



Resource Exchange and Link Formation in Decentralized Networks

Mihai Marcu

Supervisor(s): Georgios Iosifidis, Livia Chatzileftheriou

EEMCS, Delft University of Technology, The Netherlands

A Thesis Submitted to EEMCS Faculty Delft University of Technology,
In Partial Fulfilment of the Requirements
For the Bachelor of Computer Science and Engineering

June 21, 2026

Name of the student: Mihai Marcu

Final project course: CSE3000 Research Project

Thesis committee: Georgios Iosifidis, Livia Chatzileftheriou

An electronic version of this thesis is available at <http://repository.tudelft.nl/>.

Abstract

Decentralized sharing systems rely on strategic decisions of the agents, such as how much of their resources to give and to whom. Agents may benefit from creating new connections with other agents, but those additional links may involve costs and future value uncertainty caused by changes of the network in the future or other allocation decisions. We capture these ideas in a model in which agents allocate resources to their active neighbors and may unilaterally strategically create or renew links with costs when it benefits them. The model distinguishes between initial links, which are always available, and newly formed links, which may be permanent or temporary and may lose quality over time. Two allocation rules are considered: an egalitarian strategy, where agents split resources equally among neighbors, and a proportional strategy, where agents allocate more resources to neighbors from whom they previously received more. Both methods are evaluated using Eisenberg-Gale welfare and Jain’s fairness index. We observe that instead of a classic equilibrium, the systems may enter a recurring cycle of states denoted by allocations and network structure. Moreover, we prove that the egalitarian rule reaches an exact cycle under some assumptions and observe that the proportional rules converge to a cycle, in which the per-period change in allocations falls below a constant.

In simulations on sparse, dense, and small-world graphs, link formation improves fairness, especially in sparse graphs where Jain’s index rises from 0.72 to 0.88. While dense initial graphs already perform well with few additional links, sparse graphs create more links and do not reach a cycle within the simulation horizon as often. Based on strategies, the proportional ones perform better in terms of fairness, and the strategy where the new links are helped by a decreasing function reaches a cycle more often, 81.5% of the time, compared with 51.9% for the simpler proportional rule. Link formation affects welfare unevenly: it raises welfare on initially sparse graphs (Eisenberg–Gale welfare rises from 33.0 to 34.2) but can slightly reduce it on already-dense graphs (35.8 to 35.6), because links degrade and a degraded link carries less throughput.

1 Introduction

Decentralized resource-sharing systems consist of multiple agents that make decisions locally without relying on a central authority. Each agent owns a certain amount of resources and decides how to allocate them among its neighbors, such as energy sharing in microgrid communities. Because agents can exchange only over active links, the structure of the network directly determines what each agent can receive. Most existing models take this network as fixed and ask only how agents split their resource. Therefore, the paper focuses not only on how agents share resources over an existing network, but also when they actively change the network, when in addition to sharing resources they may unilaterally also form or renew links at a cost when doing so is beneficial. Our goal is to understand how this strategic link formation changes the behavior of the system compared with a fixed network.

The importance of network structure is also visible in other decentralized systems such as Bitcoin, where nodes propagate transactions and blocks by communicating with their neighbors rather than through a central authority (1). Because each node has limited bandwidth and computational capacity, it can share information only with a subset of peers, so the connections a node maintains affect how quickly it learns about the current state of the network. When information spreads slowly, different parts of the network may temporarily hold different views, and such propagation delays have been identified as one cause of blockchain forks (2). This also matters when a new node joins the network, since it must establish communication links with existing nodes in order to receive and transmit information. The example illustrates the broader motivation for our work: in decentralized sharing systems, an agent’s outcomes depend not only on the links it already has, but also on the links it can form. In this paper, we study

this idea in a specific resource-exchange setting, where agents can receive resources only through active links and may strategically create or renew those links when doing so is expected to be beneficial.

Existing models mostly study resource exchange on a fixed graph, where agents can only choose how to split their resources. Other related papers, such as the one by Tsoukatos, do consider link formation, but with a different goal, either reaching a sparse graph by dropping links from an initially complete network (3), or forming links only by bilateral consent, where an agent’s utility comes not from resource exchange but from a function of the graph structure (4). Our paper combines these two views: agents repeatedly exchange resources under egalitarian or proportional rules while unilaterally creating, renewing, or letting links expire. We expand on this related work in Section 2.1.

This gap motivates the central question of this paper, and the breakdown of the subquestions:

How does strategic, costly link formation affect the welfare, fairness, and stability of decentralized resource-exchange networks?

SQ1 What are suitable decision rules for link creation and renewal?

SQ2 How can a stable state be defined in a dynamic resource-exchange network with link creation, renewal, degradation, and expiration?

SQ3 How do different allocation strategies affect welfare, fairness, and dynamics?

SQ4 How do link cost, link lifetime, and the initial graph structure affect welfare, fairness, and dynamics?

These questions are answered through a combination of model definitions, analytical observations, and simulations. Firstly, the paper defines decision rules for link creation and renewal: an agent creates or renews a link when the expected discounted benefit from that link is at least as large as its cost.

Second, the states are defined using the allocation matrix, the remaining lifetimes of created links, and the link qualities. A state is considered stable if all agents allocate their available resources and no agent has a profitable local change, such as changing an allocation, creating a new link, renewing a link, or allowing a link to expire. However, because links can be created, renewed, expire, and degrade, the system may not settle at one fixed state, instead, it can enter a recurring cycle of states.

Third, the paper compares egalitarian and proportional allocation strategies. The egalitarian strategy is easier to analyze because allocations depend only on the active graph, and the paper proves that it eventually reaches a cycle under the stated assumptions. The proportional strategies require an additional rule for newly created links because such links have no previous exchange history. In the simulations, the proportional strategy with a temporary decreasing boost for new links performs best overall in terms of fairness and cycle formation.

Finally, the effects of link lifetime, link cost, and initial graph structure are evaluated experimentally. The results show that strategic link formation improves fairness most clearly in sparse graphs. Dense graphs already perform well without many additional links, so link formation has a smaller effect and may even reduce welfare. Lower link costs make link creation more attractive, while higher costs keep the graph closer to the fixed-network benchmark.

2 Related work, and Problem Description

The section starts with a background and related work analysis and continues by formalizing the problem and introducing the necessary notations and variables. It then presents observations

and proofs for different cases of the problem, concerning link creation, renewal, and stable states.

