 With Compliments

Ship Motions in Beam Seas

By

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Abstract

A ship laid broadside on the regular progressive waves generally performs heaving, pitching, rolling, swaying, yawing and drifting motion.

In this paper, a set of two coupled linear differential equations of the second order for heave and pitch, and a set of three coupled equations for sway, yaw and roll have been developed by making use of the Strip Method. Then we measured the amplitudes of heave, pitch, sway, roll and drifting velocity for the ship-model of Todd 60 Series $C_b=0.70$ at two conditions. The equipment for measuring six motions of ship-model has been used. Then a comparison was made between calculated heaving, pitching, swaying, rolling motions and the results of tank experiments in regular waves. Lastly unstable rolling motion in beam sea was investigated.

Main conclusions drawn are as follows.

1. Heaving and Pitching motions can be described with sufficient accuracy by the coupled equations. DOMPEN STAMPEN
2. Yawing motion is negligibly small. GIEREN VERWAARLOOZEN
3. The solutions obtained by the uncoupled swaying equation coincide well → ONGEKOPPELDE with the experimental values, and hydrodynamic coupling effects produced by rolling and yawing motions are very small.
4. The roll amplitudes obtained from the two linear coupled equations for roll and sway denote fully the experimental results in the neighbourhood of resonant period.
5. When the rolling exciting moment is small, the hydrodynamic coupling moments derived from the swaying motion have great influence upon the rolling motion.
6. The drifting velocity has two maxima at the resonant frequencies for heave and roll.
7. The records of unstable rolling motion caused by quasiharmonic rolling moment have been obtained even in case of small wave height. But in case of irregular waves the unstable roll never appeared. 2

I. Introduction

The rolling motion among ship motions in the beam sea has been investigated by many up to the present. The safety of ships can be well judged by means of the several safety regulations which were established by the studies of rolling motion and stability of a ship.

But, as for the rolling motons in following and oblique seas, we have problems yet to solve, such as coupling roll moments produced by sway and yaw motions etc.

In the beam sea, the ship is generally obliged to do heaving, pitching, swaying, yawing oscillations and drifting motion in addition to the rolling oscillation.

Considering the coupling roll moments produced by the heaving and pitching motions Yamazaki and Fukuda [1]* calculated the rolling oscillation in oblique seas. In solving the pitching and heaving motions in oblique waves they used the Watanabe's strip theory [2].

On the other hand, when Eda analysed ship motions at the horizontal plane in oblique waves, he solved coupled linear differential equations of the swaying and yawing oscillation. He assumed that swaying, heaving, pitching motions never couple to rolling motion. He also evaluated the external forces by means of the strip method developed by Watanabe [2].

As the results of his calculation, he showed that the solutions of the yawing motion were in good coincidence with the experimental results.

As for the coupling effects of the swaying motion to the rolling one, there are some investigations made by Ueno [4], O. Grim [5] and Tamura [6] etc.

Making use of the exact values of the two-dimensional hydrodynamic forces and moments generated by swaying and rolling motions Tamura calculated the rolling motion of the two-dimensional body around a horizontal fixed axis. But the results of his calculation did not coincide with the experimental results. It seems that it is mainly due to the neglection of the viscous damping moment.

Generally a ship has six degrees of freedom. In the category of linearised theory, we can analyse the ship motions by dividing them into two groups, that is, symmetric and anti-symmetric ones.

Surging, heaving and pitching motions belong to the former, and swaying, rolling and yawing motions belong to the latter.

Each of the groups is composed of three coupled linear differential equations, and the mutual coupling effects of the groups are small quantities of the 2nd order [7]. Then the six motions of a ship can be analysed as the first approximation by solving the coupled equations of pitching, heaving, surging and those of swaying, rolling, yawing motions.

As for the symmetric motions, according to the the theory [2], many investigations have been recently done neglecting the effect of the surging motion.

Therefore, it is necessary to study systematicaly the motions of the 2nd group.

In this paper, in the first place, for the analysis of the motions of a ship with zero speed in the beam sea, a set of two coupled equations of heaving and pitching motions and a set of three coupled equations of swaying, yawing and rolling motions have been developed.

Then, in the beam sea conditions, motions of the ship model of the Todd 60 Series $C_b=0.70$ were measured by means of the equipment for measuring six motions of a ship model [8] and compared with the theoretical calculations.

It was made clear as the results of these investigations that, in a special condition, the hydrodynamic moment produced by the swaying oscillation greatly affects the rolling motion.

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* Numbers in brackets designate References at the end of the paper.

On the other hand, it has been hitherto believed that the drifting motion is largest at the resonant condition of rolling motion, but our experimental results showed the drifting velocity to be very large also in the heaving resonant condition.

Paulling [9] testified that the unstable rolling motion due to non-linear static rolling moment is caused by the heaving oscillation in still water.

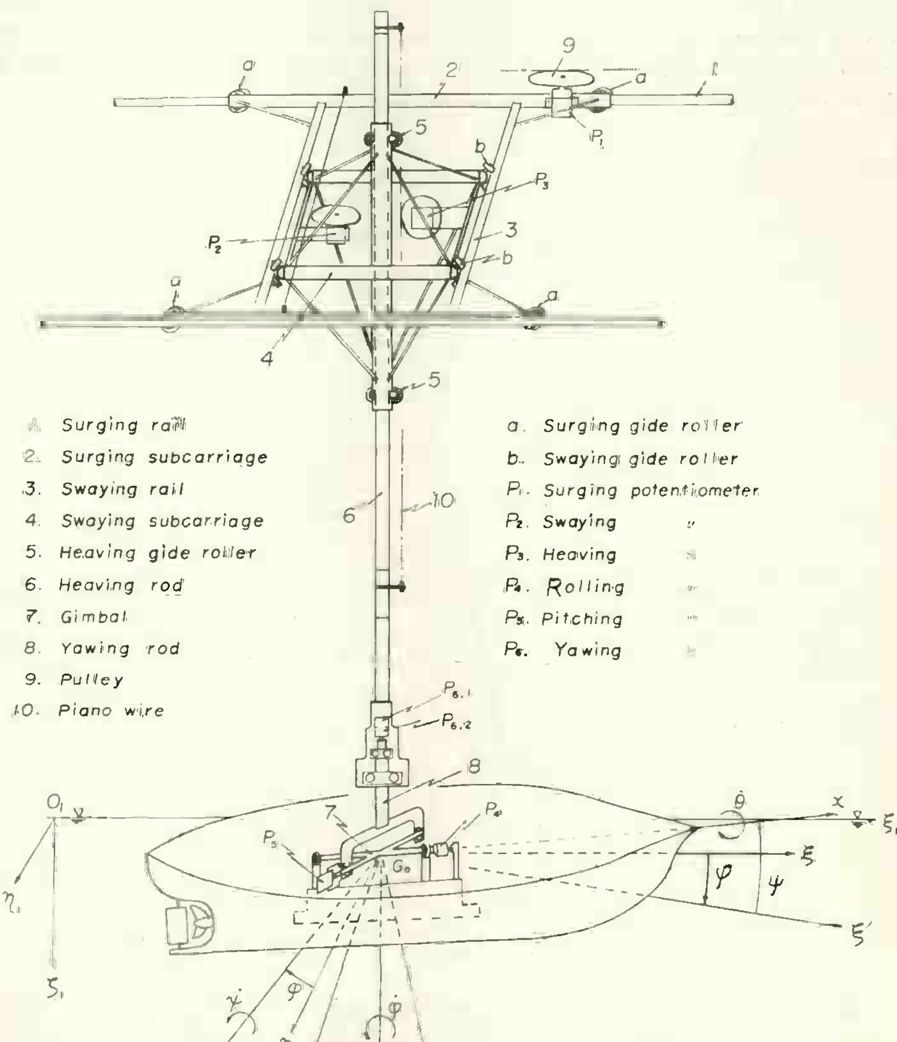


Fig. 1.

In this paper also, the coupled motion between heave and roll was investigated experimentally.

II. Equations of motions and their solutions

In Fig. 1 the equipment for measuring six motions of a ship model [8] and the coordinate axes are shown. In Fig. 1 and Fig. 2, O_1-$\xi_1\eta_1\zeta_1$ is a spatial axis, G_0-$x_1y_1z_1$ a body axis and G_0 the center of gravity of a ship. It is defined that heaving displacement is ζ, surging one ξ, swaying one η, rolling angle about the x_1-axis θ, yawing angle about the ζ axis φ and pitching angle about η^* axis ψ. Six potentiometers were used for measuring these displacements.

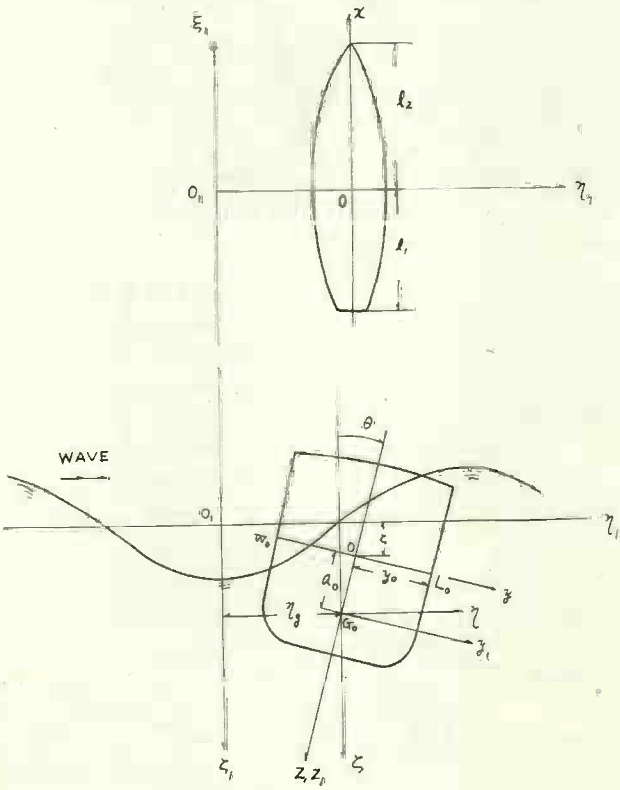


Fig. 2.

In the next place, we consider a ship laid broadside on the regular wave train which progresses in the direction of η_1 and oscillates freely. In Fig. 2, W_0OL_0 is the painted load water plane. For the sake of convenience of the theoretical calculations we take a boby axis O-xyz.

When a ship floats on the still water plane the O-xyz coincides with the O_1-$\xi_1\eta_1\zeta_1$. Now, in the beam sea, in addition to the periodic yawing motion a ship rotates about the ζ axis, that is to say, a leeway angle generally happens owing to the anti-symmetric drifting force and the gyroscopic moment.

We omit the experimental results that the leeway angle is large and therefore, it is assumed in the theoretical calculations that the orientation of a ship is always broadside on the traveling direction of the wave. And moreover we neglect all the nonlinear couples, that is, gyroscopic forces and moments by assuming that they are small.

Then we can assume reasonably that the ship motions can be approximately analysed by dividing them into two groups.

In Fig. 2, η_g is the horizontal displacement of G_0, and it is assumed that η_g is composed of the periodic swaying displacement η and the uniform drifting displacement η_d.   steady in equations of motion

Owing to this drifting velocity therefore, external forces act upon a ship with a period of encounter.

II-1. Heaving and Pitching motions

Equations of the heaving and pitching motions can be approximately obtained by means of the strip method.

Hydrodynamic inertia force acting upon a section of a ship when it dips into still water is as follows:

$$\frac{dF_\zeta}{dx} \doteqdot -\rho K_\zeta S_w \dot{v}_\zeta$$

where ρ is the density of the fluid and S_w, v_ζ, K_ζ the immersed sectional area and the dipping velocity and the coefficient of heaving added mass of the section.

In Fig. 2, the equation of the subsurface of waves progressing to the η_1 direction is $\zeta_w = h\, e^{-k\zeta_1} \cos(k\eta_1 - \sigma t)$, in which h is the amplitude of incident regular wave.

Putting λ and T_w for wave length and wave period respectively, we obtain $k = 2\pi/\lambda$ and $T_w = 2\pi/\sigma$.

Then, expressing ζ_w with the body axis we have approximately

$$\zeta_w \doteqdot h e^{-k\zeta} \cos(ky - \sigma_e t)$$

where $\sigma_e = \sigma - k v_d$ and v_d is the drifting velocity.

Therefore, downward velocity of the water particle is

$$\dot{\zeta}_w = h\sigma e^{-kz} \sin(ky - \sigma_e t)$$

On the surface of a ship $\dot{\zeta}_w$ is various in magnitude. Assuming that the breadth and draft of a ship are small as compared with the wave length, we adopt $\dot{\zeta}_w$ at $y=0$ and on the subsurface in the neighbourhood of the bottom of a ship approximately.

