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Verification of Joint Input-State Estimation by In Situ Measurements on a Footbridge

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ABSTRACT

An existing joint input-state estimation algorithm is extended for applications in structural dynamics. The estimation of the input and the system states is performed in a minimum-variance unbiased way, based on a limited number of response measurements and a system model. The noise statistics are estimated, as they are essential for the joint input-state estimation and can be used to quantify the uncertainty on the estimated forces and system states. The methodology is illustrated using data from an in situ experiment on a footbridge.

INTRODUCTION

For civil engineering structures, knowledge of the dynamic loads is crucial to design purposes. Very often these dynamic loads can hardly be obtained by direct measurements and have to be determined indirectly from the system response.

The joint input-state estimation algorithm proposed in this work is an extension of an algorithm proposed by Gillijns and De Moor [1]. The algorithm has the structure of a Kalman filter, except that the true value of the input is replaced by a minimum-variance unbiased estimate.

The noise statistics are essential when using the proposed joint input-state estimation algorithm, especially for quantification of the uncertainty on the estimation. Several methods have been proposed in the literature to identify the noise statistics, both offline [2] and online [3]. Very often, in structural dynamics applications, operational

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loads (e.g. wind loads) are modeled as stochastic noise processes. This can be directly taken into account for the noise identification procedure, which then boils down to a force identification problem.

This paper consists of a theoretical part, followed by a practical illustration. Firstly, a brief overview is given of system models as commonly used in structural dynamics. Secondly, the joint input-state estimation algorithm is presented. Thirdly, a method for the identification of noise statistics is proposed. The methodology is finally illustrated for the practical case of a footbridge, where multiple forces are identified.

MATHEMATICAL FORMULATION

System model

In structural dynamics, first principle models, e.g. finite element (FE) models, are widely used. In many cases, modally reduced order models are applied, constructed from a limited number of structural modes. When proportional damping is assumed, the continuous-time decoupled equations of motion in modal coordinates are given by:

$$\ddot{\mathbf{z}}(t) + \mathbf{\Gamma}\,\dot{\mathbf{z}}(t) + \mathbf{\Omega}^2\mathbf{z}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{S}_{\mathbf{p}}(t)\mathbf{p}(t) \tag{1}$$

where $\mathbf{z}(t) \in \mathbb{R}^{n_{\mathrm{m}}}$ is the vector of modal coordinates, with n_{m} the number of modes. The excitation force is written as the product of a selection matrix $\mathbf{S}_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{dof}} \times n_{\mathrm{p}}}$, and a time history vector $\mathbf{p}(t) \in \mathbb{R}^{n_{\mathrm{p}}}$, with n_{p} the number of forces. The number of degrees of freedom is indicated by n_{dof}. $\mathbf{\Gamma} \in \mathbb{R}^{n_{\mathrm{m}} \times n_{\mathrm{m}}}$ is a diagonal matrix containing the terms $2\,\xi_j\omega_j$ on its diagonal, with ω_j and ξ_j the natural frequency and modal damping ratio according to mode j, respectively. $\mathbf{\Omega} \in \mathbb{R}^{n_{\mathrm{m}} \times n_{\mathrm{m}}}$ is a diagonal matrix, containing the natural frequencies ω_j on its diagonal. $\mathbf{\Phi} \in \mathbb{R}^{n_{\mathrm{dof}} \times n_{\mathrm{m}}}$ is a matrix with the eigenvectors $\boldsymbol{\phi}_j$ as columns.

The output vector is generally written as:

$$\mathbf{d}(t) = \mathbf{S}_{\mathrm{a}}\mathbf{\Phi}\,\ddot{\mathbf{z}}(t) + \mathbf{S}_{\mathrm{v}}\mathbf{\Phi}\,\dot{\mathbf{z}}(t) + \mathbf{S}_{\mathrm{d}}\mathbf{\Phi}\,\mathbf{z}(t) \tag{2}$$

where $\mathbf{S}_{\mathrm{a}}$, $\mathbf{S}_{\mathrm{v}}$, and $\mathbf{S}_{\mathrm{d}} \in \mathbb{R}^{n_{\mathrm{d}} \times n_{\mathrm{dof}}}$ are selection matrices indicating the degrees of freedom corresponding to the acceleration, velocity, and displacement measurements, respectively.

