Effect of torsion on shear capacity of slabs

Valdivieso, D.A.; Sánchez, T.A.; Lantsoght, Eva

Publication date
2016

Document Version
Accepted author manuscript

Published in
Insights and Innovations in Structural Engineering, Mechanics and Computation

Effect of torsion on shear capacity of slabs

D.A. Valdivieso, & T.A. Sánchez
Universidad San Francisco de Quito, Quito, Ecuador

E.O.L. Lantsoght
Universidad San Francisco de Quito, Quito, Ecuador
Concrete Structures, Delft University of Technology, Delft, The Netherlands

ABSTRACT: A large number of existing bridges in Europe and North-America are reaching the end of their devised service life. Therefore, it is necessary to improve the methods of assessment for existing bridges. One method, suitable for existing reinforced concrete slab bridges, is the Modified Bond Model. This method, however, currently only takes the effect of torsion for loads close to the edge into account in a simplified manner. In this study, finite element models are created of a slabs with two supports, three concentrated (prestressing) loads and a distributed load, representing a truck wheel print. The load is varied along the longitudinal and transverse directions of the slab to find the bending moments (m_x and m_y) and torsional moments (m_{xy}). The results is an expression for the effect of torsion in slabs, which can be used with the Modified Bond Model for assessment and design of slab bridges.

1 INTRODUCTION

Currently, a large number of existing bridges in Europe and North-America are reaching the end of their originally devised service life (Lantsoght et al. 2013a, Johansson et al. 2014, Teworte et al. 2015). Since replacing or demolishing all these structures is economically not viable, a sharper method for assessment is necessary. In the Netherlands, a considerable subset of the ageing bridge stock consists of reinforced concrete slab bridges. In particular, the shear capacity of slab bridges is subject to discussion in the Netherlands. Therefore, over the last few years, the shear capacity of reinforced concrete slabs has been studied experimentally (Lantsoght et al. 2013c, Lantsoght et al. 2014b, Lantsoght et al. 2015b) as well as numerically (Falbr 2011, Voormeeren 2011, Doorgeest 2012, van Hemert 2012).

One analytical method that was proposed to assess the capacity of reinforced concrete slabs is the Modified Bond Model (Lantsoght et al. 2015a). However, for loads close to the edge, the effect of torsional moments reduces the capacity. Therefore, this effect needs to be quantified based on the lading position. Studying the relative effect of the torsional moments with respect to the bending moments is the topic of this paper.

2 LITERATURE REVIEW

Several methods had been developed to analyze slab behavior from using orthogonal beam systems to finite element methods to get more accurate stress fields and deflection values. For slab design purposes the Modified Bond Model currently takes into account the effect of torsion in a simplified manner.

The numerical technique applied to model and approach the deformations of the slabs is the finite element method based on dividing the slab in small symmetric areas and "the general method of analysis is to concentrate the load at the corners of the separated elements, and the restore continuity of slope and deflection at each node point…, so as to satisfy equilibrium and boundary condition requirements" (Park and Gamble 2000).

The results of the current analysis will be implemented in the Modified Bond Model (Lantsoght 2012, Lantsoght 2013a, Lantsoght 2013b, Lantsoght 2014, Lantsoght et al. 2014a, Lantsoght et al. 2015c). This model is an adaptation of the Bond Model for concentric punching shear (Alexander 1990, Alexander and Simmonds 1992, Afhami 1997), so that it can be used for slabs under concentrated loads in non-axis-symmetric situations. The model is based on the lower bound theorem of plasticity, such as the strip method for flexure (Hillerborg 1975). The geometry of the slab is taken into account by factors which reduce the capacity of the strip, which reaches its maximum, ideal capacity for the theoretical case of an infinitely large slab with a single concentrated load in the middle (i.e., the theoretical idealized case of concentric punching shear). Currently, the Modified Bond Model takes the effect of torsion into account in a simplified way.

