

An energy efficient gait for a Nao robot

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Abstract

The gait of humans is often assumed to be the most energy efficient way of walking. Srinivasan and Ruina [18] confirm this hypothesis using a simple model in which the human is a point mass with straight legs that can change in length during a step. Their results show that the inverted pendulum walk is the most energy efficient gait. The question is whether this result also holds for humanoid robots.

This paper investigate what is the most energy efficient gait for a humanoid robot such as the Nao, and what the corresponding control policy is that needs to be implemented. To answer these questions, first, the model of Srinivasan and Ruina is adapted for humanoid robots, and is used to study the energy consumption of different gaits. The model assumes a gait with dynamic stability and assumes that the torque on the knee joint provides the main contribution to the energy consumption of a gait. The former assumption implies that no energy is needed to remain stable. The latter assumption is confirmed by an experiment with a humanoid robot, namely Nao. Based on experiments with this idealize model, a gait that minimizes the energy consumption is identified. A controller for the new gait is implemented and is evaluated on a Nao robot. In the future, this controller will be the basis of an intelligent controller that can adapt to varying circumstances.

1 Introduction

Efficient energy use is an important issue in bipedal humanoid robots. Srinivasan and Ruina [18] showed that the *inverted pendulum* walk results in the most energy efficient gait for human. In their model the human is represented by a point mass with straight legs that can change in length during a step. Energy consumption is determine by the positive work while increasing the length of the leg. The spring behavior of human muscles is ignored as well as the energy required to maintain a torque on the knee joint when the leg-length does not change. The spring behavior of muscles implies that energy can be stored, for instance after the impact of the heel with the ground. The torque which plays a role when humans have to bend the knee, might be ignored while walking. When the knee is locked humans can still push off using the foot and the calf muscle. In this way humans can "increase" the length of the leg without torque on the knee joint.

A humanoid robot such as a Nao, differs in important ways from a human. First, motors of a robot often do not work as the spring. Second, our experiment with Nao shows that the energy consumption of a motor depends more on the torque than on the work that is done. Third, a robot such as a Nao cannot push off with a foot. The first difference is not an issue since humans do seem to use the spring behaviour of the muscles during walking. The second difference may not be a real difference. However, there is no torque on the knee joint of a human when its leg is stretched while the human can still stretch the leg a bit more by pushing off using the foot. So, the third difference makes it possible to ignore the torque in the human model but not in models of certain robots such as a Nao.

The reminder of the paper is organized as follows: In the next section we start with a discussion of related work. Subsequently, in Section 3 we introduce the inverted pendulum model used for this work, and how we used the model to determine the energy consumption of a gait. Section 4 will outline the experiments used to identify an energy-minimal gait of a Nao. We compare the result with the well-known *linear inverted pendulum* gait [5, 11, 12, 15]. We tested the identified energy-minimal gait on a Nao humanoid robot. Section 5 concludes the paper.

2 Related Work

A key issue in robot locomotion is energy consumption. Completely actuated systems, such as the legs of the Honda Asimo robot [4], are not very efficient because each joint has a motor and control assembly. Human-like gaits are far more efficient because movement is sustained by the natural swing of the legs instead of motors placed at each joint. Tad McGeer's 1990 paper [16] contributes an overview on the advantages of knees for walking legs. He demonstrates that knees have many practical advantages for walking systems.

Many researchers have begun to focus on efficient biped robot walking on level ground. Previous bipedal robots with human-like gait are based on the mainstream control method [3], which is precise joint-angle control. However, it requires actuators with frequent response, therefore, it requires more energy. To address these issues, McGeer's passive dynamic walking has provided inspirations to solve these problems. By applying the advantages of passive dynamic walking, human-like bipedal robots that are energy efficient have been developed [6, 14, 20].

A important problem is how to generate energy-efficient and speedy biped locomotion without taking controlling torso balance into account. Minakata and Tadakuma experimentally demonstrated that level dynamic walking could be accomplished by pumping the leg [17]. Based on these observations, Fumihiko Asano and Zhi-Wei Luo [2] proposed a simple control law for a planar telescopic legged biped model and show that level gait generation can be easily accomplished by pumping swing leg without taking torso balance into account. Jason Kulk and James Welsh [13] provided a adaptive stiffness method to achieve the balance between energy consumption and walk velocity. They give a form of stiffness control to set parameter K_s to be restricted to the motor torque which results from position feedback.

