Integrated network linear-quadratic model predictive control for disturbance on freeways

Yu Han, Yufei Yuan, Serge Hoogendoorn

Department Transport & Planning
Delft University of Technology
Delft, Netherlands
{y.han-1, y.yuan, s.p.hoogendoorn}@tudelft.nl

Abstract—**We propose an integrated approach for dynamic route guidance and ramp metering using Linear Quadratic Model Predictive Control (LQMPC). The main objective is to maximize the throughput of the network by guiding traffic flow towards to insufficiently utilized infrastructures. Due to the linear property of the LQMPC, it is fast enough for real-time online traffic control with a short control step (30 second for example). Simulation results from a case study demonstrate that the proposed traffic control approach has great potential to solve congestion when disturbances occur on freeways.**

Keywords—freeway disturbance; model predictive control; route guidance; ramp metering;

I. INTRODUCTION

Disturbances such as roadworks and accidents often occur on freeways. This will cause large capacity reductions, for instance due to temporary lane closures, speed limitation, etc. Therefore, disturbance areas will experience heavy congestion if traffic demand keeps constant.

Many control strategies have been proposed to solve freeway congestion, whereas the most widespread one is Model Predictive Control (MPC). Karimi et. al. [1] proposed an integrated model predictive control measure of dynamic route guidance and ramp metering, using METANET [11] as the simulation model. Hegyi et. al. [2] developed a model predictive control for the coordination of ramp metering and variable speed limits, the traffic flow model for their prediction is also METANET. We noticed that most of the existing model predictive control strategies are based on the second order traffic flow model such as METANET. This kind of model describe the traffic flow in a more specific way, so as to describes the traffic flow process accurately [3]. However, this feature of the model increases the complexity of computation, consequently the computation speed is low and it is not suitable for controlling severe congestion areas. The MPC approach has been proved of its effectiveness in many studies. However, it could not be used

in reality for on-line controlling of large- scale network due to its computation complexity. Therefore, how to increase the efficiency of controller becomes the main challenge for large-scale networks nowadays.

An efficient traffic flow model for road traffic control is the so-called store-and-forward model. This modeling approach describes the network traffic flow process in a simplified way. The store-and-forward model was first suggested by Gazis and Potts [4] and since then it has been used in many studies concerning urban road control. Diakaki et. al. [5] applied a multivariable feedback regulator approach to calculate the on-line signal control plan as a linear-quadratic optimal control problem, based on the store-and-forward model. Aboudolas and Papageorgiou [6] compared three store-and-forward model based strategies for an optimal control problem, which are linear quadratic formulation, quadratic programming, and nonlinear optimal control. In this paper, our prediction model is also a store-and-forward based model, we will refer it as cell based store-and-forward model. Our model is evolved from the research of Tung et. al. [7]. In their research, the whole traffic flow dynamics was viewed as an optimization problem. Here we summarize traffic dynamics and consider it as a traffic flow propagation process. The main difference between our work and the previous research is that we use integrated control measures such as routing or signal control, rather than one single control measure. Therefore, we extended the previous methodology for multiple purpose.

In this paper we propose a network wide control strategy including dynamic route guidance system (DRGS) and ramp metering signal (RMS) using Linear Quadratic Model Predictive Control (LQMPC). The LQMPC is a new control algorithm that searches the optimal control vector to minimize the predefined objective function, with quadratic programming and linear constraints. We consider networks consisting of several types of road, including freeway, major arterial road and urban road. Our controller is designed particularly for disturbances such as roadwork or accidents on freeways, to balance the pressure of the whole network by guiding redundant traffic to infrastructures that are insufficiently utilized. We apply it to a case study to validate its effectiveness.

The setup of the paper is given as follows. Section II describes methodology of this paper, including the cell-based store-and-forward model, LQPMC, and drive route choice modeling. In section III we illustrate our approach for a case study. In section IV we draw some general conclusions and propose future works.

II. METHODOLOGY

A. Cell-based store-and-forward model

The integrated road network is represented as a directed graph with cells $x \in X$ and links $j \in J$. See Fig. 1 for an example.

