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Absolute distance measurement with a gain-switched dual optical frequency comb

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Abstract: The measurement of distance plays an important role in many aspects of modern societies. In this paper, an absolute distance measurement method for arbitrary distance is proposed and demonstrated using mode-resolved spectral interferometry with a gain-switched dual comb. An accuracy of 12 µm, when compared to a He-Ne fringe counting laser interferometer, for a displacement up to 2.5 m is demonstrated by tuning the repetition frequency of the dual comb from 1.1 GHz to 1.4 GHz. The compact measurement system based on a gain-switched dual comb breaks the constraint of periodic ambiguity. The simplification and improvements are significant for further industrial applications.

1. Introduction

With the rapid development of science and technology, distance measurement plays an important role in modern life, especially in monitoring tolerances in industrial manufacturing. Time-of-flight method using laser pulses is the most direct way to measure absolute distances. However, the limited electrical bandwidth of the available high-speed photoreceivers restricts the detection resolution, between the optical pulses, to picoseconds (ps). The latter limits the detection resolution to the millimeter range [1]. Laser based interferometric techniques overcome this restriction by measuring the phase difference between the interference light. Among these techniques, the methods based on the optical frequency comb (OFC) are developing rapidly. These methods use a stabilized mode locked laser (MLL) as the OFC source and can measure long distances with extremely high accuracies [2–4]. The frequency comb was firstly applied in distance measurement by Minoshima, where the phase of the intermode beats of the frequency comb were measured to determine the distance [5]. Cross correlation measurements based on inter-pulse interference in a Michelson interferometer came thereafter [6,7]. In order to satisfy the industrial demands on the measurement precision, dispersive interferometry [8,9], spectrally resolved interferometry [3,10], time-of-flight measurement [11] and dual comb interferometry [4,12,13] were subsequently proposed and demonstrated .

Distance measurement, based on spectrally resolved interferometry, realized accurate and long distance measurements with a single frequency comb [3,10]. Here, the frequency comb is used as a multiple wavelength laser source that is fed to the input of a Michelson interferometer. The individual modes superpose after travelling from the two arms. The interference pattern is then spectrally resolved by a virtually imaged phase array (VIPA) spectrometer. The difference in distance between the two arms can be obtained after unravelling the interference pattern and extracting the phase changes. One of the main constraints in using this technique for industrial

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and outdoor measurements is the complex instrumentation. The VIPA spectrometer needs space, careful alignment and has many components. It is not suitable for low repetition rate combs due to its resolution limitation. Simplification of the instrumentation has been demonstrated by replacing the VIPA with a single grating, while using a Fabry-Pèrot (FP) cavity to filter the frequency spectrum of the laser [14]. On the other hand FP cavities add their own complexity. Some other issues that this technique has is that when the distance is an integer multiple of the pulse-to-pulse distance L_{pp}, this method fails to yield a result. At these distances the solution is to tune the repetition frequency of the comb putting higher demands on the laser.

The dual frequency comb spectrometer, having no moving parts and being able to resolve the spectra of the frequency comb is quite promising for immediate applications. The technique uses a pair of OFCs with slight differences in their repetition frequencies. The signals from the two combs interfere on a photodetector and the beat spectra yields an RF comb, which maintains the spectral information encoded in the optical domain. The dual comb technique has gained popularity as it significantly reduces the complexity of the receiver and offers high precision, fast acquisition times and potentially low bandwidth receivers [15]. Typically, OFCs employed in dual comb spectroscopy (DCS) have utilised MLLs. However, the fabrication of these devices is often costly and complex, and due to the physical nature of the devices, they possess a fixed repetition frequency. For tunability of the repetition frequency, electro-optic modulators may be a viable alternative. However, this OFC generation technique requires cascaded [16] or dual drive modulators [17] to generate broad and flat OFCs. Along with this drawback, the technique will inherently suffer from instabilities due to bias drift [18]. To combat this bias drift, a dc bias control feedback may be required to ensure that OFCs remain stable [19].

