Theoretical Calculations on the Motions, Hull Surface Pressures and Transverse Strength of a Ship in Waves*

by

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1 Introduction

There is proposed a method to analyse the motions, hull surface pressures and transverse strength of a ship in regular waves. The method of such analysis has been applied for a gigantic oil-tanker in regular waves from different directions.

Supposing that a ship goes forward in regular waves with a constant speed and a constant heading angle, the ship motions can be solved by assuming the coupled equations of heaving and pitching motions and those of swaying, yawing and rolling motions based upon the modified strip theory[1,2]. The former coupled motions are treated as linear motions and the latter as non-linear motions by introducing the non-linear roll damping.

When the solutions of ship motions are obtained, the hydrodynamic pressures induced on the hull surface can be evaluated theoretically according to Tasai's method[3,4], and the cargo oil pressures induced by the ship motions can be estimated approximately by using the solutions of motions.

Since the loads acting on a transverse section of the ship can be calculated by such methods, the transverse strength calculation can be made for the case in regular waves. The more precise method such as the three-dimensional strength calculation will be available, but here the simple two-dimensional method is applied for the transverse strength calculation of a gigantic oil-tanker.

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2 Calculation Methods

2.1 Ship Motions

Consider the case when a ship goes forward with a constant velocity V in regular waves. As shown in Fig. 1, the co-ordinate system O-XYZ is employed such that the XY-plane coincides with the still water surface and the Z-axis indicates the downward direction perpendicular to the XY-plane. The other co-ordinate system O-X'Y'Z' is determined such that the X'-axis coincides with the average course of the ship oscillating among waves coming from the negative X direction to the positive X direction. The co-ordinate system o-xyz fixed to the ship is determined such that the origin o locates at the midship on the center line of water plane and the x-axis points out ahead the longitudinal direction. The ship goes forward with the average heading angle χ to the wave direction. Then the vertical displacement of the encountered wave surface will be written as follows.

$$h = h_o\cos(kX_1-\omega t) = h_o\cos(kx\cos\chi-ky\sin\chi-\omega_e t) \qquad (1)$$

where

h_0 : wave amplitude

$k=\omega^2/g=2\pi/\lambda$

λ : wave length, g : acceleration of gravity

ω : wave circular frequency

$\omega_e=\omega-kV\cos\chi$: circular frequency of wave encounter

As for the subsurface of deep sea waves, the vertical displacement can be written as

$$h(z) = h_o e^{-kz}\cos(kx\cos\chi-ky\sin\chi-\omega_e t) \qquad (2)$$

and the vertical orbital velocity and acceleration, as

$$v_z = \omega h_o e^{-kz}\sin(kx\cos\chi-ky\sin\chi-\omega_e t) \qquad (3)$$

$$\dot{v}_z = -\omega^2 h_o e^{-kz}\cos(kx\cos\chi-ky\sin\chi-\omega_e t) \qquad (4)$$

and the horizontal orbital velocity and acceleration in the transverse direction, as

$$v_y = \omega h_o\sin\, e^{-kz}\cos(kx\cos\chi-ky\sin\chi-\omega_e t) \qquad (5)$$

$$\dot{v}_y = \omega^2 h_o\sin\, e^{-kz}\sin(kx\cos\chi-ky\sin\chi-\omega_e t) \qquad (6)$$

When the ship goes forward in regular waves, the hydrodynamic forces induced on the ship cross section can be expressed as follows by taking into account the influences of heave (ζ), pitch (ϕ), sway (η), yaw (ψ) and roll (θ) based on the momentum theory.

a) vertical force (downward: positive)

$$\frac{dF_z}{dx} = \frac{dF_{Bz1}}{dx} + \frac{dF_{Bz2}}{dx} + \frac{dF_{Bz3}}{dx} + \frac{dF_{Bz4}}{dx} + \frac{dF_{Wz1}}{dx} + \frac{dF_{Wz2}}{dx} + \frac{dF_{Wz3}}{dx} + \frac{dF_{Wz4}}{dx} \qquad (7)$$

where

$$\frac{dF_{Bz1}}{dx} = -2\rho g y_w\{\zeta - (x-x_G)\phi\}$$

$$\frac{dF_{Bz2}}{dx} = -\rho N_z\{\dot{\zeta} - (x-x_G)\dot{\phi} + V\phi\}$$

$$\frac{dF_{Bz3}}{dx} = -\rho s_z\{\ddot{\zeta} - (x-x_G)\ddot{\phi} + 2V\dot{\phi}\}$$

$$\frac{dF_{Bz4}}{dx} = V\frac{d(\rho s_z)}{dx}\{\dot{\zeta} - (x-x_G)\dot{\phi} + V\phi\}$$

$$\frac{dF_{Wz1}}{dx} = 2\rho g y_w h_e$$

$$\frac{dF_{Wz2}}{dx} = \rho N_z v_{ze}$$

$$\frac{dF_{Wz3}}{dx} = \rho s_z \dot{v}_{ze}$$

$$\frac{dF_{Wz4}}{dx} = -V\frac{d(\rho s_z)}{dx} v_{ze}$$

ρ : density of sea water, g : acceleration of gravity
y_w : half breadth of water plane
x_G : x-coordinate of the center of gravity
ρN_z : sectional damping coefficient for vertical motion
ρs_z : sectional added mass for vertical motion
$h_e \fallingdotseq C_1C_2h = C_1C_2h_0\cos(kx\cos\chi - \omega_e t)$
$v_{ze} \fallingdotseq \omega h_0 C_1 C_2 \sin(kx\cos\chi - \omega_e t)$
$\dot{v}_{ze} \fallingdotseq -\omega^2 h_0 C_1 C_2 \cos(kx\cos\chi - \omega_e t)$
$C_1 = \sin(ky_w\sin\chi)/ky_w\sin\chi$
$C_2 = e^{-kd_m}$, $d_m = (\text{sectional area})/2y_w$

b) moment about the centre of gravity due to vertical force ($\widehat{zx}$-direction: positive)

$$\frac{dM_{zx}}{dx} = -\frac{dF_z}{dx}(x-x_G) \qquad (8)$$

c) horizontal force (starboard direction: positive)

$$\frac{dF_y}{dx} = \frac{dF_{By1}}{dx} + \frac{dF_{By2}}{dx} + \frac{dF_{By3}}{dx} + \frac{dF_{By4}}{dx} + \frac{dF_{Wy1}}{dx} + \frac{dF_{Wy2}}{dx} + \frac{dF_{Wy3}}{dx} + \frac{dF_{Wy4}}{dx} \qquad (9)$$

where

$$\frac{dF_{By1}}{dx} = 0$$

$$\frac{dF_{By2}}{dx} = -\rho N_y\{\dot{\eta} + (x-x_G)\dot{\psi} - V\psi + (z_G - l_w)\dot{\theta})\}$$

$$\frac{dF_{By3}}{dx} = -\rho s_y\{\ddot{\eta} + (x-x_G)\ddot{\psi} - 2V\dot{\psi} + (z_G - l_\eta)\ddot{\theta}\}$$

$$\frac{dF_{By4}}{dx} = V\frac{d(\rho s_y)}{dx}\{\dot{\eta} + (x-x_G)\dot{\psi} - V\psi + (z_G - l_\eta)\dot{\theta}\} - V\rho s_y\frac{dl_\eta}{dx}\dot{\theta}$$

$$\frac{dF_{Wy1}}{dx} = 2\rho g h_0\int_0^d e^{-kz_s}\sin(ky_s\sin\chi)dz_s\cdot\sin(kx\cos\chi - \omega_e t)$$

$$\frac{dF_{Wy2}}{dx} = \rho N_y v_{ye}$$

$$\frac{dF_{Wy3}}{dx} = \rho s_y \dot{v}_{ye}$$

$$\frac{dF_{Wy4}}{dx} = -V\frac{d(\rho s_y)}{dx} v_{ye}$$

ρN_y : sectional damping coefficient for horizontal motion
ρs_y : sectional added mass for horizontal motion
l_w : lever of damping force due to rolling motion with respect to o
l_η : lever of added mass inertia force due to rolling motion with respect to o
d : draught of the section
y_s : y-coordinate of the section contour
z_s : z-coordinate of the section contour
$v_{ye} \doteqdot \omega h_0 \sin \chi \cdot e^{-kd/2} \cos(kx \cos \chi - \omega_e t)$
$\dot{v}_{ye} \doteqdot \omega^2 h_0 \sin \chi e^{-kd/2} \sin(kx \cos \chi - \omega_e t)$

d) moment about the centre of gravity due to horizontal force ($\widehat{xy}$-direction: positive)

$$\frac{dM_{xy}}{dx} = \frac{dF_y}{dx}(x-x_G) \tag{10}$$

e) transverse rotating moment about the centre of gravity ($\widehat{yz}$-direction: positive)

$$\frac{dM_{yz}}{dx} = \frac{dM_{B\theta 1}}{dx} + \frac{dM_{B\theta 2}}{dx} + \frac{dM_{B\theta 3}}{dx} + \frac{dM_{B\theta 4}}{dx} + \frac{dM_{W\theta 1}}{dx} + \frac{dM_{W\theta 2}}{dx} + \frac{dM_{W\theta 3}}{dx} + \frac{dM_{W\theta 4}}{dx} \tag{11}$$

where

$$\frac{dM_{B\theta 1}}{dx} = -w(z_G' - z_G)\theta - \rho g s' m_t' \theta$$

w : sectional weight of the ship
z_G : z-coordinate of the centre of gravity of ship
z_G' : z-coordinate of the centre of gravity of w
s' : sectional area, m'_t : sectional metacentric radius