Cells are built based on homogeneous road segments. Each segment is represented by one cell except for segments before bifurcations. For segments before bifurcations, cell number depends on the number of downstream branches, we call them decision cell, D (cells 2 and 3 in Fig. 1). The state of each cell, $x_k(n)$, is an instantaneous continuous count of vehicles in cell k at time step n. The vectors of cell states are defined as: $X = [x_1, x_2...x_n]'$. Each cell has a queue maximum queue length C_k, which depends on the cell length, lane number and speed limit.

Vehicles flowing out of a cell move to downstream cells through a predefined *link*. Each link represents a connection between its source cell and destination cell. Every link has its associated flow rate, f_j, which is the maximum permitted vehicles from source cell to destination cell in one time step. The value is based on traffic measurement. The vectors of link states are defined as $U = [u_1, u_2...u_n]'$. u_j is the time fraction of links, the destination of which is not a decision cell. Its value depends on the vehicle number of source cells and available spaces of destination cells. For those links with a destination cell of decision cell(u_1, u_2 in Fig. 1), u_j denotes the splitting rate, which is decided by the route choice behavior of travelers.

The network state dynamic is determined by cell state and link state. The network state evolves as follows:

$$x_k(n+1) = x_k(n) + a_k(n) + \sum_{\{j:d_j=k\}} u_j(n)f_j - \sum_{\{j:s_j=k\}} u_j(n)f_j$$

$$u_j = \min(x_{s_j} + i_{s_j}, c_{d_j} - x_{d_j} - o_{d_j}, f_j) / f_j$$

$$0 \le x_k(n) \le c_k, \forall k, n$$

$$x_k \notin D$$

(1)

where a_k denotes vehicles arrival to cell k from outside of the network, d_j is the destination cell of link j, s_j is

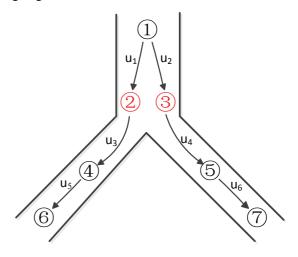the source cell of link j. i is incoming vehicles to cell x, o is outgoing vehicles to cell x.

Fig. 1. Description of cells and links of a network. Red cells are decision cells.

B. LQMPC approach for traffic control

Large amount of MPC research concerning traffic control has been proposed since the 21th century. MPC is a model based control approach that is based on the optimization of control inputs that improves a given performance criterion. The main advantages of MPC are that it takes the effect of control input on future system states, and that it is able to take both equality and inequality constraints of the manipulated and controlled variables into account [8]. However, most of the MPC approaches have the disadvantage that the computation time is so high that it is difficult to be used for on-line control. LQMPC was first proposed by Tung et. al. [7], it is fast enough for on-line control for medium sized networks, due to the linear nature of the model. Its effectiveness for urban network control has been validated. Here we extend this methodology for freeway route guidance control and ramp metering control. The objective function and constraints are defined as follows:

$$\min \sum_{i=n}^{n+N+1} X(i+1)'QX(i+1) + RU(i)$$

$$s.t.$$

$$\begin{bmatrix} B \\ H \end{bmatrix} \overline{U} \le \begin{bmatrix} g_B \\ g_H \end{bmatrix}$$

(2)

Where N is the predict time horizon, X is the network state excluding the final destination cell. Q is a constant matrix with all elements being 1. $R = -\varepsilon f$, $\varepsilon \square 1$. f is link flow rate vector. $U(i)$ is the control vector of the i time step. $\overline{U}$ is the control vector of the whole predicting time horizon.

$$B = \begin{bmatrix} Ds & 0 & \cdots & 0 \\ Dsd & Ds & & \vdots \\ \vdots & Dsd & \ddots & \\ Dsd & \cdots & Dsd & Ds \end{bmatrix} H = \begin{bmatrix} Dds & 0 & \cdots & 0 \\ Dds & Dds & & \vdots \\ \vdots & & \ddots & \\ Dds & \cdots & & Dds \end{bmatrix}$$

Ds is a matrix with elements (k, j) set to 1 if $s_j = k$ and 0 otherwise. Dd is a matrix with element (k, j) set to 1 if $d_j = k$ and 0 otherwise. $Dsd = Ds - Dd$, $Dds = Dd = Ds$.

g_B and g_H are N block column vectors, $g_B(i,1) = X(i) + \sum_{j=n}^{n+i-1} a(i), g_H(i,1) = C - X(i) - \sum_{j=n}^{n+i-1} a(i)$.