2.1 Related work

The related work can be grouped into three main parts. Firstly, in the research done in fixed networks, agents only decide how to allocate their resources among existing neighbors. Hellinga studies a model in which each agent produces a stochastic amount of resources and exchanges it, in a time-slotted, money-free sharing economy where allocation is evaluated by an Eisenberg-Gale objective (sum of agents' logarithmic utilities), which we use as a welfare metric as well (5). Our paper is also related to proportional-response dynamics, a distributed allocation rule that is known to converge to market equilibrium in related Fisher-market settings with buyers and budgets (6). Hellinga studies this problem in a more similar fixed-network exchange setting. The thesis shows that, using a mirror-descent interpretation, the proportional strategy converges to a market equilibrium where all agents allocate their available resources and no agent can improve its utility by changing its allocation (7). These works are relevant because they provide the basis for the allocation strategies used in this paper; however, they do not study how the network itself changes over time.

Secondly, a connected related work is the study of network formation, where agents decide which links to create or maintain, and the resulting network structure affects their payoff (8). Jackson discusses improving paths, stable networks, and cycles in network formation games, which are useful concepts for the stability analysis in this paper (4). Related work studies how networks evolve through local changes, often under bilateral link formation rules. In contrast, Bala and Goyal,(9), study a model where links can be created unilaterally by one agent, who also pays for the link. This unilateral link-formation rule is close to the rule used in this paper. Dynamic network formation is also studied by Song and van der Schaar (10), where agents evaluate future payoffs when deciding whether links should be formed or maintained. This is relevant to the present model because link-creation and renewal decisions are also based on expected future benefits, discounted over time. However, these network formation models usually focus on payoffs generated by the graph structure itself, rather than on repeated resource exchange over the graph.

Thirdly, the closest work to this paper is Tsoukatos' work (3), which studies reciprocity-driven sparse network formation. In that model, agents exchange resources, and sparsity penalties encourage them to maintain fewer active links, but the goal is different: it starts from a complete graph and tries to obtain sparse exchange graphs while preserving a high level of reciprocity. This paper studies the opposite direction: the network starts from an initial graph, and agents may strategically create or renew additional costly links when doing so is beneficial.

This paper combines elements that are usually studied separately: repeated decentralized resource exchange, strategic link creation and renewal, link costs, link degradation, and temporary links, bringing them together. The main contribution is to study how allowing agents to form additional links changes welfare, fairness, stability, and cycles under egalitarian and proportional allocation strategies.

2.2 Formal Problem Description

We study a decentralized resource exchange network with agents $N = \{1, \dots, n\}$. Time is divided into slots $t = 0, 1, 2, \dots$. At each time slot, agent i produces an amount of resources sampled from a normal distribution $a_i(t) > 0, a_i \approx N(\hat{a}_i, \sigma)$, with small σ , $\hat{a}_i = E[a_i(t)]$, which exchanges with other agents (5). At each time slot, every agent observes their neighbors' allocations and received resources, and will try to create or renew links based on their estimates, in order to maximize their utility.

The network at time t is represented by a graph $G_t = (N, E_t)$ where E_t is the set of active links, with E_0 being the initial set of links which are always available, and have no creation cost. There is also a set of possible new links P . A link $\{i, j\} \in P$ can be created by an agent if it is not already active. If agent i creates link $\{i, j\}$, then agent i pays a cost $c_{ij} \geq 0$ which is not subtracted from the physical resource that is exchanged; instead, it is subtracted from the utility of the agent who creates or renews the link. So, the active links are $E_t = E^0 \cup L_t$, where L_t is the set of links created by the agents.

Two cases are studied; firstly, newly created links are permanent, and secondly, newly created links are temporary and disappear after a fixed lifetime unless they are renewed. Link formation is unilateral; this means that agent i can create the link without needing agreement from agent j , and agent i pays for it. After the link is created, both agents can use it to send or receive resources. If both agents attempt to create the same inactive link in the same time step, the link is created only once, and only one agent pays for it. They will not know the decision from the neighbor agent to create or not the link, so we assume that they will not condition their decision on the possibility that the other agent may create it instead.

2.2.1 Temporal Parameters

To capture dynamic effects, the model introduces two temporal parameters. The first parameter is the discount factor $\delta \in (0, 1)$. It captures the idea that agents value current resources more than future resources. Utility obtained s periods in the future is weighted by δ^s . This is the standard discounted-utility idea introduced by Samuelson (11). So, when an agent decides whether to create or renew a link, it compares the current cost with the discounted future benefit. Similar discounting ideas are also used in dynamic network formation models with foresighted agents (12). The second parameter, the degradation factor $\beta \in (0, 1)$, captures the idea that a created link may lose quality over time. If a link has been active for s periods, its effective throughput is multiplied by β^s . In this model, degradation is used as a way to represent the weakening of created links. By having β , we define the quality of a link at time t as:

$$q_{ij}(t) = \begin{cases} 1, & \text{if } \{i, j\} \in E^0, \\ \beta^{g_{ij}(t)}, & \text{if } \{i, j\} \in L_t. \end{cases}$$

where $g_{ij}(t)$ is the age of the created link $\{i, j\}$. Initial links do not degrade, while newly formed links degrade according to β .

2.2.2 Agent Decisions

At each time slot, agent i makes two decisions. Firstly, it decides how to allocate its resources among its active neighbors, with $x_{ij}(t)$ being the amount of resource sent from agent i to agent j at time t . Each agent allocates all of its available resources:

$$\sum_{j: \{i, j\} \in E_t} x_{ij}(t) = a_i(t)$$

Secondly, agent i may decide to create or renew links. These decisions are made according to the expected effect on its own utility.

2.2.3 Received Resource and Utility

The resource received by agent i at time t is computed as the sum of all the resources received through the links from their neighbors, and the utility at time t is the resources received minus

$C_i(t)$, the cost of the links that i pays at moment t . A similar utility function is used in Jackson for link creation (8).

$$r_i(t) = \sum_{j:\{i,j\} \in E_t} q_{ij}(t)x_{ji}(t)$$

$$u_i(t) = r_i(t) - C_i(t)$$

2.2.4 Link Creation Rule

Suppose agent i considers creating link $\{i, j\}$ at time t . Let $r_i^{+ij}(\tau)$ be the estimated resource received by agent i at time τ if the link is created at time t , and $r_i^{-ij}(\tau)$ be the estimated resource received if the link is not created. The relevant periods for the estimated future gains are $\tau = t + 1, \dots, t + T_{ij}$, where T_{ij} is the assumed link lifetime, or ∞ if is permanent.