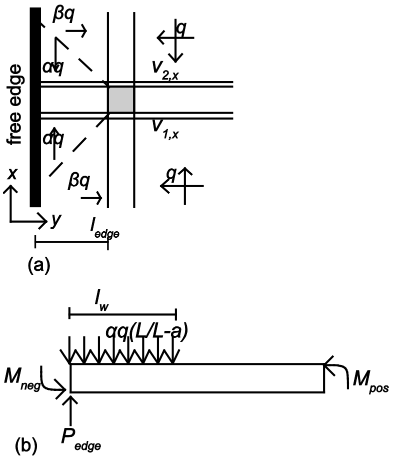

Figure 1. Modified Bond Model for loading close to a free edge: (a) resulting strips and quadrants, and loading in the quadrants; (b) loading on *y*-direction strip between load and free edge.

The applied maximum loading to determine the capacity of a strip is multiplied with a factor $\beta = 1$ if the effect of torsion is not important (for example, when the load is placed in the middle of the width), and with $\beta = 0$ when the effect of torsion is important and the capacity needs to be limited (for example, for loads close to the edge). This principle is illustrated in Figure 1.

3 DESCRIPTION OF FEM MODELS

Twelve linear elastic models of a 5000 mm x 2500 mm concrete slabs were created to evaluate the effect of torsion. The dimensions of the slab and the applied loading are based on experiment S1T1 (Lantsoght et al. 2013c) from a series of slabs tested in shear at the Stevin Laboratory of Delft University of Technology. S1T1 was used as a benchmark to develop the twelve models for this study.

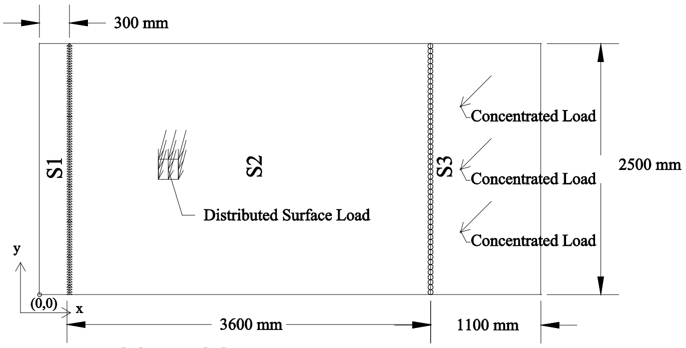

Figure 2: Slab Model Dimensions

Using SCIA Engineer 15.1 (Nemetschek-Scia 2015), a finite element software, the flexural and bending moments produced by a concentrated load over a modeled half-scale wheel print (as given by NEN-EN 1991-2:2003 (CEN 2003)) are analyzed, as well as the effect of the load close to the support and the free edge of the slab.

The structure was modeled using three 2D slab members consisting of shell elements with the dimensions as shown in Figure 2. The separate slab members are indicated as S1, S2 and S3 in Figure 2. The slab members are used to comply with the finite elements theory so each element is divided by a 100 x 100 mm mesh and the thickness of the shell elements is the total thickness of the slab, 300 mm.

The slab is supported by two line supports: the first is hinges, 300 mm from the edge of the slab to the center of the support and the second support consists of rollers, 3900 mm from the edge to the center of the support, to exemplify a simply supported structure. Three concentrated loads that represent prestressing bars creating a moment over the second support are placed 600 mm away from the second row of line supports, of magnitude 54.67 KN each.

Table 1: Models overview and load description.

Model	Magnitude (kN/m^2)	*x*-coordinate (mm)	*y*-coordinate (mm)
L1	11950	1000	1250
L2	11950	1000	850
L3	11950	1000	438
L4	11950	700	438
L5	11950	700	850
L6	11950	700	1250
L7	11950	850	438
L8	11950	850	850
L9	11950	850	1250
L10	11950	2100	438
L11	11950	2100	850
L12	11950	2100	1250

In each of the twelve models the loading is applied by a distributed surface load over a wheel print. The location along the length and width of the slab of this load is varied to study the relative effect of torsion. Table 1 describes the different load positions applied in the models. The *x*-coordinate is taken from the origin of the slab, so that the distance *a*, the center-to-center distance between the load and the support, is the value of the *x*-coordinate minus 300 mm, see Figure 2.

