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Combined line planning and train timetabling for strongly heterogeneous railway lines with direct connections

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ABSTRACT

Rail systems have been developing rapidly in recent years aiming at satisfying the growing passenger demand and shortening passenger travel time. The line planning problem (LPP) and train timetabling problem (TTP) are two key issues at the strategic level and tactical level, laying the foundation of a high-level service quality for railway operation. In this paper, a multi-frequency LPP (MF-LPP) model and a multi-period TTP (MP-TTP) model are introduced for direct connections, with consideration of both periodic and aperiodic nature to meet strongly heterogeneous train services and reduce the capacity loss of train operating companies. A combined LPP and TTP method is designed considering timetable robustness, timetable regularity, and passenger travel time. For a given line pool, a multi-objective mixed integer linear programming model for the MF-LPP is formulated to obtain a line plan with multiple line frequencies by minimizing travel time, empty-seat-hour and the number of lines. Using the acquired line plan from the previous step, a MP-TTP model is proposed to achieve the minimal travel time, the maximal timetable robustness and the minimal number of overtakings. The two models work iteratively with designed feedback constraints to find a better plan for the rail transport system. Numerical experiments are applied to verify the performance of the proposed model and solution approach.

1. Introduction

Over years, rail systems have been developing fast all over the world, with demands for more speed, punctuality and capacity improvements. Passenger demand meets a constant growth and normally features unbalanced distributions with highly heterogeneous spatial mobility patterns and flow sizes. Meanwhile, a direct service is preferable for the passengers especially for ODs with bad transfer conditions, and a regular train service is the tendency in order to attract more passengers. The rail networks also become highly utilized and experience lots of delays caused by delay propagation. A high capacity utilization of both infrastructure and trains is appealing to reduce the operation cost. Therefore, an efficient and robust plan is necessary for providing a high-level service to satisfy both passengers and operators. Railway planning is a complicated process, normally decomposed into line planning, train timetabling, rolling stock scheduling, and crew scheduling (Peeters, 2003; Liebchen, 2008; Schöbel, 2012), see Fig. 1. The line planning problem (LPP) and train timetabling problem (TTP) are two fundamental issues for rail transport systems on a strategic and tactical level respectively. The LPP is the initial step aiming to find a set of lines or paths with corresponding stop schemes and operation frequencies that satisfy travel

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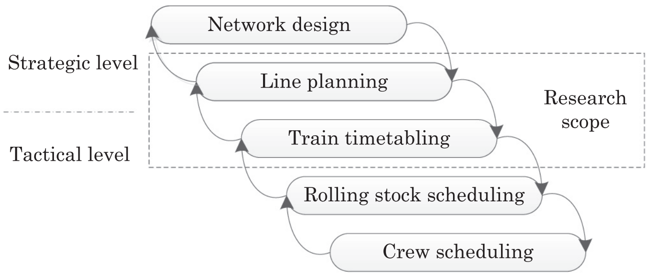

Fig. 1. Railway planning process.

demand in a given network. The TTP determines the time of departure and arrival at each station of every train within the given line structure. It could provide scheduled train times and paths for rail operators, but also give alternatives with corresponding travel time for passengers. Accordingly, the system efficiency about travel time, capacity utilization, and sensitivity to delays are highly dependent on both line plan and timetable.

Currently, two types of timetables are implemented in railway systems: periodic timetables and aperiodic timetables. In the literature, most line planning models are established to optimize the line plan specific to one timetable pattern. The four line planning models in Goerigk et al. (2013) are all proposed for periodic timetables. We categorize line plans with a periodic and aperiodic nature in this paper. A periodic timetable provides the same service pattern (arrival time, departure time and frequency) in each period, while scheduled times and train origin and destination (OD) for aperiodic timetables are defined mostly based on varying passenger demand. Periodic patterns offer regularity while aperiodic patterns offer flexibility (Robenek et al., 2017). Specifically, periodic patterns have fixed stops with high frequencies which provide a regular service for passengers in each period, while aperiodic timetables are more flexible with different stop patterns and various headways. Whereas a line plan with high frequencies could result in a low seat occupancy rate, irregular arrival/departure times are not convenient for travel planning. The strongly heterogeneous rail lines face an unbalanced passenger distribution in time and space. The question that arises in the planning is: which type of service pattern is preferable for this type of rail network, like corridors in the Chinese high-speed rail network with huge demand, unbalanced distribution and big amounts of commuters?

In practice, for railway networks with commuting passengers, periodic timetables proved to be more convenient for the railway system with years of practice in Europe. But limited stop patterns are offered, like only Sprinters (all-stop trains) and Intercities (stops at big stations) for conventional railways in the Netherlands. However, the mixed operation of express trains and all-stop trains is highly capacity consuming, and transfers are always inevitable due to the limited direct connections in the line plan. Meanwhile, train lines with many stops could lead to more time loss by dwelling, acceleration and deceleration, and this holds in particular for lines with relatively short station spacing. Especially for the Chinese high-speed rail network, more time is lost for deceleration and acceleration due to the higher running speed. Moreover, transfer times are very high due to poorly available transfer facilities. In case of a transfer, passengers typically need to check out, and then check in again with quite a long walking distance, as well as time due to the huge amounts of travelers and security check of luggages, see for details (Zhao et al., 2018). As a consequence, direct travel connections are preferred by passengers. Hence, for a strongly heterogeneous rail corridor like Chinese high-speed lines, different stop patterns with a relatively high frequency are necessary to meet the various passenger OD demands for direct traveling with respect to passenger travel time and operational cost. Robenek et al. (2016) illustrated that aperiodic timetables could perform better for high-density demand than periodic timetables by a sensitivity analysis on passenger congestion in the rail network of Switzerland. Therefore, a timetable with both periodic and aperiodic nature could be more efficient. We define a line plan and a timetable pattern with both periodic and aperiodic nature as *multi-periodic*.

In order to deal with the capacity utilization of both train and infrastructure, regular services, and direct connections, we designed a multiple frequency LPP (MF-LPP) model and a multiple period TTP (MP-TTP) model in this paper. Direct travel services for all passengers are proposed from a practical perspective. Both models take a multi-periodic pattern into account by introducing an extended period length T based on the traditional periodic timetable model. We define the problem as follows:

Multi-frequency line plan: The frequency f_l of each line l can be different and is flexible. For a traditional periodic line pattern, the period length is one hour and frequencies are always simple, like one, two and four in a period in the Netherlands. In a MF-LPP the frequencies can be anything depending on demand.

Multi-period timetable: Trains from the same line follow the same path while each train line has its own cycle time (regular interval) within the overall period length T. The cycle time of each line l can be strict T/f_l or slightly deviate between $[T/f_l - \xi, T/f_l + \xi]$ with tolerance parameter ξ, distinguishing from a traditional periodic timetable.

Fig. 2 illustrates an example of a multi-period timetable from a given multi-frequency line plan within an overall period length $T = 120$ min. Four lines l_1, l_2, l_3, and l_4 with four different frequencies of four, three, two, and one respectively, have different regular intervals $T_1 = 40$ min, $T_2 = 31$ min, $T_2' = 29$ min, $T_3 = 60$ min and $T_4 = 120$ min. l_2 has a semi-regular

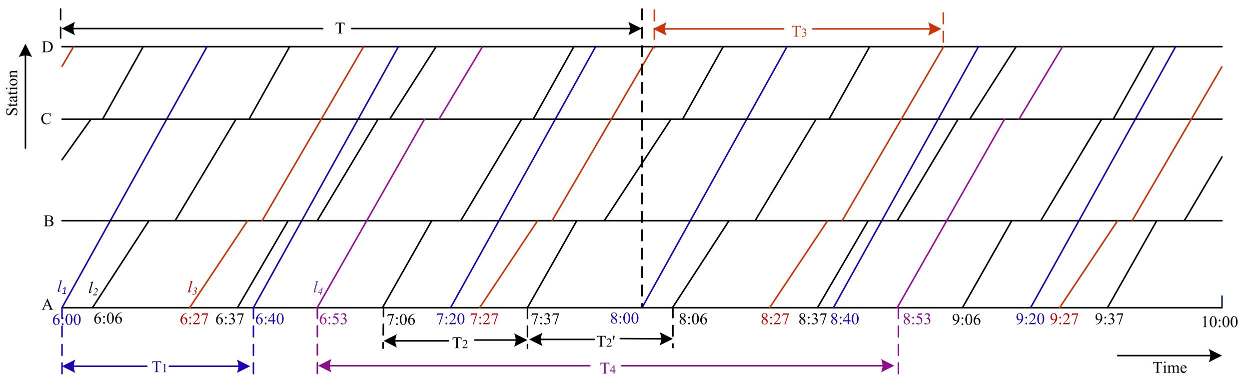

Fig. 2. Multi-period timetable from a given multi-frequency line plan. Each color of the line represents one stop pattern, and all lines have different frequencies. T is the overall period length, and T_1, T_2, T_3 and T_4 are the cycle time for each line which is smaller (frequency is more than one) or equal (frequency is one) to the overall period length.

interval T_2 and T_2'. Lines with the same color follow the same path including the same dwell time at a station and running time in a section. For example, the second train of line l_2 (black line) has an overtaking at station B, so all the other trains of line l_2 need to have a longer dwell time as well. Moreover, the regular interval might not be fixed if T/f_l is not an integer or ξ is applied due to the tight capacity utilization.

The MF-LPP model could offer a direct travel choice for ODs with fewer passengers in the aperiodic case, and also prevent travel time loss by the unnecessary dwelling of these ODs to more dedicated low-frequency lines. In addition, a high frequency is provided to offer a regular service for large commuting passengers from the periodic perspective. Except for the traditional objectives of operational cost (Claessens et al., 1998; Goossens et al., 2004) and passenger total travel time (Schöbel and Scholl, 2005; Borndörfer et al., 2007), this paper proposes the number of lines as an objective in order to control the frequency of each line, which plays an important role for the line plan with both periodic and aperiodic pattern.

When the line plan is given, the MP-TTP needs to be optimized. Since the huge impact of delay propagation could result in worse punctuality of highly-utilized rail corridors, we design a periodic timetable optimization model with consideration of robustness and overtakings based on the Periodic Event Scheduling Problem (PESP) (Serafini and Ukovich, 1989). Robustness is taken into account to mitigate the delay propagation and improve the system efficiency of rail networks. Furthermore, overtaking is allowed in this model, and the number of overtakings is proposed as objective as it could impact the capacity utilization and timetable robustness when different travel times occur, especially for a dense corridor. According to the periodic and aperiodic nature, a new regularity constraint is introduced to provide regular services for trains from the same line even though the cycle time is longer than usual, and keep all trains follow the same path.

A collaborative optimization method of MF-LPP and MP-TTP is proposed as a line plan can be infeasible in the timetable optimization model. This means that a timetable may not be found, once a line plan is determined. Although many line planning models have an upper bound on the number of trains allowed on each infrastructure section, the capacity still could be not enough while generating a timetable at the corridor level. Models for integrating line planning and timetabling have been proposed in Schöbel (2015) and Burggraeve et al. (2017) with some assumptions. Moreover, passenger travel time in line planning normally only considers the shortest path, with respect to the minimal travel time without consideration of the timetable. Whereas travel time in the timetable shows the real scheduled time.

With the purpose of less capacity loss, less time loss, regularity and robustness of the rail system, this paper proposes an iterative approach from line plan to timetable to find a high-quality service plan. The main contributions of this paper are the following:

- We develop a multi-frequency line planning optimization model.
- We develop a multi-period timetabling optimization model.
- Overtaking constraints are presented based on dwell time stretches.
- An iterative framework finds high-quality multi-periodic timetables for heterogeneous rail lines.
- A real-life case-study on a Chinese railway corridor demonstrates the method.

The rest of this paper is structured as follows. Section 2 presents the literature review. In Section 3, we elaborate the optimization models of line plan and timetable. In Section 4, numerical experiments are applied, and the result is analyzed to demonstrate the applicability of the model and approach. Some conclusions and future research work are given in Section 5.

2. Literature review

In this section, we investigate models of line planning, timetabling, and the integration of both in recent studies, and analyze the literature gaps that this paper contributes to.

2.1. Line planning

The LPP aims at finding an optimal line plan for a railway network, and has been well-studied in the literature. There are two main conflicting objectives, minimization of operation costs from the operator's perspective (cost-oriented) and minimization of generalized travel cost from the passengers' perspective (customer-oriented).

Researchers have developed a series of integer programming models to achieve both objectives. For a cost-oriented approach, the LPP aims to find a line plan serving all passengers and minimizing the costs of operators. It is always modeled concerning the lines and corresponding frequencies or train types with capacity as presented in Claessens et al. (1998),Goossens et al. (2004), and Goossens et al. (2006). For the customer-oriented approach, maximizing the number of direct travelers is proposed in Bussieck et al. (1997), and minimizing the total travel time in Schöbel and Scholl (2005) and Borndörfer et al. (2007). Branch and bound, branch-and-cut, and column generation are used for solving these mixed integer linear programs. Borndörfer et al. (2007) consider a multi-commodity flow model in which passenger paths can be freely routed and lines are generated dynamically. Schöbel and Scholl (2005) used a change-and-go graph to model travel and transfer times. However, for large scale instances the model tends to be computationally intractable. Therefore, Borndörfer and Neumann (2010) proposed a compact integer programming approach to deal with transfer minimization problems even for large instances. They incorporated penalties for transfers that are induced by "connection capacities" and compared a direct connection capacity model with a change-and-go model. Bussieck et al. (2004) proposed a fast solution combining nonlinear techniques with integer programming, and a game-theoretic model is introduced in Schöbel and Schwarze (2006), where each line acts as a player to minimize cost. For a huge rail network in China, Fu et al. (2015) described an integrated hierarchical approach to optimize a line plan, with a classification of stations and trains, and a bi-level optimization model. The stop patterns are predefined by an enumeration of higher-classified stations for higher level trains and limiting the maximal number of stops for lower level trains. The minimization of passenger's travel time in the upper level is used to estimate passenger routes, while maximization of served passengers in the lower level is designed for optimization of frequencies. Schöbel (2012) gave a review of different line planning models from a mathematical and algorithmic approach. Goerigk et al. (2013) conducted a comprehensive experimental study to evaluate the quality of line plans from four different models by travel times and robustness.

2.2. Timetabling

Normally, the resulting line plan serves as a direct input for timetable scheduling, where arrival and departure times for the given lines should be found. Two types of timetables can be distinguished: aperiodic and periodic timetables.

For aperiodic timetables, many references consider mixed integer linear programming (MILP) formulations in which the arrival and departure times are represented by continuous variables while binary variables express the order of the train departures from each station. Branch-and-bound techniques were used for solving this model by Jovanović and Harker (1991), and heuristic techniques of local search, genetic algorithms, tabu search, and hybrid algorithms were compared for finding a feasible solution by Higgins et al. (1997). Caprara et al. (2002, 2006) designed integer linear programming models based on a graph representation, which discretized time into minutes and used Lagrangian relaxation to derive bounds for the optimal solution. A comprehensive survey on aperiodic train timetabling and train platforming can be found in Cacchiani et al. (2015). To minimize total passenger waiting time, Niu et al. (2015) developed a quadratic integer programming model with linear constraints to design a train timetable with given skip-stop patterns and passenger demand. For a given scheduled timetable, Jiang et al. (2017) inserted more trains by a skip-stop method in a highly congested railway line. More precisely, Caimi et al. (2017) gives an overview of timetable models and its applications, and also provides the methods for improving timetable robustness.

