RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 52, NUMBER 4 15 JULY 1995-II

# Ballistic conductance of composite fermions

A. Khaetskii,* Yu.V. Nazarov, and G.E.W. Bauer
*Faculty of Applied Physics and DIMES, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*

We consider ballistic transport of composite fermions in a realistic microconstriction. An unambiguous signature of such transport is revealed: the linear conductance increases with increasing magnetic field near filling factor $\frac{1}{2}$ and then drops abruptly in a narrow magnetic-field interval.

In recent days many theoretical and experimental papers have considered the properties of two-dimensional (2D) electron gas in strong magnetic fields near filling factor $\nu=1/2$ ($\nu=2\pi\lambda^2 n$, $\lambda=\sqrt{\hbar c/eB}$ is the magnetic length and $n$ the electron density). It was shown theoretically[1,2] that these properties can be well described in terms of composite fermions, which is equivalent to attaching a magnetic flux tube to each electron. Within the mean field approach the average effective (external plus internal) magnetic field acting on the composite fermions is

$$\Delta B=B-2\Phi_0 n, \tag{1}$$

$\Phi_0$ being the flux quantum. For filling factor $\nu=1/2$ the average effective magnetic field vanishes and the ground state of the system is a filled Fermi sea of composite fermions. The existence of a composite-fermion Fermi surface was recently confirmed by several convincing experiments.[3]

Ballistic transport of *normal electrons* has been widely investigated using quantum point contacts (QPC). This is a tunable microconstriction created in 2D electron gas by application of a negative voltage to a split gate (see Fig. 1). The conductance of these point contacts varies in a steplike manner as function of applied gate voltage if electrons transit the constriction region ballistically.[4] Applying a magnetic field, one can approach a filling factor 1/2 where the transport through the constriction is expected to be due to *composite fermions*. In this work we reveal the characteristic properties of ballistic transport of these quasiparticles.

We calculate the linear two-terminal conductance of a microconstriction of 2D electron gas near $\nu=1/2$. It should be noted that calculations of the ballistic conductance of a microconstriction in terms of uncorrelated electrons (we refer to these as normal electrons) were given in Ref. 5. The result is that the ballistic conductance in strong magnetic fields is determined by the filling factor at the saddle point of the electron-density distribution and is independent of the width of the constriction being always of the order of the conductance quantum for the magnetic fields under consideration. For ballistic transport of composite fermions we find completely different results. In contrast to the results of Ref. 5, the magnetoconductance has a striking nonmonotonic and abrupt behavior near filling factor 1/2 (see Fig. 2). For a constriction much wider than the magnetic length the maximum conductance is much larger than the conductance quantum. Comparison of our results with experimental data should unambiguously identify the nature of transport through a constriction near $\nu=1/2$, thus distinguishing between ballistic transport of electrons and composite fermions.[6]

We concentrate on the case of small depletion, assuming that the depletion length $l_{\text{dep}}$, being much larger than $\lambda$, is much smaller than the width $2d(0)$ of the constriction: $l_{\text{dep}} \ll d(0)$. In this limit, the electrostatically induced deple-

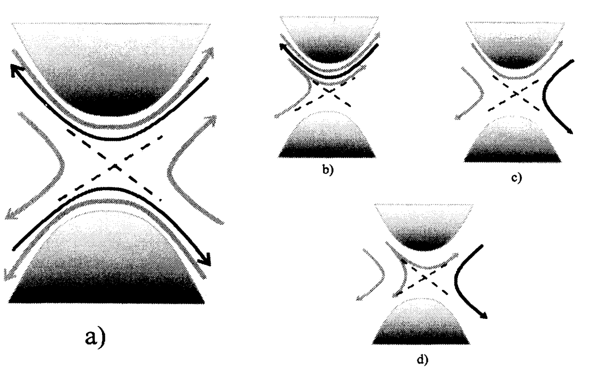


FIG. 1. The geometry of the structure considered. Black and gray solid curves with arrows show, respectively, sinusoidal and drift trajectories of composite fermions for different values of external magnetic field (a)–(d) near filling factor 1/2. The rest of the curves in (b)–(d) can be obtained by inversion with respect to the saddle point. Dashed lines denote the $y(0,0)$ curves where the density has the saddle point value.