That is to say, we use $\dot{\zeta}_w \begin{pmatrix} \text{at } y=0 \\ z=\bar{z} \end{pmatrix} = \dot{\bar{\zeta}}_w = -h\sigma\, e^{-k\bar{z}} \sin\sigma_e t$ and the following equivalent mean draft

$$\bar{z} = S_w/2y_0 \text{ or } \bar{z} = V_0/A_w = \frac{C_b}{C_w} \cdot d$$

$\doteqdot T \cdot \beta$

dus [illegible] gemeten op de bodem.

y=0 is geoorloofd omdat B e T klein zijn t.o.v. λ

In which, V_0 is the volume of the displacement, A_w is the area of waterplane, d the draft, C_b, C_w the block and waterplane coefficient respectively.
Then we also obtain

$$\ddot{\zeta}_w = -h\sigma^2\, e^{-k\bar{z}} \cos\sigma_e t$$

relative motion approach

When the heaving displacement ζ, pitching angle ψ and $\dot{\zeta}_w$ are considered, the relative dipping velocity of the section becomes $v_\zeta \doteqdot \dot{\zeta} - \dot{\zeta}_w - x\dot{\psi}$. The hydrodynamic inertia force acting on this section per unit length therefore becomes

$$\frac{dF_{\zeta_1}}{dx} = -\rho K_\zeta S_w(\ddot{\zeta} - \ddot{\zeta}_w - x\ddot{\psi}) \tag{1}$$

$K_\zeta \cdot S_w$ volume [illegible] masse

And the hydrodynamic damping force becomes

$$\frac{dF_{\zeta_2}}{dx} = -N_0 v_\zeta = -\frac{\rho g^2 \bar{A}_\zeta^{\,2}}{\sigma_e^{\,3}}(\dot{\zeta} - \dot{\zeta}_w - x\dot{\psi}) \tag{2}$$

where $\bar{A}_\zeta = h_\zeta / s$. h_ζ represents the wave amplitude generated by heaving oscillation of the two-dimensional cylinder with this sectional area S_w and s is the heaving amplitude.

In the third, we must take into consideration the Froude-Krilov's force exerted by the incident wave and the force depending on the hydrostatic pressure. Neglecting the small quantity of the 2nd-order we have

$$\frac{dF_{\zeta_3}}{dx} = -2\rho g y_0(\zeta - x\psi) + w - \rho g S_w + 2\rho g y_0\, e^{-k\bar{z}}\, h \cos\sigma_e t \tag{3}$$

where w is weight of the section per unit length.

Besides these, there exists viscous force. However, it can be neglected since it is generally small except for the high frequency region.

Then the total force acting on this section becomes as follows:

$$\frac{dF_\zeta}{dx} = \frac{dF_{\zeta_1}}{dx} + \frac{dF_{\zeta_2}}{dx} + \frac{dF_{\zeta_3}}{dx}$$

Letting m_0 denote the mass of a ship and J_y mass moment of inertia for pitching, the approximate equation of heaving motion and that of pitching motion about G_0 are given as follows:

$$m_0\, \ddot{\zeta} = \int_{-l_1}^{l_2} \frac{dF_\zeta}{dx} \cdot dx$$

$$J_y\, \ddot{\psi} = -\int_{-l_1}^{l_2} \frac{dF_\zeta}{dx} \cdot x \cdot dx$$

Putting (1), (2), (3) into the above equations we have

$$\begin{aligned} a\ddot{\zeta} + b\dot{\zeta} + c\zeta - d\ddot{\psi} - e\dot{\psi} - f\psi &= F_1 \cos\sigma_e t + F_2 \sin\sigma_e t \\ A\ddot{\psi} + B\dot{\psi} + C\psi - d\ddot{\zeta} - e\dot{\zeta} - f\zeta &= M_1 \cos\sigma_e t + M_2 \sin\sigma_e t \end{aligned} \tag{4}$$

where

$m_{zz} = \rho\nabla\cdot\bar{K}_\zeta$

$$a=m_0(1+\bar{K}_\zeta), \qquad m_0\bar{K}_\zeta=\int_{-l_1}^{l_2}\rho K_\zeta S_u dx =$$

not app

$$b=\bar{N}_\zeta=\int_{-l_1}^{l_2}N_0 dx, \qquad c=\rho g A_w$$

$$d=\int_{-l_1}^{l_2}\rho K_\zeta S_w x dx, \qquad e=\int_{-l_1}^{l_2}N_0 x dx$$

$$f=\rho g\int_{-l_1}^{l_2}2y_0 x dx, \qquad A=J_y+I_y$$

$$I_y=\int_{-l_1}^{l_2}\rho S_w K_\zeta x^2 dx, \qquad B=\bar{N}_\psi=\int_{-l_1}^{l_2}N_0 x^2 dx, \quad C=W\cdot\overline{G_0M_L}$$

In the equations (4), no three-dimensional corrections have been done for the hydrodynamic forces and moments.

As for the external forces resulted from ζ_w and $\dot{\zeta}_w$, another approximate method has been developed by Motora [10] recently.

Now, putting $\zeta=\zeta_0\cos(\sigma_e t-\varepsilon_\zeta)$ and $\psi=\psi_0\cos(\sigma_e t-\varepsilon_\psi)$ we can calculate ζ_0, ε_ζ, ψ_0 and ε_ψ from (4).

The equations obtained by putting $\psi=\pi/2$ and $V=0$ in [1] are to coincide with (4).

Making use of (4) we calculated ζ_0, ε_ζ, ψ_0 and ε_ψ for the condition 1 (See Table 2) of the ship model. These results are shown in Figs. 3 and 4.

II. 2. Swaying and Yawing motions

Swaying and Yawing motions are generally forced oscillations, for each of these has no restoring force or moment. It is considered that a floating body in beam sea makes almost the same swaying motion as water particles in an effective wave do.

By the method similar to that in the former section, following equation of swaying motion is obtained.

$$m_0(\ddot{\eta}-\ddot{\eta}_w)+m_0\bar{K}_\eta(\ddot{\eta}-\ddot{\eta}_w)+\bar{N}_\eta(\dot{\eta}-\dot{\eta}_w)=0$$

where

$\bar{K}_\eta$ = coefficient of the added mass for swaying oscillation

$\bar{N}_\eta$ = coefficient of the wave-making damping (viscous damping ...)

From the above equation we get

$$\eta\doteqdot\bar{\eta}_w\equiv\eta_w\begin{pmatrix}\text{at } y=0\\ z=\bar{z}_1\end{pmatrix}=-he^{-k\bar{z}_1}\sin\sigma_e t$$

$\bar{z}_1$ is the depth of orbital center of the effective wave. Assuming $\bar{z}_1\doteqdot d/2$ as was done by Eda [3], following solution is obtained.

$$\eta=-he^{-\frac{kd}{2}}\sin\sigma_e t \tag{5}$$

Now, in the beam sea, the forces acting on the body in the direction of η axis can be divided into three parts.

That is. $F_\eta = F_{\eta_1} + F_{\eta_2} + F_{\eta_3}$

F_{η_1} is the hydrodynamic inertia force and F_{η_2} the wave-making damping force acting on the body when it sways on the still water surface.

$F_{\eta e}$ is, the exciting force, composed of the Froude-Krilov's force and the force which is due to the reflection of waves from the restrained body in incident waves.

For the two-dimensional body, F_{η_1} and F_{η_2} have been exactly calculated by Tasai [11], Tamura [6], and also $F_{\eta e}$ by Tamura [6] as a boundary-value problem.

In Fig. 2, putting v_η the velocity of a section in the direction of η axis, we have $v_\eta = \dot{\eta} + x\dot{\varphi}$

Let K_η and N_η be the two-dimensional coefficients of the added mass and wave-making damping force.

Then forces acting on the section are expressed as follows:

$$\frac{dF_{\eta_1}}{dx} = -\rho S_w K_\eta \frac{dv_\eta}{dt} = -\rho S_w K_\eta(\ddot{\eta} + x\ddot{\varphi}) \tag{6}$$

$$\frac{dF_{\eta_2}}{dx} = -N_\eta(\dot{\eta} + x\dot{\varphi}) \tag{7}$$

$$\frac{dF_{\eta e}}{dx} = K_r \sin\sigma_e t + K_i \cos\sigma_e t \tag{8}$$

Assuming, as we did in considering heaving and pitching motions, that the viscous force is small and then neglecting it we get the following set of two coupled linear differential equations of swaying and yawing motions.

In this case also, the strip method has been used, where three-dimensional correction is left out of consideration.

Coupled equations are

$$\begin{aligned} &m_0(1+\overline{K}_\eta)\ddot{\eta} + \overline{N}_\eta \dot{\eta} + m_0\overline{K}_\eta \bar{x}_1\ddot{\varphi} + \overline{N}_\eta \bar{x}_2\dot{\varphi} = \overline{K}_r \sin\sigma_e t + \overline{K}_i \cos\sigma_e t \\ &(J_z + I_z)\ddot{\varphi} + \overline{N}_\varphi\dot{\varphi} + m_0\overline{K}_\eta \bar{x}_1\ddot{\eta} + \overline{N}_\eta \bar{x}_2\dot{\eta} = \overline{M}_{\varphi r}\sin\sigma_e t + \overline{M}_{\varphi i}\cos\sigma_e t \end{aligned} \tag{9}$$

where

$$m_0\overline{K}_\eta = \int_{-l_1}^{l_2} \rho S_w K_\eta dx, \qquad \overline{K}_r = \int_{-l_1}^{l_2} K_r \cdot dx$$

$$m_0\overline{K}_\eta \bar{x}_1 = \int_{-l_1}^{l_2} \rho S_w K_\eta x \cdot dx, \qquad \overline{K}_i = \int_{-l_1}^{l_2} K_i \cdot dx$$

$$\overline{N}_\eta = \int_{-l_1}^{l_2} N_\eta \cdot dx, \qquad \overline{M}_{\varphi r} = \int_{-l_1}^{l_2} K_r \cdot x \cdot dx$$

$$\overline{N}_\eta \cdot \bar{x}_2 = \int_{-l_1}^{l_2} N_\eta \cdot x \cdot dx, \qquad \overline{M}_{\varphi i} = \int_{-l_1}^{l_2} K_i \cdot x \cdot dx$$

J_z = mass moment of inertia of a ship for yawing motion

$$I_z = \int_{-l_1}^{l_2} \rho S_w K_\eta \cdot x^2 \cdot dx, \qquad \overline{N}_\varphi = \int_{-l_1}^{l_2} N_\eta \cdot x^2 \cdot dx$$

For each section of a ship we can evaluate K_r, K_i from the figures in [6] and N_η, K_η from [6] and [11].

Computed results of K_η, $\frac{N_\eta\sqrt{gL}}{W}$ for the model ship used in this study are shown in Fig. 5, and also I_z/J_z, $\frac{N_\varphi\sqrt{gL}}{WL^2}$ in Fig. 6, where it was assumed that $J_z=(0.25L)^2 m_0$.

According to the theory by M. D. Haskind [12] and J. N. Newman [13], the exciting force can be expressed by $\bar{A}$. Using $\xi_d=\frac{\sigma_e^2}{g}d$ we get as follows for swaying motion,

$$|F_{\eta e}| = \rho ghd\,\bar{A}_\eta/\xi_d \tag{10}$$

Then $|\bar{F}_{\eta e}|$ obtained by the stripwise integration of (10) is expected to coincide with $\sqrt{K_r^2+K_i^2}$. These values for $h=3$cm are shown in Fig. 7, both of which are in good coincidence. The small difference seen in them will probably be the reading error from [6] and [11].

The external force obtained by Froude-Krilov's method, for example, according to Dr. Watanabe's calculation [14], is approximately expressed by $W\Theta_w\Phi_\eta \sin\sigma_e t$, as shown in Fig. 8, where Θ_w is the maxmum wave slope.

The difference between the exact exciting force and the above is due to the reflection of waves.

Then, putting $\eta=\eta_0 \sin(\sigma_e t-\varepsilon_\eta')$, $\varphi=\varphi_0 \cos(\sigma_e t-\varepsilon_\varphi)$ and solving the equations (9) for the condition-1, we obtained η_0, ε_η' as shown in Fig. 9.

The dotted line in Fig. 9 also denotes the approximate solution $e^{-\frac{kd}{2}}$. Because of the φ_0 is negligibly small quantity its figure was omitted. As for this model, in the beam sea, yawing motion is very small owing to the smallness of its fore and aft anti-symmetric character.

Coupling effects by the yaw motion to the sway motion is small and therefore the solutions obtained from (9) are almost equal to the solutions of the uncoupled equation of sway.

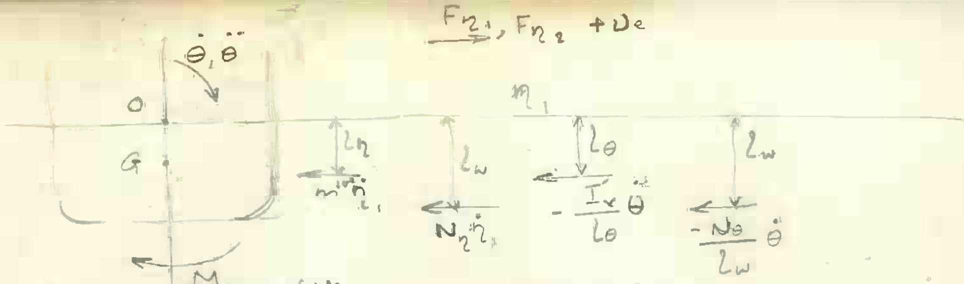
II-3. Coupled equations of swaying, yawing and rolling motions.