For periodic timetables, the Periodic Event Scheduling Problem (PESP) is frequently used for macroscopic scheduling. PESP was first proposed by Serafini and Ukovich (1989), and later it was extensively studied by many railway researchers, like Nachtigall (1996), Peeters (2003), and Liebchen (2007). Based on this model, the robustness and stability of periodic timetables were studied by Goverde (2007), Kroon et al. (2008) and Goverde et al. (2016). Kroon et al. (2008) discussed to improve the robustness by allocating buffer times between two successive trains and time supplements along train paths by a stochastic programming approach. Goverde et al. (2016) introduced a three level framework of integrated timetable construction with consideration of feasibility, stability, robustness, efficiency and energy consumption. They combine microscopic and macroscopic models of timetable design, as detailed in Bešinović et al. (2016). To address illegal overtakings (two trains occupy the same open track section at the same time), a relation between modulo parameters is presented to find a feasible timetable in Zhang and Nie (2016). Sparing and Goverde (2017) extended the PESP model with the objective function of the minimum cycle time of the periodic timetable. The model maximizes network stability while generating feasible timetables. Based on the foregoing model, Bešinović (2017, chapter 5) proposed a two-stage model to achieve a stable and robust timetable. The first stage is to find the optimal stable timetable structure by minimizing capacity utilization and journey times, and the second stage is to improve timetable robustness by optimizing allocations of the time allowance. Yan and Goverde (2017) compared several periodic timetable models, and a number of statistic indicators are defined to assess robustness.

An overview of nominal and robust timetable optimization for both periodic and aperiodic patterns is summarized in Cacchiani and Toth (2012). To assign buffer times and time supplement reasonably, Liebchen et al. (2009) combined stochastic programming and robust optimization to deal with uncertain data, and introduced the concept of *recoverable robustness*. Without consideration of infrastructure conflicts, Robenek et al. (2016) modeled a MILP timetable problem by maximizing the operation profit while maintaining a certain level of passenger satisfaction for both patterns, and the result shows aperiodic timetables are more efficient for high demand network than periodic ones. Based on the last paper, Robenek et al. (2017) proposed a hybrid timetable with both features of regularity and flexibility from periodic and aperiodic timetables respectively. Galli and Stiller (2018) discussed the new challenges of railway timetable problem: multi-period timetables and robustness, and the corresponding model and solution approach are proposed.

2.3. Integrated approach

Since the line plan has a huge impact on timetable design, an integrated model of LPP and TTP is considered to improve the quality of the service plan for both passengers and railway operators. Some scholars attempted to develop an integrated model to solve both problems at the same time. In Michaelis and Schöbel (2009), the traditional sequence of planning stages is reordered. They first determined the vehicle routes, and then optimized the line plan and timetable in a bus network with a heuristic approach. Kaspi and Raviv (2013) proposed an integrated model with consideration of passenger inconvenience and operational cost, and an evolutionary metaheuristic was introduced to achieve the goal. Yue et al. (2016) introduced a novel methodology to optimize train stopping patterns and the timetable simultaneously with the objective of train profit (based on stopping time and the number of stops), and a column-generation-based heuristic algorithm to solve the problem in a large scale network. Burggraeve et al. (2017) proposed an iterative and interactive framework to optimize the line plan and timetable. A heuristic algorithm is applied to generate a line plan with consideration of a feasible and robust timetable, while overtakings are not allowed. Yamauchi et al. (2017) designed an approach to optimize stop patterns of a railway line system. To relax congestion, both travel time and congestion rate are taken into account to find the Wardrop equilibrium. Borndörfer et al. (2017) studied the integration of periodic timetabling and routing, where four routing models are introduced to compare from a theoretical perspective. They found that different routing models could have a huge impact on the quality of the rail system. However, no exact solution approaches are given for the capacited routing problem. Due to the dependencies between different subproblem, Schöbel (2017) pointed out that integration would be more beneficial than solving the problems sequentially. She integrated line planning, timetabling and vehicle scheduling, and proposed heuristic approaches to solve the model. Pätzold et al. (2017) proposed three different ways to reduce the operation cost compared with a traditional sequential approach of the three mentioned stages. Very recently, Meng and Zhou (2019) developed an integrated optimization model considering passengers responses to service interval times, stopping patterns, arrival/departure times of trains, and capacity of infrastructure and rolling stock, and a team-based solution approach was used to synchronize demand assignment, routing, timetabling tasks.

2.4. Literature gaps

From our literature review, we have learned that periodic and aperiodic pattern were always designed by a totally different methodology. We cope with this problem by extending the period length and allow multiple frequencies in the LPP and multiple periods in the TTP. The number of lines is first introduced as objective in the line planning model in order to balance the line frequency, and a regularity constraint is designed to obtain a regular service within the extended period. So both MF-LPP and MP-TTP feature a periodic and aperiodic nature. Most robust optimization models mentioned above need an initial feasible timetable, where train orders have been fixed already, which leaves less space for improvements compared with flexible orders. As discussed in Peeters (2003), pulling apart trains using the same track could improve timetable robustness. Our paper aims at computing a robust timetable while determining train orders by spreading trains as far as possible. Overtakings are possible in our model, and the number of overtakings is introduced as objective as it would affect timetable robustness a lot. Heuristic methods are typically needed to search for the optimal solution for LPP and TTP. Therefore we developed a novel combined approach from line plan to timetable to find the optimal solution with consideration of seat capacity loss, passenger total travel time, timetable regularity and timetable robustness.

3. Model formulation

3.1. Problem statement

A line plan is a set of train lines with train ODs, train types, stop patterns and frequencies. Hence, the optimization model needs to be built from networks on three levels, as shown in Fig. 3. The first level is the railway infrastructure, which is called the physical network, including stations and tracks connecting the stations. In the second level, the train lines with stop pattern and frequency are provided to passengers, which gives for each OD an opportunity to travel. With the line plan scheduled, a service network to the passengers can be derived in the third level. Here we define the direct transport service for an OD from a line or an indirect transport service for an OD by several lines as a *service arc*. The service network is composed by service arcs from all lines. The passenger travel choice is influenced by the total travel time over different service arcs which have the same OD in the third level. This third level determines how passengers are assigned to each train, which is the network for the passenger assignment model.

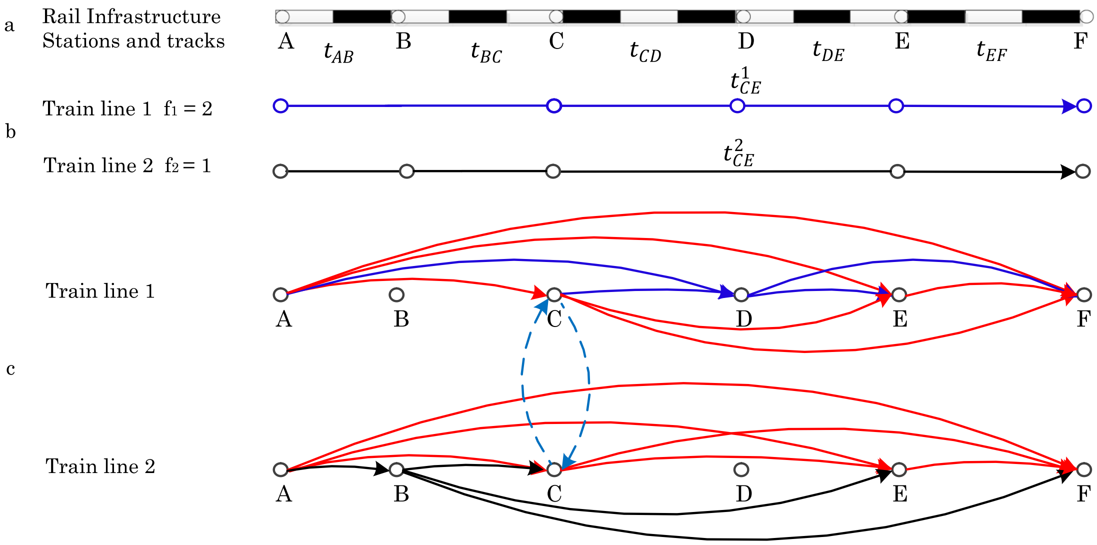

Fig. 3. Networks for the line planning optimization. Note: Red arcs represent the train services provided by both lines, and blue and black arcs stand for the train services served by the corresponding color line. The dashed blue arc denotes the transfer service between line 1 to line 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 3 also illustrates a simple network with six stations along a corridor, and sub-figures a (Rail infrastructure), b (Line system), and c (Service network) correspond to the three levels respectively. In the given rail corridor A to F, two train lines with the same OD, different stop patterns, and different frequencies are provided. One train line of two frequency ($f_1 = 2$), stops at C, D and E; and the other of one ($f_2 = 1$), stops at B, C and E. In the service network, it can be observed that both lines have 10 service arcs, and they also share five service arcs with the same OD (red arcs shown in Fig. 3). For example, both lines provide the train service for passengers from A to C, A to F, and C to E. If we assume that all lines have the same speed level and minimum train running times t_{AB}, t_{BC}, t_{CD}, t_{DE} and t_{EF} are always the same, line 1 would be more attractive for passengers from A to C because of a shorter travel time and higher frequency. For passengers from B to D, no direct service is available, so a transfer node and a connecting train needs to be selected (The dashed blue arc is a transfer arc). For passengers from C to E, line 1 offers a higher frequency but line 2 a non-stop service. The travel time difference between t^1_{CE} and t^2_{CE} is bigger than the dwell time at D, as $t^1_{CE} = t^{ac} + t_{CD} + t^{de} + t^{dwell}_D + t^{ac} + t_{DE} + t^{de}$ and $t^2_{CE} = t^{ac} + t_{CD} + t_{DE} + t^{de}$, where t^{ac} and t^{de} stand for acceleration and deceleration time loss. Hereby, the acceleration and deceleration time loss also affect the choice of the line, especially for a high-speed railway with bigger t^{ac} and t^{de}. In general, it is nontrivial which line is selected for each OD pair. Therefore, a passenger assignment model with a given objective is necessary to the line planning model to determine which train line to select for passengers.

The two main objectives for a line planning model are the operational costs and passengers generalized costs. Profit or cost is normally proposed as an objective from an operator's point. However, the monetary value from a train operating company is not always that accurate, and the empty-seat-hour could give a direct view on the capacity loss. Therefore, we choose this straightforward way to minimize the cost. From the passenger's perspective, in-vehicle time plays a primary role when passengers select a train line for a trip, so the passengers' total travel time (passengers weighted in-vehicle time) is used as the second objective. Moreover, according to the discussions in Section 1, the defined multi-frequency line plan with a multi-periodic pattern could be more efficient for heterogeneous railway lines, including both a periodic nature with higher frequencies and an aperiodic nature with more flexibility. Hence, the number of lines is proposed as an extra objective to control the flexibility of lines selected by passengers to achieve the shortest travel time, and the frequency within an appropriate range. Similar to recent research of the periodic LPP, a *line pool* is considered as a given input in this paper. A line pool is a predefined set of potential lines with different stop patterns, used to simplify the problem due to numerous combinations of all stop patterns and route choices.

With the proposed objectives, the given line pool, the vehicle seat capacity, and the day-based passenger OD demand, the passenger assignment optimization part is actually a shortest path problem with capacity constraints. To generate a multi-frequency line plan, the maximum frequency of each line should also be restricted to balance the frequencies in the line plan. Hence, the seat capacity is not only restricted by the objective of empty-seat-hour, but also by each line. A system-optimal assignment is used to find a trade-off between the total travel time, the empty-seat-hour and the number of train lines. Passengers may thus aim at the shortest travel time, but at the same time the train operator aims at minimizing empty-seat-hours and the number of different train lines (stop patterns), which may thus influence the number of trains available to passengers. Moreover, only direct connections are considered in this paper, so the transfer paths are not needed. Arcs from different lines may have different weights (in-vehicle times), so the same passenger OD could have different

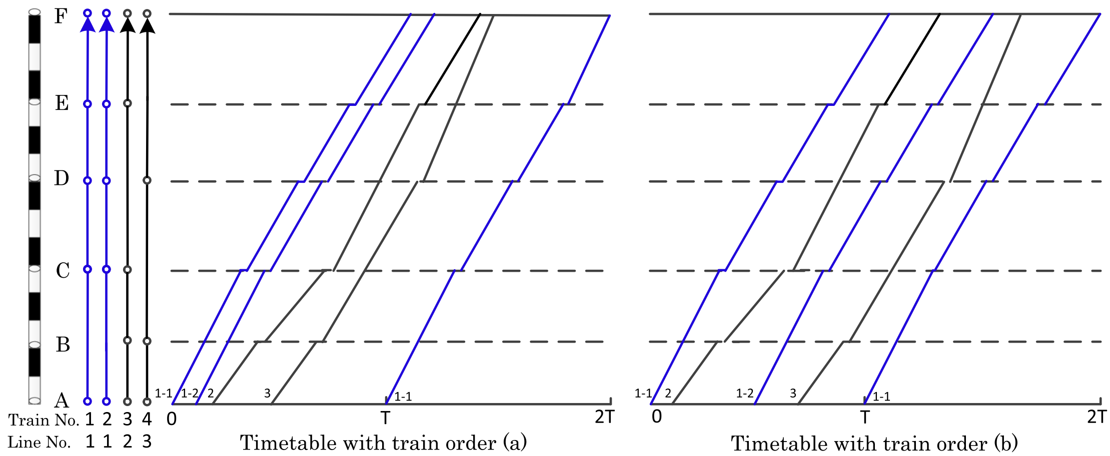

Fig. 4. Timetable structure with different train orders and headways.

travel times depending on the choice of service arcs with different weights. If a train line has the shortest travel time for several passenger ODs, then the corresponding passengers will compete with each other due to the seat capacity constraints. The optimization process will find the system-optimal solution for the model and the associated shortest paths for the passengers (in terms of travel times and constrained to seat capacity). The shortest path problem is embedded in the LPP (MILP) model, hence the system-optimal solution is our optimal line plan with stop patterns and corresponding frequencies for the LPP.

When a line plan is generated, a timetabling model is needed to compute a feasible and optimized timetable with consideration of certain objectives (like passenger travel time and timetable robustness) and constraints (infrastructure, regularity, and overtakings).

Take Fig. 4 as an illustration, where two timetables from the same line plan are displayed, with four trains of three stop patterns. Timetable (a) has a tight headway schedule but plenty of buffer at the end in one period. Train 1-1 and 1–2 with the same pattern are scheduled successively. Timetable (b) has more headway time between two departures and train line 1 has a regular schedule of a half cycle time. By observing the two timetables, it can be directly concluded that timetable (b) is more insensitive to delays and offers more regularity to passengers.

In order to obtain such a timetable with the given stop patterns and the corresponding frequencies, a robustness objective is introduced to find timetables that are less vulnerable to delays, and additionally the number of overtakings is proposed to control the increase of overtakings by the robustness objective. As our line plan is considered from a passenger's viewpoint, travel time including running times and dwell times, is also still an objective. Moreover, regularity constraints are added to lines with multiple frequencies to follow the same path, and in this sense, each train line could have its own cycle time (with slight deviations) within the overall period length. As defined before, this type of timetables can be considered as a multi-period timetable, corresponding to a multi-frequency line plan. To be specific, for our multi-frequency line plan, each train line is periodic (with possibly a slight regularity tolerance) in our model with the line cycle time determined by the frequency, while the multi-period timetable actually shows different periodic patterns for each line. With the presented objectives and constraints, a modified PESP model is proposed to generate an efficient multi-period timetable.

3.2. MF-LPP model

In order to simplify the complexity of the line planning problem, we use some assumptions.

(1) All passengers have a direct service to their destinations without any transfer, so train lines are chosen such that all passengers can travel directly.
(2) Only one direction is taken into account for optimization in our model. We assume that passenger demand is symmetric, and it can be extended to both directions.
(3) For simplicity, all train types have the same seat capacity.

The notation of parameters and variables is summarized in Tables 1 and 2 respectively.

Using the notation explained in the tables, the mathematical model of the MF-LPP is formulated as follows.

$$\text{Minimize} \quad \alpha \cdot \sum_{l\in\mathcal{L}^0}\sum_{m=1}^{N_l-1}\left(C\cdot f_l-\sum_{i=1}^{m}\sum_{j=m+1}^{N_l} q^l_{s_i,s_j}\right)\cdot t^l_{s_m,s_{m+1}} + (1-\alpha)\cdot\sum_{l\in\mathcal{L}^0}\sum_{i=1}^{N_l-1}\sum_{j=i+1}^{N_l} t^l_{s_i,s_j}\cdot q^l_{s_i,s_j} + \beta\cdot\sum_{l\in\mathcal{L}^0} x_l \tag{1}$$

This objective function is a multi-objective function, and includes three sub-objectives:

Table 1
Subscripts and parameters used in the LPP model.