Several models of bipedal walking that do not focus on energy consumption have been proposed in the literature. One such model is the Kajita et al's *Linear Inverted Pendulum Model* (LIPM) [5, 11, 12, 15]. In this model, the robot is represented as an inverted pendulum. Benjamin Stephens and Christopher Atkeson [19] developed an extension of the LIPM. Rather than following pre-determined trajectories as LIPM did, the new model can utilize reaction controller to stabilize the system from unknown perturbation. Gouallier et al.[7] have published a similar paper on the Nao. Ames et al.[1, 10] addressed the feedback control law to keep walking stable on Nao. Based on 3D-LIPM model, C.Graf and T.Röfer [8, 9] in B-Human Team of Bremen University proposed a closed-loop omnidirectional walking gait for Nao. The gait allows eliminating the double-support phase by using sensor feedback to compute a slightly modified trajectory of Center of Mass(COM) to adjust future stable steps.

3 The inverted pendulum model

The gait of humans and of humanoid (bipedal) robots is a repeating pattern consisting of two phases; a single support phase (SSP) where the body is supported by only one leg and a double support phase (DSP) where the body is supported by both legs. In the DSP the weight of the body is shifted from one leg to the other. The DSP is crucial for the sideway stability and is sometimes ignored when analyzing the gait. However, since it is impossible to implement a gait on a Nao without a double support phase, we must consider it in our model. We will start presenting a model without a DSP and subsequently extend the model with a DSP.

The goal of our model is to identify a energy minimal gait for a bipedal robot such as a Nao. To analyze the energy consumption, we need a model of the main joints and limbs of the robot. Since the walk pattern is what we are concerned about, to simplify the model, we assume that the upper part of body above the hip can be ignored. Moreover, since we are interested in a dynamically stable gait instead of the statically stable gait that is often used, we also ignore the feet and the ankle joints. Finally, we assume that legs are weightless, making it possible to use a single point mass for the whole robot. The resulting model consists of 5 links and a point m shown by the solid line in Figure 1. We can replace the 5-link model by an equivalent 2-link model consisting of two links that can change their length during a step and a point mass m , see the dashed lines in Figure 1. The two links will be denoted as (telescopic) legs.

Srinivasan and Ruina [18] describes a similar model but focuses on humans. In their model they ignore that human muscles partially work as springs that can store energy. They also ignore that carrying a weight with bended knees require more energy than carrying the same weight with stretched legs. Srinivasan and Ruina only considered the positive work that is needed to stretch a leg and claim that despite these simplifications, the most energy efficient gait that follows from their model corresponds with a human gait.

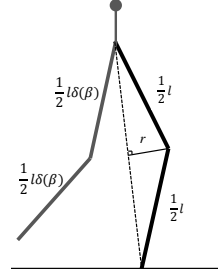
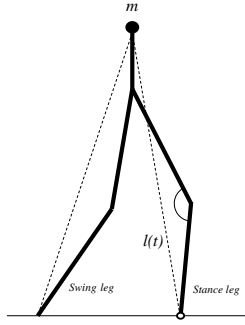


Figure 1: Simplify the 5 link model to a 2 link model Figure 2: The arm r determines the knee torque

3.1 The model without double support phase

The movement of one step If we would know the force produced by the leg, we could apply Newton's second law to derive the second order equation for the movement of the mass m . Since the ground generated a reaction force, can become infinitely large at the moment a stretched leg impacts with the ground, and since the length of the leg is bounded by l , we choose a different approach. We assume a leg-length policy $\delta(\beta)$ where $\delta : [-\pi, \pi] \rightarrow [0, 1]$ and where β is the angle between telescopic leg with vertical axis, see Figure 1. We use the leg-length policy to determine the radial force F_r on the leg and a force F_t perpendicular to F_r . Note that F_t works perpendicular to leg while the path of the mass m need not be perpendicular to the leg because the length of the leg is changing. Using the force F_t , we can determine the movement x_t perpendicular to leg which is given by:

$$F_t = ma = m \frac{d^2 x_t}{dt^2}$$

F_t is determined by the component of the gravity working perpendicular to the stance leg: $mg \sin \beta$ and the friction: $b \frac{dx_t}{dt}$. Note that we assume the friction to be linear in the speed. The air friction maybe assumed to be linear because of the low walking speed.