The dual comb in this paper is generated by two cost-efficient commercially available lasers, which have been gain switched. The gain switching technique provides a simple and flexible method for the generation of OFCs, which allows for tuning of both the wavelength and the repetition frequencies of the OFCs [20]. Some drawbacks of the gain switching technique to generate OFCs are overcome by external optical injection locking (OIL) utilizing a maser-slave configuration. The spectral flatness and the number of generated comb tones are both improved. Furthermore, external OIL results in the effective transference of the master laser's narrow linewidth to each of the individual comb lines [21]. This transference of the master laser's narrow linewidth is utilised in the dual comb architecture presented here, where both individual OFCs are injection locked using a single tunable laser (TL). This provides phase and repetition frequency locking, resulting in high phase coherence between the OFCs.

In this work, we present a distance measurement based on spectrally resolved interferometry [10] with a gain-switched dual optical comb [22]. This laser features a repetition frequency that can be tuned over 300 MHz, between 1.1 GHz to 1.4 GHz. Based on the broad tuning range of the gain switched dual comb, an absolute distance measurement method for arbitrary distance is reported along with a comparison measurement with a fringe counting helium-neon (HeNe) laser interferometer.

2. Experimental setup

A schematic of the dual comb source and the absolute distance measurement setup is shown in Fig. 1. The dual comb architecture used in this work is shown in Fig. 1 on the left side. It comprises two Fabry-Pèrot (FP) lasers, acting as slave lasers in a dual master-slave configuration. Utilizing FP lasers allows for tunable dual comb generation across the entire C-band [21]. Both FPs are gain switched [20] using sine waves, amplified to 24 dBm, at frequencies of 1.25 (signal comb) and 1.251 GHz (local oscillator) respectively. The individual OFCs are mutually injection locked, using a single master semiconductor tunable laser (STL), which provides phase synchronization between both slave (FP) lasers. A typical RF beat tone spectra measured using an electrical spectrum analyzer are shown in Fig. 2. After the dual comb is generated, the signal

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comb is sent into a Michelson interferometer, where the laser beam is split into two arms, a reference arm and a measurement arm, and recombined. The interference beam is then combined with the LO comb using a 50/50 coupler and detected using a balanced detector. An RF spectrum analyser is used to resolve the comb and record the interference signal.