4 RESULTS OF FEM ANALYSES

4.1 *Averaging the observed ratios*

The effective depth of the slab *d* is equal to 265 mm (the thickness is 300 mm minus a 20 mm of reinforcement bar diameter minus 25 mm of concrete cover). A distance 2*d* from the position of the load is taken (Fig. 3) on all the twelve models in both directions (along the *x* and *y*) to find the average of the moment's ratio m_{xy}/m_x along this distance. Also at a distance equal to *4d* this procedure is followed (Lantsoght et al. 2013b). This procedure ensures that the effect of peaks from the mesh is not affecting the results.

4.2 *Results of finite element models*

The developed finite element models are used to study the flexural and torsional moments. Then, to study the relative importance of torsion, the ratio of torsional moments to flexural moments are studied. The plots of the benchmark case S1T1, called model L1, are discussed in this paragraph. The results of the other models can be found in the background report (Valdivieso et al. 2015).

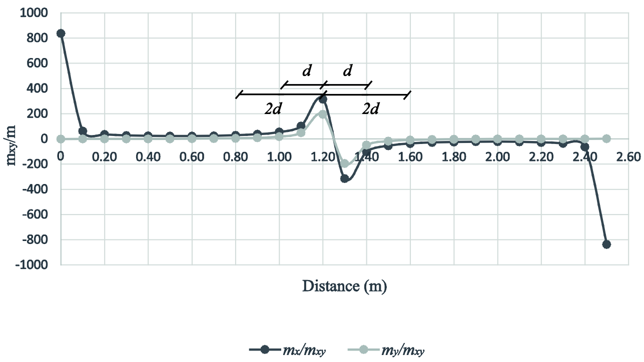

Figure 3. Averaging over a distance *d* from the center of the load in m_x/m_{xy} and m_y/m_{xy} ratios for case L1.

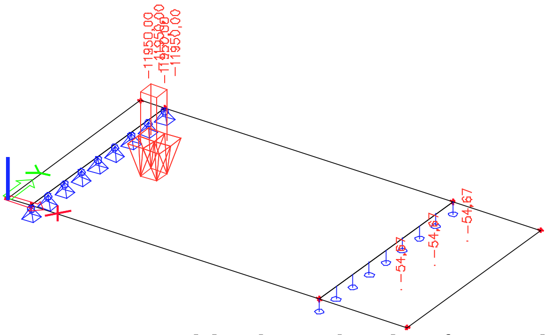

Figure 4. L1 Model with Distributed Surface Load representing wheel print with coordinates (1000 mm, 1250 mm). The concentrated loads beyond the second support create a moment over support 2.

First, the applied loading in the model, as discussed previously and given in Table 1, is shown in Figure 4. Then, the finite element software SCIA Engineer 15.1 (Nemetschek-Scia 2015), is used to determine the plot of the flexural moments in the span direction, m_x, and in the transverse direction, m_y. These plots are respectively shown in Figure 5 and Figure 6. In a next step, the plot of the torsional moments, m_{xy}, are determined, as shown in Figure 7. Figure 5, Figure 6 and Figure 7 are based on average magnitudes of the moments in the finite elements.

Once the plots of the moments in the slab are determined, the relative importance of torsion can be studied. This study is carried out by determining the ratios of the moments m_{xy}/m_x and m_{xy}/m_y. To visualize the effect of this moment ratio with positions relative from the point of application of the wheel print, the ratio is presented along a cut in the *x* and *y* direction. The cut in the *x* direction studies the longitudinal direction and has a constant y = 1250 mm. The cut in the *y* direction studies the transverse direction, and has a constant *x* through the center of the applied load. For model L1, the constant value of *x* for the transverse cut is 1000 mm, see Table 1. The resulting ratios m_{xy}/m_x and m_{xy}/m_y are then shown for the longitudinal cut in Figure 8 and for the transverse cut in Figure 9.

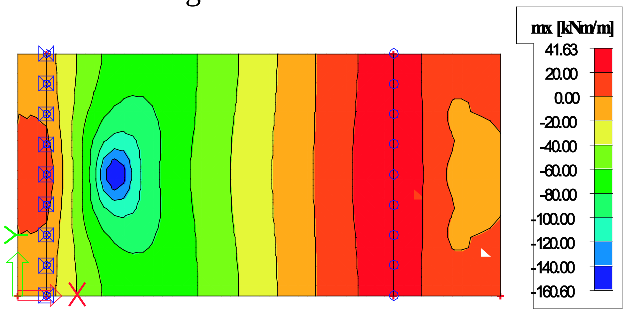

Figure 5. m_x plot of the L1 slab model analyzed using finite elements method.

