Technische Universiteit Delft
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Gelijkmatig verspreiden van het aantal benodigde bedden op de holding en recovery department

(Engelse titel: Levelling the number of required beds at the holding and recovery department)

Verslag ten behoeve van het
Delft Institute of Applied Mathematics
als onderdeel ter verkrijging

van de graad van

BACHELOR OF SCIENCE
in
TECHNISCHE WISKUNDE

door

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Delft, Nederland
Juli 2017

BSc verslag TECHNISCHE WISKUNDE

"Gelijkmatig verspreiden van het aantal benodigde bedden op de holding en recovery department

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Juli, 2017

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Abstract

In this thesis, we consider levelling the number of required beds at the holding and recovery department. Due to the decreased number of available nurses, the workload for the nurses at hospitals has increased. For this reason, it is important to level the number of required beds at the holding and recovery department. By doing so, the workload can be reduced, beds will be available for emergency surgeries, less surgeries will be cancelled due to the lack of beds and less staff will be needed.
The thesis starts with introducing an analytic calculation of the number of required beds at the holding and recovery department. These calculations consider a stochastic length of stay (LOS) distribution for each surgery type. Making use of the analytic calculation and the stochastic length of stay, several solution methods are developed. The first solution method includes a formulation of the problem as an Integer Linear Program (ILP). The exact calculation of the number of required beds is not linear, so the objective function is simplified. The ILP minimises the number of expected beds.
The other solution methods uses the start and end times of each surgery. These start and end times are spread using two algorithms called Fixed Goal Values and Flexible Goal Values.
Data from an Academic Medical Centre in the Netherlands is used to determine the LOS distributions and to find solutions for the ILP and the algorithms. This thesis shows that the number of required bed at the holding and recovery department can be reduced.

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1 Introduction

In the past few years, the number of available nurses has decreased [1]. The hospital has to provide every patient will get the best care they need. This means that the remaining nurses will have to work harder. Due to this, their workload is increased. The V&VN, 'Verpleegkundigen & Verzorgenden Nederland', published survey results on nurses' workload perception. It turned out that 9 out of 10 nurses experience a high level of stress due to a higher workload [2].
The surgery schedule influences the number of patients at the wards, which influences the workload for nurses. Nurses have to work harder when there are a lot of patients at the department. If there is an insufficient number of nurses at the department, the workload will be higher for the ones available. Influencing the surgery schedule can result in a lower number of patients at the department. If the number of patients is lower, the workload will be lower and the number of required staff can be reduced.

Two departments are highly controlled by the surgery schedule. These departments are called the holding and recovery department. The holding department is the ward where the patients arrive right before surgery. Here, they will be prepared for the surgery. After the surgery, the patients arrive at the recovery department. There, they wake up from the anaesthetics and are checked before they go to the nursing ward where they will stay for a couple of days. To reduce the workload at the holding and recovery department, the surgery schedule must be altered. The altered schedule should reduce the number of patients at the holding and recovery department.

Ward	Holding	Operating Room	Recovery	Ward

Table 1: Timetable for surgery patients

The number of required beds at the holding and recovery department is influenced by the length of stay (LOS) of each patient. The patients stay a period of time at both departments and the length of stay and number of patients determine the number of occupied beds. Also, the start and end of the scheduled surgeries influence the number of required beds. When many surgeries are scheduled at the same time, the patients will arrive at the holding ward approximately at the same time. At that moment, the number of needed beds will be high. But if the surgeries are spread over the day, the patients arrive at different times over the day, which will reduce the number of needed beds. This spreading is done by determining a new schedule, in which the start and end times of surgeries are spread over the day. This new schedule will result in a new order of the surgeries. Due to the equipment a surgery needs and an OR provides, the order of surgeries can only be changed within the OR.

Levelling the number of required beds has a couple of advantages. Firstly, the probability that a bed will be available for emergency patients at any time of the day will increase, because the bed occupancy will be levelled. Secondly, the number of cancelled surgeries due to lack of beds will be reduced. Thirdly, the workload for the nurses at the holding and recovery departments is spread over the day, because the number of patients on the departments is levelled. Finally, the third advantage ensures that less staff is needed. Therefore, it is important to level the number of required beds at the holding and recovery department.

We begin this thesis with a literature overview concerning bed occupancy at wards. In Chapter 3, we introduce the used mathematical notation. Chapter 4 provides a detailed description on calculating the number of beds at the holding and recovery department. Several solution methods are discussed in Chapter 5. In Chapter 6, the solutions are shown. Chapter 7 provides a small summary together with an overall conclusion.

2 Related literature

There are several papers available that consider improving a certain performance measure concerning a surgery schedule. Vissers et al. [10] generate a master surgical schedule (MSS) that realises a give target of patient throughput and optimises an objective function for the utilisation of resources. These resources concern the availability of nurses, the workload and the bed occupancy at the Intensive Care (IC). They investigate the influence of a stochastic length of stay (LOS) against a deterministic LOS. Vissers et al. create a cyclic MSS which repeats after a number of days. This research resulted in a better understanding of using a stochastic LOS instead of an deterministic LOS. By considering the stochastic length of stay, the MSS performed better for the utilisation of resources.

Beliën and Demeulemeester [4] create models for building surgery schedules with levelled resulting bed occupancy. The model includes two constraints. The first one concerns the demand: the surgeons or surgical group receives a specific number of OR blocks. An OR block is an certain time block in the OR schedule. The second constraint concerns the capacity: there are a limited number of blocks available per schedule. Beliën and Demeulemeester schedule OR blocks in a cyclic schedule. They use a stochastic number of patients whose surgery will be performed in the schedule and a stochastic LOS, which depends on the surgery type. Both are considered as multinomial distributions. They use a Mixed Integer Program (MIP) based heuristic and meta heuristic to minimise the expected total bed shortage. A MIP involves a problem in which some of the variables are integers and others are not. The software did not provide an overall best solution, but it gave several solutions from which the manager can decide which one to choose.

In another paper written by Beliën et al. [3], they come up with a decision support system for cyclic master surgery scheduling (MSS). Three objectives were taking into account when building the MSS. The first one was to level the bed occupancy at the wards. The second one involved assigning surgeons or surgical groups to one exclusive OR. The last objective was to make sure the MSS was as repetitive as possible with a few changes. The schedule involved the number and type of open ORs, the hours in which the ORs were opened and the surgeons who have priority in the ORs. Again, they assign surgeons or surgical groups to OR-blocks. To solve the multi objective problem, they used a penalty function, where the weights represent the importance of the objective. They used this to level the bed occupancy at the wards.

The paper written by Vanberkel et al. [9] considers how an MSS affects the workload at the wards. They consider a ward occupancy distribution and a patients admission and discharge distribution. This gives the hospital the ability to see the workload that results from an MSS. They focus on assigning surgeons or surgical groups to an OR block. Vanberkel et al. [9] came up with a new distribution in which they use the binomial distribution. This distribution equals the probability that a patient is discharged or not on a day in the planning horizon. With this probability, a distribution emerged that is used to determine how many beds are occupied or needed the days after. The process to the final distributions is explained in steps, which are used in other papers such as in the paper written by Van Essen et al. [7]. In this paper, they assign OR blocks to a surgery planning to reduce the number of required beds at the wards. They use the distribution explained in Vanberkel et al. [9] to create a cyclic planning which reduces the number of required beds and formulate an ILP to obtain solutions. The problem was solved using a local search approach and a global search approach. The paper was a request by a hospital to investigate the factors that influence the bed occupancy. They tested the approaches for a considered instance at HagaZiekenhuis and the number of required beds can be reduced by 20%.

Chow et al. [5] proposes an approach to improve scheduling practices. They use a Monte Carlo simulation model and a mixed integer optimisation model. They evaluate the impact of proposed system changes on ward congestion. The surgical block schedule (SBS) assigns an OR block, containing surgeries from one surgeon, to a cyclic OR schedule. The surgeries in the OR block are scheduled by the surgeons. The simulations enable surgical planners to predict the impact of a SBS on surgical ward occupancy. The mixed integer model schedules surgeon blocks and the patient mix within each block to help planners create surgical schedules with minimal bed requirements. A patient mix is the set of patients that will be operated in the OR block. At the end, they set up guidelines for the managers to support ongoing schedules.

A different paper in which they do not try to level the bed occupancy at the surgerical wards is written by Van Essen et al. [8]. In this paper, they change the order of surgeries in a given schedule, to minimise the expected waiting time for emergency surgeons. They order these surgeries while taking Break-In-Moments into account. Break-In-Moments are the moments when surgeries start or end in the occupied interval. The occupied interval is the interval in which all the ORs are occupied. The BIMs are spread equally over the day such that the waiting time for an emergency surgery is minimised. Spreading the BIMs could help the holding and recovery department to level the bed occupancy at these wards. If the start times and end times of the surgeries are spread over the day, the arrival of patients is spread over the day. This could help level the required numbers of beds at the holding and recovery department.