Symbol	Definition
$\mathcal{L}^0$	Line pool, the set of potential train lines with various train stop patterns
$\mathcal{S}$	Set of stations in the corridor
l	Train line from $\mathcal{L}^0$ with l-th stop pattern
$\mathcal{S}_l$, N_l	Set of stations on line l, and the number of stations on line l respectively
C	Seat capacity of rolling stock
μ	Maximal seat occupancy rate in passenger assignment model
P_{s_i,s_j}	Passenger demand between stations s_i and s_j
$f_{\max}$	Upper bound of frequency for each train line
$v^l_{s_i}$	Stop parameter which equals 1 if line l stops in station s_i, and 0 otherwise
$t^l_{s_i,s_j}$	In-vehicle time from s_i to s_j on line l, including running times (with acceleration and deceleration times), and possible intermediate dwell times
α	Weight of total train empty-seat-hours, and $1-\alpha$ is the weight of passenger total travel time, $\alpha \in [0, 1]$
β	Weight of the total number of lines in the objective function, $\beta \geq 0$

Table 2
Decision variables in the LPP model.

Symbol	Definition
f_l	Frequency of train line l.
x_l	Binary variable, which equals 1 if line l is selected in the line plan, and 0 otherwise.
$q^l_{s_i,s_j}$	The number of passengers from station s_i to s_j assigned to line l.

$$Z_1 = \sum_{l\in\mathcal{L}^0}\sum_{m=1}^{N_l-1}\left(C\cdot f_l - \sum_{i=1}^{m}\sum_{j=m+1}^{N_l} q^l_{s_i,s_j}\right)\cdot t^l_{s_m,s_{m+1}} \tag{2}$$

$$Z_2 = \sum_{l\in\mathcal{L}^0}\sum_{i=1}^{N_l-1}\sum_{j=i+1}^{N_l} t^l_{s_i,s_j}\cdot q^l_{s_i,s_j} \tag{3}$$

$$Z_3 = \sum_{l\in\mathcal{L}^0} x_l \tag{4}$$

Z_1 describes the maximum capacity utilization by minimizing the total empty-seat-hours, and Z_2 depicts the passengers' travel time, which is in-vehicle travel time. Z_3 refers to the number of lines or stop patterns in the line plan. In order to find a way to deal with multi-objectives, certain weights could be assigned to each objective. As the weight in the objective function has a great impact on the final decision, it needs to be calibrated for a certain case. If a cost-oriented line plan is requested, it is better to assign a value to α close to 1; and if a customer-oriented line plan is required, the value is better close to 0. We calibrate α by varying its value between 0 to 1 and compute the corresponding objective values without consideration of Z_3. Then the objective value as function of α could help to determine which value to choose for a certain case. For β, we aim to find a value, such that Z_3 would not have a big impact to other two objectives. The frequency of lines can be optimized and the number of passengers assigned to each train line is obtained for a given line pool and the parameters.

The constraints are described as follows in four groups, where multi-frequency constraints and line constraints are dedicatedly designed for the line plan with the multi-periodic pattern.

Passenger flow constraints:

$$\sum_{l\in\mathcal{L}^0} q^l_{s_i,s_j} = P_{s_i,s_j} \qquad \forall s_i, s_j \in \mathcal{S} \tag{5}$$

$$q^l_{s_i,s_j} \leq M\cdot v^l_{s_i}\cdot v^l_{s_j} \qquad \forall s_i, s_j \in \mathcal{S}_l, l \in \mathcal{L}^0 \tag{6}$$

$$q^l_{s_i,s_j} \in \mathbb{N} \qquad \forall s_i, s_j \in \mathcal{S}_l, l \in \mathcal{L}^0 \tag{7}$$

Constraint (5) specifies that each passenger OD demand needs to be satisfied and is equal to the sum of passengers assigned to all lines. Constraint (6) ensures that if there is no service arc from station s_i to station s_j on line l, then the corresponding passenger flow assigned to this arc equals 0. M is a very large positive number and equals $\max_{s_i,s_j} P_{s_i,s_j}$. The passenger flow on each service arc of train line l is restricted to nonnegative integers in (7). Note that we denote $\mathbb{N} = \{0, 1, 2, \ldots\}$.

Seat capacity constraints:

$$\sum_{i=1}^{m}\sum_{j=m+1}^{N_l} q^l_{s_i,s_j} \leq \mu\cdot C\cdot f_l \qquad \forall s_m \in \mathcal{S}_l, l \in \mathcal{L}^0 \tag{8}$$

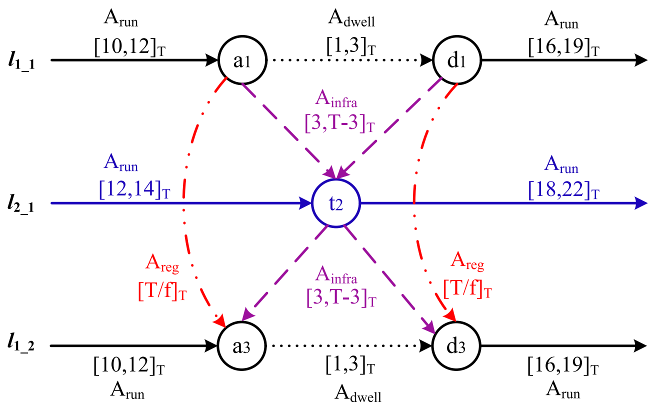

Fig. 5. Periodic Event-activity network for three trains of two lines in a station. l_{1_1} and l_{1_2} are from the same line with the same black color, and l_{2_1} has a through event at this station (the *t* means through). Solid lines represent running constraints, dotted lines are for dwell constraints, dashed for headway constraints, and dash-double-dotted for regularity constraints.

Constraint (8) ensures that the seat capacity of each line in each successive section is no less than the passengers assigned and guarantees that the service frequency between each OD is sufficient to meet all passenger demand.

Multi-frequency constraints:

$$f_l \leq f_{\max} \qquad \forall l \in \mathcal{L}^0 \tag{9}$$

$$f_l \in \mathbb{N} \qquad \forall l \in \mathcal{L}^0 \tag{10}$$

Constraint (9) allows that the frequency of each train line could only vary in a certain range, and constraint (10) assures that the frequency variable is a nonnegative integer. If the frequency is zero, then this line is not selected in the line plan. The upper bound could limit the maximal frequency of lines, and impact the number of lines when changed. A large positive number is set for the first optimization step, and would be updated in the following iterations in our model.

Line constraints:

$$x_l \in \{0, 1\} \qquad \forall l \in \mathcal{L}^0 \tag{11}$$

$$f_l / f_{\max} \leq x_l \leq f_l \qquad \forall l \in \mathcal{L}^0 \tag{12}$$

Constraint (11) restricts line selection variable x_l as a binary integer, and constraint (12) linearizes the binary variable by line frequency. If the frequency of a line l equals 0, then the upper bound ensures x_l to be 0. Otherwise, if the frequency is bigger than 0, the lower bound constraint guarantees it to be 1.

3.3. MP-TTP model

When the line plan has been generated, a timetabling model is applied to find a feasible, regular and robust timetable. Timetable models can be at the microscopic and macroscopic level, see Bešinović et al. (2016). With the given input from the line plan, our timetabling model is at a macroscopic level, where stations are considered as points, and minimum and maximum running and dwell times along the routes of each train line are provided. The PESP model is widely used for solving the macroscopic periodic timetable problem, hence we adapted this model to optimize the timetable. The PESP formulation can be represented by a direct graph $\mathcal{G} = (\mathcal{E}, \mathcal{A})$, which represents a periodic event-activity network.

With a given line plan, the model is associated with a set of train lines L. Each line $l \in L$, defines a stopping pattern $(s_1^l, .s_k^l., s_{N_l}^l)$ and a frequency f_l within a given time period T. The set $\mathcal{E}$ contains $\mathcal{E}_{\text{dep}}, \mathcal{E}_{\text{arr}}, \mathcal{E}_{\text{arr\_thr}}$ and $\mathcal{E}_{\text{dep\_thr}}$ representing departure events at s_1^l, arrival and departure events (for stops) or arrival through and departure through events at s_k^l, and arrival events at $s_{N_l}^l$ for all frequencies of all lines. We defined a through event with both arrival through and departure through in order to simplify the model in a later stage (like the overtaking model part), while it does not influence too much for the computation time. The set of activities $\mathcal{A}$ which link these events, represents the constraints on the process time between a pair of events, as shown in Fig. 5. For each event i, we determine the scheduled time $\pi_i \in [0, T)$ in a basic period while satisfying the set of constraints $\mathcal{A}$. Due to periodicity, this event would occur at times $\pi_i + z \cdot T$, where $z = 1, 2, \ldots$. Each process time a_{ij} corresponds to an activity $(i, j) \in \mathcal{A}$, where i and j are two consecutive events, which can be distinguished as running time, dwell time, headway time, and regularity interval, and each of them has a lower l_{ij} and upper bound u_{ij} modulo T, also represented as $[l_{ij}, u_{ij}]_T$.

Running activities $\mathcal{A}_{\text{run}}$ and dwell activities $\mathcal{A}_{\text{dwell}}$ are generated from the consecutive events of the same train. The lower bound for running time is the minimum running time, which equals the technical running time plus a proper time

Table 3
Sets and parameters used in the TTP model.

Symbol	Definition
T	Period length
$\mathcal{L}$	Set of line plan, index $l \in \mathcal{L}$, $\mathcal{L}^n$ represent the line plan from the $n-th$ iteration
$\mathcal{T}$	The set of achieved timetables, $\mathcal{T}^n$ is the timetable from the $n-th$ line plan if it exists
$\mathcal{E}$	Set of all events, index $i, i', j, j', n, m \in \mathcal{E}$
$\mathcal{E}_{\text{arr}}$	Set of arrival events, $\mathcal{E}_{\text{arr}} \subset \mathcal{E}$
$\mathcal{E}_{\text{dep}}$	Set of departure events, $\mathcal{E}_{\text{dep}} \subset \mathcal{E}$
$\mathcal{E}_{\text{thr}}$	Set of through events, $\mathcal{E}_{\text{thr}} \subset \mathcal{E}$
$\mathcal{A}$	Set of activities, index ij, ii', jj', $i'j'$, nm
$\mathcal{A}_{\text{run}}$	Set of running activities, $\mathcal{A}_{\text{run}} \subset \mathcal{A}$
$\mathcal{A}_{\text{dwell}}$	Set of dwell activities, $\mathcal{A}_{\text{dwell}} \subset \mathcal{A}$
$\mathcal{A}_{\text{infra}}$	Set of headway activities, $\mathcal{A}_{\text{infra}} \subset \mathcal{A}$
$\mathcal{A}_{\text{reg}}$	Set of regularity activities, $\mathcal{A}_{\text{reg}} \subset \mathcal{A}$
$\mathcal{A}_{\text{reg}}(s_k^l)$	Set of regularity activities of line l at station s_k, $\mathcal{A}_{\text{reg}}(s_k^l) \subset \mathcal{A}_{\text{reg}}$
$\mathcal{P}_l$	Set of activities from line l
a_{ij}	Scheduled process time of activity (i, j)
l_{ij}	Lower bound of activity (i, j) (from event i to event j)
u_{ij}	Upper bound of activity (i, j)
h_{ij}	Minimum headway time of activity (i, j) with two events at the same station
r_{ij}	Process time of running activity (i, j) from the line plan, considered as minimum running time
θ	The running time supplement rate, $\theta \geq 1$
d_{ij}	Process time of dwell activity (i, j) from line plan
η_{ij}	Parameter of maximal allowed dwell time buffer if an overtaking occurs
$O_l^{\max}$	The maximal overtakings allowed for train line l
ξ	Regularity tolerance, $0 \leq \xi \leq 2$
ϕ_{ij}	Robustness parameter, $\phi_{ij} = (u_{ij} + l_{ij})/2, \quad \forall (i, j) \in \mathcal{A}_{\text{infra}}$
λ_{ij}	Weight of each process (i, j)
ω_i	Weights of each objective, $\omega_i \in [0, 1]$, $i = 1, 2$ and 3

Table 4
Variables in the LPP model.

Symbol	Definition
π_i	A continuous variable, scheduled time for event i
z_{ij}	A binary variable, considered as modulo parameter of activity (i, j) deciding the order of event i and j
$c_{ii'jj'}$	An integer variable, auxiliary variable to forbid illegal overtakings during sections between activities of (i, i') and (j, j')
δ_{ij}	A continuous variable, the headway deviation to ϕ_{ij}
y_{ij}	A binary variable that equals 1 if activity (i, j) has a dwell time stretch, and 0 means no stretch and also no overtake occurring

supplement that covers various train behavior. The upper bound is the maximum running time that can be accepted by passengers. The minimum time for boarding and alighting of passengers, and the maximum time for passengers waiting at stations represent the lower bound and upper bound of the dwell process. Headway activities $\mathcal{A}_{\text{infra}}$ are generated between different train events at the same station to ensure a safety distance. The minimal safety interval is the lower bound l_{ij} for headway time, whilst $T - l_{ji}$ is the upper bound to ensure the safety between trains in the reverse order. If the frequency f_l of line l is greater than one, regularity activities $\mathcal{A}_{\text{reg}}$ are needed to ensure a regular service. Regularity activities mean that headway between successive trains from the same line are equal at each station. When strict regularity is needed, trains from the same line should have the same interval T/f_l at each station. Transfer connections and rolling stock connections are not considered in this paper.

According to the description, the notation of parameters and variables is summarized in Tables 3 and 4 respectively.

The mathematical timetabling model is defined as follows. We introduce a model for MP-TTP with multi-objective: train journey time, timetable robustness, and the number of dwell time stretches.

$$\text{PESP-MP} \quad \text{Minimize} \quad \omega_1 \sum_{(i,j)\in A_{\text{run}} \cup A_{\text{dwell}}} \lambda_{ij} \cdot (\pi_j - \pi_i + z_{ij} \cdot T) + \omega_2 \sum_{(i,j)\in A_{\text{infra}}} \delta_{ij} + \omega_3 \sum_{(i,i')\in A_{\text{dwell}}} y_{ij} \tag{13}$$

In objective function (13), ω_1, ω_2 and ω_3 stand for the weight of each objective respectively due to the different order of magnitude of each objective value. When all three objectives need to be satisfied, traditionally appropriate weights are assigned to deal with this multi-objective problem. Also, λ_{ij} represents the weight of each process. The constraints are elaborated as follows.

Periodicity constraints:

$$0 \leq \pi_i < T \qquad \forall i \in \mathcal{E} \tag{14}$$

$$z_{ij} \in \{0, 1\} \qquad \forall (i, j) \in \mathcal{A} \tag{15}$$

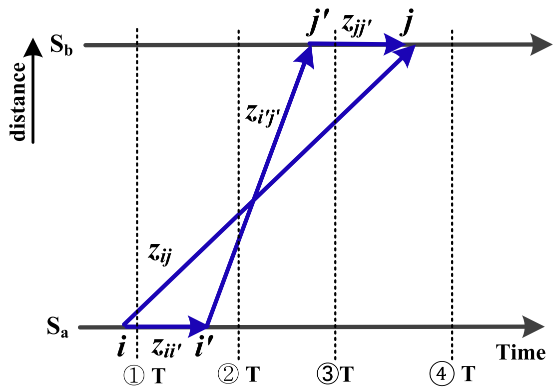

Fig. 6. Collision in a railway section. ①, ②, ③, and ④ represent four different positions of period length boundary T, and the sum of four modulo parameters equals to one or three for all boundary positions.

