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Borgart, A; de Leuw, M; Hoogenboom, JP

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The relationship of form and force in (irregular) curved surfaces

Andrew Borgart*, Marck de Leuw+, Pierre Hoogenboom+,

* Structural Mechanics, Faculty of Architecture, Delft University of Technology
Berlageweg 1, 2628 CR Delft, the Netherlands
+ Structural Mechanics, Faculty of Civil Engineering, Delft University of Technology
Stevinweg 1, 2628 CN Delft, the Netherlands

Abstract

There is a great knowledge of the mechanical behavior of geometrically regular curved surfaces like most shells structures are formed by (Flugge [1]). This is mainly because these surfaces are relatively easily described by analytical mathematical functions. For describing irregular curved surfaces, like those in Free From Architecture, there are very little analytical mathematical functions available and there for it is very hard to derive formulas to describe their mechanical behavior. One way of dealing with this problem is to calculate the stresses and strains of these irregular curved structures with computer programs based on the finite element method. The problem with that is that you only obtain quantitative information about the results (like the magnitude of the forces) but not any qualitative information. It doesn't always give clear insight into the structural behavior. For example, what is the relation between the shape of the curved surface and the flow of forces. In analytical formulas for regular curved surfaces there is a quantitative relation between the magnitude of the forces and the shape of the shell, like the radius. Because of the lack of insight it can be difficult to design irregular curved surfaces which have shell-like behavior, that is mainly extension forces and little bending moments.

The research tries the reveal some of the mysteries of the relationship between form and force of irregular curved surfaces. In 2D structures the load and the supports determine the line of thrust of the load. If the system line of a structure deviates from the line of thrust of the load it will cause "corrective" bending moments in the structure. In 3D structures like shells, for example a dome, the line of thrust of the load can be corrected by the hoop forces so to coincide with the system line of the shell so there are no bending moments in the dome. For a dome where the line of thrust of the load falls outside the system line the hoop forces are compression, and where the line of thrust of the load falls inside of the dome the hoop forces are tension (Figure 1). If we know the "3D line" (surface) of thrust of the load in regards to it's supports and we combine this with any (irregular) curved surface it is possible to determine the forces in the shell. A way of determining the flow of forces of (irregular) curved surfaces is the "rain flow" analysis of the geometry of the curved surface.

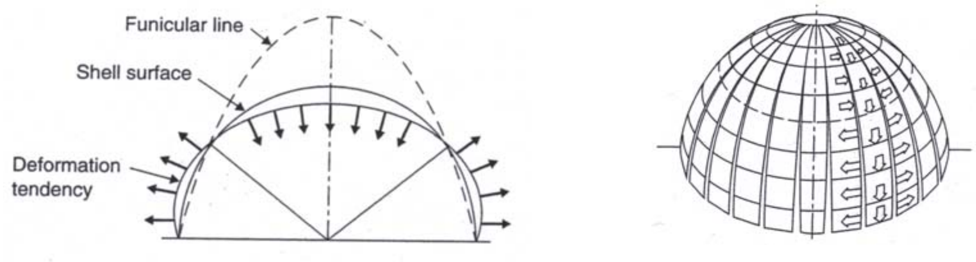

Figure 1: Line of thrust of the load (Funicular line) in relation to the Shell surface and the corrective hoopforces. (Schodek [2]).

1 Introduction

There is presently a tendency to integrate design, calculation and production driven by the possibility to exchange data between CAD-programmes (Computer Aided Design) and FEM-programmes (Finite Element Method). This has lead to a widerange of designs for buildings with complex shapes (Free Form Architecture), which sometimes seem to have shell-like behaviour. Shell-like behaviour consists of mainly extension forces and little bending moments, due to the curvature of the surface. These buildings are often calculated by first importing the data of the geometry of the shape made by the CAD-programmes to the FEM-programmes, where the structional model is build and calculated. For describing irregular curved sufaces, there are very little analytical mathematical functions available and therefore it is very hard to derive formulas to describe their mechanical behaviour. Because of this only quantitative information about the results (like the magnitude of the forces) but not any qualitative information is obtained. This quantitative information doesn't always give clear insight into the structural behaviour. Therefore it is convenient to get insight in the mechanical behaviour of (irregular) curved surfaces, without use of FEM-programmes or very complicated analytical formulas.

2 Some well-known analytical and graphical solutions for shell structures

Many shell theories have been developed to analyse the mechanical behaviour of shell structures. A well-known theory is the membrane theory for thin shallow shells. In the membrane theory it is assumed that the thickness of a shell is far smaller than the overal dimensions. Due to this the flexural rigidity is far smaller than the extensional rigidity. A thin shell subjected to external applied loads therefore mainly produces membrane forces, which are actually resultants of the inplane normal and shear stresses that are uniformly distributed across the thickness. In the regions where the membrane theory will not hold, because for example edge disturbances, some (or all) of the bending field components are needed to compensate the shortcomings of the membrane field in the disturbed zone. These disturbances have to be described by a more complete analysis, which leads to a bending theory of thin elastic shells. Because of its simplicity the membrane theory gives a direct insight into the structural behaviour and the order of magnitude of the expected response without elaborate computations.