Overall, this short literature review provides us an overview of interesting papers on changing the surgery schedule and creating a cyclic master surgery schedule. Several papers address the consequence of these surgery schedules on the surgical ward but none of the papers considers the problem to level the number of required beds at the holding and recovery department.
Every paper explained above, creates a cyclic surgery schedule to level or to minimise the bed occupancy at wards. They all assign surgery blocks to surgery days. In most of the papers, the surgeries in the OR blocks were not assigned in a specific order. In this paper, we do not create a surgery schedule from scratch, but we determine the order of the surgeries. These surgeries are already assigned to a specific day and OR. The new order is obtained by the solutions of the ILP and by using two algorithms explained in van Essen et al. [8]. The ILP uses the LOS distributions to determine a new order, which reduces the number of required beds at both departments.
The two algorithms in the paper written by van Essen et al. [8] are used to determine a new schedule, such that the start and end times of the surgeries are spread. As discussed before, spreading these start and end times can help level the number of required beds at both departments. These algorithms only spread the start and end times and do not involve the LOS distributions.
In papers written by Vissers et al. [10] and Vanberkel et al. [9], they also make use of stochastic LOS distribution and show that it resulted in a better schedule than using a deterministic LOS. Therefore, we use an stochastic LOS distribution. This stochastic LOS distribution is provided by data given from a hospital in the Netherlands.

3 Used notation

In this section, we introduce the notations used throughout this thesis. We consider three time horizons, T_h, T_r, T_k. The first one, $T_h = \{s_h, ..., e_h\}$, is the time horizon in which the holding department is open. Here s_h and e_h are the opening and closing times of the holding department. The second one is T_r, which is the same as T_h but for the recovery department. The last time horizon is T_k, the opening hours of the ORs. The set of ORs is given by K, with index $k \in K = \{1, ..., R\}$. The time horizon for the ORs $k \in K$ is $T_k = \{s_k, ..., e_k\}$.
The set of patients whose surgeries are scheduled in the given schedule is given by $I = \{1, ..., M\}$. The set of patients in OR k is given by I_k. This gives us: $\sum_{k=1}^{R} |I_k| = W$. In this thesis, we use the index $i \in I$ for patient i. But i is also used to refer to the surgery belonging to patient i.
The OR in which surgery $i \in I$ is scheduled is given by $O(i)$. The expected duration of surgery $i \in I$ is d_i, which depends on the surgery type. A surgery type is the type of surgery performed on patient i. We assume that each patient has a surgery type.
The start and end time of OR $O(i)$ in which surgery $i \in I$ is scheduled is given by: $s_{O(i)}$ and $e_{O(i)}$ respectively. The start and end time of surgery i is given by s_i and e_i. The start and end time of the occupied interval, the interval in which all the ORs are occupied, is equal to s and e. The entire list of the definitions is given below:

Definitions

I The set of patients {1, ..., W}.

d_i The expected duration of the surgery performed on patient i.

K The set of ORs {1, ..., R}.

$O(i)$ The OR the surgery belonging to patient i will be performed.

I_k The set of patients whose surgeries will be performed in OR k.

s_c The start time of department or OR c, where c is h for holding, r for recovery or k for $k \in K$.

e_c The end time of department or OR c, where c is h for holding, r for recovery or k for $k \in K$.

T_c The time horizons for the department of OR c, where c is h for holding, r for recovery or k for $k \in K$.

$s_{O(i)}$ The start time of the OR where the surgery belonging to patient i will be performed.

$e_{O(i)}$ The end time of the OR where the surgery belonging to patient i will be performed.

s_i The start time of the surgery belonging to patient i.

e_i The end time of the surgery belonging to patient i.

s The start time of the occupied interval.

e The end time of the occupied interval.

When a schedule is provided, the number of required beds can be calculated for the holding and recovery department. This provides the hospital an indication of the workload at the departments. In the next chapter, the calculation of the number of required beds at the holding and recovery department is presented.

4 Calculating the number of required beds at the holding and recovery department

There are two possibilities to calculate the number of required beds at a department. The first one is to assume that patients have the same preparation time and the same recovery time per surgery type. From historical data, we can determine the preparation and recovery time per surgery type. This of course would be an average, because the preparation time and recovery time is different for each patient. With the preparation time, recovery time and OR schedule, we know at what time the patient arrives and leaves both departments. With this, the number of required beds can be calculated every m minutes.

However, this is not realistic. Because the preparation time and recovery time is different for each patient, it is better to calculate the number of required beds using a probability distribution. This distribution represents the probability that a patient spends m minutes at the holding or recovery department.

In this chapter, we describe a way to calculate the number of required beds at the holding and recovery department by using length of stay (LOS) distributions. This distribution is given by the probability that a patient stays m minutes at the holding or recovery department. The distribution will be determined from historical data.
To determine the number of required beds, we use a modification of the approach introduced by Vanberkel et al. [9]. In this paper, they came up with a way to determine the number of required beds by using the distribution and a binomial distribution to describe the LOS.
For the holding department, we determine the calculation by looking back in time. For the recovery department, we determine the number of required beds after the surgery is performed.

In the next section, we calculate the number of required beds for a given schedule. The calculation will involve the set of patients, i.e., I, and the set of patients whose surgery will be performed in OR k, i.e., I_k.

4.1 Holding department

In order to calculate the number of required beds at the holding and recovery department, we need the LOS distributions. The probability that patient $i \in I$, stays m minutes at the holding department has to be determined. Historical data given by a hospital can provide this distribution.

We denote the probability that patient i, whose surgery will be performed in OR k, needs between $[n, n+m)$ minutes of preparation as p_n^i, where m is the length of each interval and n the time. With this, we can determine the probability that patient i occupies one bed at the holding department. We denote this probability as $g_n^{ik}(1)$ which is given by,

$$g_n^{ik}(1) = \sum_{j=n}^{M_i - m} p_j^i. \tag{1}$$

where M_i equals the maximum length of stay at the holding department. There is also the possibility that patient i occupies 0 beds. Because a patient can only occupy one bed or zero beds, $g_n^{ik}(0)$ equals $1 - g_n^{ik}(1)$. The probability that patient i, whose surgery will be performed in OR k, occupies zero beds is given by

$$g_n^{ik}(0) = 1 - \sum_{j=n}^{M_i - m} p_j^i = \sum_{j=1}^{n-1} p_j^i. \tag{2}$$

We have to shift the distribution in such a way that the probability is in the right order. We want the probability that patient i occupies one or zero beds between $[n, n+m)$ on a specific time t before surgery. This is done by shifting the distribution:

$$\overline{G}_t^{ik}(x) = g_{s_i - t}^{ik}(x) \quad t \leq s_i \tag{3}$$

where s_i is the start time of the surgery performed on patient i. The approach introduced by Vanberkel et al. [9] calculated the number of required beds on a daily basis for a cyclic schedule. We only consider the bed occupancy distribution for one day which is given per time period.
Next, the combined probabilities of all the patients in OR k is obtained by taking discrete convolutions. This is the probability distribution of the bed occupancy at each moment t over the whole day resulting from OR $k \in K$ and is given by G_t^k:

$$G_t^k(x) = \overline{G}_t^{1k}(x) * \overline{G}_t^{2k}(x) * \overline{G}_t^{3k}(x) * ... * \overline{G}_t^{|I_k|k}(x) \tag{4}$$

Because we took the discrete convolutions over all the patients in OR k, the variable x can take up to the number of patients $|I_k|$. The only thing left to do is to combine all ORs. This is done by taking the discrete convolutions of all the $G_t^k(x)$ for all $k \in K$. The obtained distribution is given by,

$$\mathscr{G}_t(x) = G_t^1(x) * G_t^2(x) * G_t^3(x) * ... * G_t^R(x) \tag{5}$$

The previous function gives us the probability that there are x beds occupied on time t, but we need the number of required beds. We want a sufficient number of beds available at the holding department. A sufficient number of beds at the department means that there should be at least x beds available at time t. We want the number of required beds at the holding department to be higher than a given probability. This is the probability that there is a sufficient number of beds available at the holding department. We denode this probability as $\frac{p}{100}$. The p can be for example 95%, which means, there is a 95% chance that there will be a sufficient number of beds available. We determine the number x as followed:

$$\gamma = \max_{t \in T_h} \min \left\{ x \,\middle|\, \sum_{y=0}^{x} \mathscr{G}_t(y) \geq \frac{p}{100} \right\} \quad \forall t \in T_h \tag{6}$$

After calculating the number of required beds at the holding, the same is done for the recovery department.