$$\frac{d^2 x_t}{dt^2} + \frac{b}{m} \frac{dx_t}{dt} - g \sin \beta = 0 \quad (1)$$

We can transform this movement in a change of the angle β using:

$$d\beta = \frac{dx_t}{\delta(\beta)l}$$

Therefore,

$$\begin{aligned} \frac{dx_t}{dt} &= \delta(\beta)l \frac{d\beta}{dt} \\ \frac{d^2 x_t}{dt^2} &= l \frac{d\delta(\beta)}{dt} \frac{d\beta}{dt} + \delta(\beta)l \frac{d^2 \beta}{dt^2} \\ &= l \frac{d\delta(\beta)}{d\beta} \left(\frac{d\beta}{dt} \right)^2 + \delta(\beta)l \frac{d^2 \beta}{dt^2} \end{aligned}$$

This result in:

$$\frac{d^2 \beta}{dt^2} + \frac{1}{\delta(\beta)} \frac{d\delta(\beta)}{d\beta} \left(\frac{d\beta}{dt} \right)^2 + \frac{b}{m} \frac{d\beta}{dt} - \frac{g}{\delta(\beta)l} \sin \beta = 0$$

To solve this second order differential equation using a program such as MATLAB, the equation must be reformulated into a system of first order differential equations.

$$\begin{aligned} \frac{d\omega}{dt} &= -\frac{1}{\delta(\beta)} \frac{d\delta(\beta)}{d\beta} \omega^2 - \frac{b}{m} \omega + \frac{g}{\delta(\beta)l} \sin \beta \\ \frac{d\beta}{dt} &= \omega \end{aligned} \quad (2)$$

The length of the stance leg at the beginning and the end of a step, denoted by l_1 and l_2 respectively, need not be the same. We therefore need to know the angle β at the beginning and the end of a step, denoted by β_1 and β_2 respectively, given a fixed step size s . The leg policy should describe the changes in the leg-length between these angles. To determine the angles β_1 and β_2 , we apply the cosine-rule, which gives us $\alpha = 0.5\pi - \beta$. Hence,

$$\beta_1 = \arcsin \frac{s^2 + l_1^2 - l_2^2}{2sl_1} \quad \beta_2 = \arcsin \frac{s^2 + l_2^2 - l_1^2}{2sl_2}$$

Forces on the stance leg The solution of the above presented system of differential equations enables us to determine the radial force the stance leg. This force together with the leg-length policy $\delta(\beta)$ determines the energy consumption of the knee joint of the stance leg. The radial force consists of a gravitational component and a component needed to accelerate the mass in the direction of the radius. The former is a reaction force equal to: $G \cos \beta$, and the latter is determined by the second derivative of the leg-length, that is, it is determined by the the second derivative of the leg policy:

$$\begin{aligned}\frac{dl\delta(\beta)}{dt} &= l \frac{d\delta(\beta)}{d\beta} \frac{d\beta}{dt} \\ \frac{d^2 l\delta(\beta)}{dt^2} &= l \left(\frac{d^2 \delta(\beta)}{d\beta^2} \left(\frac{d\beta}{dt} \right)^2 + \frac{d\delta(\beta)}{d\beta} \frac{d^2 \beta}{dt^2} \right)\end{aligned}$$

So,

$$F_r = mg \cos \beta + ml \left(\frac{d^2 \delta(\beta)}{d\beta^2} \left(\frac{d\beta}{dt} \right)^2 + \frac{d\delta(\beta)}{d\beta} \frac{d^2 \beta}{dt^2} \right)$$

When the foot of the robot impacts with the ground at the beginning of a step, the direction in which the mass m is moving may change. Since the change in direction is instantaneous, no energy is transferred to the mass. The conservation law of kinetic energy now implies that the speed of the mass may not change the moment the foot impacts with the ground. For a gait without acceleration, this implies that the speed of m at the beginning and end of a step must be the same.