There are several ways to make a classification of surfaces: by using the definition of Gaussian curvature and by using the way the surface is generated. The Gaussian curvature is defined as the product of the two principal curvatures of a surface in a point. When the Gaussian curvature is positive both curvatures are pointing in the same direction and the surface is called synclastic. If the Gaussian curvature is negative both curvatures are pointing in another direction and the surface is called anticlastic. When both the curvatures are zero the surface is flat and is called zeroclastic. When one curvature is zero the surface is called monoclastic. There are several ways to develop surfaces. The main ways are revolution, translation and ruling. Surfaces of revolution are generated by the revolution of a plane curve, called the meridional curve, about an axis, called the axis of revolution. Surfaces of translation are generated by sliding a plane curve along another plane curve, while keeping the orientation of the sliding curve constant. Ruled surfaces are generated by sliding each end of a straight line on their own generating curve, while remaining the straight line parallel to a prescribed direction or plane. It is also possible to combine several surfaces.

For several well known shells the mechanical behaviour has been formulated by using the membrane theory. The bending moments caused by the edge disturbances can be calculated separately and superimposed with the result of the membrane solution. The predominantly load case is most often its own weight. For the surfaces of revolution shells subjected to its own weight it is always possible to determine the mechanical behaviour with a graphical solution, to give more insight in the flow of forces. By this graphical method it is easily to construct a polygon of forces. This polygon represents the “corrected” line of thrust, whereby the hoop forces correct the line of thrust of the load to coincide with the system line of the shell. A nice example of this graphical method is used to calculate the forces in a masonry dome (Figure 2).

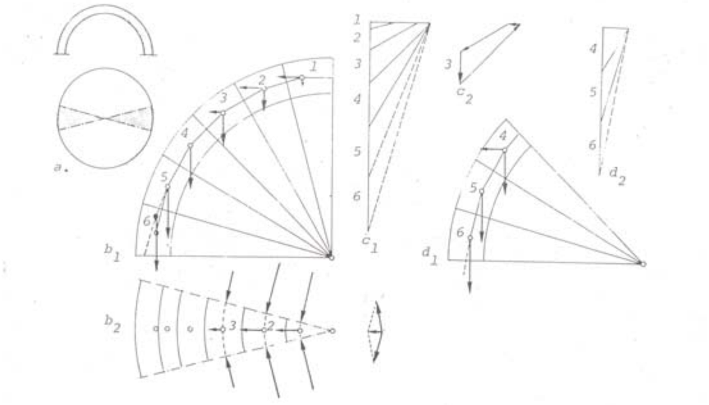

Figure 2: Graphical determination of the forces in masonry dome due to it’s own weight (Beranek [3]).

3 Hypothesis

The next hypothesis for the flow of forces in shell structures has been the basis of this research:
- Like a rain flow loads will flow along curves with the steepest ascent on the shell surface to it’s supports.

These curves can be derived from a gradiëntplot of a surface, which plots the normal vectors of the surface in a view from above. These curves are always orthogonal to the vertical contours of the surface, which are easily to plot. The gradiënt forces in the vertical sections between curves can than be obtained in the same way as with the masonry dome through the vertical equilibrium of forces. The hoop forces have to be in equilibrium with the horizontal forces, which have to ensure that the pressure surface (line of thrust of the load) converges with the system (line) surface of the shell. The hypothesis has been formulated from studying the way plates transmit there loads. The maximum shear force in a plate is a vector with the magnitude of the carried load and points in the direction of the flow of the shear force to the supports. The vector of the maximum shear force can be derived from the gradiëntplot of the surface which represents the sum of the curvatures because the shear force is the derivative of the sum of the bending moments (M):

$$\overline{M} = \Delta w = \partial^2 w / \partial x^2 + \partial^2 w / \partial y^2$$

As an analogy (the rainflow analogy (Beranek [4]) this surface also represents an air inflated membrane, on which the rain flows along curves with the steepest ascent to the supports. In the case for the flow of forces of shells (the hypothesis) the surface of the sum of the curvatures for plates or the surface of the air inflated membrane is replaced by the surface of the shell itself.