4.2 Recovery department

We denote the LOS distribution as l_n^i, i.e., the probability that patient i whose surgery will be performed in OR k is leaving the recovery department in the interval $[n, n+m)$ after surgery. Here, m is the length of the interval in which we want to know the number of required beds.

With this, we can determine the probability that patient i occupies 1 or 0 beds at the recovery department between the interval $[n, n+m)$ after the surgery is performed. We denote this as $h_n^i(x)$, where x is either one or zero. This is given by

$$h_n^{ik}(1) = \sum_{j=n}^{N_i - m} l_j^i, \quad \text{for } n > 0, \tag{7}$$

$$h_n^{ik}(0) = 1 - \sum_{j=n}^{N_i - m} l_j^i = \sum_{j=1}^{n-1} l_j^i, \quad \text{for } n > 0, \tag{8}$$

where N_i is equal to the maximum length of stay at the recovery department of the patient belonging to surgery i.
Next, we have to shift the distribution $h_n^{ik}(x)$, such that patient i is admitted at the recovery department at time t after surgery. In the following function, e_i is the end time of patient i's surgery. The shifted probability distribution is given by $\overline{H}_t^{ik}$.

$$\overline{H}_t^{ik}(x) = h_{t-e_i}^{ik}(x) \quad t \geq e_i \tag{9}$$

Now, we have to take the discrete convolution $\overline{H}_t^{ik}(x)$ over all $|I_k|$ patients in OR k. This is the probability distribution of the bed occupancy at each moment t over the whole day resulting from OR $k \in K$, denoted by H_t.

$$H_t^k(x) = \overline{H}_t^{1k}(x) * \overline{H}_t^{2k}(x) * \overline{H}_t^{3k}(x) * ... * \overline{H}_t^{|I_k|k}(x) \tag{10}$$

Because we use the shifted distribution, x is no longer just 0 and 1, but can take up to the number of the patient scheduled in OR $k \in K$. The next step is to combine the probabilities over all the ORs.

$$\mathscr{H}_t(x) = H_t^1(x) * H_t^2(x) * H_t^3(x) * ... * H_t^R(x) \tag{11}$$

Now, we can calculate the number of required beds at the recovery department for a given surgery schedule, which is the same as in equation (6):

$$\gamma = \max_{t \in T_r} \min \left\{ x | \sum_{y=0}^{x} \mathscr{H}_t(y) \geq \frac{p}{100} \right\} \quad \forall t \tag{12}$$

With these equations, the number of required beds at the holding and recovery department can be calculated for a given OR schedule. In the next chapter, we introduce several solution methods to level the number of required beds given by equations (6) and (12).

5 Solution methods

The obtained functions given in Chapter 4 can be used to calculate the number of required beds for a given schedule. In this chapter, we propose several solution methods which provide such a schedule. The schedule is obtained by using the LOS distributions of each patient and involves the spreading of BIMs. In the first section an ILP formulation is presented, which minimises the maximum number of required beds at the holding and recovery department. The ILP uses the LOS distributions to find a new order of surgeries to reach the goal. Because the number of required beds at the recovery department is higher than at the holding department, we use a weighted sum method. This weight makes sure that the number of required beds at both departments are minimised.
The other solution methods consider the start and end time of the surgeries. These start and end times influence the number of patient arriving at both departments. Therefore, it can help to level the number of required beds. The start and end times are spread by using two algorithms. These algorithms ensures that the time between one surgery ending and the other one starting is equal.

5.1 ILP formulation

In this section, we introduce an formulation that levels the number of required beds at both departments. As discussed before, we want to create a new order of surgeries such that the number of required beds is levelled over the day. This can be formulated as minimising the maximum number of required beds at both departments. The problem can be written as an Integer Linear Program (ILP), which is presented in the following.

The formulation of the ILP consists some of the restrictions used in the paper written by Van Essen et al. [8]. The solution represents a new order of surgeries, i.e., a new surgery schedule.
The solution is defined by binary variables Y_{ij} for each OR k, which will be one if surgery $i \in I$ is scheduled before surgery $j \in I$ in OR k. We also need to make sure that Y_{ij} and Y_{ji} are not both equal to one. Both are satisfied in the following restriction:

$$Y_{ij} + Y_{ji} = 1 \quad \forall i \neq j \in I_k, \quad \forall k \in K \tag{13}$$

For the calculation of the number of required beds, we need to know when each surgery starts and ends. This is given by the start and end time resulting from the schedule, i.e., s_i and e_i.

$$s_i = S_{O(i)} + \sum_{j \in I_{O(i)}} d_j Y_{ji}, \quad \forall i \in I_k \tag{14}$$

$$e_i = S_{O(i)} + \sum_{j \in I_{O(i)}} d_j Y_{ji} + d_i, \quad \forall i \in I_k \tag{15}$$

We also want the start time of each surgery to be increasing for every surgery j scheduled after surgery i. This is given by:

$$s_i - \mathscr{M}(1 - Y_{ij}) \leq s_j \quad \forall i \neq j \in I_k, \quad \forall k \in K, \tag{16}$$

where $\mathscr{M}$ is a sufficient large number. The equation only becomes active when $Y_{ij} = 1$.

In Chapter 4, we determined the calculation for the numbers of required beds given by equations (6) and (12). The objective function is to minimise the number of required beds. The objective function resulting from using the equations in (6) and (12) is not linear. For this reason, the equations in (6) and (12) have to be linearised. We follow the approach in the paper written by Beliën and Demeulemeester [4] and Van Essen et al. [6].

First, we introduce the two binary variables w_{it} and v_{it}. The variable w_{it} is one if surgery i starts at time t and is zero otherwise. The variable v_{it} is one if surgery i ends at time t and zero otherwise. Every surgery $i \in I$ has exactly one start and end time. The following constraint ensures this:

$$\sum_{t \in T_k} w_{it} = 1 \quad \forall i \in I_k, \quad \forall k \in K, \tag{17}$$

$$\sum_{t \in T_k} v_{it} = 1 \quad \forall i \in I_k, \quad \forall k \in K. \tag{18}$$

Next, we have to link the start time s_i and end time e_i to the binary variables in equations (17) and (18). This link is given by:

$$s_i = \sum_{t \in T_k} t \cdot w_{it} \quad \forall i \in I_k, \quad \forall k \in K, \tag{19}$$

$$e_i = \sum_{t \in T_k} t \cdot v_{it} \quad \forall i \in I_k, \quad \forall k \in K. \tag{20}$$

With this, the following functions describe the expected numbers of required beds at time $t \in T_c$ at the holding and recovery department respectively:

$$\gamma_t^1 = \sum_{k \in K} \sum_{i \in I_k} \left(\sum_{\hat{t}=t}^{t+M_i} p^i_{\hat{t}-t} w_{i\hat{t}} \right) \quad \forall t \in T_h, \tag{21}$$

$$\gamma_t^2 = \sum_{k \in K} \sum_{i \in I_k} \left(\sum_{\hat{t}=t-N_i}^{t} l^i_{t-\hat{t}} v_{i\hat{t}} \right) \quad \forall t \in T_r. \tag{22}$$

The equations in (21) and (22) are an approximation of the existing equations given in (6) and (12). Therefore, we do not calculate the number of required beds, but the number of expected beds.

We want to minimise the maximum number of required beds during the day at the holding and recovery department. The ILP has two objective functions, i.e., one objective function to minimise the number of beds at the holding department and the other objective function to minimise the number of beds at the recovery department. Taking one γ that represents the maximum number of required beds at both departments, does not guarantee that both departments will reach their goal. For this reason, we first denote two variables, γ^1 and γ^2, given by:

$$\begin{aligned} \gamma^1 &\geq \gamma_t^1, \quad \forall t \in T_h, \\ \gamma^2 &\geq \gamma_t^2, \quad \forall t \in T_r. \end{aligned} \tag{23}$$

With the equations in (23), we use a weighted sum method, which is defined as followed:

$$F = q_1 \gamma^1 + q_2 \gamma^2, \tag{24}$$

where the following holds:

$$q_1 + q_2 = 1, \quad q_1, q_2 > 0. \tag{25}$$

The ILP formulation for the number of required beds at the holding and recovery department becomes:

$$\begin{aligned} &\min F \\ &\text{s.t. } (13) - (24) \\ &Y_{ij}, w_{it}, v_{it} \in \{0, 1\}, \quad \forall i, j \in I_k, \quad \forall k \in K, \quad \forall t \in T_c, \\ &e_i, s_i \in \mathbb{R}. \end{aligned} \tag{26}$$

In Chapter 6, the results of this ILP are presented. In the following chapter, we introduce solution methods, which are used to spread the start and end time of the surgeries. Spreading the start and end times with equal length can influence and can help to level the number of required beds.

