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Selecting the number and location of sources and receivers for non-linear time-domain inversion

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***Abstract*—The design of ultrasound scanning systems for breast cancer detection is a challenging task. To decide on the number of transducers and where to place them, several approaches could be followed. A simple and straightforward approach is to compute for several configurations the energy distribution in the region of interest, and treat each receiver as if it is a transmitter. In that case, the assumption is made that the effect a source or a receiver has on the resulting image is on an equal level. This assumption is mainly based on reciprocity; the observation that the response R_{AB} measured by a receiver at the location B and for a transmitter at location A is identical to the response R_{BA} obtained after interchanging the source and receiver. This is true for linear imaging methods, where a source can be interchanged with a receiver without affecting the resulting image. However, for the non-linear imaging method named Contrast Source Inversion (CSI), it does matter. In the past we showed that interchanging sources with receivers does affect the image. In this work, we evaluate the reciprocity property for the non-linear time-domain inversion (TDI). We show that the reciprocity principle is satisfied when using TDI for imaging, by using a synthetic experiment design.**

***Index Terms*—non-linear inversion, reciprocity, breast cancer detection**

I. Introduction

Ultrasound has enabled breast imaging for the detection and identification of malignant lesions. Different ultrasound imaging techniques can be used to reconstruct the profile of inhomogeneous objects from the scattered field measurements. Traditionally, the object profile reconstruction have been formulated as a linear inverse problem because of its simplicity and relatively good results when the objects have a small speed of sound variation. Non-linear methods for breast imaging are becoming more popular because they retrieve images of higher quality in comparison to the linear methods [1].

The design of ultrasound scanning systems is not trivial when the profile reconstruction is formulated as a non-linear problem. We approach the design of the ultrasound scanning system by evaluating if the principle of reciprocity holds for the inverse problem. The principle of reciprocity states that for a fixed configuration with a constant object, the response R_{AB} measured by a receiver at the location B and for a transmitter at location A is identical to the response R_{BA} obtained after interchanging the source and receiver. Previously, we have shown that for the linear Synthetic Aperture Focussing Technique (SAFT) and the Conjugate Gradient (CG) method, the sources and receivers can be interchanged and the principle of reciprocity holds, resulting in the same images after reconstruction [2]. However, for the non-linear contrast source inversion (CSI) method, the results show that it is preferred to have more receivers than sources, despite the fact the measured signals are reciprocal [1], [2].

In this paper, we evaluate the principle of reciprocity for the method referred to as time-domain inversion (TDI). This method is based in a finite-difference time-domain approximation of the acoustic and isotropic wave equation, which uses the Gauss-Newton method to obtain the gradient via the first order adjoint state approach to reconstruct the model parameters [3], [4]. The major difference between TDI and CSI relies on the formulation of the inverse problem. TDI establishes the inverse problem as the minimization of a ℓ_2-error functional over the data, whereas CSI is based on the minimization of an error functional containing two terms; one reflecting the mismatch between the field measured and computed from the contrast sources, and one relating the contrast sources to the sound speed contrast profiles.

II. Theoretical Background

Consider a scattering object with speed of sound profile $c(\mathbf{x})$ contained in the two-dimensional spatial domain $\mathbb{D}$ with Cartesian coordinates $\mathbf{x} = (x, y)$. The object is imaged using a set of sources and receivers located in the spatial domain $\mathbb{S} \notin \mathbb{D}$. The time-domain pressure wavefield denoted by $p_j(\mathbf{x})$ satisfies the scalar wave equation with constant density

$$\nabla^2 p_j(\mathbf{x}) - \frac{1}{c^2(\mathbf{x})} \frac{\partial^2 p_j(\mathbf{x})}{\partial t^2} = -S_j(\mathbf{x}), \qquad (1)$$

where t is the time variable and $S_j(\mathbf{x})$ is the j^{th} primary source generating the wavefield.

The inverse problem consists in reconstructing the speed of sound $c(\mathbf{x})$ for known primary sources $S_j(\mathbf{x})$, and known pressure field $p_j(\mathbf{x})$ measured by the receivers located in $\mathbb{S}$. This is an ill-posed problem because the speed of sound and the pressure field are both unknown inside the spatial domain $\mathbb{D}$. Despite the fact that the wave equation in equation (1)

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is linear with respect to the sources because it satisfies the superposition principle, the inverse problem is non-linear because $c(\mathbf{x})$ and $p_j(\mathbf{x})$ are not linearly related, as can be seen in equation (1).

