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Optimal Bayesian Linear Estimator and Active Learning for the Wiener Model

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1 Abstract

We propose a Bayesian estimation for the Wiener model, where the main objective is to learn a nonlinear output map under known linear state dynamics. The optimal linear estimator and its corresponding optimal estimation error are explicitly computed. Utilizing this explicit optimal error, we further propose an efficient first-order optimization algorithm to synthesize a sequence of inputs for active learning.

2 Problem Statement

The Wiener model is a class of nonlinear dynamical systems characterized by a linear process followed by a static nonlinear observation model [1]. Unlike existing Wiener system identification techniques [2, 3], we focus solely on Bayesian estimation of static nonlinear observation parameters. Consider a *known* discrete-time linear time-varying dynamical system described by $\mathrm{x}_{t+1} = \mathrm{A}_t\mathrm{x}_t + \mathrm{B}_t\mathrm{u}_t + \mathrm{w}_{t+1}$, where the states $\mathrm{x}_t \in \mathbb{R}^{n_\mathrm{x}}$ are observed through an *unknown* observation model: $\mathrm{y}_t = h(\mathrm{x}_t) + \mathrm{v}_t$. The independent random variables include the initial state x_0, process noise $\mathrm{w}_{t+1} \in \mathbb{R}^{n_\mathrm{x}}$, and measurement noise $\mathrm{v}_t \in \mathbb{R}$, all drawn from distributions with known first and second moments. The function $h\colon \mathbb{R}^{n_\mathrm{x}} \to \mathbb{R}$ is defined as a finite combination of *known* basis functions $\phi_n\colon \mathbb{R}^{n_\mathrm{x}} \to \mathbb{C}$, expressed as $h(\cdot) = \sum_{n=1}^{\mathrm{N}} \theta_n\phi_n(\cdot)$, where the unknown parameters $\theta_n \in \mathbb{R}$ have known priors with first and second moments μ_{θ_n} and $\sigma^2_{\theta_n}$, respectively. Given these models and input-output measurement data over a trajectory $t = \{0,\ldots,\mathrm{T}\}$, we aim to perform a noncausal mapping, i.e., $\mathrm{y}_{0:\mathrm{T}} \mapsto \hat{\theta}_n$, where $\hat{\theta}_n$ are the estimates of the unknown parameters θ_n.

3 Main Result

By representing the models in *lifted* form, we denote the vectors of input and measurement over the entire trajectory as $\bar{\mathrm{u}}$ and $\bar{\mathrm{y}}$, with the moments of the prior distribution represented by μ_θ and Σ_θ. Considering an affine estimator of the form $\hat{\theta}_B(\bar{\mathrm{y}}) = \Psi\bar{\mathrm{y}} + \psi$, we derive an analytical expression for the Bayesian Minimum Mean Squared Error (MMSE) estimation. The obtained optimal parameters and estimation error, $\Psi^\star(\bar{\mathrm{u}},\mu_\theta,\Sigma_\theta)$, $\psi^\star(\bar{\mathrm{u}},\mu_\theta,\Sigma_\theta)$, and $\mathscr{J}^\star_B(\bar{\mathrm{u}},\mu_\theta,\Sigma_\theta)$, are all functions of the input vector and prior information.

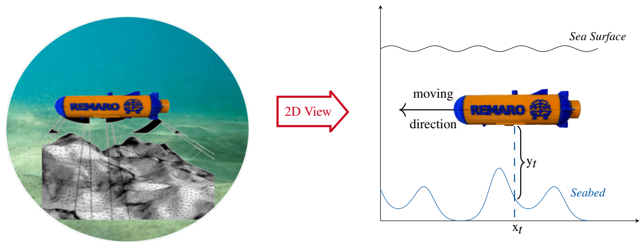

Figure 1: AUV Seabed Mapping

Leveraging the use of priors in Bayesian estimation, we further minimize the estimation error through two approaches. First, we employ an active learning method to identify optimal input strategies using the analytical first-order gradient of $\mathscr{J}^\star_B$. Second, we estimate θ from multiple batches of data by iteratively updating the moments of the prior distribution with estimated values from the previous batch. This combination of active learning and prior updates enables us to optimize inputs over smaller batches while significantly reducing estimation error. We illustrate the efficacy of our method via Monte Carlo experiments with varying process noise covariances.

4 Motivating Example

Our proposed solution can be applied to various scenarios where an unknown nonlinear function is influenced by time-varying correlated noise, such as robot mapping in unknown environments. As illustrated in Fig. 1, Autonomous Underwater Vehicles (AUVs) mapping the seabed in deep-sea environments face challenges due to the interplay of vehicle dynamics and the unknown nonlinear seabed observation model.

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