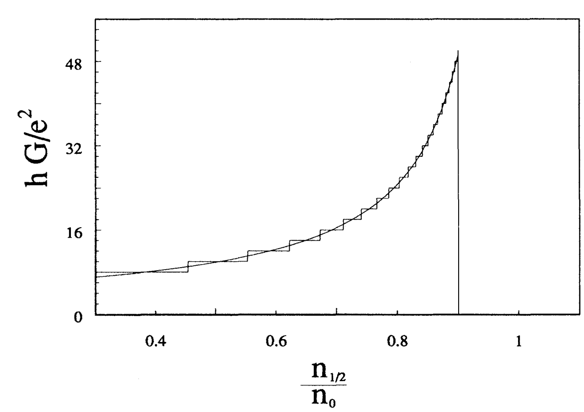


FIG. 2. Dependence of the conductance of microconstriction vs magnetic field described by Eq. (7) (smooth curve). Steplike curve describes the conductance quantization. The parameters are $\lambda=80$ Å, $d(0)=1$ $\mu$m, $l_{\text{dep}}=1000$ Å. The abrupt drop corresponds to the condition $n_{1/2}=n(0,0)$.

 

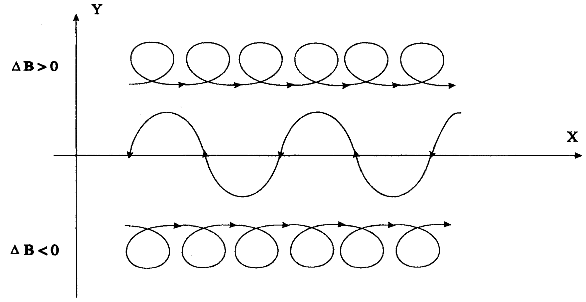


FIG. 3. Sinusoidal and drift trajectories of composite fermions. Adopted from Ref. 9.

tion of the electron density is small. Surprisingly, the effective magnetic field produced by this density deviation is strong enough to localize the propagating modes of composite fermions (CF's) near the lines $y_{1/2}$ of zero effective magnetic field. Indeed, the fermion propagates only if its cyclotron radius $r_c \approx \Phi_0/B(\Delta y)\lambda$ exceeds the distance $\Delta y$ from the line $y_{1/2}$ (Ref. 7). Equating this we obtain $\Delta y \approx \sqrt{k_F/n'}$, $n'$ being the density gradient at $y_{1/2}$. Electrostatics gives us a typical value of $n' \approx l_{\rm dep}/\lambda^2 d(0)^2$, thus $\Delta y \approx d(0)\sqrt{\lambda/l_{\rm dep}}$. Since $\lambda \ll l_{\rm dep}$, $\Delta y \ll d(0)$ and all propagating modes are concentrated in a narrow channel at $y_{1/2}$. There are two such channels in the constriction. Each channel is similar to a common constriction for normal electrons without magnetic field containing the equal number of modes propagating from the left to the right and vice versa. The conductance is simply given by the number of modes. By changing the external magnetic field, we can move the line $y_{1/2}$ in the constriction and change the number of propagating modes. We will see below that this increases the conductance with increasing magnetic field. When the filling factor in the center of the constriction becomes smaller than 1/2 [$n_{1/2} > n(0,0)$, $n_{1/2} = 1/4\pi\lambda^2$, $n(0,0)$ being the electron density at the saddle point], the lines $y_{1/2}$ do not pass through the constriction but turn back to the same lead. The number of the propagating modes drops abruptly, and the conductance vanishes. This is reminiscent of the phenomenon predicted in Ref. 8.

Let us consider first an auxiliary problem of a straight channel where the electron density depends only on the transverse ($y$) coordinate. As mentioned above, the conductance is mainly determined by the region near line $y_{1/2}$, where the effective magnetic field is zero and $\nu = 1/2$. The width of this region is much smaller than the channel width, and we approximate the effective magnetic field near the line $y_{1/2}$ as follows:

$$\Delta B = 2\Phi_0 n'(y - y_{1/2}), \quad n' = \left.\frac{dn}{dy}\right|_{y=y_{1/2}}. \tag{2}$$

The Schrödinger equation for CF's reads

$$\frac{\hbar^2}{2}\frac{d^2\chi(y)}{dy^2} + \left[E - \frac{\hbar^2}{2}(k - 2\pi n' y^2)^2\right]\chi(y) = 0, \tag{3}$$