$$\frac{dM_{B\theta 2}}{dx} = -\rho N_y (z_G - l_w)\{\dot{\eta} + (x - x_G)\dot{\psi} - V\psi + z_G\dot{\theta}\} + \rho N_\eta l_w (z_G - l_w)\dot{\theta}$$

$$\frac{dM_{B\theta 3}}{dx} = -\rho s_y (z_G - l_\eta)\{\ddot{\eta} + (x - x_G)\ddot{\psi} - 2V\dot{\psi} + z_G\ddot{\theta}\} + \rho s_y l_\eta (z_G - l_\theta)\ddot{\theta}$$

$$\frac{dM_{B\theta 4}}{dx} = V\frac{d\{\rho s_y (z_G - l_\eta)\}}{dx}\{\dot{\eta} + (x - x_G)\dot{\psi} - V\psi + z_G\dot{\theta})\} - V\frac{d\{\rho s_y l_\eta (z_G - l_\theta)\}}{dx}\dot{\theta}$$

$$\frac{dM_{W\theta 1}}{dx} = \frac{dF_{Wy1}}{dx}(z_G - l_1)$$

$$\frac{dM_{W\theta 2}}{dx} = \frac{dF_{Wy2}}{dx}(z_G - l_w)$$

$$\frac{dM_{W\theta 3}}{dx} = \frac{dF_{Wy3}}{dx}(z_G - l_\eta)$$

$$\frac{dM_{W\theta 4}}{dx} = -V\frac{d\{\rho s_y (z_G - l_\eta)\}}{dx} v_{ye}$$

$l_\theta = \rho i / \rho s_y l_\eta$, ρi : added mass moment of inertia

$$l_1 = \frac{\int_0^d e^{-kz_s} \sin(k y_s \sin \chi) z_s dz_s - \int_0^{y_w} e^{-kz_s} \sin(k y_s \sin \chi) y_s dy_s}{\int_0^d e^{-kz_s} \sin(k y_s \sin \chi) dz_s}$$

The coupled equations of heaving and pitching motions and those of swaying, yawing and rolling motions can be obtained by putting as follows.

$$\left.\begin{aligned} \frac{W}{g}\ddot{\zeta} &= \int_L \frac{dF_z}{dx}dx \\ \frac{I_\phi}{g}\ddot{\phi} &= \int_L \frac{dM_{zx}}{dx}dx \end{aligned}\right\} \quad (12)$$

$$\left.\begin{aligned} \frac{W}{g}\ddot{\eta} &= \int_L \frac{dF_y}{dx}dx \\ \frac{I_\psi}{g}\ddot{\psi} &= \int_L \frac{dM_{xy}}{dx}dx \\ \frac{I_\theta}{g}\ddot{\theta} &= \int_L \frac{dM_{yz}}{dx}dx \end{aligned}\right\} \quad (13)$$

where the integrations should be carried out from the after end to the fore end along the water line length, and

W/g : mass of the ship
I_ϕ/g : moment of inertia of the ship for pitching motion
I_ψ/g : moment of inertia of the ship for yawing motion
I_θ/g : moment of inertia of the ship for rolling motion

Eqs. (12) and (13) will be written in the following forms.

$$\left.\begin{aligned} A_{11}\ddot{\zeta} + A_{12}\dot{\zeta} + A_{13}\zeta + A_{14}\ddot{\phi} + A_{15}\dot{\phi} + A_{16}\phi &= F_\zeta \\ A_{21}\ddot{\zeta} + A_{22}\dot{\zeta} + A_{23}\zeta + A_{24}\ddot{\phi} + A_{25}\dot{\phi} + A_{26}\phi &= M_\phi \end{aligned}\right\} \quad (14)$$

$$\left.\begin{aligned} a_{11}\ddot{\eta} + a_{12}\dot{\eta} + a_{13}\eta + a_{14}\ddot{\psi} + a_{15}\dot{\psi} + a_{16}\psi & \\ + a_{17}\ddot{\theta} + a_{18}\dot{\theta} + a_{19}\theta &= F_\eta \\ a_{21}\ddot{\eta} + a_{22}\dot{\eta} + a_{23}\eta + a_{24}\ddot{\psi} + a_{25}\dot{\psi} + a_{26}\psi & \\ + a_{27}\ddot{\theta} + a_{28}\dot{\theta} + a_{29}\theta &= M_\psi \\ a_{31}\ddot{\eta} + a_{32}\dot{\eta} + a_{33}\eta + a_{34}\ddot{\psi} + a_{35}\dot{\psi} + a_{36}\psi & \\ + a_{37}\ddot{\theta} + a_{38}\dot{\theta} + a_{39}\theta &= M_\theta \end{aligned}\right\} \quad (15)$$

A_{11}, A_{12}, --- - ; A_{21}, A_{22}, - - - ; F_ζ and M_ϕ in Eq. (14) can be determined by using the added mass and damping coefficent according to Tasai's method[5] and a_{11}, a_{12}, - - - ; a_{21}, a_{22}, - - - ; a_{31}, a_{32}, - - - ; F_η, M_ψ and M_θ in Eq. (15) by using those according to Tasai's method[6] or Tamura's one[7] (see Appendix). The roll damping coefficient a_{38} in Eq. (15), however, should be determined by taking into consideration the non-linear viscous damping in addition to the linear wave-making resistance. Here the non-linear roll damping is introduced by using "N-coefficent" according to Watanabe-Inoue's method[8] and the speed influence estimated from model experiments carried out by Takahashi[9] as follows.

$$a_{38} = 2\alpha_e a_{37} [1 + 0.8(1 - e^{-10Fr})(\omega_n/\omega_e)^2] \qquad (16)$$

where

$$2\alpha_e = \frac{2}{\pi}\omega_n\{a_1 + b_1(\omega_e/\omega_n)\theta_0\}$$

$$\omega_n = \sqrt{a_{39}/a_{37}}$$

θ_0 : rolling amplitude, F_r : Froude number

a_1 and b_1 are the coefficients in the formula of extinction curve for the free rolling in still water as follows.

$$\Delta\theta = a_1\theta_m + b_1\theta_m^2$$

On the other hand, $\Delta\theta$ will be approximately given by follows for large θ_m.

$$\Delta\theta = N\theta_m^2$$

For the cases of θ_m=10° and θ_m=20°, $N_{10°}$ and $N_{20°}$ can be estimated according to Watanabe-Inoue's methd. Then, a_1 and b_1 will be determined by the following equations.

$$\left.\begin{aligned} N_{10°} &= (a_1/10°) + b_1 \\ N_{20°} &= (a_1/20°) + b_1 \end{aligned}\right\}$$

Thus the non-linear roll damping is replaced by the equivalent linear damping coefficient a_{38} and Eqs. (15) should be solved by the iteration method.

The solutions of heave, pitch, sway, yaw and roll will be obtained as follows.

$$\left.\begin{aligned} \zeta &= \zeta_o\cos(\omega_e t-\varepsilon_\zeta) \\ \phi &= \phi_o\cos(\omega_e t-\varepsilon_\phi) \\ \eta &= \eta_o\cos(\omega_e t-\varepsilon_\eta) \\ \psi &= \psi_o\cos(\omega_e t-\varepsilon_\psi) \\ \theta &= \theta_o\cos(\omega_e t-\varepsilon_\theta) \end{aligned}\right\} \qquad (17)$$

2.2 Hydrodynamic Pressures

When the solutions of ship motions are known in the forms of (17), the hydrodynamic pressures induced on the hull surface can be evaluated theoretically according to Tasai's method[3,4] in the form of

$$P = P_o \cos(\omega_e t - \varepsilon_p) = P_c \cos\omega_e t + P_s \sin\omega_e t \qquad (18)$$

and this pressure will be expressed as follows.

$$P = P_V + P_H + P_R + P_W \qquad (19)$$

where

P_V : pressure due to vertical motion

P_H : pressure due to horizontal motion with respect to o

P_R : pressure due to rolling motion with respect to o

P_W : pressure due to regular wave

$$P_V = \rho g h_0 \{\bar{P}_{VC} \cos\omega_e t + \bar{P}_{VS} \sin\omega_e t\} \qquad (20)$$

$$P_H = \rho g h_0 \{\bar{P}_{HC} \cos\omega_e t + \bar{P}_{HS} \sin\omega_e t\} \qquad (21)$$

$$P_R = \rho g h_0 \{\bar{P}_{RC} \cos\omega_e t + \bar{P}_{RS} \sin\omega_e t\} \qquad (22)$$

$$P_W = \rho g h_0 \{\bar{P}_{WC} \cos\omega_e t + \bar{P}_{WS} \sin\omega_e t\} \qquad (23)$$

$$\left.\begin{matrix}\bar{P}_{VC}\\ \bar{P}_{VS}\end{matrix}\right\} = \frac{\zeta_0}{h_0}\left[(1+P''_{aH})\left\{\begin{matrix}\cos\varepsilon_\zeta\\ \sin\varepsilon_\zeta\end{matrix}\right\} - P''_{dH}\left\{\begin{matrix}\sin\varepsilon_\zeta\\ -\cos\varepsilon_\zeta\end{matrix}\right\}\right]$$
$$-(x-x_G)\frac{\phi_0}{h_0}\left[(1+P''_{aH})\left\{\begin{matrix}\cos\varepsilon_\phi\\ \sin\varepsilon_\phi\end{matrix}\right\} - P''_{dH}\left\{\begin{matrix}\sin\varepsilon_\phi\\ -\cos\varepsilon_\phi\end{matrix}\right\}\right]$$
$$-(V/\omega_e)\frac{\phi_0}{h_0}\left[2P''_{aH}\left\{\begin{matrix}\sin\varepsilon_\phi\\ -\cos\varepsilon_\phi\end{matrix}\right\} + P''_{dH}\left\{\begin{matrix}\cos\varepsilon_\phi\\ \sin\varepsilon_\phi\end{matrix}\right\}\right] \qquad (24)$$