5.2 Constructive heuristics

In this chapter, we discuss two solution methods. These methods involve the spreading of Break-In-Moments. As discussed before, a Break-In-Moment is the moment when a surgery starts or ends. In a paper written by Van Essen et al. [8], they determined a couple of solution methods in spreading the start and end times of surgeries over the day. The method uses the interval between two consecutive BIMs and makes sure this interval is equal over the day. Such an interval is called a Break-In-Interval (BII). Spreading the BIMs equally over the day equals spreading the start and end times over the day. The spreading of BIMs does not guarantee the numbers of required beds being levelled, but it may help. There has not been a method presented before, in which the BIMs are spread over the day combined with the LOS. In this thesis, we only use the methods from Van Essen et al. [8], to see if the result helps levelling the number of required beds over the day.

5.2.1 Fixed Goal Values

The 'Fixed Goal Values' is an algorithm that uses a given goal λ to approximate the new BII. This λ is the length of each BII if the start and end times over all the ORs are equally distributed, i.e.,

$$\lambda = \frac{e - s}{1 + W - R}. \tag{27}$$

Here, $e - s$ equals the length of the occupied interval, e the end time of the occupied interval, s the start time of the occupied interval, W the number of surgeries over the whole day and R the number of ORs used over the day.
The algorithm calculates in every step for each patient i in OR k, the end time e_i if it is placed in the schedule, i.e., $e_i = B_k + d_i$, with B_k the end time of the previous surgery in OR k. In the first step, it will calculate the end time if the surgery starts at the beginning of the day, at $B_k = s_k$. Then, the algorithm finds the end time e_i of surgery i that approximates the start time of the occupied interval $s + \lambda$ the best. The start time of OR k will be updated: $B_k = e_i$. In the next steps the end times of each surgery i will be calculated again and the surgery i with the end time e_i that approximates $s + 2\lambda$ the best, will be placed next in the schedule. The entire algorithm is given below, where I is the set of patients.

Step 1. Determine λ, set $z = 1$, $I' = I$ and $B_k = s_k$.

Step 2. Determine for each $i \in I'$ the end time e_i if it is scheduled next, i.e., $e_i = B_{O(i)} + d_i$.

Step 3. Find i such that $e_i \leq |e_j - z\lambda| \quad \forall j \in I' \backslash \{i\}$, i.e., $e_i = \min_{j \in I'} |e_j - z\lambda|$.

Step 4. Determine the new start time $B_{O(i)}$ for OR $O(i)$, i.e., $B_{O(i)} = e_i$. Delete patient i from I', and set $z = z + 1$.

Step 5. If $I' = \emptyset$ then stop, else go to Step 2.

5.2.2 Flexible Goal Values

The Flexible Goal Values algorithm is similar to the Fixed Goal Values. The only difference is that the λ changes in every step. In each step, the occupied interval becomes smaller, so changing the λ in each step could resolve in a better schedule. The λ becomes:

$$\lambda = \frac{e - B}{1 + W - R - Z}. \tag{28}$$

Here, $e - B$ equals the length of the occupied interval, e the end time of the occupied interval, B the maximum end time over all ORs, W the number of surgeries over the whole day, R the number of ORs used over the day and Z the number of surgeries that are scheduled in the previous step. The variable Z will be updated every step. The entire algorithm is given below:

Step 1. Set $I' = I$, $Z = 1$ and $B_k = s_k$.

Step 2. Set $B = \max_{k \in K} B_k$ and determine λ.

Step 3. Determine for each $i \in I'$ the end time e_i if it is scheduled next, i.e., $e_i = B_{O(i)} + d_i$.

Step 4. Find i such that $e_i \leq |e_j - \lambda - Z| \quad \forall j \in I' \backslash \{i\}$, i.e., $e_i = \min_{j \in I'} |e_j - \lambda - Z|$.

Step 5. Determine the new start time $B_{O(i)}$ for OR $O(i)$, i.e., $B_{O(i)} = e_i$. Delete patient i from I', and set $Z = Z + 1$.

Step 6. If $I' = \emptyset$ then stop, else go to Step 2.

6 Results

In this chapter, we discuss the results from the solution methods explained in Chapter 5. We have developed two different types of solution methods. The first method considers an ILP, which is solved using MATLAB and the function 'intlinprog'. The number of required beds are calulated for every 10 minutes. The second type of method consists of two methods and considers the spreading of BIMs. Fixed Goal Values uses a fixed λ to spread the start and end time of the surgeries between the interval in which all the ORs are occupied. The Flexible Goal Value uses an λ which changes every step, because the length of the occupied interval of the ORs change.

First, we discuss the data set provided by an Academical Medical Centre in the Netherlands. Then, we discuss the results.

6.1 Data

The historical data was provided by the medical centre and the data set contained around 13750 surgeries over a period of months, from January 1st, 2015 until March 31th, 2016. We used the data to calculate the number of required beds of the original data and for the new obtained solutions. The data is also used to determine the LOS distributions at the holding and recovery department and the surgery duration.

To determine the number of required beds, we needed the amount of time a patient spends at the holding, OR and at the recovery. Unfortunately, the data was incomplete, i.e., some surgeries did not have an end or start time. For the calculation of the LOS distributions at the holding, we calculated the differences between the time of the patients arrive at the holding and the time the surgery starts. For the recovery department, we calculated the difference between the end time of the surgery and the time the patient leaves the recovery. If these differences could not be calculated, due to incompleteness of the data, we set this difference to zero and did not use if for further calculations.
To alter the given schedule, we needed the time patients spend in the OR. We calculated this by subtracting the end time from the start time of the surgery for each surgery type. Some of the data was incomplete, so we set this difference to zero and did not use this for further calculations.
We also had the 'Snijtijd', which is the time from the first incision to closure. Whenever a surgery ended before the start time of the surgery, we used the Snijtijd to determine the duration of the surgery.

The data also showed an operating room dedicated for emergency surgeries. Emergency surgeries are not planned before hand, so we did not use this operating room for further calculations. A couple of emergency surgeries were scheduled after a surgery ended in an OR different from the emergency OR. In these ORs, the surgeries were performed back-to-back. We considered these surgeries as normal surgeries and used this in further calculations.

The data was used to obtain the surgery duration, the LOS distribution at the holding and recovery department and was used to obtain solutions for the ILP and the algorithms described in Chapter 5. When calculating the LOS distribution, there was a possibility that there were no LOS distributions for some surgery types. This was due to the possibility that there are no other surgeries performed with the same surgery type or that for these surgeries, the data was incomplete. This happened a couple of times. From historical data, we assumed that these patient stay at least 10 minutes at both department. Therefore, we gave these surgeries a LOS of 10 minutes.

To determine a new schedule using the ILP and the algorithms, the start time of ORs were needed. Also, this data was sometimes incomplete. When this was the case, we set the start time at 8 o'clock. Most of the ORs opened at 8 o'clock, so this time seemed reasonable.

With this data, we calculated the number of required beds at the holding and recovery data for a certain time period. During this time period, all the elective surgeries were scheduled on weekdays. In the weekends only emergency surgeries were performed, which were not scheduled before hand. On a weekday there are 18 available ORs in which surgeries can be performed. This includes the emergency OR, in which emergency surgeries are performed. On such a day around 37 surgeries are scheduled. With this information and this chosen time period, the results are presented in the next section.

6.2 Results

In this section, we discuss the results given by solving the ILP and solving the two algorithms. Firstly, we discuss the results from solving the ILP. We do this by comparing a F value, which we explain in the following parts. Next, we compare the number of required beds for both departments with the original schedule. Then, we compare the solutions given by the time limit set to 10 minutes with the ones with time limit set to 30 minutes. This is done for two types of ILP solutions, equal weight and performance based weight. The same is done for the solutions given by the algorithms. At last, we compare the solutions given by the ILP and the algorithms.

