Play My Math: Design and Implementation of a Fraction Generator Tool Within a Mathematical Musician Digital Learning Platform

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Abstract. In this paper, we present and describe a Fraction Generator tool, which is one of the components of our mathematical musician digital learning platform, Play My Math. Teachers and students can use the Fraction Generator tool to discuss, practice, and reinforce conceptual and procedural knowledge of fractions. To that end, the tool offers customizable settings that randomly render exercises in six different categories (i.e., identifying fractions, identifying notations, comparison of fractions, arithmetic applied to fractions, least common multiple, and quantity of a fraction). Additionally, we offer an extra module called the Worksheet Demonstration that teachers can use to guide the students when working with paper-based worksheets. The rationale for the Fraction Generator came from feedback given by teachers during a study we conducted in Mexico in 2024. For designing our tool, we considered principles of technology-enhanced learning and tangible learning interfaces. Finally, to further develop our Fraction Generator initial design, we analyzed qualitative data coming from UK teachers' feedback as part of an intervention we conducted in the UK primary education context at the beginning of 2025.

Keywords: Mathematical musician curricula · music and mathematics · educational technology · fraction education

1 Introduction

Play My Math (PMM) is a digital learning platform that merges music and mathematics within the same learning space to enhance mathematics education in primary school settings [1, 2]. In a previous design-based research cycle [2], teachers expressed a desire for

W.-S. Wang et al. (Eds.): ICITL 2025, LNCS 15913, pp. 264–273, 2026.
https://doi.org/10.1007/978-3-031-98185-2_28

a dedicated component of the platform for generating exercises to practice and reinforce fractions-based conceptual and procedural knowledge without necessarily using music, as their students would still need to face traditional mathematics tests. To address these teachers' request, we built a prototype Fraction Generator (FG) tool that considered the following subtopics of fractions: equivalence of fractions, translation between graphical bar representation of fractions and different mathematical notations (i.e., algebraic, percentage, and decimal notations), and three least common multiple cases (i.e., same denominator, different denominator but direct multiple, and different denominator but no direct multiple). However, when preparing for research done in the UK, where teachers delivered a ten-week fraction program using PMM, we realized that the FG prototype was insufficient. Hence, we iterated and expanded our FG prototype to cover the list of subtopics of fractions that we describe in detail below.

2 Theoretical Background

2.1 What is a Fraction?

In mathematics education, definitions of a fraction can vary in complexity depending on the audience. Definitions can range from "a piece broken off something" [3] to more technical, such as "a numerical representation indicating the quotient of two numbers" [4]. From a mathematical perspective, a fraction can be defined as any number expressed in an a/b format where both a (i.e., numerator) and b (i.e., denominator) are whole numbers and are not equal to zero. The taxonomy of fractions includes the following types of fractions: unit fraction, proper fraction, improper fraction, and mixed fraction. The latter types of fractions vary depending on the numbers assigned to the numerator and denominator. Additionally, when working with two or more fractions, mathematics education distinguishes between having equivalent or nonequivalent fractions, as well as like or unlike fractions. Regardless of the chosen definition and types, Beyranevand [5] posits that the topic of fractions (belonging to rational numbers) plays five targeted functions students should be able to grasp: parts of a whole, ratios, operators, indicated quotients, and measures.

2.2 Common Students' Misunderstandings About Fractions

Learning fractions is notoriously challenging due to the necessary conceptual reorganization, as understanding fractions requires a significant shift from natural number concepts to more abstract ideas [6]. Common errors in fraction tasks, such as treating fractions as whole numbers or misapplying procedures, indicate deep-seated misconceptions due to a lack of conceptual understanding and procedural knowledge [7]. For example, previous research posits that one of the main challenges in learning fractions is students' tendency to directly transfer their knowledge of procedures and concepts around natural numbers to fractions [7, 8]. For instance, in relation to the parts of a whole function, students tend to attribute more value if the fraction has bigger numbers in the numerator and denominator [9]. Another misunderstanding is in relation to the so-called "density of a fraction", which requires students to understand that there is an infinite number of

fractions that are equivalent to a given fraction, and that between two given fractions, there is also an infinite number of fractions [10]. Regarding operations with fractions misconceptions, students tend to apply algorithms from natural numbers that do not necessarily apply when dealing with fractions. For instance, parallel direct addition or subtraction is applied to the numerator and denominator [11], or the understanding bias is that multiplying should make a quantity or number bigger whereas dividing should make it smaller [12]. Regarding the indicating quotient function, this presents less challenge as general division rules are applied. Finally, when fractions function as a measure, the most challenging part is that this function requires students to understand that the fractions could also stand for specific numbers (e.g., when specifying a point in a number line) and not necessarily as a relationship between numbers [5].

