

Exact failure criterion of wood

Theory extension and synthesis of all series A publications

Final extension of the derivation and explanation of:

- the failure criterion of wood (with hardening) and the meaning of the constants;
- the necessary data fitting conditions by the derived relations between the constants;
- the orthotropic extension of the critical distortional energy principle of yield;
- the generalized and extended Hankinson equations; the Norris- Tsai Wu- Hoffmann- Coulomb- and Wu- fracture criteria.
- the extended Tresca criterion for matrix failure at final yield;
- the determination of all constants of the failure criterion from simple uniaxial, oblique-grain compression and tension tests.
- the hypothesis of the replacement of the normality rule by the intrinsic minimum work for dissipation, with hardening state constants, for orthotropic materials like wood.

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Extension and discussion of A(2005) and of A(1982), A(1993), A(2009)

Tensor-polynomial failure criterion of wood - A(2005)

A general failure criterion for wood – A(1982)

Discussion and proposal of a general failure criterion of wood – A(1993)

A continuum failure criterion applicable to wood A(2009) All by T.A.C.M. van der Put

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1. Introduction

An overview is needed of the development during 3 decades of: A(1982), [1], to A(1993), [7] and to A(2005), up to now, to this Section A, of the general failure criterion for wood. A general failure criterion for wood was for the first time derived in A(1982), what was necessarily based on a tensor polynomial format. This followed from a first derivation of the orthotropic extension of the critical distortional energy principle, in paragraph 2.1.2 of A(1982), showing this basic principle to have the general form of the second degree tensor polynomial and further by the derivation of paragraph 1.2.4 of A(1982), showing the third degree tensor polynomial terms to represent hardening behavior up to the exact fracture mechanics mode I - II strength. The further extension to a higher degree tensor polynomial represents the polynomial expansion of the failure criterion, because the measured data points represent points of the exact failure criterion, while these points also are the polynomial points, which thus represents the polynomial expansion of the exact failure criterion and as such, as many polynomial terms and data points can be chosen, as necessary for a fit of the wanted precision.

In the introduction and paragraph 1.1, of A(1982), the concept of the yield surface of classical plasticity theory is discussed with the conditions of orthotropic symmetry in the main planes. All transformation laws of the stress tensors: σ and of the strength tensors: F_{ij} are given, making it possible to define e.g. the uniaxial strength in any direction. This is shown in paragraph 1.2 of A(1982), by the fit to test results of tension compression and shear of clear wood. The initial flow properties perpendicular to grain are fully and precisely described by the second degree polynomial, confirming the critical distortional energy principle for initial yield. In the longitudinal direction, compressional hardening is possible in the radial plane after this initial yield. This is discussed in paragraph 1.2.4, of A(1982) leading to the derivation of the Wu-equation of Fracture Mechanics, which also applies for micro-cracks of clear wood as is explained in [9] and is discussed in Section C, about fracture mechanics. In paragraph 1.2.5. of A(1982), the uniaxial off-grain-axis strength is discussed, leading to the derivation of the Hankinson and Norris equations as initial yield equations. It is shown that the, usually applied, von Mises- Hill- Hoffmann- Hankinson- and Norris criteria are special forms of the critical distortional energy principle of yield and are not generally valid. The Hill- and Norris- criteria only apply for materials with equal compression and tension strengths. Only the Hoffmann criterion accounts for such different strengths. However the Hill- and Hoffmann criteria contain a cyclic symmetry of the stresses in the quadratic terms, as applies for the isotropic case what causes a fixed, not free, orientation of the failure ellipse in stress space [1]. These criteria thus cannot apply generally for the orthotropic case. The same prescribed orientation is given by the theoretical Norris equation, being far from wood behavior that shows a zero, or nearly zero, slope of the ellipsoid with respect to the main direction. This explains why the older empirical Norris equation, with zero slope, applied for wood in Europe, is less worse than the later theoretical Norris equation.

A further derivation in A(1993) provided the extended Hankinson equations, extended by one hardening parameter, which is able to fit precisely different test results, at different hardening states, by different test methods and the fact that different values of one parameter are able to precisely fit whole curves of different hardening states of different test types, is the proof that the polynomial third degree terms F_{ijk} determine the hardening state as part of the exact criterion based on a theoretical necessity. This theoretical necessity is explained by the exact mixed mode Wu-equation of fracture mechanics, which is shown to represent these third degree coupling terms.

The tensor transformations of F_{ijk} ' tensors were only given in A(1982), because the choice was made, in later publications, for the in general more simple stress tensor approach of strengths in the main planes, by expansion of the stresses into the main material planes, providing the less number of polynomial terms. For information, the F_{ijk} ' transformations are also given here in Appendix 2.

Paragraph 2 of A(1982) delivers general information. The method of paragraph 2.2 of determination of hardening rules should not be followed. The method is too complicated and only descriptive (phenomenological) and determination of the initial response with gradual “plastic” flow with hardening is not needed for the determination of the ultimate state, which follows from the elastic full plastic approximation of limit analysis.

Extensions of the derivations of A(1982), are given in [7] and [10], where also an alternative derivation was given of the critical distortional energy criterion of initial yield of orthotropic wood. However the final, exact derivation is given in Appendix 1 of this Section. A further discussion is given in A(1993) of the third degree terms representing the Wu-equation with special hardening effects due to micro-crack arrest by strong layers occurring after initial yield. It followed, that for a precise fit, without meaningless higher degree polynomial terms, separate criteria are necessary for tension and for compression. This is obvious, because of the different failure mechanisms of tension and compression. This is applied to resolve the initial yield equation, eq.(2.14), into 2 factors, giving a factor for compression and a factor for tension failure, leading to the product of the Hankinson equations for tension and for compression. In A(1993), also the derivation was given of the exact modified Hankinson criterion and of the general form of the higher degree constants and how they can safely be determined from uniaxial tests.

An extension of the tensor polynomial method was given [3] by a general approach for anisotropic, not orthotropic, behavior of joints, (as punched out metal plates) and the simplification of the transformations by 2 angles as variables.

A confirmation of the results of [1] by means of coherent measurements (only in the radial-longitudinal plane) of [4] provided the generalization to an equivalent, quasi homogeneous, polynomial failure criterion for timber, (wood with small defects). These measurements also show a determining influence of hardening (by hindered micro-crack propagation) on the equivalent main strengths and on the failure criterion of wood. This follows from the theoretical explanation [9], of the Wu fracture mechanics criterion for layered composites. The mentioned main developments and further developments to A(2005) and A(2009) are subject of this Section A in order to provide an overview of the final derived theory.

Design and control calculation have to be based on the exact theory of limit analysis e.g. by finding an equilibrium system that satisfies the boundary conditions and nowhere surmounts the failure criterion. Essential for design thus is the derivation of the exact failure criterion for wood, which is the subject of this Section A. The Influence of temperature, moisture content, creep and loading rate on the behavior at “flow” and failure is given in Section B or in B(1989a) or [6] (see e.g. in general fig. 5.6 of [6]). The molecular deformation kinetics rate equations [6], provide the physical constitutive equations for wood and other materials.

2. The general failure criterion for wood polymers

2.1. General properties

A yield- or flow-criterion gives the combinations of stresses whereby flow occurs in an elastic-plastic material like wood in compression. For more brittle failure types in

polymers with glassy components like wood at tensile loading, there is some boundary where above the gradual flow of components at peak stresses and micro-cracking may have a similar effect on stress redistribution as flow especially for long term loading. It is discussed in [10] and later that these flow and failure boundaries may be regarded as equivalent elastic-full- plastic flow surfaces of limit analysis. The initial loading line shows gradual flow and hardening and stable micro-cracking up to final “flow” at the top. The following unloading is elastic and reloading shows a linear elastic loading up to flow at the same top. This is independent of the loading history (by unaltered geometry) and the linear elastic loading up to full plastic failure can be chosen to determine the ultimate state. The full plastic state is a line in a cross section of stress space and the flow- or failure criterion is a closed surface in the stress space i.e. a more dimensional space with coordinates

$$\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6.$$

A cut, e.g. according to figure 2.1 through the plane of the coordinate axes $x = \sigma_1$ and $y = \sigma_2$, will show a closed curve and such a curve always can be described by a polynomial in x and y like:

$$ax + by + cx^2 + dy^2 + exy + fx^3 + gy^3 + hx^2y + ixy^2 + \dots = k \quad (2.1)$$



Figure 2.1. Failure ellipsoid and definition of positive stresses.

whereby as much as terms can be accounted for as is necessary for the wanted precision. The surface will be concave because of the normality principle, and higher degree terms, causing local peaks on the surface (and thus causing inflection points) are only possible by local hardening effects depending on the loading path and are outside the initial flow-criterion. It can also be seen that the constants f and g are indeterminate and have to be taken zero because, for $y = 0$, eq.(2.1) becomes: $ax + cx^2 + fx^3 = k$, having the real roots x_0 , $-x_1$, $-x_2$ and thus can be written:

$$(x - x_0) \cdot (x + x_1) \cdot (x + x_2) = 0 \quad (2.2)$$

Because there are only two points of intersection possible of a closed surface with a line, there are only two roots by the intersecting x -axis e.g. $x = x_0$ and $x = -x_1$ and the part $(x + x_2)$, being never zero within or on the surface and thus is indeterminate, has to be omitted. For a real concave surface “ f ” thus is necessarily zero.

The same applies for g : $g = 0$ following from the roots of y when $x = 0$.

The equation can systematically be written as stress-polynomial like:

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j + F_{ijk} \sigma_i \sigma_j \sigma_k + \dots = 1 \quad (i, j, k = 1, 2, 3, 4, 5, 6) \quad (2.3)$$

In [1] it is shown that clear wood can be regarded to be orthotropic in the main planes and the principal directions of the strengths are orthogonal (showing the common tensor transformations) and higher degree terms, which are due to hardening, normally can be neglected so that eq.(2.3) becomes:

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (2.4)$$

In [10], and as discussed in Appendix 1, it is shown that this equation represents the critical distortional energy criterion for initial flow or failure.

In eq.(2.4) is, for reasons of energetic reciprocity, $F_{ij} = F_{ji}$ ($i \neq j$) and by orthotropic symmetry in the main planes (through the main axes along the grain, tangential and radial) there is no difference in positive and negative shear-strength and terms with uneven powers in σ_6 thus are zero or: $F_{16} = F_{26} = F_6 = 0$; and there is no interaction between normal- and shear-strengths or: $F_{ij} = 0$ ($i \neq j$; $i, j = 4, 5, 6$).

Thus eq.(2.4) becomes for a plane stress state in a main plane:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \quad (2.5)$$

For a thermodynamic allowable criterion (positive finite strain energy) the values F_{ii} must be positive and the failure surface has to be closed and cannot be open- ended and thus the interaction terms are constrained to:

$$F_{11}F_{22} > F_{12}^2 \quad (2.6)$$

($F_{11}F_{22} = F_{12}^2$ gives a parabolic surface and $F_{11}F_{22} < F_{12}^2$ is hyperbolic, both open ended)

For the uniaxial tensile strength $\sigma_1 = X$ ($\sigma_2 = \sigma_6 = 0$) and eq.(2.5) becomes:

$$F_1\sigma_1 + F_{11}\sigma_1^2 = 1 \quad \text{or: } F_1X + F_{11}X^2 = 1 \quad (2.7)$$

and for the compression strength $\sigma_1 = -X'$ this is:

$$-F_1X' + F_{11}X'^2 = 1 \quad (2.8)$$

and it follows from eq.(2.7) and (2.8) that F_1 and F_{11} are known:

$$F_1 = \frac{1}{X} - \frac{1}{X'}, \quad \text{and } F_{11} = \frac{1}{XX'} \quad (2.9)$$

In the same way is for $\sigma_1 = \sigma_6 = 0$, in the direction perpendicular:

$$F_2 = \frac{1}{Y} - \frac{1}{Y'}, \quad \text{and } F_{22} = \frac{1}{YY'} \quad (2.10)$$

Further it follows for $\sigma_1 = \sigma_2 = 0$ (pure shear), for the shear strength S , that:

$$F_{66} = \frac{1}{S^2} \quad (2.11)$$

$$\text{and according to eq.(2.6) is: } -1/\sqrt{XX'YY'} < F_{12} < +1/\sqrt{XX'YY'} \quad (2.12)$$

It can be shown (as discussed in [1]) that the restricted values of $2F_{12}$, based on assumed coupling according to the deviator stresses, as given by Norris [13], Hill or Hoffmann [14] as: $2F_{12} = -1/2XY$, or: $F_{12} = -(1/X^2 + 1/Y^2 - 1/Z^2)$ are not general enough for orthotropic materials and don't apply for wood. There also is no reason to restrict F_{12} according to e.g. Tsai and Hahn [15] as: $2F_{12} = 1/\sqrt{XX'YY'}$ or according to Wu and Stachurski [16] as: $2F_{12} \approx -2/XX'$. These chosen values suggest that F_{12} is ~ 0.2 to 0.5 times the extreme value of eq.(2.12).