Generally, when a ship sways, there is inevitably generated hydrodynamic roll moment as well as swaying force, and when she rolls the rolling moment and swaying force are created.

O. Grim [5] developed coupled equations of the swaying and rolling motion about O, and Tamura [6] induced the equation of rolling motion about a fixed point on the z-axis for a two-dimensional body. In [6] the swaying motion was restrained.

In this paper, we will first discuss the coupled equations of swaying and rolling motions about G_0 in the case of two-dimensional body.

In Fig. 10, the hydrodynamic force and the moment which are generated by the swaying displacement η and rolling motion θ about G_0, are nearly the same as those generated by swaying displacement η, and rolling motion θ about O.



If e.g. $\overline{OG} > l_{\eta}$
then
$M_1 = -m''\ddot\eta_1(\overline{OG} - l_\eta) +$
etc.
$M_2 = \frac{I_x'}{l_\theta}\ddot\theta(\overline{OG} - l_\theta) +$ etc
$= \ddot\theta\left(I_x'\frac{\overline{OG}}{l_\theta} - I_x'\right) +$ etc.

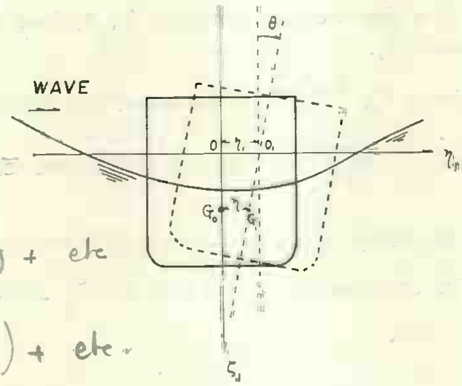


Fig. 10.

Swaying forces can be therefore obtained from [6] and [11].
Let $F_{\eta 1}'$ and $F_{\eta 2}'$ be the force which is due to sway η_1 and rolling motion θ about O respectively.

Then we have

$$F_{\eta 1}' = -m''\ddot\eta_1 - N_\eta \dot\eta_1$$

$$F_{\eta 2}' = \frac{I_x'}{l_\theta}\ddot\theta + \frac{N_\theta}{l_w}\dot\theta$$

= coupling force

where m'' = added mass of swaying motion for the two-dimensional body
I_x' = added mass moment of inertia of rolling about O
N_η = coefficient of damping force of swaying motion
$= \rho g^2 \bar{A}_\eta^2/\sigma_e^3$
N_θ = coefficient of damping force of rolling motion
$= \rho g^2 (B/2)^2 \bar{A}_0^2/\sigma_e^3$

Letting denote m and $F_{\eta e}'$ the mass and the external force for the two-dimensional body, we obtain following equation of swaying motion,

$$m\ddot\eta = -m''\ddot\eta_1 - N_\eta\dot\eta_1 + \frac{I_x'}{l_\theta}\ddot\theta + \frac{N_\theta}{l_w}\dot\theta + F_{\eta e}'$$

$l_w = \alpha_{\varphi y} \approx \alpha_{y\varphi}$

or

$$m\ddot\eta + m''\ddot\eta_1 + N_\eta\dot\eta_1 - I_x'\ddot\theta/l_\theta - N_\theta\dot\theta/l_w = F_{\eta e}' \tag{11}$$

In the next place, let M_1 and M_2 be the rolling moment about G_0 which is due to the swaying displacement η_1 and the rolling motion θ about O respectively, M_3 the linear restoring momont and $M'_{\theta e}$ the exciting moment generated by waves. M_1, M_2 etc. are given as follows:

$N_\eta \cdot l_w \cdot \dot y$

$$M_1 = -m''\ddot\eta_1(\overline{OG_0} - l_\eta) - N_\eta\dot\eta_1(\overline{OG_0} - l_w)$$

$$M_2 = (I_x'\overline{OG_0}/l_\theta - I_x')\ddot\theta + (N_0\overline{OG_0}/l_w - N_\theta)\dot\theta$$

$$M_3 = -W\overline{G_0M}\,\theta$$

$N_{24} = l_w \cdot N_{22}$

too many l_ws !

$$M_{0e}' = F_{\eta e}' \, (\overline{OG_0} - l_{\theta})$$

Putting now J_0 for the mass moment of inertia of rolling motion about G_0, we obtain the following equation of the rolling motion about G_0.

$$J_0\ddot{\theta} + W\overline{G_0M}\theta + \left(I_x' - \frac{I_x'}{l_\theta}\overline{OG_0}\right)\ddot{\theta} + \left(N_\theta - \frac{N_\theta}{l_w}\overline{OG_0}\right)\dot{\theta}$$
$$+ m''(\overline{OG_0} - l_\eta)\ddot{\eta}_1 + N_\eta(\overline{OG_0} - l_w)\dot{\eta}_1 = F_{\eta e}'(\overline{OG_0} - l_w) \tag{12}$$

In the above equations, l_θ, l_η and l_w are respectively the same as h_{R_r}, h_{S_r} and h_w given in [6].

As for a two-dimensional body, there exists the following relation between the hydrodynamic moments generated by swaying motion and the forces by rolling motion. (See [16]).

$$\boxed{\frac{N_{44}}{N_{22}} = l_w^2} \qquad N_\theta/l_w = N_\eta l_w \ \text{and} \ I_x'/l_\theta = m'' l_\eta \qquad \text{see Grim [5]} \tag{13}$$

Making use of (13) the equations (11) and (12) are expressed as follows:

$$\left.\begin{aligned} & m\ddot{\eta} + m''\ddot{\eta}_1 + N_\eta\dot{\eta}_1 - m'' l_\eta\ddot{\theta} - N_\eta l_w\dot{\theta} = F_{\eta e}' \\ & (J_0 + I_x' - m'' l_\eta\overline{OG_0})\ddot{\theta} + N_\eta l_w(l_w - \overline{OG_0})\dot{\theta} + W\overline{G_0M}\,\theta \qquad l_e\,? \\ & + m''(\overline{OG_0} - l_\eta)\ddot{\eta}_1 + N_\eta(\overline{OG_0} - l_w)\dot{\eta}_1 = F_{\eta e}'(\overline{OG_0} - l_w) \end{aligned}\right\} \tag{14}$$

Using $\eta_1 = \eta + \overline{OG_0}\cdot\theta$ we can eliminate η_1 from (14). Then, the resulting equations are

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$$(m + m'')\ddot{\eta} + N_\eta\dot{\eta} + \underline{m''(\overline{OG_0} - l_\eta)\ddot{\theta}} + \underline{N_\eta(\overline{OG_0} - l_w)\dot{\theta}} = F_{\eta e}' \tag{15}$$

$$(J_0 + I_0)\ddot{\theta} + N_\eta(l_w - \overline{OG_0})^2\dot{\theta} + W\cdot\overline{G_0M}\cdot\theta$$
$$+ \underline{m''(\overline{OG_0} - l_\eta)\ddot{\eta}} + \underline{N_\eta(\overline{OG_0} - l_w)\dot{\eta}} = F_{\eta e}'(\overline{OG_0} - l_w) \tag{16}$$

where

$$I_0 = I_x' - 2m'' l_\eta\cdot\overline{OG_0} + m''\cdot\overline{OG_0}^2 \tag{17}$$

In (15) and (16) underlined terms are coupled forces and moments. Suppose now that a ship makes swaying oscillation η, yawing one φ and rolling one θ about G_0–x_1 axis.

In this case, we can express that

$$\eta_1 = \eta + \overline{OG_0}\cdot\theta + x\cdot\varphi \tag{18}$$

Then, by substituting (18) into (14), the hydrodynamic swaying force acting on a section in the distance x from G_0 will become

$$\frac{dF_\eta}{dx} = -m''\ddot{\eta} - N_\eta\dot{\eta} - m''x\ddot{\varphi} - N_\eta x\dot{\varphi} - m''(\overline{OG_0} - l_\eta)\ddot{\theta}$$
$$- N_\eta(\overline{OG_0} - l_w)\dot{\theta} + F_{\eta e}' \tag{19}$$

And the hydrodynamic yawing moment which is due to the above force will lead to

$$\frac{dM_\varphi}{dx}=x\cdot\frac{dF_\eta}{dx}$$

Using the 2nd equation of (14) and (18), we will have the hydrodynamic rolling moment about G_0-x_1 axis as follows,

$$\begin{aligned}\frac{dM_\theta}{dx}=&\{-m''\overline{OG_0}(\overline{OG_0}-l_\eta)+m''\,l_\eta(\overline{OG_0}-l_\theta)\}\ddot{\theta}\\&+N_\eta(l_w-\overline{OG_0})(\overline{OG_0}-l_w)\dot{\theta}-m''\,(\overline{OG_0}-l_\eta)\ddot{\eta}-N_\eta(\overline{OG_0}-l_w)\dot{\eta}\\&-m''\,(\overline{OG_0}-l_\eta)\,x\ddot{\varphi}-N_\eta(\overline{OG_0}-l_w)\,x\dot{\varphi}+F_{\eta e}'\,(\overline{OG_0}-l_w)\end{aligned}$$

By the use of the strip method the following equations of motions are obtained,

$$\left.\begin{aligned}m_0\ddot{\eta}&=\int_{-l_1}^{l_2}\left(\frac{dF_\eta}{dx}\right)dx\\J_z\ddot{\varphi}&=\int_{-l_1}^{l_2}\left(\frac{dF_\eta}{dx}\right)xdx\\J_x\ddot{\theta}+W\overline{G_0M}\theta&=\int_{-l_1}^{l_2}\left(\frac{dM_\theta}{dx}\right)dx\end{aligned}\right\}\qquad(20)$$

Now, we introduce the following nomenclature.

$$\left.\begin{aligned}&\int_{-l_1}^{l_2}m''\,(\overline{OG_0}-l_\eta)dx=m_0\overline{K_\eta}\overline{x_4}\ ,\ \int_{-l_1}^{l_2}N_\eta(\overline{OG_0}-l_w)dx=\overline{N_\eta}\overline{x_5}\\&\int_{-l_1}^{l_2}m''(\overline{OG_0}-l_\eta)xdx=m_0\overline{K_\eta}\overline{x_6}^2,\quad\int_{-l_1}^{l_2}N_\eta(\overline{OG_0}-l_w)\,xdx=\overline{N_\eta}\overline{x_7}^2\\&\int_{-l_1}^{l_2}(m''\cdot\overline{OG_0}^2-2m''\cdot\overline{OG_0}\cdot l_\eta+m''\,l_\eta l_\theta)dx=I_x\\&\int_{-l_1}^{l_2}N_\eta(\overline{OG_0}-l_w)^2dx=\overline{N_\theta}\\&\int_{-l_1}^{l_2}F_{\eta e}'\,dx=F_{\eta e},\quad\int_{-l_1}^{l_2}F_{\eta e}'\ x\ dx=M_{\varphi e}\\&\int_{-l_1}^{l_2}F_{\eta e}'\,(\overline{OG_0}-l_w)dx=M_{\theta e}\end{aligned}\right\}\qquad(21)$$

Making use of (21) the equations (20) become

$$m_0(1+\overline{K_\eta})\ddot{\eta}+\overline{N_\eta}\dot{\eta}+m_0\overline{K_\eta}\overline{x_1}\ddot{\varphi}+\overline{N_\eta}\overline{x_2}\dot{\varphi}+m_0\overline{K_\eta}\overline{x_4}\ddot{\theta}+\overline{N_\eta}\overline{x_5}\dot{\theta}=F_{\eta e}\qquad(22)$$

$$(J_z+I_z)\ddot{\varphi}+\overline{N_\varphi}\dot{\varphi}+m_0\overline{K_\eta}\overline{x_6}^2\ddot{\theta}+\overline{N_\eta}\overline{x_7}^2\dot{\theta}+m_0\overline{K_\eta}\overline{x_1}\ddot{\eta}+\overline{N_\eta}\overline{x_2}\dot{\eta}=M_{\varphi e}\qquad(23)$$

$$\begin{aligned}&(J_x+I_x)\ddot{\theta}+\overline{N_\theta}\dot{\theta}+W\cdot\overline{G_0M}\cdot\theta\\&\qquad+m_0\overline{K_\eta}\ \overline{x_4}\ddot{\eta}+\overline{N_\eta}\overline{x_5}\dot{\eta}+m_0\overline{K_\eta}\overline{x_6}^2\ddot{\varphi}+\overline{N_\eta}\overline{x_7}^2\dot{\varphi}=M_{\theta e}\end{aligned}\qquad(24)$$

In the above developement of the coupled equations the viscous effect and three dimensional correction were also neglected.