The objective function quadratically penalizes the number of vehicles in the network while linear penalizes the network propagation vector, so the traffic flow of the network propagates the same way as in our cell-based store-and-forward model.

C. Driver route choice modeling

To simulate the effect of our control measures, first we should assume the route choice behavior of drivers. Traveler's route choice behavior includes pre-trip route choice and en-route route choice [9]. In this paper, we only consider the pre-trip route choice behavior of travelers. Logit model is a well-known behavior model, which determines the choice of consumer based on the cost of several alternatives [10]. It is widely used for pre-tip route choice of traveler. Assume that we have two possible choice m_1 and m_2 . For the calculation of the splitting rate out of travel time difference between two alternatives the logit model results in

$$\beta^m = \frac{\exp(-\sigma \times t^m)}{\exp(-\sigma \times t^{m1}) + \exp(-\sigma \times t^{m2})} \quad (3)$$

for $m = m_1$ or $m = m_2$. Where t is the route travel time. The parameter σ describes how drivers react on a travel time difference between two alternatives.

We assume travelers' pre-trip route choice behavior follows the logit model, and they have perfect prediction of the travel time on each route due to their day to day experience, so the splitting rate u' of every time step could be known. LQMPC provides the optimal splitting rate for the maximum throughput of the network. However, drivers still have their own route preference, so only part of drivers will follow the guidance. Here we assume a compliance rate a , the final splitting rate after route guidance would be:

$$v = ((x_s \cdot u' - x_s \cdot u) \cdot (1 - a) + x_s \cdot u) / f \quad (4)$$

III. CASE STUDY

A. Network set up

The conceptual network is shown in Fig. 2. There are five heterogeneous links in the network. The detail network description is shown in Fig. 3, it contains two origins and one destination. Numbers on solid lines are cell number and those on dashed lines are link number. Cells on the left path denote freeway cells, on the middle path denotes urban road cells and on the right one represent major arterial road cells. Horizontal cells are ramp cells. Cell 25, 26 and 29 are on-ramps, 27 and 28 are off-ramps.
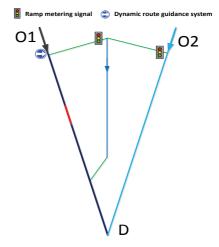

Fig. 2. Conceptual network of the case study. There are two origins and one destination in this network. The network contains five heterogeneous links. The black line represents freeway; red line represents disturbance areas; light blue line represents major arterial road, dark blue line is urban road and green line are ramps. There are two RMSs and one DRGS in this network control system.
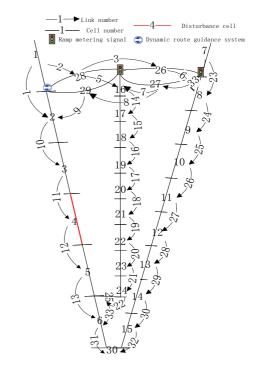
Fig. 3. The traffic network of the case study consist of two origins 1, 7 and one destination 30. Three paths from left to right are freeway, urban road and major arterial road. Numbers on the solid line denotes cell number; numbers on the dashed line denotes link number.

The free flow speeds of freeway, major arterial road, urban road and ramp are chosen respectively as 120km/h, 90km/h, 60km/h and 30km/h. The simulation step is set to 30s. Vehicles are not allowed to travel across one cell within one simulation step. Therefore, according to the free flow speed, the lengths of each kind of cells are 1 km, 750 m, 500 m and 250 m. Only one direction is considered, which is from 1, 7 to 30, so all links can be considered to be unidirectional.

For each O-D pairs, drivers can choose whether they travel via the freeway or the major arterial road or the urban road. A DRGS is installed at the end of cell 1, to guide freeway traffic flows when disturbance happens. Two RMSs are installed at the end of cell 26 and the end of cell 7. The role of the left one is to optimize the bifurcated flow from cell 28 to 16 and 26. The role of the right one is to limit the flow from the major arterial road to the freeway, from cell 7 to cell 27.