Constraint (14) requires periodicity of events by bounding to $[0, T)$. The modulo parameter z_{ij} determines the order of event i and j within the defined period T. Here we assume $u_{ij} - l_{ij} \in [0, T-1]$ and $l_{ij} \in [0, T-2]$. Then z_{ij} is a binary variable in our model constrained in (15). $z_{ij} = 1$ if $\pi_i > \pi_j$, and it is 0 otherwise.

Running time and headway time constraints:

$$h_{ij} \le \pi_j - \pi_i + z_{ij} \cdot T \le T - h_{ij} \qquad \forall (i,j) \in \mathcal{A}_{\text{infra}} \tag{16}$$

$$r_{ij} \le \pi_j - \pi_i + z_{ij} \cdot T \le \theta \cdot r_{ij} \qquad \forall (i,j) \in \mathcal{A}_{\text{run}} \tag{17}$$

Constraint (16) ensures that all headway process times are within the given safety bounds. Constraint (17) allows that running activities could vary in a certain range, and the upper bound is defined by a given proper time supplement rate based on the minimal running time.

Non-collision constraints:

$$z_{ij} + z_{i'j'} + z_{ii'} + z_{jj'} = 2 \cdot c_{ii'jj'} \qquad (i,j),(i',j') \in \mathcal{A}_{\text{run}}, (i,i'),(j,j') \in \mathcal{A}_{\text{infra}} \tag{18}$$

$$0 \le c_{ii'jj'} \le 2 \qquad (i,j),(i',j') \in \mathcal{A}_{\text{run}}, (i,i'),(j,j') \in \mathcal{A}_{\text{infra}} \tag{19}$$

$$c_{ii'jj'} \in \mathbb{N} \qquad (i,j),(i',j') \in \mathcal{A}_{\text{run}}, (i,i'),(j,j') \in \mathcal{A}_{\text{infra}} \tag{20}$$

Constraints (18)–(20) guarantee that no collision can arise (see Fig. 6), when the sum of four modulo parameters of related running and infra processes equals zero, two or four, see for details in Zhang and Nie (2016).

Regularity constraints:

$$\left\lfloor \frac{T}{f_l} \right\rfloor - \xi \le \pi_j - \pi_i + z_{ij} \cdot T \le \left\lceil \frac{T}{f_l} \right\rceil + \xi \qquad \forall (i,j) \in \mathcal{A}_{\text{reg}}(s_1^l), l \in L \tag{21}$$

$$\pi_n - \pi_m + z_{mn} \cdot T = \pi_j - \pi_i + z_{ij} \cdot T \qquad \forall (m,n) \in \mathcal{A}_{\text{reg}}(s_k^l), (i,j) \in \mathcal{A}_{\text{reg}}(s_1^l), k \in [2,N], l \in L \tag{22}$$

Regularity constraints are formulated in (21) and (22). Flexible frequencies are proposed in this paper, so f_l could be any number given from a line plan. A parameter ξ is introduced to provide a certain tolerance in case T/f_l is not integer or to express the tolerance from strict regularity. We assume that all trains from the same line have the same trajectory (train path), which means the same running time between two successive stations and dwell time at each station.

Robustness constraints:

$$-\delta_{ij} \le \pi_j - \pi_i + z_{ij} \cdot T - \phi_{ij} \le \delta_{ij} \qquad \forall (i,j) \in \mathcal{A}_{\text{infra}} \tag{23}$$

The robustness constraint is displayed in (23), where the variable δ_{ij} and parameter ϕ_{ij} are introduced (see details in Peeters, 2003) as

$$\phi_{ij} = \frac{u_{ij} + l_{ij}}{2} \tag{24}$$

$$\delta_{ij} = \left| \pi_j - \pi_i + z_{ij} \cdot T - \phi_{ij} \right| \tag{25}$$

Parameter ϕ_{ij} is the middle of $[l_{ij}, u_{ij}]$. For a certain process $(i,j) \in \mathcal{A}_{\text{infra}}$, if $\pi_j - \pi_i + z_{ij} \cdot T = \phi_{ij}$, it could provide the best robust solution for event i and j since the two events are distributed as far as possible. Formulation (23) is the linearized version of (25) since $\pi_j - \pi_i + z_{ij} \cdot T - \phi_{ij}$ can be positive or negative.

Overtaking constraints:

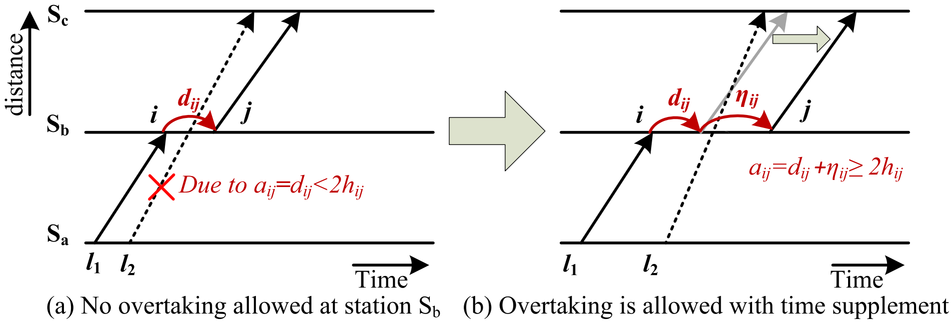

Fig. 7. Overtaking at stations. (i, j) represents a dwell activity of line l_1 at station S_b, where a_{ij} is the activity time and d_{ij} is the minimum dwell time.

$$d_{ij} \leq \pi_j - \pi_i + z_{ij} \cdot T \leq d_{ij} + \eta_{ij} \cdot y_{ij} \qquad \forall (i, j) \in \mathcal{A}_{\text{dwell}} \tag{26}$$

$$\sum_{(i,j)\in\mathcal{P}_l, (i,j)\in\mathcal{A}_{\text{dwell}}} y_{ij} \leq O_l^{max} \qquad \forall l \in \mathcal{L} \tag{27}$$

The number of overtakings might increase when robustness is appealing to. In order to analyze the trade-off between robustness and overtakings, the number of overtakings are considered to be optimized. Overtaking constraints are proposed in (26) and (27). Constraint (27) guarantees the lower bound and upper bound of dwell activities, and constraint (27) restricts the maximal number of overtakings for line l. In this model, we use the number of "dwell time stretches" to represent the overtakings, which is explained in more detail next. The minimum dwell time d_{ij} (the lower bound l_{ij} of dwell activity (i, j)) of a station is always 1 or 2 min for passenger trains which is smaller than twice the minimum headway h_{ij}. Fig. 7(a) displays that overtaking is not possible when there is no time supplement for a dwell activity, and Fig. 7(b) shows that overtaking could occur when enough time supplement η_{ij} is given. Traditionally η_{ij} can be predefined by a maximum waiting time at stations. Here we assume $d_{ij} + \eta_{ij}$ is exactly equal to $2h_{ij}$, which means that only one overtaking could occur for each dwell activity.

Meanwhile, total dwell time would increase a lot if time supplements are directly added for every dwell activity. The minimum dwell time at stations would be difficult to achieve even without an overtaking. In order to ensure that a dwell activity has a minimum dwell time when no overtaking occurs, an extra binary variable y_{ij} is proposed to the dwell time supplement η_{ij}. The upper bound of the dwell activity then becomes

$$u_{ij} = d_{ij} + \eta_{ij} \cdot y_{ij} \tag{28}$$

When y_{ij} equals one, it can be interpreted as "a stretch of dwell time" and overtaking could occur, since the dwell time is stretched to twice the minimum headway. So if a dwell activity time is larger than the corresponding minimum dwell time, then it is a *dwell time stretch*. Otherwise, the dwell time equals the minimum dwell time, and overtaking cannot happen as the time is not enough for infrastructure constraints. The minimization of the number of dwell time stretches could also reduce overtakings. Therefore, y_{ij} represents whether an overtaking occurs for activity (i, j), and the model can be considered as minimizing the number of overtakings.

In general, we made modifications in both the objectives and constraints of the traditional PESP-based models (with PESP being just a feasibility problem) to find an efficient timetable. First, we extended the feasibility model to a multi-objective timetabling model with extra binary variables, PESP constraints including a binary variable in the upper bound, and extra constraints on the binary variables. For the objective of robustness, we chose to minimize the sum of headway deviation to half the cycle time for all infrastructure activities (all headways between trains), which tries to pull apart headways as far as possible in order to have more buffer times between trains. This absorbs or reduces disturbances and avoids or reduces propagation of delays to other trains. The most significant contribution are the overtakings constraints introducing "dwell time stretches with additional binary variables and constraints (26) and (27). Overtakings affect the capacity utilization and timetable robustness for mixed train traffic (with different running times and stopping patterns). The existing methods to express an overtaking in the timetabling model brings at least three variables (see also Zhang and Nie, 2016; Sparing and Goverde, 2017), while our model simplifies this part by introducing one dwell time stretch variable. Due to the MF-LPP model, the train lines have flexible frequencies. A strict regularity constraint could easily lead to an infeasible problem in the timetabling model. Hence we introduced regularity constraints with some tolerance to the strict regularity between trains from the same line but still following the same path.

The proposed robustness objective helps to find a robust timetable, however also leads to a bigger and harder optimization problem due to the huge increase of soft constraints (21). The regularity and overtaking constraints reduce the search space to find the optimal solution more efficiently.

Moreover, in order to improve the computation time, we also define two sub-models. The PESP model only aiming at minimizing passenger travel time is defined as PESP-PTT, and the model with consideration of regularity, passenger travel time and overtaking is PESP-REG. Each sub-model is formulated as follows.

$$\text{PESP-PTT} \quad \text{Minimize} \sum_{(i,j)\in\mathcal{A}_{\text{run}}\cup\mathcal{A}_{\text{dwell}}} \lambda_{ij}\cdot(\pi_j-\pi_i+z_{ij}\cdot T) \tag{29}$$

subject to (14)–(20), (26) and (27).

$$\text{PESP-REG} \quad \text{Minimize} \;\; \omega_1 \sum_{(i,j)\in\mathcal{A}_{\text{run}}\cup\mathcal{A}_{\text{dwell}}} \lambda_{ij}(\pi_j-\pi_i+z_{ij}\cdot T)+\omega_3\sum_{(i,j)\in\mathcal{A}_{\text{dwell}}} y_{ij} \tag{30}$$

subject to (14)–(22), (26), and (27).

3.4. Iterative optimization

The framework of the iterative optimization process is illustrated in Fig. 8. The circled numbers represent the iteration processes, and are designed for specifying the iteration path. Here we assume that the railway infrastructure capacity is enough to operate all trains when seat occupancy is one (i.e., 100%).

MF-LPP: For the line planning models, Schöbel (2012) has already discussed that most of the line planning models are NP-hard even if both the upper and lower bound on frequency for each edge equals one in the LP-Basic model. Our MF-LPP model is similar to the one in Schöbel and Scholl (2005), which is proved NP-hard, while we have extra objective and constraints for the number of lines. Therefore, the optimization problem is also associated with an NP-hard feasibility problem and has long computation time for big cases. For the given input of passenger demand, line pool, infrastructure data and corresponding parameters, an initial line plan $\mathcal{L}$ is obtained by solving the MF-LPP model until the optimization gap is achieved. In Fig. 8, n represents a given number of iterations for the MF-LPP model.

MP-TTP: The basic PESP model is a feasibility problem without any objectives. It is proved NP-complete for a fixed period time $T \geq 3$ by several researchers (Serafini and Ukovich, 1989; Peeters, 2003). When an objective is applied to PESP, the related feasibility problem becomes NP-hard as discussed in Nachtigall (1996). Whereas, multi-objective is introduced to this MP-TTP with extended period length $T = 180$. Moreover, we added non-collision constraints (the relation between modulo parameters) and overtaking constraints in our multi-objective model which differ from the basic PESP constraints. Hence it is a hard problem even on corridors and a challenge to achieve exact optimality with limited computation time. Our previous research (Yan and Goverde, 2017) showed that a similar model could find the optimal solution within two seconds with a smaller case, if the infrastructure capacity is enough. With the input of the previous optimized line plan, the MP-TTP model is applied to find the optimal timetable. But a timetable might not exist for a given line plan as mentioned before. By testing our TTP model with a couple of line plans, it could be observed that the model might be infeasible or no solution occurs within the given computation time limitation. We observed that the infrastructure capacity and the regularity constraints could directly lead to an infeasible solution, and the model size plays a crucial role on the infeasible issue when it comes to computation time limitation. The following steps are designed to deal with the infeasibility in MP-TTP and the feedback loop to MF-LPP for exploring a better timetable. The model in each step could find a feasible solution within one minute, but could not achieve a zero optimality gap until the computation time limitation (60 min). Hence, the optimization gap is analyzed in our study instead of computation time.

- Step 1 PESP-PTT: A feasibility check of capacity utilization for the newly generated line plan is applied to the PESP-PTT model by setting the optimization gap to 100%. So when a feasible solution is found, it stops computing. Otherwise, if no solution could be found, we increase the seat occupancy rate to one in the MF-LPP model. As higher seat occupancy rate leads to fewer trains, as well as fewer constraints in the TTP model, this makes the model easier to solve. A feasible solution would be obtained by running the MF-LPP model again. Note that we assume that the capacity is enough for seat occupancy $\mu = 1$.
- Step 2 PESP-REG: This step solves the line plan (feasible in Step 1) by PESP-REG with a given optimization gap and computation time limitation. PESP-REG considers minimization of passenger travel time and the number of overtakings as well as regularity constraints with initial regularity tolerance $\xi = 0$. If the model is infeasible, the regularity tolerance is relaxed to $\xi + 1$ until a feasible solution is found with $\xi \leq 2$. If no solution could be found when $\xi = 2$, reduction of the frequency bound is applied as a lower frequency relaxes the regularity constraints. The upper bound of the frequency in the MF-LPP model is decreased by $f^n_{\max} = F^{n-1}_{\max} - 1$ ($f^n_{\max}$ is the upper bound of the frequency in the MF-LPP model, and $F^{n-1}_{\max}$ is the maximal frequency from the obtained line plan in the previous iteration). Then the MF-LPP model is solved with the updated frequency constraints, and a new line plan $\mathcal{L}^n$ is generated.
- Step 3 PESP-MP: If model PESP-REG in step 2 could find a feasible solution within the given regularity tolerance bound, this solution is set as the initial solution to PESP-MP. Then we optimize the timetable by PESP-MP with consideration of passenger travel time, timetable robustness and the number of overtakings until the optimization gap or computation time limitation.