$$\left.\begin{matrix}\bar{P}_{HC}\\ \bar{P}_{HS}\end{matrix}\right\} = \frac{\eta_0}{h_0}\left[P''_{as}\left\{\begin{matrix}\cos\varepsilon_\eta\\ \sin\varepsilon_\eta\end{matrix}\right\} - P''_{ds}\left\{\begin{matrix}\sin\varepsilon_\eta\\ -\cos\varepsilon_\eta\end{matrix}\right\}\right]$$
$$+(x-x_G)\frac{\psi_0}{h_0}\left[P''_{as}\left\{\begin{matrix}\cos\varepsilon_\psi\\ \sin\varepsilon_\psi\end{matrix}\right\} - P''_{ds}\left\{\begin{matrix}\sin\varepsilon_\psi\\ -\cos\varepsilon_\psi\end{matrix}\right\}\right]$$
$$+(V/\omega_e)\frac{\psi_0}{h_0}\left[2P''_{as}\left\{\begin{matrix}\sin\varepsilon_\psi\\ -\cos\varepsilon_\psi\end{matrix}\right\} + P''_{ds}\left\{\begin{matrix}\cos\varepsilon_\psi\\ \sin\varepsilon_\psi\end{matrix}\right\}\right]$$
$$+z_G\frac{\theta_0}{h_0}\left[P''_{as}\left\{\begin{matrix}\cos\varepsilon_\theta\\ \sin\varepsilon_\theta\end{matrix}\right\} - P''_{ds}\left\{\begin{matrix}\sin\varepsilon_\theta\\ -\cos\varepsilon_\theta\end{matrix}\right\}\right] \qquad (25)$$

$$\left.\begin{matrix}\bar{P}_{RC}\\ \bar{P}_{RS}\end{matrix}\right\} = y_s\frac{\theta_0}{h_0}\left\{\begin{matrix}\cos\varepsilon_\theta\\ \sin\varepsilon_\theta\end{matrix}\right\} + y_w\frac{\theta_0}{h_0}\left[P''_{aR}\left\{\begin{matrix}\cos\varepsilon_\theta\\ \sin\varepsilon_\theta\end{matrix}\right\} - P''_{dR}\left\{\begin{matrix}\sin\varepsilon_\theta\\ -\cos\varepsilon_\theta\end{matrix}\right\}\right] \qquad (26)$$

$$\bar{P}_{WC} = -e^{-kz_s}\cos(kx\cos\chi - ky_s\sin\chi)$$
$$-e^{-kz_s}\left(\frac{\omega}{\omega_e}\right)^2 P''_{aH}\cos(kx\cos\chi - ky_s\sin\chi)$$
$$+e^{-kz_s}\left(\frac{\omega}{\omega_e}\right) P''_{dH}\sin(kx\cos\chi - ky_s\sin\chi)$$
$$+e^{-kz_s}\sin\chi\left(\frac{\omega}{\omega_e}\right)^2 P''_{as}\sin(kx\cos\chi - ky_s\sin\chi)$$
$$+e^{-kz_s}\sin\chi\left(\frac{\omega}{\omega_e}\right) P''_{ds}\cos(kx\cos\chi - ky_s\sin\chi)$$

$$\bar{P}_{WS} = -e^{-kz_s}\sin(kx\cos\chi - ky_s\sin\chi)$$
$$-e^{-kz_s}\left(\frac{\omega}{\omega_e}\right)^2 P''_{aH}\sin(kx\cos\chi - ky_s\sin\chi)$$
$$-e^{-kz_s}\left(\frac{\omega}{\omega_e}\right) P''_{dH}\cos(kx\cos\chi - ky_s\sin\chi)$$
$$-e^{-kz_s}\sin\chi\left(\frac{\omega}{\omega_e}\right)^2 P''_{as}\cos(kx\cos\chi - ky_s\sin\chi)$$
$$+e^{-kz_s}\sin\chi\left(\frac{\omega}{\omega_e}\right) P''_{ds}\sin(kx\cos\chi - ky_s\sin\chi) \qquad (27)$$

Accordingly, P_c, P_s, P_o and ε_p can be calculated as follows.

$$\left.\begin{aligned} P_c &= \rho g h_o(\bar{P}_{Vc} + \bar{P}_{Hc} + \bar{P}_{Rc} + \bar{P}_{Wc}) \\ P_s &= \rho g h_o(\bar{P}_{Vs} + \bar{P}_{Hs} + \bar{P}_{Rs} + \bar{P}_{Ws}) \\ P_o &= (P_c^2 + P_s^2)^{1/2} \\ \varepsilon_p &= \tan^{-1}(P_s/P_c) \end{aligned}\right\} \qquad (28)$$

The calculaton methods of P''_{aH}, P''_{dH}, P''_{aS}, P''_{dS}, P''_{aR} and P''_{dR} are given in detail by Tasai[3,4,5,6].

2.3 Dynamic Pressures of Cargo Oil due to Ship Motions

When the solutions of ship motions are known in the forms of (17), the dynamic pressures of cargo oil in a tank can be estimated approximately. Consider the case, for example, when the centre tank is filled up with cargo oil and the both side tanks are empty. Assuming that the motion of a particle of oil would be just similar to that of ship body, namely, there would be no ralative motion between a particle of oil and the tank, the dynamic pressures due to the ship motions would be approximately given by follows in the transverse section shown in Fig. 2.

a) pressure due to vertical motion

The dynamic pressure is in proportion to the vertical acceleration and to the depth of tank. On the bottom of tank, the dynamic pressure is given by

$$\Delta p_1 = -\rho_c \bar{d} Z = -\rho_c \bar{d}[\zeta - (x-x_G)\phi] \qquad (29)$$

where

ρ_c: density of cargo oil

$\bar{d}$: depth of centre tank

b) pressure due to horizontal motion

The dynamic pressure is in proportion to the horizontal acceleration and to the horizontal distance from the tank centre line. On the longitudinal bulkhead, the dynamic pressure is given by

$$\Delta p_2 = -\rho_c \frac{\bar{b}}{2} Y = -\rho_c \frac{\bar{b}}{2} [\eta + (x-x_G)\psi] \qquad (30)$$

where

$\bar{b}$: breadth of centre tank

c) pressure due to rolling motion

For the practical purpose, it is sufficient to consider only the increase of pressure due to heel angle, because the influence of rolling acceleration amounts to only 10∿20 per cent of the former[10]. The pressure increase due to heel is in proportion to the rolling angle and to the horizontal distance from the longitudinal bulkhead of the opposite side to heel. On the longitudinal bulkhead of the heeled side, the pressure increase is given by

$$\Delta p_3 = \rho_c g \bar{b} |\theta| \tag{31}$$

2.4 Transverse Strength Calculation

The hydrodynamic pressures induced on the hull surface and the dynamic pressures of cargo oil on the inside of a tank can be evaluated by the methods described above, Hence, the dynamical loads on a transverse section of ship body can be determined at any time during an encountered period in regular waves. Under such a load condition, the two-dimensional transverse strength calculation can be performed by assuming appropriate support conditions. The more precise method such as the three-dimensional strength calculation will be applicable, but here the simple two-dimensional method is applied under the support conditions shown in Fig. 3.

The transverse load distribution on a section of ship body is symmetric in longitudinal waves but not in oblique waves. Therefore, two kinds of support condition are supposed for those cases as shown in Fig. 3. Under such support conditions, transverse loads are given by superposing the static water pressures and the hydrodynamic pressures on the hull surface and also by superposing the static cargo oil pressures and the dynamic oil pressures on the inside of the tank.

The method of transverse strength calculation is based on "Stress analysis of Plane Frame Structure by Digital Computer" by Fujino and Ohsaka[11].

3 Results of Calculation

According to the methods described avobe, the calculations on the motions, hydrodynamic pressures and transverse strength were carried out for a gigantic oil tanker in regular waves. The main particulars of the ship is shown in Table 1.

3.1 Ship Motions

The ship motions were investigated for the following cases.

a) Heading angle

χ = 0, 45, 90, 135, 180° (χ = 0° : following waves)

b) Ship speed

Fr = 0.10, 0.15 (Fr : Froude number)

c) Wave length

$\sqrt{L/\lambda}$ = 0.3 1.5 (L : ship length, λ : wave length)

d) Wave height

H_w = 10m ($H_w = 2h_o$: wave height)

In the coupled equations of heaving and pitching motions of (14), the hydrodynamic coefficients and the wave exciting forces and moments were evaluated by using the sectional added mass and damping accoding to Tasai[5].

And, in the coupled equations of swaying, yawing and rolling motions, the hydrodynamic coefficients except a_{38} and the wave exciting forces and moments were derived by Tamura's method[7] where the influence of ship speed was not introduced into the wave exciting rolling moment. The roll damping a_{38} was estimated by Eq. (16) including non-linear viscous damping so that Eqs. (15) should be solved by the iteration method.

Amplitudes of the non-linear motions are shown in Fig. 4 as functions of wave height, which are obtained by solving Eqs. (15) according to the iteration method.

In order to investigate the influence of non-linear roll damping, the comparative calculations were made for the cases shown in Table 2.