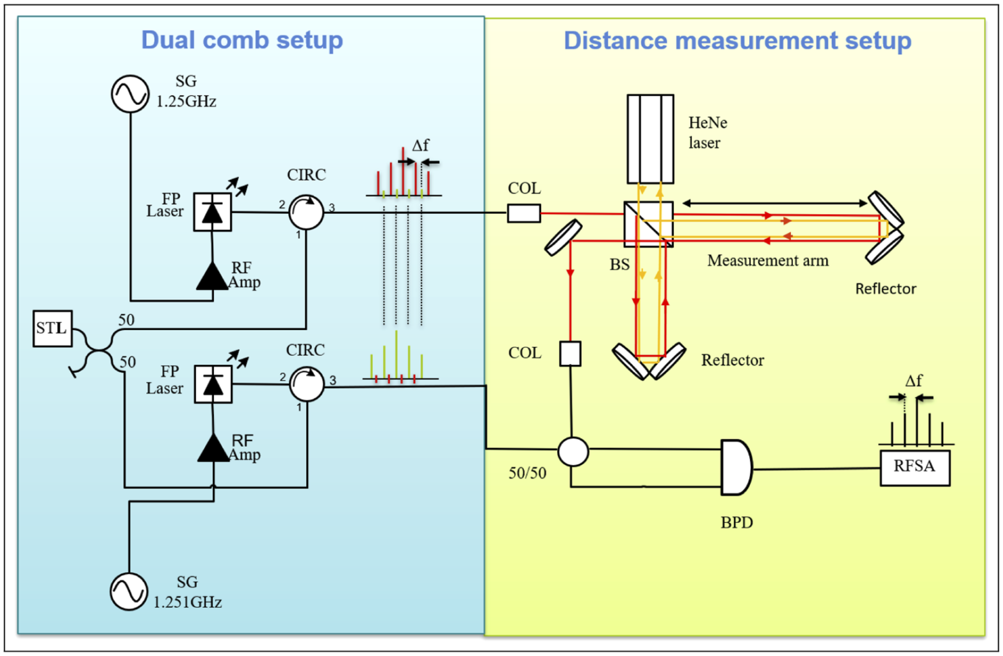

Fig. 1. Schematic overview of the measurement setup. The dual comb is generated by a semiconductor tunable laser (STL) and two Fabry Pèrot (FP) lasers. The FP lasers are gain switched by two RF signals from by two signal generators (SG) and amplified by two RF amplifiers (Amp). The signal comb (red), which is the output of an optical circulator (CIRC), is sent to a Michelson interferometer by optical fibers and a collimator (COL). The signal is sent to a balanced photodetector (BPD) by a 50/50 coupler after being split and recombined by the beam splitter (BS). The LO comb is directly sent to the BPD by optical fibers, a CIRC, and the same 50/50 coupler. The resulting baseband beat signals are recorded by an RF spectrum analyzer (RFSA). Both the HeNe laser (yellow line) and dual comb laser (red line) measure the displacement quasi-simultaneously.

One of the key components for the distance measurement setup is the Michelson interferometer. The measurement arm of the Michelson interferometer can be moved over a distance of 2.5 m with a motor driven carriage carrying the retroreflector. In order to verify the distance measurement ability, the reading of the HeNe displacement interferometer and frequency comb measurement are compared for each measurement point.

3. Measurement principles

3.1. *Spectral interferometry with a dual comb*

Mathematically, the reference comb, measurement comb and the LO comb could be described as below:

$$E_{mea} = \sum_m A_m \cos\left[2\pi(mf_{rep,sig} + f_{sig})t - \phi_m\right] \tag{1}$$

$$E_{ref} = \sum_m A_m \cos\left[2\pi(mf_{rep,sig} + f_{sig})t\right] \tag{2}$$

$$E_{LO} = \sum_m B_m \cos\left[2\pi(mf_{rep,LO} + f_{LO})t\right] \tag{3}$$

where E_{ref} and E_{mea} are the electric fields of the signal pulses reflected from the reference arm and the measurement arm, respectively, $f_{rep,sig}$ and f_{sig} are the repetition frequency and the frequency of the first mode of the comb, respectively. ϕ is the phase difference between the reference arm and the measurement arm. E_{LO} represents the electric field of the local oscillator (LO) pulse, where the repetition rate and the frequency of the first mode of the comb are $f_{rep,LO}$ and f_{LO}. There is a slight difference between $f_{rep,sig}$ and $f_{rep,LO}$, which is represented by Δf_{rep}. The signal pulse and the LO pulse are slaves to the same master oscillator, thus f_{sig} and f_{LO} are equal [22]. Therefore the output intensity is $|(E_{mea} + E_{ref}) + E_{LO}|^2$. The detection is done using a balanced detector so the DC terms of the intensity are canceled, and only the correlative terms $(E_{mea} + E_{ref})E_{LO}$ remain. The output intensity can be written as:

$$I_{out} = \sum_m 2A_m B_m \cos\frac{\phi_m}{2} \cos\left[2\pi(mf_{rep,sig} + f_{sig})t - \frac{\phi_m}{2}\right] \cos\left[2\pi(mf_{rep,LO} + f_{LO})t\right] \tag{4}$$

The limited bandwidth of the detector ensures that the high frequency components are filtered out. The intensity without the high frequency part can be written as:

$$I_{out} = \sum_m I_m = \sum_m A_m B_m \cos\frac{\phi_m}{2} \cos(2\pi m \Delta f_{rep} t - \frac{\phi_m}{2}) \tag{5}$$

Here I_m is the intensity of the single frequency mode. A spectrum analyzer is used to analyse the output signal. The power of RF signals with different frequencies are extracted separately. The measurement with spectrum analyzer improves the signal to noise ratio and is similar to using lock-in detection [23]. This is in contrast to the fast Fourier transform method [4] which cannot be used here since the repetition frequency needs to be swept continuously in order to realize absolute arbitrary distance measurement and a frequency change during a sampling period would lead to error. Using a spectrum analyzer precludes these errors due to the frequency change. The output voltage can be written as:

$$P_m = \frac{(I_m \Re)^2}{r} = {A_m}^2 {B_m}^2 \Re^2 (\frac{1}{2} - \cos\phi_m). \tag{6}$$