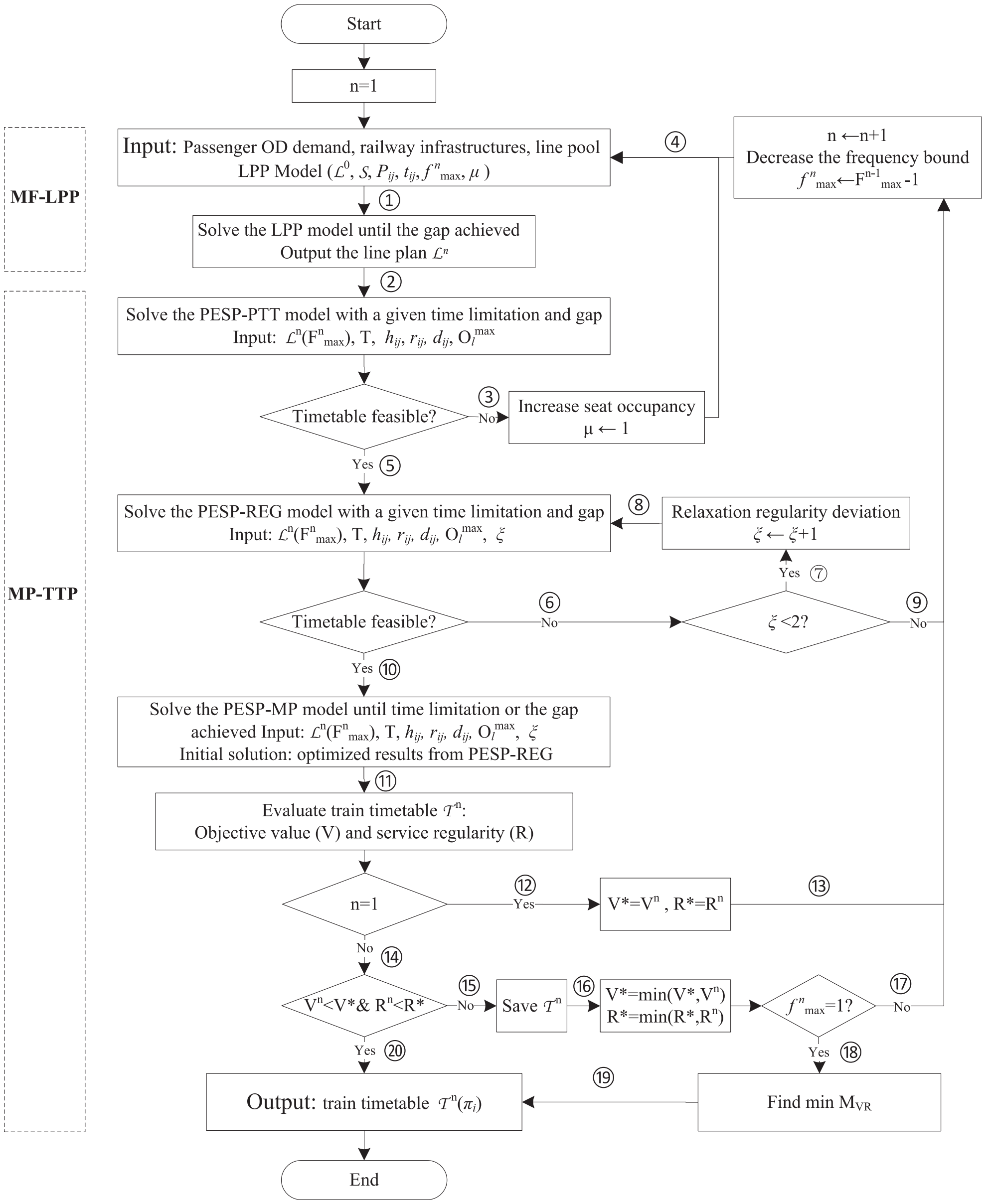

Fig. 8. Framework of optimization process.

Evaluation: In this framework, two evaluation indicators are used: The objective value V and regularity condition R. V can be directly obtained during the optimization process, but R could only be calculated when the timetable is generated. R not only considers the periodic lines (with multiple frequency) but also aperiodic ones (frequency is one). The number of passenger ODs (N_{OD}) is known when the passenger demand is given. When the timetable is obtained, the train order is also determined. We define $I^{\tau,\tau'}_{s_i,s_j}$ as the departure interval between two successive trains (τ and τ') of which both have service arcs from s_i to s_j (or both have stops at station s_i and s_j). The sum of the service frequency $F(s_i,\ s_j)$ from s_i to s_j can be calculated by

$$F(s_i, s_j) = \sum_{l \in \mathcal{L}} f_l \cdot v^l_{s_i} \cdot v^l_{s_j} \tag{31}$$

Hence the average service interval is

$$\bar{I}_{s_i,s_j} = \frac{T}{F(s_i, s_j)} \tag{32}$$

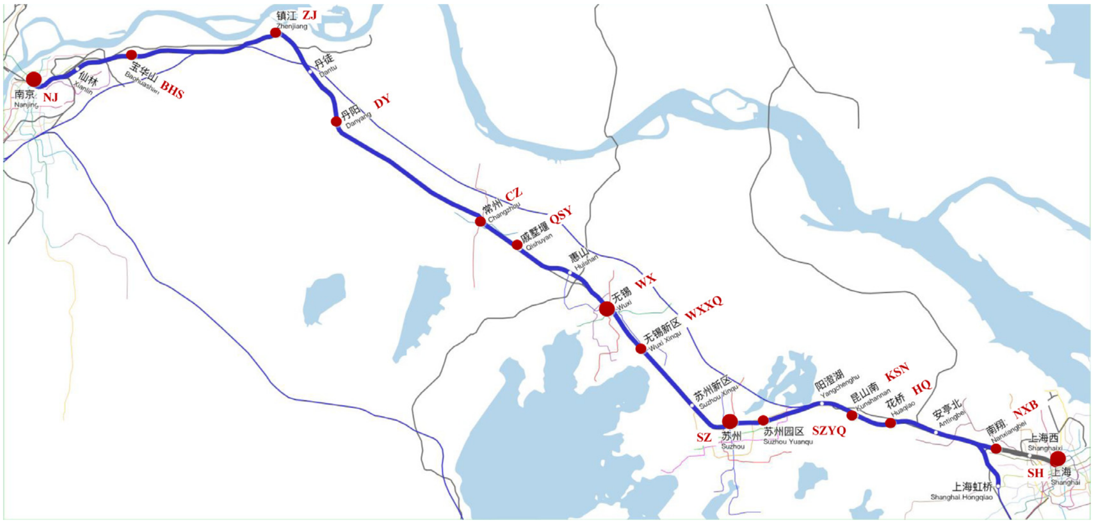

Fig. 9. Railway corridor of Shanghai–Nanjing high-speed railway (Source: https://en.wikipedia.org/wiki/Shanghai%E2%80%93Nanjing_intercity_railway).

Then the regularity deviation for the OD from s_i and s_j can be obtained by Eq. (33) and the average regularity condition of the timetable by (34).

$$R(s_i, s_j) = \frac{1}{F(s_i, s_j)} \sum_{\tau=1}^{F(s_i,s_j)} \frac{|I^{\tau,\tau'}_{s_i,s_j} - \bar{I}_{s_i,s_j}|}{\bar{I}_{s_i,s_j}} \tag{33}$$

$$R = \frac{1}{N_{OD}} \sum_{s_i \in \mathcal{S}} \sum_{s_j \in \mathcal{S}: i<j} R(s_i, s_j) \tag{34}$$

The first achieved V and R is marked as the best (minimal) objective value V^* and regularity condition R^*, and the upper bound of frequency in the **MF-LPP** model is modified by $f^n_{\max} = F^{n-1}_{\max} - 1$. Then we solve the model as in the previous steps, and check whether the new obtained V and R are lower than V^* and R^* respectively. If only one lower value occurs, V^* or R^* would be replaced by the corresponding lower value. If both V and R are lower than V^* and R^*, the iteration to the line plan model would terminate. Then the timetable and line plan are achieved with the lowest existing V and R. Moreover, if the lowest V and R could not be found until $f_{\max} = 1$, we find the timetable with the total minimal

$$M_{VR} = \frac{V^n - V^*}{V^*} + \frac{R^n - R^*}{R^*} \tag{35}$$

This obtained timetable and the corresponding line plan could be treated as the optimal solution.

4. Computational experiments

In this section, a real network case in China is applied to verify the proposed approach for designing a better service plan with shorter travel times, less capacity loss, high regularity and robustness. Fig. 9 illustrates the corridor layout of Shanghai to Nanjing high-speed railway (Shanghai on the right side, and Nanjing on the left.), 14 stations (red dots) are taken into account in our model, of which four major stations (big red dots) with technical facilities to be origin and destination of a train line. Passenger OD demand is predicted flow data in 2015. 268 types of stop patterns are provided with maximal 9 stops and minimal 3 stops in the given *line pool*, which is based on the technical condition of stations, passenger demand and time loss of all-stop line. The nominal period length is defined as 180 min (3 h) in this paper (after comparing with 2 h and 4 h with sensitivity analysis of passenger travel time and seat occupancy).

The values of the fixed parameters for the line planning model and timetabling model are depicted in Tables 5 and 6 respectively. Both optimization models are implemented using Matlab R2016b, Gurobi version 7.0.2, and the Yalmip toolbox (Löfberg, 2004).

4.1. Parameter calibration

With both models elaborated in Section 3, firstly, we need to calibrate the parameters in both objective functions.

4.1.1. Weights in the MF-LPP model

In the line planning model, the weight α for empty-seat-hour (Z_1), $1-\alpha$ for the total travel time (Z_2), and the weight β for the number of lines (Z_3) need to be determined.

Table 5
Input parameters of line planning model.

Parameter	Initial value	Value in iteration n
Acceleration time loss t_{ac}	2 min	–
Deceleration time loss t_{de}	1 min	–
Seat capacity C	620 seats/train-set	–
Weight of the number of lines β	300	–
Maximal optimization gap	3.5%	–
Seat occupancy rate μ	0.85	0.85(1*)
Upper bound of frequency $f_{\max}$	100	$F_{\max}^{n-1} - 1$

* If no feasible timetable can be obtained from the initial optimization even without regularity and robustness constraints, it increases to 1.

Table 6
Input parameters of timetabling model.

Parameter	Value
Minimal headway time h_{ij}	3 min
Maximal running time supplement rate θ	1.3
Minimal dwell time	1 or 2 min
Regularity tolerance ξ	0 min (initial),1 or 2 min (iteration)
Maximal dwell time buffer η_{ij}	$2h_{ij}$
Maximal overtakings allowed $O_l^{\max}$	4
Optimization gap	5.0%
Computation time limitation	60 min

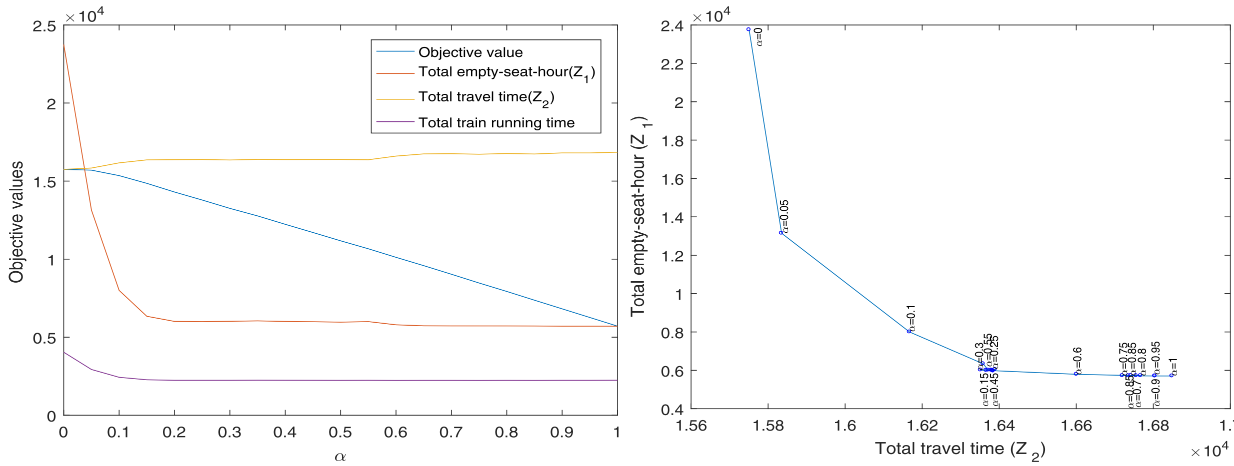

Fig. 10. Variation curves for the model with objective of total empty-seat-hour Z_1 (yellow line) and passenger total travel time Z_2 (red line) for $\alpha \in [0, 1]$. Left: Variation of objective values for α from 0 to 1; right: Correlation curve between passenger total travel time and empty-seat-hour (The corresponding value of α is displayed nearby the intersection point). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

For α, we try to explore the correlation between empty-seat-hour Z_1 and the total travel time Z_2 by varying α from 0 to 1 (Interval: 0.05) with only these objectives. The left side of Fig. 10 illustrates the variation of the total objective value, total travel time, total empty-seat-hour, and total train running time. It can be observed that Z_2 grows slightly from 0 to 0.15, and almost remains the same between 0.15 to 0.55. Then with a marginal increase to 0.65, it turns relatively stable until $\alpha = 1$. Z_1 has a remarkable fall from 0 to 0.2, and becomes almost steady between 0.2 and 1, except a slight decrease from 0.55 to 0.65. Meanwhile, the total train running time keeps identical after 0.2. The right side of Fig. 10 depicts the correlation between Z_1 and Z_2, and the values of α are shown nearby the intersection points. We could find that there are two main centralized distribution groups. One group is the area with $\alpha \in [0.15, 0.55]$ where Z_1 is around 1.64×10^4, and the other is $\alpha \in [0.65, 0.95]$ with a higher value of Z_1 about 1.68×10^4 and almost the same Z_2. For the purpose of line planning, a line plan with lower total travel time and total empty-seat-hour is more attractive to both passengers and operators. Therefore, α between 0.15 and 0.55 could be more reasonable for the line planning model.

Now the interval of α is determined, we choose one value to calibrate β with all objectives. After obtaining β, we return to check the feasibility of the selected α by implementing the whole model. For the sake of less disturbance compared to the boundary value, 0.4 is selected for α in our model as it is almost in the middle of this interval. Fig. 11 displays the variation of the objective related values when all objectives are considered. The left side of Fig. 11 reveals the changes of

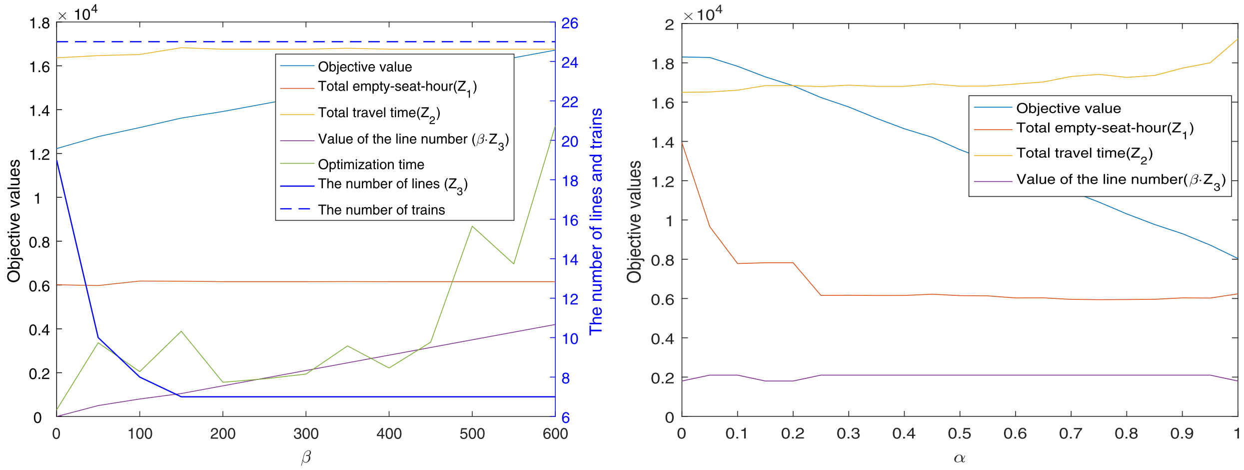

Fig. 11. Variation curves for all objectives. Left: Variation of objective values when α is 0.4 and β changes from 0 to 600; right: Variation of objective values when β is 300 and α changes from 0 to 1.

the objective value, total empty-seat-hour Z_1 (yellow line), passenger total travel time Z_2 (red line), objective value of the number of the line ($\beta \cdot Z_3$) (purple line), optimization time (green line), the number of lines Z_3 (blue line, right axis) and the number of trains (dashed blue line, right axis) when β varies from 0 to 600 (Interval:50). Z_1 almost remains the same with a slight rise from 0 to 100, and Z_2 increases from 0 to 150, and becomes stable afterward. To the opposite side, Z_3 decreases to 7 from 0 to 150, then keeps constant. In addition, the number of trains is 25 during all the iterations, and in general, the optimization time has a tendency to rise when β becomes larger even though it fluctuates. Thus, [200, 450] can be summarized as a reasonable interval of β. In this paper, we choose 300 in the model. Then we verify the feasibility of both values by varying α from 0 to 1 again, illustrated in the right side of Fig. 11. It shows that Z_1, Z_2, and Z_3 (as the value of line number equals $\beta \cdot Z_3$) are all in the stable area when α equals 0.4. To sum up, the weight parameters in the line planning model are calibrated as $\alpha = \mathbf{0.4}$ and $\beta = \mathbf{300}$.