The calculated results of heave, pitch, sway, yaw and roll in regular waves of 10 meter wave height are shown in Figs. 5$\sim$9. It was found that the rolling amplitude was large in beam and quartering waves.

3.2 Hydrodynamic Pressures

By using the solutions of motions, the hydrodynamic pressures induced on the surface of hull sections were calculated for the following cases.

a) Heading angle

χ = 0, 45, 90, 135, 180°

b) Ship speed

Fr = 0.10, 0.15

c) Wave length

λ/L = 0.50, 0.75, 1.00

d) Wave height

H_w = 10m

e) Hull section

Midship, S.S.$7\frac{1}{2}$ (0.25L forwards from midship)

The calculated results of the pressure amplitude are shown in Figs. 10 ~ 18.

Large hydrodynamic pressures are found in beam waves and in quartering waves on the weather side water line.

For the cases when the larger hydrodynamic pressures are found, the variations of pressure during an encountered period in waves are shown in Figs. 19 ~ 22. In those figures, the pressures include the still water pressure and the hydrodynamic pressure.

3.3 Transverse Strength Calculation

The calculations on the transverse strength were performed for the cases when the large hydrodynamic pressures were induced on the surface of hull sections.

The distributions of transverse load were determined by taking the water pressures along the hull section contour at the time when the pressure on the water line took the maximum value and also the cargo oil pressures in the centre tank including the static pressures and the dynamic pressures due to ship motions at the same instant. The both side tanks are assumed to be empty and the center tank to be filled up with cargo oil. The loads on deck were assumed by taking the water head in excess of the freeboard distributed linearly from the weather side to the other side.

By assuming such transverse loads, the two-dimensional strength calculations were carried out for the hull section with ring structure under the support conditions as shown in Fig. 3 for the cases in longitudinal waves and in oblique waves.

The calculated results of shearing force and bending moment are shown in Figs. 23～30.

The large bending moments are found at the deck corner, bilge and bottom of longitudinal bulkhead in the transverse ring structure in beam and quartering waves where the large hydrodynamic pressures are induced.

The stress calculations will be possible by using the distributions of shearing force and bending moment as shown in Figs. 23～30.

4 Concluding Remarks

A method to analyse the motions, hull surface pressures and transverse strength of a ship in regular waves has been proposed here with the examples of calculation. This is the first stage of study on the transverse strength design. The final goal will be reached by means of the statistical prediction of transverse wave loads in ocean waves.

In this paper, though the problem is dealed with for the cases in regular waves, the important characters of transverse wave loads and strength of a gigantic oil tanker reveals themselves.

Due to the rolling charaterristic of a large tanker, the large hydrodynamic pressures are induced on the hull surface in beam and quartering waves, which produce the large bending moment at the deck corner, bilge and bottom of longitudinal bulkhead in the transverse ring structure when the centre tank is filled up with cargo oil and the both side tanks are empty.

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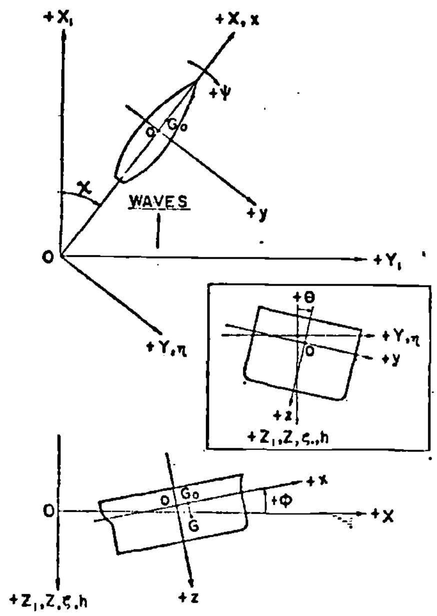


Fig. 1 Coordinates

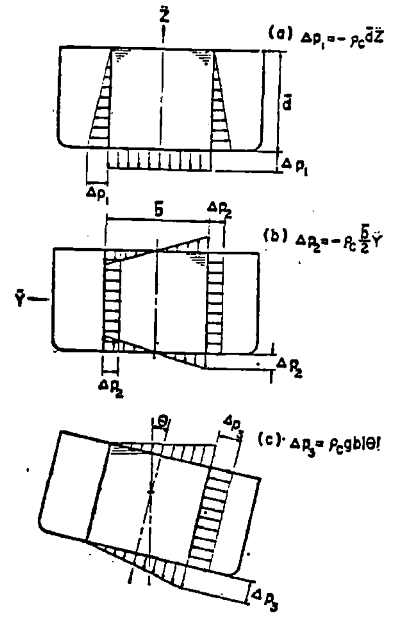


Fig. 2 Fluctuations of Cargo Oil Pressure due to Ship Motions

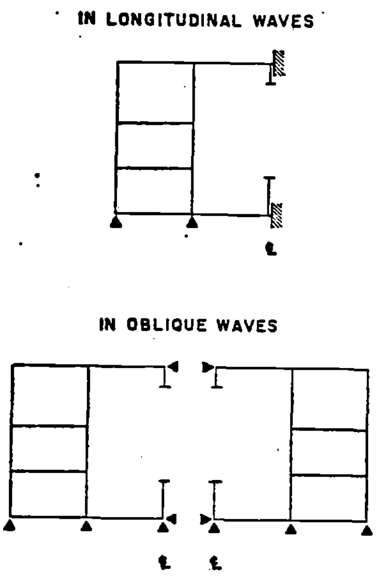


Fig. 3 Support Conditions for Transverse Strength Calculation

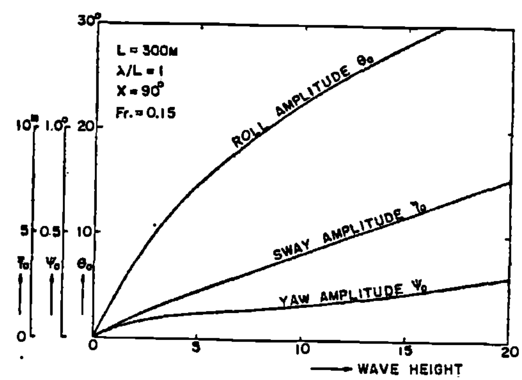


Fig. 4 Amplitudes of Non-Linear Motions as Functions of Wave Height

Table 1 Main Particulars of a Gigantic Oil Tanker

Length between Perpendiculars (L)	310.00 m
Breadth Moulded (B)	48.40 m
Depth Moulded (D)	23.60 m
Draught Moulded (d)	17.80 m
Displacement (W)	230,048 t
Length/Breadth (L/B)	6.4050
Breadth/Draught (B/d)	2.7191
Block Coefficient (C_b)	0.8403
Centre of Gravity from Midship (x_G)	0.0326 L
Centre of Gravity from Water Line (z_G)	0.3202 d
Metacentric Radius (GM)	0.4264 d
Longitudinal Gyradius (κ_l)	0.2336 L
Transverse Gyradius (κ_t)	0.3469 B

Table 2 Comparisons of Rolling Amplitude Calculations

λ/L	F_r	Rolling Amplitude in Degree		
		(a)	(b)	(c)
0.7	0.10	13.9	13.9	13.6
	0.15	13.8	13.8	13.5
1.0	0.10	58.3	36.8	30.4
	0.15	58.2	36.7	28.4
1.5	0.10	15.6	15.6	15.4
	0.15	15.4	15.6	15.4

(a) : Calculated by using linear roll damping without speed influence
(b) : Calculated by using non-linear roll damping without speed influence
(c) : Calculated by using non-linear roll damping with speed influence
in regular beam waves. (Wave Height $=\lambda/20$)

