A STUDY OF FLUID FLOW AND HEAT TRANSFER OF THREE-DIMENSIONAL PLATE-FIN AND TUBE HEAT EXCHANGERS BY THE LEAST-SQUARES FINITE ELEMENT METHOD

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Abstract.

A numerical calculation procedure based on the least-squares finite element method (LSFEM) is employed to study the fluid flow and heat transfer in a 3-D heat exchangers with in-lined and staggered multiple-row (4 rows) tubes.

In this study, the fin pitch of the heat exchanger is 8 fins per inch and the fluid flow is assumed incompressible, and laminar with Reynolds number ranging from 200 to 600.

In this paper the pressure drop, pressure coefficient, heat transfer coefficient, local Nusselt number and average Nusselt number for different geometric arrangements have been examined in detail.

The numerical results demonstrate that the average heat transfer coefficient of staggered arrangement is 10%-30% higher than that of the in-line one; also, it is effected more at low Reynolds number than at the high Reynolds number. The distribution of pressure drop of staggered array is higher than that of in-lined array. The variation of pressure coefficient at tube surface is dramatically for both the staggered and in-line arrangements for the angle less than 90 degree. The local Nusselt number of staggered array is higher 30%-80% than that of in-lined array for the tube row 2 to 4. Overall, the numerical results are in good agreement with the experimental measurement.

1 INTRODUCTION

The application of plate-fin and tube heat exchangers in the gas heaters, air conditioning system, coolers and compressors is very important in mechanical or chemical engineering. Since the heat transfer inside the tube structure is a major concern in these kinds of design.

The process of a heat exchanger is to exchange the heat of fluids with different temperatures. Its major purpose is to heat up or cool down the temperature of fluid. In our daily life, due to the development of high technology, some electronic products become much smaller and much more efficient. The heat dissipation inside the electronic parts will cause a significant effect on the stability of their usable life period. Therefore, how to increase the heat dissipation rate is getting more and more attention in the design of electronic products. Also, for different geometric arrangements of the tubes and fins the pressure drop, heat distribution and transfer coefficient will be totally different. There have been many studies¹ in related heat transfer in tube banks for the past three decades.

For researches of 2-D problems, Thom and Apelt² used the conformal mapping technique to solve the flow field past a 2-D tube bundle. Le Feuver³ applied the nonuniform grids to solve the heat flow field of a in-lined tube bank. Wung and Chen.⁴⁻⁵ used a boundary-fitted coordinate system to study the flow field and heat transfer of in-lined and staggered tube system. A hybrid Cartesian-ploar coordinate system for both in-lined and staggered tube bank has been successfully applied to exam the flow field and heat transfer by Launder and Massey⁶ and Fujii et al.⁷ Kundu et al.⁸ conducted both numerical and experimental study on a 2-D heat flow confined by two-parallel plates. A naphthalene mass transfer method was applied to measure the coefficients for different rows of plate-fin and tube heat exchangers by Saboya and Sparrow.⁹⁻¹¹ The effects on pressure drop and heat transfer of heat exchangers for different number of staggered tube rows were studied through experiments by Rich^{12,13}. McQuiston¹⁴, Gary and Webb¹⁵ established the relationships of Colburn and friction factors with Reynolds number for plain fins on staggered tubes.

For 3-D flows, Yamashita^{16,17} studied the flow and heat fields of a pairs of parallel plates with a square cylinder situated perpendicularly through the plates. Bastani et al.¹⁸ simulated the heat and flow fields of in-line tube arrays by employing one circular tube as the computation domain. Jang and Wu¹⁹ numerically and experimentally studied the fluid flow and heat transfer over a multi-row(1-6 rows) plate-fin and tube heat exchanger. They used the finite difference method to solve the governing equations and discussed the simulated results of pressure drop, heat transfer coefficients and the Nusselt number in the computational domain with the experimental measurements. The simulation results showed that the average heat transfer coefficient of staggered arrangement is 15-27% higher than that of in-lined one. The pressure drop of in-lined setup is 20-25% lower than that of staggered setup. Zdravistch et al.²⁰ simulated the heat flow field in laminar and turbulent conditions. The computational process was done with the element-by-element method.