Eq. (1) and Eq. (2) can be written into state space form. After time discretization and adding process noise and measurement noise to the state equation (3), and the output equation (4), respectively, the following discrete-time combined deterministic-stochastic state space description of the system is obtained:

$$\mathbf{x}_{[k+1]} = \mathbf{A}\,\mathbf{x}_{[k]} + \mathbf{B}\,\mathbf{p}_{[k]} + \mathbf{w}_{[k]} \tag{3}$$
$$\mathbf{d}_{[k]} = \mathbf{G}\,\mathbf{x}_{[k]} + \mathbf{J}\,\mathbf{p}_{[k]} + \mathbf{v}_{[k]} \tag{4}$$

where $\mathbf{x}_{[k]} = \mathbf{x}(k\Delta t)$, $\mathbf{p}_{[k]} = \mathbf{p}(k\Delta t)$ and $\mathbf{d}_{[k]} = \mathbf{d}(k\Delta t)$ $(k = 1, \dots, N)$, Δt is the sampling time step, and N is the total number of samples. The state vector $\mathbf{x}_{[k]}$ consists of the modal displacements and modal velocities: $\mathbf{x}_{[k]} = \left[\mathbf{z}_{[k]}^{\mathrm{T}}\ \dot{\mathbf{z}}_{[k]}^{\mathrm{T}}\right]^{\mathrm{T}}$.

Joint input-state estimation

An existing joint input-state estimation algorithm for linear systems with direct feedthrough [1] is extended in order to include the correlation between the process noise $\mathbf{w}_{[k]}$ and the measurement noise $\mathbf{v}_{[k]}$. As will be illustrated in the following, this correlation becomes important if operational loads are modeled as stochastic noise processes, as is often the case in structural dynamics. The system under consideration is described by Eq. (3) and Eq. (4). The noise processes $\mathbf{w}_{[k]} \in \mathbb{R}^{2n_\mathrm{m}}$ and $\mathbf{v}_{[k]} \in \mathbb{R}^{n_\mathrm{d}}$ are assumed to be zero mean and white, with known covariance matrices $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{S}$:

$$\mathbb{E}\left[\begin{pmatrix}\mathbf{w}_{[k]}\\ \mathbf{v}_{[k]}\end{pmatrix}\begin{pmatrix}\mathbf{w}_{[k]}^\mathrm{T} & \mathbf{v}_{[k]}^\mathrm{T}\end{pmatrix}\right] = \begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^\mathrm{T} & \mathbf{R}\end{bmatrix}\boldsymbol{\delta}_{[k-l]} \tag{5}$$

with $\mathbf{R} > 0$, $\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^\mathrm{T} & \mathbf{R}\end{bmatrix} \geq 0$, and $\delta_{[k]} = 1$ for $k = 0$ and 0 otherwise.

Joint input-state estimation consists of estimating the forces $\mathbf{p}_{[k]}$ and states $\mathbf{x}_{[k]}$, from a set of response measurements $\mathbf{d}_{[k]}$. A state estimate $\hat{\mathbf{x}}_{[k|l]}$ is defined as an estimate of $\mathbf{x}_{[k]}$, given the output sequence $\left\{\mathbf{d}_{[n]}\right\}_{n=0}^{l}$. The corresponding error covariance matrix, denoted as $\mathbf{P}_{[k|l]}$, is defined as $\mathbb{E}\left[\left(\mathbf{x}_{[k]} - \hat{\mathbf{x}}_{[k|l]}\right)\left(\mathbf{x}_{[k]} - \hat{\mathbf{x}}_{[k|l]}\right)^\mathrm{T}\right]$. An input estimate $\hat{\mathbf{p}}_{[k|l]}$ and its error covariance matrix $\mathbf{P}_{\mathrm{p}[k|l]}$ are defined similarly.