The binary link-creation decision variable $y_{ij}(t) \in \{0, 1\}$ equals one if agent i creates link $\{i, j\}$ at time t , which happens only when the discounted future gain from creating the link is at least as large as the creation cost c_{ij} (12).

$$y_{ij}(t) = \begin{cases} 1, & \text{if } \sum_{\tau=t+1}^{T_{ij}+t} \delta^{\tau-t} (r_i^{+ij}(\tau) - r_i^{-ij}(\tau)) \geq c_{ij}, \\ 0, & \text{otherwise.} \end{cases}$$

2.2.5 Estimating utility from links

Calculating the exact expected benefit of creating a link is difficult and not practically feasible because the expected benefit from adding a link $\{i, j\}$ depends on the benefit in future states, and these future states depend themselves on future allocations, future link formations, renewals, expiration, and changes in link quality. Because of that, to compute the exact value of a link, an agent would have to reason about a sequence of future states that are themselves affected by the current link-creation decision. To solve this issue, approximations need to be introduced to ease the decision for creating or not the link. Those approximations will be used in case the exact value cannot be determined. For example, in the case of the egalitarian strategy, an exact amount of resources can be defined by the total number of resources divided by the number of neighbors the agent has. However, in the case of the proportional strategy, the sharing ratio will be used.

One approximation is to use the sharing ratio of the candidate neighbor j : $\rho_j(t) = \frac{r_j(t)}{a_j(t)}$ meaning the resource received by j divided by the resource allocated by j . A high sharing ratio means that the agent receives a great amount of resources compared to the amount it gives away. So due to this fact, an agent i with a big sharing ratio is likely to share less with a newly connected agent. The reason behind that is that if agent i gives amount $x_{ij}(t)$ of resource to agent j , then agent i expects to receive $x_{ji}(t+1) = \frac{x_{ij}(t)}{\rho_j(t)+\varepsilon}$ units of resource back from agent j , where $\varepsilon > 0$ is a small constant used to avoid division by zero. A similar sharing ratio is used in Hellenga (5). For simplicity, the decision to create a new link only considers the direct resource that agent i expects to receive through that link. Indirect effects on the allocation of resources to other neighbors are ignored. This simplification is used because the graph structure may change while the value of the link is being estimated, making the full future effect of the link difficult to predict reliably.

If $x_{ij}(t)$ denotes the initial amount assigned to the newly created link, then the ratio-based estimated benefit is

$$E_{ij}(t) = \sum_{s=0}^{T_{ij}-1} (\delta\beta)^s \frac{x_{ij}(s+t)}{\rho_j(s+t) + \varepsilon}.$$

For $\delta\beta \neq 1$, this becomes

$$E_{ij}(t) = \frac{x_{ij}(t)}{\rho_j(t) + \varepsilon} \cdot \frac{1 - (\delta\beta)^{T_{ij}}}{1 - \delta\beta}.$$

The link is created if

$$E_{ij}(t) \geq c_{ij}.$$

2.2.6 Temporary Links and Renewal

A temporary link is active only for a fixed number of time slots, T_{ij} , the lifetime of link $\{i, j\}$, but can also be renewed before its expiry, and the age of the link is reset to zero. The link will be renewed only if the discounted gain from renewal is at least as large as the renewal cost. Let $r_i^R(t)$ be the estimated resource received by agent i if the link is renewed, and let $r_i^E(t)$ be the estimated resource received if the link is not renewed and continues until it expires. If $g_{ij}(t)$ is the age of the link, and $l_{ij} = T_{ij} - g_{ij}(t)$ the remaining life, the ratio-based estimated renewal gain is therefore

$$E_{ij}^R(t) = \sum_{s=0}^{l_{ij}-1} \delta^s (r_i^R(t+s) - r_i^E(t+s)),$$

Using the sharing-ratio approximation, this becomes

$$E_{ij}^R(t) = \sum_{s=0}^{l_{ij}-1} \delta^s \left(\beta^s \frac{x_{ij}^R(t)}{\rho_j(t) + \varepsilon} - \beta^{g_{ij}(t)+s} \frac{x_{ij}^E(t)}{\rho_j(t) + \varepsilon} \right).$$

For $\delta\beta \neq 1$, this gives the closed form

$$E_{ij}^R(t) = \frac{x_{ij}^R(t) - \beta^{g_{ij}(t)} x_{ij}^E(t)}{\rho_j(t) + \varepsilon} \cdot \frac{1 - (\delta\beta)^{l_{ij}}}{1 - \delta\beta}.$$

The renewal rule becomes

$$y_{ij}^R(t) = \begin{cases} 1, & \text{if } E_{ij}^R(t) \geq c_{ij}^R, \\ 0, & \text{otherwise.} \end{cases}$$

2.2.7 Efficiency and Fairness

Efficiency is measured after the agents make their decisions. It is not the rule used by agents to create links. The welfare measure that is used is the Eisenberg-Gale welfare:

$$W(t) = \sum_{i \in N} a_i(t) \log(r_i(t) + \epsilon),$$

The fairness of the whole network can also be measured by Jain's fairness index. This measure is equal to 1 when all agents receive the same amount of resource, and decreases as the distribution becomes more unequal(13)

$$J(t) = \frac{(\sum_i r_i(t))^2}{n \sum_i r_i(t)^2}.$$

In the next sections, we will observe that the system does not converge to a stable state or configuration, all the time. Therefore, those metrics, such as $W(t)$ or $J(t)$, cannot be applied to only a state defined by a timestamp. In the experiments, we will average this $W(t)$ and $J(t)$ on a number of states.

2.2.8 Allocation Strategies

Two allocation strategies are considered: egalitarian and proportional. In the egalitarian strategy, agent i splits its resource equally among all active neighbors. In the proportional strategy, agent i sends more resources to agents from which it received more effective resources in the previous time slot. These will be studied in more detail in the next chapter.

2.2.9 Stable States and Equilibrium

Since the graph can change over time, and since created links can expire, be renewed, and degrade, the system state must contain the information needed to determine the next period. We define the state at time t as

$$S_t = (X(t), \ell(t), q(t)),$$

where $X(t)$ is the matrix whose entry $X_{ij}(t)$ denotes the proportion of agent i 's resource allocated to agent j . $\ell(t)$ is the vector of remaining lifetimes of created links, and $q(t)$ is the vector of link qualities. For each possible created link $\{i, j\} \in P$, the remaining lifetime is defined as

$$\ell_{ij}(t) = \begin{cases} 0, & \text{if the link is inactive,} \\ T_{ij} - g_{ij}(t), & \text{if the link is active and temporary,} \\ \infty, & \text{if the link is active and permanent.} \end{cases}$$

The link quality is defined as

$$q_{ij}(t) = \begin{cases} 1, & \text{if } \{i, j\} \in E^0, \\ 0, & \text{if } \{i, j\} \in P \text{ is inactive,} \\ \beta^{g_{ij}(t)}, & \text{if } \{i, j\} \in P \text{ is active.} \end{cases}$$

An adjacent state is a state that differs from the current state by one or more changes in the allocations, the creation of one or more new links, the renewal of one or more temporary links, or the expiration of one or more temporary links. A state S' defeats a state S for agent i if S' can be reached from S by one or more links and if agent i 's utility is higher in S' than in S . (4). A state S_t is stable if two conditions hold. First, every agent allocates all of its available resources: $\sum_{j:\{i,j\} \in E_t} x_{ij}(t) = a_i$. Second, there is no adjacent state that defeats S_t . In other words, no agent can improve its own utility by changing its allocation, creating a new link, renewing an existing link, or choosing not to renew a temporary link. Thus, stability is defined locally: the system is stable when no profitable admissible local change remains.