In this study, a three-dimensional (3D) flow condition is considered to be laminar with Reynolds number less than 2000. The numerical calculation procedures based on the least-squares finite element method (LSFEM)²¹ are employed to study the fluid flow and heat transfer in a 3-D heat exchangers with in-lined or staggered multiple–row (4 rows) tubes. In this paper the pressure drop, pressure coefficient, heat transfer coefficient, local Nusselt number and average Nusselt number for different geometric arrangements have been examined in detail.

2 MATHEMATICAL MODEL

2.1 Governing equations

In this study, the heat flow is considered to be Newtonian fluid, incompressible fluid, 3D laminar flow. The governing equations are continuity, momentum and energy equations, they represented in the form of $u - p - \omega - \Theta - q$ as follows:

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\frac{\partial V}{\partial t} + \vec{V} \cdot \nabla \vec{V} + \frac{1}{\rho} \nabla p - \upsilon \nabla^2 \vec{V} = 0$$
⁽²⁾

$$\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T - \alpha \nabla^2 T = 0 \tag{3}$$

Based on the following dimensionless parameters,

$$x^{*} = \frac{x}{H}, y^{*} = \frac{y}{H}, z^{*} = \frac{z}{H}, u^{*} = \frac{u}{w_{in}}, v^{*} = \frac{v}{w_{in}}, w^{*} = \frac{w}{w_{in}}, p^{*} = \frac{p}{\rho w_{in}^{2}}, \Theta = \frac{T - T_{w}}{T_{in} - T_{w}}, \operatorname{Re}_{H} = \frac{w_{in}H}{v}, \operatorname{Pr} = \frac{v}{\alpha}, \operatorname{P} e = \operatorname{Pr} \cdot \operatorname{Re}_{H}$$

where *H* is the distance between two parallel plates, w_{in} is the inlet velocity, *Tin* is the temperature at the inlet section, *Tw* is the wall temperature; Re_H is the Reynolds number, Pr is the Prandtl number and Pe is the Pelect number. Equations (1)-(3) can be expressed in the following dimensionless forms by dropping the * sign for simplicity

$$\nabla \cdot \vec{V} = 0 \tag{4}$$

$$\frac{\partial V}{\partial t} + \vec{V} \cdot \nabla \vec{V} + \nabla p - \frac{1}{\operatorname{Re}_{H}} \nabla^{2} \vec{V} = 0$$
(5)

$$\frac{\partial \Theta}{\partial t} + \vec{V} \cdot \nabla \Theta - \frac{1}{Pe} \nabla^2 \Theta = 0 \tag{6}$$

2.2 Least-squares finite element method

In oreder to obtain the first-order differential form for applying the LSFEM, we add the vorticity vector, $\vec{\omega}$, and the temperature gradient, \vec{q} into (5) and (6)

$\vec{\omega} = \nabla \times V$	(7)
$\nabla \cdot \bar{\omega} = 0$	(8)

$$\bar{q} = \nabla \Theta \tag{9}$$

The general form of the governing equations of (1)–(9) can be expanded as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{10}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial p}{\partial x} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{z}}{\partial y} - \frac{\partial \omega_{y}}{\partial z} \right) = 0$$
(11)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial p}{\partial y} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{x}}{\partial z} - \frac{\partial \omega_{z}}{\partial x} \right) = 0$$
(12)

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial p}{\partial z} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{y}}{\partial x} - \frac{\partial \omega_{x}}{\partial y} \right) = 0$$
(13)

$$\omega_x + \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} = 0 \tag{14}$$

$$\omega_y + \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = 0 \tag{15}$$

$$\omega_z + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \tag{16}$$

$$\frac{\partial \omega_x}{\partial x} + \frac{\partial \omega_y}{\partial y} + \frac{\partial \omega_z}{\partial z} = 0$$
(17)

$$\frac{\partial\Theta}{\partial t} + u\frac{\partial\Theta}{\partial x} + v\frac{\partial\Theta}{\partial y} + w\frac{\partial\Theta}{\partial z} - \frac{1}{Pe}\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) = 0$$
(18)

$$q_x - \frac{\partial \Theta}{\partial x} = 0 \tag{19}$$

$$q_{y} - \frac{\partial \Theta}{\partial y} = 0 \tag{20}$$

$$q_z - \frac{\partial \Theta}{\partial z} = 0 \tag{21}$$

where Θ is the dimensionless temperature; u, v and w means the dimensionless velocity in x, y and z direction, respectively, and $\omega_x \cdot \omega_y \cdot \omega_z$ represent the vorticity components. There are eleven equations (10)-(21) for solving the eleven unknowns.