Here $\Re$ is responsivity of the photodiode and r is the RF input impedance of the spectrum analyzer. Once the power spectrum is obtained, the signal of the spectrum is filtered with FFT filter, which can remove the DC terms [24]. The phase of the respective frequency comb modes can then be extracted by the Hilbert transform [25], and can be written as:

$$\phi_m = \frac{4\pi L(mf_{rep,sig} + f_{sig})n}{c}, \tag{7}$$

with L the distance difference between the reference arm and measurement arm. Here c is the speed of light in vacuum and n is the air refractive index. The derivative of phase with respect to m can be written as:

$$\frac{d\phi}{dm} = \frac{4\pi L f_{rep,sig}}{c}\left(n + (mf_{rep,sig} + f_{sig})\frac{dn}{d(mf_{rep,sig} + f_{sig})}\right) = \frac{4\pi L f_{rep,sig}}{c} \mathrm{n_g}, \tag{8}$$

with n_g the group refractive index. The distance can be written as:

$$L = \frac{d\phi}{dm}\frac{c}{4\pi f_{rep,sig} n_g}. \tag{9}$$

After unwrapping the phases of each mode, a simple linear fit of the phases gives the slope $K = \frac{d\phi}{dm}$. Plots of a typical experimental data, at different stages of the data processing procedure described above, is shown in Fig. 3.

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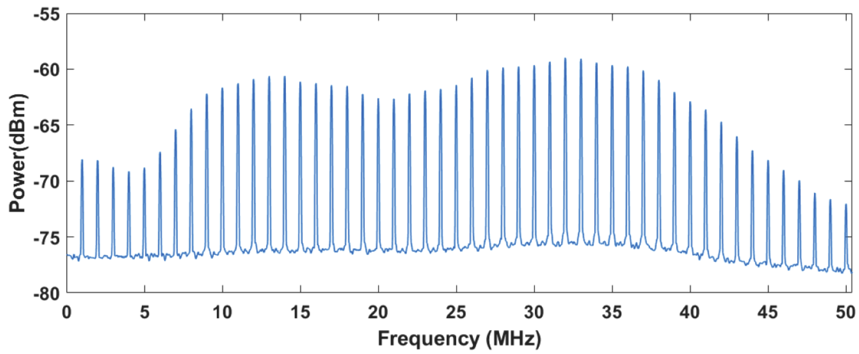

Fig. 2. RF beat tone spectra measured using an electrical spectrum analyzer.

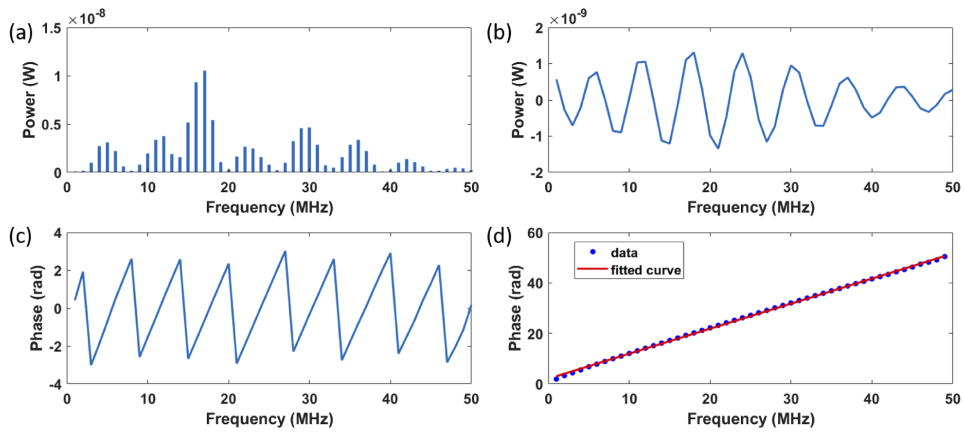

Fig. 3. Data processing procedure to obtain distance L; (a) typical interference spectrum captured by the RF spectrum analyzer, (b) interference spectrum filtered by FFT and inverse FFT method, (c) the wrapped phase obtained by Hilbert transform and (d) the unwrapped phase (blue points) and linear fit result (red line).

