On the Effectiveness of Boundary Dampers for Strings or Beams

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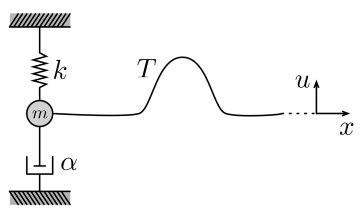
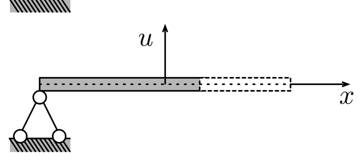
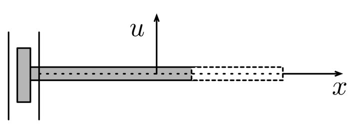
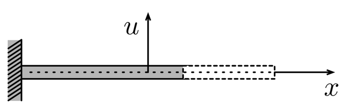
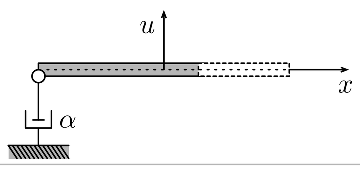
Introduction

Due to low structural damping of a bridge, a wind-field containing raindrops may excite a galloping type of vibrations. For instance, the Erasmus bridge in Rotterdam (Photo: courtesy of Massimo Catarinella) started to swing under mild wind conditions, shortly after it was opened to traffic in 1996. To suppress the undesired oscillations of the bridge, dampers were installed as can be seen in the following (Photo: courtesy of TU Delft)

Aim: understanding of how effective boundary damping is for string and beam equations.

Schematic Models

We consider the following boundary conditions for strings (s) and beams (b):

Type of system	Left end condition	B.C. at $x = 0$
Mass-spring-dashpot (s)		$mu_{tt} = Tu_x - ku - \alpha u_t$
Pinned (b)		$u = 0, \quad u_{xx} = 0$
Sliding (b)		$u_x = 0, \quad u_{xxx} = 0$
Clamped (b)		$u = 0, \quad u_x = 0$
Damper (b)		$u_{xxx} = \alpha u_t, \quad u_{xx} = 0$

Mathematical models

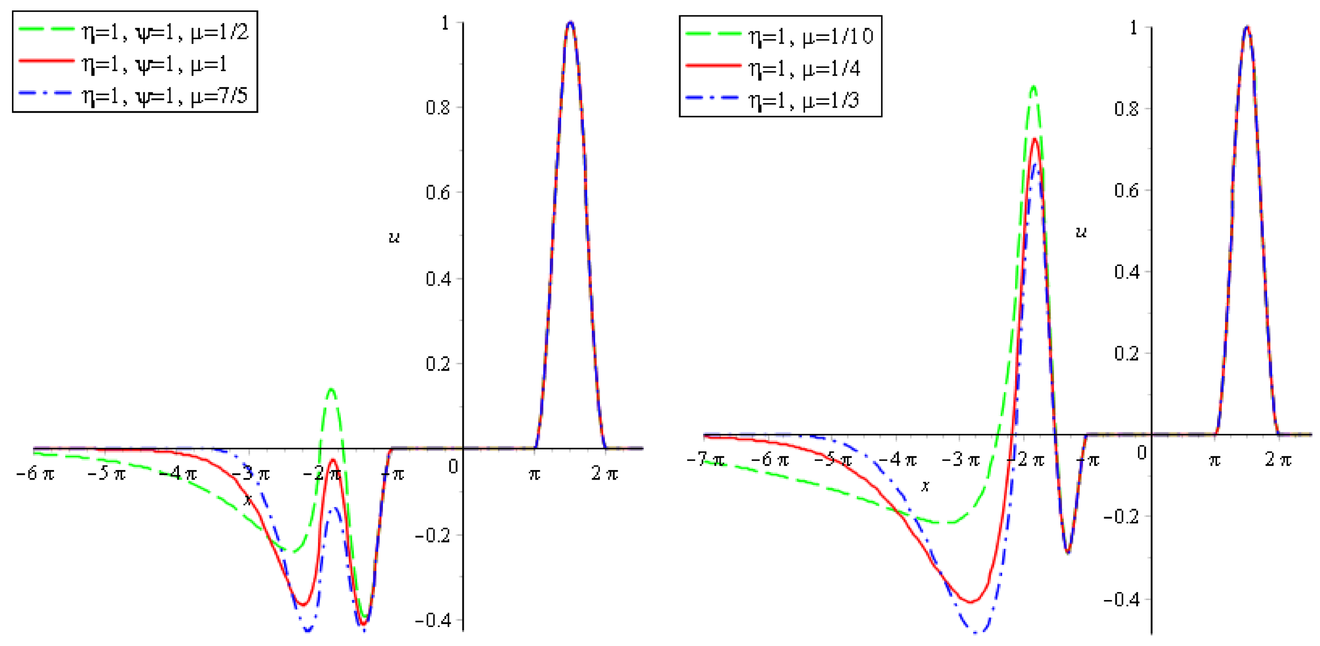
String-like problem: The D'Alembert method

