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Technical Committee No. 19: Load Factor Design

State of Art Report No. 3

LOADING FUNCTION

J. Strating
Stevin Laboratory
Delft University of Technology
Delft, The Netherlands

archief exemplaar

1. INTRODUCTION

Loads acting on structures are generally caused by a large number of causes which cannot be controlled; therefore, loads are variable and of random nature (1). Loads defined as those elements of the environment which cause stresses in the structure can be divided in (2)

1. imposed loads
2. loads resulting from forced deformation
3. loads resulting from restraint of deformations

Another distinction between loads is possible. They dan be regarded as simple loads, varying only in magnitude, and complex loads varying in magnitude, location and distribution. Furthermore, loads can be considered random in their simultaneous occurence.

2. THEORY

In classical reliability analysis the probability of failure P_F is obtained from (3)

$$P_F = \int_0^{\infty} \int_0^{S} f_S(s) \cdot f_R(r) \, ds.dr. \qquad (1)$$

$$P_F = \int_0^\infty f_S(s)\, F_R(s)\, ds = \int_0^\infty f_R(r) \left[1 - F_S(r)\right] dr \qquad (2)$$

$$f_S(s)\, ds = \text{prob}\, [s < S < s + ds]$$

$$f_R(r)\, dr = \text{prob}\, [r < R < r + dr]$$

$$F_S(s) = \text{prob}\, [S < s]$$

$$F_R(r) = \text{prob}\, [R < r]$$

Equation (1) is derived assuming that the load and resistance are independent. Application of Eq. (1) assumes complete knowledge of the distribution functions of loads and resistances. This, however, is impossible to achieve. The true failure probability cannot be determined because of limited load data and knowledge of the resistance function. Therefore, approximate solutions are necessary.

In load factor design calculations are done for a specific safety level say, 10^{-6}, 10^{-5} or 10^{-4}. The probability of failure following from Eq. (1) or (2) holds for the same time interval for which the distribution functions of the loads and resistances are defined (4). It is generally assumed that the resistance is constant in time; this assumption seems justified for steel structures, except for fatigue problems. This leaves the influence of the time interval on the load distribution to be considered. To each loading a basic time interval or natural period is assigned (5). In this interval the load can be considered constant. The length of this interval depends on the type of loading. It may be a 5 min period for wind or a yr period for occupancy loading. This problem of basic time intervals is closely related to the problem of load combinations. If the basic period is large, the probability of simultaneous occurence of maximum load values is large.

In the basic time interval concept, the distribution function of the load can be regarded as the distribution function of the values which the load can take in the successive basic time intervals. The distribution functions are taken as continuous functions.

The function of interest is

$p(S_N|S_1)$, the probability that the load is between S_N and S_{N+1} in the Nth interval given the first load was S_1 (6).

The complete loading history is taken into account by defining

$$p(S_N \mid S_1, S_2, \ldots S_{N-1}, R_1, \ldots R_N).$$

If the structure fails in the Nth interval, it is necessary that

$S_N > R_N$ but $S_1 < R_1$; $S_2 < R_2$; $S_3 < R_3 \ldots S_{N-1} < R_{N-1}$.

The probability of all combinations is given

$$P_{F_N} = \int_0^\infty \int_0^{R_1} \ldots \int_0^\infty \int_{R_N}^\infty p(R_1).p(S_1).p(R_2|R_1,S_1) \, p(S_2|S_1,R_1) \ldots \ldots dS_N, dR_N . dS_{N-1} \, dR_{N-1} \ldots\ldots dS_1 \, dR_1 \qquad (3)$$

P_{F_N} is prob. of failure in the Nth interval

If the resistance does not change with time (that is from one interval to another) and if the loads and resistances are independant

$$p(S_2|R_1, S_1) = p(S_1)$$

$$p(R_2|R_1, S_1) = 1 \quad \text{if } R_2 = R_1$$

$$= 0 \quad \text{if } R_2 \neq R_1.$$

This reduces Eq. (3) to:

$$P_{F_N} = \int_0^\infty p(R) \left[\int_0^R p(S)ds \right]^{N-1} . \left[1 - \int_0^R p(S)ds \right] dR \qquad (4)$$

Eq. (4) describes the probability of failure at exactly the Nth interval.