The properties of a real physical surface in stress space have to be independent on the orientation of the axes and therefore the tensor transformations apply for the stresses σ of eq.(2.5). These transformation are derivable from the equilibrium of the stresses on an element formed by the rotated plane and on the original planes, or simply, by the analogous circle of Mohr construction. For the uniaxial tensile stress then is:

$$\sigma_1 = \sigma_t \cos^2 \theta \quad \sigma_2 = \sigma_t \sin^2 \theta \quad \sigma_6 = \sigma_t \cos \theta \sin \theta$$

Substitution in eq.(2.5) gives:

$$F_1 \sigma_t \cos^2 \theta + F_2 \sigma_t \sin^2 \theta + F_{11} \sigma_t^2 \cos^4 \theta + (2F_{12} + F_{66}) \sigma_t^2 \cos^2 \theta \sin^2 \theta + F_{22} \sigma_t^2 \sin^4 \theta = 1 \quad (2.13)$$

and substitution of the values of F:

$$\begin{aligned} & \sigma_t \cos^2 \theta \left(\frac{1}{X} - \frac{1}{X'} \right) + \sigma_t \sin^2 \theta \left(\frac{1}{Y} - \frac{1}{Y'} \right) + \frac{\sigma_t^2 \cos^4 \theta}{XX'} + 2F_{12} \sigma_t^2 \sin^2 \theta + \frac{\sigma_t^2 \sin^4 \theta}{YY'} + \\ & + \frac{\sigma_t^2 \cos^2 \theta \sin^2 \theta}{S^2} = 1 \end{aligned} \quad (2.14)$$

It can be seen that for $\theta = 0$ this gives the tensile- and compression strength in e.g. the grain direction: $\sigma_t = X$ and $\sigma_t = -X'$, and for $\theta = 90^\circ$, the tensile and compression strength perpendicular to the grain: $\sigma_t = Y$ and $\sigma_t = -Y'$, and that a definition is given of the tensile and compression strengths in every direction. These are the points of intersection of the rotated axes with the failure surface. Eq.(2.13) thus can be read in this strength component along the rotated x-axis: $\sigma_t = \sigma_1$ according to:

$$F'_1 \sigma_1 + F'_{11} \sigma_1^2 = 1 \quad (2.15)$$

and eq.(2.13) gives the definition of the transformations of F'_1 and F'_{11} . The same can be done for the other strengths. The transformation of F'_{ij} thus also is a tensor-transformation (of the fourth rank) that follows from the rotation of the symmetry axes of the material. Transformation thus is possible in two manners. The stress-components can be transformed to the symmetry directions according to eq.(2.5). Or the symmetry axes can be rotated, leaving the stresses along the rotating axes unchanged. For this case the general polynomial expression eq.(2.16) applies:

$$F'_1 \sigma_1 + F'_2 \sigma_2 + F'_{11} \sigma_1^2 + 2F'_{12} \sigma_1 \sigma_2 + F'_{22} \sigma_2^2 + F'_{16} \sigma_1 \sigma_6 + F'_{26} \sigma_2 \sigma_6 + F'_{66} \sigma_6^2 = 1 \quad (2.16)$$

These transformations of strength tensors F' are e.g. given in [1] and in Appendix 2.

2.2. Initial yield criterion and derivation of the Hankinson and extended Hankinson equations

As mentioned, eq.(2.5) or eq.(2.14) for the off-grain-axis tensile- and compression strengths, represents the initial yield condition, being the extended orthotropic critical distortional energy principle derived in Appendix 1..

This "initial yield" equation, eq.(2.14), can be resolved into factors as follows:

$$\left(\frac{\sigma_t \cos^2 \theta}{X} + \frac{\sigma_t \sin^2 \theta}{Y} - 1 \right) \cdot \left(\frac{\sigma_t \cos^2 \theta}{X'} + \frac{\sigma_t \sin^2 \theta}{Y'} + 1 \right) = 0 \quad (2.17)$$

giving the product of the Hankinson equations for tension and for compression, (where X and X' are the tensile and compressional strengths in grain direction). This applies when:

$$2F_{12} + 1/S^2 = 1/X'Y + 1/XY' \quad (2.18)$$

In this equation, derived in [1], $(1/X'Y + 1/XY')$ is of the same order, and thus about equal to $1/S^2$ so that $2F_{12}$ is of lower order with respect to $1/S^2$. In [2] eq.(2.18) was used as a measure for F_{12} what is a difference of two higher order quantities and thus can not give a precise information of the value of F_{12} , that also can be neglected as first estimate.

In [5], wrongly the sum of $1/S^2$ and $(1/X'Y + 1/XY')$ is taken to be equal to $2F_{12}$, being of higher order with respect to the real value of $2F_{12}$ and it is evident that this value did not satisfy eq.(2.12) for a closed surface.

Eq.(2.17) shows that the exponent "n" of the generalized Hankinson formula eq.(2.19):

$$\frac{\sigma_t \cos^n \theta}{X} + \frac{\sigma_t \sin^n \theta}{Y} = 1 \quad (2.19)$$

is: $n = 2$ for tension and compression at initial yield when there are no higher degree terms. A value of n , different from $n = 2$ thus means that there are third degree terms due to hardening after initial yield as in eq.(2.21).

The initial yield criterion eq.(2.14) or eq.(2.17), being the, for orthotropy, extended critical distortional energy principle, should satisfy both the elastic and the yield conditions at the same time. Because the Hankinson equation with $n = 2$ also applies for the axial modules of elasticity and because this modulus is proportional to the strength, the Hankinson equations with $n = 2$, eq.(2.17), satisfies this requirement. Thus $n = 2$ is necessary for initial yield. Thus after some strain in the elastic stage, the initial yield is reached and because the modulus of elasticity follows the Hankinson equation with $n = 2$, also the yield criterion, eq.(2.17), containing the Hankinson equations, follows this and has the quadratic form and no higher degree terms. This also is measured. It is mentioned in [8], that for glulam and for clear wood in bending and in tension, $n \approx 2$. The combined compression with shear tests (of Keylwerth by the "Schereisen", allowing only shear-deformation in one plane) show that for off-axis longitudinal shear, also in the radial plane, $n = 2$, showing no higher degree terms for the shear strength. According to fig. 2.4.1, this also applies for the tangential plane, but not for the radial plane. The value of n thus depends on the type of test and it is mentioned e.g. by Kollmann [19], that $n \approx 2.5$ for compression of clear wood, showing that hardening was possible in the tests and the third degree terms of the yield criterion are not zero [10]. The test method of [4] shows that F_{112} , F_{166} and F_{266} in the radial plane have an influence, (what is shown to be the hardening effect due to crack arrest). Thus the test method (early instability by loss of equilibrium of the test, or not) has influence on whether only initial yield ($n = 2$), or a more stable failure will occur (n different from $n = 2$). Thus, when $n \neq 2$, higher degree terms are not zero in the failure criterion and eq.(2.21) applies.

An equation of the fourth degree (eq.(2.21) in σ_t) can always be written as the product of two quadratic equations, eq.(2.20). For a real failure surface the roots will be real and because the measurements show that one of the quadratic equations is determining for compression- and the other for tension- failure mechanisms and must be valid for zero values of C_t and/or C_d as well, this factorization leads as the only possible solution to be the product of extended Hankinson equations for tension and compression as follows:

$$\left(\frac{\sigma_t \cos^2 \theta}{X} + \frac{\sigma_t \sin^2 \theta}{Y} - 1 + \sigma_t^2 \sin^2 \theta \cos^2 \theta \cdot C_t \right) \cdot \left(\frac{\sigma_t \cos^2 \theta}{X'} + \frac{\sigma_t \sin^2 \theta}{Y'} + 1 + \sigma_t^2 \sin^2 \theta \cdot \cos^2 \theta \cdot C_d \right) = 0 \quad (2.20)$$

Performing this multiplication, eq.(2.20) thus is in general:

$$F_1 \sigma_t \cos^2 \theta + F_2 \sigma_t \sin^2 \theta + F_{11} \sigma_t^2 \cos^4 \theta + (2F_{12} + F_{66}) \sigma_t^2 \cos^2 \theta + F_{22} \sigma_t^2 \sin^4 \theta + 3(F_{112} + F_{166}) \sigma_t^3 \cos^4 \theta \cdot \sin^2 \theta + 3(F_{122} + F_{266}) \sigma_t^3 \sin^4 \theta \cdot \cos^2 \theta + 12F_{1266} \sigma_t^4 \cos^4 \sin^4 \theta = 1 \quad (2.21)$$

giving the third degree tensor polynomial, applied in [1] and [4], where it appeared that F_{1122} and other possible higher degree terms can be neglected except F_{1266} .

The values C_t and C_d can be found by fitting of the modified "Hankinson equations" eq.(2.20), for uniaxial off-axis tension and compression test results, giving the constants:

$$2F_{12} = 1/X'Y + 1/XY' - 1/S^2 + C_t - C_d; \quad 3(F_{112} + F_{166}) = C_t/X' + C_d/X; \\ 3(F_{122} + F_{266}) = C_t/Y' + C_d/Y; \quad \text{and} \quad 12F_{1266} = C_t C_d - 12F_{1122} \approx C_t C_d \quad (2.22)$$

A fit of the Hankinson power equation, eq.(2.19) always is possible and different n values for tension and compression from $n = 2$ in that equation means that there are higher degree terms and that C_t and C_d are not zero, as follows from eq.(2.20).

For timber with defects and grain and stress deviations, the axial strength is determined by combined shear and normal stress perpendicular to the grain. This may cause some stable crack propagation and a parabolic curve of the effective shear strength (according to the Mohr- or Wu-equation, eq.(2.27) with $c = 1$) given by a third degree term. For timber n can be as low as $n \approx 1.6$ in eq.(2.19) for tension, showing higher degree terms to be present. This also follows from $n \approx 2.5$ for compression. The data of [4], show that F_{166} , F_{266} and F_{112} of the radial plane have influence showing (see fig. 2.4.1, 2.5.2 and 2.4.4), the parabolic like curves, different from elliptic curves of 2nd degree, at longitudinal tension side, of fig. 2.4.3. It could be expected for clear wood that $F_{166} = 0$ and $F_{122} = 0$ because the longitudinal stress σ_1 is in the plane of the crack and not influenced by the crack tip.

However collinear crack propagation is not possible at shear failure and also due to grain deviations in timber there is an influence on F_{166} and F_{122} .

It was shown in [1] that F_{12} is small and can not be known with a high accuracy. Small errors in the strength values (X, X', Y, Y', S) may switch F_{12} from its lower bound to its upper bound, changing its sign and the value thus is not important and thus negligible for a first estimate. The data of [4] of the principal stresses in longitudinal tension, being close to initial yield, show F_{12} to be about zero at initial yield, thus when $C_d = C_t = 0$ and thus

$$\text{when: } 1/S^2 = 1/X'Y + 1/XY' \quad (2.23)$$

$$\text{Then eq.(2.22) suggests that: } 2F_{12} = C_t - C_d \quad (2.24)$$

due to hardening when C_t and C_d are not zero. This is tested in A(1993) and it appears that, because $F_{12} \approx 0$ for longitudinal tension, S follows, (according to eq.(2.22), from:

$$1/S^2 = 1/X'Y + 1/XY' + C_t - C_d$$

and S should not be measured separately by a different type of test, but follows, (as the other strength values) from the uniaxial off-axis tension- and compression tests.