2.3 PMM Fraction Generator for Technology-Enhanced Learning as a Tangible User Interface

Our FG tool leverages on technology-enhanced learning (TEL) principles included in the conceptualization of Kirkwood and Price [14] who present three affordances of TEL, namely operational improvement to streamline learning activity, quantitative change to improve students affective and emotional engagement, and qualitative change to improve students' cognitive engagement with the learning activity. For example, leveraging on technology for automated feedback in a reinforcement learning activity can execute such an activity faster and more efficiently (operational improvement), gamification elements of such a reinforcement activity tap into motivational aspects (quantitative change) and the adaptable levels of such an activity support deepening the necessary conceptual and procedural knowledge, bringing qualitative change.

In addition to the general TEL affordances, the FG adopts principles of tangible user interfaces (TUI). In mathematics education, TUIs evolved from tangible physical materials, for which rationale and theory posit that cognition is activated by embodiment, model simulations, and situated actions [15]. For instance, when teachers ask the students to count with their fingers (i.e., situated action), the students' finger movement (i.e., embodied element) serves for them to count a quantity (i.e., model representation). Tangible teaching and learning material aids are vast (e.g., fake coins, Lego bricks, etc.). Piaget advocated for using tangible materials in mathematics education to support the development of students' ability to perceive abstract mathematical concepts [16]. TUIs are built upon the same principles of conventional physical, tangible materials in that both account for the multimodal nature of how humans interact with and make sense of their environment [5]. The factors making a TUI effective for education are physical, social, and emotional affordances [17]. In Sect. 3.1, we explain further how these factors have been considered in our design.

3 Method

Overall, in developing the PMM educational platform, we have been following a design-based research approach, which considers multiple iterations to achieve the intended design goals [18]. Next, we describe the three iterations that lead to the current design and state of the FG.

3.1 First Cycle: An Initial Fraction Generator Prototype

As mentioned in the introduction, we initially conceptualized the FG based on our previous intervention study in Mexico 2024 [2]. Back then, from a focus group analysis, we realized that teachers wanted the possibility of generating fraction exercise so that the students could practice their conceptual and procedural fraction skills without necessarily a musical application (i.e., performing or hearing the musical representation of generated fractions). This necessity was driven by the teachers' concern for preparing their students for traditional and/or standardized mathematics tests. To address this, we designed and implemented a FG covering three main subtopics of fractions that the Mexican students struggled most with: equivalence of fractions, translation between graphical bar representation of fractions and different mathematical notations (i.e., algebraic, percentage, and decimal notations), and three least common multiple cases (i.e., same denominator, different denominator but direct multiple, and different denominator but no direct multiple).

We then moved to account for the factors making a TUI effective for education (i.e., physical, social, and emotional). Regarding the physical factor, we decided to 1) enable teachers and students to manipulate all relevant parameters for generating exercises and 2) incorporate configurable bar models so that the students can connect the abstract concept with a responsive graphical representation when solving the exercises. Regarding the social factor, we decided to 1) enable the teachers to incorporate and save any FG module as part of their lesson planning to engage with their students at the individual, group, or classroom level and 2) enable the students to challenge their peers in the PMM platform by configuring and sending them exercises to advance in their mathematical musician progress as part of PMM's gamification narrative [1, 2]. Finally, on the emotional factor, we hypothesized that students would experience positive emotions as a result of solving the FG exercises collectively or by practicing their math skills in a formative assessment manner (e.g., using the FG to practice and gain confidence/feel calmer before a summative test).