2.3 Pressure and heat transfer coefficients and Nusselt number

From the simulation, there are three parameters are examined in this study: pressure coefficient, local heat transfer coefficient and local Nusslet number. The pressure coefficient,

$$c_p$$
, is defined as $c_p = \frac{p - p_{in}}{\frac{1}{2}\rho w_{in}^2}$, where the p_{in} is the pressure at the inlet section. The local

heat transfer coeffcient, h, is defined as $h = \frac{q''}{T_w - T_b}$, where $q'' = -k \cdot \nabla T = -k \cdot \frac{\partial T}{\partial n}\Big|_{wall}$, q'' is

the heat flux per unit area, k is theheat conduction coefficient of fluid and n is the unit normal vector of the wall.

The local Nusselt number (Nu) is the dimensionless heat transfer coefficient and is

defined as
$$Nu = \frac{h \cdot H}{k} = \frac{\partial \left(\frac{\Theta}{\Theta_b}\right)}{\partial n} |_{wall}$$
, where $\Theta_b = \frac{\left(T_b - T_w\right)}{\left(T_{in} - T_w\right)}$.

3. NUMERICAL METHOD

3.1 Discretization

The discretization of equations is discussed in this section. The time derivative is approximated by the finite difference forward scheme. (10)-(21) can be expressed as follows:

$$\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y} + \frac{\partial w^{n+1}}{\partial z} = 0$$

$$(22)$$

$$\frac{u^{n+1} - u^{n}}{\Delta t} + \theta \left[u^{n+1} \frac{\partial u^{n+1}}{\partial x} + v^{n+1} \frac{\partial u^{n+1}}{\partial y} + w^{n+1} \frac{\partial u^{n+1}}{\partial z} + \frac{\partial p^{n+1}}{\partial x} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{z}^{n}}{\partial y} - \frac{\partial \omega_{y}^{n}}{\partial z} \right) \right]$$

$$+ (1 - \theta) \left[u^{n} \frac{\partial u^{n}}{\partial x} + v^{n} \frac{\partial u^{n}}{\partial y} + w^{n} \frac{\partial u^{n}}{\partial z} + \frac{\partial p^{n}}{\partial x} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{z}^{n}}{\partial y} - \frac{\partial \omega_{y}^{n}}{\partial z} \right) \right] = 0$$

$$(23)$$

$$\frac{v^{n+1} - v^{n}}{\Delta t} + \theta \left[u^{n+1} \frac{\partial v^{n+1}}{\partial x} + v^{n+1} \frac{\partial v^{n+1}}{\partial y} + w^{n+1} \frac{\partial v^{n+1}}{\partial z} + \frac{\partial p^{n+1}}{\partial y} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{z}^{n}}{\partial z} - \frac{\partial \omega_{z}^{n}}{\partial x} \right) \right] = 0$$

$$(24)$$

$$\frac{w^{n+1} - w^{n}}{\Delta t} + \theta \left[u^{n+1} \frac{\partial w^{n+1}}{\partial x} + v^{n+1} \frac{\partial w^{n+1}}{\partial y} + w^{n+1} \frac{\partial w^{n+1}}{\partial z} + \frac{\partial p^{n+1}}{\partial z} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{z}^{n}}{\partial z} - \frac{\partial \omega_{z}^{n}}{\partial x} \right) \right] = 0$$

$$(24)$$

$$\frac{w^{n+1} - w^{n}}{\Delta t} + \theta \left[u^{n+1} \frac{\partial w^{n+1}}{\partial x} + v^{n+1} \frac{\partial w^{n+1}}{\partial y} + w^{n+1} \frac{\partial w^{n+1}}{\partial z} + \frac{\partial p^{n+1}}{\partial z} + \frac{\partial p^{n+1}}{\partial z} + \frac{1}{\operatorname{Re}_{H}} \left(\frac{\partial \omega_{z}^{n}}{\partial x} - \frac{\partial \omega_{z}^{n}}{\partial x} \right) \right] = 0$$