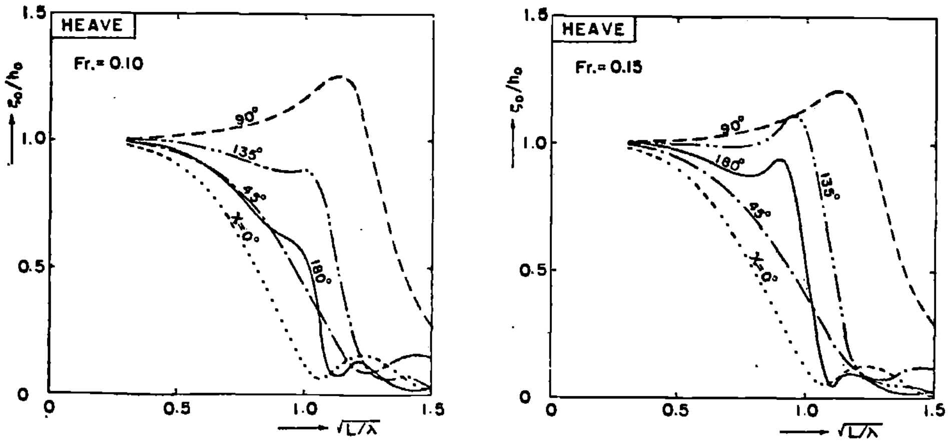


Fig. 5 Heaving Amplitudes in Regular Waves from Different Directions

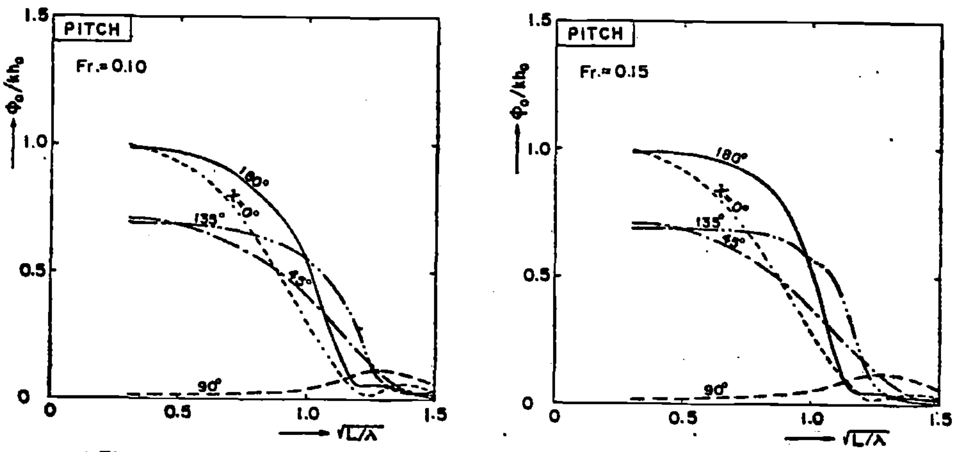


Fig. 6 Pitching Amplitudes in Regular Waves from Different Directions

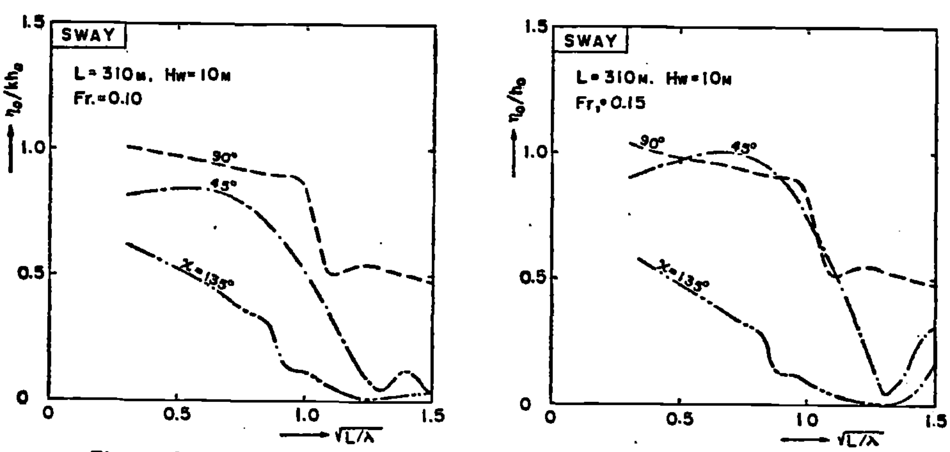


Fig. 7 Swaying Amplitudes in Regular Waves from Different Directions

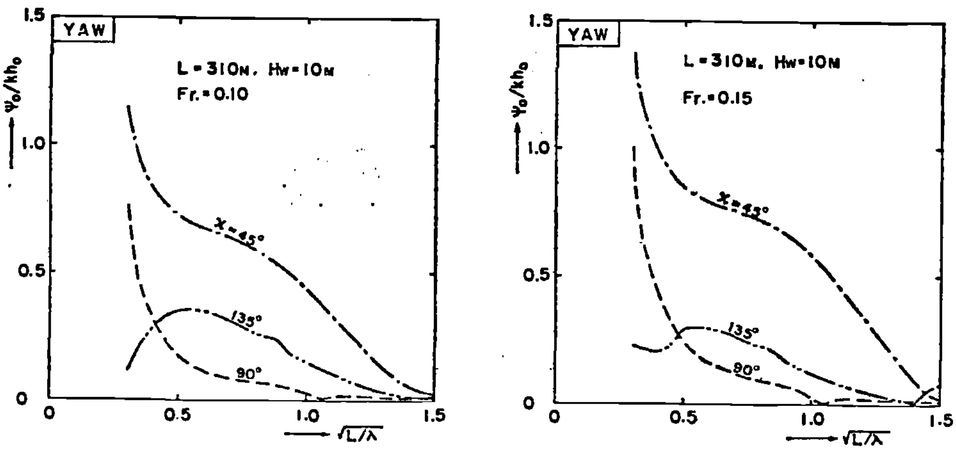


Fig. 8 Yawing Amplitudes in Regular Waves from Different Directions

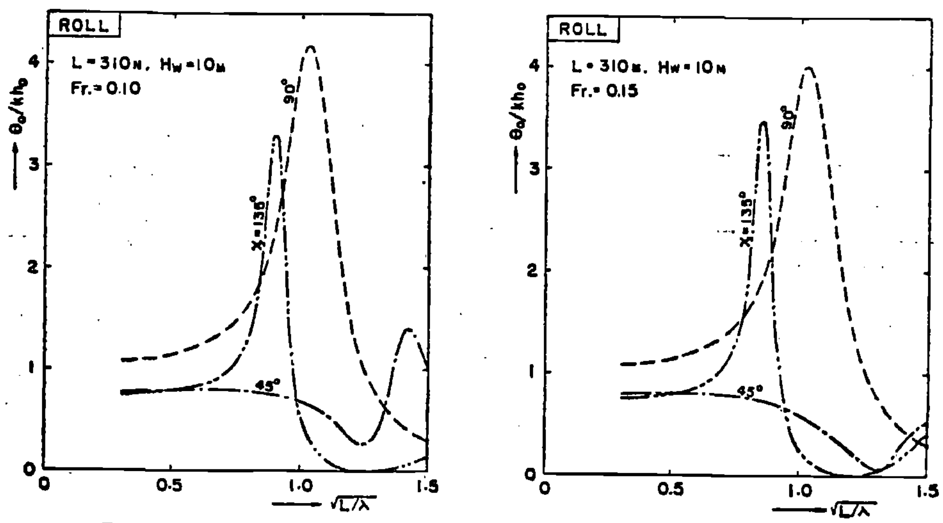


Fig. 9 Rolling Amplitudes in Regular Waves from Different Directions

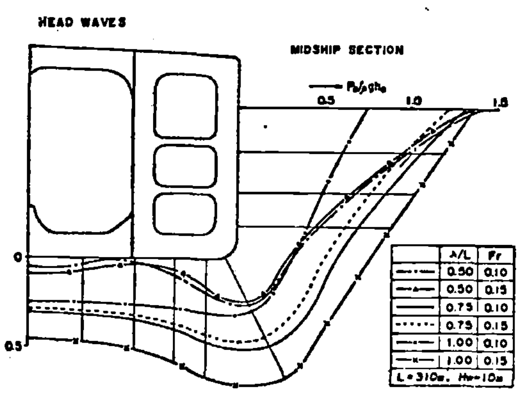


Fig. 10 Amplitudes of Hydrodynamic Pressure in Regular Head Waves ($\chi=180°$), Midship Section

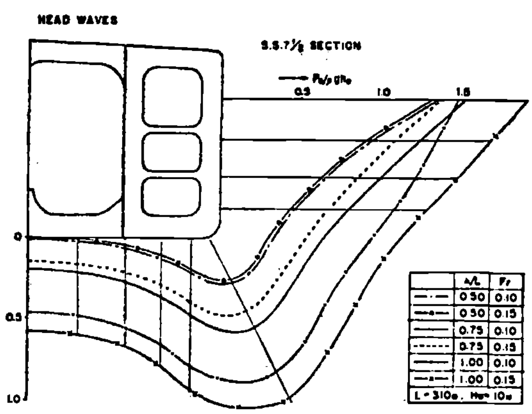


Fig. 11 Amplitudes of Hydrodynamic Pressure in Regular Head Waves ($\chi=180°$), S.S. 7½ Section

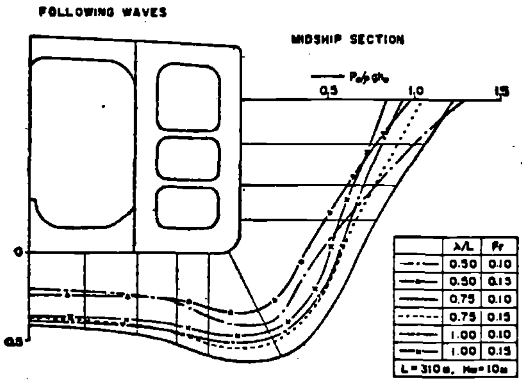


Fig. 12 Amplitudes of Hydrodynamic Pressure in Regular Following Waves ($\chi=0°$), Midship Section

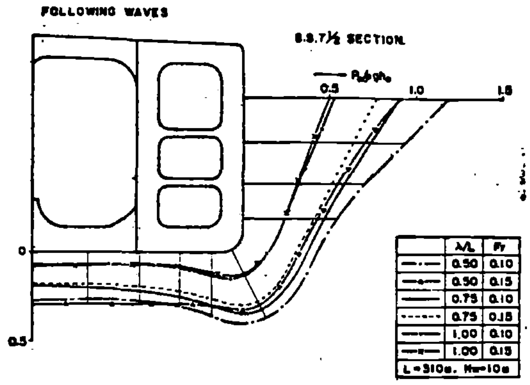


Fig. 13 Amplitudes of Hydrodynamic Pressure in Regular Following Waves ($\chi=0°$), S. S. $7^1/_2$ Section

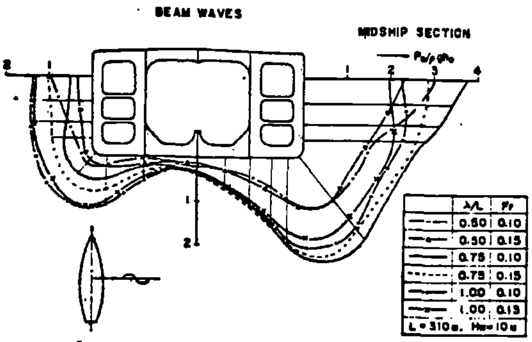


Fig. 14 Amplitudes of Hydrodynamic Pressure in Regular Beam Waves ($\chi=90°$), Midship Section