In terms of affordances for TEL, the FG supports operational improvement as teachers can instantly generate a vast number of customized exercises with options for automatic feedback. Qualitative change, in the form of targeted improvement in conceptual and procedural understanding is, in addition to applying the needed knowledge, further supported by different visualizations and representations. Quantitative change is leveraged by the gamification narrative, as well as by diverse instructional designs such as the option of solving the exercises collectively.

3.2 Second Cycle: Expanding the Fraction Generator to Cover 4th Grade Fractions Curricula in the UK

In preparation for ongoing research in collaboration with one primary school in London, teachers shared with us the ten-week fraction learning material (i.e., lesson slides) they normally use for teaching fractions. Next, we list the subtopics of fractions that we used to compare and further expand our first Fraction Generator prototype coverage (Table 1).

Table 1. Our UK Partner School's 10-Week Lesson Plan (Based on UK Curricula Year 4).

Week	Lesson	Learning objectives or key concepts
W1	What is a fraction?	Numerator and denominator
W2	Equivalent fractions part 1	Relating fractions in terms of $=$, $>$, or $<$
W3	Equivalent fractions part 2	Relating fractions in terms of $=$, $>$, or $<$
W4	Fractions greater than 1	Improper and mixed fractions
W5	Counting in fractions	Arithmetic applied to fractions
W6	Add two or more frac tions	Arithmetic applied to fractions
W7	Subtract two fractions	Arithmetic applied to fractions
W8	Subtract from whole amounts	Arithmetic applied to fractions
W9	Fractions of a quantity	Arithmetic applied to fractions
W10	Calculate quantities	Arithmetic applied to fractions

3.3 Third Cycle: Teacher's Feedback from Our Ongoing Research in the UK

At the end of January 2025, we began our research collaboration with one UK primary school where two teachers delivered an adapted version of PMM's fraction program to groups of students. To facilitate teachers' delivery of the PMM program, we designed and implemented a Course Creator tool [19]. Using this tool, we created ten lesson templates mirroring the topics listed in Sect. 3.2. Finally, on a weekly basis, we held online meetings with the UK teachers to gather their feedback after delivering the corresponding lesson. Similarly to the Mexican intervention described in Sect. 3.1 and reported in previous work [2], we organized and analyzed the teacher's weekly feedback using thematic analysis to fine-tune the FG tool. To inform the further development of the FG tool, we considered all of the teachers' feedback belonging to the grading and streamline teaching processes category of our thematic analysis (e.g., "… and it would be nice if they could practice or solve exercises with more graphical representation than just bars").

4 Result

The current version of the PMM FG considers the following modules: Identifying Fractions, Identifying Notations, Comparing Fractions, Arithmetic Applied to Fractions, Least Common Multiple, Quantity of a Fraction, and Worksheet Demonstration. The first six modules were built to generate random exercises based on parameters the users (i.e., teachers and students) can adjust to generate exercises on the desired difficulty level and fractions subtopic. The Worksheet Demonstrator module is a module the teachers requested to practically guide their students when working with paper-based worksheets. Finally, in the PMM Course Creator that we described in another paper [19], teachers can

design and print exercises or worksheets using any combination of the above-mentioned FG modules. Next, we elaborate on the FG modules.

4.1 Identifying Fractions

The Identifying Fractions module, depicted in Fig. 1, considers two sub-exercise configuration paths: Identify Model and Identify Type. The objective of these exercises' paths is for students to practice fractions functioning as a part of a whole and ratios. The Identify Model settings allow the user to configure and generate random exercises with a variety of graphical representations (i.e., bars, circles, collections of objects) using the denominator range and numerator percentage, while incorporating/varying improper and mixed fractions, multiple choice answers, and number of exercises. When multiple choice is not selected, the students need to input the corresponding fraction. The Identify Type path is a multiple-choice, pre-configured exercise generator where users can select among unit, proper, improper, and mixed fractions.

4.2 Identifying Notations

The Identifying Notation module, depicted in Fig. 1, objective is for users to configure and generate exercises to practice translating between graphical representations of fractions and the following notations: numerator/denominator (n/d), algebraic (where the variable S stands for silence), decimal, percentage, and music notation. We decided to keep the music notation exercise generation option for the teachers to decide on incorporating (or not). To that end, the user can configure the number of bars, the denominator range, the numerator percentage, select among the above-mentioned notation options, and the number of exercises to be rendered.