$k$ being the wave vector along the $x$ axis. Equation (3) has been discussed in Ref. 9 (see also Ref. 7). It was revealed that there are two different types of states which cross the Fermi energy and have opposite velocities $\partial E/\hbar\partial k$ in the $x$ direction (see Fig. 3). The states of the first type are localized near the line $y_{1/2}$ and have negative velocities. The others having positive velocities are spatially separated from the line $y_{1/2}$ and drift due to the gradient of the magnetic field. They come in pairs corresponding to the possibilities of being on the left and on the right of $y = y_{1/2}$. In order to calculate the conductance of the system, we need to know the total number $N_t$ of subbands below the Fermi energy. We have solved Eq. (3) semiclassically, obtaining this number with the help of the Bohr-Sommerfeld quantization rule. It reads

$$N_t = \left[1.85\frac{n_0^{3/4}}{(n')^{1/2}} + \frac{1}{2}\right], \tag{4}$$

where $n_0$ is the value of background density and the square brackets denote the integer part of the number. Since we are interested in the case of small depletion, we have not distinguished between $n_0$ and $n_{1/2}$ in Eq. (4) and used the following relation between the Fermi wave vector and density: $k_F^2 = 4\pi n_0$, which holds for the spin-polarized system. Since $N_t \gg 1$, in the following we will ignore the term 1/2 in Eq. (4) compared to the first one, except for the cases when we discuss the conductance quantization.

For a symmetric contact we have two identical lines of zero effective magnetic field that doubles the conductance

$$G = 2N_t\frac{e^2}{2\pi\hbar}. \tag{5}$$

Straight channel geometry is not realistic. In reality, experiments on ballistic transport are carried out with point contacts similar to the one shown in Fig. 1. We show now that the results obtained above can be immediately applied to a realistic constriction where the electron density depends on both coordinates. Indeed, as it follows from Eq. (4), the conductance is determined by the local value of the density derivative at the line of zero effective magnetic field. The value of the latter quantity far from the constriction (at infinity) is always larger than its value in the central section ($x=0$). This means that the number of transverse modes of CF's far from the saddle point is smaller than in the central section and the ballistic conductance of the system is determined by the number of modes at infinity. As a result, the conductance of a sufficiently smooth constriction does not depend on its shape. This situation is in sharp contrast to the common one.[4] We calculate now the density derivative entering Eq. (4) far from the saddle point (at infinity) where line $y_{1/2}$ lies much closer to one of the boundaries than to the other. The electron-density deficit (due to an applied negative voltage at the split gate) at large (compared to the depletion length) distance $s$ from the boundary is[10]

$$\delta n(x,y) = n_0 - n(x,y) = n_0\frac{l_{\rm dep}}{2s}. \tag{6}$$

At the curve $y_{1/2}$ where $\delta n_{1/2} = n_0 - n_{1/2}$ we find for the derivative of the density $\delta n'(y = y_{1/2}) = 2\delta n_{1/2}^2/n_0 l_{\rm dep}$, which is independent of the constriction shape. It determines the conductance of the system:

$$G(n_0-n_{1/2})=\left(\frac{e^2}{2\pi\hbar}\right)2.62\,\frac{n_0\sqrt{n_0^{1/2}l_{\text{dep}}}}{(n_0-n_{1/2})}\,. \tag{7}$$

Equation (7) is valid under the condition: $n_0-n_{1/2}\ll n_0$ which guarantees that the line $y_{1/2}$ lies at distances from the boundaries which are much larger than the depletion length.

The conductance can be calculated in the more general case when $n_0-n_{1/2}\simeq n_0$ and the separation of the line $y_{1/2}$ from the boundary is of the order of the depletion length. It reads

$$G(n_0-n_{1/2})=\left(\frac{e^2}{2\pi\hbar}\right)5.24\,\frac{z^{5/4}\sqrt{n_0^{1/2}l_{\text{dep}}}}{(1-z^2)}\,,\qquad z=n_{1/2}/n_0\,. \tag{8}$$

When deriving Eq. (8), we substituted $n_{1/2}$ for $n_0$ in Eq. (4) and in the calculation of the density derivative we used the formula for the density profile obtained in Ref. 10.

The conductance increases when the filling factor in the bulk approaches 1/2 [see Eq. (7)] since the line $y_{1/2}$ lies in the region of an increasingly flat density profile. This is only valid if $n_{1/2}$ is smaller than the density value at the saddle point $n(0,0)$. When the filling factor at the saddle point becomes smaller than 1/2, the line $y_{1/2}$ cannot pass the bottleneck of the constriction and the conductance should decrease. The magnetic-field dependence of the conductance given by Eq. (7) is shown in Fig. 2. The maximum value of the conductance is equal to $G_{\max}\simeq(e^2/h)\times[d(0)/l_{\text{dep}}]\sqrt{n_0^{1/2}l_{\text{dep}}}\gg e^2/h$.