$$(25)$$

$$\omega_x^{n+1} + \frac{\partial v^{n+1}}{\partial z} - \frac{\partial w^{n+1}}{\partial y} = 0$$
(26)

$$\omega_{y}^{n+1} + \frac{\partial w^{n+1}}{\partial x} - \frac{\partial u^{n+1}}{\partial z} = 0$$
(27)

$$\omega_z^{n+1} + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$
(28)

$$\frac{\partial \omega_x^{n+1}}{\partial x} + \frac{\partial \omega_y^{n+1}}{\partial y} + \frac{\partial \omega_z^{n+1}}{\partial z} = 0$$
(29)

$$\frac{\Theta^{n+1} - \Theta^{n}}{\Delta t} + \theta \left[u^{n+1} \frac{\partial \Theta^{n+1}}{\partial x} + v^{n+1} \frac{\partial \Theta^{n+1}}{\partial y} + w^{n+1} \frac{\partial \Theta^{n+1}}{\partial z} - \frac{1}{Pe} \left(\frac{\partial q_{x}^{n+1}}{\partial x} + \frac{\partial q_{y}^{n+1}}{\partial y} + \frac{\partial q_{z}^{n+1}}{\partial z} \right) \right] + \left(1 - \theta\right) \left[u^{n} \frac{\partial \Theta^{n}}{\partial x} + v^{n} \frac{\partial \Theta^{n}}{\partial y} + w^{n} \frac{\partial \Theta^{n}}{\partial z} - \frac{1}{Pe} \left(\frac{\partial q_{x}^{n}}{\partial x} + \frac{\partial q_{y}^{n}}{\partial y} + \frac{\partial q_{z}^{n}}{\partial z} \right) \right] = 0$$
(30)

$$q_x^{n+1} - \frac{\partial \Theta^{n+1}}{\partial x} = 0 \tag{31}$$

$$q_{y}^{n+1} - \frac{\partial \Theta^{n+1}}{\partial y} = 0 \tag{32}$$

$$q_z^{n+1} - \frac{\partial \Theta^{n+1}}{\partial z} = 0 \tag{33}$$

In (22)-(33), "n+1" means at the present time step and "n" means the previous time step, and θ is the weighting parameter for representing different schemes such as the explicit $(\theta = 0)$, implicit or $(\theta = 1)$ Crank-Nicloson $(\theta = 1/2)$. The nonlinear terms in the governing equations are linearized by the following way, assuming that the nonlinear form can be approximated by the known form to simplify the computation, ex:

$$u^{n+1}\frac{\partial u^{n+1}}{\partial x}\approx u^n\frac{\partial u^{n+1}}{\partial x}+u^{n+1}\frac{\partial u^n}{\partial x}-u^n\frac{\partial u^n}{\partial x},$$

then we can express (22)-(33) into the standard first-order differential form (34):

$$A_{1}\frac{\partial U}{\partial x} + A_{2}\frac{\partial U}{\partial y} + A_{3}\frac{\partial U}{\partial z} + A_{0}U = F$$
(34)

where

	1	0	0	0	0	0	0	0	0	0	0
	θu"	0	0	θ	0	0	0	0	0	0	0
	0	θv^n	0	0	0	0	$-\frac{\theta}{\operatorname{Re}_{H}}$	0	0	0	0
	0	0	θw^n	0	0	$\frac{\theta}{\operatorname{Re}_{H}}$	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0
$A_1 =$	0	-1	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	θu^n	$-\frac{\theta}{Pe}$	0	0
	0	0	0	0	0	0	0	-1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	- 0	1	0	0	0	0	0	0	0	0	0
	θv^n	0	0	0	0	0	$\frac{\theta}{\operatorname{Re}_{H}}$	0	0	0	0
	0	θv^n	0	θ	0	0	0	0	0	0	0
	0	0	θv^n	0	$-\frac{\theta}{\operatorname{Re}_{H}}$	0	0	0	0	0	0
	0	0	-1	0	0	0	0	0	0	0	0
<i>A</i> ₂ =	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	θv^n	0	$-\frac{\theta}{Pe}$	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	-1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0