4.3 Comparing Fractions

The Comparing Fractions module, depicted in Fig. 1, considers three sub-exercise configuration paths: Comparison, Identify Equivalent, and Create Equivalent. The objective of these exercises' paths is for students to practice fractions functioning as a part of a whole and ratios when comparing two or more fractions. All three exercise paths allow the users to configure and generate random exercises using the denominator range, numerator percentage, and number of exercises. The Comparison path exercises require the user to select between $=$, $<$, and $>$ when comparing a pair of fractions. The Identify Equivalent path requires the user to answer exercises in two steps. First, selecting the equivalent fractions among three different bar configurations, and then writing all three fractions in an n/d notation format. Finally, in the Create Equivalent path, users are given a fraction with its graphical bar configuration as well as either the numerator or numerator on a second configurable bar. Then, users need to find the missing value with the factor needed to create the equivalent fraction.

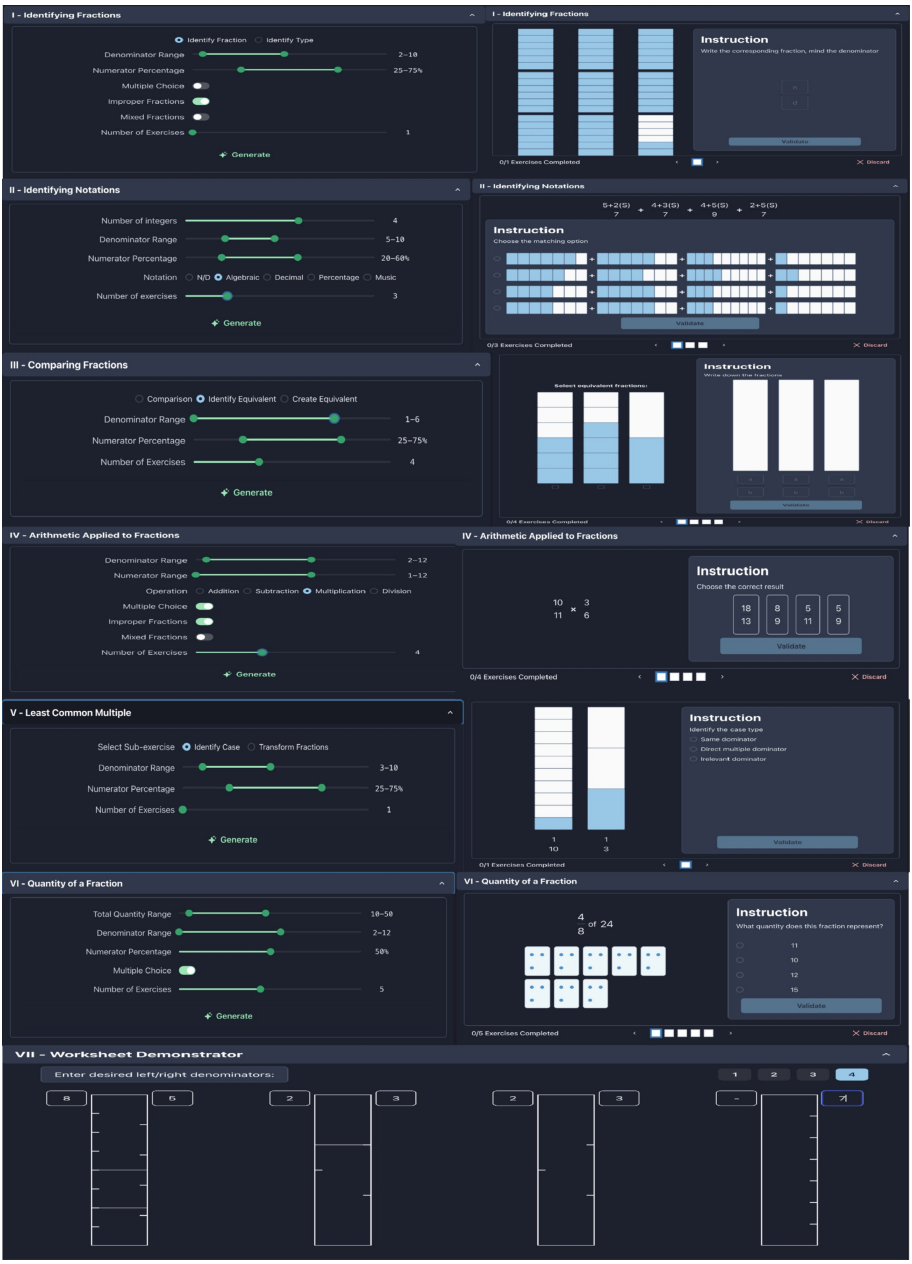

Fig. 1. PMM Fraction Generator modules examples.

4.4 Arithmetic Applied to Fractions

The Arithmetic Applied to Fractions module, depicted in Fig. 1, allows the user to configure and generate random exercises for practicing fractions using addition, subtraction,

multiplication, and division. The configurable settings are similar to those of the Identifying Fractions module. When multiple choice is not selected, the students need to input the resulting fraction in a numerator and denominator notation (n/d). To support students with their procedural knowledge, we added a help button they can refer to for remembering procedural considerations when solving these types of exercises.

4.5 Least Common Multiple

The Least Common Multiple module, depicted in Fig. 1, considers two sub-exercise configuration paths: Identify Case and Transform Fractions. The objective of these exercises' paths is for students to practice finding equivalence between two given fractions using their least common multiple. Both types of paths allow the user to configure and generate random exercises using the denominator range, numerator percentage, and number of exercises. The Identify Case is a pre-configured multiple choice exercise generator where users need to identify the correct least common multiple cases when two fractions are given (i.e., same denominator, different denominator but direct multiple, different denominator but no direct multiple). The Transform Fractions path requires the users to input the corresponding fractions resulting from finding the least common multiple from two given fractions.

4.6 Quantity of a Fraction