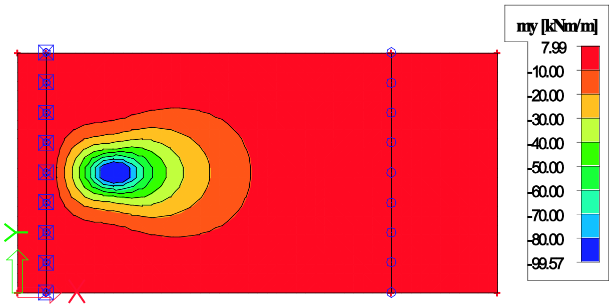

Figure 6. m_y plot of the L1 slab model analyzed using finite elements method.

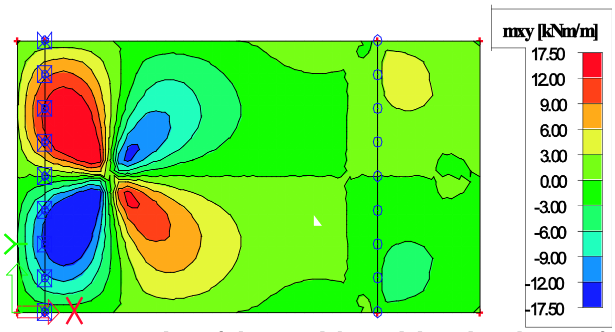

Figure 7. m_{xy} plot of the L1 slab model analyzed using finite elements method.

For the other cases (L1, L6, L9, L12) where the load is applied in the center of the slab in the *y* direction (along width) the m_x/m_{xy} and my/m_{xy} plots have a peak just where the distributed load is applied due to the large flexural moments and small (approximately zero) torsional moments. In the *x*-direction (along length) as the distributed area load gets far from the first row of continuous supports the plots are asymptotic to the ordinates which means that the flexural moments m_y are small close to the edges and decrease while the distance increases from the positon of the load in both directions. For the cases where the load gets closer to the free edges the resultant torsional moments get larger and the flexural moments are also larger and the moment ratios are low, so the plots have the x axis as an asymptote but have a small peak in the position of the load because it creates small torsional moments

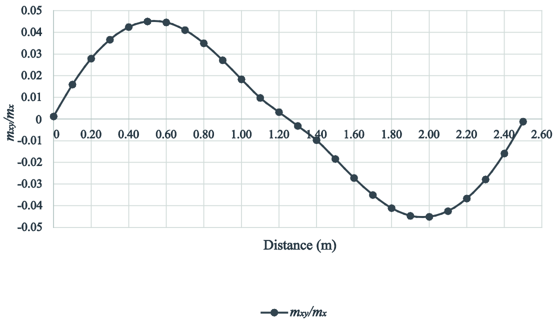

Figure 8. Torsional moment ratios m_{xy}/m_x along the width of the L1 model.

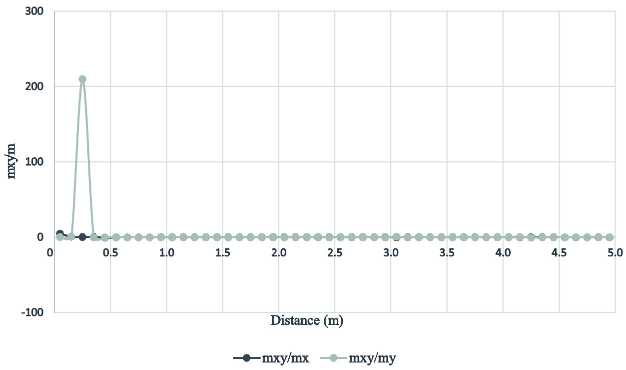

Figure 9. Flexural moment's ratio along the length of the L1 model.

