TABLE 1: GEOMETRIC AND MATERIAL PROPERTIES [5]**

Model	Carcass	Liner	Pressure armor
Internal diameter (mm)	101.60	114.40	Var.
Layer thickness(mm)	6.40	5.00	5.86
Young's Modulus (GPa)	200	-	207
Poisson's Ratio	0.3	0.45	0.3
Tangent Modulus (GPa)	2.02	-	62.92
Yield stress (MPa)	600	-	650



**FIGURE 12: STRESS-STRAIN CURVE OF THE LINER [5]**

The strain energy-based equivalent layer method was employed to provide equivalent properties for the analytical models [24]. The equivalent thickness, Young's Modulus and Yield stress of the carcass is 4.5 mm, 158 GPa and 473 MPa, individually<sup>1</sup>. The elastic-plastic collapse pressure of both analytical and numerical models described above is summarized in Table 2 and 3.

From the results listed in Table 2 and 3, it can be seen that the ovalization and layer gap weaken the collapse resistance of the carcass significantly. The analytical predictions agree well with the data from the literature in the ovalization cases. As for initial gap, the analytical model gives a good prediction for a tightly-fitted carcass; however, the analytical results deviate gradually from the numerical data with the increase of the gap value.

<sup>1</sup> A 2D FE ring model was constructed with those equivalent properties for the purpose of checking reliability, which gives a collapse pressure of 23.51 MPa that correlates well with 23.60 MPa from the 3D full FE model.

**TABLE 2: COMPARISO OF COLLAPSE PRESSURE BETWEEN FE AND ANALYTICAL MODELS FOR DIFFERENT OVALIZATION**

Ovalization		0.5%	1%	2%
Numerical results (MPa)	Gay Neto, et al.	25.56	23.96	22.33
	Edmans	24.10	23.55	22.65
Analytical results (MPa)		23.38	22.12	20.45

**TABLE 3: COMPARISO OF COLLAPSE PRESSURE BETWEEN FE AND ANALYTICAL MODELS FOR DIFFERENT GAP WIDTH**

Gap width (mm)	0	0.05	0.1	0.2	0.3
Numerical results (MPa)	23.60	23.32	23.04	22.51	21.99
Analytical results (MPa)	23.38	20.37	18.91	16.89	16.82

## 5. CONCLUSIONS

Ovalization and gap are two initial imperfections which may reduce the anti-collapse strength of flexible risers significantly. This paper proposes an analytical approach to predict the wet critical pressure of the flexible riser with initial ovality and gap. Numerical simulation was adopted to check the proposed model. Due to the scope of this paper, only the ovalization collapse mode of the carcass was considered. The investigation results showed that the collapse pressure is very sensitive to the ovalization and radial gap.

The proposed analytical approach gives a conservative prediction of collapse pressure compared with the numerical simulation. The reasons for this phenomenon come from two sources: the definition of elastic-plastic collapse as well as the boundary condition that used in the analytical models. In this analytical approach, the definition of the elastic-plastic collapse given by Timoshenko and Gere is adopted, which defined the plastic collapses of the carcass occurs at the onset of its material yielded. Although the practical collapse of the carcass occurs with a compressive stress higher than the material yield stress, this definition makes the prediction always on the safe side. As for the boundary conditions, this approach regards the pressure armor as springs to provide the radial reaction force at the ends of the detached portion of the carcass only. However, the actual pressure armor also provides rotation constraints to the ends to some degree. This factor is neglected in the proposed analytical model. As a result, discrepancy occurs between the analytical and numerical data.

Up to now, there are few analytical models that developed for addressing those imperfections. The collapse pressure for an actual flexible pipe is not easy to be predicted since it is affected by multiple factors, e.g. the value of the imperfections, stiffness of the liner and pressure armor, etc. This paper attempts to propose an analytical approach to address the initial imperfection issues. According to the comparison with the numerical simulation, it could be a useful tool in the design of flexible risers.

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