3.2. Absolute distance measurement

We define L_{pp} as the pulse-to-pulse distance in the medium, which is written as:

$$L_{pp} = \frac{c}{f_{rep,sig} n_g}. \tag{10}$$

When the measured distance is less than $\frac{1}{4}L_{pp}$, the phase is determined with no ambiguity. If the distance being measured is between $\frac{1}{4}L_{pp}$ and $\frac{1}{2}L_{pp}$, oversampling occur in which case $\frac{d\phi}{dm} = 2\pi - K$ and L can be written as:

$$L = (2\pi - K)\frac{c}{4\pi f_{rep,sig} n_g}. \tag{11}$$

When L is larger than $\frac{1}{2}L_{pp}$, the interference patterns repeat for each $\frac{1}{2}L_{pp}$. We define $L0$ = L mod L_{pp}. The arbitrary distance can be written as:

$$L = \begin{cases} (2N\pi + K)\dfrac{c}{4\pi f_{rep,sig} n_g} & L_0 <= \dfrac{1}{2}L_{pp} \\ (2N\pi - K)\dfrac{c}{4\pi f_{rep,sig} n_g} & L_0 > \dfrac{1}{2}L_{pp} \end{cases}. \tag{12}$$

In order to realise an absolute distance measurement, N must be determined properly. Here we use repetition frequency tuning method, which can be easily implemented with the gain-switched frequency comb. After scanning over a certain range of repetition frequencies, a series of K values for different frequencies is obtained. The slope $C = \frac{dK}{df_{rep,sig}}$ is determined with a simple linear fit. A typical sweeping result from 1.1 GHz to 1.4 GHz is shown in Fig. 4. In order to avoid aliasing, the repetition frequency of both of the lasers are tuned simultaneously to keep the frequency difference equal to 1 MHz. After the C is calculated, the previous measurement of the distance L_r is written as:

$$L_r = |C|\frac{c}{4\pi n_g}, \tag{13}$$

and the integer number N is calculated as:

$$N = round(\frac{L_r}{L_{pp}}). \tag{14}$$

After N and K are determined, the absolute distance can be calculated by Eq. (12).

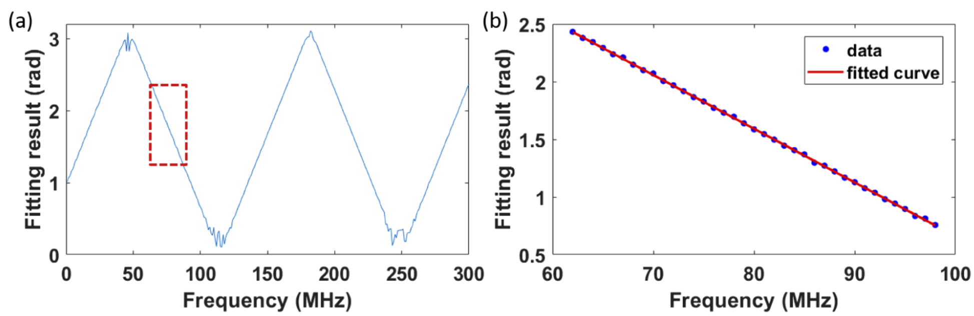

Fig. 4. (a) A typical sweeping result from 1.1 GHz to 1.4 GHz. (b) The zoom-in figure shows the fitting result, which is used to calculate the absolute distance and determine the integer number N.

3.3. Arbitrary distance measurement

It is obvious from Fig. 4 that the fitting results are not correct when K is near zero or π. This problem is also mentioned in [10,24]. It means that when L is equal to a multiple of $\frac{1}{4}L_{pp}$, a fixed repetition frequency cannot meet measurement requirements. It also means that the measurement accuracy is influenced by distance. In order to realize arbitrary distance measurement, we use Eq. (12) at the points, at which K is near π, to calculate the distance. After tuning the repetition frequency over a certain bandwidth we can always find these points and use the interference patterns with a similar period to calculate the distance. This makes the accuracy independent of the distance when the influence of refractive index is ignored. It should be noted that in order to make sure there are enough suitable points for measurement the tuning bandwidth should be larger than $\frac{c}{L}$. Tuning bandwidth should be larger than the period of the fitting result, to ensure that the points, at which K is close to π, can be found.