The filtering algorithm is initialized using an initial state estimate vector $\hat{\mathbf{x}}_{[0|-1]}$ and its error covariance matrix $\mathbf{P}_{[0|-1]}$, both assumed known. Hereafter, it propagates by computing the force and state estimates recursively in three steps, i.e. the input estimation step, the measurement update and the time update:

Input estimation

$$\widetilde{\mathbf{R}}_{[k]} = \mathbf{G}\,\mathbf{P}_{[k|k-1]}\mathbf{G}^\mathrm{T} + \mathbf{R} \tag{6}$$

$$\boldsymbol{\mathcal{M}}_{[k]} = \left(\mathbf{J}^\mathrm{T}\widetilde{\mathbf{R}}_{[k]}^{-1}\mathbf{J}\right)^{-1}\mathbf{J}^\mathrm{T}\widetilde{\mathbf{R}}_{[k]}^{-1} \tag{7}$$

$$\hat{\mathbf{p}}_{[k|k]} = \boldsymbol{\mathcal{M}}_{[k]}\left(\mathbf{d}_{[k]} - \mathbf{G}\,\hat{\mathbf{x}}_{[k|k-1]}\right) \tag{8}$$

$$\mathbf{P}_{\mathrm{p}[k|k]} = \left(\mathbf{J}^\mathrm{T}\widetilde{\mathbf{R}}_{[k]}^{-1}\mathbf{J}\right)^{-1} \tag{9}$$

Measurement update

$$\mathbf{L}_{[k]} = \mathbf{P}_{[k|k-1]}\mathbf{G}^\mathrm{T}\widetilde{\mathbf{R}}_{[k]}^{-1} \tag{10}$$

$$\hat{\mathbf{x}}_{[k|k]} = \hat{\mathbf{x}}_{[k|k-1]} + \mathbf{L}_{[k]}\left(\mathbf{d}_{[k]} - \mathbf{G}\,\hat{\mathbf{x}}_{[k|k-1]} - \mathbf{J}\,\hat{\mathbf{p}}_{[k|k]}\right) \tag{11}$$

$$\mathbf{P}_{[k|k]} = \mathbf{P}_{[k|k-1]} - \mathbf{L}_{[k]}\left(\widetilde{\mathbf{R}}_{[k]} - \mathbf{J}\mathbf{P}_{\mathrm{p}[k|k]}\mathbf{J}^\mathrm{T}\right)\mathbf{L}_{[k]}^\mathrm{T} \tag{12}$$

$$\mathbf{P}_{\mathrm{xp}[k|k]} = \mathbf{P}_{\mathrm{px}[k|k]}^\mathrm{T} = -\mathbf{L}_{[k]}\mathbf{J}\mathbf{P}_{\mathrm{p}[k|k]} \tag{13}$$

Time update

$$\hat{\mathbf{x}}_{[k|k-1]} = \mathbf{A}\,\hat{\mathbf{x}}_{[k|k]} + \mathbf{B}\,\hat{\mathbf{p}}_{[k|k]} \tag{14}$$

$$\boldsymbol{\mathcal{L}}_{[k]} = \mathbf{L}_{[k]}\left(\mathbf{I} - \mathbf{J}\,\boldsymbol{\mathcal{M}}_{[k]}\right) \tag{15}$$

$$\boldsymbol{\mathcal{S}}_{[k]} = \mathbf{S}\left(\mathbf{A}\boldsymbol{\mathcal{L}}_{[k]} + \mathbf{B}\boldsymbol{\mathcal{M}}_{[k]}\right)^\mathrm{T} \tag{16}$$

$$\mathbf{P}_{[k+1|k]} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{P}_{[k|k]} & \mathbf{P}_{\mathrm{xp}[k|k]}\\ \mathbf{P}_{\mathrm{px}[k|k]} & \mathbf{P}_{\mathrm{p}[k|k]}\end{bmatrix}\begin{bmatrix}\mathbf{A}^\mathrm{T}\\ \mathbf{B}^\mathrm{T}\end{bmatrix} + \mathbf{Q} - \boldsymbol{\mathcal{S}}_{[k]} - \boldsymbol{\mathcal{S}}_{[k]}^\mathrm{T} \tag{17}$$