The probability of surviving N intervals is equal to

$$P_{SN} = \int_0^\infty p(R) \left[\int_0^R p(S)ds \right]^N dR \qquad (5)$$

The probability of failure in the first N intervals:

$$P_{FN} = 1 - P_{SN} = \int_0^\infty p(R) \left[1 - \left\{\int_0^R p(S)ds\right\}^N\right] dR$$

$$= \int_0^\infty p(R) \left[1 - F_S(R)^N\right] dR \qquad (6)$$

$$\text{since} \int_0^R p(R)\, dR = 1$$

The same result was derived by Loof (5).

The above formula also applies to probability of failure after N load applications.

Load combinations are important in reliability analysis. Little is known, however, about combinations of instantaneous load values. The distribution functions of combined loads cannot be derived from the distribution functions of the individual loads if the covariance functions are not available, as is generally true.

Combined loads are greatly influenced by the duration of each load. High wind loads occur in short time intervals. The probability of simultaneous occurrence of high wind load and high live load depends mainly on the duration of the live load. The basic time interval (or natural period) concept provides a means of taking this into account. The basic time intervals should be synchronized in such a way that a number of smaller intervals fits into one larger interval. The loading in the larger interval may then be regarded as permanent. The probability of failure in the larger interval is determined from Eq. (6). Next the procedure is repeated for a number of larger intervals. The important feature of this procedure is that first the loads are superposed and then the transition to a larger interval is done.

It has been suggested (3,4) that if the probability of surviving the first interval is

$P_{S1} = 1 - P_{F1}$, then the probability of surviving N intervals is found from

$$P_{SN} = (1 - P_{F1})^N \rightarrow P_{FN} = 1 - (1 - P_{F1})^N \qquad (7)$$

and if P_{F1} is small: $P_{FN} \approx N\, P_{FN}$

This, however, implies that at the beginning of each time interval a new resistance is chosen. This is definitely not true. The resistance might be changed by the previous loadings and time (fatigue) but is not chosen independently at each time interval.

The first time interval (or load application) plays a dominant role in reliability analysis. If a structure is put into service, survival of the first interval means that the strength of the structure is not extremely low. A structure that does not fail in the first time interval has a much larger probability of surviving the next intervals.

An improvement to Eq. (7) would be to emphasize the importance of the first interval.

$$P_{FN} = P_{F1} + \sum_{n=2}^{N} P_{F|S1} \qquad (8)$$

$P_{F|S1}$ is the probability of failure given survival for $S = S_1$

The loads in each time interval (or application) have been considered independently; if, however, the loads are ideally dependent, survival of the first interval implies survival of all intervals. Therefore:

$$P_{FN} = P_{F1}$$

C.A. Cornell has given these bounds on the probability of failure (7)

$$P_{F1} \leq P_{FN} \leq P_{F1} + \sum_{n=2}^{N} P_{F}|S_1 \qquad (9)$$

3. First Order, Second Moment Theory

Because of the almost impossible task of collecting enough data to derive a complete distribution function for the loading, methods have been developed which are, in a probabilistic sense, less severe.

Especially Cornell's first order, second moment theory has been widely discussed (8, 9, 10, 11). The only parameters in this theory are the first two moments of the distribution function or mean value m, and (if taken around the mean) variance σ^2. The relations between the variables are linearized. A distinction is made between load and load effect. The latter resulting from the first after structural analysis. Load effects are described in terms of characteristic values.

$$S^* = m_2 + k\sigma_s$$

$$= m_s \left[1 + k.V_s\right] \qquad (10)$$

m_s = mean value

σ_s = stand. dev.

$V_s = \frac{m_s}{\sigma_s}$ = coefficient of variation (C.O.V.)