Because F_{1122} is negligible, is, according to eq.(2.22): $12 F_{1266} \approx C_t C_d$,

what also is small and negligible.

F_{166} will have a similar bound as F_{266} , as follows from eq.(2.27) what is given in fig. 2.4.1 and follows by replacing the index 2 by 1 and Y by X . However the determining bound of F_{166} follows from eq.(2.22), when F_{112} is known. F_{112} is not discussed in [1], but a general method to determine the bounds of F_{112} is given in [1], for F_{266} . the followed estimation, in § 2.4, of F_{112} , based on σ_1 and σ_2 alone, $\sigma_6 = 0$ also is sufficient.

It appears not possible to have one failure criterion for the different failure types of longitudinal tension and longitudinal compression. For the longitudinal tension fit, the hardening constants F_{112} , F_{12} and F_{122} are zero by no hardening. For the longitudinal compression fit, these constants are not zero and F_{112} , thus hardening, dominates. For tension, the early instability of the test, by splitting, determines the strength, while for compression the late instability after hardening defines failure. It thus is necessary for a precise fit, to fit both regions (longitudinal tension and longitudinal compression separately and not to apply one overall criterion for longitudinal tension and compression. With the estimates of F_{266} and F_{112} to be close to their bounds for compression, and with zero

normal coupling terms for tension, all constants are known, according to eq.(2.22), depending on C_d and C_t from uniaxial off-axis tension and compression tests. (see § 2.4).

2.3. Transverse strengths

In [1] it is shown that for rotations of the 3-axis, when this axis is chosen along the grain, eq.(2.5) and (2.16) may precisely describe the peculiar behavior of the compression-tension- and (rolling) shear-strength perpendicular to the grain and the off-axis strengths without the need of higher degree terms. These measured lines of the off-axis uniaxial transverse strength of fig. 2.2, follow precisely from eq.(2.15):

$$F'_1 \sigma_1 + F'_{11} \sigma_1^2 = 1$$

When for compression the failure limit is taken to be the stress value after that the same, sufficient high, amount of flow strain has occurred, then the differences between radial-tangential- and off-axes strengths disappear and one, directional independent, strength value remains (see fig. 2.2). For tension perpendicular to the grain, only in a rather small region (around 90° , see fig. 2.2) in the radial direction, the strength is higher and because in practice, the applied direction is not precisely known and avoids this higher value, a lower bound of the strength will apply that is independent of the direction. The choice of these limits means that:

$$F_1 - F_2 = 0 \text{ and } F_{11} - F_{22} = 0$$

and that also F_{12} is limited according to:

$$2F_{12} = F_{11} + F_{22} - F_{66}$$

Further then also is:

$$F'_6 = 0 \text{ and } F'_{66} = F_{66} = 1/\tau_{\text{rol}}^2 \quad (2.25)$$

From measurement it can be derived that F_{12} is small leading to:

$$F_{66} \approx F_{11} + F_{22} \text{ or } \tau_{\text{rol}} \text{ is bounded by:}$$

$$\tau_{\text{rol}} = \sqrt{XX'/2} = \sqrt{YY'/2} \quad (2.26)$$

and the ultimate behavior can be regarded to be quasi isotropic in the transverse direction.

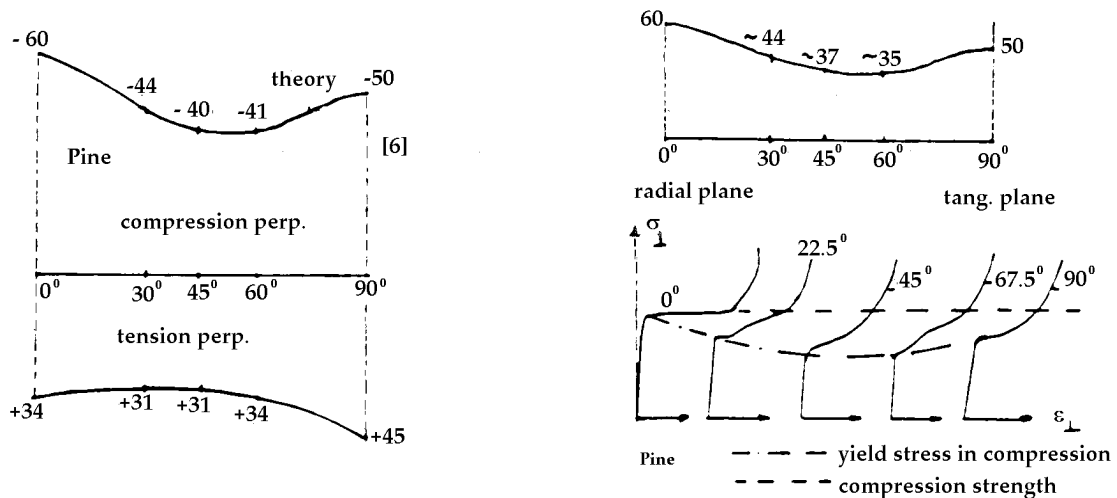


Figure 2.2 - Yield stresses and hardening

The measurements further show for this rotation around the grain-axis that the "shear

strengths" in grain direction in the radial- and the tangential plane, F_{44} and F_{55} , are uncoupled or $F_{45} = 0$, as is to be expected from symmetry considerations.

2.4. Longitudinal strengths

When now the 3-axis is chosen in the tangential or in the radial directions, the same relations apply (with indices 1, 2, 6) as in the previous case. The equations for this case then give the strengths along and perpendicular to the grain and the shear-strength in the grain direction.

In [1] it is shown that this longitudinal shear strength in the radial plane increases with compression perpendicular to this plane according to the coupling term F_{266} (direction 2 is the radial direction" direction 1 is in the grain direction):

$$F_2\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 3F_{266}\sigma_2\sigma_6^2 = 1$$

$$\text{or: } \frac{\sigma_6}{S} = \sqrt{\frac{(1-\sigma_2/Y) \cdot (1+\sigma_2/Y')}{1+c\sigma_2/Y'}}$$
(2.27)

with: $c = 3F_{266}Y'S^2 \approx 0.9$ (0,8 to 0.99, see fig. 2.4.1).

When c approaches $c \approx 1$ (measurements of project A in fig. 2.4.1), eq.(2.27) becomes:

$$\left(\frac{\sigma_6}{S}\right)^2 + \frac{\sigma_2}{Y} \approx 1$$
(2.28)

which is the mixed I– II mode Wu- equation of fracture mechanics, showing that micro-crack and macro crack extensions are the same. The same can be done at the tensile side giving the same equation with Y replaced by $-Y'$. The exact derivation of this equation, in orthotropic stresses, is given in C(2011), paragraph 2.3:

$$1 = \frac{\sigma_2}{\xi_0\sigma_t/2} + \frac{\sigma_6^2}{\xi_0^2\sigma_t^2n_6^2} = \frac{\sigma_2\sqrt{\pi c}}{\sigma_t\sqrt{\pi r_0}/2} + \frac{(\sigma_6\sqrt{\pi c})^2}{(\sigma_t n_6\sqrt{2\pi r_0})^2} = \frac{K_I}{K_{Ic}} + \frac{K_{II}^2}{K_{IIc}^2}$$
(2.29)

because by the transformation from elliptical to circular coordinates: $\xi_0 = \sqrt{2r_0/c}$. Critical small crack propagation occurs at a critical crack density, when the crack distance is about the crack-length and is thus independent of the crack length and crack tip radius r_0 , which can be chosen to have a standard value and the second part of eq.(2.29) can be written as:

$$1 = \frac{\sigma_2\sqrt{\pi c}}{\sigma_t\sqrt{\pi r_0}/2} + \frac{(\sigma_6\sqrt{\pi c})^2}{(\sigma_t n_6\sqrt{2\pi r_0})^2} = \frac{\sigma_2}{\sigma_{2c}} + \frac{\sigma_6^2}{\sigma_{6c}^2}$$
(2.30)

thus in deterministic ultimate strength values: σ_{2c}, σ_{6c} .

The value of F_{266} of eq.(2.27), depends on the stability of the test, thus is not a constant, but a hardening factor, determining the amount of hardening at the, by the testing instability determined, ultimate state. This is shown e.g. by the following Fig. 2.5.1, where parameter values according to more stable torsion tube tests, are used to predict the oblique grain compression strength values. Because of more hardening in the torsion tube test, the peak of 1.1, at 10^0 , is predicted, which can not be measured in the oblique grain test, due to earlier instability due to lack of equilibrium, of this test setup, after "initial flow".

As derived in [9], eq.(2.27) does not only apply for tension with shear but also for shear with compression σ_2 perpendicular to the flat crack. For a high stress σ_2 the crack is

closed at: $\sigma_2 = \sigma_c = \sigma_c$ and the crack tip notices only the influence of $\sigma_2 = \sigma_c$ because for the higher part of σ_2 , the load is directly transmitted through the closed crack and eq.(2.28) becomes:

$$\frac{\sigma_6}{S} = \frac{-\mu(\sigma_2 - \sigma_c)}{S} + \sqrt{1 - \frac{\sigma_c}{Y}} \quad \text{or:} \quad \sigma_6 = C + \mu |\sigma_2| \quad (2.31)$$

where σ_2 and σ_c are negative, giving the Coulomb-equation with an increased shear capacity due to friction: $\mu |\sigma_2|$. However, inserting the measured values of [4], it appears that the frictional contribution is very small. The micro-crack closure stress σ_c will numerical be about equal to the tensile strength: $\sigma_c \approx -Y$. The shear strength will be maximal raised, at high compression of $\sigma_c \approx -0.9Y'$, by a factor:

$$\left(1 + \mu(0.9Y' - Y)/S\sqrt{2}\right) = \left(1 + 0.3(0.9 \cdot 5.6 - 3.7)/9.8 \cdot \sqrt{2}\right) = 1.03$$

Thus the combined shear- compression strength is mainly determined by an equivalent hardening effect, caused by crack arrest in the critical direction by the strong layers. At higher σ_2 stresses, compression plasticity perpendicular to the grain (project A of [11], see fig. 2.4.1) or instability of the test (project B of [11] with oblique-grain compression tests) may become determining, showing a lower value of c of eq.(2.27) than $c = 1$.

Because the slopes of the lines (at small σ_2) of project A and B of [11] are the same, there is no indication, for clear wood, of an influence of the higher degree terms: F_{112} , F_{122} and F_{166} of project B. When for longitudinal tension F_{12} , F_{122} and F_{112} are zero, then, when $F_{166} = 0$, also $F_{266} = 0$ according to eq.(2.22). and then also $C_t = C_d = 0$. Further, the line of B is below the line of A and the c -value of B is lower, closer to the elliptic failure criterion. This is an indication that hardening after initial yield (thus departure from the elliptic equation) of project B, the oblique-grain compression test, is less than that of project A and thus that the test is less stable. (Project C of [11] follows the elliptic failure criterion because of the influence of transverse failure due to rolling shear that is shown before, (§2.3), to be elliptic).

The high value of F_{266} , in the radial plane, (measured with $\sigma_1 = 0$), indicates that for clear wood, F_{122} has to be small according to eq.(2.22). It further follows from published Hankinson lines, with $n \approx 2$, of clear wood that third degree terms are zero in the tangential plane, confirming the results of projects A and B of [11], mentioned before. There is an indication that this is a general property of timber [11], because when shear failure is free to occur in the weakest plane, as usually in large timber beams and glulam, it occurs in the tangential plane and $n = 2$, showing no higher degree terms.