With TDI, the problem of reconstructing the unknown speed of sound profile is formulated as

$$\min_{c} \|\mathbf{R}\{\mathbf{L}(c)[p_j(\mathbf{x})] + S_j(\mathbf{x})\} - \mathbf{d}_j^{obs}\|_{\mathbb{S}}^2, \quad (2)$$

where $\mathbf{R}\{\cdot\}$ is the time-domain finite-difference operator used to obtain the modeled data $\mathbf{d}_j^{mod}$, $\|\cdot\|_{\mathbb{S}}^2$ is the ℓ_2-norm over the data domain, $\mathbf{L}(c)[\cdot] = \nabla^2[\cdot] - \frac{1}{c^2(\mathbf{x})}\frac{\partial^2[\cdot]}{\partial t^2}$ is the d'Alembertian operator, and $\mathbf{d}_j^{obs}$ is the observed data in the spatial domain $\mathbb{S}$. The gradient of the cost function is obtained using the adjoint state method, which uses the Fréchet derivative of equation (2), and it is given by

$$\mathbf{L}^{\dagger}(c)[\lambda_j(\mathbf{x})] = -(\mathbf{R}\{\mathbf{L}(c)[p_j(\mathbf{x})] + S_j(\mathbf{x})\} - \mathbf{d}_j^{obs})^T, \quad (3)$$

where $\mathbf{L}^{\dagger}(c)[\cdot]$ is the adjoint operator of $\mathbf{L}(c)[\cdot]$ and $(\cdot)^T$ represents a flip on the time direction. For the acoustic wave equation that models isotropic media with constant density, the operator is self-adjoint, hence $\mathbf{L}^{\dagger}(c)[\cdot] = \mathbf{L}(c)[\cdot]$. Therefore equation (3) resembles a wave equation where $(\mathbf{R}\{\mathbf{L}(c)[p_j(\mathbf{x})] + S_j(\mathbf{x})\} - \mathbf{d}_j^{obs})^T$ represents a source and $\lambda_j(\mathbf{x})$ the resulting wavefield.

The sound speed model can be updated using the Gauss-Newton method, which finds an update of the sound speed at iteration $k+1$, as

$$c^{k+1} = c^k - \alpha \cdot \mathbf{g}(c^k), \quad (4)$$

For weak contrast, the initial speed of sound model c^0 can be set to the speed of sound of the embedding, and the step length α is set to one. The iterative TDI method stops when a maximum number of iterations or a tolerance error is reached.

Note in equation (3) that TDI method minimizes a ℓ_2-functional over the data only. Therefore, the update of the speed of sound model (or the contrast) generates a wave field that satisfies the wave equation, in the ℓ_2-error sense. Extra penalty terms can be added to the problem formulated in equation (3), as constraints, in order to reduce the ill-possedness of the inverse problem, to increase the convergence rate or to represent the data in a certain domain [5].

III. Experimental Results

The reciprocity property of TDI is evaluated using the reconstruction of the 2-D speed of sound profile of a synthetic breast model, which is built in terms of inhomogeneities representing two different type of tissues of different sizes: tumor ($c = 1572$ m/s) and fat ($c = 1437$ m/s); inside homogeneous medium representing breast tissue ($c = 1540$ m/s). The synthetic model is submerged in water ($c_0 = 1520$ m/s). The measured data represents a full 2-D scan using a circular array with diameter 259 mm, containing a configuration of sources and receivers. The synthetic model is shown in Figure 1, with a circular array having a particular configuration of transducers of 100 receivers (blue crosses) and 10 sources (red dots). The spatial domain contains 128×128 elements of size $\Delta x \times \Delta y \approx 2.2$ mm$\times 2.2$ mm. Each transducer acts as a point source, and generates a Gaussian-modulated wavefield with a central frequency $f_0 \approx 90$ kHz. The central frequency is obtained as $f_0 = \frac{c_0}{N_\lambda \cdot \Delta x}$, where $N_\lambda = 8$ is the number of points per wavelength.

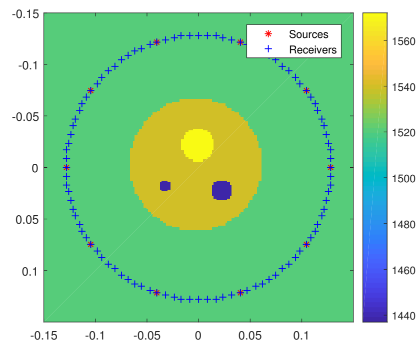

Fig. 1. Synthetic speed of sound profile and the circular array having a particular configuration of transducers: 100 receivers (blue crosses) and 10 sources (red dots).