6.2.1 ILP

The ILP described in Chapter 5 was implemented in MATLAB and the function 'intlinprog' was used to obtain the solutions. In Chapter 5, we described an ILP, in which the function $F = q_1\gamma^1 + q_2\gamma^2$ is minimised. To solve the problem, we need to determine q_1 and q_2. If $q_1 = 1$ and $q_2 = 0$, the objective function F minimises the number of required beds at the holding department. If $q_1 = 0$ and $q_2 = 1$ the number of required beds at the recovery department is minimised. It is easy to use $q_1 = 0.5$ and $q_2 = 0.5$. Unfortunately, this means that we put equal weight on both departments, while we know that the recovery department has a higher number of required beds than the holding department. For this reason, we choose two types of weights. For the first type, we choose equal weight, i.e., $q_1 = 0.5$ and $q_2 = 0.5$. For the second type, we first determine the number of required beds at the holding department with $q_1 = 1$ and $q_2 = 0$ and for the recovery department with $q_1 = 0$ and $q_2 = 1$. With these results, we determine right values for the q's. We call this, performance based weight.

6.2.1.1 Equal weight

As described before, we use $q_1 = 0.5$ and $q_2 = 0.5$. This makes sure that the number of required beds at both departments gets reduced equally. We know that this is not fair, because of the difference in the number of required beds at the departments.
With the chosen q's, the ILP was solved for 5 months, between the 5th of January and the 5th of June 2015, without the weekends (103 days). In test runs, we noticed that the solver took a lot of time to provide a solution. The problem we are solving is NP-hard. This means that it could take hours, days or even weeks, before the optimal solution is found. For this reason, we decided to interrupt the solver after 10 minutes. Because not all the found solutions were optimal, we checked whether the found solutions had a optimal solution within 30 minutes. We discuss both time limits.

When determining the number of required beds, we observed that the number of required beds at the holding department is very high around 8 o'clock. This is due to the opening times of the ORs. Almost every OR opens around 8 o'clock and every patient whose surgery starts at 8 is at the holding department before 8. The patient whose surgery starts after the first surgeries ends, could also be at the holding department, so the number of required beds around 8 is high. For this reason, we determined the number of required beds after 8 o'clock. The peak around 8 o'clock ended around 8.30 am, so for this reason, we choose to determine the number of required beds after 8.30 am. Therefore, equation (23) becomes:

$$\begin{aligned} \gamma^1 &\geq \gamma^1_t, \quad \forall t > 8.30 \text{ am}, \\ \gamma^2 &\geq \gamma^2_t, \quad \forall t \in T. \end{aligned} \tag{29}$$

For the 103 days, the solver provided 28 instances with a solution within the 10 minutes. The relative gaps were between the 0 and the 40.7 %. For two days, we had an optimal solution.

As discussed before, we want to minimise the number of required beds for each day. We compared the ILP solutions with the original schedule to see whether the obtained solutions are better than the original schedules. To compare the solutions with the original schedules, we calculated the resulting objective function F by using the number of required beds. The value of F is calculated as follows:

$$\begin{aligned} F = &q_1 \cdot \{\text{number of required beds at the holding department}\} \\ &+ q_2 \cdot \{\text{number of required beds at the recovery department}\} \end{aligned} \tag{30}$$

An example: the number of required beds for an obtained solution at the holding and recovery department are 9 and 5 respectively. For the original schedule, the number of required beds at the holding and recovery departments are 6 and 11 respectively. The values of F are given by

$$\begin{aligned} F_{\text{Solution ILP}} &= 0.5 \cdot 9 + 0.5 \cdot 5 = 7 \\ F_{\text{Original}} &= 0.5 \cdot 6 + 0.5 \cdot 11 = 8.5 \end{aligned} \tag{31}$$

In this example, we observed that the obtained solution from the ILP is better than the original schedule provided.

Another way to compare the ILP solutions with the original schedule is to compare the number of required beds at each department. All the results are shown in the following.

Comparison based on objective function

We calculated the value F for every obtained solution and the corresponding original schedules. The results are presented in Tables 2 and 3. Table 2 shows the number of times one schedule has a lower F value than the other. Table 3 shows the number of times the F value between the two schedules are equal. ILP EW means the ILP solutions with equal weight.

$<$	ILP EW	Original
ILP EW		12
Original	12	

(a) Time limit: 10 minutes

$<$	ILP EW	Original
ILP EW		14
Original	10	

(b) Time limit: 30 minutes

Table 2: Comparison F value ILP EW solutions and the original schedule

		Number of times equal
Original	ILP EW 10 min	4
Original	ILP EW 30 min	4

Table 3: Equal F value for the ILP solution and the original schedule

In Table 2a, we compared the original schedule with the ILP EW when the time limit was set to 10 minutes. The number 12 in the right upper corner is the number of times the ILP EW has a lower F value than the original schedule provides. The original schedule is also 12 times better than the ILP EW. With this, we conclude that ILP EW performs as good as the original schedule.
Table 2b has a different outcome. We compared the original schedule with the results of the ILP EW when the time limit was set to 30 minutes. This table shows that the ILP EW performs better than the original schedule, which is shown in the number 14.
When comparing Tables 2a and 2b, we observe that the ILP EW has at least two solution that gets better when comparing them with the original schedule. This is shown in the change from the number 12 in the left under corner of Table 2a to the number 10 in Table 2b and the change of the 12 in the right upper corner in Table 2a to the number 14 in Table 2b.

We also compared the number of required beds at the holding and recovery department with the ILP solutions and the original schedule. The results are shown in Tables 4 to 9.

Holding

Table 4a shows that the original schedule and the ILP EW perform equally as good. This follows from the the fact that the number 9 and 10 in Table 4a are close together. When the time limit is set to 30 minutes, we observe that there is a small change in the number of required beds. The number of times the original schedule has less beds at the holding department than the ILP EW went from 10 in Table 4a to 8 in 4b. The difference between these numbers is two. In Table 5, the number of times both schedules are equal increases from 9 to 11 when the time limit changes from 10 to 30 minutes. With this, we observe that the number of required beds for both schedules is more often equal. This is shown in the number 11 in Table 5 and the number 8 and 9 in 4b. With this, we conclude that both schedules perform equally as good.

<	ILP EW	Original
ILP EW		9
Original	10	

(a) Time limit: 10 minutes

<	ILP EW	Original
ILP EW		9
Original	8	

(b) Time limit: 30 minutes

Table 4: Comparison number of required beds ILP EW solutions and the original schedule: holding

		Number of times equal
Original Schedule	ILP EW 10 min	9
Original Schedule	ILP EW 30 min	11

Table 5: Equal number of required beds for the ILP EW solution and the original schedule: holding

Type Schedule	Average	95% interval	[min,max]
ILP EW 10 min	5.46	[5.00, 5.92]	[3, 8]
ILP EW 30 min	5.36	[4.92, 5.79]	[3, 8]
Original Schedule	5.43	[5.09, 5.76]	[3, 7]

Table 6: Number of required beds at the holding department after 8.30 am

Table 6 shows that the the averages are close together, but the ILP with EW 30 minutes has the lowest average.

Recovery

The same tables are made for the recovery department, which is shown in the tables 7 to 9.

<	ILP EW	Original
ILP EW		14
Original	11	

(a) Time limit: 10 minutes

<	ILP EW	Original
ILP EW		14
Original	10	

(b) Time limit: 30 minutes

Table 7: Comparison number of required beds ILP EW solutions and the original schedule: recovery

		Number of times equal
Original Schedule	ILP EW 10 min	3
Original Schedule	ILP EW 30 min	4

Table 8: Equal number of required beds for the ILP solution and the original schedule: recovery

In Table 7a, the ILP EW is more often lower than the original schedule, which is shown in the number 14 compared with 11. When the time limit is set to 30 minutes, the ILP EW is again more often lower. We also observe that the ILP EW gets lower for at least one solution, which is shown in the change from the number 11 in Table 7a to the number 10 in Table 7b. In Table 8, we observe that the number of times the ILP EW and the original schedule have a equal number of required beds increases. Which tells us that the number of required beds at the recovery department gets at least for one solution reduced.

Type Schedule	Average	95% interval	[min,max]
ILP EW 10 min	11.2	[10.46,11.89]	[8,15]
ILP EW 30 min	11.1	[10.3,11.8]	[8,15]
Original Schedule	11.2	[10.7, 11.7]	[9,14]

Table 9: Number of required beds at the recovery department

In Table 9, the averages are quite the same, but the ILP EW when the time limit is set to 30 minutes has the lowest average.

Comparison time limit 10 minutes and 30 minutes

In this section, we discuss the results and the differences between the solutions with time limit 10 minutes and solutions with time limit 30 minutes. We made the same type of tables as before, in which we compare the number of times one schedule is lower than the other schedule. In this case the schedules are the obtained ones with time limit 10 minutes and the other ones are the obtained schedules with time limit 30 minutes.