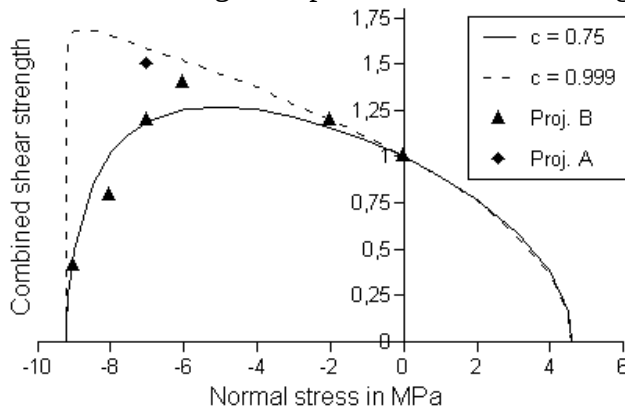


Figure 2.4.1 a - Combined shear-tension and shear-compression strengths. F_{266}

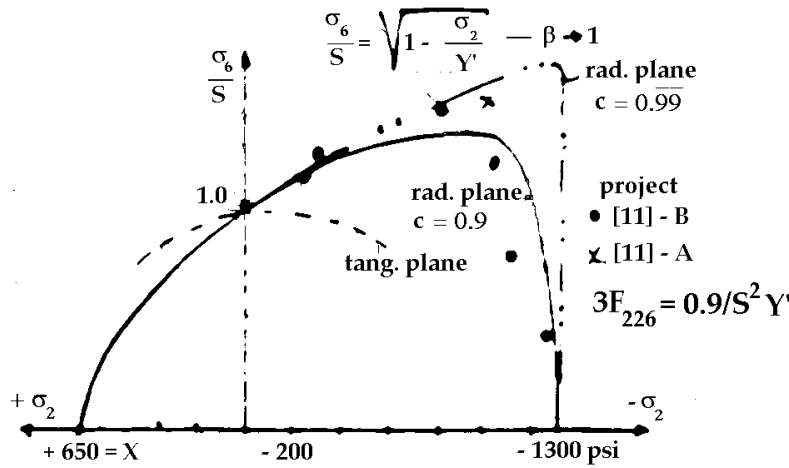


Figure 2.4.1 b.: Fig.10 of A(1982) of Combined shear-tension -compression strengths. F_{266}

Determining for compression failure, in the radial plane, is the microscopic kinks formation in the cell walls, which is a buckling and plastic shearing mechanism. The kinks multiply and unite in kink-bands and kink-planes at fiber misalignments. Known, by everyone, is the slip-plane of the prism compression test showing a horizontal crease (shear line, slip line) on the longitudinal radial plane, while on the longitudinal tangential plane the crease is inclined at 45° to 60° with the vertical axis (depending on the species), The cause are the rays in the radial planes, which are the main disturbances of the alignment of the vertical cells. For this bi-axial compression fracture, the same fracture mechanism occurs as for combined mode I-II fracture, discussed above. The shear loading of the micro-cracks is now due to the misalignment component of the normal stress. The general equation now becomes: $F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 3F_{112}\sigma_1^2\sigma_2 = 1$ (2.32)

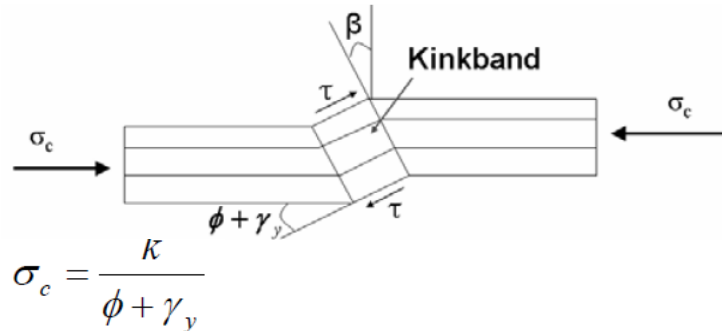
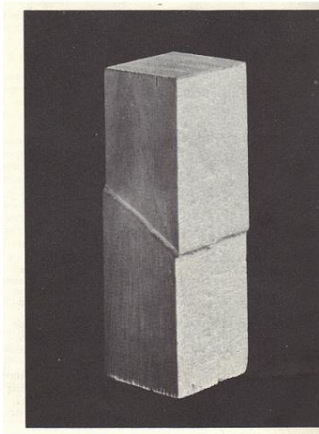


Fig. 2.4.2. Kinkband formation, where K is the plastic shear strength of the matrix (e.g. 11.3 Mpa), $\phi = 15^\circ$ is the misalignment (e.g. for Spruce) γ_y is the longitudinal shear yield strain.

Because $F_{12} = \sigma_6 = 0$ and the contribution of the term with F_{122} is of lower order, not visible in Fig. 2.4.3. The choice of $\sigma_6 = 0$ is made because then any high value of F_{112} is most determining. Eq.(2.32) can be written:

$$\sigma_1 \left(\frac{1}{X} - \frac{1}{X'} \right) + \sigma_2 \left(\frac{1}{Y} - \frac{1}{Y'} \right) + \frac{\sigma_1^2}{XX'} + \frac{\sigma_2^2}{YY'} + 3F_{112}\sigma_2\sigma_1^2 = 1 \quad (2.33)$$

Thus::

$$\sigma_1 (X' - X) + \sigma_1^2 (1 + 3F_{112}\sigma_2 XX') = (1 - \sigma_2 / Y) \cdot (1 + \sigma_2 / Y') \cdot XX' \quad (2.34)$$

The critical value of F_{112} , to just have a closed surface, will occur at high absolute values of σ_1 and σ_2 , thus in the neighborhood of $\sigma_1 \approx -X'$. Inserting safely this value in the smallest term of eq.(2.34) gives:

$$\sigma_1^2 (1 + 3F_{112}\sigma_2 XX' + (X' - X)/(-X')) = (1 - \sigma_2 / Y) \cdot (1 + \sigma_2 / Y') \cdot XX'$$

or: $\frac{\sigma_1}{X'} = -\sqrt{\frac{(1 - \sigma_2 / Y)(1 + \sigma_2 / Y')}{1 + c\sigma_2 / Y'}} \approx -\sqrt{1 - \sigma_2 / Y}$ where: $c = 3F_{112}Y'X'^2$ (2.35)

Thus when the hardening constant c approaches one: $c \approx 1$, the curve reduces to a parabola and the requirement to have a closed curve is $c < 1$, or: $3F_{112} < 1/Y'X'^2$ (2.36)

More general when F_{12} and F_{122} are not fully negligible, the bound: $c < 1$ becomes:

$$c \approx 3F_{112}X'^2 Y' - 2F_{12}Y'X' + 3F_{122}Y'^2 X' < 1 \quad (2.37)$$

for longitudinal compression, where besides $\sigma_1 \approx -X'$, also $\sigma_2 \approx -Y'$ is substituted in the contribution of the smallest term, as determining point to just have a closed surface.

The same could be expected to apply for longitudinal tension, giving the same equation (2.35) with X' replaced by X . However, because of an other type of failure, F_{112} and F_{122} are zero for longitudinal tension, see fig. 2.4.3 which is an ellipse at the longitudinal tension side, thus is a second degree equation, according to eq.(2.33) with $F_{112} \approx 0$ (and with $F_{12} \approx 0$ by the zero slope of the ellipse).

The found (cut-off) parabola eq.(2.35) (for c close to $c = 1$) is, as eq.(2.27), equivalent to the mixed I–II mode Wu-fracture equation for shear with tension or with compression perpendicular. For wood in longitudinal compression, this failure mechanism acts in the radial plane giving high values of F_{266} and F_{112} close to their bounds of $c \approx 0.8$ to 0.9 .

The parabolic Eq.(2.35) is shown in Fig. 2.4.3, by the data points outside the points on the ellipse of the longitudinal compression side and is shown as fitted to the theoretical Wu-parabola in fig. 2.4.4. As mentioned, this hardening of the torsion tube tests, is not found in the uniaxial oblique grain tests, which is earlier unstable, thus showing less hardening.

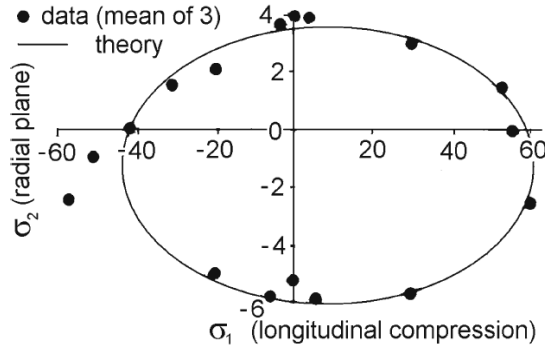
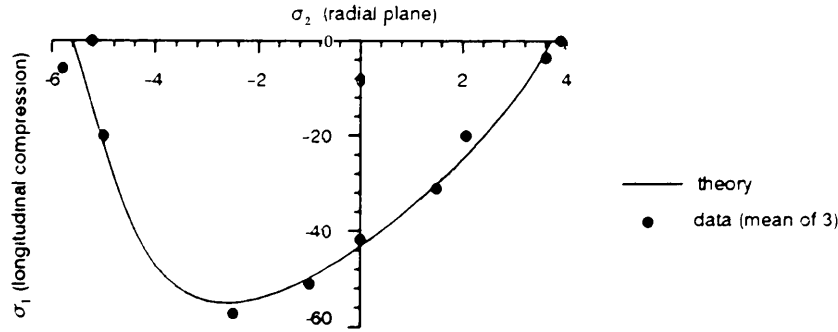
According to fig. 2.4.3, below, is F_{122} -term of lower order with respect to F_{112} -term and not visible in the figure. Determining is F_{112} , representing hardening by kinking and slip-plane formation (see Fig. 2.4.2). As to be expected, and according to fig.2.4.3, is F_{112} zero at the longitudinal tension side (as F_{122} and F_{12}).

In A(1993) is shown that all data may show a different amount of hardening at failure. Because tests in longitudinal compression show other and more hardening than tests in tension, separate data fits for longitudinal tension and longitudinal compression are necessary, as given by eq.(2.43) and eq.(2.44). For the parameter estimation by the uniaxial oblique grain tests, is in eq.(2.22):

$$F_{12} = F_{122} = F_{166} = 0; \quad 3F_{112} \approx 0.9 / ((X')^2 Y'); \quad 3F_{266} \approx 0.9 / (S^2 Y') \quad (2.38)$$

Because hardening is mostly not guaranteed in real structures and test situations, the initial flow criterion applies for the Codes according to:

$$\frac{\sigma_6^2}{S^2} + \frac{\sigma_1}{X} - \frac{\sigma_1}{X'} + \frac{\sigma_1^2}{XX'} + \frac{\sigma_2}{Y} - \frac{\sigma_2}{Y'} + \frac{\sigma_2^2}{YY'} = 1 \quad (2.39)$$


 Fig. 2.4.3. Initial yield for $F_{12} = 0$ and $\sigma_6 = 0$

 Figure 2.4.4. Yield criterion for compression F_{112} ($\sigma_1 < 0$) for $\sigma_6 = 0$.

2.5. Estimation of the polynomial constants by uniaxial tests

Based on data fitting of uniaxial tension- and compression tests of [4], the values of C_d and C_t are determinable and by eq.(2.22) the polynomial constants are known. This can be compared with the data and fit of the biaxial measurements of [4].

In fig. 2.5.1, a determination of C_d and of C_t is given. In this figure of [4], the compression- strength perpendicular to the grain measurement $Y'/X' = 0.204$ is reduced to obtain a value of $Y'/X' = 0.13$ (at 90°) to be able to use the measured constants of the bi-axial tests. It is not mentioned how that possibly can be done but the drawn lines in the figure give the prediction of the uniaxial values based on the measured constants according to the general eq.(2.21) (given in [4], as in [1], in the strength tensor form of eq.(2.15)). For comparison the fits of the Hankinson equations are given following these drawn lines. For tension the extended Hankinson equation (2.20) becomes (by scratching the non zero compression factor of the extended Hankinson product: eq.(2.20)):

$$\frac{\sigma_t \cos^2 \theta}{X} + \frac{\sigma_t \sin^2 \theta}{Y} + \sigma_t^2 \sin^2 \theta \cos^2 \theta \cdot C_t = 1 \quad (2.40)$$

and this equation fits the line for tension in fig. 2.5.1 when $C_t \approx 11.9/X^2$. The Hankinson equation (2.19) fits in this case for $n \approx 1.8$ and all 3 equations (2.21), (2.40) and (2.19) give the same result although for the Hankinson equations only the main tension- and compression strength have to be known and the influence of all other quantities are given by one parameter: n or by C_t .