5 DESCRIPTION OF THE EFFECT OF TORSION

5.1 *Discussion of the observed flexural and torsional moments*

An analysis of the dependence of the load positon in the m_{xy}/m_x and m_{xy}/m_y ratios shows that as the load approaches the center of the slab (y-direction), the effect of torsion becomes less at the position of the load. If the load is closer to the first support, the torsional moments are larger but the flexural moments m_x and m_y are smaller. This result is expected, as the span is smaller, thus the effect of shear becomes more important relative to bending. As the load gets closer to the free edge (y-direction) the torsional moments are larger. This means that the moment ratios m_{xy}/m_x and m_{xy}/m_y are the largest for a load close to the support and close to the free edge as shown in Figures 8 and 9.

As the distributed load moves away from the vicinity of the support into the span along the longitudinal direction, the torsional moments (m_{xy}) become smaller; in the width direction, when the load is placed in the center, the torsional moments are approximately zero due to the symmetry, but as the position of the load moves closer to the edges the torsional moments increase.

5.2 *Derivation of an expression for the effect of torsion to be used with the Modified Bond Model*

In a next step, the results of the different models are brought together. Now, the relative effect of torsion is shown for different loading positions, see Figures 10 and 11. Figure 10 shows the effect of torsional moments as the load is placed at different positions in the longitudinal direction and Figure 11 shows the results with different transverse directions on the x-axis. In each of the plots of Figures 10 and 11 there is an approximately linear relation between the position of the load along the width and length of the slab models.

The effect of torsion will now be derived as a function of the position of the load in the x- and y-direction. To find a generally valid expression, the position of the load is expressed based on the distance a, the center-to-center distance between the load and the support, and b_r the distance from the free edge to the center of the load. These positions are indicated in Figure 12. As can be seen from Figure 12, the value of b_r lies between 0 and $b/2$, $0 \leq b_r \leq b/2$, with b the total width of the slab, as a result of symmetry in the transverse direction.

The distances a and b_r describe the position of the load and are used to derive a new expression for the factor β that can be used in the Modified Bond Model. As shown in Figure 1, the bounds that need to be respected for this value are 0 (lower bound, case for which torsion is important and the capacity needs to be reduced) and 1 (upper bound, case for which torsion can be neglected and does not affect the capacity). To mathematically express the dependence of the reduction factor β on the x and y coordinates, the following expression is used:

$$\beta(x, y) \rightarrow \beta(a/d, b_r/b) \quad (1)$$

From the results in Figures 10 and 11, a linear relation between the moments ratios m_{xy}/m_x and the loading position can be observed. This observation is valid for a position of the load between a = (0 – 700 mm). After this position, the effect of the vicinity of the support fades out. As such, the effect needs to be only considered for shear span to depth ratios a/d up to 2.5.

First, the effect of torsion for different positions of the load in the longitudinal direction is assessed based on the span-to-depth ratio a/d. Assuming a linear approximation for the moment ratios m_{xy}/m_x (as observed in Figures 10 and 11), the resulting expression is then:

$$\beta\left(\frac{a}{d}\right) = 0.4\frac{a}{d} \quad \text{for } 0 \leq \frac{a}{d} \leq 2.5 \quad (2)$$