<i>A</i> ₃ =	0	0	1	0	0	0	0	0	0	0	0
	θw^n	0	0	0	0	$-\frac{\theta}{\operatorname{Re}_{H}}$	0	0	0	0	0
	0	θw^n	0	0	$\frac{\theta}{\operatorname{Re}_{H}}$	0	0	0	0	0	0
	0	0	θw^n	θ	0	0	0	0	0	0	0
	0	-1	0	0	0	0	0	0	0	0	0
	-1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	θw^n	0	0	$-\frac{\theta}{Pe}$
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	-1	0	0	0

$$F = \begin{bmatrix} -(1-\theta)\cdot\left(\frac{\partial u^{n}}{\partial x} + \frac{\partial v^{n}}{\partial y} + \frac{\partial u^{n}}{\partial z}\right) \\ \frac{u^{n}}{\Delta t} + \theta\left(u^{n}\frac{\partial u^{n}}{\partial x} + v^{n}\frac{\partial u^{n}}{\partial y} + w^{n}\frac{\partial u^{n}}{\partial z}\right) - (1-\theta)\left[u^{n}\frac{\partial u^{n}}{\partial x} + v^{n}\frac{\partial u^{n}}{\partial y} + w^{n}\frac{\partial u^{n}}{\partial z} + \frac{\partial p^{n}}{\partial x} + \frac{1}{\mathrm{Re}_{u}}\left(\frac{\partial w^{n}_{z}}{\partial y} - \frac{\partial w^{n}_{z}}{\partial z}\right)\right] \\ \frac{v^{n}}{\Delta t} + \theta\left(u^{n}\frac{\partial v^{n}}{\partial x} + v^{n}\frac{\partial v^{n}}{\partial y} + w^{n}\frac{\partial v^{n}}{\partial z}\right) - (1-\theta)\left[u^{n}\frac{\partial v^{n}}{\partial x} + v^{n}\frac{\partial v^{n}}{\partial y} + w^{n}\frac{\partial v^{n}}{\partial z} + \frac{\partial p^{n}}{\partial y} + \frac{1}{\mathrm{Re}_{u}}\left(\frac{\partial w^{n}_{z}}{\partial z} - \frac{\partial w^{n}_{z}}{\partial x}\right)\right] \\ \frac{w^{n}}{\Delta t} + \theta\left(u^{n}\frac{\partial w^{n}}{\partial x} + v^{n}\frac{\partial w^{n}}{\partial y} + w^{n}\frac{\partial w^{n}}{\partial z}\right) - (1-\theta)\left[u^{n}\frac{\partial w^{n}}{\partial x} + v^{n}\frac{\partial w^{n}}{\partial y} + w^{n}\frac{\partial w^{n}}{\partial z} + \frac{\partial p^{n}}{\partial z} + \frac{1}{\mathrm{Re}_{u}}\left(\frac{\partial w^{n}_{z}}{\partial x} - \frac{\partial w^{n}_{z}}{\partial x}\right)\right] \\ - (1-\theta)\left(w^{n}_{z} + \frac{\partial v^{n}}{\partial z} - \frac{\partial w^{n}}{\partial z}\right) \\ - (1-\theta)\left(w^{n}_{z} + \frac{\partial v^{n}}{\partial z} - \frac{\partial w^{n}}{\partial z}\right) \\ - (1-\theta)\left(w^{n}_{z} + \frac{\partial w^{n}}{\partial z} - \frac{\partial w^{n}}{\partial z}\right) \\ - (1-\theta)\left(\frac{\partial w^{n}_{z}}{\partial x} + v^{n}\frac{\partial w^{n}}{\partial z} + w^{n}\frac{\partial \Theta^{n}}{\partial z} - \frac{1}{\mathrm{Re}_{u}}\left(\frac{\partial w^{n}_{z}}{\partial x} + \frac{\partial q^{n}_{z}}{\partial y} + \frac{\partial q^{n}_{z}}{\partial z}\right) \\ - (1-\theta)\left(w^{n}_{z} + \frac{\partial w^{n}}{\partial z} - \frac{\partial w^{n}}{\partial z}\right) \\ - (1-\theta)\left(w^{n}_{z} + \frac{\partial w^{n}}{\partial z} + w^{n}\frac{\partial w^{n}}{\partial z}\right) \\ - (1-\theta)\left(\frac{\partial w^{n}_{z}}{\partial x} + v^{n}\frac{\partial \Theta^{n}}{\partial y} + w^{n}\frac{\partial \Theta^{n}}{\partial z}\right) \\ - (1-\theta)\left(\frac{\partial w^{n}_{z}}{\partial x} + v^{n}\frac{\partial \Theta^{n}}{\partial y} + w^{n}\frac{\partial \Theta^{n}}{\partial z}\right) \\ - (1-\theta)\left(q_{z}^{n} - \frac{\partial W^{n}}{\partial y}\right) \\ - (1-\theta)\left(q_{z}^{n} - \frac{\partial \Theta^{n}}{\partial y}\right)$$