The dependence of the conductance in the narrow interval near $n(0,0)$ actually has a rich additional structure due to the fact that CF's trajectories of different types behave differently in the bottleneck region. Let us investigate this behavior near $\nu=1/2$ in more detail. We notice that not all the modes of CF's existing far from the saddle point can propagate through the constriction. Some of them will be reflected when $n_{1/2}$ approaches $n(0,0)$. This will first occur for drift trajectories which at infinity lie at the higher density side of the line $y_{1/2}$. When the drift trajectories move towards the bottleneck region their separation from the line $y_{1/2}$ increases since the density derivative $n'$ decreases. As a result, these trajectories have a small radius and drift along the lines of constant density in the bottleneck region. Those which lie on the higher density side of the line $y(0,0)$ will be reflected [see Fig. 1(b)]. [At the line $y(0,0)$ the density has the saddle point value $n(0,0)$.] The number $N_{\text{ref}}$ of reflected drift trajectories for a given value of $n_{1/2}$ can be found from

$$\frac{e\hbar}{c}2\Phi_0[n(0,0)-n_{1/2}]N_{\text{ref}}=\frac{\hbar^2k_F^2}{2}\,. \tag{9}$$

The number of transmitted modes equals $2N_t-N_{\text{ref}}$ and the conductance is described by the formula

$$G=\frac{e^2}{2\pi\hbar}\left\{2.62\frac{n_0\sqrt{n_0^{1/2}l_{\text{dep}}}}{n_0-n_{1/2}}-\frac{n_0}{2[n(0,0)-n_{1/2}]}\right\}. \tag{10}$$

The dependence given by Eq. (10) holds if the number of reflected modes is smaller than $N_t/2$ which is just the total number of the drift trajectories lying at infinity at the higher density side of the line $y_{1/2}$. This number is equal to a fourth

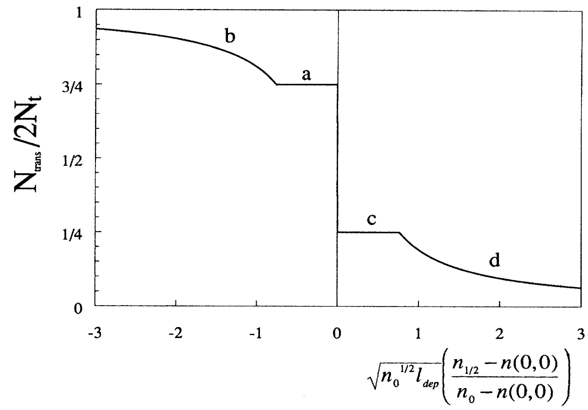


FIG. 4. The number of transmitted modes through the constriction divided by the total number of incoming CF modes as a function of magnetic field near filling factor 1/2. Different parts of this dependence (a)–(d) correspond to the situations shown in Fig. 1.

of the total number $2N_t$ of CF's, trajectories at infinity (see Fig. 3). The last drift trajectory coming from infinity is reflected at the magnetic-field value determined by

$$n(0,0)-n_{1/2}^{(1)}=0.76\frac{n_0-n(0,0)}{(n_0^{1/2}l_{\text{dep}})^{1/2}}\,. \tag{11}$$

Since a fourth of the total number of modes coming from infinity has been already reflected [the corresponding situation is shown in Fig. 1(a)], starting from this magnetic field, the ratio of the number of transmitted modes to the total number of incoming modes remains equal to 3/4 in the finite interval of magnetic fields. This interval corresponds to the first plateau in Fig. 4. The width of the plateau is given by Eq. (11).

With further increase of the external magnetic field we reach a situation when $n_{1/2}=n(0,0)$. For larger values of magnetic field the snake trajectories coming from infinity cannot propagate through the constriction and give no contribution to the conductance. The number of snake states at infinity is a half of the total number of CF's states. As a result, the conductance drops by the value $G_{\max}/2$ in a narrow interval near $n_{1/2}=n(0,0)$ (see Fig. 4). The width of this interval can be estimated as

$$n(0,0)-n_{1/2}^{(2)}\simeq\frac{n_0-n(0,0)}{(n_0^{1/2}l_{\text{dep}})^{4/5}} \tag{12}$$

and therefore is much smaller than the plateau width Eq.(11). With further increase of the magnetic field $[n_{1/2}>n(0,0)]$ only the drift trajectories, which at infinity lie at the low density side of the $y_{1/2}$ curve, will propagate through the constriction [see Fig. 1(c)]. Again, in the finite interval of the magnetic fields the ratio of transmitted to incoming modes is constant (1/4) which corresponds to the second plateau in Fig. 4. The width of this second plateau coincides with the width of the first given by Eq. (11). For larger values of the magnetic field these propagating drift trajectories will also be reflected [Fig. 1(d)]. It is easy to understand that the number $N_{\text{trans}}$ of the modes which can transit through the constriction is given by Eq. (9) with the substitution $N_{\rm trans}$ for $N_{\rm ref}$. As a result, we obtain for the conductance in this magnetic-field regime

$$G=\left(\frac{e^2}{2\pi\hbar}\right)\frac{n_0}{2[n_{1/2}-n(0,0)]}\,. \qquad (13)$$