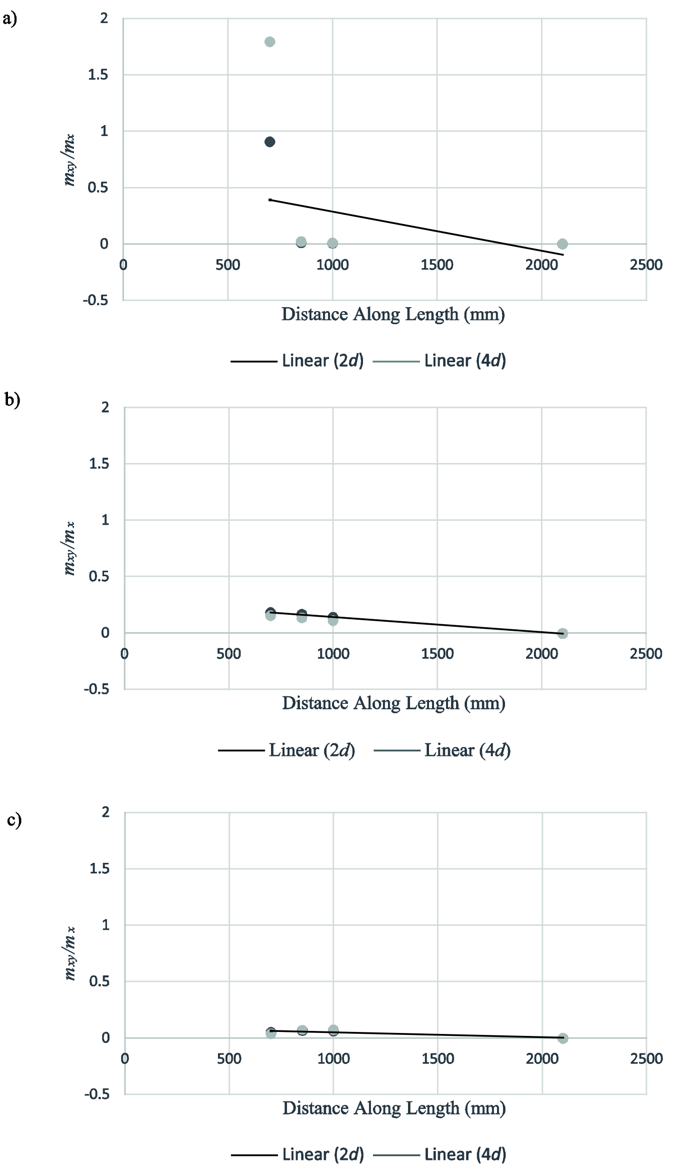

Figure 10. Moment ratios (m_{xy}/m_x) for the twelve models at distances: a) 1250 mm, b) 438 mm and c) 850 mm along the width.

For loads placed along the y direction (Fig. 11) the values of the m_{xy}/m_x ratios of the slab models are approximated to a linear relation between those values and the position of the load along the length of the slab model.

In a next step of the derivation, the transverse direction is considered. The value of b_r goes from 0 to $b/2$, because of symmetry around the x-axis. The position of the load is thus taken from one free edge to the middle of the slab. Then, the expression becomes:

$$\beta\left(\frac{b_r}{b}\right) = 2\frac{b_r}{b} \text{ for } 0 \leq \frac{b_r}{b} \leq \frac{1}{2} \quad (3)$$

Finally, equations (2) and (3) are combined into the general type of expression from equation (1) to get the final expression for β. In this expression, the distance from the load to the center of the support a and the distance b_r from the free edge to the center of the load are used to express the loading position. The final equation becomes:

$$\beta = 0.8\frac{a}{d}\frac{b_r}{b} \text{ for } 0 \leq \frac{a}{d} \leq 2.5 \text{ and } 0 \leq \frac{b_r}{b} \leq \frac{1}{2} \quad (4)$$

The upper and lower bounds that were previously already used in the Modified Bond Model are respected by this equation, as shown for the two extreme cases:

Case 1: β = 1 for b_r/b = 1/2 and a/d = 2.5
Case 1: β = 1 for b_r/b = 1/2 and a/d = 2.5