Estimation of noise statistics

In structural dynamics applications, operational loads, such as wind loads, are very often modeled as stochastic noise processes. The noise statistics $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{S}$, under the assumption of white noise processes, can then be estimated from a preliminary vibration experiment where only noise sources are present ($\mathbf{p}_{[k]} = 0$).

The noise sources are modeled as a set of stochastic forces $\mathbf{p}_{\mathrm{s}[k]}$, concentrated in a limited number of structural nodes. The power spectral density (PSD) of the forces $\mathbf{S}_{\mathrm{p_s p_s}}(\omega) \in \mathbb{C}^{n_{\mathrm{p_s}} \times n_{\mathrm{p_s}}}$ at frequency ω is obtained from the PSD of the measured response $\mathbf{S}_{\mathrm{d_s d_s}}(\omega) \in \mathbb{C}^{n_{\mathrm{d_s}} \times n_{d_s}}$ and the frequency response function (FRF) matrix $\mathbf{H}(\omega) \in \mathbb{C}^{n_{\mathrm{d_s}} \times n_{\mathrm{p}_s}}$, as follows:

$$\mathbf{S}_{\mathrm{p_s p_s}}(\omega) = \mathbf{H}^{+}(\omega)\mathbf{S}_{\mathrm{d_s d_s}}(\omega)\mathbf{H}^{*+}(\omega) \tag{18}$$

where $\mathbf{H}^{+}(\omega)$ and $\mathbf{H}^{*}(\omega)$ denote the Moore-Penrose pseudo-inverse and the Hermitian transpose of the matrix $\mathbf{H}(\omega)$, respectively. Defining the PSD of a sampled time series as:

$$\mathbf{S}_{\mathrm{pq}}(\,\omega) := \sum_{k=0}^{N-1} \mathbf{R}_{\mathrm{pq}[k]} \exp(-i\omega k \Delta t)\,, \tag{19}$$

where $\mathbf{R}_{\mathrm{pq}[k]} := \mathbb{E}\{\mathbf{p}_{[l+k]}\mathbf{q}_{[l]}^{\mathrm{T}}\}$.

The force PSD $\mathbf{S}_{\mathrm{p_s p_s}}(\omega)$, obtained from Eq. (18), equals the force covariance matrix $\mathrm{Cov}\big(\mathbf{p}_{s[k]}\big) := \mathbb{E}[\mathbf{p}_{s[k]}\mathbf{p}_{s[k]}^{\mathrm{T}}]$, under the assumption of a stationary discrete-time white noise process. This holds for each frequency. The noise covariance matrices are calculated as:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^{\mathrm{T}} & \mathbf{R} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{\mathrm{s}} \\ \mathbf{J}_{\mathrm{s}} \end{bmatrix} \mathrm{Cov}\big(\mathbf{p}_{\mathrm{s}[k]}\big)[\mathbf{B}_{\mathrm{s}}^{\mathrm{T}} \quad \mathbf{J}_{\mathrm{s}}^{\mathrm{T}}] + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\mathrm{m}} \end{bmatrix} \tag{20}$$

where the matrices $\mathbf{B}_{\mathrm{s}} \in \mathbb{R}^{2n_{\mathrm{m}} \times n_{\mathrm{p_s}}}$ and $\mathbf{J}_{\mathrm{s}} \in \mathbb{R}^{n_{\mathrm{d}} \times n_{\mathrm{p_s}}}$, occurring in Eq. (3) and Eq. (4), respectively, correspond to the $n_{\mathrm{p_s}}$ stochastic forces assumed for the noise identification procedure and the n_{d} output measurements used for the joint input-state estimation procedure. $\mathbf{R}_{\mathrm{m}}$ is the “true” measurement noise covariance matrix, which is chosen as a diagonal matrix with the squared value of the sensor resolution on its diagonal.