The synthetic noise-free measurement data used to test the TDI is computed by solving the forward model problem using finite-difference time-domain approximation of the full-wave equation in (1), with a second and eighth order stencil for time and space derivatives, respectively. The temporal step size is $\Delta t = 0.276$ μs to meet the Courant's stability criterion and to reduce numerical dispersion.

A. Less receivers than sources

We first test a transducer configuration, where the number of receivers is lower than the number of sources. In particular, we locate 100 sources equally spaced at the circular array. On the other hand, only 10 receivers (which corresponds to 10% of the number of sources) are located at the same circular array, and also equally spaced.

The images obtained with TDI are shown in the top-row of Figure 2, for different number of iterations. After a few number of iterations (~ 32), the image converges to a good quality image, despite of using only 10 receivers.

B. Less sources than receivers

We now test the reciprocal transducers configuration, where the number of sources is lower than the number of receivers. This means, we locate 10 sources and 100 receivers, both equally spaced at the circular array.

The images obtained with TDI for the reciprocal configuration are shown in the bottom-row of Figure 2, for the same number of iterations as before. It can be noticed

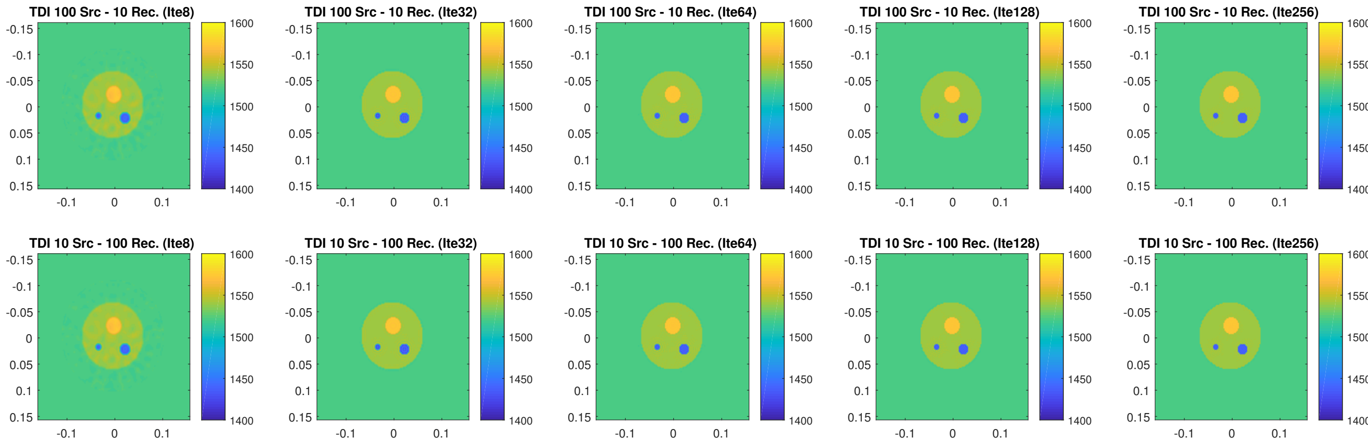

Fig. 2. Reconstruction results for different number of iterations. Top-row shows the results for 100 sources and 10 receivers. Bottom-row shows the results for 10 sources and 100 receivers.

Iteration	Relative error (%)
8	0.13
32	0.06
64	0.06
128	0.02
256	0.01

TABLE I
MAXIMUM ERROR RELATIVE TO THE CORRECT SPEED OF SOUND VALUE, PER ITERATION.

that the rate of convergence for the reciprocal configuration of sources and receivers is the same.

The difference between the resulting images for both source/receivers configurations in all iterations is small. The maximum difference between the images obtained with both configurations is shown in Table I. Note that the largest relative difference is found at iteration 8^{th} and is only 0.13% of the correct speed of sound value, which is 1437 m/s for this case. The maximum numerical difference between the images obtained with both configurations is negligible, and therefore exchanging sources and receiver retrieves the same image for time-domain inversion.

IV. Conclusions

In previous work, we showed that the sources and receivers can not be interchanged for the CSI method, and that having more receivers than sources is desirable in order to obtain a higher quality image. This occurs since the CSI method simultaneously minimizes functionals over the data and object equations by alternating conjugate-gradient updates of the contrasts and the fields. Interchanging sources with receivers can be done when only a functional over the data space is minimized. This is the case for the time-domain inversion method, studied in this paper. We showed that despite the non-linear relation between sources/receivers and the speed of sound model (or contrast), the principle of reciprocity is satisfied by the TDI method. Thus, interchanging the sources and receivers in the design of an ultrasound imaging system give the same result when TDI is used as imaging technique. Including an additional constraint in the TDI method, such as an error functional in the object domain, requires to evaluate if the sources and receivers still can be interchanged.

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