For compression, the same line as found in [4] was found in [1], (see fig. 11 of [1]), by the

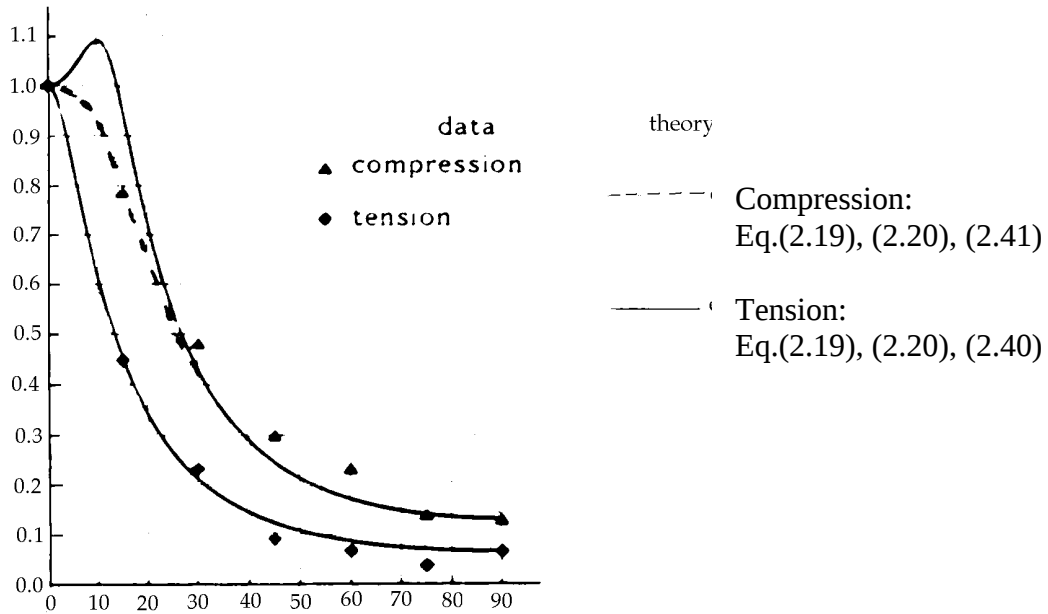


Figure 2.5.1 - Uniaxial tension- and compression strengths

second degree polynomial with the minimal possible value of F_{12} (according to eq.(2.12)), showing that except a negative F_{122} (as used in [4]) also a high negative value of F_{12} may cause the strong peak at small angles. Because such a peak never is measured, the drawn line of [4] is only followed here for the higher angles by the Hankinson equation. For the small angles, the line (dashed) is drawn through the measured point at 15° , giving the expectable Hankinson value of $n = 2.4$ in eq.(2.19) and for eq.(2.31): $C_d \approx 4/X'^2$.

Because of this low measured value, the predicted peak at 10° in fig. 2.5.1 is not probable, although the peak-factor of 1.1 is theoretically possible, for a high shear strength, to occur at 18° in stead of 10° with $C_d \approx 7.6/X'^2$ in the extended Hankinson equation:

$$\frac{\sigma_t \cos^2 \theta}{X'} + \frac{\sigma_t \sin^2 \theta}{Y'} + \sigma_t^2 \sin^2 \theta \cos^2 \theta \cdot C_d = 1 \quad (2.41)$$

This shows that the fit of the polynomial constants, based on the best fit of the measurements of [4], is not well for the oblique grain test. The explanation of this deviation is the different state of hardening of the data that can be more or less strong, depending on the equilibrium stability of the type of test what is less in the uniaxial Hankinson test. This, for instance, follows from the ratio of the compression strengths perpendicular to the grain and along the grain of 0.2 in the uniaxial tests and 0.1 in the biaxial tests showing more hardening in the biaxial tests. Further, because the local peak is not occurring in the oblique grain test, the stability is less than in the biaxial test. An analogous behaviour occurs in the oblique grain test of clear wood [1] where the tensile test shows $C_t = 0$ in eq.(2.20) and the compression test shows C_d to be not zero. The tensile test shows unstable failure at yield what needs not to be so for the compression test that may show additional hardening. For the different hardening states in the different possible types of tests, the lowest always possible value should be used for practice thus

$C_t = C_d = 0$. It thus has to be concluded that the strong hardening in the biaxial test in the radial plane will not occur in other circumstances and the hardening parameters should be omitted for a safe lower bound criterion (in accordance with the oblique grain test). As generally found in [1] for spruce clear wood, a fit is possible for off-axis tension by a second degree polynomial with $F_{12} = 0$. This also applies for wood with defects, as follows from a fit of the data of [4] by the second degree polynomial (ellipse) in the principal stresses σ_1 and σ_2 ($\sigma_6 = 0$), for longitudinal tension ($\sigma_1 > 0$; $F_{12} = 0$), see fig. 2.4.3. This fit means that F_{112} and F_{122} are also zero (for $\sigma_1 > 0$) in the radial plane and because the Hankinson value for tension n is different from $n = 2$, there must be higher degree terms for shear (F_{166} , F_{266}). For fitting these parameters, several starting points are possible. A first hypothesis of A(1993) was rejected. :It was concluded that C_t and C_d are coupling terms between longitudinal tension and compression and that the different types of failure in longitudinal tension and in compression should be given in separate failure criteria for these cases. Because of the small values of F_{122} and F_{12} , the best fit for longitudinal tension $\sigma_1 > 0$ is, as hypothesis 2, chosen as fit for the total criterion for practice..

In table 1, column hyp. 2, this fit is given for $F_{12} = F_{112} = F_{122} = 0$. Because the fit does not change much when data above the uniaxial compression strength: $X' = 41.7$ are neglected, the fit applies for longitudinal compression too, given in column hyp.2, providing the same hardening state as in the oblique grain test (where the strong compression hardening does not occur). Based on the strength values of [4], the constants for this case, eq.(2.44), are:

$$C_t = 11.9 / X^2 = 11.9 / 59.5^2 = 0.00336; \quad C_d = 4 / X'^2 = 4 / 41.7^2 = 0.00230 \quad \text{and by eq.(2.22)} \\ 3F_{266} = C_t / Y' + C_d / Y = 0.00332 / 5.95 + 0.0023 / 3.5 = 0.00122 \quad \text{or } c \text{ of eq.(2.27) is:}$$

$$c_{266} = 0.00122 \cdot 9.7^2 \cdot 5.95 = 0.68 \quad \text{and:}$$

$$3F_{166} = C_t / X' + C_d / X = 0.00336 / 41.7 + 0.0023 / 59.5 = 0.000119, \text{ or:}$$

$$c_{166} = 0.000119 \cdot 9.7^2 \cdot 41.7 = 0.47.$$

$$F_1 = \frac{1}{X} - \frac{1}{X'} = 1/59.5 - 1/41.7 = -0.0072; \quad F_{11} = \frac{1}{XX'} = 1/(59.5 \cdot 41.7) = 0.00040,$$

$$F_2 = \frac{1}{Y} - \frac{1}{Y'} = 1/3.5 - 1/5.95 = 0.092; \quad F_{22} = \frac{1}{YY'} = 1/(3.5 \cdot 5.95) = 0.048 \quad \text{and:}$$

$$F_{66} = \frac{1}{S^2} = 1/9.7^2 = 0.0106; \quad F_{12} = F_{112} = F_{122} = 0.$$

Eq.(2.44) thus also applies for longitudinal compression as follows from fig. 2.4.3 and Table 1, hyp. 2, showing a better overall fit than according to [4] and to hyp. 4.

To correct the best fit of [4], to obtain a closed curve, the shear strength had to be reduced and a reduced factor 0.8 in stead of 0.9 for F_{112} , was necessary giving:

$$3F_{122} = 0.8 / (5.6 \cdot 43.1) = 0.000077; \text{ and}$$

$$3F_{166} = C_t / X' + C_d / X - 3F_{122} = 0.000128 - 0.000077 = 0.000051,$$

Thus giving the c-values: $c_{166} = 0.000051 \cdot 9.4^2 \cdot 43.1 = 0.2$ and: $c_{266} = 0.9$ (starting point).

This corrected fit is given in table 1, column 4 (compression fit), and it is seen that the corrected fit is not better than column [4] and needs further improvement. For $\sigma_6 = 0$, the fit for F_{112} is given in fig. 2.4.4. For longitudinal compression eq.(2.21) then becomes:

$$F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + 2F_{12} \sigma_1 \sigma_2 + F_{22} \sigma_2^2 + F_{66} \sigma_6^2 + 3F_{112} \sigma_1^2 \sigma_2 + 3F_{122} \sigma_2^2 \sigma_1 +$$

Section A, Failure criterion of wood and wood like polymers

$$+ 3F_{166}\sigma_6^2\sigma_1 + 3F_{266}\sigma_6^2\sigma_2 = 1 \quad (2.42)$$

Table 1. Shear strength σ_6 for combined normal stresses

σ_1	σ_2	σ_6 test	factor: $\sigma_{6,theory} / \sigma_{6,test}$				
			[4]	hyp. 1	hyp. 2 tens.	3 compr.	hyp. 4 compr.
30	1.5	5.8	1.07		1.03		1.02
30	0	8.5	0.88		0.91		0.92
30	-2.5	7.9	0.99		1.10		1.29
7.3	0	9.2	1.04		1.03		1.01
0	2.9	3.7	1.38		1.13		1.19
0	1.5	8.5	0.96		0.89		0.86
0	0	9.0	1.11		1.08		1.04
0	-2.5	10.9	0.96		1.05		1.07
0	-5.4	6.8	0.53		1.12		1.12
-7.7	0	9.6	1.05		1.01		0.96
-20	1.5	7.7	0.84		0.83		0.68
-20	0	9.6	0.99		0.96		0.88
-30	-2.5	11.3	1.04		0.90		0.94
mean factor			0.99		1.0		1.0

Inserting F-values in eq.(2.42), this equation becomes:

$$\frac{\sigma_6^2}{S^2} \cdot \left(1 + 0.9 \cdot \frac{\sigma_2}{Y'} + 0.2 \cdot \frac{\sigma_1}{X'} \right) = \left(1 - \frac{\sigma_1}{X} \right) \cdot \left(1 + \frac{\sigma_1}{X'} \right) + \left(1 - \frac{\sigma_2}{Y} \right) \cdot \left(1 + \frac{\sigma_2}{Y'} \right) +$$

$$- \left(1 + 0.8 \cdot \frac{\sigma_2 \sigma_1^2}{Y' X'^2} - 0.77 \cdot \frac{\sigma_1 \sigma_2^2}{X' Y'^2} - 0.41 \cdot \frac{\sigma_1 \sigma_2}{X' Y'} \right) \quad (2.43)$$

This equation thus only applies for the torsion tube test for failure in the radial plane, when it is assumed that negative values of F_{12} and F_{122} (by confined dilatation) are possible.

This however is questionable because its fit [4], in Table 1 is not well enough.

For longitudinal tension ($\sigma_1 \geq 0$), eq.(2.21) becomes:

$$\frac{\sigma_6^2}{S^2} \cdot \left(1 + 0.68 \cdot \frac{\sigma_2}{Y'} + 0.47 \cdot \frac{\sigma_1}{X'} \right) = \left(1 - \frac{\sigma_1}{X} \right) \cdot \left(1 + \frac{\sigma_1}{X'} \right) + \left(1 - \frac{\sigma_2}{Y} \right) \cdot \left(1 + \frac{\sigma_2}{Y'} \right) - 1 \quad (2.44)$$

As mentioned, this equation also applies for compression failure in the tangential plane.

Because the compression hardening F_{112} , F_{122} , according to eq.(2.43) occurs for low values of σ_6 only, and only in the torsion tube test in the radial plane, eq.(2.44) more

generally represents the failure criterion for both tension and compression and shear.