4.1.2. Weights in the MP-TTP model

In the train timetabling model, three objectives are introduced: passenger travel time, timetable robustness, and the number of dwell time stretches. Due to the huge magnitude difference among the objective values, reasonable weights need to be calibrated to apply for the case study. Scaling the activities of passenger travel time and timetable robustness could lead to the same order of magnitude of all objectives, which makes it easier to assign weights. Hence, our MP-TTP model is reformulated as

$$\text{Minimize} \quad \sum_{i=1}^{3} \omega_i \cdot V_i \tag{36}$$

$$V_1 = \sum_{(i,j)\in\mathcal{A}_{\text{run}}\cup\mathcal{A}_{\text{dwell}}} \frac{\lambda_{ij}\cdot(\pi_j - \pi_i + z_{ij}\cdot T)}{N_{\text{run}} + N_{\text{dwell}}} \tag{37}$$

$$V_2 = \sum_{(i,j)\in\mathcal{A}_{\text{infra}}} \frac{\delta_{ij}}{N_{\text{infra}}} \tag{38}$$

$$V_3 = \sum_{(i,j)\in\mathcal{A}_{\text{dwell}}} y_{ij} \tag{39}$$

V_1 and V_2 is the scaled value of passenger travel time and timetable robustness, and V_3 is the number of dwell time stretches. N_{run}, N_{dwell} and N_{infra} stand for the number of run activities, dwell activities, and infra activities respectively. We assume λ_{ij} equals one in order to compare travel time, dwell time and time supplement with different line plans.

As there are three objectives, we choose to find the correlation between V_1 and V_3 first. The left of Fig. 12 illustrates the variation of V_1, V_2, and V_3 when ω_1 changes from 0 to 1 and $\omega_3 = 1 - \omega_1$. When $\omega_1 = 1$, V_3 becomes 80 at this point. It can be observed that V_1 and V_3 are not sensitive when $\omega_1 \in [0.1, 0.8]$, and have an obvious variation from 0.8 to 1. Meanwhile, V_2 changes quite irregular, as it represents the headway deviation of half period length for all infrastructure activities, which is not quite stable when searching for solutions without taking it into account. The aim of this model is to find a timetable with less travel time and best robustness, with the least dwell time stretches. Therefore, the weight for each of the objectives could not be too big to balance the objectives. Considering the non-sensitive area of V_1 and V_3, and the change tendency of V_3, we choose ω_1 as **0.7**, and ω_3 would be the **0.3** accordingly. The right of Fig. 12 shows the variation of V_2 for ω_2 from 0

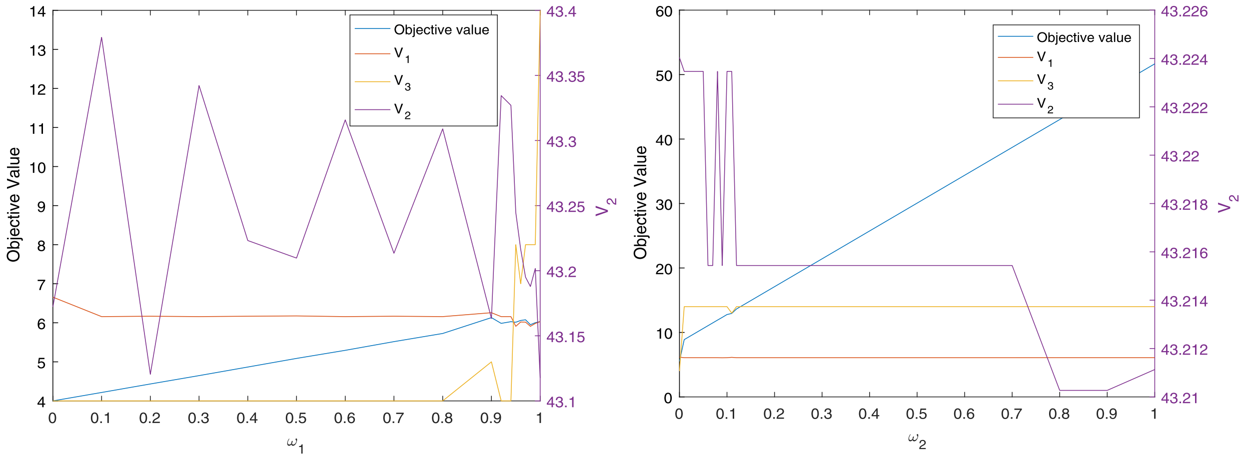

Fig. 12. Variation curves for model with objective of V_1 and V_3 for ω_1 from 0 to 1. Left: Variation of all objective values for ω_1 from 0 to 1; Right: Variation of V_2 with ω_2 from 0 to 1 and selected ω_1 and ω_3.

Table 7
Initial optimized values of line planning model.

Obj val	Z_1	Z_2	Z_3	#line	#train	Max freq	#stop	Av occup	Opt time	Opt gap
14,586.06	6149.2	16,710.63	2100	7	25	14	92	73.16%	1828.47s	1.9%

to 1 with previous selected ω_1 and ω_3. V_1, V_2 and V_3 are totally stable from 0.12 to 0.7, and they all have some fluctuation from 0 to 0.11. Hence, **0.2** is chosen for ω_2 which is in the stable area and nearby the sensitive area. Moreover, with these selected weights, all three objectives would vary in close values.

4.2. Case study

The MF-LPP model and MP-TTP model will be tested in this section with the determined parameters. Using the framework of Section 3.4 to find a solution for the given network.

4.2.1. Line plan

Since α and β are selected, an initial optimized line plan can be computed. Table 7 reports the obtained objective value (Obj val), total empty-seat-hour (Z_1), total passenger travel time (Z_2), total value of the line number (Z_3), the number of lines (#line), the number of trains (#train), maximal frequency (Max freq), the number of stops (#stop), average seat occupancy (Av occup), and optimization gap (Opt gap). The seat occupancy for line l between station s_m and s_{m+1} ($s_i \in \mathcal{S}_l$), and the average seat occupancy for line l could be obtained by the following formulas respectively:

$$\gamma_l^{s_m,s_{m+1}} = \frac{\sum_{i=1}^{m}\sum_{j=m+1}^{N_l} q_{s_i,s_j}^l}{C \cdot f_l}$$

$$\gamma_l = \frac{\sum_{s_i,s_j \in \mathcal{S}_l : i<j} q_{s_i,s_j}^l \cdot b_{s_i,s_j}}{C \cdot f_l \cdot b_l}$$

where b_{s_i,s_j} and b_l represent the distance between station i and station j, and the total travel distance of line l respectively. Hence, for a line plan, the average seat occupancy rate can be calculated by

$$Av\ occup = \frac{\sum_{l \in \mathcal{L}} \gamma_l \cdot f_l}{\sum_{l \in \mathcal{L}} f_l}$$

With the optimization model, seven lines are selected from *line pool* with a total number of 25 trains, and the maximal line frequency is 14. In total, 92 stops are achieved to provide alighting and boarding services for all passenger ODs on stations between origin and destination. The average seat occupancy of this line plan is 73.16% which is quite high with consideration of the designed maximal rate 85.0%. Table 8 gives the achieved line plan with the line number from the line pool, stop patterns (The number represents the order of the corresponding station along the corridor), corresponding frequency and average seat occupancy. Three lines have a frequency more than one, line 11 has the maximal frequency of 14 with seat occupancy 71.75%. Fig. 13 depicts the correlation between passenger flow and stop frequency at each station, where the passenger flow includes both arrival and departure passengers due to the symmetry passenger flow assumption. It indicates that the stop frequency follows the variation of passenger flow.

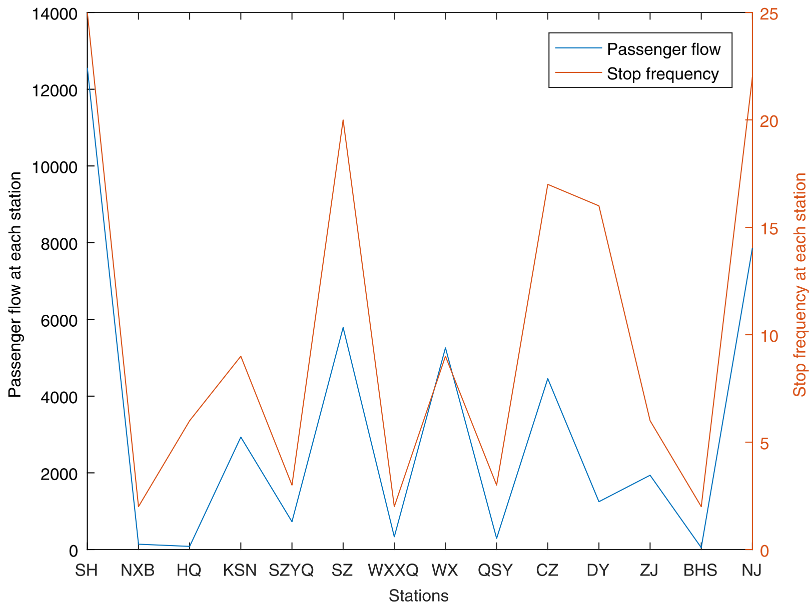

Fig. 13. The correlation between passenger flow and stop frequency at each station.

Table 8
Initial line plan.

Line number	Lines (stop patterns)	Frequency	#Stop	Av occup
1	1- 4- 6- 8	3	2	79.68%
11	1- 6-10-11-14	14	3	71.75%
127	1- 3- 4- 8-12-14	4	4	80.20%
94	1- 4- 5- 6- 9-10-12-13-14	1	7	59.64%
124	1- 3- 5- 7- 8-10-11-13-14	1	7	54.53%
215	1- 2- 4- 6- 7- 9-11-12-14	1	7	77.64%
266	1- 2- 3- 5- 6- 8- 9-10-14	1	7	72.74%

Table 9
Settings of optimization model.

#Integer variable	#Binary variable	#Continuous variable	#Constraint
3486	7653	7586	56,240

4.2.2. Timetable

With the obtained line plan, the extended PESP model is applied. Table 9 indicates the size of the model by the number of variables and constraints. As mentioned in Section 3.4, a feasible solution might not be found until the given computation time limitation if the model size is big or some constraints are too strict. When all objectives and constraints are applied in this model, an infeasible solution is returned. Therefore, we optimize the model using in the framework of Fig. 8.

First, we solve PESP-PTT to check whether the seat capacity is enough. A feasible timetable is achieved, as shown in Fig. 14. Each color represents a certain train stop pattern, where the black lines are the train line with the maximal frequency. As robustness and regularity are not included, trains of the same line always departure successively with the minimal headway interval. Second, PESP-REG is applied with the initial regularity tolerance $\xi = 0$ and optimization gap of 100%, and an infeasible solution occurs. By relaxing $\xi = 1$ still no feasible solution exists. When updating ξ to the upper bound two, a feasible solution is found, as displayed in Fig. 15. With this solution as the initial solution, the PESP-MP model is solved. The time-distance diagram of the optimized timetable is depicted in Fig. 16, and the results are shown in Table 10, including the objective value, the value of the three objectives (V), V_1, V_2, V_3, passenger travel time (PTT), timetable robustness (TRO), the number of overtakings (NOV), the average dwell time (T_{dwell}), regularity indicator (R), optimization gap and time. By comparing the three timetables, it can be identified that PESP-PTT could result in the least travel time, but with the worst regularity and robustness. PESP-MP has better robustness and fewer overtakings compared to PESP-REG ($\xi = 2$), but with certain travel time and regularity increase. In Fig. 15, the red rectangular boxes display the extra

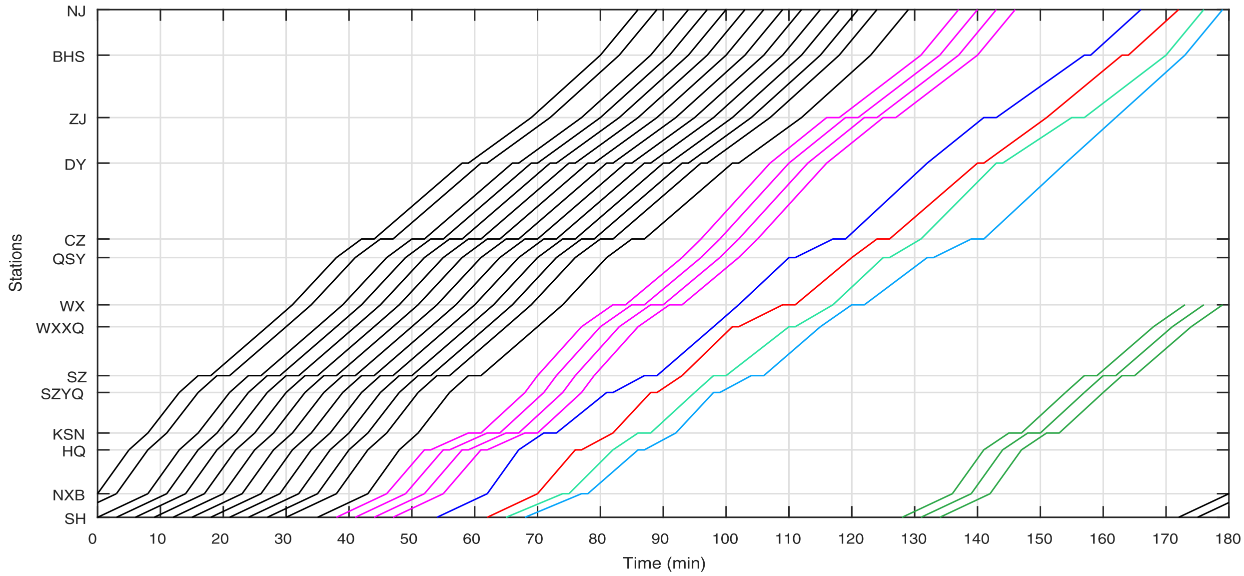

Fig. 14. Time-distance diagram of a feasible timetable of PESP-PTT for the initial line plan.

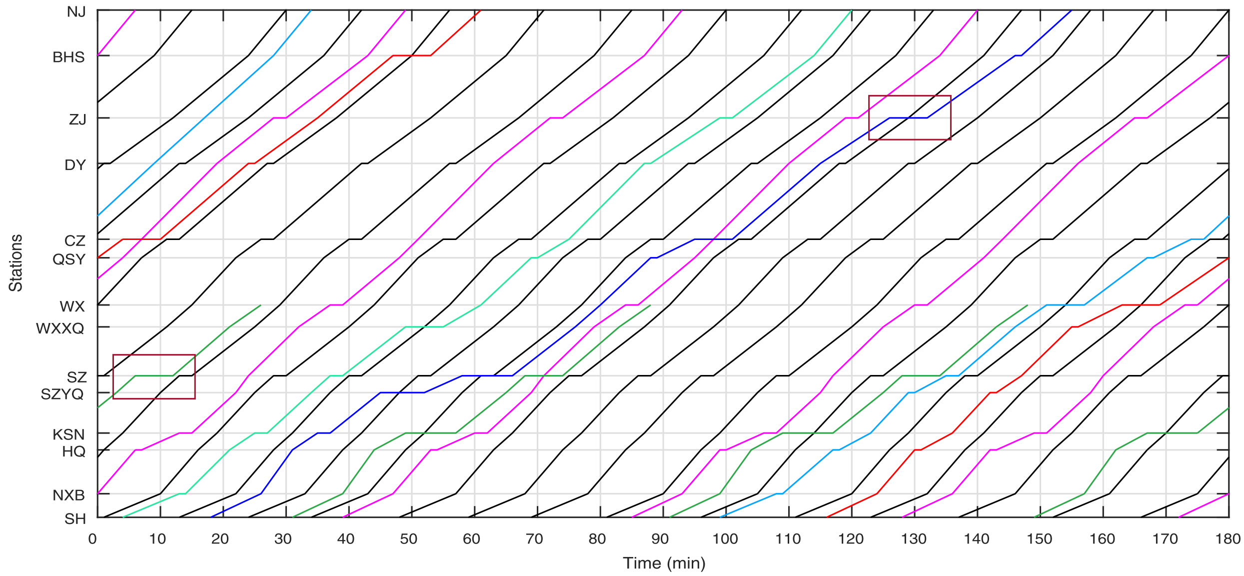

Fig. 15. Time-distance diagram of the optimized timetable of PESP-REG for the initial line plan.