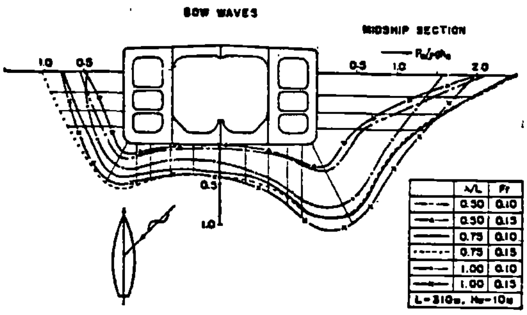


Fig. 15 Amplitudes of Hydrodynamic Pressure in Regular Bow Waves ($\chi=135°$), Midship Section

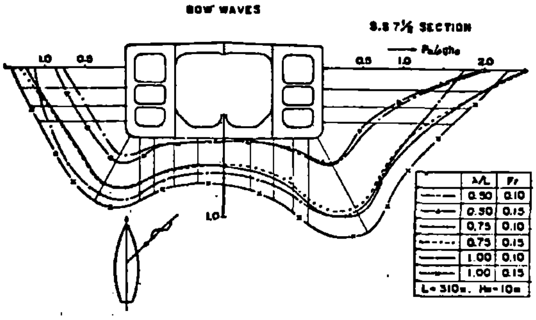


Fig. 16 Amplitudes of Hydrodynamic Pressure in Regular Bow Waves ($\chi=135°$), S. S. $7^1/_2$ Section

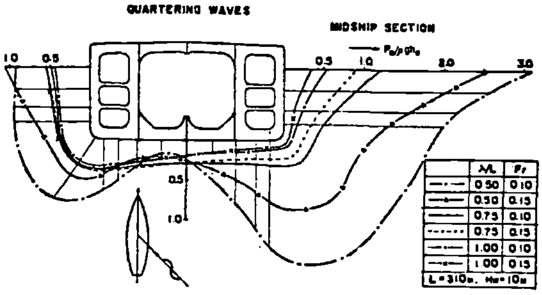


Fig. 17 Amplitudes of Hydrodynamic Pressure in Regular Quartering Waves ($\chi=45°$), Midship Section

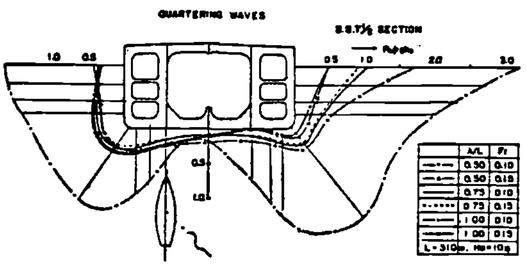


Fig. 18 Amplitudes of Hydrodynamic Pressure in Regular Quartering Waves ($\chi=45°$), S. S. $7^1/_2$ Section

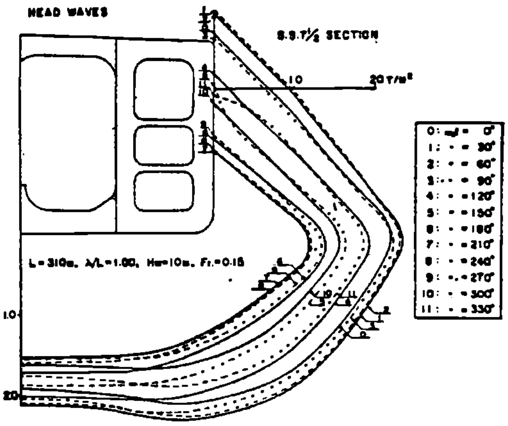


Fig. 19 Pressure Distributions at Time Intervals of $T_e/12$ during an Encountered Period in Regular Head Waves ($\chi=180°$), S. S. $7^1/_2$ Section

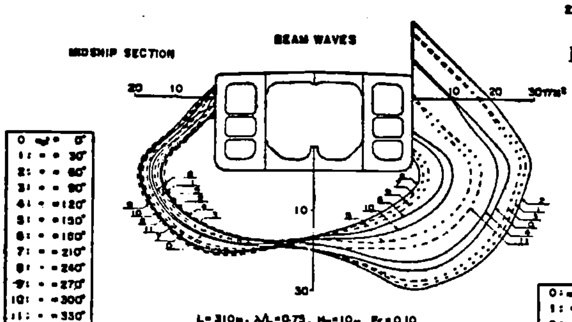


Fig. 20 Pressure Distributions at Time Intervals of $T_e/12$ during an Encountered Period in Regular Beam Waves ($\chi=90°$), Midship Section

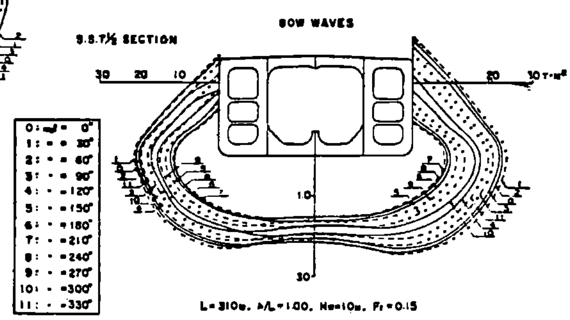


Fig. 21 Pressure Distributions at Time Intervals of $T_e/12$ during an Encountered Period in Regular Bow Waves ($\chi=135°$), S. S. $7^1/_2$ Section

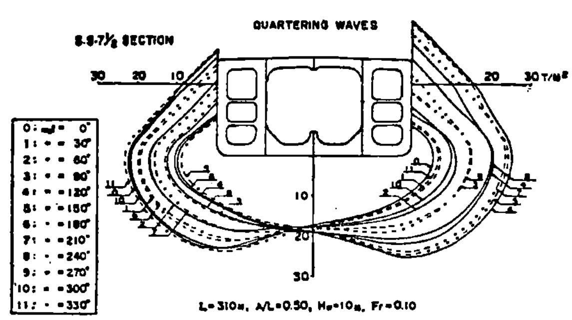
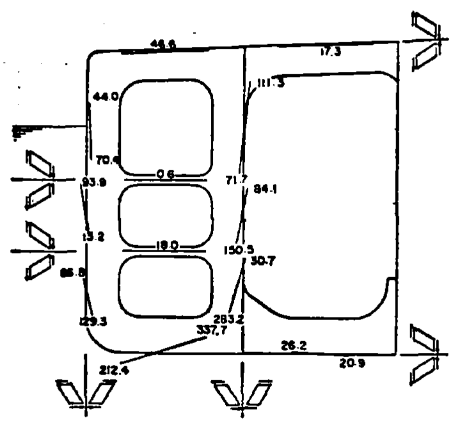


Fig. 22 Pressure Distributions at Time Intervals of $T_e/12$ during an Encountered Period in Regular Quartering Waves ($\chi=45°$), S. S. $7^1/_2$ Section

IN STILL WATER

MIDSHIP SECTION



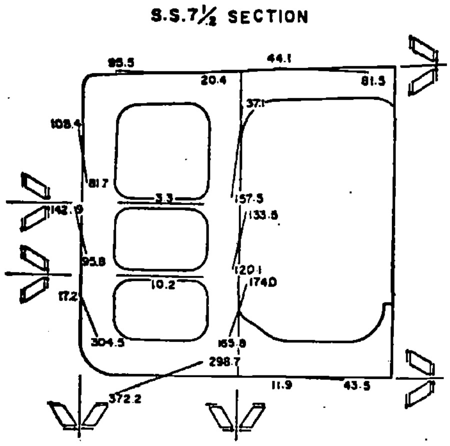
SHEARING FORCE IN TON

Fig. 23 Shearing Forces Calculated in Still Water, Midship Section

HEAD WAVES

L=310M, λ/L=1.00, H_w=10M, Fr=0.15
$\omega_e t$=30°

S.S.7½ SECTION

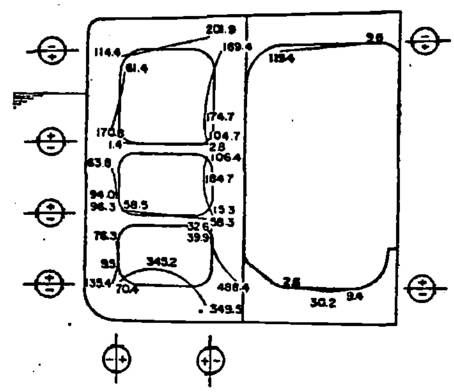


SHEARING FORCE IN TON

Fig. 25 Shearing Forces Calculated in Regular Head Waves (χ=180°), S. S. 7½ Section

IN STILL WATER

MIDSHIP SECTION



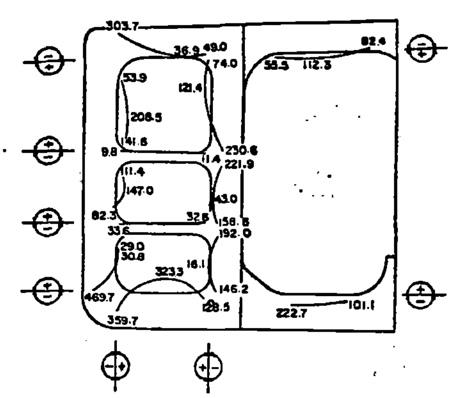
BENDING MOMENT IN T-M

Fig. 24 Bending Moments Calculated in Still Water, Midship Section

HEAD WAVES

L=310M, λ/L=1.00, H_w=10M, Fr=0.15
$\omega_e t$=30°

S.S.7½ SECTION



BENDING MOMENT IN T-M

Fig. 26 Bending Moments Calculated in Regular Head Waves (χ=180°), S. S. 7½ Section