$$u_{tt} - c^2\, u_{xx} = 0, \quad 0 < x < \infty, \quad t > 0, \qquad (1)$$

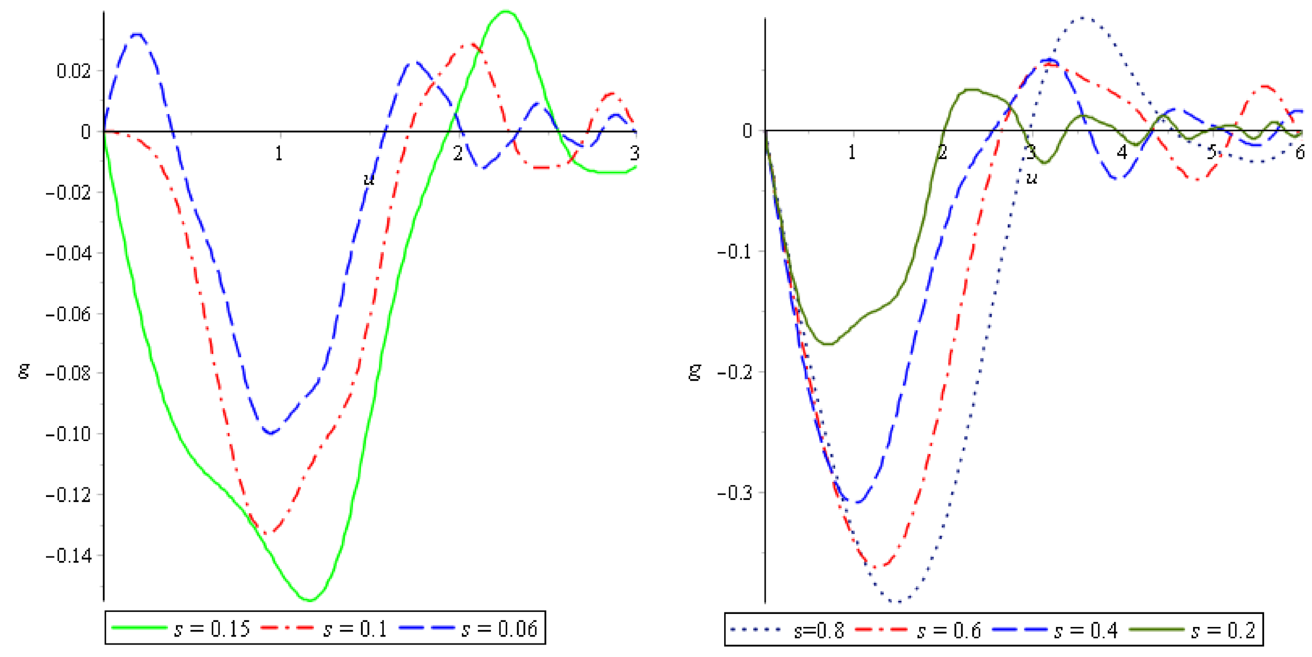
Beam-like problem: The method of Laplace tranforms