The term $\bar{N}_\theta\,\dot{\theta}$ in (24) is wave-making damping moment. As viscous damping moment is considerably large in the case of roll, it is reasonable to consider that the damping takes a form of $\bar{N}_{\theta_1}\dot{\theta}\pm\bar{N}_{\theta_2}\,\dot{\theta}^2$

Moreover, substituting the above by an equivalent linear damping $2\bar{N}_{\theta e}\,\dot{\theta}$ we obtain, instead of (24)

$$(J_x+I_x)\ddot{\theta}+2\bar{N}_{\theta e}\dot{\theta}+W\overline{G_0M}\theta+m_0\bar{K}_\eta\bar{x}_4\ddot{\eta}+\bar{N}_\eta x_5\ddot{\eta}$$
$$+m_0\bar{K}_\eta\bar{x}_6^{\,2}\ddot{\varphi}+\bar{N}_\eta\bar{x}_7^{\,2}\dot{\varphi}=M_{\theta e} \qquad (24)'$$

Now, the uncoupled rolling equation is obtained from (24)' by putting $\eta=\varphi=0$.

The resulting equation is

$$(J_x+I_x)\ddot{\theta}+2\bar{N}_{\theta e}\dot{\theta}+W\overline{G_0M}\theta=M_{\theta e} \qquad (25)$$

When the external moment $M_{\theta e}$ is calculated from the equations (8) and (22), it becomes

$$M_{\theta e}=W\overline{G_0M}\Theta_w(C_R'\sin\sigma_e t+C_i'\cos\sigma_e t)$$

Then, for the coodition-1 of the model we computed $C^*=\sqrt{C_R'^2+C_i'^2}$ and the γ derived from Dr. Watanabe's theory [17]. As is clearly seen from Fig. 11, the above two are in good coincidence in the range of $T_w>1.0$sec, whereas in the case of condition-2, these two are very different. As for this matter we will discuss in chapter III.

Put $\quad 2\bar{N}_{\theta e}/(J_x+I_x)=2\alpha_e,\quad W\overline{G_0M}/(J_x+I_x)=\sigma_n^2,\quad \sigma_e/\sigma_n=T_\theta/T_e=\Lambda$

Assuming further $\theta=\theta_0\sin(\sigma_e t-\varepsilon_\theta')$ and solving the equation (25) we obtain

$$\theta_0/\Theta_w=c'/\sqrt{(1-\Lambda^2)^2+\left(\frac{2\alpha_e}{\sigma_n}\right)^2\Lambda^2} \qquad (26)$$

In solving the non-linear equation of motion

$$\ddot{\theta}+2\alpha\dot{\theta}\pm\beta\dot{\theta}^2+\sigma_n^2\theta=W\overline{G_0M}\gamma\Theta_w\sin\sigma_e t,$$

Dr. Watanabe [17] substituted the above non-linear damping by the following equivalent linear damping, that is,

$$2\alpha_e=\frac{2}{\pi}\sigma_n(a_1+b_1\Lambda\theta_0) \qquad (27)$$

Where a_1 and b_1 are coefficients in the equation $\Delta\theta=a_1\theta_m+b_1\theta_m^2$, which can be determined from the extinction curve of the free rolling of a ship.

Making use of (27), we can reduce (26) to

$$\theta_0/\Theta_w=c'/\sqrt{(1-\Lambda^2)^2+\left\{\frac{2}{\pi}(a_1+b_1\Lambda\theta_0)\Lambda\right\}^2} \qquad (26)'$$

and then the phase difference ε_0' will become

$$\varepsilon_0' = \tan^{-1}\left\{\frac{-(1-A^2)C_i' + (2\alpha_e/\sigma_n)\,AC_R'}{(1-A^2)C_R' + (2\alpha_e/\sigma_n)\,AC_i'}\right\} \tag{28}$$

Now, (22), (23) and (24)′ can be written as follows:

$$\left.\begin{aligned} &a\ddot{\eta}+b\dot{\eta} \qquad +c\ddot{\varphi}+d\dot{\varphi}+e\ddot{\theta}+f\dot{\theta}=F_{ne}\\ &A\ddot{\varphi}+B\dot{\varphi} \qquad +p\ddot{\theta}+q\dot{\theta}+c\ddot{\eta}+d\dot{\eta}=M_{\varphi e}\\ &E\ddot{\theta}+F\dot{\theta}+G\theta+e\ddot{\eta}+f\dot{\eta}+p\ddot{\varphi}+q\dot{\varphi}=M_{\theta e}\end{aligned}\right\} \tag{29}$$

If only the real part is assumed to be considered, we can put

$$F_{ne}=\overline{F}e^{i\sigma et},\quad M_{\varphi e}=\overline{M}e^{i\sigma et},\quad M_{\theta e}=\overline{N}e^{i\sigma et} \qquad \overline{F}=|F|\cdot e^{i\varepsilon_F}$$

As we take into consideration only the particular solutions, we will put

$$\eta=\overline{\eta}e^{i\sigma et}=\eta_0 e^{i(\sigma et-\varepsilon\eta)}$$

$$\varphi=\overline{\varphi}e^{i\sigma et}=\phi_0 e^{i(\sigma et-\varepsilon\varphi)}$$

$$\theta=\overline{\theta}e^{i\sigma et}=\theta_0 e^{i(\sigma et-\varepsilon\theta)}$$

By subsituting these into (29), we obtain following equations.

$$(-a\sigma_e^2+ib\sigma_e)\overline{\eta}+(-c\sigma_e^2+id\sigma_e)\overline{\varphi}+(-e\sigma_e^2+if\sigma_e)\overline{\theta}=\overline{F}$$

$$(-A\sigma_e^2+iB\sigma_e)\overline{\varphi}+(-p\sigma_e^2+iq\sigma_e)\overline{\theta}+(-c\sigma_e^2+id\sigma_e)\overline{\eta}=\overline{M}$$

$$(-E\sigma_e^2+iF\sigma_e+G)\overline{\theta}+(-e\sigma_e^2+if\sigma_e)\overline{\eta}+(-p\sigma_e^2+iq\sigma_e)\overline{\varphi}=\overline{N}$$

These can be reduced further as follows,

$$\left.\begin{aligned} P\overline{\eta}+X\overline{\varphi}+Y\overline{\theta}&=\overline{F}\\ X\overline{\eta}+Q\overline{\varphi}+Z\overline{\theta}&=\overline{M}\\ Y\overline{\eta}+Z\overline{\varphi}+R\overline{\theta}&=\overline{N}\end{aligned}\right\} \tag{29$'$}$$

where

$$P=-a\sigma_e^2+ib\sigma_e,\qquad X=-c\sigma_e^2+id\sigma_e$$

$$Q=-A\sigma_e^2+iB\sigma_e,\qquad Y=-e\sigma_e^2+if\sigma_e$$

$$R=-E\sigma_e^2+iF\sigma_e+G,\qquad Z=-p\sigma_e^2+iq\sigma_e$$

One may solve the equations (29)′ and finds

$$\overline{\eta}=\frac{\begin{vmatrix}\overline{F} & X & Y\\ \overline{M} & Q & Z\\ \overline{N} & Z & R\end{vmatrix}}{\Delta_0},\quad \overline{\varphi}=\frac{\begin{vmatrix}P & \overline{F} & Y\\ X & \overline{M} & Z\\ Y & \overline{N} & R\end{vmatrix}}{\Delta_0},\quad \overline{\theta}=\frac{\begin{vmatrix}P & X & \overline{F}\\ X & Q & \overline{M}\\ Y & Z & \overline{N}\end{vmatrix}}{\Delta_0} \tag{30}$$

Provided that,

$$\varDelta_0=\begin{vmatrix} P & X & Y \\ X & Q & Z \\ Y & Z & R \end{vmatrix} \neq 0$$

From (30) amplitudes η_0, φ_0, θ_0 and phase difference ε_η, ε_φ, ε_θ can be obtained.

II-4 Cross coupling effect between Roll and Sway

As the anti-symmetrical property of the model dealt with in this paper is very small, yawing motion in beam seas is negligibly small.

We therefore consider the sway-roll system which is resulted from setting $\varphi=0$.

The solutions of this system are given by

$$\bar{\theta}=\frac{P\bar{N}-Y\bar{F}}{PR-Y^2}, \quad \bar{\eta}=\frac{R\bar{F}-Y\bar{N}}{PR-Y^2} \tag{31}$$

It is assumed that F in (29) can be approximately substituted by the following equivalent linear damping, that is

$$F=\frac{2}{\pi}(J_x+I_x)(a_1\sigma_n+b_1\sigma_e\theta_0) \tag{32}$$

Then, introducing

$$\left.\begin{array}{l} \bar{F}/\Theta_w\equiv\bar{F}_0=f_{10}+if_{20},\ \bar{N}/\Theta_w\equiv\bar{N}_0=n_{10}+in_{20} \\ P=p_1+ip_2,\ Y=y_1+iy_2,\ R=r_1+ir_2 \end{array}\right\} \tag{33}$$

We obtain

$$\theta_0'=\theta_0/\Theta_w=\sqrt{\frac{L_1^2+L_2^2}{N_1^2+N_2^2}}, \quad \varepsilon_0=\tan^{-1}\left(\frac{L_1N_2-L_2N_1}{L_1N_1+L_2N_2}\right) \tag{34}$$

where

$$\left.\begin{array}{l} L_1=p_1n_{10}-p_2n_{20}-y_1f_{10}+y_2f_{20} \\ L_2=p_1n_{20}+p_2n_{10}-y_1f_{20}-y_2f_{10} \\ N_1=p_1r_1=p_2r_2-y_1^2+y_2^2 \\ N_2=p_1r_2+p_2r_1-2y_1y_2 \end{array}\right\} \tag{35}$$

And

$$\eta_0'=\eta_0/h=k\sqrt{\frac{T_1^2+T_2^2}{N_1^2+N_2^2}}, \quad \varepsilon_\eta=\tan^{-1}\left(\frac{T_1N_2-T_2N_1}{T_1N_1+T_2N_2}\right) \tag{36}$$

where

$$\left.\begin{array}{l} T_1=r_1f_{10}-r_2f_{20}-y_1n_{10}+y_2n_{20} \\ T_2=r_1f_{20}+r_2f_{10}-y_1n_{20}-y_2n_{10} \end{array}\right\} \tag{37}$$

As θ_0' is included in $r_2=F\sigma_e$, θ_0' can be obtained by solving the following form of equation

$$K_0+K_1\theta_0'+K_2\theta_0'^2=(L_1^2+L_2^2)/\theta_0'^2$$

and consequently η_0', ε_η' and ε_θ are obtained from (34), (36) successively.

III. Comparison between model experiments and calculations

The experiments were carried out in a large tank (80m in length, 8m in breadth and 3.5m in depth) at the Research Institute for Applied Mechanics in Kyushu University.

The model was set under the beam sea condition and in the same manner as followed in rolling experiments except the drifting motion which was not restrained.

Wave periods were changed in the range of 0.8 sec.~2.4sec. and the steepness $2h/\lambda$ was 1/50~1/55. The amplitudes of sway, heave, roll, pitch and the drifting velocity were mainly measured.

When a model happens to have a large leeway angle, experimental results are omitted. The wave height was surveyed in advance to the experiments. An ultrasonic type wave probe which had been developed cooperatively by the members of our laboratory and the KAIJŌ DENKI Company, was used. Of course, the roll shaft of the equipment of measuring six motions passes through the center of gravity G_0 of the model. Several examples of the oscillogram are shown in Fig. 12.

In the Table 1, 2, principal details on the conditions of the model and expe-

Table 1. Principal Particulars of a ship model.

Model: Todd 60 series $C_b=0.70$

$L_{pp}=3.0$m	$C_b=0.70$
$B=0.428$m	$C_p=0.710$
$L/B=7.0$	$C_{\boxtimes}=0.985$
$D=0.267$m	$C_w=0.785$
$d=0.171$m	$\boxtimes B=0.013$m fore
$W=153.74$Kg	$KB=9.02$cm

Radius of gyration of pitching in air $k=0.24L_{pp}$
even keel, without Bilge keel and propeller, with Rudder.

Table 2. Model conditions

Condition 1.

$G_0M=2.78$cm,	$KG_0=14.66$cm
$a_1=0.0385$,	$b_1=0.0022$ (1/deg)
$\boxtimes B=0.0314$m aft, even keel.	$d=0.171$m
Natural Rolling Period	$T_\theta=1.61$sec
Natural Pitching Period	$T_\psi=1.00$sec
Natural Heaving period	$T_\zeta=1.03$sec

Condition 2.

$G_0M=1.40$cm,	$T_\theta=2.18$sec (at $\theta_0\doteqdot10°$)
$a_1=0.0530$,	$b_1=0.0017$ (1/deg)
$\boxtimes B=0.013$m fore,	$d=0.171$m

Other particulars are the same with the condition 1, riments are given.

Now, for the condition 1, comparison of the experimental values and the calculated results of ζ_0/h, ψ_0/Θ_w, η_0/h are shown in Fig. 13, 14 and 15.