BEAM WAVES MIDSHIP SECTION

L = 310M, λ/L = 0.75, H_W = 10M, Fr. = 0.10
$\omega_e t$ = 60°

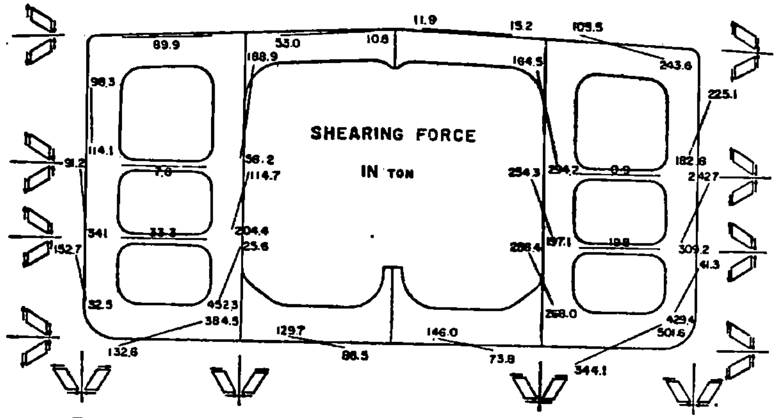


Fig. 27 Shearing Forces Calculated in Regular Beam Waves (χ=90°), Midship Section

BEAM WAVES MIDSHIP SECTION

L = 310M, λ/L = 0.75, H_W = 10M, Fr. = 0.10
$\omega_e t$ = 60°

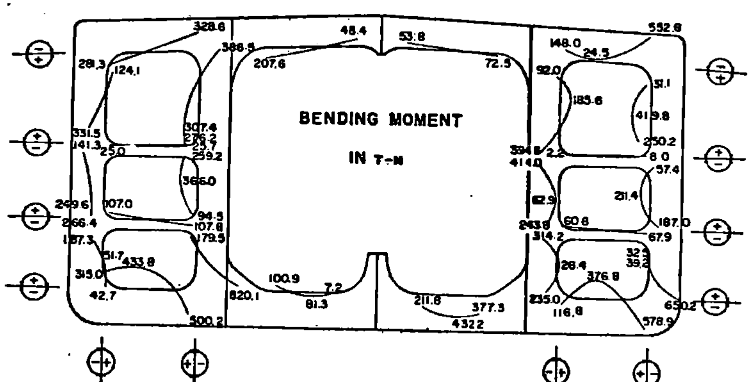


Fig. 28 Bending Moments Calculated in Regular Beam Waves (χ=90°), Midship Section

BOW WAVES S.S.7½ SECTION

L = 310M, λ/L = 1.00, H_W = 10M, Fr. = 0.15
$\omega_e t$ = 30°

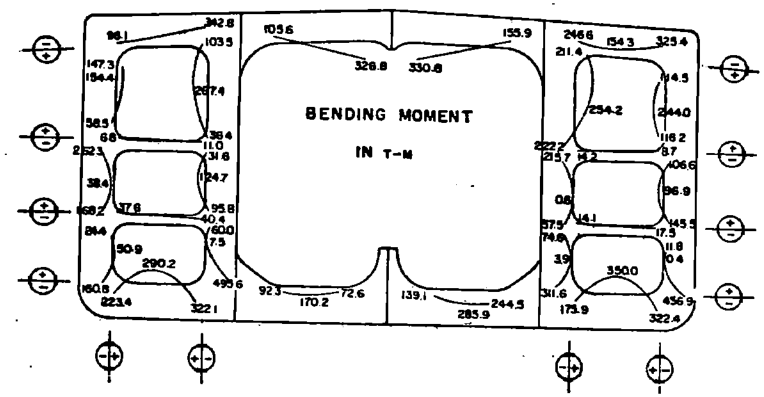


Fig. 29 Bending Moments Calculated in Regular Bow Waves (χ=135°), S. S. 7½ Section

QUARTERING WAVES S.S.7½ SECTION

L = 310M, λ/L = 0.50, H_W = 10M, Fr. = 0.10
$\omega_e t$ = 180°

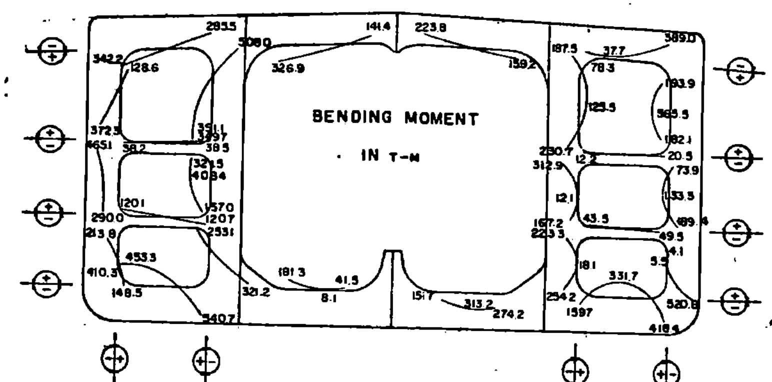


Fig. 30 Bending Moments Calculated in Regular Quartering Waves (χ=45°), S. S. 7½ Section

Appendix

Hydrodynamic Coefficients and Exciting Forces and Moments

In the appendix, the hydrodynamic coefficients and the wave exciting forces and moments are given based upon the linear strip theory.

a) Heaving and Pitching Motions

In the coupled equations of heaving and pitching motion :

$$\left.\begin{aligned} A_{11}\ddot{\zeta} + A_{12}\dot{\zeta} + A_{13}\zeta + A_{14}\ddot{\phi} + A_{15}\dot{\phi} + A_{16}\phi &= F_\zeta \\ A_{21}\ddot{\zeta} + A_{22}\dot{\zeta} + A_{23}\zeta + A_{24}\ddot{\phi} + A_{25}\dot{\phi} + A_{26}\phi &= M_\phi \end{aligned}\right\} \qquad (14)$$

A_{11}, A_{12}, - - - , A_{21}, A_{22}, - - - , F_ζ and M_ϕ are derived as follows.

$$\left.\begin{aligned} A_{11} &= \frac{W}{g} + \int \rho s_z dx \\ A_{12} &= \int \rho N_z dx \\ A_{13} &= 2\rho g \int y_w dx \\ A_{14} &= -\int \rho s_z (x - x_G) dx \\ A_{15} &= -\int \rho N_z (x - x_G) dx + V \int \rho s_z dx \\ A_{16} &= -2\rho g \int y_w (x - x_G) dx + V A_{12} \end{aligned}\right\} \qquad (14.1)$$

$$\left.\begin{aligned} A_{21} &= A_{14} \\ A_{22} &= -\int \rho N_z (x - x_G) dx - V \int \rho s_z dx \\ A_{23} &= -2\rho g \int y_w (x - x_G) dx \\ A_{24} &= \frac{I_\phi}{g} + \int \rho s_z (x - x_G)^2 dx \\ A_{25} &= \int \rho N_z (x - x_G)^2 dx \\ A_{26} &= 2\rho g \int y_w (x - x_G)^2 dx + V A_{22} \end{aligned}\right\} \qquad (14.2)$$

$$\left.\begin{aligned} F_\zeta &= F_{\zeta c} \cos \omega_e t + F_{\zeta s} \sin \omega_e t \\ M_\phi &= M_{\phi s} \cos \omega_{et} + M_{\phi s} \sin \omega_e t \end{aligned}\right\} \qquad (14.3)$$

$$\left.\begin{matrix} F_{\zeta c} \\ F_{\zeta s} \end{matrix}\right\} = h_0 \left\{\begin{matrix} f_{1c} + f_{2c} + f_{3c} \\ f_{1s} + f_{2s} + f_{3s} \end{matrix}\right\}, \quad \left.\begin{matrix} M_{\phi c} \\ M_{\phi s} \end{matrix}\right\} = h_0 \left\{\begin{matrix} m_{1c} + m_{2c} + m_{3c} \\ m_{1s} + m_{2s} + m_{3s} \end{matrix}\right\} \qquad (14.4)$$

$$\left.\begin{aligned} \left.\begin{matrix} f_{1c} \\ f_{1s} \end{matrix}\right\} &= 2\rho g \int C_1 C_2 y_w \left\{\begin{matrix} \cos k^* x \\ \sin k^* x \end{matrix}\right\} dx \\ \left.\begin{matrix} f_{2c} \\ f_{2s} \end{matrix}\right\} &= \omega \int C_1 C_2 \rho N_z \left\{\begin{matrix} \sin k^* x \\ -\cos k^* x \end{matrix}\right\} dx \\ \left.\begin{matrix} f_{3c} \\ f_{3s} \end{matrix}\right\} &= -\omega\omega_e \int C_1 C_2 \rho s_z \left\{\begin{matrix} \cos k^* x \\ \sin k^* x \end{matrix}\right\} dx \end{aligned}\right\} \qquad (14.5)$$