The Quantity of a Fraction module, depicted in Fig. 1, allows users to practice fractions functioning as an operator and measure. The users can configure and generate random exercises using the quantity range values, denominator range, numerator percentage, multiple choice answer, and number of exercises options. When multiple choice is not selected, the students need to input the quantity the given fraction stands for. In this module, the randomized quantity range value is rendered as small circles that get distributed in the generated denominator. Then, the user needs to answer how many objects (i.e., small circles) the numerator stands for in the presented fraction. The teachers can use this graphical analogy to work with different measuring cases.

4.7 Worksheet Demonstrator

The Worksheet Demonstrator module, depicted in Fig. 1, is a practical and effective way for teachers to guide students when using physical PMM worksheets. This module allows users to work with up to four bars where each bar can have two desired denominators (left and right) with a partial dividing line. Then, teachers can decide which partial line of the respective denominator to complete.

5 Discussion, Conclusion and Future Work

In this paper, we described the iterative design and implementation of the FG tool of our PMM mathematical musician educational technology. The tool comprises seven modules (i.e., Identifying Fractions, Identifying Notations, Comparing Fractions, Arithmetic

Applied to Fractions, Least Common Multiple, Quantity of a Fraction, and Worksheet Demonstration) with teaching and learning affordances to cover subtopics of fractions. The teachers can use this tool to showcase fraction functions (i.e., part of a whole, ratios, operators, indicated quotients, and measures) [5], generate exercises for students to practice their conceptual and procedural knowledge of fractions, and engage in class discussions about common misunderstandings coming from natural number generalizations in the context of fractions [6–12]. As a next step, we are analyzing pre-post data from a research collaboration with one public school in London. In this study, two teachers delivered a ten-week fraction program using the latest PMM platform including the FG iterations described herein (i.e., experimental condition), while three other groups received the standard school fractions program (i.e., control conditions). We expect the FG tool to, in part, positively contribute to students' learning outcomes; however, we acknowledge that it will not be possible to differentiate between the effects resulting from students experiencing mathematics with music from the specific effects of interacting with the FG tool. Future studies are needed to contrast the unique contributions of distinct PMM components.

Acknowledgments. This study was partially funded by Validierungsförderung 2021-2027 (100737267) and by the British Academy's BA/Leverhulme Small Research Grants Programme (grant reference number: SRG2324\241835).

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

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