In the literature, several methods have been proposed to estimate the cross-PSD of two sampled time series $\mathbf{p}_{[k]}$ and $\mathbf{q}_{[k]}$ [4]. For the remainder of this paper, the periodogram approach is used to calculate the output PSD $\mathbf{S}_{\mathrm{d_s d_s}}(\omega)$ from a set of output measurements. The FRF matrix $\mathbf{H}(\omega)$ can be obtained from an updated finite element model of the structure or can be obtained from system identification techniques.

The number of modes significantly contributing to the response at a frequency ω can become less than the number of stochastic forces to be estimated. The problem of estimating the force PSD from a set of output measurements then becomes ill-posed and rank deficient. The stochastic force covariance matrix is estimated hereafter by averaging the force PSD over a number of frequencies where the ill-posedness of the problem is minimal. The accuracy of the result at a frequency ω is therefore assessed based on a criterion proposed by Fabunmi [5].

IN SITU EXPERIMENT ON A FOOTBRIDGE

In this section, the proposed methodology is illustrated for an in situ experiment on a real structure, in the presence of modeling errors, measurement errors and ambient excitation. The structure under consideration is a footbridge, located in Ninove (Belgium). It is a two-span cable-stayed steel bridge (Figure 1) with a main and secondary span of 36 m and 22.5 m, respectively.

A system model is constructed from a FE model of the bridge. The FE model has been updated using a set of experimental modal parameters, which have been obtained through a combined output-only [6] and input-output system identification procedure [7]. TABLE I presents a comparison between the experimentally identified modal characteristics and those calculated from the updated FE model. The MAC value [8] indicates the correspondence between the measured mode shapes and those obtained from the FE model. For nearly all vertical bending modes and torsional modes, a good correspondence is obtained between the identified modal characteristics and the modal characteristics obtained from the FE model (natural frequency, mode shape).

A reduced-order discrete-time state-space model is constructed from the updated FE model of the footbridge, applying a zero order hold assumption on the force. The model includes the 15 modes listed in TABLE I. The mass normalized mode shape is assumed to be known from the FE model. The natural frequency as well as the modal damping ratio are taken as the experimentally identified values.

Figure 1. The footbridge in Ninove, Belgium.

TABLE I. COMPARISON BETWEEN THE EXPERIMENTALLY IDENTIFIED MODAL CHARACTERISTICS AND THE MODAL CHARACTERISTICS OF THE UPDATED FE MODEL (f_{id}: identified natural frequency, ξ_{id}: identified modal damping ratio, f_{fem}: undamped natural frequency updated FE model, ϵ: error f_{fem} w.r.t. f_{id}, MAC: MAC-value).

No.	f_{id} [Hz]	ξ_{id} [%]	f_{fem} [Hz]	ϵ [%]	MAC [−]	Description
1	2.98	0.4	2.93	1.59	0.99	1st vertical bending main span
2	3.08	0.67	3.15	-2.55	0.97	1st lateral bending main span
3	3.81	0.58	3.77	1.13	0.95	1st combined lateral bending
4	5.84	0.89	5.56	4.79	(*)	1st lateral bending secondary span
5	6.00	0.67	5.92	1.36	0.99	1st vertical bending secondary span
6	6.92	0.29	7.23	-4.44	0.95	1st torsional main span
7	8.00	0.76	7.77	2.79	0.99	2nd vertical bending main span
8	9.84	0.48	10.06	-2.29	0.76	2nd combined lateral bending
9	10.98	0.87	11.01	-0.27	0.89	1st torsional secondary span
10	12.52	1.62	12.97	-3.56	0.94	3rd combined lateral bending
11	13.55	0.52	13.24	2.27	0.97	3rd vertical bending main span
12	14.02	0.16	14.25	-1.61	0.89	3rd lateral bending main span
13	14.71	0.57	14.29	2.92	0.97	2nd vertical bending secondary span
14	17.29	0.14	17.3	-0.04	(*)	4th lateral bending main span
15	18.57	0.46	18.16	2.26	0.91	4th vertical bending main span