Also the conservation law of momentum applies. The sum of the momentum before and after the foot impacts with the ground must be 0. This does not imply that the impulse generated by the reaction force of the leg when it impacts with the ground is 0. This impulse causes the change in the direction of m . The impulse generated by the reaction force of the leg is determined by the change in speed of m in the direction of the *new* stance leg:

$$I_r = v_{r,b}m - v_{r,e}m$$

The subscript r denotes the radial direction of the leg, the subscript b denotes the beginning of a step and the subscript e the end. After the foot impacts with the ground, the speed in the direction of the leg is:

$$v_{r,b} = \frac{dl\delta(\beta)}{dt}(t_b)$$

Before the foot impacts with the ground we have to calculate the component of the speed in the direction of the new stance leg. We first calculate the speed in the x and z direction of the Cartesian coordinate system.

$$v_t = l\delta(\beta) \frac{d\beta}{dt} \quad v_r = \frac{dl\delta(\beta)}{dt} = l \frac{d\delta(\beta)}{d\beta} \frac{d\beta}{dt} \quad v_x = v_t \cos(\beta) + v_r \sin(\beta) \quad v_z = v_t \sin(\beta) + v_r \cos(\beta)$$

So, impulse produced by the leg becomes:

$$I_r = m(v_{r,b} - (v_{x,e} \sin(\beta_b) + v_{z,e} \cos(\beta_b)))$$

The impulse is also equal to: $I_r = \int F_r dt$, which enables us to calculate the force on the leg. In an ideal situation the impact time with the ground is infinitely small implying an infinitely large reaction force produced by the leg on the mass m . In practice material always bends or compresses somewhat. This increases the impact time and thereby reduces the reaction force of the leg. Although the force will now be finite, even if we would know the impact time, we cannot calculate it. We therefore make the simplifying assumption that the reaction force is constant during the impact. Moreover, we assume that the impact time is 10 ms. These assumptions make it possible to calculate the force on the leg over time.

The energy consumption To calculate the energy consumption of the robot, we make use of the fact that the robot has to bend its leg at the knee joint in order to shorten the leg. The energy consumption is assumed to be proportional with the torque of these joints. So, a stretched leg requires no energy while an almost completely bended leg requires a maximum amount of energy. The experiment with a Nao described in Sub-section 4.1, confirmed our consumption that the torque on the knee joint determines its energy consumption. We will use this observation to determine the energy consumption in the model.

Let $\delta(\beta) \in [0, 1]$ be the percentage of shortening the leg. Then we must determine the arm r (see Fig.3) in order to calculate the torque. This gives us: $r = \frac{1}{2}l\sqrt{1 - \delta(\beta)^2}$.

The radial force F_r acting on the mass m can become infinitely high if the is completely stretched while impacting with the ground. So, keeping the leg stretched which implying $r = 0$ minimizes the energy consumption. Also no energy is provided to the system, and therefore, because of the friction, the robot will not continue walking. To provide energy, the leg must bend and stretch again. The energy is used to provide the necessary torque and the positive work of lifting the mass m when stretching the leg. We conducted an experiment in which we measured the energy consumption while bending and stretching both legs is the upright standing position for different torque values. Based on experiments with the Nao robot, we will assume that the work can be ignored. Therefore, we define the energy consumption as:

$$E = \int_{-\frac{1}{2}T}^{\frac{1}{2}T} \frac{1}{2} l \sqrt{1 - \delta(\beta)^2} F_r dt$$

In this equation, the shortening of the leg $\delta(\beta) \in [0, 1]$ is the policy that is used to control the gait. This policy should ensure the Nao keeps walking with minimal energy consumption.

3.2 Adding the double support phase

We extended the model described in the previous subsection with double support phase. We stop the single support phase at 75% of a step; i.e., at the angle $\beta_1 + 0.75(\beta_2 - \beta_1)$ where β_1 and β_2 are the begin and the end angle, respectively, of the stance leg during a step. From this moment the swing leg will also be on the ground, thereby influencing the movement of the mass m .