$$u_{tt} + a^2\, u_{xxxx} = 0, \quad 0 < x < \infty, \quad t > 0, \qquad (2)$$

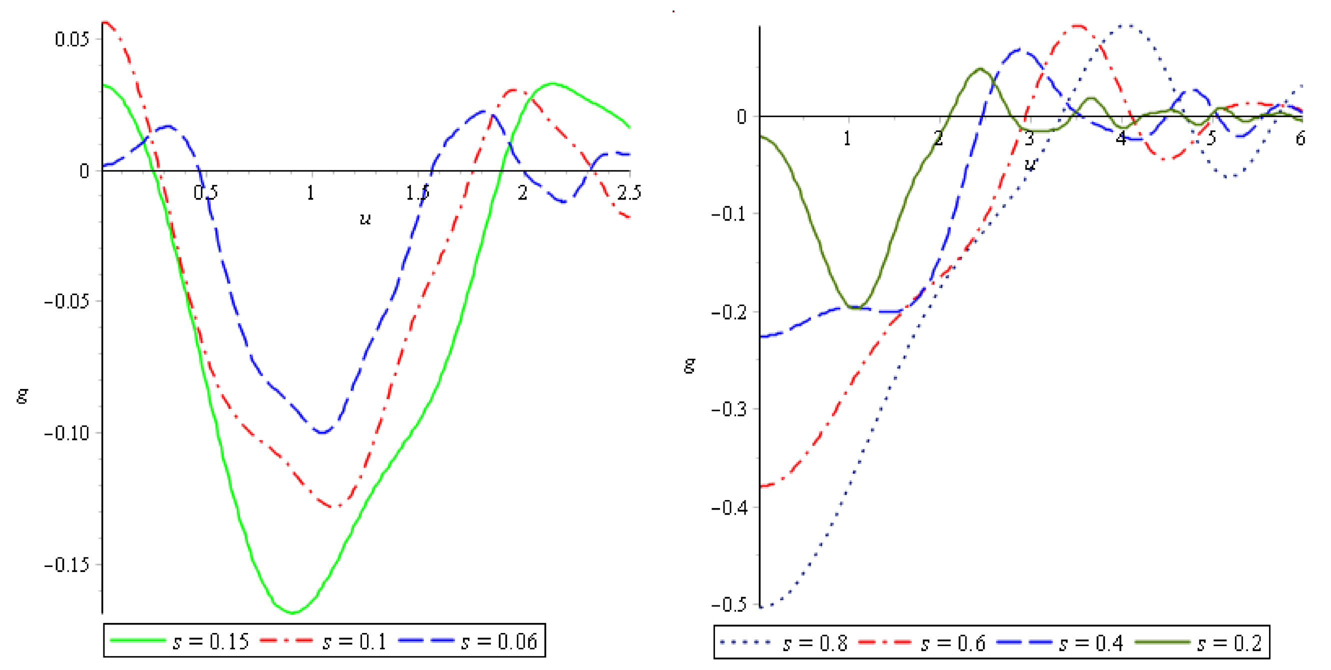
Results



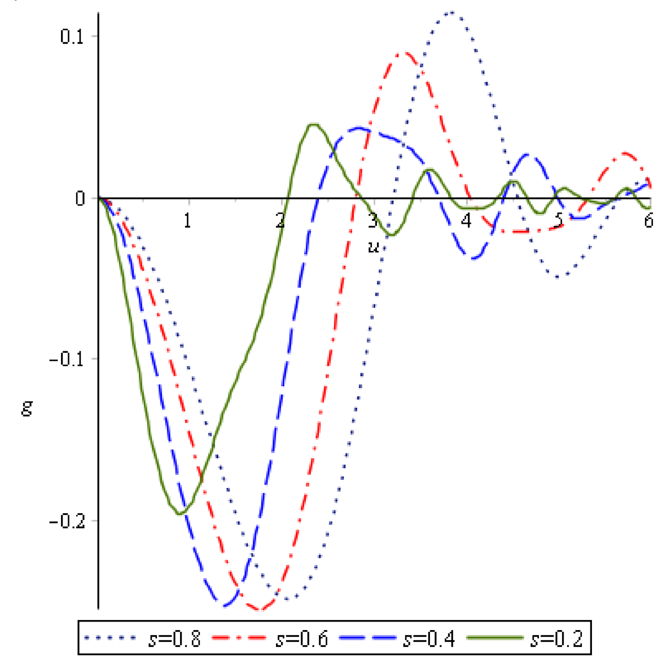
Some reflected waves for tension η and/or damper ψ fixed, and varying spring coefficient μ.



Green's function g for a semi-infinite one-sided pinned beam, where u is the displacement and s is the time. (1st) initial phase of the wave; (2nd) fading-out wave.



Green's function for a semi-infinite one-sided sliding end beam. (1st) initial phase of the wave; (2nd) fading-out wave.



Fading-out waves for the Green's function of a semi-infinite one-sided clamped end beam.

[1]: T. Akkaya and W.T. van Horssen. "Reflection and damping properties for semi-infinite string equations with non-classical boundary conditions". Journal of Sound and Vibration, 336(3), 2015.

[2]: N. Ortner and P. Wagner. "The Green's functions of clamped semi-infinite vibrating beams and plates". International Journal of Solids and Structures, 26(2), 1990.