The theoretical results for ζ_0/h are generally in good coincidence with those of the experiments, but in the small period range the former are lower than the latter because of the approximate theoretical treatment. On the other hand, it is also with η_0/h that the solutions according to the equation (9) coincide well with the experimental results. In the part of a long period, however, the former is lower than the latter.

These circumstances may be ascribed to the three-dimensional effect for the added mass and damping.

The experimental values of ψ_0/Θ_w are, in spite of the large scattering, in accord with the theoretical calculations in their tendency.

The comparison among the solutions obtained from (26)', the ones from tne coupled equations (34) and the experimental values are shown in Fig. 16. In calculating (26)' and (34), J_x+I_x was evaluated by the approximate relation $2\pi\sqrt{(J_x+I_x)/W\cdot G_0M} \doteqdot T_\theta$, where T_θ is the natural rolling period measured in still water.

Moreover, we made a simple assumption that J_x+I_x has a constant value in the experimental frequency range.

Considering the case of rolling motion from the results of [6] and [11], it is probable that we may not make a large error.

As is readily seen, the solutions by coupled equations in the neighbourhood of resonance are a little smaller than those by uncoupled equation.

This is due to the reduction of the exciting moment and the increase of the damping moment due to the coupling effect caused by swaying oscillation. On the other hand, the swaying motion is hardly affected by the rolling motion and therefore the solutions obtained by (34) are almost in coincidence with the ones by (9) (Fig. 15).

In the next place, from $M_{\theta e}=\int_{-l_1}^{l_2} F_{ne}'(\overline{OG_0}-l_w)dx$ we can calculate the position of G_0 which makes the exciting moment as small as possible. In condition 2, we raised G_0 slightly so as to make $M_{\theta e}$ as small as possible but check T_θ from becomming too large.

In Fig. 17, the comparison between the theoretical calculations and the experimental results is shown.

In this case, the solutions obtained from the uncoupled equation are much smaller than those from the coupled equations. And also the latter is close to the experimental values in the neighbourhood of resonance.

It is supposed now that the experimental values became larger than the theoretical ones in the range of $\Lambda > 1.0$ owing to the non-linear restoring moment and the quasi-harmonic moment.

For the case of condition 2, the γ obtained from Dr. Watanabe's calculation [14] and C' from the author's calculation are shown in Fig. 18.

The difference between the two is due to the effect of the reflectd waves. In the neighbourhood of resonance, C' is so small that the values of the solutions obtained from the uncoupled equation are small.

However, the coupled moment $-m_0K_\eta \overline{x_4}\ddot{\eta}$ produced by swaying motion was too large effective to get small θ_0/Θ_w.

In Fig. 19, the comparison between the solutions obtained from the uncoupled swaying equation and the experimental results for the case of condition-2 is shown.

The coincidence of the two was as good as the condition-1. In the condition -2 also, coupled effect by the rolling motion was extremely small.

Therefore, it will be easily seen from the above results that the coupling effect to the sway by roll is small. However, when M_{0e} is very small the coupling effect to the roll by sway is very large.

On the other hand, the solutions by Dr. Watanabe's theory were a little larger than the experimental results in the vicinity of resonance, but in the other periods the both generally well coincided with each other.

IV. Drifting

Dr. Suehiro found that the drifting force has a maximum value in the roll resonance, more than forty years ago [18].

In Fig. 20, drifting velocity obtained from the experiments for the condition -1 is shown. As is easily seen from this figure, the drifting velocity becomes maximum at the resonance of roll and also heave.

Moreover, the maximum value at the heave resonance is larger than the one at the roll resonance. This tendency was the same also in the condition-2.

Dr. Watanabe proved using the Froude-Krilov's theory that the drifting force is due to the phase difference between wave exciting force and ship motion, and therefore, it becomes maximum at roll resonance [17].

Considering not only the rolling motin θ but also heave ζ and sway η, we can obtain the following force acting on the hull in the η direction according to [17].

$$F_\eta = W\Theta_w \Phi_\eta \sin\sigma_e t + W\Theta_w H_2 \frac{\zeta}{B} \sin\sigma_e t - W\Theta_w k\overline{G_0M}\cdot\gamma\cdot\theta\cos\sigma_e t$$
$$- W\Theta_w \Phi_\eta k\eta \cos\sigma_e t \qquad (38)$$

where Φ_η and H_2 are given in [14].

Put now $\theta=\theta_0 \sin(\sigma_e t-\varepsilon_\theta)$, $\zeta=\zeta_0 \cos(\sigma_e t-\varepsilon_\zeta)$

$\eta=-\eta_0 \sin(\sigma_e t+\varepsilon_{\eta 1})$

where $\varepsilon_{\eta 1}=\pi-\varepsilon_\eta{}'$

Substituting these into (38), it becomes

$$F_\eta = W\Theta_w \left\{ \Phi_\eta \sin\sigma_e t + \frac{\zeta_0}{2B} H_2 \sin(2\sigma_e t-\varepsilon_\zeta) \right.$$
$$\left. -\frac{k\cdot\overline{G_0M}}{2}\cdot\gamma\theta_0 \sin(2\sigma_e t-\varepsilon_\theta) + \frac{\Phi_\eta k\eta_0}{2} \sin(2\sigma_e t+\varepsilon_{\eta 1}) \right\}$$

$$+\frac{W\Theta_w}{2}\left\{\zeta_0\frac{H_2}{B}\sin\varepsilon_\zeta+k\overline{G_0M}\gamma\theta_0\ \sin\varepsilon_\theta+\Phi_n k\eta_0\ \sin\varepsilon_{\eta_1}\right\}$$

Note As ε_ζ, ε_θ, ε_{η_1}, are the magnitude in the range of 0°~180°, $\sin\varepsilon_\zeta$, $\sin\varepsilon_\theta$ and $\sin\varepsilon_{\eta_1}$ are all positive.

Therefore the drifting force is always positive and becomes maximum at the condition of $\varepsilon_\zeta=\varepsilon_\theta=\frac{\pi}{2}$, and as the ε_{η_1} is very small it seems that the effect of swaying motion η is small.

Maruo [19] proved that the drifting force is due to the scattered waves from a moving ship and that Dr. Watanabe's theory [17] covers only a part of drifting force with its value overestimated.

Both of these theories however, indicate that the drifting force becomes maximum at resonant conditions of heave and roll.

According to [19], drifting force acting on the two-dimensional body becomes $D=\frac{1}{2}\rho g|A^-|^2$, where $A^-$is the amplitude of the wave reflected from the body in the direction adverse to the incident wave.

In the heaving motion, we obtain from the relation $|A^-|\doteqdot|\zeta-\overline{\zeta}_w|\overline{A}_\zeta$

$$D=\frac{1}{2}\rho g|\zeta-\overline{\zeta}_w|^2\overline{A}_\zeta^2 \tag{30}$$

Since it is difficult to calculate the exact drifting force for a three-dimensional ship hull, we made an approximate computation depending on the strip method.

That is, $D_\zeta=\frac{1}{2}\rho g|\zeta-\overline{\zeta}_w|^2\int_L\overline{A}_\zeta^2\,dx$

Making use of ζ_0 given in II-1, D_ζ was computed and the results are shown in Fig. 21.

The dotted line in the figure shows the non-dimensional values of D_ζ for the steepness of 1/50.

Moreover, D_θ caused by the roll was also computed by using θ of (20)'.

As is seen from Fig. 21, both of these D_ζ and D_θ have maximum in the resonant conditions respectively, and in the case of constant steepness the maximum of D_ζ is larger than that of D_θ.

Note. Now, the theories introduced so far are applicable to the case where mean position of G_0 is constant but the oscillating body never drift. And the results obtained from them don't indicate the drifting force acting on the body that is drifting with a constant velocity. It is therefore impossible for us to confirm theoretically the state of affairs of Fig. 20 from the calculations of Fig. 21.

However, we can deduce the following two points from the above theories and the results of Fig. 21, namelly,

(1) In the resonant conditions of heave and roll, v_d becomes large, possessing maximum.

(2) Since $\overline{A}_\zeta$ is generally larger than $\overline{A}_\theta$ in the ship-shape sections in the case

of constant steepness the maximum in heave resonance seems to be larger than that in roll resonance.

V. Unstable Roll

J. R. Paulling and R. M. Rosenberg [9] discussed on the unstable roll in the first unstable region of the Mathieu equation, exemplifying it. That is, when a ship performs forced heaving oscillation in still water, there is caused an unstable roll at $T_\zeta \fallingdotseq T_\theta/2$ because of periodic variation of the restoring roll moment.

On the other hand, there are some researches made on the unstable roll in longitudinal waves by Dr. Watanabe [20], Manabe [21], O. Grim [22], Kerwin [23] and Ogawara, Miura [24] and the other by Yamazaki, Fukuda [1] who treated of oblique waves.

Considering the linear term only, the rolling restoring moment of a ship performing the heaving oscillation ζ in still water will be

$$-(W\ \overline{G_0M}+C_1\zeta+C_2\zeta^2)\theta \tag{40}$$

As for the model used in this paper, we obtained $C_1=32$Kg, $C_2=5$Kg/cm and $W.G_0M \fallingdotseq 215$ Kg. cm (Condition-2).

The term $C_1\zeta$ in (40) is the one used by Paulling and Manabe.

In condition-2, $T_\theta=2.2$ sec for $\theta=5°$ and $T_\zeta=1.03$ sec.

Therefore, when $T_e=\frac{1}{2}T_\theta$, that is, at the condition $\Lambda=2$ the rolling angle θ_0 is extremely small, but as the heave is in the neighbourhood of resonance heaving motion is very large. Now after we heel the model slightly, the rolling amplitude increases gradually and comes to keep up a considerable large value. In Fig. 22 examples of the oscillogram are shown. This is an unstable roll in the first unstable region of Mathieu equation.

Because of the third term in (40), equivalent G_0M becomes large when a ship heaves. Therefore, when the heaving amplitude is large at $\Lambda=2$, the natural rolling period after the instant when a ship is given a small initial heel is smaller than that in still water. For example when $T_w=1.02$ sec and $\Theta_w=7.5°$, T_θ was reduced about 10 %.

Taking the various Θ_w and T_w we investigated the appearance of the unstable roll in the neighbourhood of $\Lambda=2$. Even at $\Theta_w=2°$, that is, $2h/\lambda=1/90$ there was clear appearance, but in case of $2h/\lambda>1/25$ it never appeared as shown in Fig. 23.

As the drifting velocity is large when Θ_w is large, the judgement of the stable or unstable becomes very difficult, for the model ship can drift only 1.70 m. At any rate, however, we could not obtain a clear unstable roll in the occasion of large steepness.

To pursue this cause further researches are continued at present.

We also investigated the same phenomenon in irregular waves. The periods of successive waves were in the neighbourhood of the period where an unstable roll clear appeared at $2h/\lambda=1/50$.

In irregular waves, however, the initial heels were all damped and the unstable motion never occurred.

VI. Conclusions and Remarks

Coupled motions of heave and pitch as well as sway, yaw and roll in the beam sea were calculated using a model of Todd 60 Series $C_b=0.70$. And the results of calculation were compared with experimental values.

From these investigations following conclusions can be drawn.

1. Heaving and Pitching motions, except for the case of large B/λ, can be well explained by the approximate method as discussed in this paper.
2. For ships with a small anti-symmetrical property in the fore and aft directions, the yawing amplitudes are very small in the beam sea.
3. The solutions of the uncoupled equations of sway obtained by using the Tamura's two-dimensional hydrodynamic forces and moments are in good coincidence with the experimental results.
4. For the swaying motion, the cross coupling effects derived from the yaw and roll are very small.
5. The solutions of roll obtained by the coupled equations of sway and roll almost coincided with the experimental results in the neighbourhood of $\Lambda=1.0$, but however in $\Lambda<1.0$, that is, in the region of long wave length the former has given smaller values than the latter.
6. When the exciting roll momet is large at $\Lambda \fallingdotseq 1.0$ the coupling effects caused by sway are small. On the contrary, when the exciting roll moment is very small the coupled roll moment produced by sway works a great influence on the roll motion, and this moment is mainly due to the swaying inertia force. Similar result was obtained by Bessho [25]. Therefore, even in the case of small exciting roll moment, we could not minimize the roll amplitude.
7. On the other hand, the solutions given by the uncoupled equation of motion obtained by using Watanabe's γ have given values a little larger than the experimental ones in resonance, but in the other range of period, the coincidence of the both were fairly good.
8. The drifting velocity in beam seas becomes maximum in roll and heave resonance. Moreover, it is supposed that the maximum drifting velocity in heave resonance will be larger than the other, because the wave-making damping is generally larger in heave.
9. The unstable roll caused by the periodic variation of the restoring moment of roll clearly appeared in case of $\Lambda \fallingdotseq 2$ and small steepness. However, at large steepness or in irregular waves this phenomenon never appeared.

The model dealt with in this paper has no bilge keel. In case of a ship with bilge keel, the coupling action between sway and roll may be different from the present case.