4 Example

It possible to combine straight edge hyppars to get a shell as shown in Figure 3.1. It is assumed that the separate hyppars act as like a single hyppar with diagonal compression and tension parabola. But according to (Lauletta [5]) however the compression forces are dominate and the tension froces merely distribute the loads towards the compression trajectories. This gives in a stress distribution (a results of tests) as shown in Figure 3.2, which resembles the contourplot and gradiëntplot of the flow of forces according to the hypothsis, shown in Figure 3.3.

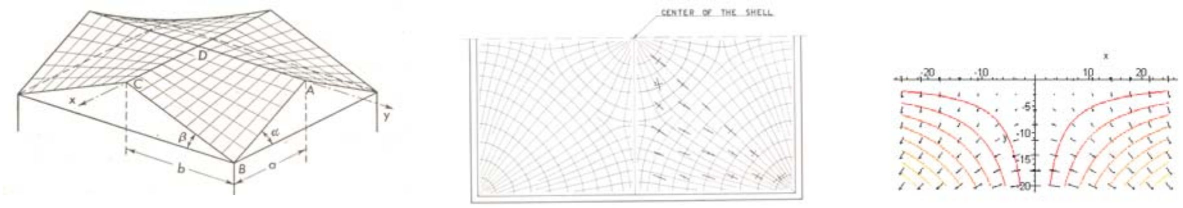

Figure 3.1: Shell combined of several straight edge hyppars / 3.2: Principal stresses in a shell model combined of several straight edge hyppars (Lauletta [5]) / 3.3: Contourplot and gradiëntplot of the flow of forces of a shell combined of several straight edge hyppars

5 Case study

This hypothesis has been tested on a Free Form design of a student from the Faculty of Architecture (Hanselaar [6]) and comprehends of a shell-like structure for a indoor ski-slope (
Figure 4), and will be extensively presented at the conference. The structure spans in the short dierection, so that the long sides are pin fixed.
A gradiëntplot of the surface (

Figure 5) represents the flow of the forces (loads) according to the hypothesis. There a two different kinds of free edges on the short sides. On the left side a raised edge is visible. Because of this raised edge the loads run away form the edge towards the support direction, which leads to a desirable membrane stress distribution. On the other side however the loads run towards the free edge, which then has to transfer the loads towards the supports. This leads to an undesirable situation. In the gradient plot also a couple of drain curves are visible. Loads from curves leading to these drain curves can be transferred by these drain curves like membrane forces under certain conditions. However when the curves leading to these drain curves are orthogonal to them large bending moments can be expected. This is visible by the dark bleu color representing large vertical deformations as shown in Figure 4. Looking along a curve large hoop forces can be expected at places where the slope changes rapidly. This is due to the sudden change of the horizontal force that has to ensure that the pressure surface converges with the system surface. These large hoop forces can also result in bending moments.

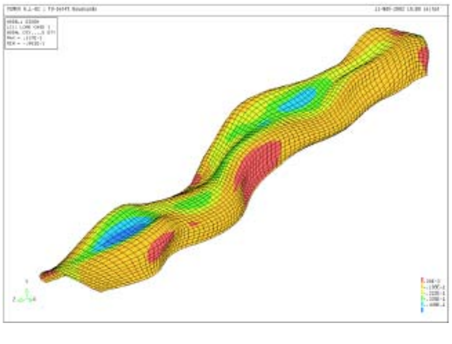

Figure 4: Shell-like structure with contourplot of the vertical displacements determined with an elastic calculation (FEM).

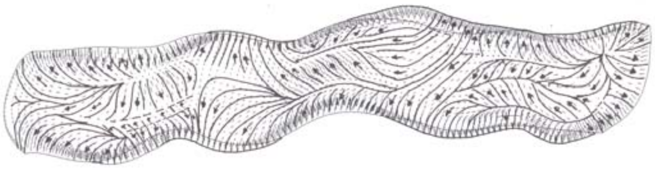

Figure 5: Sketch of gradiëntplot in top view.

References

[1] Flügge, W, Stresses in Shells. *Springer Verlag, Berlin.* 1960.

[2] Schodek, D.L, Structures, *Prentice Hall*, 1998

[3] Prof.ir W.J. Beranek, Understanding of structures, *Proceedings of the InternationalUpdating Course on Structural Consolidation edited by,* Prof.dr.ir. R.M. Lemaire / ir. K. van Balen, Stable-Unstable? Structural Consolidation Of Ancient Buildings, Center For The Conservation Of Historic Towns And Buildings, Leuven Universtity, 1988.

[4] Prof.ir W.J. Beranek, Berekening van platen (in Dutch, translation: Calculating Plates), TU Delft, 1976

[5] E. Lauletta, Statics Of Hyperbolic Paraboloidical Shells Studied By Means Of Models, *Proceedings of the symposium on shell research*, Delft, 1961; 47-57.

[6] H. Hanselaar, Snowplace, *MSc thesis Delft University of Technology*, 2003