If the improving process reaches such a state, the system has reached a stable state. However, because links can be created, renewed, expire, and degrade over time, the process may also fail to settle at one fixed state. Instead, it may enter a repeating sequence of states (4). The system reaches an improving cycle of period p if there exist states $S^1, S^2, \dots, S^p$ such that each state is defeated by the next one, $S^1 \rightarrow S^2 \rightarrow \dots \rightarrow S^p \rightarrow S^1$. Equivalently, the system satisfies $S_{t+p} = S_t$ for some $p > 0$ and for all sufficiently large t . While in a stable state, no agent has a profitable local deviation; in an improving cycle, the system keeps moving because at each state, at least one agent has a profitable change.

2.2.10 Link lifetime threshold assumption

For simplicity in the experiments and in the proofs we will assume that a link has died once $\beta^{g_{ij}} \hat{a}_i < \eta$, where η is a small constant. Meaning that if a link can't effectively transport a non-negligible amount of resources, they should be considered dead. Pruning these negligible links prevents asymptotic, near-zero interactions from unnecessarily complicating the network topology.

2.3 Examples

2.3.1 Small example

Consider a network with nodes 1, 2, 3, 4 which share their resources with the egalitarian strategy. The initial links are $\{1, 4\}$ and $\{2, 3\}$. The possible temporary links are $\{1, 2\}$, $\{1, 3\}$, and $\{3, 4\}$. The creation costs are $c_{12} = c_{13} = 0.5$, $c_{34} = 0.8$. Each agent produces one unit of resource, the discount factor and degradation factor are both 0.9, and all newly created links have lifetime $T = 2$. The links $\{1, 2\}$, $\{1, 3\}$, and $\{3, 4\}$ are created at time 1, they age to time 2 and after that they disappear and get to the third timestamp which has the same state as the first one which means that arrives at a cycle of states.

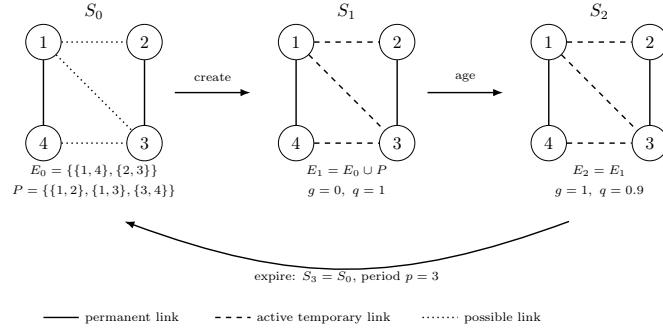
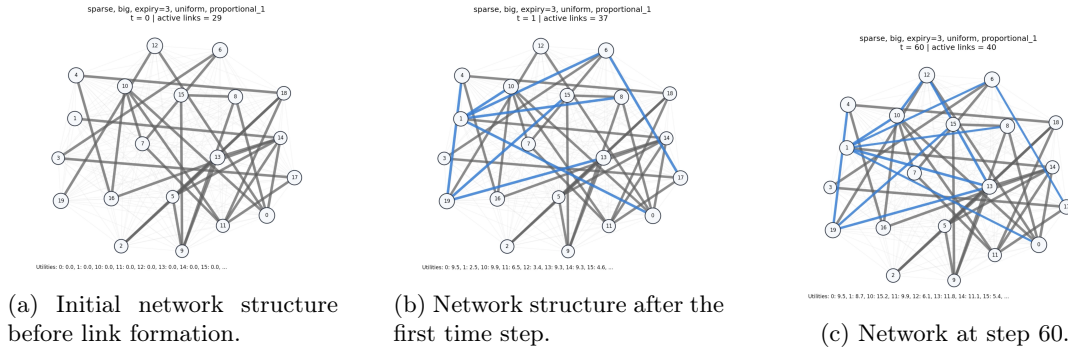


Figure 1: State cycle under the egalitarian strategy with $a_i = 1$, $\delta = \beta = 0.9$, temporary-link lifetime $T = 2$, and costs $c_{12} = c_{13} = 0.5$, $c_{34} = 0.8$.

2.3.2 Image examples



3 Analysis of Allocation Strategies and Link Formation

This section studies how strategic link formation affects the resulting graph structure, equilibrium formation, welfare, and fairness under different decentralized allocation strategies. We first study the egalitarian strategy, where each agent splits its resource equally among all active neighbors, after that, we study the proportional strategy, where agents allocate more resources to neighbors from whom they received more resources in the previous time slot.

3.1 Egalitarian Strategy

Let $d_i(t)$ be the number of active neighbors of agent i . Under the egalitarian strategy, agent i splits its resource equally among all active neighbors. Therefore,

$$x_{ij}^{\text{eg}}(t) = \begin{cases} \frac{a_i(t)}{d_i(t)}, & \text{if } j \in \mathcal{N}_i(t), \\ 0, & \text{otherwise.} \end{cases}$$

This strategy is useful because the allocation rule is simple and does not depend on resources received in previous time slots. If agent i creates a link to agent j , the direct benefit for agent i comes from the additional resource that agent j sends through this new link. Suppose agent i considers creating the link $\{i, j\}$ at time t . If the link is created, then the degree of agent j becomes $d_j(t) + 1$. Because in theory, the time horizon is infinite and the agent i can estimate the $\hat{a}_j$ of their neighbor, it can use that in approximating the received resources. Thus, under the egalitarian strategy, agent j sends $\frac{\hat{a}_j}{d_j(t)+1}$ units of resource to agent i . So in the case of the egalitarian strategy, the link formation rule becomes:

$$E_{ij}(t) = \sum_{s=t+1}^{t+T_{ij}} (\delta\beta)^s \frac{\hat{a}_j}{d_j(s) + 1}.$$

Some observations can be made about the egalitarian strategy.