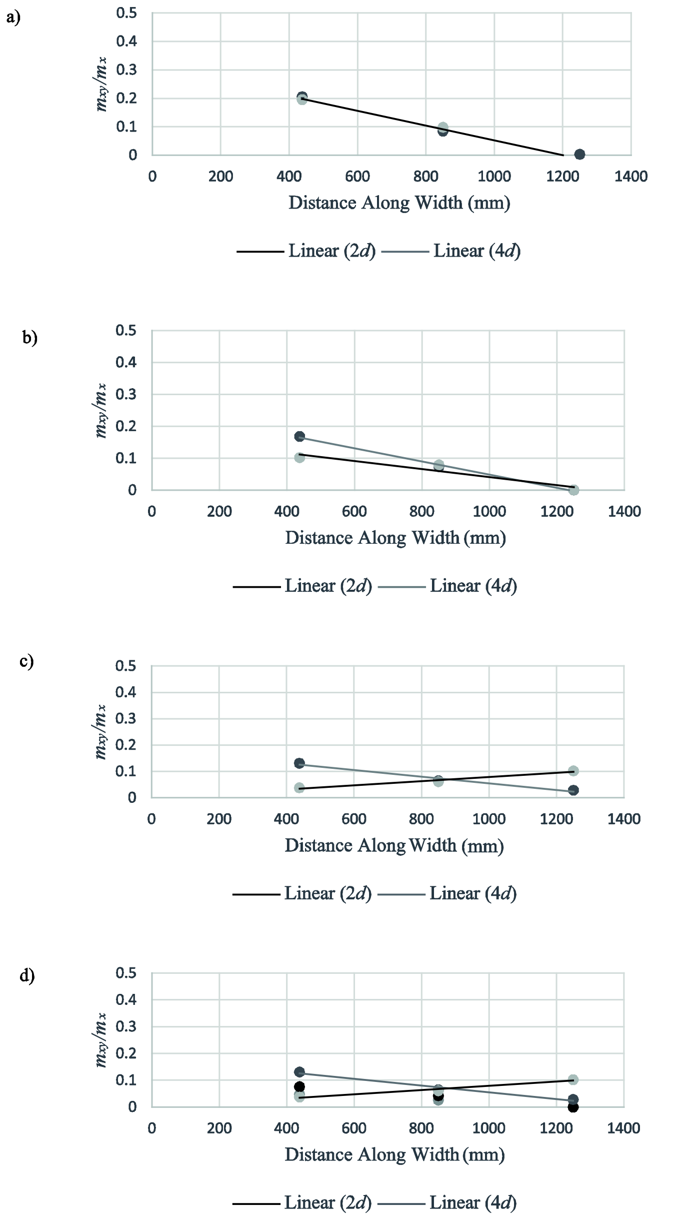

Figure 11. Moment ratios (m_x/m_{xy}) for the twelve models at distances: a) 700 mm, b) 850 mm, c) 1000 mm and d) 2100 mm along the length.

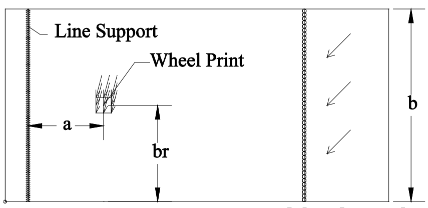

Figure 12. Distances a and b_r along the length and width.

6 SUMMARY AND CONCLUSIONS

In this paper, an expression for the effect of torsion on the shear and bending moment capacity of slabs is derived. The method used to quantify the ultimate capacity of slabs is the Modified Bond Model. This model needs a better expression for the effect of the torsional moments, which reduce the ultimate capacity. Therefore, the relative magnitude of the torsional moments with respect to the bending moments is studied by means of linear finite element models. In the models, different loading positions are considered. The relative effect of torsion is then linked to the distance along the width and length of the slab. The resulting expression for implementation in the Modified Bond Model is:

$$\beta\left(\frac{a}{d},\frac{b_r}{b}\right) = 0.8\frac{a}{d}\frac{b_r}{b} \text{ for } 0 \leq \frac{a}{d} \leq 2.5 \text{ and } 0 \leq \frac{b_r}{b} \leq \frac{1}{2}$$

7 ACKNOWLEDGEMENT

This research was financed by Universidad San Francisco de Quito USFQ (Ecuador) by means of the program of Chancellor Grants (grant no. xxx). The experiments that were used to benchmark model L1 were carried out at Delft University of Technology, with funding of the Dutch Ministry of Infrastructure and the Environment (Rijkswaterstaat). The authors want to express their gratitude to the funding institutions.

REFERENCES

AFHAMI, S. 1997. *Strip model for capacity of flat plate-column connections.* Thesis (Ph D), University of Alberta, 1997.

ALEXANDER, S. D. B. 1990. *Bond model for strength of slab-column joints.* Thesis (Ph D), University of Alberta, 1990.

ALEXANDER, S. D. B. & SIMMONDS, S. H. 1992. Bond Model for Concentric Punching Shear. *Aci Structural Journal,* 89**,** 325-334.

CEN 2003. Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges, NEN-EN 1991-2:2003. Brussels, Belgium: Comité Européen de Normalisation.

DOORGEEST, J. 2012. *Transition between one-way shear and punching shear.* MSc, Delft University of Technology.

FALBR, J. 2011. *Shear redistribution in solid concrete slabs.* MSc, Delft University of Technology.