and

$$U^{T} = \left[u v w p \omega_{x} \omega_{y} \omega_{z} \Theta q_{x} q_{y} q_{z} \right]^{n+1}$$

3.2 Boundary conditions

The boundary conditions are specified as follows:

- (1) Inlet section: u = 1, v = w = 0, $\Theta = 1$
- (2) Outlet section: v = w = 0, p = 0, $q_x = 0$
- (3) At the fin surface: u = v = w = 0, $\Theta = 0$

(4) Center plane, symmetric plane in the y-direction, v = 0, $\omega_x = \omega_z = 0$, $q_y = 0$

- (5) Symmetric plane in the z-direction, w = 0, $\omega_x = \omega_y = 0$, $q_z = 0$
- (6) Tube surface: u = v = w = 0, $\Theta = 0$

In Fig.1 and Fig. 2, the boundary conditions for the in-linedand and staggered heat exchangers are depicted. In this study, the Reynolds number is set in the range between 200 and 600; Prandtl numer is equal to 0.736. The tolerance for the convergent requirement of the numerical simulation is 10^{-7} . The steady results are presented in the following section.

4 RESULTS AND DISCUSSION

The results simulated by the LESFM are discussed in this section, we only choose the results under the laminar condition at Reynolds number equal to 400 with inlet velocity equal to $2m/sec^{-1}$.

In Fig. 3, the pressure distribution at the central plate (z=0.5H) of in-lined heat arrangement is shown. The maximum pressure occurs at the stagnation point in front of the first tube. The pressure decreases gradually from the stagnation point downward to the outlet section. In Fig. 4, the pressure distribution in the middle plate (z=0.5H) of staggered arrangement is shown. The pressure distribution in Fig. 4 is similar to that of in-line arrangement near the first tube, but quite different from the second to the fourth tubes. There are much higher pressure in 2-3 rows than that of the in-lined tubes.

For the temperature distribution, the simulated results at z=0.5H are shown in Fig. 5 and 6 for in-lined and staggered arrangements, respectively. For the in-lined arrangement case, the temperature distribution like the pressure distribution declines gradually downward to the outlet section. In the recirculation zone behind each row (2-4 rows), due to the small velocity distribution in the vortex, the temperature is much lower than the temperature at the main stream section. For the staggered arrangement, the temperature distribution is similiar to the flow velocity distribution (not shown here). The recirculation zones of the staggered arrangement are smaller than that of in-lined arrangement. In the first row, the temperature distribution is similiar for both in-lined and staggered arrangement is higher than the one in the in-lined one. Overall, due to the geometric effect, the flow configuration is effected so much that the staggered arrangement has better ability to carry the heat to the downstream. Also,

from the results presented, it can be concluded qualitatively that the staggered heat exchanger has stonger ability to transfer the heat from upstream to downstream in the domain.

The calculated pressure coefficient of the tube surface of the in-lined and staggered arrangements are shown in Fig. 7 (a) and (b), respectively. From these figures, both the inlined and staggered arrangements, the pressure coefficients have the similiar form of distributions. In the first row, the value of pressure coefficient is maximum at the stagnation point, where the angle is zero shown in the figures for both cases. The value of pressure coefficient decreases along the surface of the tube to the minimum value at about 80 degree, then the value increase a little from the angle between 80 to 180. For in-lined arrangement, the value of rows 2-4 has the same trend and almost the same magnitude. From row 2 to row 4, the value reduce very small amout. For the staggered case, the value of pressure coefficient all have the same trend of distribution for four rows. The value of pressure coefficient decreases along the surface of the tube to the minimum value at about 80 degree, then the value increase a little from the angle between 80 to 180. When the angle is less than 80, the value reduce much more in the staggered arrangement than in the in-lined one.