Remark 1. It can be proven that the egalitarian strategy will always arrive at a cycle as in A.1 using the assumption from 2.2.10. This follows a proof by demonstrating that the effective number of states that can be formed is finite. Because there are only finitely many possible states, the system cannot keep producing new states forever, and, eventually, one state must repeat. Since the next state is determined by the current state, this repeated state implies a recurring cycle.

Remark 2. Sparse candidates will be preferred by an agent i because E_{ij} increases as d_j decreases. This is the reason why, for the experiments, sparse initial graphs generate the most link formations. This observation has another consequence: as only the initial links exist in the first time slot, it means that $d_j(0) \leq d_j(t)$. So if $d_j(t)$ increases, $\frac{\hat{a}_j}{d_j(t)+1}$ which is used for approximating the value transported by the $\{i, j\}$ link decreases. Therefore, the expected benefit is highest at the first time slot. If the link is not profitable when the benefit is highest, then it will not become profitable later. Also the images in section 2.3 2a, 2b, 2c show that in the first time slot a lot more edges are created comparing the other slots. The number of edges increase from the initial 29 links to 37 in the first time slot to 40 in the 60th time slot.

Remark 3. If all temporary links have the same lifetime and no link is renewed before its death, at time $p = T + 1$, where T is the lifetime of all links, the system will arrive at cycle of states. This is because the links are created at the first time slot, persist T slots, expire simultaneously and the system returns to the initial state S_0 . This happens in the example 1.

3.2 Proportional Strategy

The proportional strategy states that an agent i sends resources to their connected neighbors.

$$x_{ij}(t+1) = a_i(t) \times q_{ij}(t) \frac{x_{ji}(t)}{\sum_{k \in \mathcal{N}_i(t)} x_{ki}(t)}$$

A problem with the proportional strategy in our case is that, when creating a new link at time t , there is no previous resource received through that link. Therefore, the standard proportional rule would give $x_{ij}(t) = 0$. This means that, without an additional rule, there would forever be zero resources on that link. This is related to the usual initialization problem in proportional-response dynamics, where initial allocations are needed before the proportional update can start. In the literature, Tsoukatos states that at the start, “peers allocate resources $x(0)$ either equally or randomly” (3). In our paper, we found 2 suitable allocation rules:

Initial equal allocation rule. Each agent starts by allocating a proportion of its produced resource to the new link. One solution would be the following. If agent i has $d_i(t)$ active neighbors after the new link has been created, then the new link receives $x_{ij}(t) = \frac{a_i(t)}{d_i(t)}$ and the remaining resource is then shared according to the proportional strategy.

Temporary exploration boost for new links. A second solution is to add a temporary exploration boost for newly created links. Under the proportional strategy, allocations depend on the resource received in the previous time slot. A newly created link has no previous exchange history, so without an additional term it would receive very little resource at the beginning and would not have a fair chance to prove whether it is useful. Therefore, the new link is given an initial boost that decreases with the age of the link. This idea is motivated by time-weighting methods, where recent observations receive a larger weight than older observations. For example, Ding and Li (14) use an exponential time-weighting function to give recent data a higher weight while gradually decreasing the weight of older data. A similar idea is used here: a newly created link receives the largest boost when it is new, and this artificial support decreases as the link becomes older. If the age of link $\{i, j\}$ is $g_{ij}(t)$, a possible decreasing boost is

$$b(g_{ij}(t)) = \lambda e^{-\alpha g_{ij}(t)}, \quad \lambda > 0, \alpha > 0.$$

Here, λ controls the initial size of the boost, while α controls how fast the boost decreases over time. The proportional strategy can then be written as

$$x_{ij}(t) = a_i(t) \frac{q_{ij}(t-1)x_{ji}(t-1) + b(g_{ij}(t))}{\sum_{m \in \mathcal{N}_i(t)} (q_{im}(t-1)x_{mi}(t-1) + b(g_{im}(t)))}.$$

For the proportional strategy, the fixed-graph case is related to proportional-response dynamics. In such settings, previous work has shown that proportional-response dynamics can converge to a market equilibrium (6) (7). However, the setting studied in this paper is different because the graph itself changes over time: links can be created, renewed, expire, and degrade. Therefore, the fixed-graph convergence result cannot be applied directly. In the simulations, the proportional strategies often do not return to exactly the same state, because the allocation values may continue to change by very small amounts. For this reason, cycles are detected approximately. A run is considered to have reached a cycle if, for some period $p > 0$, the difference between the current state and a previously visited state satisfies $|S_{t+p} - S_t| < \varepsilon$ for all the states that form the first viewed cycle. In the experiments, the tolerance is set to $\varepsilon = 0.0001$. Thus, for the proportional strategies, cycle detection should be interpreted as approximate recurrence rather than as a formal proof of convergence to an exact cycle. With this setup, similarly to the proof in Appendix A.1, it can be shown that the proportional strategy will arrive close to a state

that has previously been observed. Despite this, what makes the proportional strategy different is that, for the egalitarian strategy, with the stated assumptions, arriving at a state that was previously observed meant that a cycle was made. Here, in the proportional strategy, this is not enough, because the allocation matrix $X(t)$ is not fully dependent on the graph structure. Therefore, arriving close to a previous state does not mean that the following states will also be close. Also, arriving at the same structural state does not imply that the next full states are: $|S_t - S_{t+p}| < \epsilon \not\Rightarrow |S_{t+1} - S_{t+p+1}| < \epsilon$. This can be seen in Appendix A.2. So, for the proportional strategy, this doesn't mean a permanent cycle.

4 Experimental Setup and Results

For the experiment, a Python program was written, the pseudo code of it can be found in appendix, and the full code on GitHub. For all of the timestamps in the experiment horizon, all the agents allocate their resources at once, the agents receive the resource and decide on the link renewals and the link formation. After that, the state will be marked as visited and will be checked if a cycle occurred.

What is more interesting about the algorithm is the cycle finding. The observation done in Section 3.2 for the proportional strategy is that even if 2 states belong to the same bucket, so that the difference between all the elements of the states is smaller than *epsilon*, the next states can be in different buckets. To make the chance of flagging false positive cycles, when simulating, if $|S_{t+p} - S_t| < \varepsilon$ then we will check for all of the elements from $k = t + 1$ until $k = t + p - 1$ that $|S_k - S_{k-p}| < \varepsilon$. To make the cycle check faster, previously visited states are stored in a hash map. Each state is hashed by rounding its elements with a precision determined by ε to put them in buckets with the difference smaller than ε . For example, if $\varepsilon = 10^{-4}$, four decimal places are kept. The map stores the time indices at which each rounded state appeared. When a new state has the same hash as a previous one, it is treated only as a candidate cycle. The actual element-wise condition is still checked afterwards, so the hash is only used to speed up the search.