V_s not only represents the variability in the load itself but also uncertainties in structural analysis. It is proposed that the load effect is a function of the following parameters

$$S = c \,.\, T \,.\, E \qquad (11)$$

c = constant that follows from analysis

T = environmental load

E = factor reflecting professional engineering uncertainties

If these factors are assumed independent

$$V_s^2 = V_T^2 + V_E^2$$

The characteristic load is written as

$$S^* = m_s \left[1 + k \sqrt{V_T^2 + V_E^2}\right]$$

V_T can be found from load analysis while V_E can be found comparing calculated load effects with measured load effects (12)

(V_E is a function of the assumptions in structural analysis)

Load combinations (if both occur in the same time interval) can be easily accounted for. As an example consider the combined effect of dead load and live load.

$$S = S_{DL} + S_{LL}$$

$$\sigma_S = \sqrt{\sigma^2_{DL} + \sigma^2_{LL}} = \alpha\,(\sigma_{DL} + \sigma_{LL})$$

α is a factor necessary to carry out the linearization. The value of α depends on the ratio $\frac{\sigma_{DL}}{\sigma_{LL}}$. A value $\alpha = 0.75$ seems most appropriate. (13)

$$S^* = m_s \left[1 + k_s V_s\right] \qquad m_s = m_{DL} + m_{LL}$$

$$= m_s \left[1 + k_s \cdot \frac{\alpha(\sigma_{DL} + \sigma_{LL})}{m_{DL} + m_{LL}}\right] \qquad (12)$$

The same procedure is followed for the resistance function. The probability of failure is calculated from

$$R^* \geqq S^*$$

Rosenblueth and Esteva (11) have applied Cornell's theory to combinations of dead and live load. Their modified load effect is described as:

$$S_{io} = w_{oi} \int \alpha_i\, f_o\, dA$$

i = DL; LL

$w_{oi}\alpha_i$ = load intensity

f_o = influence function

α_i is a function of the location.

$$\bar{S}_{io} = \bar{w}_{oi} \int f_o\, dA$$

$$V(S_{io}) = \left[\frac{V^2(\alpha_i)}{A} \cdot u^2 + V^2(w_{oi})\right]^{1/2}$$

A = tributary area

$$u^2 = \frac{\overline{f_o^2}}{(\bar{f}_o)^2} \text{ taken over } A$$

For most cases $f_o > o \rightarrow u = 1.2$ is a good approximation. If the total live load is considered over A $f_o = 1$ and $u^2 = 1$.

In their original paper Rosenblueth and Esteva suggest that if the probability of having no live load on a given area is much greater than the probability of failure (which is generally true) then the maximum load is computed assuming that load is acting only on those portions of the tributary area which have the same sign.

Mean value and C.O.V. are respectively:

$$\bar{S}_{LO} = \Sigma w_{OL} \int f_o \, dA$$

$$V(S_{LO}) = \left[\frac{V^2(\alpha_i)}{\Sigma A} u^2 + V^2(w_{OL}) \right]^{1/2}$$

The summation is over every portion of the tributary area where f_o has the same sign.

4. Loading as a random process

It has been assumed in the previous sections that a structure will fail if $S > R$. It is possible, however, that failure occurs gradually due to damage accumulation. The amount of damage at the end of a preset time interval is the limiting state. The probability of reaching this state should be sufficiently small. In order to carry out such an analysis, the loading as a function of time must be available (1).

It should be understood that the concept of limit states and loading functions discussed before are special cases of this general theory. If the loading is considered a function of time, so is the process of damage accumulation. This kind of analysis applies mainly to fatigue and seismic problems.

In seismic analysis the event of interest is the excursion of the load over a certain level, the strength of the structure. To ease the calculations the strength is regarded as a deterministic value;

This, however, is no drawback of the general theory because it is simple to introduce the distribution function of R. In fatigue analysis the strength function is defined by a "cumulated damage theory". Such a theory relates the damage to a number of times a certain load value occurs. Therefore, the following events are of interest. Let the loading function be defined as:

$S = s(t)$

$\bar{P}\ (s > S \mid T)$ = prob. of at least one excursion of s over level S in the lifetime T