Therefore, for the above problem and moreover for various type of ships further studies should be performed.

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(March, 1965)

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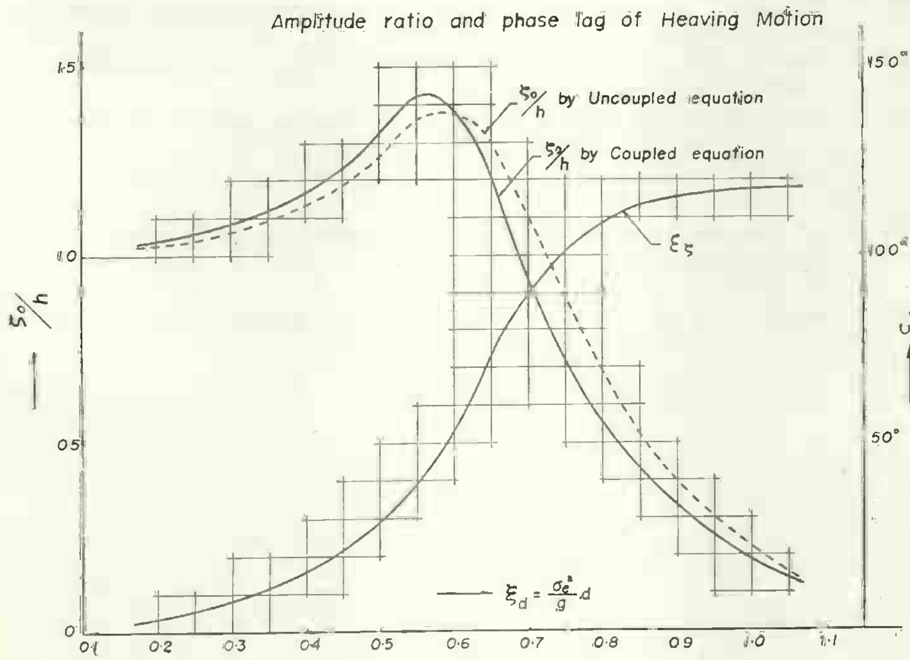

Amplitude ratio and phase lag of Heaving Motion
ζ_0/h by Uncoupled equation
ζ_0/h by Coupled equation
ε_ζ
ζ_0/h
ε_ζ
$\xi_d = \frac{\sigma_e^2}{g} d$
150°
100°
50°
1.5
1.0
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1

Fig. 3.

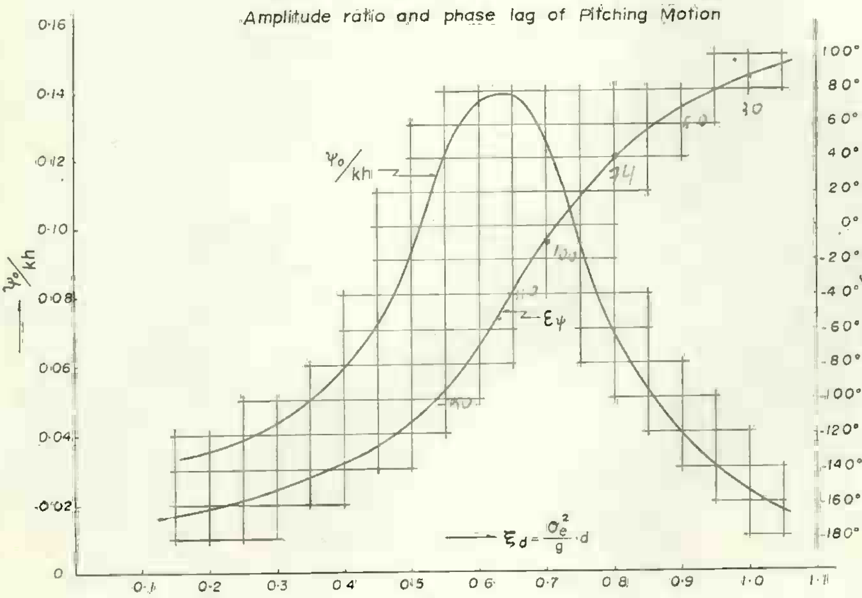
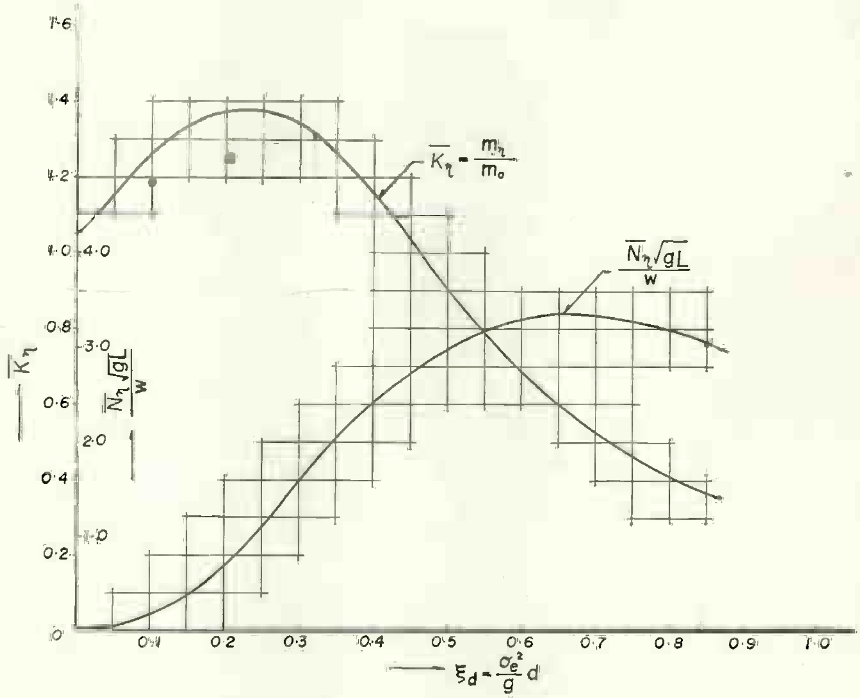
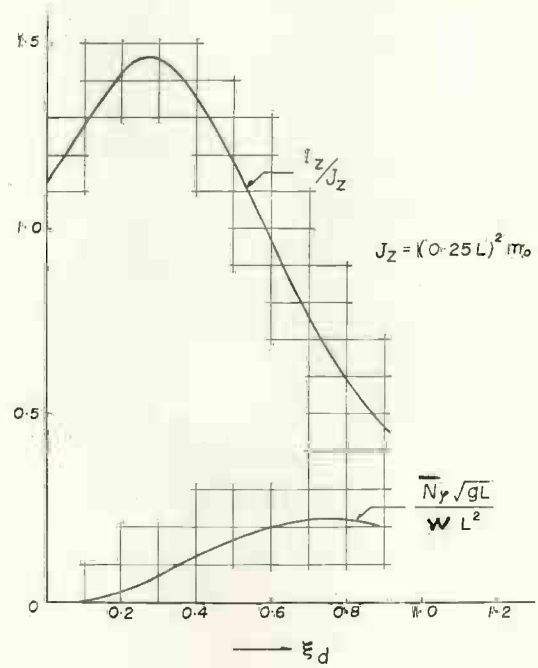

Amplitude ratio and phase lag of Pitching Motion
ψ_0/kh
ε_ψ
$\xi_d = \frac{\sigma_e^2}{g} \cdot d$
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1

Fig. 4.



$\overline{K}_\eta = \frac{m_\eta}{m_o}$
$\frac{\overline{N}_\eta\sqrt{gL}}{W}$
$\overline{K}_\eta$
$\xi_d = \frac{\sigma_e^2}{g} d$

Fig. 5.



I_z/J_z
$J_Z = (0\cdot25\,L)^2\,m_o$
$\frac{\overline{N}_\gamma\sqrt{gL}}{W\,L^2}$
ξ_d

Fig. 6.

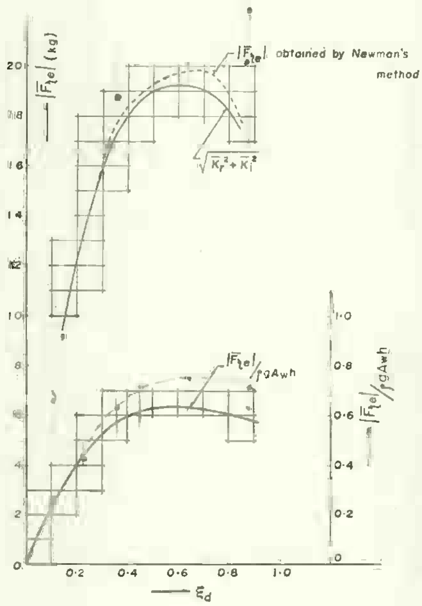

$|\bar{F}_{\zeta e}|$ (kg)
$|\bar{F}_{\zeta e}|$ obtained by Newman's method
$\sqrt{\bar{K}_r^2+\bar{K}_i^2}$
$|\bar{F}_{\zeta e}|/\rho g A_w h$
0·2
0·4
0·6
0·8
1·0
ξ_d

Fig. 7.

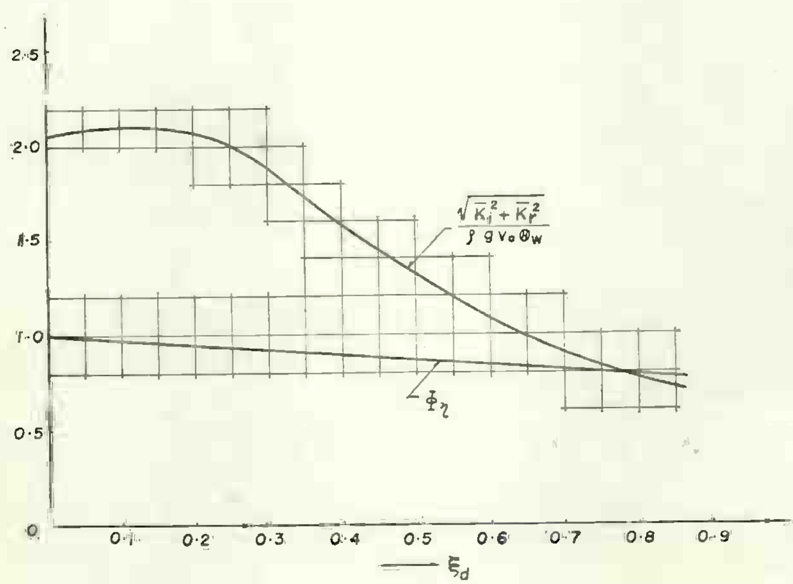
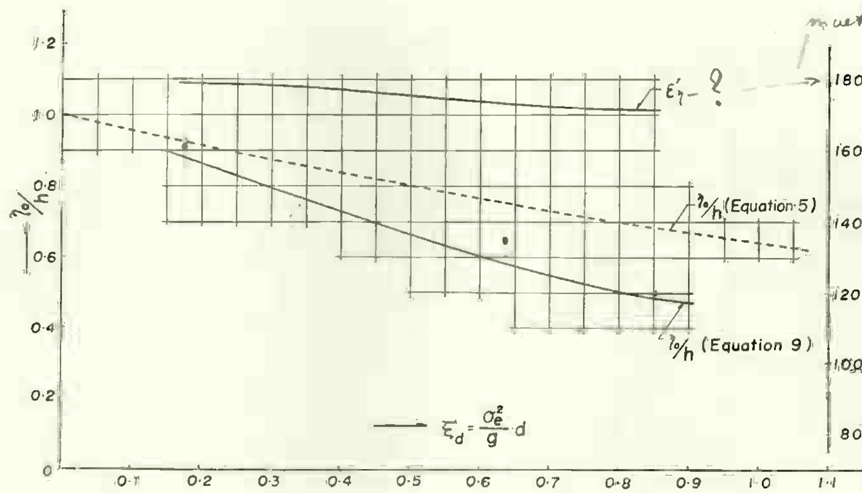
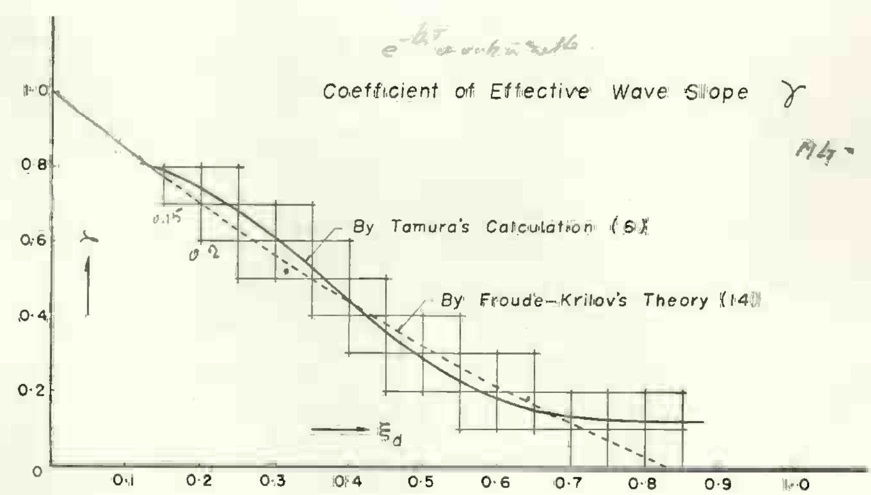

2·5
2·0
1·5
1·0
0·5
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
$\frac{\sqrt{\bar{K}_i^2+\bar{K}_r^2}}{\rho g V_0 \Theta_w}$
Φ_η
ξ_d

Fig. 8.