4.1 Experimental setup

The experiments were run on networks with $n = 16$ agents and three initial graph structures: sparse graphs, dense graphs, and small-world graphs. For each graph type, the simulations tested three link-cost settings: small, medium, and large costs. Small costs refer to link creation and renewal costs sampled from $[2, 6]$, medium costs from $[7, 14]$, and large costs from $[16, 28]$. Three lifetime ranges were considered: small lifetimes lasting between 1 and 3 steps, medium lifetimes between 3 and 7 steps, and large lifetimes between 7 and 12 steps. In addition, two fixed-lifetime cases were tested, where created links last exactly 3 steps or exactly 7 steps, together with a mixed-lifetime setting in which different created links may have different expiration times. This mixed case is useful because it better represents dynamic networks in which links do not all expire simultaneously.

The simulations used the model constants $\beta = 0.95$ for the degradation factor and $\delta = 0.9$ for the discount factor. The initial network links are permanent baseline links, so they do not expire or degrade. Degradation and lifetimes apply only to links that are created or renewed during link formation. Since the proportional strategies may change their allocations by very small amounts before reaching a cycle, an experiment was considered to have reached an cycle when the maximum absolute difference between the current allocation-proportion matrix and a previously observed allocation-proportion matrix was below $\epsilon = 10^{-4}$.

The experiments compared the egalitarian strategy with two proportional strategies under

different link-dynamic conditions. For each parameter combination, the same setting was also evaluated without link formation. In the no-link-formation benchmark, the network remains fixed and agents only allocate resources over the initial graph. The comparison evaluates how link costs, graph structure, link lifetimes, and strategy choice affect welfare, fairness, and the stability of the exchange system. The full experiment grid consists of 162 runs with link formation, and 162 matched benchmark runs without link formation. The results table aggregates these outcomes by link cost, graph structure, link lifetime, and allocation strategy. For each simulation run, welfare and fairness are reported as average values. If the system reaches a cycle, welfare and fairness are calculated as the mean over all states in the detected cycle. If no cycle is detected within the simulation horizon, they are calculated as the average over the final 100 steps of the simulation.

4.2 Results

Table 1: Aggregated simulation results with and without link formation

Aggregation	Category	Setting	Cycle rate	Welfare	Fairness
Link cost	Small	With link formation	37.0%	35.07	0.976
	Small	Without link formation	100.0%	34.82	0.877
	Medium	With link formation	66.7%	35.02	0.958
	Medium	Without link formation	100.0%	34.82	0.877
	Large	With link formation	100.0%	34.82	0.877
	Large	Without link formation	100.0%	34.82	0.877
Graph structure	Sparse	With link formation	57.4%	34.15	0.880
	Sparse	Without link formation	100.0%	33.05	0.721
	Dense	With link formation	85.2%	35.59	0.970
	Dense	Without link formation	100.0%	35.84	0.969
	Small-world	With link formation	61.1%	35.17	0.961
	Small-world	Without link formation	100.0%	35.56	0.942
Link lifetime	Small lifetime	With link formation	74.1%	35.68	0.928
	Small lifetime	Without link formation	100.0%	35.22	0.867
	Medium lifetime	With link formation	59.3%	34.80	0.943
	Medium lifetime	Without link formation	100.0%	34.61	0.882
	Large lifetime	With link formation	55.6%	34.50	0.943
	Large lifetime	Without link formation	100.0%	34.61	0.882
	Mixed lifetime	With link formation	55.6%	34.71	0.943
	Mixed lifetime	Without link formation	100.0%	34.61	0.882
	Fixed lifetime 3	With link formation	88.9%	35.53	0.921
	Fixed lifetime 3	Without link formation	100.0%	35.22	0.867
	Fixed lifetime 7	With link formation	74.1%	34.59	0.944
	Fixed lifetime 7	Without link formation	100.0%	34.61	0.882
Strategy	Egalitarian	With link formation	70.4%	34.72	0.928
	Egalitarian	Without link formation	100.0%	34.60	0.835
	Proportional 1	With link formation	51.9%	34.82	0.937
	Proportional 1	Without link formation	100.0%	34.77	0.885
	Proportional 2	With link formation	81.5%	35.36	0.947
	Proportional 2	Without link formation	100.0%	35.08	0.911

Notes: The table reports aggregated values for each category separately for the setting with link formation and the benchmark without link formation. The cycle rate is reported as the share of runs that reached a cycle.

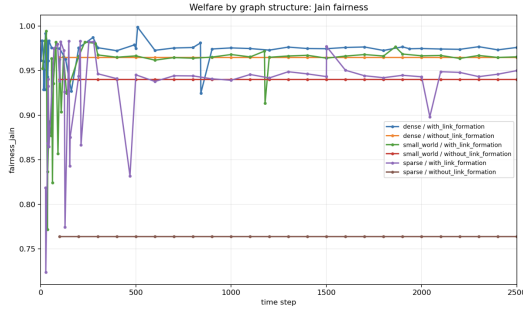


Figure 3: Jain fairness over time grouped by initial graph structure.



Figure 4: Jain fairness over time grouped by allocation strategy.

4.3 Results Interpretation

This section interprets the aggregated results in Table 1, together with the fairness dynamics shown in Figures 3 and 4. The comparison focuses on cycle rate, average welfare, and fairness. If the system reaches a cycle, welfare and fairness are calculated as the mean over all states in the detected cycle. If no cycle is detected within the simulation horizon, they are calculated as the average over the final 100 steps of the simulation.

Overall, link formation improves fairness most clearly in sparse graphs. Table 1 and Figure 3 show that Jain fairness in sparse graphs increases from 0.721 without link formation to 0.880 with link formation. This improvement is much larger than in dense graphs, where fairness changes only from 0.969 to 0.970. This suggests that sparse graphs benefit more from additional links because agents initially have fewer exchange opportunities.

The effect on welfare depends on the graph structure. Link formation increases welfare in sparse graphs, from 33.05 to 34.15, but slightly decreases welfare in dense graphs, from 35.84 to 35.59. One explanation is that additional links improve access to resources in sparse networks, but in dense networks, they may introduce extra link costs and degradation effects without adding much new exchange value. Therefore, more links are not always better; their value depends on the initial network structure.