$P\ (S \mid T)$ = prob. density function of overloads S in T

$N_o(S)$ = mean number of overloads per unit time

S.O. Rice has shown that the mean number of excursions of a time function s (t) through interval $dS.d\dot{s}$ per unit time equals (14)

$$|\dot{s}|\ p(S,\ \dot{s},\ t)\ d\dot{s}$$

$$\dot{s}(t) = \frac{ds(t)}{dt}$$

The mean number of excursions of s (t) over level S per unit time is:

$$N_o(S) = \int_o^\infty \dot{s}p(S,\ \dot{s},\ t)\ d\dot{s}$$

The mean number of excursions over S in time T is:

$$N(S|T) = \int_o^T dt \int_o^\infty \dot{s}p(S,\ \dot{s},\ t)\ d\dot{s}$$

For stationary random processes this reduces to

$$N(S|T) = T\ .\ p(S) \int_o^\infty \dot{s}\ p(\dot{s})\ dt$$

because for stationary processes the correlation between s (t) and $\dot{s}$ (t) is zero. $p(s,\ \dot{s}) = p(s).p(\dot{s})$.

J.R. Rice derived that (15)

$$p(S|t)\ dt = N_o(S)\ \exp\left[-\ N_o(S).t\right]$$

$p(S|t)\ dt$ = prob. that the first excursion of S take place in time interval $[t,\ t + dt]$

The probability of no excursion of S in T

$$F(S|T) = 1 - \int_o^T p(S|t)\ dt$$

$$= \exp\left[-\ N_o(S).T\right]$$

$$P(S|T) = \frac{\partial(F|T)}{\partial S} = \frac{\partial}{\partial S}\left[\exp\left\{-\ N_o(S)|T\right\}\right]$$

Moses has shown that in reliability analysis the influence of a particular random model is slight (16). It is suggested, therefore, to choose a Gaussian random process to carry out the above analysis.

If $$p(s) = \frac{1}{\sqrt{2\pi.\sigma_s^2}}\exp\left[-\frac{(S-a)^2}{2\sigma_s^2}\right]$$

$$p(\dot{s}) = \frac{1}{\sqrt{2\pi\sigma_{\dot{s}}^2}}\exp\left[-\frac{\dot{s}^2}{2\sigma_{\dot{s}}^2}\right]$$

then $$N_o(S) = \frac{1}{2\pi}\left[\frac{-\sigma_{\dot{s}}}{\sigma_s}\right]\exp\left[-\frac{(S-a)^2}{2\sigma_s^2}\right] = \frac{1}{T_e}\exp\left[-\frac{(S-a)^2}{2\sigma_s^2}\right]$$

and $$p(S|T) = \frac{(S-a)}{\sigma_s^2}\cdot\frac{T}{T_e}\exp\left[-\frac{(S-a)^2}{2\sigma_s^2}\right]\exp\left[-\frac{T}{T_e}\exp\left\{-\frac{(S-a)^2}{2\sigma_s^2}\right\}\right]$$

$$T_e = \frac{2\pi\sigma_s}{\sigma_{\dot{s}}}$$

A Gaussian random process is completely determined by its power spectrum. This spectrum is easily determined by measurements on structures.

5. Gravity loads and temperature effects

Awaiting results of Committee 5.

6. Earthquake loading and response

Awaiting results of Committee 6.

7. Wind loading and wind effects

Awaiting results of Committee 7.

8. Conclusions

BIBLIOGRAPHIC REFERENCES

1. Bolotin, V.V.

   STATISTICAL METHODS IN STRUCTURAL MECHANICS, Holden-Day Inc., 1968.

2. Yao, J.T.

   STRUCTURAL SAFETY - A LITERATURE REVIEW, Task Committee on Structural Safety, Structural division, A.S.C.E., March 1971.

3. Freudenthal, A.M., Garrelts, J.M. and Shinozuka, M.

   THE ANALYSIS OF STRUCTURAL SAFETY, Proceedings A.S.C.E., Feb., 1966.

4. Manusio, C.

   A RATIONAL DETERMINATION OF LOAD FACTORS (POUR UNE DETERMINATION RATIONNELLE DES COEFFICIENTS DE PONDERATION), C.E.C.M.-1-GT Pondération 69-3.

5. Loof, H.W.

   THE RELATION BETWEEN PROBABILITY OF DAMAGE AND THE TIME INTERVAL CONSIDERED, C.E.C.M.-1-GT Pondération 70-3.

6. Turkstra, C.J.

   THEORY OF STRUCTURAL DESIGN DECISIONS, S.M. Study No. 2, 1970 Solid Mechanics Division, University of Waterloo, Waterloo, Ontario, Canada.