$$\left.\begin{aligned}
&\left.\begin{matrix} m_{1c} \\ m_{1s} \end{matrix}\right\} = -2\rho g \int C_1 C_2 y_w \begin{Bmatrix} \cos k^*x \\ \sin k^*x \end{Bmatrix} (x - x_G)\,dx \\
&\left.\begin{matrix} m_{2c} \\ m_{2s} \end{matrix}\right\} = -\omega \int C_1 C_2 \rho N_z \begin{Bmatrix} \sin k^*x \\ -\cos k^*x \end{Bmatrix} (x - x_G)\,dx \\
&\left.\begin{matrix} m_{3c} \\ m_{3s} \end{matrix}\right\} = \omega\omega_e \int C_1 C_2 \rho s_z \begin{Bmatrix} \cos k^*x \\ \sin k^*x \end{Bmatrix} (x - x_G)\,dx \\
&\qquad - \omega V \int C_1 C_2 \rho s_z \begin{Bmatrix} \sin k^*x \\ -\cos kx^* \end{Bmatrix} dx \\
&\qquad\qquad k^* = k \cos \chi
\end{aligned}\right\} \quad (14.6)$$

b) Swaying, Yawing and Rolling Motions

In the coupled equations of swaying, yawing and rolling mot motions:

$$\left.\begin{aligned}
&a_{11}\ddot{\eta} + a_{12}\dot{\eta} + a_{13}\eta + a_{14}\ddot{\psi} + a_{15}\dot{\psi} + a_{16}\psi \\
&\qquad + a_{17}\ddot{\theta} + a_{18}\dot{\theta} + a_{19}\theta = F_\eta \\
&a_{21}\ddot{\eta} + a_{22}\dot{\eta} + a_{23}\eta + a_{24}\ddot{\psi} + a_{25}\dot{\psi} + a_{26}\psi \\
&\qquad + a_{27}\ddot{\theta} + a_{28}\dot{\theta} + a_{29}\theta = M_\psi \\
&a_{31}\ddot{\eta} + a_{32}\dot{\eta} + a_{33}\eta + a_{34}\ddot{\psi} + a_{35}\dot{\psi} + a_{36}\psi \\
&\qquad + a_{37}\ddot{\theta} + a_{38}\dot{\theta} + a_{39}\theta = M_\theta
\end{aligned}\right\} \quad (15)$$

a_{11}, a_{12}, - - - , a_{21}, a_{22}, - - - , a_{31}, a_{32}, - - - , F_η, M_ψ and M_θ are derived as follows.

$$\left.\begin{aligned}
&a_{11} = \frac{W}{g} + \int \rho s_y\,dx \\
&a_{12} = \int \rho N_y\,dx, \qquad a_{13} = 0 \\
&a_{14} = \int \rho s_y (x - x_G)\,dx \\
&a_{15} = \int \rho N_y (x - x_G)\,dx - V \int \rho s_y\,dx \\
&a_{16} = -Va_{12} \\
&a_{17} = \int \rho s_y (z_G - l_\eta)\,dx \\
&a_{18} = \int \rho N_y (z_G - l_w)\,dx \\
&a_{19} = 0
\end{aligned}\right\} \quad (15.1)$$

$$\left.\begin{aligned}
&a_{21} = a_{14} \\
&a_{22} = \int \rho N_y (x - x_G)\,dx + V \int \rho s_y\,dx \\
&a_{23} = 0 \\
&a_{24} = \frac{I_\phi}{g} + \int \rho s_y (x - x_G)^2\,dx \\
&a_{25} = \int \rho N_y (x - x_G)^2\,dx \\
&a_{26} = -Va_{22} \\
&a_{27} = \int \rho s_y (z_G - l_\eta)(x - x_G)\,dx \\
&a_{28} = \int \rho N_y (z_G - l_w)(x - x_G)\,dx + V_{a17} \\
&a_{29} = 0
\end{aligned}\right\} \quad (15.2)$$

$$\left.\begin{aligned}
&a_{31}=a_{17},\ a_{32}=a_{18},\ a_{33}=0,\ a_{34}=a_{27}\\
&a_{35}=\int \rho N_y(z_G-l_w)(x-x_G)dx-V_{a17}\\
&a_{36}=-V_{a18}\\
&a_{37}=\frac{I_\theta}{g}+\int \rho i dx+2z_G a_{17}-z_G^2\int \rho s_y dx\\
&a_{38}=\int \rho N_y'(z_G-l_w)^2 dx\\
&a_{39}=W m_t
\end{aligned}\right\} \qquad (15.3)$$

$\rho i=\rho s_y l_\eta l_\theta$: added mass moment of inertia

m_t : metacentric radius

$$\left.\begin{aligned}
F_\eta&=F_{\eta c}\cos\omega_e t+F_{\eta s}\sin\omega_e t\\
M_\psi&=M_{\psi c}\cos\omega_e t+M_{\psi s}\sin\omega_e t\\
M_\theta&=M_{\theta c}\cos\omega_e t+M_{\theta s}\sin\omega_e t
\end{aligned}\right\} \qquad (15.4)$$

$$\left.\begin{aligned}
\begin{Bmatrix}F_{\eta c}\\F_{\eta s}\end{Bmatrix}&=h_0\sin\chi\begin{Bmatrix}f_{\eta 1c}+f_{\eta 2c}+f_{\eta 3c}+f_{\eta 4c}\\f_{\eta 1s}+f_{\eta 2s}+f_{\eta 3s}+f_{\eta 4s}\end{Bmatrix}\\
\begin{Bmatrix}M_{\psi c}\\M_{\psi s}\end{Bmatrix}&=h_0\sin\chi\begin{Bmatrix}m_{\psi 1c}+m_{\psi 2c}+m_{\psi 3c}+m_{\psi 4c}\\m_{\psi 1s}+m_{\psi 2s}+m_{\psi 3s}+m_{\psi 4s}\end{Bmatrix}\\
\begin{Bmatrix}M_{\theta c}\\M_{\theta s}\end{Bmatrix}&=h_0\sin\chi\begin{Bmatrix}m_{\theta 1c}+m_{\theta 2c}+m_{\theta 3c}+m_{\theta 4c}\\m_{\theta 1s}+m_{\theta 2s}+m_{\theta 3s}+m_{\theta 4s}\end{Bmatrix}
\end{aligned}\right\} \qquad (15.5)$$

$$\left.\begin{aligned}
\begin{Bmatrix}f_{\eta 1c}\\f_{\eta 1s}\end{Bmatrix}&=\rho g\int S_1\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}dx\\
\begin{Bmatrix}f_{\eta 2c}\\f_{\eta 2s}\end{Bmatrix}&=\omega\int \rho N_y C_3\begin{Bmatrix}\cos k^*x\\\sin k^*x\end{Bmatrix}dx\\
\begin{Bmatrix}f_{\eta 3c}\\f_{\eta 3s}\end{Bmatrix}&=\omega^2\int \rho s_y C_3\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}dx\\
\begin{Bmatrix}f_{\eta 4c}\\f_{\eta 4s}\end{Bmatrix}&=-\omega k^* V\int \rho s_y C_3\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}
\end{aligned}\right\} \qquad (15.6)$$

$$\left.\begin{aligned}
\begin{Bmatrix}m_{\psi 1c}\\m_{\psi 1s}\end{Bmatrix}&=\rho g\int S_1\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}(x-x_G)dx\\
\begin{Bmatrix}m_{\psi 2c}\\m_{\psi 2s}\end{Bmatrix}&=\omega\int \rho N_y C_3\begin{Bmatrix}\cos k^*x\\\sin k^*x\end{Bmatrix}(x-x_G)dx\\
\begin{Bmatrix}m_{\psi 3c}\\m_{\psi 3s}\end{Bmatrix}&=\omega^2\int \rho s_y C_3\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}(x-x_G)dx\\
\begin{Bmatrix}m_{\psi 4c}\\m_{\psi 4s}\end{Bmatrix}&=\omega V\int \rho s_y C_3\begin{Bmatrix}\cos k^*x\\\sin k^*x\end{Bmatrix}dx\\
&\quad-\omega k^* V\int \rho s_y C_3\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}(x-x_G)dx
\end{aligned}\right\} \qquad (15.7)$$

$$\left.\begin{aligned}
\begin{Bmatrix}m_{\theta 1c}\\m_{\theta 1s}\end{Bmatrix}&=\rho g\int S_1\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}(z_G-l_1)dx\\
\begin{Bmatrix}m_{\theta 2c}\\m_{\theta 2s}\end{Bmatrix}&=\omega\int \rho N_y C_3\begin{Bmatrix}\cos k^*x\\\sin k^*x\end{Bmatrix}(z_G-l_w)dx\\
\begin{Bmatrix}m_{\theta 3c}\\m_{\theta 3s}\end{Bmatrix}&=\omega^2\int \rho s_y C_3\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}(z_G-l_\eta)dx\\
\begin{Bmatrix}m_{\theta 4c}\\m_{\theta 4s}\end{Bmatrix}&=-\omega k^* V\int \rho s_y C_3\begin{Bmatrix}\sin k^*x\\-\cos k^*x\end{Bmatrix}(z_G-l_\eta)dx
\end{aligned}\right\} \qquad (15.8)$$

$$S_1=\frac{2}{\sin\chi}\int_0^d e^{-kz_s}\sin(ky_s\sin\chi)dz_s$$

$C_3=\exp(-kd/2)$, l_1 : (11)

In those equations, the integrations should be carried out from the after end to the fore end along the water line length.

In the actual calculations, a_{38} was evaluated by Eq. (16)

introducing the non-linear viscous damping, and M_θ was calculated according to Tamura's method[7] where the influence of ship speed corresponding to $m_{\theta 4c}$ and $m_{\theta 4s}$ in Eqs. (15.8) was not included.