We need a way to describe the influence of the swing leg on the mass in DSP. But We can not do that by just simply applying leg-length policy for the swing leg in the double support phase. Given the step size s and the leg-length policy $\delta(\beta)$ of the stance leg, the length leg of the swing leg is fixed. Prescribing this length of the swing leg by a policy, creates a rigid triangle in which the mass m can no longer mover freely. We therefore choose to let the mass m move freely given the leg-length policy of the stance leg and use a force policy for the swing leg in the double support phase. So the length of the swing leg is determined by the leg-length policy of the stance leg and the step size, but force the swing leg executes on the mass is determined by the force policy of the swing leg. This force may influence the forward speed of the mass m .

The force policy $\gamma(\beta')$ where $\gamma : [-\pi, \pi] \rightarrow [0, 1]$ and where β' is the angle of the swing leg with the vertical axis, will be expressed as a percentage of the force F_r on the stance leg. F_r is the force on the stance leg caused be gravity and by changes in the length of the stance leg. The force F_s generated by the swing leg causes a force F_o in the opposite direction of F_r and a force F_p perpendicular to the swing leg. The force policy $\gamma(\beta')$ determines $F_o = \gamma(\beta')F_r$, and thereby the force generated by the swing leg F_s and the force F_p . Note that the angle β of the stance leg with the vertical axis, the leg-length policy $\delta(\beta)$, and the step size, determines the angle β' of the swing leg with the vertical axis.

We can now derive the following differential equations for the movement of the mass m in the DSP:

$$\frac{d^2\beta}{dt^2} + \frac{1}{\delta(\beta)} \frac{d\delta(\beta)}{d\beta} \left(\frac{d\beta}{dt} \right)^2 + \frac{b}{m} \frac{d\beta}{dt} - \frac{1 - \gamma(\beta) \tan(\beta + \beta')}{\delta(\beta)l} g \sin \beta = 0 \quad (3)$$

4 Experiments

Torque As mentioned in the introduction, we assume that the energy consumption of a gait is determined by the torque on the knee joint. We can ignore the torque on the ankle joint, because we assume a gait with dynamic stability. Since the upper part of body is balanced above the hip joint, ideally, no torque is required in the hip joint. Experiment on the Nao by Kulk and Welsh [13] confirms the assumption about the ankle and hip joints. Moreover they show that the knee joint provides the main contribution to the energy consumption of a walk.

In an experiment with a Nao, we investigated the relation between the torque and the energy consumption of the knee joint. We changed the position of the Nao from standing upright (knee angle = 180 °) to a position where the angle is around 90°. We kept the Nao for a while in this position after which we let the Nao move back to the upright position. During this exercise, we monitored the current of a knee joint. When standing upright it is around 0.2A, and decreases to 0.1A during the bending of the knees. In the fixed bended position, the current is 0.7A. Finally, when the Nao moves back to the upright position, the current gradually drops from 0.7A to 0.2A. During these experiment “Smart Stiffness” was switched on. Based on this experiment, we conclude that the torque determine energy consumption and that the positive work can be ignored.

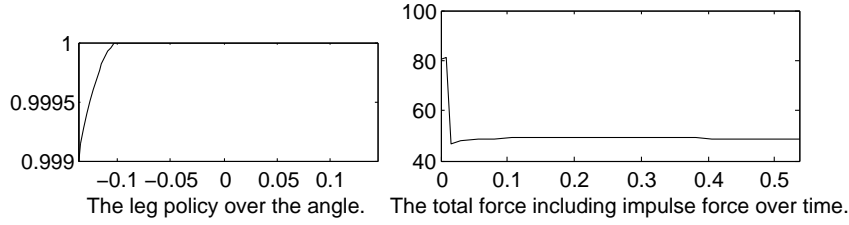


Figure 3: The "energy optimal" leg-length policy which can minimize the energy consumption to 0.011.

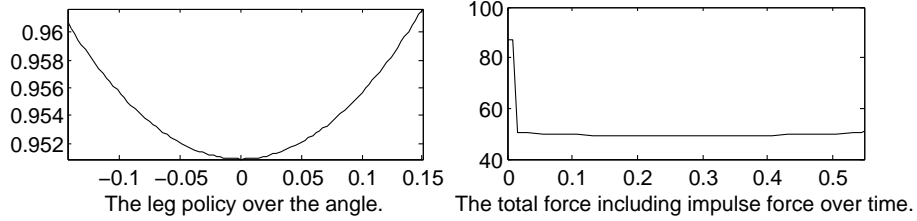


Figure 4: Minimal energy consumption on Linear Inverted Pendulum. The total energy consumption is 0.98

Optimal leg-length policy in the absence of a double support phase In the first series of experiment, we generated a large number of leg-length policies. We divided a step into eight intervals and chose a leg-length for the beginning (end) of each interval. Next we determined a polynomial for the leg-length policy through these points using the cubic spline method. We repeat the process to investigate all possible policies with a resolution of eight intervals, to find an optimal policy that minimizes the power usage.