7. Cornell, C.A.

   BOUNDS ON THE RELIABILITY OF STRUCTURAL SYSTEMS, Proceedings A.S.C.E., Feb., 1967.

8. Cornell, C.A.

STRUCTURAL SAFETY SPECIFICATIONS BASED ON SECOND-MOMENT RELIABILITY ANALYSIS, I.A.B.S.E. International Symposium on Concepts of Safety of Structures and Methods of Analysis, London, Sept., 1969.

9. Cornell, C.A. and Peir, J.C.

ILLUSTRATION OF APPLICATION OF SECOND-MOMENT CODE FORMAT, Memo to A.S.C.E. Task Committee on Structural Safety, Feb., 1970.

10. Cornell, C.A.

IMPLEMENTING PROBABILITY BASED STRUCTURAL DESIGN, Presented at the 1971 Annual Convention A.C.I., Denver, March, 1971.

11. Rosenblueth, R. and Esteva, L.

RELIABILITY BASIS FOR SOME MEXICAN CODES, Presented at A.C.I. Annual Convention, Denver, March, 1971.

12. Ferry Borges, Castanheta, M.

STRUCTURAL SAFETY, Laboratorio Nacional de Engenharia Civil, Lisbon, Nov., 1968.

13. Lind, N.C.

CONSISTENT PARTIAL SAFETY FACTORS, S.M. Report No. 36, Solid Mechanics Division, University of Waterloo, Waterloo, Ontario, Canada.

14. Rice, S.O.

MATHEMATICAL ANALYSIS OF RANDOM NOISE, Selected Papers on Noise and Stochastic Processes, ed. N. Wax, Dover, New York, 1954.

15. Rice, J.R.

THEORETICAL PREDICTION OF SOME STATISTICAL CHARACTERISTICS OF RANDOM LOADINGS RELEVANT TO FATIGUE AND FRACTURE, Lehigh University, Bethlehem, Pa., 1964, Ph.D.

16. Lind, N.C. (editor)

STRUCTURAL RELIABILITY AND CODIFIED DESIGN, S.M. Study No. 3, 1970, Solid Mechanics Division, University of Waterloo, Waterloo, Ontario, Canada.

Additional

17. Manusion, C.

TOTAL LOAD FACTORS CALCULATED FOR A LOADING RESULTING FROM THE SIMULTANEOUS ACTION OF LOADS WITH A GAUSSIAN AND EXTREME TYPE OF DISTRIBUTION FUNCTION, (Détermination des coefficients de pondération globeaux pour une charge résultant de la somme d'une loi normale et d'une loi extrême), C.E.C.M. 1-GT Pondération 71-2.

18. Anton, J.M.

SOME RESULTS OF THE PROBALISTIC THEORY OF SAFETY (Quelques résultats des theories probabilistes sur la sécurité), C.E.C.M.-1-GT Pondération, Anton 6, Oct., 1970.

19. Delesques, R.

ESSAIS DE COEFFICIENTS DE PONDERATION PARTIELS EN CAS DE DISTRIBUTIONS AUTRES QUE NORMALES, C.E.C.M.-1-GT Pondération 71-1.

20. Ang, A.H.S. and Amin, M.

SAFETY FACTORS AND PROBABILITY IN STRUCTURAL DESIGN, Proceedings A.S.C.E., July, 1969.

21. Mitchell, G.R. and Woodgate, R.W.

FLOOR LOADINGS IN OFFICE BUILDINGS - THE RESULTS OF A SURVEY, Building Research Station, CP-3/71, Jan., 1971.

STRUCTURAL ANALYSIS WITH MORE THAN ONE LOADFACTOR

Loadfactors are determined according to the limit state design philosophy. Their value depends on the limit state considered; for example, stability limit state, maximum strength limit state or deformation limit state. The strength function of a compression element differs from the strength function of a bending element. The loading function, however, will be the same. Therefore, loadfactors of various magnitude must be assigned to the limit states.

In a structure, more than one limit state is possible. Each limit state must be provided for. This means that a structure should be analysed for each loading system corresponding to one of the limit states. The question arises whether it is possible to pass, in a simple way, from one loadfactor to another, without repeating the complete analysis.