HILLERBORG, A. 1975. *Strip Method of Design,* Wexham Springs, Slough, England, Viewpoint Publications, Cement and Concrete Association.

JOHANSSON, C., NUALLÁIN, N. Á. N., PACOSTE, C. & ANDERSSON, A. 2014. A methodology for the preliminary assessment of existing railway bridges for high-speed traffic. *Engineering Structures,* 58**,** 25-35.

LANTSOGHT, E., VEEN, C. V. D. & BOER, A. D. 2015a. Modified Bond Model for shear in slabs under concentrated loads. *Concrete – Innovation and Design, fib Symposium.* Copenhagen, Denmark

LANTSOGHT, E. O. L. 2012. Background to Modified Bond Model. Delft University of Technology.

LANTSOGHT, E. O. L. 2013a. Modified Bond Model: Analysis of edge effect and influence of bearings.

LANTSOGHT, E. O. L. 2013b. *Shear in Reinforced Concrete Slabs under Concentrated Loads Close to Supports.* Delft University of Technology; PhD Thesis.

LANTSOGHT, E. O. L. 2014. Modified Bond Model: Analysis and behavior of strips. Delft University of Technology.

LANTSOGHT, E. O. L., VAN DER VEEN, C. & DE BOER, A. 2014a. Predicting the Shear Capacity of Reinforced Concrete Slabs subjected to Concentrated Loads close to Supports with the Modified Bond Model. *IABSE 2014, Engineering for Progress, Nature and People.* Madrid, Spain.

LANTSOGHT, E. O. L., VAN DER VEEN, C., DE BOER, A. & WALRAVEN, J. 2014b. Influence of Width on Shear Capacity of Reinforced Concrete Members. *ACI Structural Journal,* 111**,** 1441-1450.

LANTSOGHT, E. O. L., VAN DER VEEN, C., DE BOER, A. & WALRAVEN, J. 2015b. One-way slabs subjected to combination of loads failing in shear. *ACI Structural Journal,* 112**,** 417-426.

LANTSOGHT, E. O. L., VAN DER VEEN, C., DE BOER, A. & WALRAVEN, J. C. 2013a. Recommendations for the Shear Assessment of Reinforced Concrete Slab Bridges from Experiments *Structural Engineering International,* 23**,** 418-426.

LANTSOGHT, E. O. L., VAN DER VEEN, C., WALRAVEN, J. & DE BOER, A. 2013b. Peak shear stress distribution in finite element models of concrete slabs. *Structural Engineering, Mechanics and Computation 2013.* Cape Town, South Africa.

LANTSOGHT, E. O. L., VAN DER VEEN, C. & WALRAVEN, J. C. 2013c. Shear in One-way Slabs under a Concentrated Load close to the support. *ACI Structural Journal,* 110**,** 275-284.

LANTSOGHT, E. O. L., VEEN, C. V. D. & BOER, A. D. 2015c. Modified Bond Model for Shear in Slabs under Concentrated Loads. *In:* STANG, H. & BRAESTRUP, M. (eds.) *fib Symposium Copenhagen: Concrete Innovation and Design.* Kopenhagen.

NEMETSCHEK-SCIA 2015. Scia Engineer. Version 15.1 ed. Herk-de-Stad, Belgium.

PARK, R. & GAMBLE, W. L. 2000. *Reinforced Concrete Slabs,* New York, John Wiley & Sons, INC.

TEWORTE, F., HERBRAND, M. & HEGGER, J. 2015. Structural Assessment of Concrete Bridges in Germany — Shear Resistance under Static and Fatigue Loading. *Structural Engineering International,* 25**,** 266-274.

VALDIVIESO, D., LANTSOGHT, E. O. L. & SANCHEZ, T. 2015. Effect of Torsion on Shear Capacity of Slabs. Politecnico, Universidad San Francisco de Quito.

VAN HEMERT, P. 2012. *Shear Capacity of Reinforced Concrete Slabs under Line and Wheel Load Close to the Support.* MSc, Delft University of Technology.

VOORMEEREN, L. 2011. *Extension and Verification of Sequentially Linear Analysis to Solid Elements.* M.Sc., Delft University of Technology.