1·2
1·0
0·8
0·6
0·4
0·2
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
1·1
180°
160°
140°
120°
100°
80°
η_0/h (Equation 5)
η_0/h (Equation 9)
$\xi_d = \frac{\sigma_e^2}{g} d$

Fig. 9.



Coefficient of Effective Wave Slope γ
By Tamura's Calculation (6)
By Froude–Krilov's Theory (14)
ξ_d
1·0
0·8
0·6
0·4
0·2
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0

Fig. 11.

$T_w = 1.115$ sec

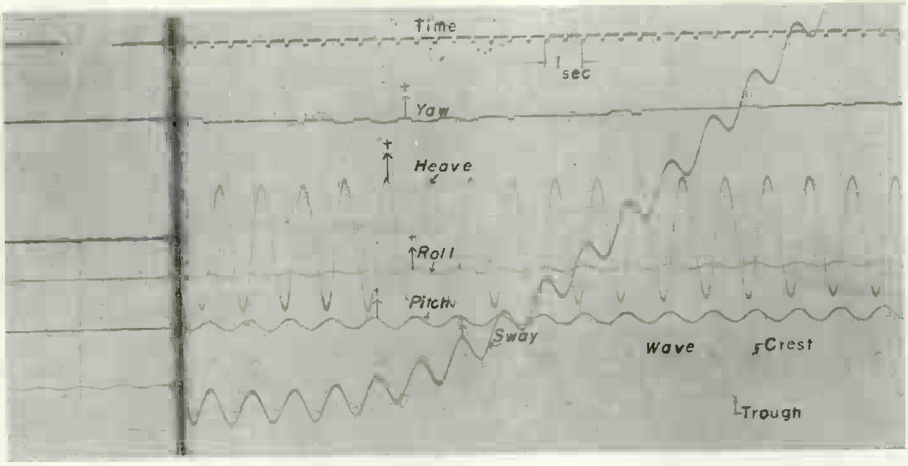


Fig. 12. (a)

$T_w = 1.325$ sec

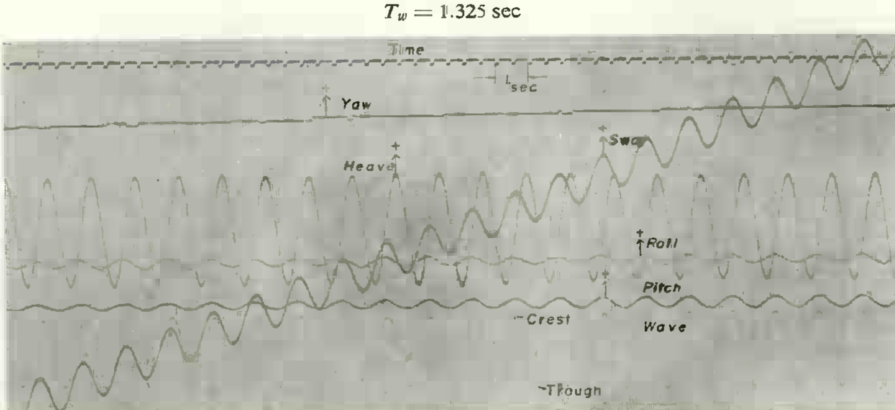


Fig. 12. (b)

$T_w = 1.550$sec

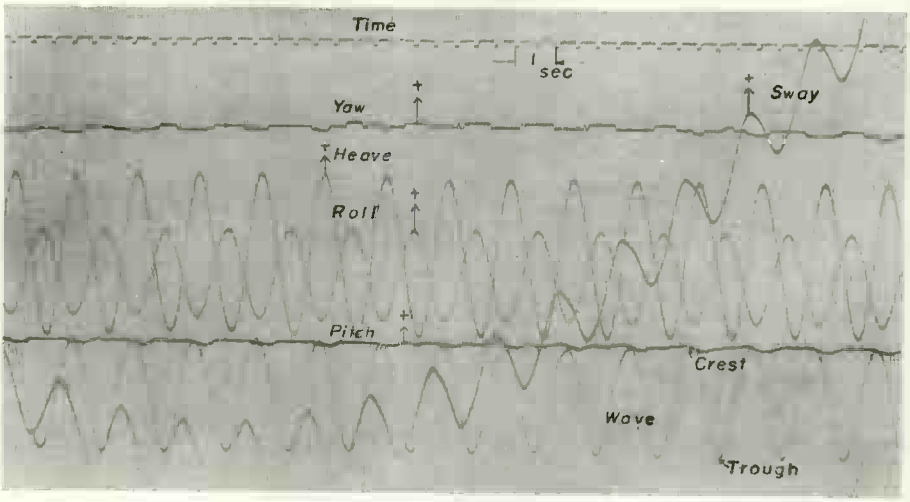


Fig. 12. (c)

$T_w = 1.900$ sec

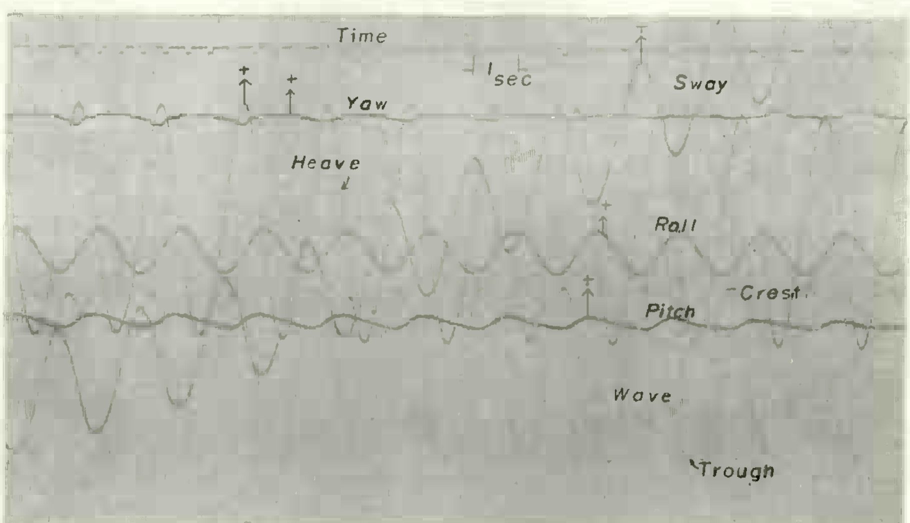


Fig. 12. (d)

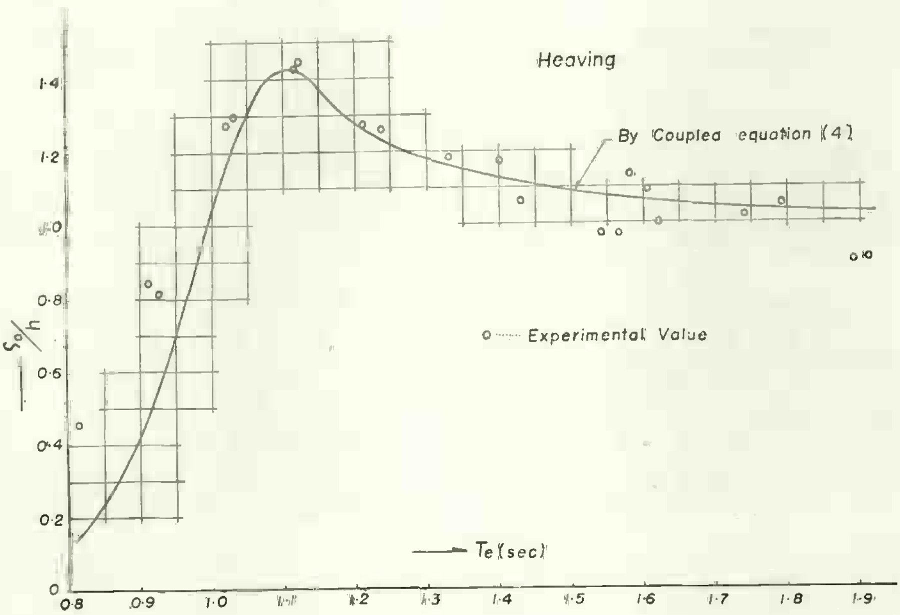

Heaving
By Coupled equation (4)
Experimental Value
ζ_0/h
Te (sec)
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
0·8
0·9
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·7
1·8
1·9

Fig. 13.

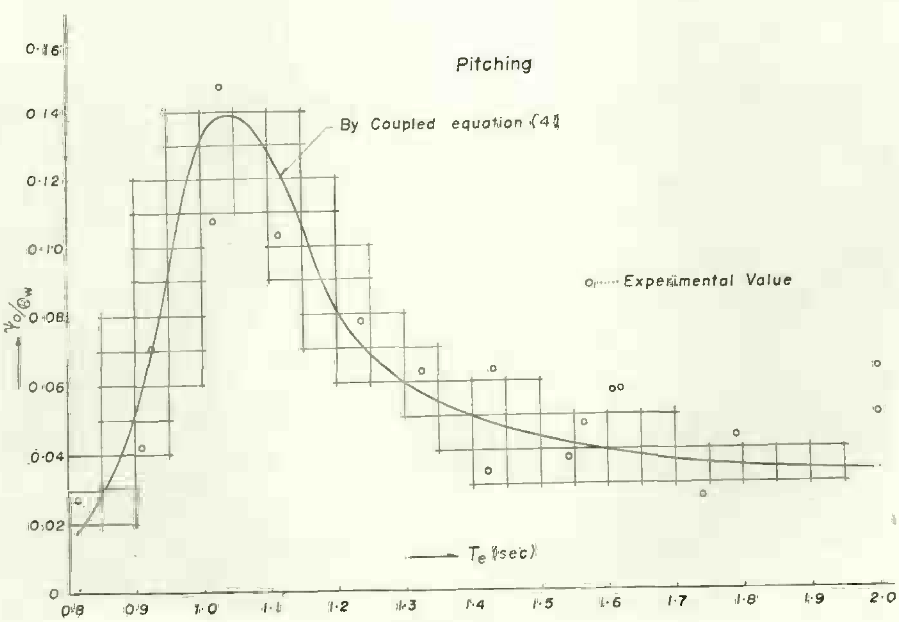

Pitching
By Coupled equation (4)
Experimental Value
γ_0/Θ_w
T_e (sec)
0
0·02
0·04
0·06
0·08
0·10
0·12
0·14
0·16
0·8
0·9
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·7
1·8
1·9
2·0

Fig. 14.

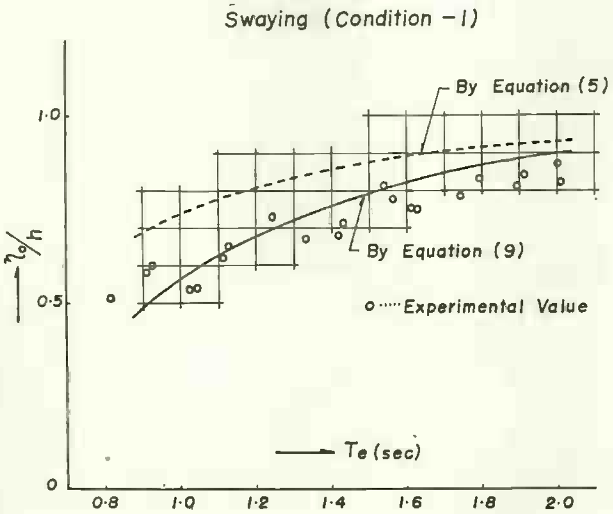

Swaying (Condition - I)
By Equation (5)
By Equation (9)
o····· Experimental Value
η_0/h
Te (sec)
1·0
0·5
0
0·8
1·0
1·2
1·4
1·6
1·8
2·0

Fig. 15.