This depends greatly on the complexity of the analysis. Generally the analysis is of the first order. The results (load actions or deformations) are linear functions of the design load. A transformation to another loadfactor is simple.
Second order analysis proofs more difficult. There is no linear relationship between load actions, deformations and design loads. It is necessary, therefore, to repeat the complete analysis for each loading system.

An exception can be made, if analytical expressions are available relating loads and load actions (or deformations).
The revised Dutch Code includes the following relation for design of columns in braced frames:

$$\frac{\omega F}{A} + \beta . \frac{n}{n-1} . \frac{M}{W} \leq \sigma_v$$

ω = factor for centrally loaded simple columns

β = factor accounting for different end moments $0,4 \leq \beta \leq 1$

σ_v = yield stress

F = load action resulting from factored loading

= load factor

M = load action " " " "

W = section modulus

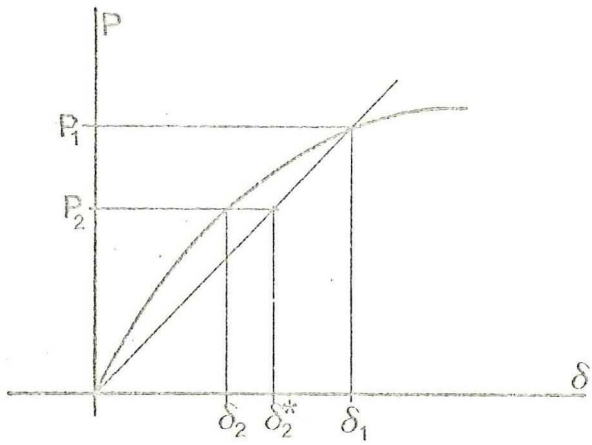
A = area

$n = \frac{F_{cr}}{F}$

F_{cr} = critical column loading $(= \frac{\pi^2 EI}{\ell_k^2})$

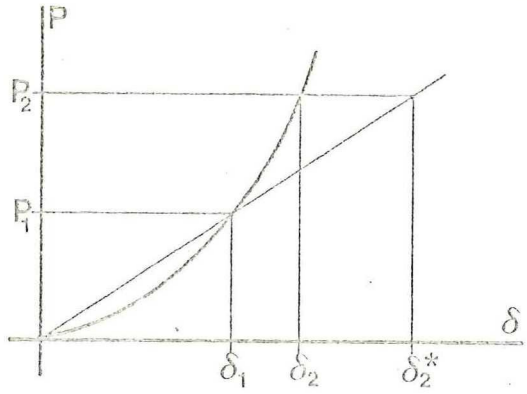
The amplification factor $\frac{n}{n-1}$ accounts for non-linear effects. Because F and M are first order terms, another load factor can be introduced by simply changing the value of F and M in the formula. It is not necessary to recalculate the structure.
If such analytical expressions are not available, another simplification is possible.

Suppose it is known that the relation between P and δ (deformation) is of the shape shown in fig. 1 (convex type)



Analysis has shown that for $P = P_1 \rightarrow \delta = \delta_1$. $P = \alpha_1 . P_0$ with P_0 = characteristic load.
Now the result is desired for another limit state with $\alpha_2 < \alpha_1$. It is clear that $P_2 = \alpha_2 P_0 = \frac{\alpha_2}{\alpha_1} P_1$ and $P_2 < P_1$. The deformation δ_2 corresponding to P_2 in shown in fig. 1. If one does not want to repeat the analysis it is possible to linearise the P-δ diagram starting from the highest P value. The deformation corresponding to P_2 will than be δ_2^*, which result overestimates the true δ_2-value. The amount of overestimation depends on the degree of non linearity.

If the P-δ relation is known to be of the concave shape shown in fig. 2, the first analysis should be done for the lowest P value.



Again the true deformation is overestimated.

One saves calculation time but pays with a less economic structure.

It should not be forgotten however, that one of the reasons to introduce loadfactors is to account in a simple manner for non-linear effects.

ACTION: It is suggested that this workinggroup does not trouble itself with this problem but to leave it to the judgement of the engineer.