(*) Low MAC-value due to irregular identified mode shape.

Estimation of noise statistics

The data used in the following analysis contain the response of the footbridge to ambient excitation. The measurement setup is shown in Figure 2. The response data consist of the vertical (z-direction) and lateral (y-direction) acceleration measurements for each of the five sensor locations. A time period of 400 s is considered.

The response data and the system model are now used to estimate the ambient force covariance matrix. For each acceleration signal a stochastic force is assumed, acting at the same node and along the same direction. In this way, a set of 10 stochastic forces is estimated from 10 acceleration measurements. The averaged force PSD values are shown in Figure 3. Since the force covariance matrix has real elements, the averaging is performed for the real part of the PSD values over the frequency range where the ill-posedness of the problem is minimal [5]. In general, the force covariance of the lateral forces is larger than the force covariance of the vertical forces. The largest force variance occurs at node 27. The covariance matrix obtained is now used to calculate the noise covariance matrices.

Force identification

The data used in the following analysis are obtained during the excitation of the footbridge by two vertical hammer forces at the bridge deck, one at node 27 and one at node 48. The measurement setup is shown in Figure 4. A time period of 25 seconds is considered, containing one impact at both nodes, see Figure 5.

The force identification is performed using the proposed joint input-state estimation algorithm. During the actual experiment, only accelerations have been measured. Displacement signals, however, are required for the stability of the joint input-state estimation algorithm and the uniqueness of the estimated quantities. They are obtained

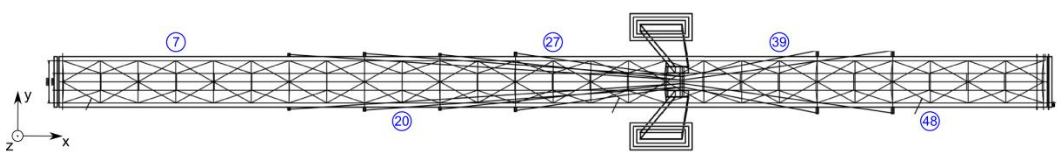

Figure 2. Overview of the measurement setup used for the estimation of the noise statistics. Accelerometer positions are indicated in blue.

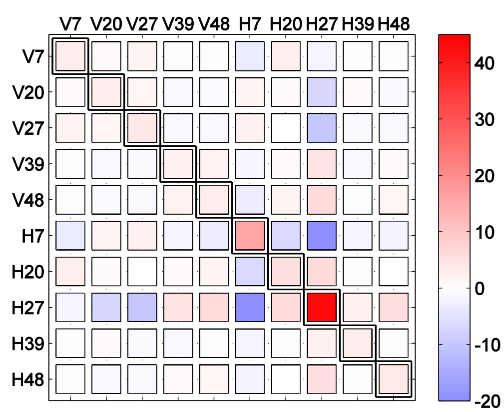

Figure 3. Estimated stochastic forces covariance matrix in [N^2] (Vi: vertical and Hi: lateral force node i).

by integrating the corresponding acceleration signals. We do not suggest this as the way to follow, but rather as a way to illustrate the proposed methodology for this set of measurements. The output vector consists of two acceleration signals, i.e. the vertical accelerations at nodes 27 and 48, and two displacement signals, i.e. the vertical displacements at nodes 20 and 39 (Figure 4). The integration of the acceleration signals is performed in the frequency domain. The displacement signals obtained are passed through a fifth order Chebyshev type I high-pass filter with a cutoff frequency of 1 Hz and a filter ripple of 0.1 dB.