Table 10
Results of optimized timetables of three PESP models for the initial obtained line plan.

Model	ξ	Obj	V	V_1	V_2	V_3	PTT	TRO	NOV	T_{dwell}	R	Opt gap	Opt time
PESP-PTT	0	4.02	15.05	5.74	55.16	0	2291	384,598	0	1.63	63.42%	0.00%	4.37
PESP-REG	2	8.81	17.44	6.15	43.16	13	2454	300,919	15	2.41	21.53%	54.35%	104.39
PESP-MP	2	17.20	17.20	6.25	43.15	12	2492	300,856	14	2.39	21.68%	43.57%	3600.04

overtaking compared with Fig. 16 and the dwell time stretches without overtaking. For the overall objective, the timetable from PESP-MP is better than PESP-REG with a lower $V = 17.20$. In general, in order to find a feasible timetable for the initial line plan, we have performed feedback loop ⑥ → ⑦ → ⑧ (circled numbers are shown in Fig. 8) twice sequentially, while following the mainstream in the framework. To be specific, the path for this iteration from the very beginning is ① → ② → ⑤ → ⑥ → ⑦ → ⑧ → ⑥ → ⑦ → ⑧ → ⑩ → ⑪.

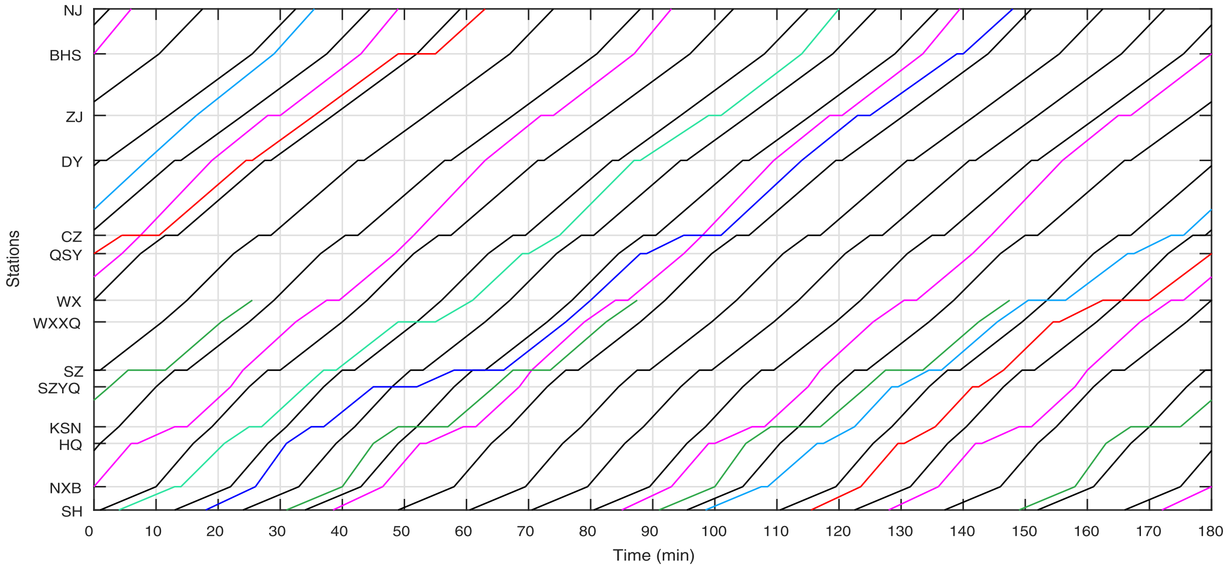

Fig. 16. Time-distance diagram of the optimized timetable of PESP-MP for the initial line plan.

Table 11
Iterative results of line plans from the MF-LPP model.

n	Obj val	Z_1	Z_2	Z_3	Train time	F_{max}	# line	# train	# stop	Av occup	Opt time	Opt gap
1	14,586.06	6149.20	16,710.63	2100	2291	14	7	25	92	73.16%	1828.47	1.92%
2	14,603.58	6179.25	16,719.80	2100	2296	13	7	25	93	73.09%	1663.46	1.93%
3	14,631.79	6169.15	16,773.55	2100	2301	12	7	25	94	72.72%	2884.06	1.72%
4	14,677.25	6171.75	16,847.58	2100	2310	11	7	25	95	72.74%	2572.14	1.98%
5	14,699.21	6165.55	16,888.32	2400	2319	10	8	25	97	72.68%	2713.41	1.99%
6	14,712.59	6181.05	16,900.28	2100	2316	9	7	25	97	73.24%	2695.08	1.82%
7	14,752.30	6254.80	16,917.30	2100	2335	8	7	25	101	72.68%	2195.98	1.85%
8	14,822.02	6189.65	16,576.93	2400	2291	7	8	25	92	72.90%	2279.57	1.97%
9	14,894.82	6183.55	16,702.33	2400	2301	6	8	25	95	72.62%	2297.31	1.97%
10	15,057.82	6252.80	16,927.83	2400	2327	5	8	25	102	72.72%	10,168.69	1.99%
11	15,173.79	6196.65	16,658.55	2700	2299	4	9	25	94	72.72%	2863.07	2.00%
12	15,429.16	6358.55	16,976.23	2700	2347	3	9	25	106	72.72%	808.10	1.98%
13	16,417.21	6133.28	16,773.17	3900	2302	2	13	25	99	73.85%	3941.65	2.00%
14	19,732.89	6027.93	16,369.53	7500	2247	1	25	25	86	73.29%	31.90	1.93%

4.2.3. Iteration results

As discussed in Section 3.4, we use an iterative method to find the optimal solution with the given network and passenger demand. Table 11 describes the iterative MF-LPP results when the maximal frequency f_{max} changes from 14 to 1, and Table 12 lists the TTP optimization results of PESP-MP with the corresponding line plans. Since model constraints are similar as unimodular matrix, we found that the solutions for event times are all integer even though defined as continuous variables. The double horizontal lines in both tables indicate the termination iteration as the optimal solution is found. The lowest objective value ($V = 13.82$) and regularity indicator ($R = 18.82\%$) appear in iteration 3, and the corresponding line plan is depicted in Table 13 and the time-distance diagram is shown in Fig. 17. The iterative path for finding this optimal solution is as following: ① → ② → ⑤ → ⑥ → ⑦ → ⑧ → ⑥ → ⑦ → ⑧ → ⑩ → ⑪ → ⑫ → ⑬ → ④ → ① → ② → ⑤ → ⑥ → ⑦ → ⑧ → ⑥ → ⑦ → ⑧ → ⑩ → ⑪ → ⑭ → ⑮ → ⑯ → ⑰ → ④ → ① → ② → ⑤ → ⑩ → ⑪ → ⑭ → ⑳ (circled numbers shown in Fig. 8). The stop patterns in iteration 3 are the same as iteration 1 except a frequency decrease of two for line 11 and an increase of two for line 127. The time related objective variation in both MF-LPP and MP-TTP from iteration 1 to 3 is illustrated in Fig. 18. The objective values from the MF-LPP model changed in the MP-TTP model, and the variation tendency is also different. In the MF-LPP model, the passenger weighted total travel time and train running time increase due to the updated frequency constraint, with a largest overall objective value in iteration 3. All three values grow in iteration 2 and drop to smaller values in iteration 3. Hence, the integration of the LPP and TTP is quite important to find a more efficient timetable as only minimal running times are considered in LPP without infrastructure constraints. The best line plan might have the worst timetable performance.

Table 12
Iterative results of timetables from the MP-TPP model.

n	ξ	F_{max}	Obj(V)	V_1	V_2	V_3	PTT	TRO	NOV	T_{dwell}	R	Opt gap	M_{VR}
1	2	14	17.20	6.25	43.15	14	2492	300,856	12	2.39	21.68%	43.57%	51.21%
2	2	13	15.24	6.37	43.42	7	2547	302,709	7	1.97	25.02%	35.82%	53.46%
3	0	12	13.82	6.15	43.07	3	2466	300,315	3	1.82	18.82%	17.32%	**9.27**%
4	2	11	16.84	6.15	43.20	13	2472	301,189	10	2.35	19.06%	42.43%	34.48%
5	2	10	15.95	6.17	43.19	10	2491	301,102	9	2.22	20.73%	40.19%	36.31%
6	0	9	14.20	6.22	43.21	4	2513	301,294	4	1.88	20.74%	20.81%	22.48%
7	0	8	22.73	6.38	43.33	32	2602	302,084	13	3.11	21.33%	54.43%	93.10%
8	1	7	21.15	6.31	43.18	27	2518	301,034	8	3.01	20.92%	55.71%	78.45%
9	2	6	16.69	6.29	43.47	12	2527	303,064	6	2.33	23.82%	46.37%	58.61%
10	1	5	21.31	6.50	43.29	27	2660	301,810	16	2.82	22.47%	60.40%	87.92%
11	0	4	21.39	6.22	43.16	28	2495	300,887	12	2.78	19.89%	53.80%	74.79%
12	0	3	12.65	5.70	43.27	0	2356	301,674	0	1.55	20.93%	27.58%	11.21%
13	0	2	15.24	6.04	43.06	8	2460	300,240	2	1.89	38.49%	50.53%	125.07%
14	0	1	14.74	6.10	52.37	0	2402	365,102	0	1.57	48.06%	65.43%	171.96%

Table 13
Line plan of iteration 3.

Line number	Lines (stop patterns)	Frequency	#Stop	Av occup
1	1- 4- 6- 8	3	2	73.44%
11	1- 6-10-11-14	12	3	74.04%
127	1- 3- 4- 8-12-14	6	4	74.91%
94	1- 4- 5- 6- 9-10-12-13-14	1	7	60.17%
124	1- 3- 5- 7- 8-10-11-13-14	1	7	50.11%
215	1- 2- 4- 6- 7- 9-11-12-14	1	7	76.64%
266	1- 2- 3- 5- 6- 8- 9-10-14	1	7	72.83%

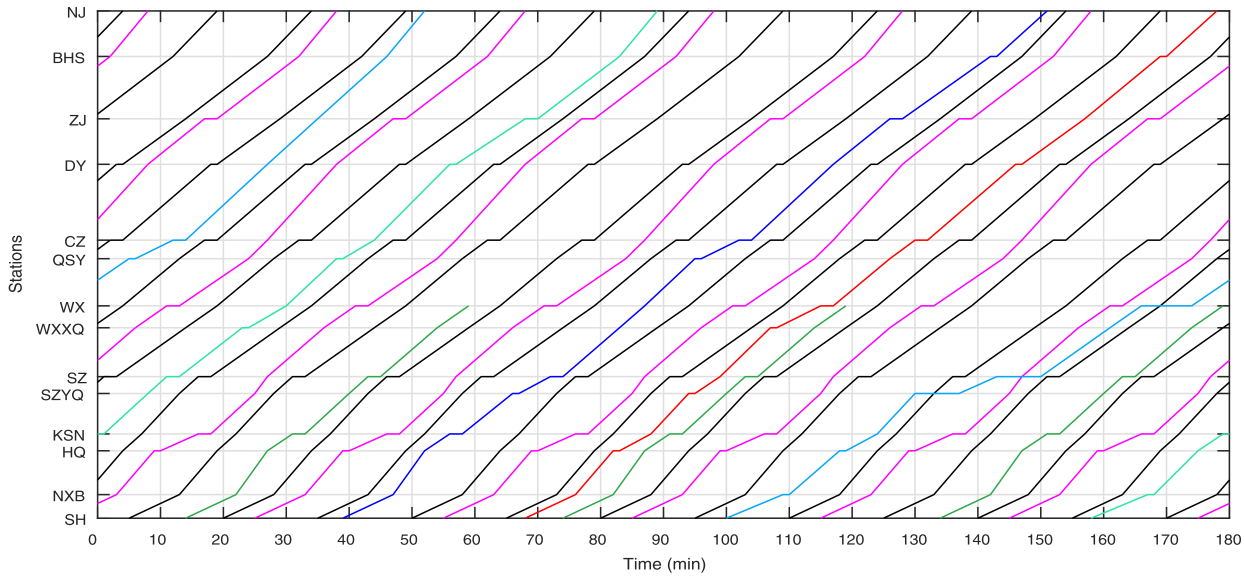

Fig. 17. Time-distance diagram of timetable from iteration 3.

Fig. 19 shows the variation of objective values in TTP from iteration 1 to 3, including the overall objective value (V), average travel time (V_1), average robustness (V_2), the number of dwell time stretches (V_3), the number of overtakings (NOV), regularity condition, and the optimization gap. It can be identified that the overall objective value, and regularity both achieved the lowest value in iteration 3. The values of V_1 and V_2 do not vary too much, and V_3 also comes to the lowest in iteration 3, as well as the optimization gap. Moreover, it also indicates that dwell stretches could represent the overtakings from the slight difference between V_3 and NOV.

In order to verify our method, we also computed all of the remaining iterations and found the lowest objective value (V^*) and regularity indicator (R^*). Meanwhile, M_{VR} is calculated from Eq. (35) after all iterations were completed, as shown in the last column in Table 12. It can be discovered that iteration 3 is still the best solution with the lowest M_{VR}, even

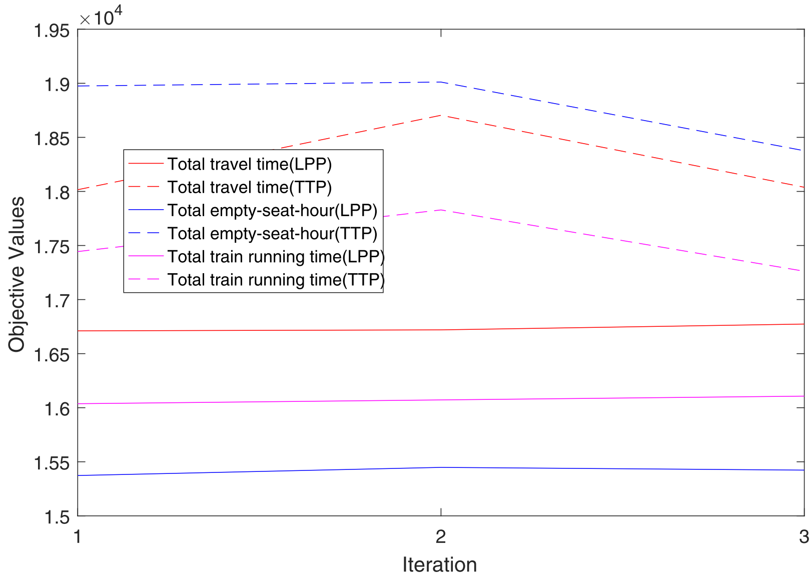

Fig. 18. Time related value changes of both MF-LPP and MP-TTP models from iteration 1 to 3. The solid lines represent values obtained from the MF-LPP model, while the dashed lines are from the MP-TTP model. (In order to make variation more obvious, total empty-seat-hour and total train running time are multiplied by 2.5 and 8 respectively).