Figure 3 shows that a slightly bended leg which is subsequently stretched to the maximum length results in the energy efficient gait. The optimal policy keeps the leg stretched during the remainder of the step. The energy consumption of this policy for one step is 0.011. Figure 3 also shows a relatively large peak force on the leg, which results from the impulse when leg hits the ground.

Figure 4 shows the energy consumption of the inverted pendulum model for one step. The total energy consumption is 0.98, which is 89 times more than the energy consumption of the optimal gait.

Optimal leg-length and force policy in the presence of a double support phase In the series of experiment, we generated a large number of leg-length and force policies. We divided the a step into eight intervals and chose a leg-length percentage for the beginning (end) of each interval. Next we determined a polynomial for the leg-length policy through these points using the cubic spline method. Furthermore, we divided the double support phase in to four intervals and chose a force percentage or the beginning (end) of each interval. Next we determined a polynomial for the force policy through these points using the cubic spline method. We repeat the process to investigate all possible combination of leg-length and force policies to find an optimal combination that minimizes the power usage.

The experiments showed that the optimal leg-length policy is the same as the leg-length policy for the gait without DSP, see Figure 3. The optimal force policy is the policy that sets the force generated by the swing leg to zero till the end of the DSP. This implies that the force generate by the swing leg in the DSP will only be determined by requirement of the sideway stability.

An implementation on a Nao humanoid robot We implemented the energy optimal gait on the Nao and compared the energy consumption of this new gait with the energy consumption of the standard gait of the Nao. The step size of the new gait was set to 6cm. We did not yet address the sideway stability and therefore used an ad hoc solution. We investigated two versions of the new gait. In the first version of the new gait the Nao walks 4.4 times faster than the standard gait. However, the sideway stability is extremely problematic. Next we implemented another and more stable version of the new gait in which the Nao walks only 1.5 times faster than the standard gait. Table 1 shows the energy consumption of the knee joints, the ankle joints and the total energy consumption of the standard gait and the two versions of the new gait. The table shows that the new fast gait reduces the energy consumption of the knee joints with 25%, the energy consumption of the ankle joints with 29%, and the total energy with 27%. Note that the ankle joint still consumes energy in the new gait. For stability issues it was necessary to execute some force with the ankle joints in the SSP. The

energy consumption of the new slow gaits is 18.7% higher than fast gait, but still reduces the total energy consumption with 11.1% w.r.t. the standard gait. A video of the experiment showing the new slow gait can be found on Youtube:<http://youtu.be/TaBaENZRo2s>.

gait	energy knee joint	energy ankle joint	total energy	walking speed (m/s)
new gait (fast)	0.5292	0.3565	0.8857	0.075
new gait (slow)	0.6013	0.4211	1.089	0.025
standard gait	0.7091	0.5011	1.2102	0.017

Table 1: The average energy consumption walking speed of the new and the standard gait of the Nao robot.

5 Conclusion

We introduced a refined inverted pendulum biped model with telescopic legs for humanoid robots such as a Nao robot. We used this model to identify a leg-length policy that minimizes the energy consumption of a gait. We compare the energy consumption of the energy optimal gait with the energy consumption of gait based on the linear inverted pendulum model. We also implemented the new gait on a Nao robot and compared the energy consumption with the energy consumption of the standard gait. In all cases, the new gait is more efficient. Moreover, the Nao walks almost five times faster with the new gait at full speed mode.

Future Work In our future work we will fine-tune the implementation of the new gait on the Nao. Moreover, we will apply a similar analysis as described in this paper for the sideway stability. Finally we will develop an adaptive controller for the gait on the sideway stability that learns to fine-tune its parameters and can cope with uneven surfaces.

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