Our results are formally valid in the limit of large values $n_0^{1/2}l_{\rm dep}$. In this case, the decrease of conductance is abrupt, as it is shown in Fig. 2. In reality, this parameter takes only moderately large values, $n_0^{1/2}l_{\rm dep}\simeq 3-5$. The conductance is still given by Eqs. (10,13) but the decreasing part of the curve is smooth and the maximum value is decreased. Another restriction on the validity of the results arises from the fact that the probability of transitions (due to finite curvature radius) between different adiabatic modes of composite fermions having opposite velocities (see Fig. 3) must be small. This requires $k_F l_{\rm dep}\gg 1$. This holds independently of the constriction shape.

In conclusion, we have predicted the characteristic ballistic transport properties of composite fermions. The dependence of the conductance of microconstriction versus magnetic field is strongly nonmonotonous around filling factor 1/2. This dependence is specific for composite fermions and can be used to identify these quasiparticles.

We thank C.W.J. Beenakker and A.K. Geim for discussion of the results and D.B. Chklovskii who drew our attention to Ref. 7. This work is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie (FOM)."

*Permanent address: Institute of Microelectronics Technology, Russian Academy of Sciences, 142432 Chernogolovka, Moscow District, Russia.

[1] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[2] V. Kalmeyer and S. C. Zhang, Phys. Rev. B **46**, 9889 (1992).

[3] R. L. Willett *et al.*, Phys. Rev. Lett. **71**, 3846 (1993); W. Kang *et al.*, *ibid.* **71**, 3850 (1993); V.J. Goldman *et al.*, *ibid.* **72**, 2065 (1994).

[4] B. J. van Wees *et al.*, Phys. Rev. Lett. **60**, 848 (1988); D. A. Wharam *et al.*, J. Phys. C **21**, L209 (1988); L. I. Glazman *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 238 (1988)].

[5] D. B. Chklovskii, K. A. Matveev, and B. I. Shklovskii, Phys. Rev. B **47**, 12 605 (1993).

[6] We assume also that even if the contacts supply the normal electrons, there is an effective way of transforming normal electrons into composite fermions.

[7] D. B. Chklovskii and P. A. Lee, Phys. Rev. B **48**, 18 060 (1993).

[8] A. Khaetskii and G. E. W. Bauer (unpublished); Phys. Rev. B **51**, 7369 (1995); L. Brey and C. Tejedor (unpublished).

[9] J. E. Müller, Phys. Rev. Lett. **68**, 385 (1992). See also R. I. Shekhter and A. S. Rozhavskii, Zh. Eksp. Teor. Fiz. **65**, 772 (1974) [Sov. Phys. JETP **38**, 383 (1974)]; V. L. Fal'ko and F. G. Bass, Fiz. Nizk. Temp. **6**, 60 (1981) [Sov. J. Low Temp. Phys. **6**, 29 (1980)].

[10] D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B **46**, 4026 (1992).

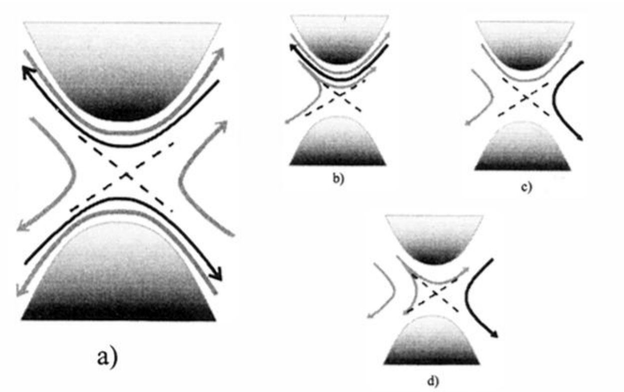


FIG. 1. The geometry of the structure considered. Black and gray solid curves with arrows show, respectively, sinusoidal and drift trajectories of composite fermions for different values of external magnetic field (a)–(d) near filling factor 1/2. The rest of the curves in (b)–(d) can be obtained by inversion with respect to the saddle point. Dashed lines denote the $y(0,0)$ curves where the density has the saddle point value.