B. Model and control parameters

The capacities of the four kinds of cells are chosen respectively equal to 2000, 1800, 1200 and 1200 veh/h/lane. Freeway cells have three lanes, major arterial road and urban road have two lanes, the ramp has one lane. Therefore, the corresponding flow rate of each link is 50, 30, 20, 10 (veh/time step). We assume the average car's length is 5m and the minimum distance between cars in a congested road is 2.5m. Hence, the maximum queue lengths of four kinds of cell are 405, 200, 135, 35 (veh/cell).

The main advantage of linear-quadratic MPC is the efficiency, so we can let the control step to be very short. The following section will also show the advantage of short control step. We set the simulation step T_1 to be 30s, while the control step $T_2 = NT_1$, N is an integer. The prediction horizon $T_3 = 10T_1$ corresponds to a prediction of 5min ahead.

C. Scenario

We deploy the same model, the cell-based store-and-forward model, for both validation and assessment. We do not use more complicated traffic flow models for the assessment because our model can capture the main features of traffic dynamics such as spillbacks.

We simulated the morning peak from 6:00 to 11:00. Demands of the two origins are shown in Fig. 4. We have five scenarios for our case study.
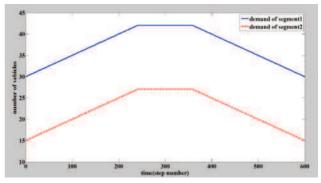

Fig. 4. Demand of origin 1 and 2. The peak demand of cell 1 is 42veh/step and cell 2 is 27veh/step.

1. User equilibrium of the network without disturbance. We assumed travelers have perfect knowledge of which route to choose due to their day-to-day experience. Route preference was also considered. Traffics from cell 1 always followed the path of the freeway, traffics from cell 7 chose freeway or major arterial road based on the logit model. Travel time was free flow travel time. The value of σ is set to 60, based on reasonable experiment results. The urban road was never used.

2. System optimal of the network without disturbance. We assumed all travelers followed the guidance of our MPC, so we would know the improvement of network performance in contrast with scenario 1.

3. System optimal of network with disturbance. We assumed disturbance like roadwork or accident, which lead to large capacity reduction happen on cell 4, the flow rate of link 3 and 4 dropped to 30veh/step. The maximum queue length of cell 4 will drop to 270.

4. No guidance of network with disturbance. We assumed travelers did not know the disturbance, they followed their previous route choice at the bifurcation node, so the splitting rate of scenario 1 was used as the input to this scenario. We assumed that travelers would change their route only when the queue of the forward cell was very large (300 for freeway and 150 for arterial road).

5. Integrated control of network with disturbance. Our MPC provided the optimal splitting rate at every node, different compliance rates (0.2, 0.5, 0.8) to the DRGP were considered.

D. Simulation result

We have simulated the network of the case study for scenarios given above. Below we discuss some most relevant results.

In our experiment, the whole simulation process time (600 time step) of each of the five scenarios is less than 12 seconds, by using a normal computer. This is much faster than other MPC approaches.

Fig. 5 shows the network throughput difference with and without MPC. The blue line indicates the network throughput difference between scenarios 1 and 2, while the red line indicates the network throughput difference between scenarios 3 and 4. We can easily conclude that in this case, our MPC improves the network performance much better in disturbance condition than in normal condition.
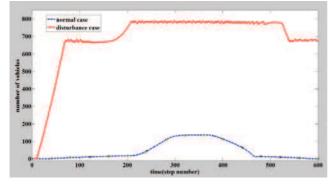

Fig. 5. Difference of vehicle number that reach the destination with and without traffic control.

Fig. 6 and 7 shows the cell density of scenario 3 and 4. In scenario 3, no control measure was implemented into the network. The congestion first emerged on cell 3, and then it spilled back to cell 2. Afterwards more traffic flows chose the major arterial route, so cell 8 began to become congested. The biggest difference between scenario 3 and scenario 4 is that scenario 4 makes full use of the insufficient utilized infrastructures at the beginning of the simulation process. From the picture we notice that the utilization of major arterial road and urban road are earlier than scenario 3, so the pressure of bottleneck (cell 3) will be lower.
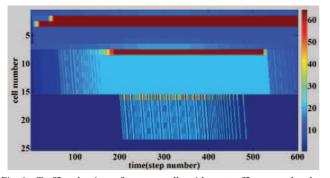