The noise covariance matrices **Q**, **R**, and **S** are calculated from the estimated stochastic force covariance matrix, according to Eq. (20), and assuming a resolution of 10^{-6} ms^{-2} for the acceleration measurements and of 10^{-6} m for the displacement signals. The initial state estimate vector $\hat{\mathbf{x}}_{[0|-1]}$ is assumed zero and its error covariance matrix $\mathbf{P}_{[0|-1]}$ is assigned a diagonal matrix with values of 10^{-10} on its diagonal.

The reconstructed forces are characterized by a low frequency drift. This is due to inaccuracies in the low frequency content of the displacement signals, but also to the large influence of ambient excitation at low frequencies. For frequencies up to 2.6 Hz, the identified force signals are characterized by a large error. The low frequency drift is removed by applying a fourth order Butterworth high-pass filter with a cutoff frequency of 2.6 Hz to the identified force signals. The measured force signal is filtered using the same filter. The uncertainty on the identified force signals is quantified by means of the force error covariance matrix $\mathbf{P}_{\mathrm{p}[k|k]}$ (see Eq. (9)). The diagonal elements of this matrix are a measure for the variance of the estimation error and are used to define an uncertainty bound on the results obtained.

The results of the force identification are shown in Figure 6 and Figure 7 for the hammer forces at node 27 and node 48, respectively. Both for the force applied at node 27 and at node 48 a very good correspondence between the measured and the identified force signal is obtained. In addition, the uncertainty bound on the results gives a good indication of the true estimation error. The first few seconds after the impact, the uncertainty interval, however, does not contain the measured force signal. This is due to low frequency ambient excitation and modeling errors, which are not directly accounted for.

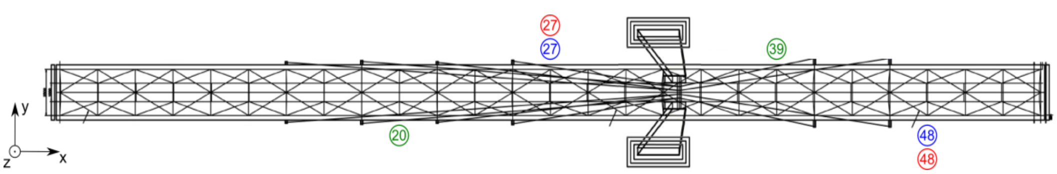

Figure 4. Overview of the measurement setup used for the force identification (red: force locations, blue: acceleration location, green: displacement location).

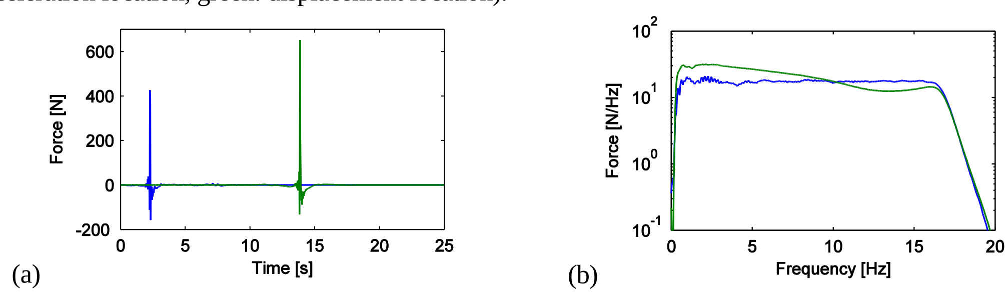

Figure 5. (a) Time history and (b) frequency content up to 20 Hz, of the hammer forces applied vertically to the bridge deck (blue: node 27, green: node 48).