However, for tests and structures, showing early instability at initial flow, the higher degree hardenings terms will be zero, causing the Hankinson value of $n = 2$ for timber and glulam. Because this is to be expected in most situations in practice, the determining criterion becomes:

$$\frac{\sigma_6^2}{S^2} - \left(1 - \frac{\sigma_1}{X} \right) \cdot \left(1 + \frac{\sigma_1}{X'} \right) - \left(1 - \frac{\sigma_2}{Y} \right) \cdot \left(1 + \frac{\sigma_2}{Y'} \right) + 1 = 0,$$

or worked out, identical to eq.(2.5) with $F_{12} = 0$:

$$\frac{\sigma_6^2}{S^2} + \frac{\sigma_1}{X} - \frac{\sigma_1}{X'} + \frac{\sigma_1^2}{XX'} + \frac{\sigma_2}{Y} - \frac{\sigma_2}{Y'} + \frac{\sigma_2^2}{YY'} = 1 \quad (2.45)$$

It therefore is necessary to use eq.(2.45) in the Codes in all cases for timber and clear wood to replace the now commonly used, not valid Norris-equations. This criterion is a critical strain energy condition of the reinforcements leading to eq.(3.9) for equal tension and compression strengths and to eq.(3.11) with $F_{12} = 0$, for wood.

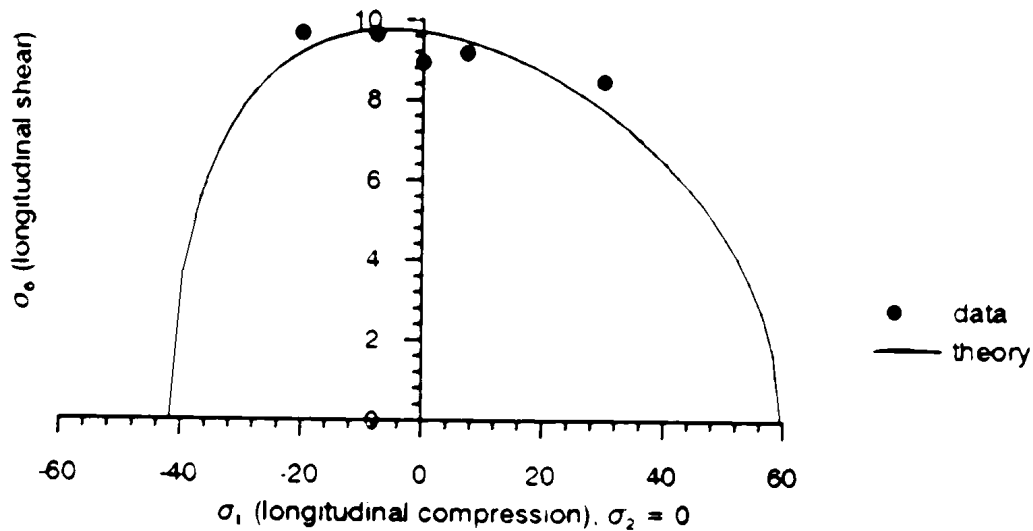


Figure 2.5.2 – Combined longitudinal shear with normal stress in grain direction. F_{166}

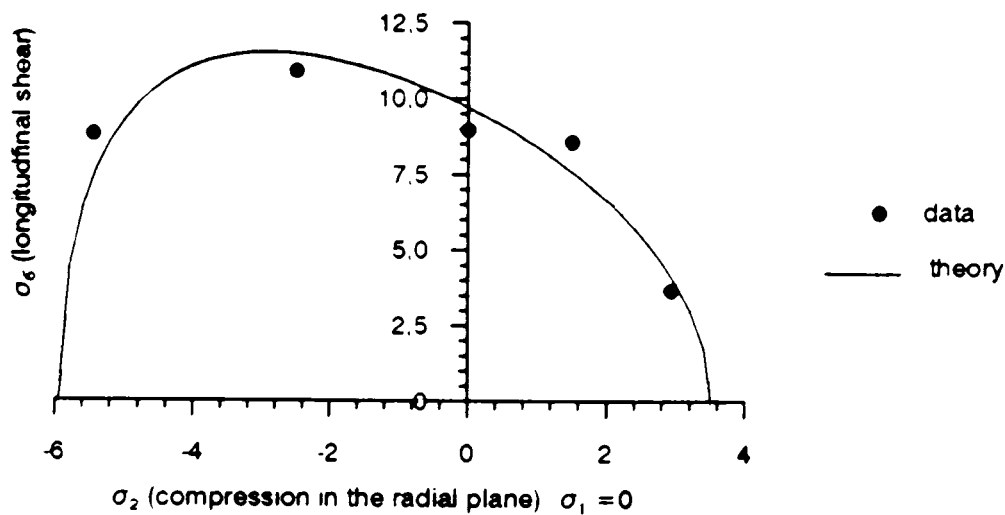


Figure 2.5.3 - Longitudinal shear strength ($\sigma_1 = 0$) depending on the normal stress. F_{266}

3. Discussion of applied failure criteria

3.1 Yield criterion.

A yield- or flow-criterion gives the combinations of stresses whereby flow occurs in an elastic-plastic material. For more brittle failure types in polymers with glassy components like wood at tensile loading, there is some boundary where below the behaviour is assumed to be elastic and where above the gradual flow of components at peak stresses and micro-cracking may have a similar effect as plastic flow with hardening (like metals with gradual plasticity and no yield point).

The loading, damage and hardening behaviour up to failure can fully be described by deformation kinetics [6]. There are several processes acting causing early local flow and stable micro-crack propagation, while the main part of the material is elastic and different inelastic strain rate equations are necessary depending on the loading type and material zone.

The failure criterion depends on the ultimate damage process and failure occurs when the standard test becomes unstable (due to loss of equilibrium).

3.2. Critical distortional energy of the isotropic matrix

It is not necessary to describe the whole initial loading curve with gradual flow and hardening to describe the ultimate state of flow. The unloading from this ultimate state is linear elastic and on again reloading, the loading line is linear elastic up to flow. Thus the geometry is unaltered and the loading history has no effect on the ultimate state and the linear elastic – full plastic approach of limit analysis is applicable and the initial yield criterion gives the boundary where below the behaviour is elastic.

Because an isotropic matrix of a material may sustain very large hydrostatic pressures without yielding, yield can be expected to depend on a critical value of the distortional energy. This energy is found by subtracting the energy of the volume change from the strain energy. Thus for the isotropic matrix material this is (expressed in matrix stresses):

$$\begin{aligned} & \left(\frac{1}{2E} \cdot (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E} \cdot (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x) + (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right) + \\ & - \left(\frac{1-2\nu}{6E} \cdot (\sigma_x + \sigma_y + \sigma_z)^2 \right) = \\ & = \frac{1+\nu}{6E} \cdot \left((\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \right) + \frac{1}{2G} \cdot (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \end{aligned} \quad (3.1)$$

For plane stress, the distortional energy thus is with $2G = E/(1 + \nu)$:

$$\frac{1+\nu}{3E} \cdot (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau^2) \quad (3.2)$$

When σ_x , σ_y and τ are the nominal stresses of a material, having a reinforcement in x and y direction that takes a part of the loading, then the distortional energy of the matrix becomes:

$$\frac{1+\nu}{3E} \cdot \left((1-c_x) \sigma_x^2 - \sigma_x \sigma_y + (1-c_y) \sigma_y^2 + 3(1-c_{tx} - c_{ty}) \tau^2 \right), \quad (3.3)$$

where the reinforcement parts are subtracted from the total load. For the reinforcement, taking only normal force and shear, this is:

$$\frac{1+\nu'}{3E_a} \cdot (\sigma_{ax}^2 + 3\tau_a^2) \quad \text{with:} \quad \sigma_{ax} = \left(\frac{E_a}{E} - 1 \right) \cdot \omega_x \sigma_x$$

where ω_x is the area of the reinforcement per unit area, giving:

$$c_x = \frac{1+\nu'}{1+\nu} \cdot \left(1 - \frac{E}{E_a}\right)^2 \cdot \omega_x^2 \cdot \frac{E_a}{E} \quad (3.4)$$

The other values of c_i are analogous.

When the distortional energy is constant at yield then eq.(3.3) gives:

$$(1-c_x)\sigma_x^2 - \sigma_x\sigma_y + (1-c_y)\sigma_y^2 + 3(1-c_{tx}-c_{ty})\tau^2 = C \quad (3.5)$$

For $\sigma_y = \tau = 0$, this gives the yield stress in x-direction $\sigma_x = X'$. In the same way

$\sigma_y = Y'$, when $\sigma_x = \tau = 0$ and is $\tau = S$ when $\sigma_x = \sigma_y = 0$, giving the equation:

$$\frac{\sigma_x^2}{X'^2} - 2F_{12}\sigma_x\sigma_y + \frac{\sigma_y^2}{Y'^2} + \frac{\tau^2}{S^2} = 1 \quad (3.6)$$

The Norris equation follows from eq.(3.6) when $2F_{12} = 1/X'Y'$. This however is a special value of $2F_{12}$ that need not to apply in general.

For the special case that: $c_{tx} = c_{ty} = 0$, when, as for concrete, it is assumed that the reinforcement takes no shear, eq.(3.5) becomes:

$$\frac{\sigma_x^2}{X'^2} - \frac{\sigma_x\sigma_y}{3S^2} + \frac{\sigma_y^2}{Y'^2} + \frac{\tau^2}{S^2} = 1 \quad (3.7)$$

and because $3S^2 \approx X'Y'$, as applies for isotropy and isotropy is assumed by Norris for the cell walls in his derivation (what also is measured), so that this equation becomes:

$$\frac{\sigma_x^2}{X'^2} - \frac{\sigma_x\sigma_y}{X'Y'} + \frac{\sigma_y^2}{Y'^2} + \frac{\tau^2}{S^2} = 1 \quad (3.8)$$

giving the Norris equation as critical distortional energy equation of the matrix when the reinforcement “flows” and thus only may carry a normal force.

Wood shows early failure of the matrix. Then the reinforcement carries the total load by the normal- and shear forces and the coupling term disappears and the equation gives the apparent critical distortional energy of the reinforcement:

$$\frac{\sigma_x^2}{X'^2} + \frac{\sigma_y^2}{Y'^2} + \frac{\tau^2}{S^2} = 1 \quad (3.9)$$

being the older empirical Norris equation.

The Norris equations (3.8) and (3.9) give the possible extremes of F_{12} between zero and the maximal value. Although the Norris-equations are used for wood, they only apply for materials with equal compression and tension strengths.

When these yield strengths are not equal, as for wood, different apparent critical distortional energies should be applied for tension and compression as first approximation.

3.3 Hankinson equations

The Hankinson equations apply for the off-axis uniaxial strengths and has to satisfy the Critical distortional energy equation for initial yield:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \quad (3.10)$$

where for uniaxial tensile stress is:

$$\sigma_1 = \sigma_t \cos^2 \theta \quad \sigma_2 = \sigma_t \sin^2 \theta \quad \sigma_6 = \sigma_t \cos \theta \sin \theta$$

Substitution of these stresses gives eq.(2.14) which can be resolved into factors giving eq.(2.17), what is the product of the Hankinson equation for tension and for compression. As discussed before, this is possible because::

$$2F_{12} + 1/S^2 \approx 1/X'Y + 1/XY' \quad (3.11)$$

In the generalized Hankinson equation, eq.(2.19):

$$\frac{\sigma_t \cos^n \theta}{X} + \frac{\sigma_t \sin^n \theta}{Y} = 1 \quad (3.12)$$

is the exponent $n = 2$ for the initial yield equation. Measured is also $n = 2$ for the strengths in bending and in tension of clear wood, also for veneer and for shear in the radial plane measured with the "Schereisen"-device. The measurements thus indicate that also in the radial plane $n = 2$ applies for initial yield. For $n \neq 2$, as may apply for compression, the extended Hankinson equations, eq.(2.20), apply.

3.4. Rankine criterion

The Hankinson equation (2.19) for $n = 2$,

$$\frac{\sigma_t \cos^2 \theta}{X} + \frac{\sigma_t \sin^2 \theta}{Y} = 1 \quad (3.13)$$

contains the maximum stress condition (or Rankine criterion) of failure for very low and for high angles (see fig. 3.2).