The local Nusselt number (Nu) distribution on the surface of the four in-lined rows of tube is shown in Fig. 8(a), and the result by Jung et al.¹⁹ is shown in Fig. 8(b). The computed results demostrate that the local Nusselt number has the maximum value 32 occured at about 30 degree in the first row of tube, then it reduce to near zero at about 120 degree and maintain the value to 180 degree. For rows 2-4, the distribution of Nu values are almost the same for the three tubes. The maximum value of the local Nu of rows 2-4 is about 12 occured at the 75 degree.. The simulation results obtained by Jung et al.¹⁹ have the similiar trend for the local Nusselt number distribution even with lower maximum value for the first row but with the higher maximum values for the rest of rows.

The local Nusselt number distribution on the surface of the four staggered rows of tube is shown in Fig. 9(a), and the results obtained by Jung et al.¹⁹ is shown in Fig. 9(b). For the first row of tube, the local nusselt number distribution obtained from the LSFEM and Jung et al.¹⁹ is very closed to each other. By comparing the results from Fig. 9 (a) and (b), it is

found that the local Nusselt number of staggered array is higher 30%-80% than that of inlined array for the tube row 2 to 4.

The averaged heat transfer coefficient distribution of staggered arrangement is plotted in Fig. 10, in comparison with the numerical outcomes by Jung et al.¹⁹, we can see that the numerical results by the LSFEM are much closed to the experimental measurements.

The distribution of averaged heat transfer coefficient of in-lined and staggered arrangements is depicted in Fig. 11. At the Reynolds number Re_H=200, w_{in} is 1 m/s, the averaged heat coefficient of staggered arrangement is about 10%-30% higher than that of in-lined arrangement. The discrepancy of averaged heat transfer coefficient will be gradually reduced as the Reynolds number increases up to the value of 600 (w_{in} is 3 m/s). It means that the influence of geometric arrangement of the tube exchangers on the averaged heat transfer coefficient will be very small when the Reynolds number is greater than 600.

5 CONCLUSIONS

From this simulation, there are some points can be depicted as follows:

- (1) The average heat transfer coefficient of staggered arrangement is 10%-30% higher than that of the in-line one at Reynolds number equal to 200.
- (2) The average heat transfer coefficient is effected more at low Reynolds number than at the high Reynolds number.
- (3) The distribution of pressure drop of staggered array is higher than that of in-lined array. The variation of pressure coefficient at tube surface is dramatically for both the staggered and in-line arrangements for the angle less than 90 degree.
- (4) The local Nusselt number of staggered array is higher 30%-80% than that of inlined array for the tube row 2 to 4.

Overall, the numerical results are in good agreement with the experimental measurement.

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Fig. 1 The geometry and boundary setup of in-lined heat exchangers



Fig. 2 The geometry and boundary setup of staggered heat exchangers



Fig.4 Pressure distribution at z=0.5Hof staggered arrangement ($Re_H = 400$)



Fig. 5 Temperature distribution at z=0.5Hof in-lined arrangement ($\text{Re}_H = 400$)



Fig. 6 Temperature distribution at z=0.5Hof staggered arrangement ($Re_H = 400$)



(a)



(b) Fig. 7 Pressure coefficient distributions of tube surface (a) in-lined arranement (b) staggered arrangement







Fig. 8 Local Nusslet number distribution of staggered arrangement, (a) LSFEM (b) Result from Jang et al.¹⁹



(a)



Fig. 9 Local Nusslet number distribution of in-lined arrangement, (a) by LSFEM
 (b) Result from Jang et al¹⁹



Fig. 10 Averaged heat transfer coefficient distribution of staggered arrangement exchanger, results are compared with those of Jang et al¹⁹.



Fig. 11 Comparison of averaged heat transfer coefficient distribution of in-lined and staggered arrangement exchanger.