Link cost strongly affects how much the network changes. With small link costs, link formation is more attractive, leading to higher welfare and fairness, but also to a lower detected cycle rate of 37.0%. This is because more profitable link-creation decisions keep changing the graph, making recurrence harder to detect within the simulation horizon. With large link costs, the results with and without link formation are close. This indicates that high costs prevent most additional links from being created, so the system behaves like the fixed-network benchmark.

Graph structure also affects stability. Dense graphs reach a detected cycle in 85.2% of the runs with link formation, while sparse graphs do so in only 57.4%. The sparse graphs create more opportunities for profitable link formation, which makes the system more dynamic. Dense graphs are already well connected, so fewer new links are created and the system is more likely to return to a previous state.

The lifetime results show that shorter and fixed lifetimes make cycles easier to detect. Fixed lifetime 3 has the highest cycle rate, 88.9%, because links expire in a more synchronised way. When links have equal lifetimes, the system is more likely to return to a previous combination of active links and remaining lifetimes.

Finally, the allocation strategy has a visible effect on fairness and cycle formation. Proportional 2 achieves the highest welfare and fairness among the strategies with link formation, with welfare 35.36 and Jain fairness 0.947. It also reaches a detected cycle in 81.5% of runs, compared with 51.9% for Proportional 1.

5 Responsible Research

5.1 Reproducibility of results

In a paper is important that the results obtained can be verified and reproduced. In order to do that, the code used to obtain the simulation results is publicly available and is based on the formulas and setup presented in this paper. It can be run on a normal computer and produces data regarding cycle, utility welfare, and fairness. Also, all visual representations and the tables from the paper are solely based on the code.

To make the experiments reproducible, the base random seed was set to 1000, with derived seeds 1037, 1091, and 1131 used for stochastic graph generation, link costs, and link lifetimes, respectively.

5.2 Ethical considerations

This research is based on simulations of abstract agents in decentralized resource exchange networks. It doesn't include human participants or real user behavior. Despite this, the topic still has ethical relevance. Decentralized systems can affect how resources, information, or communication capacity are distributed among participants. For this reason, the paper evaluates not only utility and welfare, but also fairness and cycles.

5.3 Usage of LLM

AI tools, including large language models, were used to support language polishing, identify writing mistakes, create the LaTeX figure shown in Figure 1, assist with code writing, and support literature search and brainstorming. All of the writing, resources, and data produced by AI were verified and did not substitute for thinking.

6 Conclusions and Future Work

This paper studied how strategic link formation affects decentralized resource-exchange networks. The main research question was how costly additional links influence welfare, fairness, stability, and graph structure when agents can both allocate resources and change the network through local link-creation and renewal decisions.

The analytical contribution is the model choices and the analysis for the egalitarian strategy. We define link creation and renewal through discounted expected gains, and define the system state by the allocation matrix, remaining link lifetimes, and link qualities. This allows us to introduce the recurring cycle of states. For the egalitarian strategy, where allocations depend only on the active graph, we show that the system reaches an exact cycle under the stated assumptions. The analysis also shows why sparse candidate neighbours are more attractive: the expected benefit of a link is higher when the candidate has fewer active neighbours. Therefore, the value of a new link is highest when the graph is still close to its initial structure.

The remaining conclusions are based on simulations. This is the case for the proportional strategies, because their allocations depend on previous exchange outcomes and newly created links have no prior exchange history. In the simulations, it has been observed that these strategies arrive at cycles of close states, meaning that the system returns to states that are within the chosen tolerance of earlier states.

The simulations show that link formation is most useful when it removes real structural constraints. For example, in sparse graphs, Jain fairness increases from 0.721 without link formation to 0.880 with link formation, while welfare increases from 33.05 to 34.15. This shows

that additional links can help poorly connected agents access more exchange opportunities. In dense graphs, however, the effect is much smaller, and welfare can even decrease slightly because new links may degrade and redirect resources through lower-quality connections. Thus, the result is not simply that dense graphs perform better, but that link formation can also have diminishing returns. It is useful when it improves access, but not necessarily when the network is already well-connected.

Finally, the allocation strategy matters. The egalitarian strategy is easier to analyse, but the proportional strategies perform better in the simulations. The best tested strategy is the proportional rule with the decreasing boost for new links. Proportional 2 reaches a detected cycle in 81.5% of runs, compared with 51.9% for Proportional 1, and also achieves higher welfare and fairness.

Overall, strategic link formation affects decentralized exchange in a non-trivial way. More links are not always better. Their benefit depends on where they are added, how costly they are, how long they last, whether they degrade, and how agents allocate resources through them.

Future work could strengthen the theoretical analysis of proportional strategies, especially approximate cycle formation. It could also improve link-benefit estimation, test larger networks, other types of initial network structures, and study more strategies, including how agents using different strategies may interact.

A Proofs and examples

A.1 Proof for the egalitarian strategy

For the egalitarian strategy, assume that $0 < \beta < 1$ and $0 < \delta < 1$. Under this strategy, the allocation only depends on the active graph because the allocation matrix is directly affected by the number of neighbors. If $d_i(t)$ is the number of active neighbors of agent i , then

$$x_{ij}^{\text{eg}}(t) = \begin{cases} \frac{a_i}{d_i(t)}, & \text{if } \{i, j\} \in E_t, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, if the active graph is known, the allocation matrix is also known.

Since created links degrade over time, the quality of a created link $\{i, j\}$ is

$$q_{ij}(t) = \beta^{g_{ij}(t)}.$$

We introduce a small constant $\eta > 0$. If the effective resource that can be received through a created link becomes smaller than η , then the link is treated as inactive. In other words, link $\{i, j\}$ is removed once

$$\beta^{g_{ij}(t)} a_j < \eta.$$

This means that a created link can only remain active while

$$\beta^{g_{ij}(t)} a_j \geq \eta.$$

Solving this condition gives a maximum effective age for every created link:

$$g_{ij}(t) \leq \frac{\log(\eta/a_j)}{\log(\beta)}.$$

Since $0 < \beta < 1$, we have $\log(\beta) < 0$, and this bound is finite. Thus, we define

$$T_{ij}^{\max} = \left\lceil \frac{\log(\eta/a_j)}{\log(\beta)} \right\rceil.$$

After this age, the link is no longer considered active.

Therefore, every created link has a finite maximum lifetime. Temporary links have their given lifetime T_{ij} , while permanent degrading links have the effective lifetime $T_{ij}^{\max}$. Since the number of possible links is finite, and each link can only have finitely many possible ages, the number of possible states is finite.

Finally, assume that all decisions are deterministic. If the system reaches the same state twice, then the next state will also be the same. Therefore, by the pigeonhole principle, the sequence of states must eventually repeat. Hence, under the egalitarian strategy with the threshold rule above, the system eventually enters a cycle.