$<$	ILP EW 10 min	ILP EW 30 min
ILP EW 10 min		0
ILP EW 30 min	3	

Table 10: Comparison F value ILP EW with time limit set to 10 minutes and 30 minutes

In this table, we observe that the ILP only gets better. When we checked the number of required beds at these days, we saw that only for 3 days, the number of required beds changed when compared with the solutions with the 30 minutes time limit. The number of required beds at both departments for original schedule, the solutions for the 10 minutes and 30 minutes time limit are shown in Table 11.

Day	Original		10 min		30 min	
	H	R	H	R	H	R
1	6	12	7	12	6	12
2	6	12	5	13	5	12
3	5	11	7	10	5	8

Table 11: Comparison number of required beds ILP EW with time limit set to 10 minutes and 30 minutes

6.2.1.2 Performance based weight

The number of required beds at the holding and recovery department are different. This is because of the LOS at the holding and the recovery department. We know that the LOS at the recovery department is longer than at the holding department, which results in more beds at the recovery department. Therefore, it seems fair to put more weight on q_1. We determine these q's by finding the number of required beds when one q is equal to one.

In the first run, we put $q_1 = 1$ and for the second run $q_2 = 1$. We determined the number of required beds for the instances which had a solution. The solver had a time limit of 10 minutes. For some instances, the solver did not provide a solution yet.

Because the patients stay longer at the recovery department, the number of required beds at the recovery department is higher than at the holding department. For this reason, we want to put more weight at the holding department, such that reducing a bed at the holding department is equal to reducing a bed at the recovery department. For each day, we calculate q_1 and q_2 as follows:

$$q_1 = \frac{\text{the required number of beds at the recovery department}}{\text{sum of the required numbers at the holding and recovery department}} \tag{32}$$

$$q_2 = \frac{\text{the required number of beds at the holding department}}{\text{sum of the required numbers at the holding and recovery department}} \tag{33}$$

Day	Holding	Recovery	q_1	q_2
1	5	10	$^{10}/_{15}$	$^{5}/_{15}$
2	5	11	$^{11}/_{16}$	$^{5}/_{16}$
3	6	13	$^{13}/_{18}$	$^{6}/_{18}$
4	5	16	$^{16}/_{21}$	$^{5}/_{21}$
5	7	8	$^{8}/_{15}$	$^{7}/_{15}$

Table 12: Required number of beds to determine q_1 and q_2

We need one q_1 and one q_2. A small example of the q's are shown in Table 12 and is used to explain how the right values for the q's are determined. We observe that the number of required beds are quite similar. To determine the q's for all the data, we took an average of the q's in Table 12. First, we took the sum of the obtained q's and added them up. This was equal to: $3.6216 + 1.6839 = 5.3055$. Then, we divided the sum of all the q's by 5.3055. The q's then become $q_1 = 0.6826$ and $q_2 = 0.3174$. The q's add up to one as was needed.
For our data, we had 30 days in which the number of required beds at both department was obtained. The obtained q's are

$$q_1 = 0.6767 \tag{34}$$

$$q_2 = 0.3233 \tag{35}$$

The obtained solutions were used to calculate the number of required beds at each department. Because the solver was still trying to reach the optimal solution for the ILP, there is a gap between the obtained solution and the optimal solution, i.e., the relative gap.

For the 103 days, only 30 days had a solution in under 10 minutes with relative gaps between the 0% and 38.63 %. Only two solutions were optimal.

The solutions were compared with the original schedule in the same way as before, where we compared the values of F and the number of required beds at the holding and recovery department. The results are presented in Tables 13 & 14. In these tables, ILP PBW means ILP with performance based weight.

Comparison based on objective value

In Table 13, we observe that the ILP performs better than the original schedule, which is seen in the number 17. The number 17 means the number of times the value F provided by the ILP solution was lower than the original schedule. We also observe that when the time limit is set to 30 minutes that the Tables 13a and 13b have no differences.

$<$	ILP PBW	Original
ILP PBW		17
Original	10	

(a) Time limit: 10 minutes

$<$	ILP PBW	Original
ILP PBW		17
Original	10	

(b) Time limit: 30 minutes

Table 13: Comparison F value ILP PBW solutions and the original schedule

		Number of times equal
Original Schedule	ILP PBW 10 min	3
Original Schedule	ILP PBW 30 min	3

Table 14: Equal F values for the ILP solution and the original schedule

The ILP is written in such a way that the value F is minimised. This value F together with the q's represents the number of required beds. Because we did not try to create a schedule in which the number of required beds is lower than the original schedule, it is possible that the number of beds at one of the departments is higher, while the solution of the ILP is optimal and the value of F is lower. We also compared the number of required beds which are shown in Tables 15 to 20.

Holding

$<$	ILP PBW	Original Schedule
ILP PBW		16
Original Schedule	5	

(a) Time limit: 10 minutes

$<$	ILP PBW	Original
ILP PBW		16
Original	6	

(b) Time limit: 30 minutes

Table 15: Comparison number of required beds ILP EW solutions and the original schedule: holding

		Number of times equal
Original Schedule	ILP PBW 10 min	9
Original Schedule	ILP PBW 30 min	8

Table 16: Equal number of required beds for the ILP solution and the original schedule: holding

Type Schedule	Average	95% interval	[min,max]
ILP PBW 10 min	5.10	[4.73, 5.44]	[3, 7]
ILP PBW 30 min	5.10	[4.74, 5.46]	[3, 7]
Original Schedule	5.53	[5.17, 5.90]	[3, 8]

Table 17: Number of required beds at the holding department after 8.30 am

In the tables 15a and 15b, we observe that the ILP with performance based weight performs better than the original schedule, which is shown in the numbers 16. But there is a change when comparing those two tables. We observe that one solution gets worse, which is shown in the number 5 in Table 15a and the number 6 in Table 13b. We see that the original schedule gets better. This is possible, because of the objective function for the ILP. We discuss this in the part 'comparison time limit 10 minutes and 30 minutes PBW'.

In Table 17, both ILPs have a lower average than the original schedule.

Recovery

Here, we observe that the original schedule is better, which is shown in Table 18 and in Table 20. In these tables, the original schedule is more often lower than the ILP and the average of the original schedule is lower. This is shown in the number 13 in 18a and the number 11 in 18b. With this, we conclude that the original schedule provides a better schedule to reduce the number of beds at the recovery department. We also observe that when the time limit is set to 30 min, the average number of required beds get lower. So it seems like, when the time limit is set higher, the number of required beds at the recovery gets lower. It is possible that when the time limit is set to a higher limit, the number of required beds get lower, which could gets lower than the original schedule.

For the holding department, the schedule obtained by the ILP is better. Hereby, we can not conclude which provided schedule is better when comparing the number of required beds. But, when comparing the values of F, we can conclude that the ILP solution is better than the original schedule.

$<$	ILP PBW	Original
ILP PBW		7
Original	13	

(a) Time limit: 10 minutes

$<$	ILP PBW	Original
ILP PBW		7
Original	11	

(b) Time limit: 30 minutes

Table 18: Comparison number of required beds ILP PBW solutions and the original schedule: recovery

		Number of times equal
Original Schedule	ILP PBW 10 min	10
Original Schedule	ILP PBW 30 min	12

Table 19: Equal number of required beds for the ILP solution and the original schedule: Recovery

Type Schedule	Average	95% interval	[min,max]
ILP PBW 10 min	11.3	[10.9,11.8]	[8,13]
ILP PBW 30 min	11.2	[10.7,11.7]	[9,13]
Original Schedule	11.1	[10.6, 11.6]	[8,14]

Table 20: Number of required beds at the recovery department

Comparison time limit 10 minutes and 30 minutes

In this section, we discuss the results and the differences between the solutions with time limit 10 minutes and solutions with time limit 30 minutes. We made the same type of tables as before, in which we compare the number of times one schedule is lower than the other schedule. In this case the schedules are the obtained ones with time limit 10 minutes and the other ones are the obtained schedules with time limit 30 minutes.

$<$	ILP PWB 10 min	ILP PBW 30 min
ILP PBW 10 min		2
ILP PBW 30 min	3	

Table 21: Comparison 10 minutes with 30 minutes: PBW

In Table 21, we observe that the ILP gets worse for 2 days and gets better for 3 days. When we checked what happened at both departments, we observed that for one day the number of required beds at the recovery department gets reduced with one bed and the number of required beds at the holding departments gets increased with one bed. The F value got worse. This is due to the differences in objective function. In Chapter 3, we described an ILP which minimises the following objective function:

$$F = q_1\gamma^1 + q_2\gamma^2. \tag{36}$$

The F used in equation (36) is not the same as the F described in equation (30) in which we used the number of required beds. The solver minimises the F in (36) making use of the γ's which are in fact the sum of probabilities, given in equations (21) and (22). That's why it is possible that the ILP gets a better, i.e., F value in equation (36) got reduced, but the F value in equation (30) got increased.