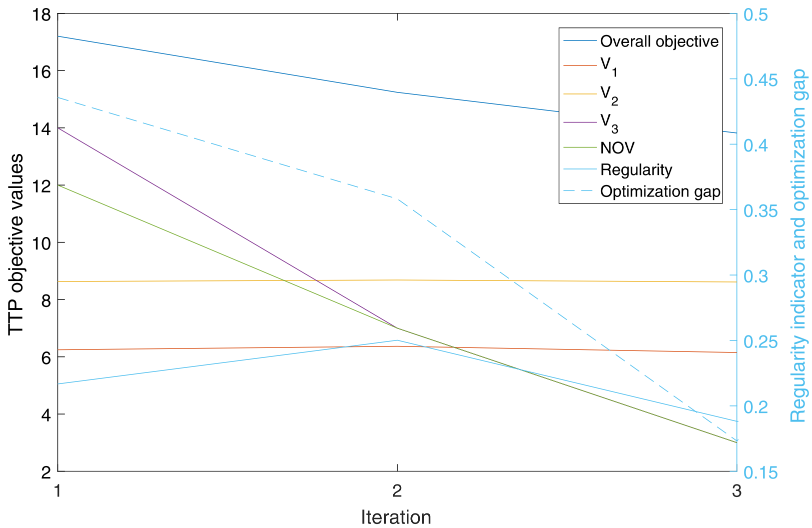

Fig. 19. Objective value variation of the TTP from iteration 1 to 3. Lines in light blue color use the right axis, the other use the left one. (In order to make variation more obvious, average robustness V_2 is divided by 5). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

though a lower $V = 12.65$ occurs in iteration 12. In the optimal solution, R and the optimization gap are the overall lowest; the timetable robustness and average dwell time are the second lowest.

For all iterations of LPP and TTP, most objective values change without convergence. However, we observed that the number of overtakings (NOV) almost has the same fluctuation as the overall objective value of TTP, seen in Fig. 20. Meanwhile, M_{VR} also follows the same variation except the last iteration. This means that the number of overtakings plays an important role in the final timetable result. Moreover, a timetable in which the maximal frequency (shown in the upper X-axis in Fig. 20) is a divisor (i.e., 12, 10, 9...) of the period length 180 min has a lower overall objective value, NOV, and M_{VR} (except iteration 14), especially when the maximal frequency is a multiple of three (see the corresponding frequency of dashed vertical lines). The best solution we found has a frequency of 12. This is the reason that we defined the regularity interval in the regularity constraints to be an integer. This indicates that the extended period length T also affects the timetable optimization.

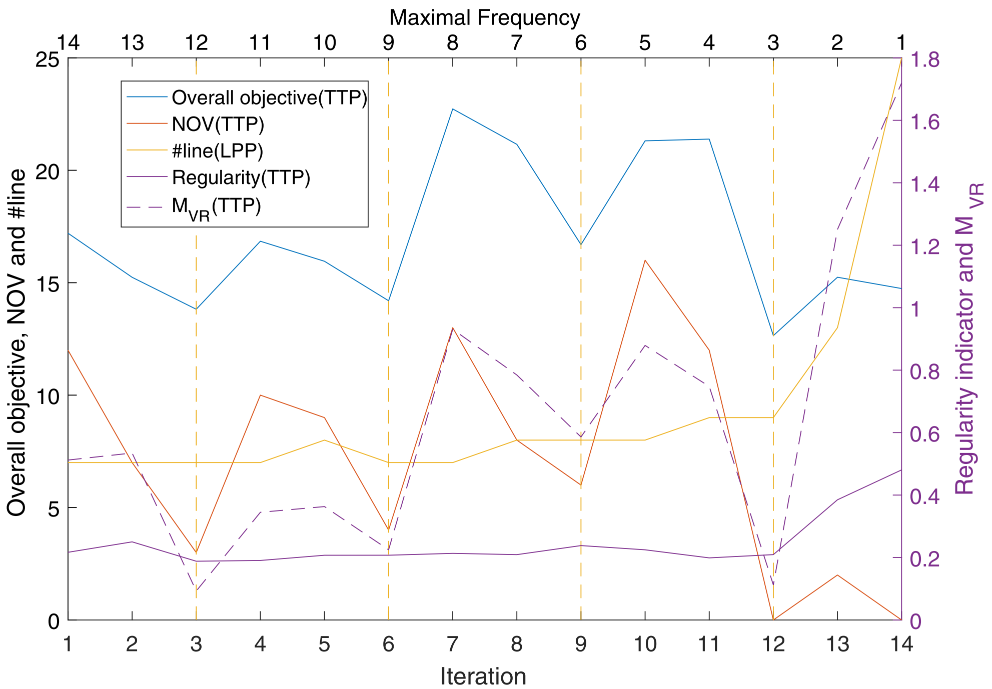

Fig. 20. The overall objective value of TTP, the number of overtaking (NOV), the number of lines, regularity and M_{VR} variations within all iterations. Regularity and M_{VR} are depicted by purple lines with right axis, and the upper X-axis represents the maximal frequency corresponding to the iteration on lower X-axis. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 14
Line plan of iteration 12.

Line number	Lines (stop patterns)	Frequency	#Stop	Av occup
1	1- 4- 6- 8	3	2	73.44%
8	1- 6-11-12-14	3	3	76.48%
17	1- 6- 7- 12–14	3	3	84.83%
35	1- 5- 8- 9–14	3	3	84.89%
94	1- 4- 5- 6-9-10-12-13-14	3	7	47.64%
130	1- 3- 4- 8- 10–14	3	3	80.76%
192	1- 2- 4- 7- 8- 10- 11-13-14	3	7	48.95%
255	1- 2- 3- 8- 12–14	3	4	84.79%
268	1- 2- 3- 5- 6- 7- 9-11-14	1	7	72.69%

In addition, both MF-LPP and MP-TTP are designed to have both periodic and aperiodic pattern, so we compare our approach with both patterns to validate the model. Iteration 12 with maximal frequency of three, has almost the same frequency of each line which could be treated as a periodic pattern with cycle time one hour, except for one extra train per 3 h, as seen from the line plan and timetable in Table 14 and Fig. 21 respectively. Iteration 14 is a totally aperiodic pattern since the maximal frequency is one. Fig. 22 depicts the layout of this timetable, with some train departures with small headways resulting in several big time windows.

By comparing the three time-distance diagrams, it can be directly observed that Fig. 17 (iteration 3) has a better regularity, and this also can be verified by the value of R in Table 12 with 18.82% being less than 20.93% and 48.06%. The left of Fig. 23 displays the objective values of iteration 3, 12 and 14 from the MF-LPP model. It can be found that iteration 14 has the lowest (best) Z_1, Z_2, train running time and the number of stops, and iteration 12 has the highest of all. The values from the optimal solution are in-between. Meanwhile, the optimal solution has four lines with frequencies of one and three lines with frequencies larger than two. This indicates that our model could achieve a multi-periodic pattern with a certain increase of the objective values compared with the aperiodic pattern, but much lower than the periodic pattern. Moreover, iteration 12 is the second optimal solution in our case, and it has almost totally different stop patterns with respect to iteration 1 and 3. This means that the best line plan depends on a set of lines, and it is difficult to prove that a certain line has a high quality. For the almost periodic solution 12, several lines have almost the highest seat occupancy rate with parameter $u = 85\%$, and the ones with many stops have quite low values due to dwell time loss. Only one line has a frequency of one, indicating that the objective for the number of lines and the upper bound frequency constraint in the MF-LPP could help to

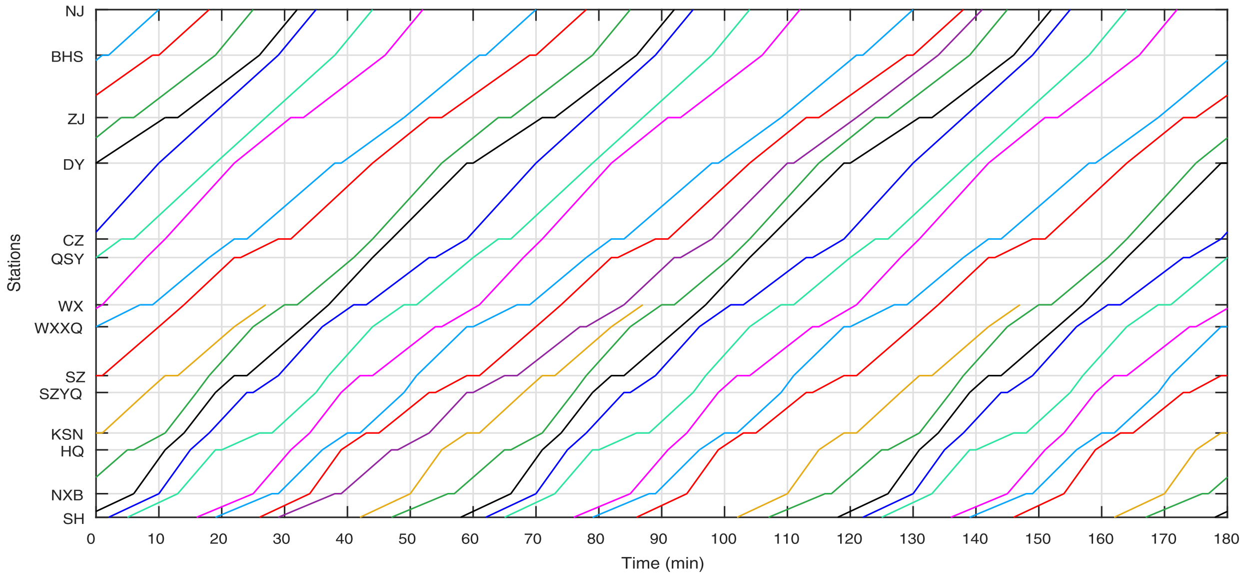

Fig. 21. Time-distance diagram of timetable from iteration 12.

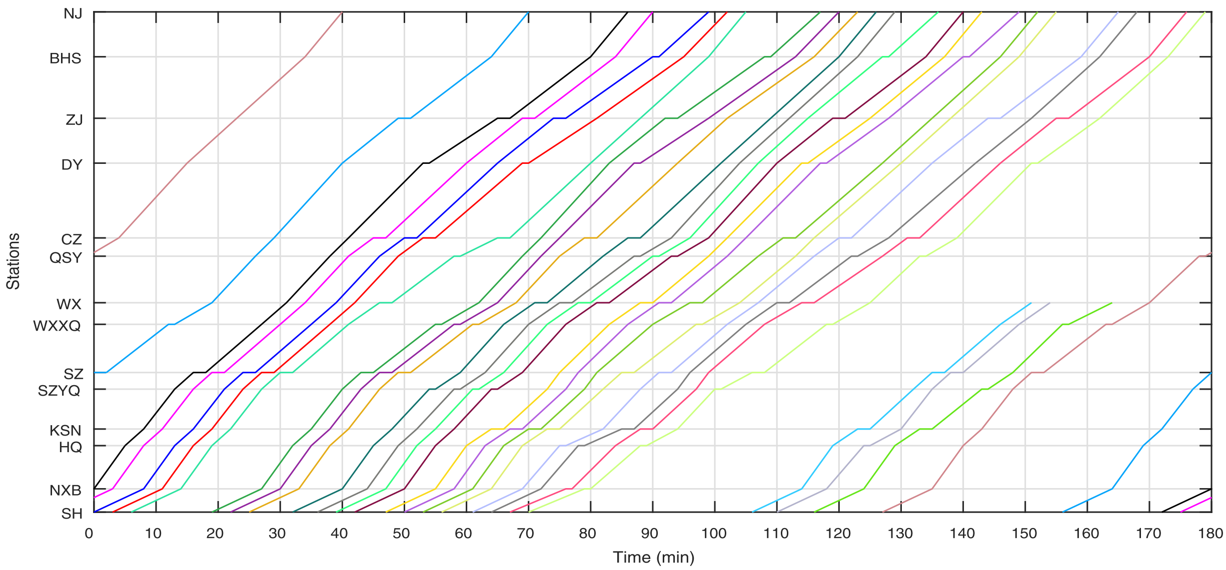

Fig. 22. Time-distance diagram of timetable from iteration 14.

equalize the frequency of lines. Therefore, we conclude that the MF-LPP model can provide a line plan with both periodic and aperiodic natures with reasonable objective values.

The right of Fig. 23 depicts the objective values of iteration 3, 12 and 14 from the MP-TTP model. It could be identified that our optimal solution has the best regularity and timetable robustness, whereas iteration 14 has the worst of both. The lowest total travel time, as well as average dwell time (T_{dwell} in Table 12), occurs in iteration 12, which is the opposite result with the LPP. These can be explained by two reasons. One is that regularity constraints could help to spread train lines in the whole period (improve robustness) but need more time supplement (increase travel time) to be satisfied, especially for multiple frequencies. The other is that the objective of robustness could increase time supplement, which leads to higher total travel time in iteration 14. This implies that the multi-periodic timetable from our MP-TTP could acquire better robustness and regularity than either a periodic and aperiodic timetable with a certain increase of total travel time. Note that our approximate periodic timetable has one extra aperiodic train, so the comparison result is a bit different from the full periodic case: best regularity of periodic pattern and least travel time of aperiodic pattern.

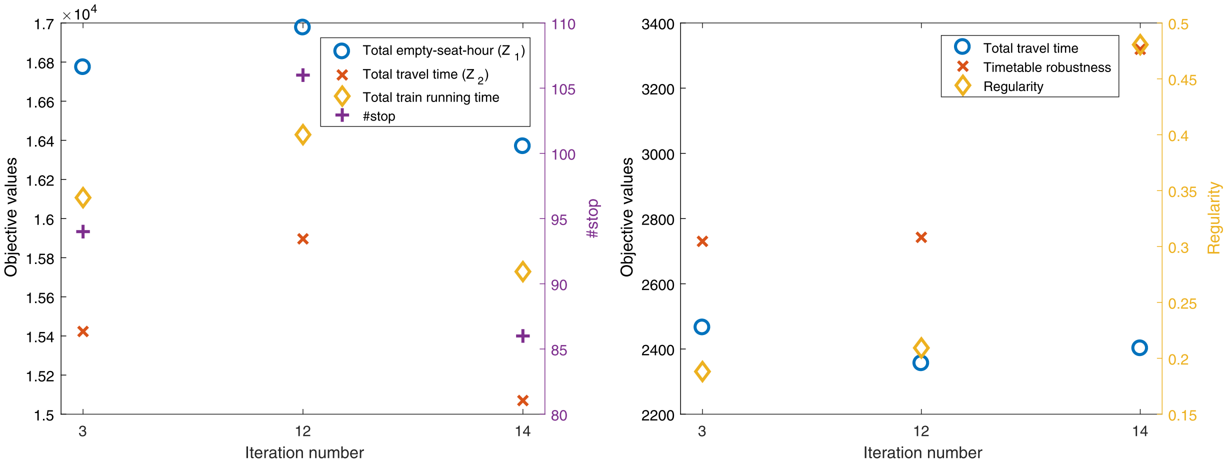

Fig. 23. Comparison of objective values of iteration 3, 12 and 14 from MF-LPP (left) and MP-TTP (right). (In order to make the variation more obvious, timetable robustness is scaled by a factor of 1/110).

5. Conclusions

This paper presented an MF-LPP model with consideration of passenger travel time, empty-seat-hour, the number of lines and frequency bounds within the extended cycle time. A corresponding MP-TTP model was proposed taking into account passenger travel time, timetable regularity, timetable robustness and the number of overtakings. Both models help to find a plan of both periodic and aperiodic nature that performs better for strongly heterogeneous lines. The multiple frequency constraints and the number of lines in the objective of MF-LPP control that the number of lines does not increase too much to have a more balanced frequency of each line. The regularity constraints in MP-TTP ensure a regular interval of trains for each train line, while the overtakings are used to find feasible timetables with acceptable capacity utilization of the rail corridors. An iterative framework of line planning and timetabling has been designed to find an overall optimal solution, with feedback loops for regularity constraints in TTP and frequency constraints to LPP from TTP.

The MF-LPP model computes a multi-periodic line plan without the competitive total travel time and empty-seat-hours compared to an aperiodic pattern. The MP-TTP model optimizes a multi-periodic timetable with better robustness than the periodic timetable with an hour pattern. Moreover, the combined design of MF-LPP and MP-TTP with extended period length could support rail transport planners to find a timetable with the best regularity and robustness.

Future research will consider the trade-off between robustness and overtaking. Also, a Pareto front could be computed to deal with multiple objective functions in the TTP, as it is quite difficult to calibrate reasonable weights for the objectives with different perspectives as applied in Section 4.1. Also, new approaches and measures for selecting the best timetable could be an interesting topic since different situations have different requirements for timetables.

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