4. Results and discussion

In the experiment, the distances were measured simultaneously with our dual comb system and a fringe counting HeNe interferometer for comparison. Firstly, the carriage was positioned at a distance which is 1 meter longer than the reference arm. This is to ensure that there are enough suitable points for measurement. Subsequently the carriage was moved to 0.5 m. This procedure was repeated for other distances, from 1 m to 2.5 m in steps of 0.5 m. At every position, five measurements were recorded. The environmental conditions (temperature, air pressure and humidity) were recorded simultaneously, and were used for the refractive index calculation using the Edlén Equation [26].

For each repetition frequency around 50 modes are used to calculate the slope K. After tuning the frequency from 1.1 GHz to 1.4 GHz with a step of 1 MHz, about 100 points are used to estimate the final distance with a normal estimation. Figure 5 shows the outcome of this analysis. For each individual measurement the agreement between the dual comb and the HeNe interferometer is within 26 µm. When averaged over five measurements, the largest difference is 12 µm. The standard deviation doesn't show an obvious distance dependence and is on average 6 µm.

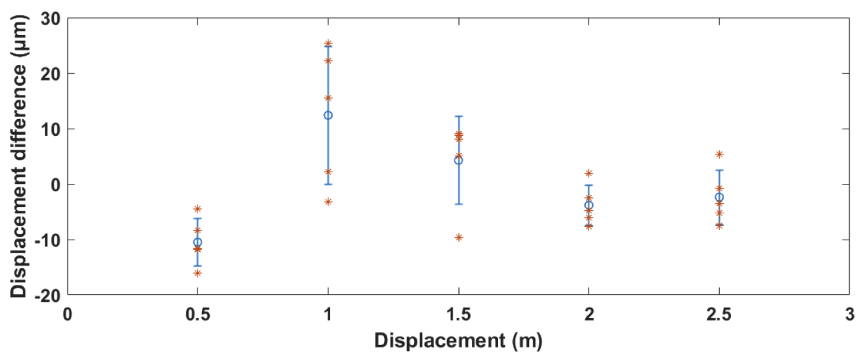

Fig. 5. The differences between distance measurements taken using a dual comb laser and a HeNe laser interferometry for displacement up to 2.5 m. The error bars show the standard deviation of the measurements.

The linear fitting uncertainty of K is the main limitation for accurate distance measurement. This error mostly arises from the limited number of modes available from the gain switched dual comb. For the dual comb used in this work the spectral bandwidth is around 62.5 GHz (50 modes) in comparison to the measurement with several thousands of modes [10] using a Ti:Sapphire laser. Reducing the repetition frequency of the dual comb is also a method to increase the number of modes, but this leads to a larger L_{pp}, which in turn reduces the accuracy of the measurement. A smaller L_{pp} means using a finer ruler to measure the distance. Therefore, the best way to improve the accuracy is to extend the bandwidth of the frequency comb. A method of extending the bandwidth of a single gain-switched optical frequency comb has been published [27]. However, this method needs to be further researched to improve the bandwidth of the dual comb.

5. Conclusion

An absolute distance measurement method for arbitrary distance was presented utilising a discrete component simple cost-efficient dual comb architecture. A comparison measurement with a fringe counting helium-neon (HeNe) laser interferometer has also been implemented. The technique demonstrated absolute arbitrary distance measurement by tuning the repetition frequency of the laser from 1.1 GHz to 1.4 GHz and achieved an accuracy of 12 µm when compared to a

He-Ne fringe counting laser interferometer, for a displacement up to 2.5 m. Further improvement on spectral width of the laser source will allow the technique to reach the accuracy obtained with traditional frequency comb sources. This architecture has been photonically integrated [22]. Integration of the required components on to a single chip not only reduces the physical footprint of the device, but simplifies coupling and removes polarisation dependencies. This results in greater device stability and in turn can lead to improvements in the absolute distance measurements.

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