A.2 Example for the proportional strategy

This example shows that if 2 states are close within a constant, it doesn't mean for the proportional strategy that the next state will be within the same interval.

Consider three agents $N = \{1, 2, 3\}$, where each agent has one unit of resource. Let $\epsilon = 0.1$, and consider the two allocation matrices

$$X_t = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.01 & 0 & 0.99 \\ 0.0001 & 0.9999 & 0 \end{pmatrix}, \quad X_{t+p} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.0001 & 0 & 0.9999 \\ 0.01 & 0.99 & 0 \end{pmatrix}.$$

$$\|S_t - S_{t+p}\| < 0.1 = \epsilon$$

So the two states are in the same bucket. After one proportional update, agent 1's allocation changes in the two states. In the first state, agent 1 receives 0.01 from agent 2 and 0.0001 from agent 3. Therefore,

$$x_{12}(t+1) = \frac{0.01}{0.01 + 0.0001} = 0.9901, \quad x_{13}(t+1) = \frac{0.0001}{0.01 + 0.0001} = 0.0099.$$

In the second state, agent 1 receives 0.0001 from agent 2 and 0.01 from agent 3. Therefore,

$$x_{12}(t+p+1) = \frac{0.0001}{0.0001 + 0.01} = 0.0099, \quad x_{13}(t+p+1) = \frac{0.01}{0.0001 + 0.01} = 0.9901.$$

$$|x_{12}(t+1) - x_{12}(t+p+1)| = |0.9901 - 0.0099| = 0.9802 > 0.1 = \epsilon.$$

Therefore, even though the two current states are close, the next states are not necessarily close:

$$\|S_t - S_{t+p}\| < \epsilon \not\Rightarrow \|S_{t+1} - S_{t+p+1}\| < \epsilon.$$

B Pseudocode

Algorithm 1 Decentralized resource exchange with link formation

```

1: Input: agents  $N$ , initial links  $E^0$ , possible links  $P$ , resources  $a_i$ , link costs, discount factor
    $\delta$ , degradation factor  $\beta$ , horizon  $T$ 
2: Initialize: start from the initial graph  $E^0$  and an empty set of created links
3: for  $t = 0, 1, \dots, T$  do
4:   Update the active graph  $E_t$ 
5:   Update link qualities using link ages and the degradation factor  $\beta$ 

6:   Resource allocation
7:   for all agents  $i \in N$  do
8:     Allocate resource  $a_i$  among active neighbors using the selected strategy
9:   end for

10:  Received resources
11:  for all agents  $i \in N$  do
12:    Compute the effective resource received by agent  $i$ 
13:  end for

14:  Link renewal
15:  for all active temporary links do
16:    Renew the link if the discounted benefit is larger than the renewal cost
17:    Remove the link if it expires and is not renewed
18:  end for

19:  Link formation
20:  for all agents  $i \in N$  do
21:    Create profitable inactive links from  $P$ , if their discounted benefit is larger than their
    creation cost
22:  end for

23:  Cycle check
24:  if the current state has already appeared before then
25:    return detected cycle
26:  end if
27: end for
28: return final state

```

References

- [1] S. Nakamoto, “Bitcoin: A peer-to-peer electronic cash system,” <https://bitcoin.org/bitcoin.pdf>, 2008.
- [2] C. Decker and R. Wattenhofer, “Information propagation in the bitcoin network,” in *Proceedings of the 13th IEEE International Conference on Peer-to-Peer Computing*. IEEE, 2013, pp. 1–10, doi: 10.1109/P2P.2013.6688704.
- [3] K. P. Tsoukatos, “Reciprocity-driven sparse network formation,” in *Proceedings of the 11th EAI International Conference on Performance Evaluation Methodologies and Tools*, ser. VALUETOOLS 2017. ACM, 2017, pp. 90–97, doi: 10.1145/3150928.3150930.
- [4] M. O. Jackson and A. Watts, “The evolution of social and economic networks,” *Journal of Economic Theory*, vol. 106, no. 2, pp. 265–295, 2002.
- [5] R. Hellinga, G. Iosifidis, and N. Yorke-Smith, “Towards strengthening decentralised exchange,” in *Proceedings of the 25th International Conference on Autonomous Agents and Multiagent Systems*, ser. AAMAS 2026. Richland, SC: International Foundation for Autonomous Agents and Multiagent Systems, 2026, pp. 3128–3130. [Online]. Available: <https://dl.acm.org/doi/10.65109/RDIS7876>
- [6] F. Wu and L. Zhang, “Proportional response dynamics leads to market equilibrium,” in *Proceedings of the 39th Annual ACM Symposium on Theory of Computing*, ser. STOC ’07. Association for Computing Machinery, 2007, pp. 354–363, doi:10.1145/1250790.1250844.
- [7] R. Hellinga, “Algorithms for the sharing economy: An economic modelling perspective on distributed exchange,” Master’s thesis, Delft University of Technology, Delft, The Netherlands, Jul. 2025, available at TU Delft Repository.
- [8] M. O. Jackson and A. Wolinsky, “A strategic model of social and economic networks,” *Journal of Economic Theory*, vol. 71, no. 1, pp. 44–74, 1996, doi: 10.1006/jeth.1996.0108.
- [9] V. Bala and S. Goyal, “A noncooperative model of network formation,” *Econometrica*, vol. 68, no. 5, pp. 1181–1229, 2000, doi:10.1111/1468-0262.00155.
- [10] Y. Song and M. van der Schaar, “Dynamic network formation with incomplete information,” *Economic Theory*, vol. 59, no. 2, pp. 301–331, 2015, doi:10.1007/s00199-015-0858-y.
- [11] P. A. Samuelson, “A note on measurement of utility,” *The Review of Economic Studies*, vol. 4, no. 2, pp. 155–161, 1937.
- [12] Y. Song and M. van der Schaar, “Dynamic network formation with foresighted agents,” *International Journal of Game Theory*, vol. 49, no. 2, pp. 345–384, 2020, doi: 10.1007/s00182-020-00714-4.
- [13] R. K. Jain, D.-M. W. Chiu, and W. R. Hawe, “A quantitative measure of fairness and discrimination for resource allocation in shared computer systems,” *DEC Research Report TR-301*, 1984.
- [14] Y. Ding and X. Li, “Time weight collaborative filtering,” in *Proceedings of the 14th ACM International Conference on Information and Knowledge Management*, ser. CIKM 2005. ACM, 2005, pp. 485–492, doi:10.1145/1099554.1099689.