The number of required beds at both departments for the 5 days are shown in the Table 22.

Day	Original		10 min		30 min	
	H	R	H	R	H	R
1	6	13	5	12	5	10
2	6	11	6	13	7	12
3	4	10	6	11	6	10
4	4	10	3	8	3	9
5	6	10	5	11	4	10

Table 22: Comparison 10 minutes and the 30 minutes time limit

6.2.1.3 Comparison equal weight and performance based weight

We solved the ILP for two types of q's. The first one is the equal weight and the second one is the performance based weight. The question now is, which one performs better. To determine which one performs better, we compared the number of required beds at both departments for each ILP type.
The two types each had a different set of solution days. There were only 19 days left to compare. We only compare the number of beds with the time limit set to 30 minutes.

We begin comparing the number of required beds at the holding department. The results are presented in Tables 23 and 24.

$<$	ILP PBW	ILP EW
ILP PBW		7
ILP EW	6	

Table 23: Comparison ILP solutions and original schedule: holding

		Number of times equal
ILP PBW	ILP EW	6

Table 24: Equal number of required beds for the ILP solutions: holding

In Table 23, we observe that the ILP solution with EW and with PBW performs just as good. This is also seen in the Table 25, where we observe that the average, 95% interval and the minimum and maximum of both types are close together.

Type Schedule	Average	95% interval	[min,max]
ILP PBW	4.95	[4.50,5.40]	[3, 7]
ILP EW	5	[4.47, 5.53]	[3, 8]
Original Schedule	5.42	[5.00, 5.84]	[3, 7]

Table 25: Number of required beds at the holding department after 8.30 am

Next, we compared the number of required beds at the recovery department. The results are shown in Table 27, 26 and 28.

<	ILP PBW	ILP EW
ILP PBW		4
ILP EW	9	

Table 26: Comparison ILP solutions and original schedule: recovery

		Number of times equal
ILP PBW	ILP EW	6

Table 27: Equal number of required beds for the ILP solutions: recovery

Type Schedule	Average	95% interval	[min,max]
ILP PBW	11.1	[10.5,11.7]	[9,13]
ILP EW	10.7	[9.94, 11.5]	[8, 15]
Original Schedule	11.4	[10.7, 12.0]	[9, 14]

Table 28: Number of required beds at the recovery

We observe that the ILP with equal weight performs better, which is shown in the number 9 in Table 26. This was expected because the number of required beds at the recovery department is high. The ILP with equal weight made sure that the number of required beds at the recovery department was reduced more, than with the performance based weight. Therefore, the ILP with equal weight has a lower outcome than the performance based weight.

Overall, we observe that the number of required beds at the holding department is quite similar between both ILP types. At the recovery department, the ILP with equal weight performs better. This was expected, due to the chosen q_2 and the high number of required beds at the recovery department. Although, it seems that the ILP with equal weight performs slightly better, we do not use the ILP with equal weight for further comparisons. All due to the fact that the number of required beds at the department are very different. Therefore, it is better to use the ILP with performance based weight. In the next sections, when we discuss 'ILP', we mean the ILP with performance based weight.

6.2.2 Constructive heuristics

The Fixed Goal Values and Flexible Goal Values were also implemented in MATLAB and were used to alter the schedule for the same time period as for the ILP. For each of these days, the algorithms determined a new schedule and for the resulting schedule, we calculated the number of required beds at the holding and the recovery department. We compared the original schedule with the results from the algorithms.
In Chapter 5, we explained that we wanted to minimise the number of required beds per day. The algorithms spreads the start and end time over the whole day. It does not ensure the desired result, which is to minimise the number of required beds, but it may help.

Because of this, we compared the number of required beds at the holding and recovery department for each algorithm with the original schedule and with each other. First, we investigate the holding department.

For the first row in Table 29, 5 is the number of times the number of required beds resulting from the Fixed Goal Values was lower than the results of Flexible Goal Values. In Table 30, the number of times the results of the algorithms were equal to each other or the original schedule is presented.

<	Fixed Goal Values	Flexible Goal Values	Original Schedule
Fixed Goal Values		5	24
Flexible Goal Values	8		22
Original Schedule	25	22	

Table 29: Comparison algorithms with original schedule at the holding department

		Number of times equal
Original Schedule	Fixed Goal Values	54
Original Schedule	Flexible Goal Values	59
Fixed Goal Values	Flexible Goal Values	90

Table 30: Equal number of required beds for each algorithm and the original schedule

The mean of the number of required beds for each algorithm is given in Table 31, together with a 95 % interval and the minimum and maximum number of required days over all considered days.

Type Schedule	Average	95% interval	[min,max]
Fixed Goal Values	12.1	[11.8,12.4]	[9,17]
Flexible Goal Values	12.1	[11.8, 12.4]	[9,17]
Original Schedule	12.1	[11.9, 12.4]	[9,16]

Table 31: Number of required beds at the holding department over all considered days

Table 29 shows that the original schedule, the Fixed Goal Values and the Flexible Goal Values performs equally as good. This is also shown in Table 17, in which the average, 95% interval and the minimum and maximum are quite similar.

Almost every OR opens around 8.00 am. As discussed before, the number of beds around 8 o'clock is very high. In Table 32, the average of the time when the number of required beds is at its maximum is shown. The holding department is at its busiest at 8 o'clock. This is around the time the first surgeries start. An example to minimise the number of required beds at the holding department is to change the opening times of the ORs, such that the peaks are more levelled.

We can not do anything about the peak at 8 o'clock for the holding department, so we try to minimise the number of beds after 8 o'clock. Therefore, the maximum number of beds at the holding department after 8.30 was also determined. This is shown in Tables 33, 34 and 35.

Department	Algorithm	Average Time
Holding	Fixed Goal Values	7.52
	Flexible Goal Values	7.52
	Original Schedule	7.52

Table 32: Average time when number of beds at the holding is at its maximum

<	Fixed Goal Values	Flexible Goal Values	Original Schedule
Fixed Goal Values		10	10
Flexible Goal Values	36		16
Original Schedule	70	67	

Table 33: Comparison algorithms with original schedule at the holding department after 8.30 am

		Number of times equal
Original Schedule	Fixed Goal Values	23
Original Schedule	Flexible Goal Values	20
Fixed Goal Values	Flexible Goal Values	57

Table 34: Equal number of required beds for each algorithm and the original schedule after 8.30 am

Type Schedule	Average	95% interval	[min, max]
Fixed Goal Values	7.2	[6.88,7.47]	[4,11]
Flexible Goal Values	6.89	[6.62,7.16]	[4,11]
Original Schedule	5.97	[5.76 6.18]	[3,9]

Table 35: Number of required beds at holding department over all considered days after 8.30 am

Looking at Table 33, we see that there is a huge difference in comparison with Table 29. These tables show that the original schedule is much better than the created schedules. This result is also shown in Table 35, in which we see that the average number of beds for the original schedule is lower than the algorithms. To see whether the original schedule is also better for the recovery department, the same tables are made. These are shown in Tables 36, 37 and 38.

<	Fixed Goal Values	Flexible Goal Values	Original Schedule
Fixed Goal Values		13	3
Flexible Goal Values	38		9
Original Schedule	88	81	

Table 36: Comparison algorithms with original schedule at the recovery department

Table 36 shows us that the original schedule is much better for the number of required beds at the recovery department. The Flexible Goal Values is slightly better than the Fixed Goal values. Flexible Goal Values provided solutions which was 38 times lower than the solutions provided by Fixed Goal Values provided.

		Number of times equal
Original Schedule	Fixed Goal Values	12
Original Schedule	Flexible Goal Values	13
Fixed Goal Values	Flexible Goal Values	52

Table 37: Number of required beds equal for the algorithms and the original schedule

Type Schedule	Average	95% interval	[min,max]
Fixed Goal Values	14.8	[14.3,15.3]	[9,23]
Flexible Goal Values	14.3	[13.8, 14.9]	[9,23]
Original Schedule	11.7	[11.4, 12.0]	[8, 16]

Table 38: Number of required beds at the recovery department over all considered days

Overall, we observe that the original schedule is better than the obtained schedules from the Fixed and Flexible Goal Values. These algorithms spread the start and end times of the surgeries. Because the results of the algorithms are worse than the original schedule, we conclude that the start and end time of the surgeries do not level the number of required beds.