Fig. 6. Traffic density of every cell without traffic control when disturbance happens
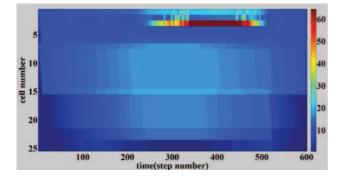

Fig. 7. Traffic density of every cell with traffic control when disturbance happens

Fig. 8 shows the total number of vehicles in the network (exclude cell 30, the destination cell), with all scenarios. Our MPC can solve the congestion within about 200 time steps if all travelers follow the guidance. If 80 percent travelers follow the guidance, the disappearance of congestion will be 400 time steps. If 50 percent of travelers follow the guidance, more than 400 time steps will be needed. Therefore in this case, to solve the congestion within the morning peak, the road authority should take measures to make more than 80% travelers comply with the guidance, this is what we will research afterwards.
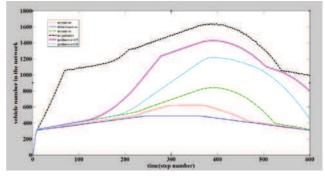

Fig. 8. Total vehicle number in the network of every scenario

Fig. 9 shows the queue length of the bottleneck (cell 4). The result is similar as Fig. 8. It also indicates that 80% compliance would result in an acceptable result.
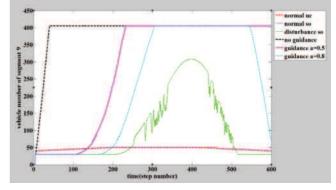

Fig. 9. Queue length of the bottleneck of every scenario

We also demonstrate the advantage of short control step by changing the value of the prediction time step N in scenario 2. Fig. 10 shows the different performance of the network when $N = 1, 2, 5$. Fig. 11 shows time fraction values of link 2 (the off ramp link) when $N = 1, 2, 5$. In figure 10, the performance of $N = 1, 2$ is much better than that $N = 5$. Even though the blue line is close to the red line, it also makes some difference especially when demands beyond the capacity (step 200 to 300). The red line is slightly better. Therefore, the shorter the control step, the better performance of the network will be. Computation time is a great challenge to shorten the control step. In our case, when $N = 1$, the whole simulation time is 11.88 seconds; $N = 2$ corresponds to 11.81 seconds; $N = 5$ corresponds to 11.67

seconds. Thus our controller is fast enough to make the step short. Fig. 11 explains why the performance is different in Fig. 10. The red line indicates the most suitable splitting rate of link 2. If the route guidance deviates this most suitable value, such as the blue line and black line, the network performance will be poor.
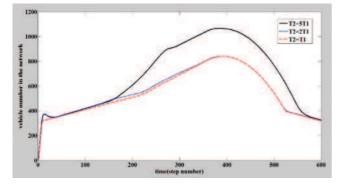

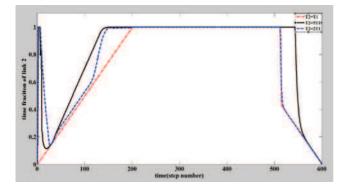Fig. 10. Total vehicle number in the network of scenario 2 when N =1, 2, 5.

Fig. 11. Time fraction of link 2 of scenario 2 when N =1, 2, 5.

IV. CONCLUSION AND FUTURE RESEARCH

In this work, we have proposed an integrated approach for dynamic route guidance and ramp metering control using Linear Quadratic Model Predictive Control. The aim of our approach is to maximize network throughput. We have simulated five scenarios in a road network with constant demand. Simulation results show that LQMPC has great potential for real-time network-wide traffic control. The computation is fast enough to ensure on-line traffic control. Disturbance on freeways will cause heavy congestion if no control measures are implemented. Our approach succeeds to relieve congestions by guiding traffic flow to insufficient utilized facilities through DRGS, and limiting the entering vehicles of every path via RMS during every cycle. We also consider travelers' compliance behavior, which could provide effective information to the road authority.

In future research, we will model the urban traffic network in a more detailed way, put urban intersections into consideration; consider trip generation inside networks; make this approach more generic by extending it to a large network with multiple destinations; optimize the route guidance through anticipating travelers' behavior.

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