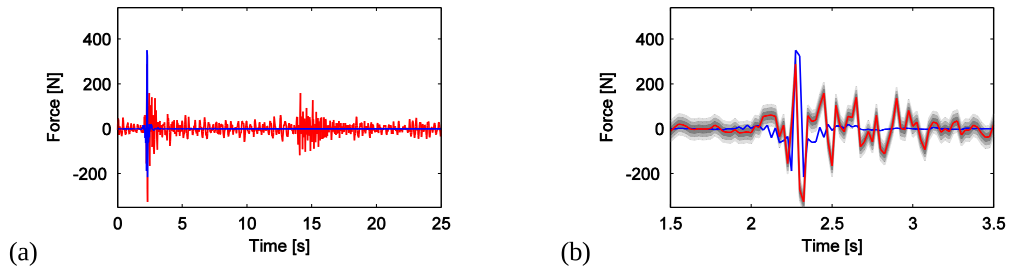

Figure 6. (a) Complete time history and (b) detail of the time history of the identified hammer force at node 27 (blue: measured, red: identified, dark grey to light grey: 1**σ** – 3**σ** uncertainty interval).

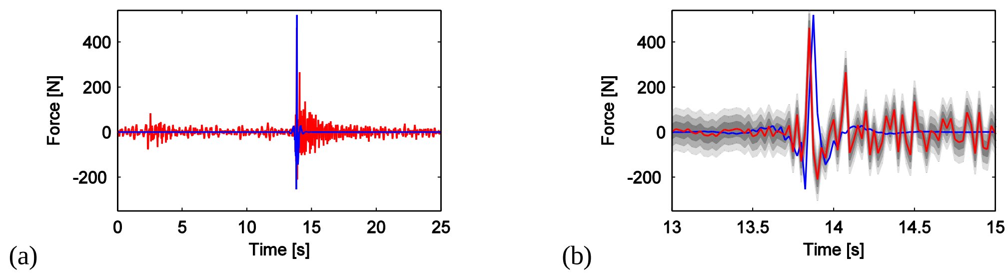

Figure 7. (a) Complete time history and (b) detail of the time history of the identified hammer force at node 48 (blue: measured, red: identified, dark grey to light grey: 1**σ** – 3**σ** uncertainty interval).

CONCLUSIONS

An existing joint input-state estimation algorithm was extended for applications in structural dynamics. In addition, a method was proposed to identify the process noise and measurement noise characteristics. The methodology was illustrated for a set of data collected from an in situ experiment on a footbridge. Multiple hammer forces have been identified from a limited set of acceleration data. The identified force signals are a very good estimate of the true applied forces. In addition, taking the identified noise statistics into account leads to an indication of the estimation error by means of an uncertainty interval.

REFERENCES

1. S. Gillijns, B. De Moor. 2007. "Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough," *Automatica*, 43(5): 934–937.
2. R.K. Mehra. 1970. "On the identification of variances and adaptive Kalman filtering," *IEEE Transactions on Automatic Control*, 15(2): 175–184.
3. K. Myers, B. Tapley. 1976. "Adaptive sequential estimation with unknown noise statistics," *IEEE Transactions on Automatic Control*, 21(4): 520–523.
4. J.S. Bendat, A.G. Piersol. 2000. *Random data: analysis and measurement procedures*. John Wiley & Sons, 3rd edition.
5. J.A. Fabunmi. 1986. "Effects of structural modes on vibratory force determination by the pseudoinverse technique," *AIAA Journal*, 24(3): 504–509.
6. B. Peeters, G. De Roeck. 1999. "Reference-based stochastic subspace identification for output-only modal analysis," *Mechanical Systems and Signal Processing*, 13(6): 855–878.
7. E. Reynders, G. De Roeck. 2008. "Reference-based combined deterministic-stochastic subspace identification for experimental and operational modal analysis," *Mechanical Systems and Signal Processing*, 22(3): 617–637.
8. R.J. Allemang, D.L. Brown. 1982. "A correlation coefficient for modal vector analysis," In: *Proc. 1st Int. Modal Analysis Conf.* Orlando, 110–116.