For θ in the neighborhood of $\theta = 90^\circ$, eq.(3.13) is about:

$$\frac{\sigma_t \sin^2 \theta}{Y} = 1 \quad (3.14)$$

the maximal stress criterion for tension perpendicular to the grain. This also applies down to e.g. 45° , because $1/X$ is of lower order with respect to $1/Y$ and thus the difference of eq.(3.14) with eq.(3.13) is of lower order then. In the same way, for very small values of θ , the ultimate tensile strength criterion in grain direction, eq.(3.15) applies:

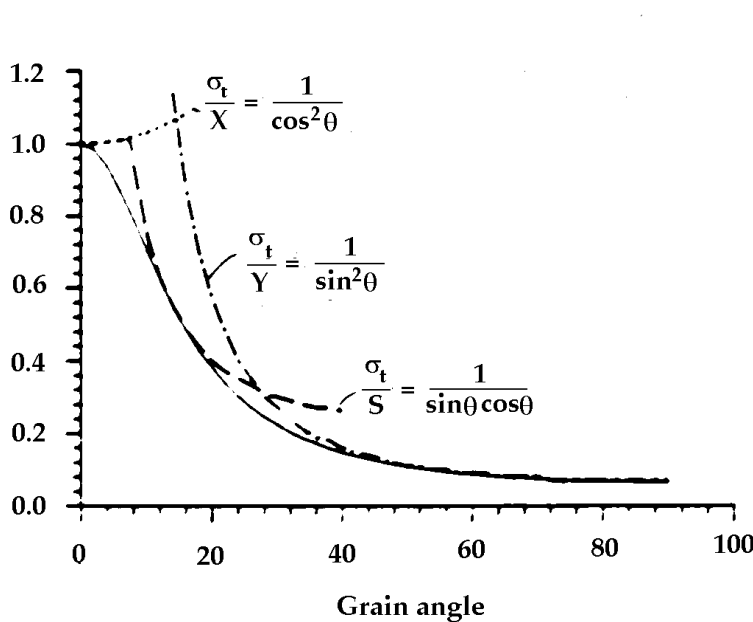


Figure 3.2. - Hankinson and Maximal stress criteria

$$\frac{\sigma_t \cos^2 \theta}{X} = 1 \quad (3.15)$$

For values of θ , where the first two terms of eq.(3.13) are equal or: $\cos \theta / \sqrt{X} = \sin \theta / \sqrt{Y}$,

the deviations of eq.(3.14) and (3.15) from eq.(3.13) are maximal (50%). In the neighborhood of this value of θ is:

$$(\cos \theta / \sqrt{X} - \sin \theta / \sqrt{Y})^2 \approx 0 \text{ or: } \cos^2 \theta / X + \sin^2 \theta / Y - 2 \sin \theta \cos \theta / \sqrt{XY} = 0$$

or with eq.(3.13):

$$\frac{\sigma_t \sin \theta \cdot \cos \theta}{\sqrt{XY} / 2} = \frac{\sigma_t \sin \theta \cdot \cos \theta}{S} = 1 \quad (3.16)$$

giving the ultimate failure criterion for shear by the fictive shear-strength: $S = \sqrt{XY} / 2$.

It is easy to show that this value of S is the point of contact of the lines eq.(3.16) and eq.(3.13). Although eq.(3.16) fits precisely at this point where $\tan \theta = \sqrt{Y/X}$, the difference of equations (3.14) to (3.16) with eq.(3.13) is too high at their intersects for application (see fig. 3.2). This also follows from figure 3.3 for wood and for other comparable polymers.

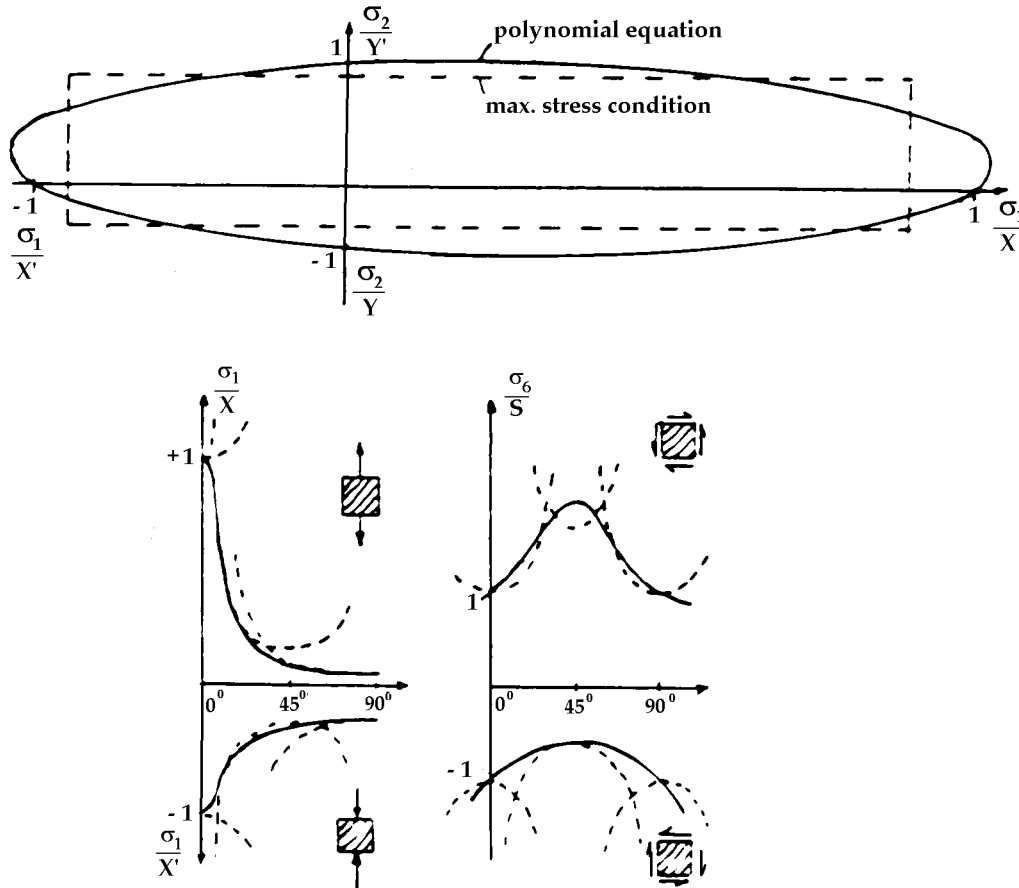


Figure 3.3. - Maximal stress failure conditions.

3.5. Norris equations

The Norris equations follow from the yield equation, eq.(3.10), when compression and tension strengths are equal: $X = X'$ and $Y = Y'$ and thus different equations should be used in each stress quadrant with the strengths X, Y ; X', Y ; X, Y' ; X', Y' .

When this is done, fig. 3.4 shows that the Norris equations still do not apply.

The success of these equations follows from the uniaxial applications (in the first and third quadrant) when the Hankinson equations apply.

After substitution of $X = X'$ and $Y = Y'$, the yield equation, eq.(2.14), can be resolved in factors, like eq.(2.17) as:

$$\left(\frac{\sigma_t \cos^2 \theta}{X'} + \frac{\sigma_t \sin^2 \theta}{Y'} - 1 \right) \cdot \left(\frac{\sigma_t \cos^2 \theta}{X'} + \frac{\sigma_t \sin^2 \theta}{Y'} + 1 \right) = 0$$

showing the Hankinson equations to apply and leading to:

$$\frac{\sigma_t^2 \cos^4 \theta}{X'^2} + \frac{\sigma_t^2 \sin^4 \theta}{Y'^2} + \frac{2\sigma_t^2 \sin^2 \theta \cos^2 \theta}{X'Y'} = 1 \quad (3.17)$$

This is equal to the Norris-criterion:

$$\frac{\sigma_t^2 \cos^4 \theta}{X'^2} + \frac{\sigma_t^2 \sin^4 \theta}{Y'^2} - \frac{\sigma_t^2 \sin^2 \theta \cos^2 \theta}{X'Y'} + \frac{\sigma_t^2 \sin^2 \theta \cos^2 \theta}{S^2} = 1 \quad (3.18)$$

when: $1/S^2 \approx 3/X'Y'$.

This value of S is measured and can be found in literature (see [1]), showing that the Norris equations are the same as the Hankinson equations for the uniaxial stress case.

For tension (replacing X' by X and Y' by Y in eq.(3.18)), it follows in the same way that

$S^2 = XY/3$, what may be different from the value for compression, showing that fictive values of S is needed in the other quadrants. Further, the yield criterion eq.(3.10) is an ellipsoid, having a small, (or zero) slope with respect to the σ_1 -axis and thus a negligible F_{12} . The centre of the ellipse in the 1 – 2 – plane is the point: $((X - X')/2; (Y - Y')/2)$.

When the part of this ellipse in e.g. the compression – compression quadrant has to be approximated by an ellipse with the centre at the point (0,0), (as applies for the Norris equation), then F_{12} of that ellipse has a pronounced value. In the tension – compression quadrant the apparent F_{12} even has the opposite sign. An improvement of eq.(3.18) thus will be to have a free slope of the ellipses and to use eq.(3.6) in stead as an extended Norris equation.

From eq.(3.17) it follows that:

$$\frac{\sigma_t^2 \cos^4 \theta}{X'^2} + \frac{\sigma_t^2 \sin^4 \theta}{Y'^2} + \frac{\sigma_t^2 \sin^2 \theta \cos^2 \theta}{S'^2} = 1 \quad (3.19)$$

when $1/S'^2 \approx 2X'Y'$ in eq.(3.17), giving the older empirical Norris equation, that has a zero F_{12} and fits better than the later proposed equation (3.18), but still does not fit in all quadrants (see fig. 3.4) because of the assumed equal compression and tension strengths. Further in all four stress quadrants an other, fictive shear strength has to be used.

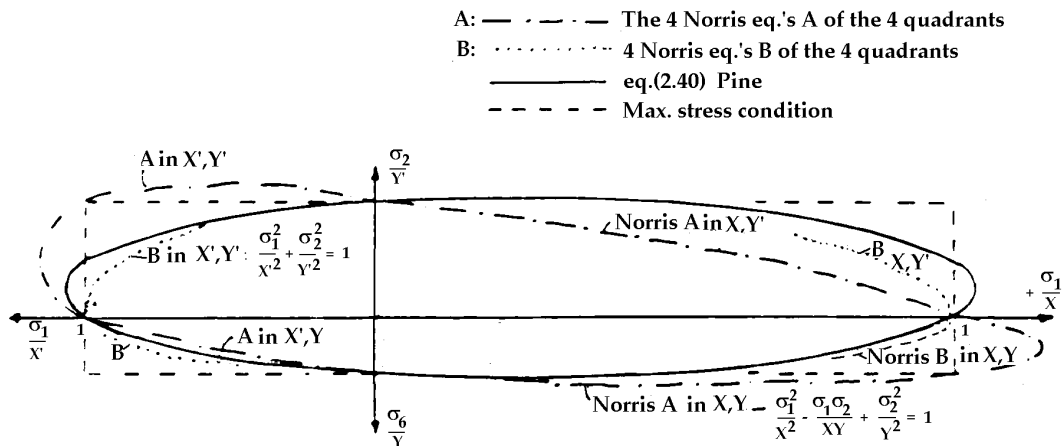


Figure 3.4. - Norris equations for $\sigma_6 = 0$.

It can be concluded that the Norris equations only can be applied for uniaxial stress being equivalent to the Hankinson equations for initial yield.

Because the Norris equations in the general form are not right, they should not be used any more.

As discussed before the hardening is mostly not present in tests and structures and a lower bound should be used where also F_{12} can be neglected. Thus for plane stress is:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \quad (3.20)$$

in all cases, what is more easy to use than the not valid Norris criteria.

In general eq.(3.21) applies for the 3-axial stress state, as also is discussed in [1]:

$$\begin{aligned} \sigma_1 \left(\frac{1}{X} - \frac{1}{X'} \right) + (\sigma_2 + \sigma_3) \cdot \left(\frac{1}{Y} - \frac{1}{Y'} \right) + \frac{\sigma_1^2}{XX'} + 2F_{12}(\sigma_1\sigma_2 + \sigma_1\sigma_3) + \frac{\sigma_2^2 + \sigma_3^2 + 2\sigma_4^2}{YY'} + \\ + \frac{\sigma_5^2 + \sigma_6^2}{S^2} = 1 \end{aligned} \quad (3.21)$$

In this equation σ_4 is the rolling shear and, σ_2 and σ_3 are the normal stresses in the tangential and radial planes. In this equation too, $F_{12} = 0$ should be assumed.