6.2.3 Comparison algorithms based on objective function ILP

In this section, we compare the algorithms with the original schedule and with each other based on the objective function from the ILP. As discussed before, the objective function minimises the number of required beds at both departments. We calculate the value of F by using equation (30). These values were then compared and the results are shown in Tables 39 and 40.

<	Fixed Goal Values	Flexible Goal Values	Original
Fixed Goal Values		16	14
Flexible Goal Values	44		25
Original Schedule	71	60	

Table 39: Comparison algorithms with original schedule based on objective function ILP

		Number of times equal
Original Schedule	Fixed Goal Values	18
Original Schedule	Flexible Goal Values	18
Fixed Goal Values	Flexible Goal Values	43

Table 40: Number of required beds equal for the algorithms and the original schedule

In these tables, we again observe that the original schedule has a better outcome than the schedule provided by the algorithms. The original schedule is therefore better than the schedules obtained from the algorithms.

6.2.4 ILP compared with Fixed and Flexible Goal Values

In this section, we compare all the solutions, i.e., the solutions obtained from the ILP with the performance based weight and the algorithms. We compare them using the objective function in equation (30) given in Section 6.2.1.

<	Original Schedule	ILP Solution	Fixed Goal Values	Flexible Goal Values
Original Schedule		10	26	25
ILP Solution	17		29	28
Fixed Goal Values	3	1		7
Flexible Goal Values	4	2	7	

Table 41: Comparison obtained solutions and original schedule based on objective function

		Number of times equal
Original	ILP Solution	3
Original	Fixed Goal Values	1
Original	Flexible Goal Values	1
ILP Solution	Fixed Goal Values	0
ILP Solution	Flexible Goal Values	0
Fixed Goal Values	Flexible Goal Values	16

Table 42: Number of required beds equal for the original schedule, the algorithms and the ILP solution

Type Schedule	Average	95% interval	[min,max]
Original Schedule	7.33	[6.98, 7.69]	[4.94, 8.97]
ILP	7.07	[6.74, 7.41]	[4.84, 8.62]
Fixed Goal Values	8.85	[8.35, 9.35]	[6.59, 12.3]
Flexible Goal Values	8.69	[8.25, 9.13]	[6.59, 11.9]

Table 43: Number of required beds at the recovery department over all considered days

In Table 41, we observe that the number of required beds compared with the original schedule is the most reduced by the solutions obtained from the ILP. This is given in the number 17 in the first column. For the obtained solutions, the ILP solution is also the best. This is provided by the number of times the ILP solution had a lower value of F. We see that the ILP solution is 29 times better than the Fixed Goal Values and 28 times better than the Flexible Goal Values. This result is also shown in Table 43. In this table, the ILP has the lowest average.

Overall, we observe the ILP provides a better schedule than the original schedule and the algorithms. This is provided by Tables 41 and 43.

The number of required beds is determined for three types of obtained schedules. The first schedules are the solutions from the ILP. The other solutions are the ones obtained from the algorithms, i.e., Fixed Goal Values and Flexible Goal Values. In these results, we observe that spreading the start or end time of these surgeries does not provide a better outcome when compared to the original schedule.

The ILP only provided 30 solution within the maximum time limit of 10 min. These 30 solutions were solved again with a time limit of 30 min. There were only two days in which the ILP solution was optimal. These two solutions had a lower number of required beds at both departments when compared to the original schedule.
From all the obtained solutions, 17 out 30 solutions were better than the original schedule. It is possible that the ILP provided more solutions, if the time limit was set higher.

7 Conclusion

In this thesis, we developed several solution methods to level the number of required beds at the holding and recovery department. These are the departments where a patient arrives right before the surgery starts and right after the surgery ends. Due to the nurses' workload and the decreasing number of available nurses, it is important to develop methods to level the number of required beds. The number of required beds at the holding and recovery department depend not only on what time the surgeries start or end, but also on the length of stay (LOS) of each patient.

Firstly, an exact calculation of the number of required beds was derived, which uses the LOS distributions per surgery type. We obtained solutions from an Integer Linear Program, in which the goal was to minimise the number of expected beds at both departments. The ILP formulation was implemented in MATLAB and was used to find solutions for 103 instances. These instances were provided by an Academic Medical Centre in the Netherlands. We developed two types of ILP solutions, i.e., ILP with equal weight and ILP with performance based weight.
From the 103 days, the ILP with equal weight only provided 28 solutions in the set time limit of 10 minutes. The 28 solutions were again solved with the time limit set to 30 minutes. From these 28, the objective function value F was calculated for the obtained solution and the original schedule. Within the 10 minutes time limit, the ILP and original schedule were equally as good. Within the 30 minutes time limit, the ILP with equal weight was better than the original schedule.
For the performance based weight, the solver provided 30 solutions in the set time limit of 10 minutes and were again solved with time limit set to 30 minutes. These solutions were compared with the original schedule. With the results, we concluded that the ILP solution with performance based weight was better than the original schedule.
We also compared both types of solutions with each other, in which we can not really conclude which one is better. The ILP with performance based weight uses the information that the recovery department has a higher number of required beds than the holding department. Therefore, we decided to use this ILP in further comparisons. We only compared the ILP solution with performance based weight with the other solutions and the original schedule. The name 'ILP' was used to refer to it.
Both types of solutions had two optimal solutions. For these days the value of F was indeed lower when compared with the original schedule. It is likely that if the time limit was set to a couple of hours, more solutions and more optimal solutions could have been found. The time issues of the ILP are caused by the problem itself. The problem is NP-hard and it could take weeks before the optimal solution is found. Nonetheless, the outcomes of the obtained solutions, did reduce the number of required beds at both departments.

The number of required beds at the holding and recovery not only depends on the length of stay of each patient, but also on the start and end times of the surgeries. Due to this, we investigated whether the spreading of Break-In-Moments was useful in levelling the number of required beds. Break-In-Moments are the moments when a surgery starts or ends when all the ORs are occupied. We used two algorithms which spread the start and end time of the surgeries over the day. The output of the algorithms Fixed Goal Values and Flexible Goal Values, were compared with the F values and the number of required beds for the original schedule. With this, we observed that the number of required beds are not levelled by spreading the start and end times.

The ILP did not provide a solution for many of the considered days. Although the obtained solutions reduced the number of required beds at the holding or recovery department, the number of solutions were too little to make an overall conclusion on the problem. The results of both algorithms showed that the spreading of the start and end time does not result in a better schedule.

Another method that can reduce the number of required beds at the holding and recovery department is to alter the ORs opening time. For the used data, the peak at the holding department was at its highest around 8.00 am. This is due to the start time of the first surgeries. Almost every OR starts at 8.00 am. The number of beds is at least the number of ORs opening at 8.00 am. Therefore, it may be useful to alter the start time in order to reduce the peak at 8.00.
With the provided ILP, we did not consider the maximum LOS. When scheduling surgeries, it could help to schedule surgeries with a long LOS at the recovery department at the beginning of the day such that the recovery department does not have to be open for a long time.

To obtain more solutions, the MATLAB code that calculated the solutions, could have been simplified. In the MATLAB code, the opening times for the ORs were set to a large time interval, to make sure a solution was found in between. This resulted in a larger problem with more variables. Unfortunately, due to the problem, which is NP-hard, and the opening times, it took a lot of time to compute a solution. For this reason, only a few optimal solutions were found. For further research, this could be simplified by changing the OR opening times. This should be changed to the opening time of the holding department and the end time of the surgery plus LOS of the surgery with the longest LOS.

In test runs for the ILP solver, we noticed that the solver obtains an optimal solution within seconds when the number of ORs is below 10. For this reason, it could also be useful to create schedules for a group of ORs. In these groups the number of required beds is reduced and at the end the groups are combined. The ILP solver solves the problem faster, which in return leads to more solutions than the ILP provided in this thesis.

We formulated an ILP which uses an approximation for the number of required beds. Therefore, the solution obtained from the ILP may not be better than the original schedule. Because we compared the solutions with the original schedules, it may lso be better to use this original schedule to obtain more solutions. Using this original schedule can make sure that the obtained solutions are better than the original. This can be done by using heuristics such as Simulated Annealing.

It is also possible to set up a maximum number of beds at the holding and recovery department in the written ILP, such that number of required beds obtained from the solutions is lower than the maximum given.

Overall, this thesis showed that the number of required beds at the holding and recovery department can be reduced. Although the ILP did not provide many solution, the provided solutions did reduce the number of required beds at the departments. This thesis also showed that the number of required beds at the holding and recovery department does not depend on the start and end times of the surgeries. With this, a hospital has the knowledge that the number of required beds can be reduced.

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