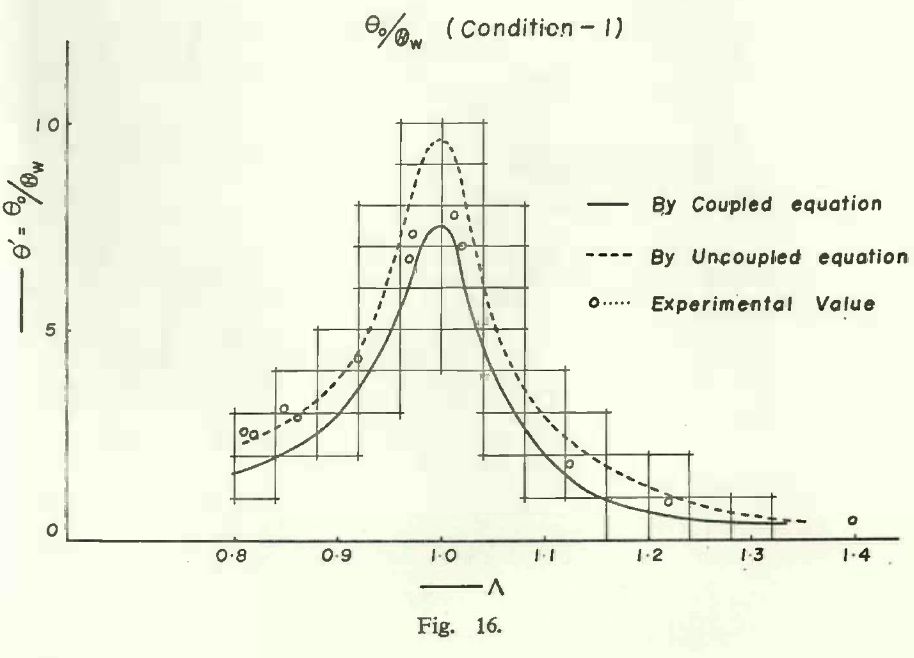

θ_0/Θ_w (Condition - I)
$\theta' = \theta_0/\Theta_w$
By Coupled equation
By Uncoupled equation
o····· Experimental Value
10
5
0
0·8
0·9
1·0
1·1
1·2
1·3
1·4
Λ

Fig. 16.

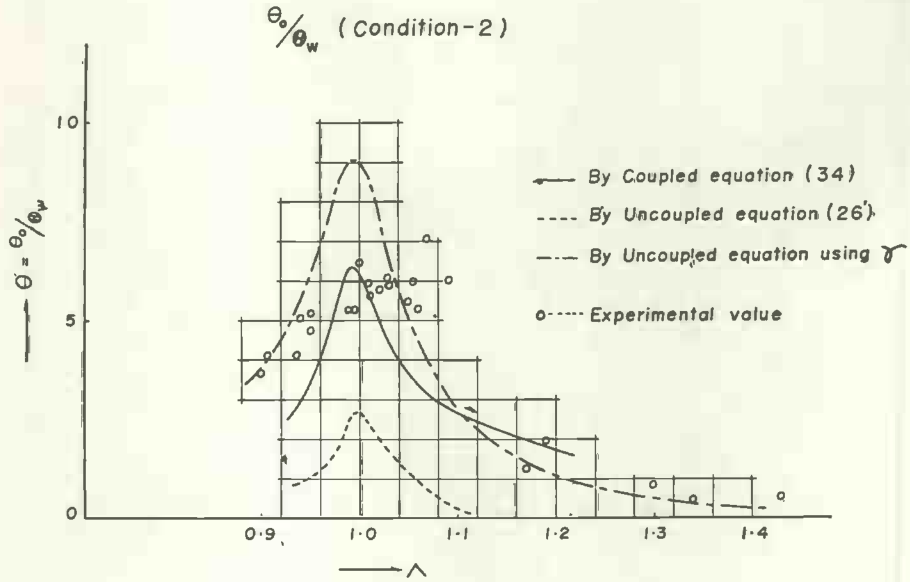

θ_0/θ_w (Condition-2)
$\theta = \theta_0/\theta_w$
10
5
0
By Coupled equation (34)
By Uncoupled equation (26')
By Uncoupled equation using γ
Experimental value
0·9
1·0
1·1
1·2
1·3
1·4
Λ

Fig. 17.

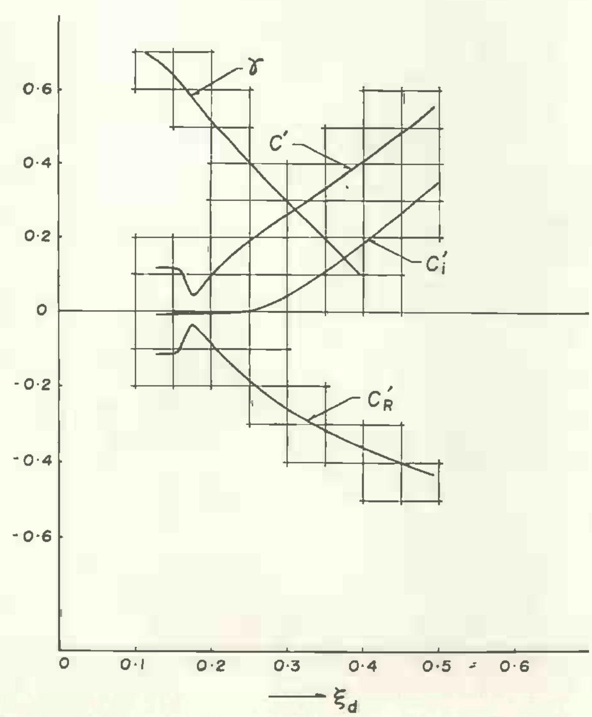

0·6
0·4
0·2
0
-0·2
-0·4
-0·6
γ
C'
C_i'
C_R'
0
0·1
0·2
0·3
0·4
0·5
0·6
ξ_d

Fig. 18.

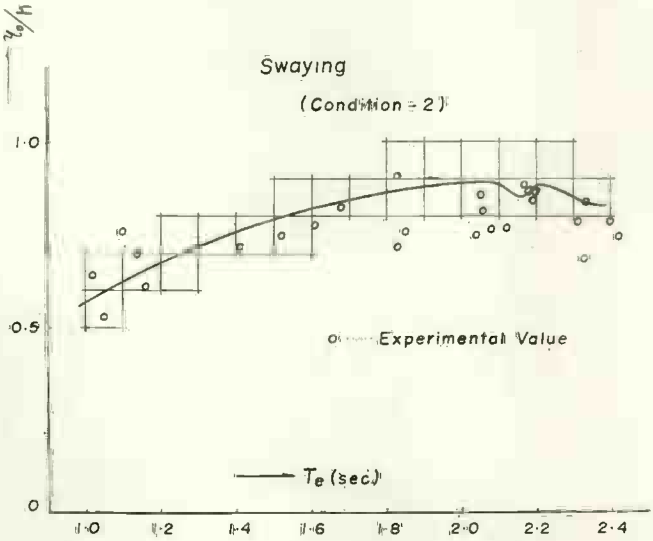

Swaying
(Condition - 2)
Experimental Value
Te (sec)
1·0
0·5
0
1·0
1·2
1·4
1·6
1·8
2·0
2·2
2·4

Fig. 19.

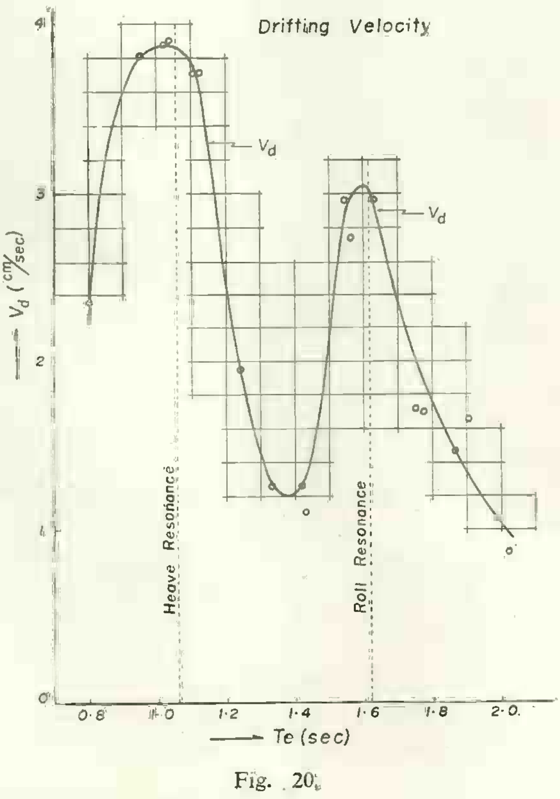

Drifting Velocity
Vd
Vd
Vd (cm/sec)
Heave Resonance
Roll Resonance
Te (sec)
4
3
2
1
0
0·8
1·0
1·2
1·4
1·6
1·8
2·0

Fig. 20.

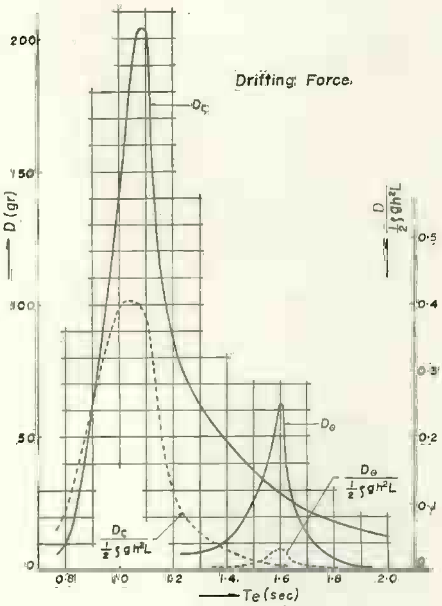


Fig. 21.

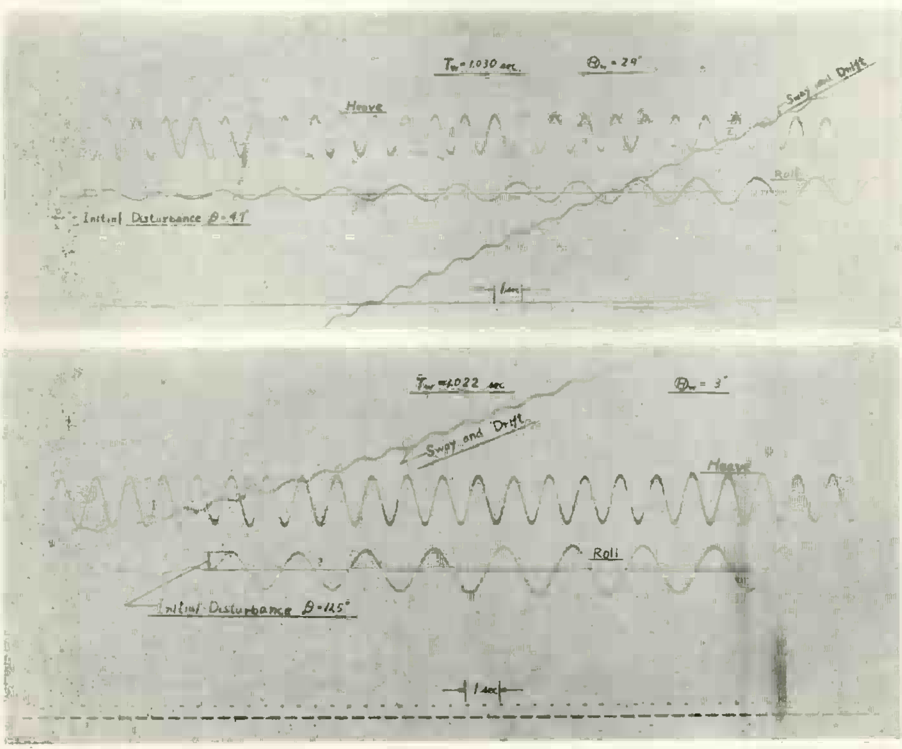


Fig. 22.

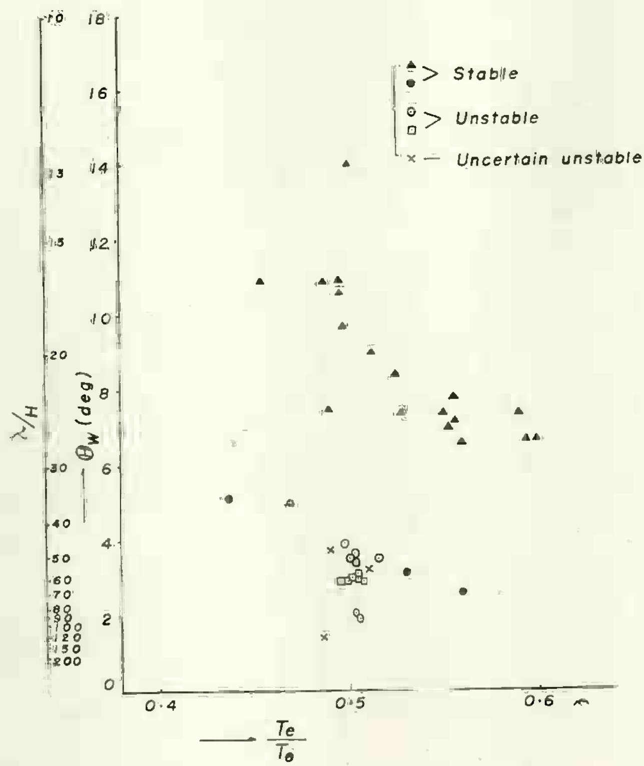

Stable
Unstable
Uncertain unstable
λ/H
θ_w (deg)
0·4
0·5
0·6
Te/Tθ

Fig. 23.