It thus can be concluded that the critical distortional energy criterion, reduced when $F_{12} = 0$, to the critical strain energy criterion, also has to be used as a lower bound of the ultimate failure condition.

4. Conclusions

- The tensor polynomial failure criterion is shown to be regarded as a polynomial expansion of the real failure criterion.
- It is also shown (Appendix 1), that the second degree tensor-polynomial yield criterion represents the critical distortional energy principle for initial yield.
- Initial flow in transverse direction, follows the second degree polynomial eq.(2.15). For compression, (perpendicular), strong hardening is possible leading to the isotropic strength behavior (independent of orientation), at the strain where all empty spaces are pressed away.
- For longitudinal initial yield in the radial plane, the second degree polynomial eq.(2.42), (with $F_{12} = F_{122} = 0$), applies in a stable test, while in the tangential plane $F_{122} = F_{112} = F_{12} = 0$. When early failure instability occurs in the test, at initial crack extension, as for instance in the oblique-grain tension test, or for shear with compression in the “Schereisen” test, there are no third and higher degree terms, also not in the radial plane. Higher degree terms thus are due to hardening, depending on the type of test, due to stable crack propagation and crack arrest after initial yield.
- The third degree polynomial hardening terms of the failure criterion are shown to represent the, in C(2011) theoretical derived, Wu-mixed-mode I-II fracture equation, showing hardening to be based on hindered micro-crack extension and micro-crack arrest. This also applies for kinkband and slip line formation of compression failure, eq.(2.35), which is a variant of shear failure according to the mixed mode Wu-equation. Important is the conclusion that the failure criterion shows that micro-crack extension is always involved in fracture processes. The derivation of the new fracture mechanics theory, is therefore based on micro-crack extension. In C(2014), the exact derivation is given of the

geometric correction factor for small crack extension towards the macro-crack tip. This correction factor appears to be numerical the same as for macro-crack extension.

- Because in limit analysis, the extremum variational principle applies for initial “flow” and thus the virtual work equations apply, the variations are sufficient small to get a linear irreversible process, and then the plastic potential function exists, which is identical to the yield function at flow, and for which the normality rule applies. This thus applies for the derived orthotropic critical distortional energy criterion, making complete exact solutions possible.

- Wood behaves like a reinforced polymer. The absence of coupling term, $F_{12}=0$, between the normal stresses in the main planes, means that the reinforcement takes only normal loading, causing the matrix to carry the whole shear loading. Therefore also $F_{122}=0$. The reinforcement then is the most effective, as when flow of the reinforcement occurs.

- Failure of the matrix occurs before flow of the reinforcement. This follows e.g. from Balsa wood, which is highly orthotropic, but shows the isotropic ratio of the critical stress intensities of the isotropic matrix material at failure at initial flow. For dense, strong, (thus with a high reinforcement content) clear wood, this is shown by the oblique crack extension, according to Fig. 1 of C(2011), showing the isotropic oblique angle at the start of shear crack extension, and thus shows the matrix to be determining for initial failure. It is therefore a requirement for an exact orthotropic solution, applicable to wood, to satisfy the equilibrium condition for the total orthotropic stresses, as well as for the isotropic stresses in the matrix, at failure. This last condition is not satisfied in all other existing fracture mechanics models.

- Early failure of the matrix causes stress redistribution of mainly shear with compression in the matrix and increased tensile stress in the fibres. The measured negative contraction for creep in tension indicates this mechanism. As in reinforced concrete, truss action is possible, as noticeable by the strong negative contraction coefficient (swelling instead of contraction) in the bending tensile zone of the beam. Failure in compression is determined by the difference in the principal compression stresses. Thus the maximal shear stress or Tresca criterion applies. The necessary validity of the Tresca criterion is confirmed by D(2008b) and D(2008a), where the strongly increased (sixfold) compression strength under the load of locally loaded blocks and the increased embedding strength of dowels is explained by the construction of the equivalent slip line field in the specimen based on the Tresca criterion. In addition, the many apparent contradictions of the different investigations are explained by this theory. This strong increase of the compression strength is due to confined dilatation by real hardening (when the empty spaces in wood are pressed away).

- The initial yield equation for uniaxial loading can be resolved into factors containing the Hankinson equation for tension and compression for $n = 2$. Thus when the Hankinson parameter n in eq.(2.19) is $n = 2$, in tension and in compression, all higher degree terms are zero. This applies for clear wood, depending on the type of test. It also is probable that this is a general property for timber [11], due to preferred failure of the tangential plane.

- The yield equation for uniaxial loading, containing higher degree terms, can be resolved in factors of the extended Hankinson equations, eq.(2.46) for tension and compression when n in eq.(2.19) is different from $n = 2$.

- For wood, at least in the radial plane, after hardening in a stable test, the combined compression - shear strength depends on the third degree coupling term F_{266} , or F_{166} , giving the parabolic Mohr- or Wu- equation of fracture. This is theoretically explained in [9] by micro-crack propagation in grain direction. This increase of the shear strength is an equivalent hardening effect due to crack arrest in the worst direction by strong layers.

It is shown that the increase of the shear strength, by compression perpendicular to the shear plane, is not due to Coulomb friction, being too small for wood.

- Because of the grain deviations from the regarded main directions, there always is combined shear-normal stress loading in the real material planes where eq.(2.27) applies.

F_{112} is due to misalignment of the vertical cells by rays in the radial planes.

- Therefore, for wood in longitudinal compression in the radial plane this micro-crack failure mechanism is determining, giving high values of F_{266} and F_{112} , close to their bounds of $c \approx 0.8$ to 0.9 .

- The same as found for F_{266} as function of σ_2 , is to be expected for F_{166} as function of σ_1 . This is given in fig. 2.5.2.

- For wood in longitudinal tension, F_{12} , F_{112} and F_{122} are zero and only F_{166} and F_{266} remain in the radial plane as higher degree terms, in stable tests, showing an other type of failure than for longitudinal compression.

For longitudinal compression, at $\sigma_6 = 0$, equivalent slip line hardening, (high F_{112}) as well hardening by confined dilatation (showing a negative F_{122} and F_{12}) may occur. This last type of hardening occurs only in the torsion tube test, because the negative F_{122} and F_{12} of [4] predict the compression peak of fig. 2.5.1 in the oblique grain test, that does not occur by the lack of hardening in the oblique grain test. This also will be so for structural elements and the lower bound criterion with only F_{166} and F_{266} (and zero F_{12} , F_{112} and F_{122}) is probably more reliable (hyp 2 fits better than hyp 4 in Table 1) for longitudinal compression failure in the radial plane. In the tangential plane also F_{166} and F_{266} are zero, making the second degree criterion determining.

- In general thus eq.(3.21) applies for the 3-axial stress state, as is discussed in [1]:

$$\sigma_1 \left(\frac{1}{X} - \frac{1}{X'} \right) + (\sigma_2 + \sigma_3) \cdot \left(\frac{1}{Y} - \frac{1}{Y'} \right) + \frac{\sigma_1^2}{XX'} + \frac{\sigma_2^2 + \sigma_3^2 + 2\sigma_4^2}{YY'} + \frac{\sigma_5^2 + \sigma_6^2}{S^2} = 1$$

where σ_4 is the rolling shear and σ_2 and σ_3 are the normal stresses in the tangential and radial planes and where it is assumed that $F_{12} = 0$ as applies for longitudinal tension.

- Equations (2.28) and (2.44) can be used for analyzing test data. Because it is questionable that the hardening by confined dilatation or crack arrest may occur in all circumstances, because it depends on the type of test, the hardening contained by the third degree terms should be omitted for a general application.

- Therefore the second degree polynomial, eq.(3.20) or eq.(2.45), for plane stress:

$$\frac{\sigma_6^2}{S^2} + \frac{\sigma_1}{X} - \frac{\sigma_1}{X'} + \frac{\sigma_1^2}{XX'} + \frac{\sigma_2}{Y} - \frac{\sigma_2}{Y'} + \frac{\sigma_2^2}{YY'} = 1$$

should be used for initial yield and for ultimate failure for the Codes and as initial yield equation, it applies for the 5th percentile of the strength as well.

- Only this derived extension of the von Mises criterion contains the, for orthotropic materials, necessary independent value of the interaction constant as F_{12} and accounts for different tension- and compression strengths and is able to give the strength in any direction in the strength tensor form.

- The ultimate stress principle for failure, eq.(3.14), (3.15) and (3.16), does not apply for the general loading case and only applies locally and approximately for only uniaxial loading. These equations also are predicted by the fracture mechanics singularity method [9], showing thus that this method, that always is applied in fracture mechanics for all materials, is not right and should not be used.

- The Norris equations are not generally valid and are only for uniaxial loading identical to the Hankinson equation with $n = 2$, when the right (mostly) fictive shear-strength is used. This equation thus should not be used any more.
- There thus is no reason to not apply this exact general criterion, also for the future Codes, for all cases of combined stresses. Only this criterion gives the possibility of a definition of the off-axis strength of anisotropic materials.
- It was for the first time shown in A(1982) that the tensor polynomial failure criterion applies to wood. Also is shown, that the fourth-degree and higher-degree polynomial terms have no physical meaning and thus are zero. Only the third-degree polynomial part is identical to the real initial flow criterion, while the third degree terms represent deviations from orthotropic behavior and represent post initial flow hardening behavior, which numerical value depends on the stability of the test specimen and testing device.
- For uniaxial loading, the failure criterion can be resolved in factors, leading to the derivation of extended Hankinson equations. This provides a simple method to determine all strength parameters by simple uniaxial, oblique grain compression and tension tests. Based on this, the numerical failure criterion is given with the simple lower bound criterion for practice and for the codes,
- The existence of an isotropic matrix in wood (lignin with branched hemicellulose) follows not only from material analysis, but also, as mentioned, from the high compression strength at confined dilation with the absence of failure by triaxial hydrostatic compression, (what is not the case for orthotropy, because then, for equal triaxial stresses, the strains then are not equal and yield remains possible).
- Plastic flow in wood starts with propagation of empty spaces by segmental jumps, just as the dislocation propagation in steel and the possibility should be accounted that there is no change in density at initial flow (as for steel) and the plastic incompressibility condition should be accounted as possibility, and as follows from the normality rule of flow in combination with perfect plasticity, the Tresca criterion (maximal shear stress criterion) then also should apply. By the dissipation according to the incompressibility condition, the minimum energy principle is followed providing the lowest possible upper bound and therefore the closest to the exact flow criterion. Limit analysis of the matrix therefore has to be based on incompressibility and the Tresca criterion.
- It has to be stressed, for the virtual work equations of limit analysis, that neither the chosen equilibrium, nor the compatible strain and displacement set need not be the actual state, nor need the equilibrium and compatible sets to be related in any way to each other.
- The loading curve up to yield and failure also should be described by deformation kinetics [6] to adapt for temperature, time and loading rate influences.

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Appendix 1

Derivation of the orthotropic critical distortional energy principle

It was for the first time shown for wood, in A(1982), that the second degree tensorpolynomial describes initial “flow”, what is shown in the following, to represent the orthotropic extension of the critical distortional energy criterion providing an exact flow criterion as necessary basis for exact solutions according to limit analysis. Because the matrix of wood material is isotropic and therefore may sustain large hydrostatic pressures without yielding, yield depends on a critical value of the distortional energy. This energy W_d is found by subtracting the energy of the volume change from the total strain energy. Thus for the isotropic matrix material this is:

$$\begin{aligned}
 W_d &= \left(\frac{1}{2E} \cdot (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E} \cdot (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x) + \frac{1+\nu}{E} \cdot (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right) + \\
 &\quad - \left(\frac{1-2\nu}{6E} \cdot (\sigma_x + \sigma_y + \sigma_z)^2 \right) = \\
 &= \frac{1+\nu}{6E} \cdot \left((\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \right) + \frac{1}{2G} \cdot (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \quad (a.1)
 \end{aligned}$$

where σ_i are the normal matrix stresses; τ_i the shear stresses; E the modulus of elasticity,

G the shear modulus and ν Poisson's ratio of the matrix material following $2G = E/(1 + \nu)$. Wood has to be regarded as a reinforced material and initial failure is due to failure of the isotropic matrix. This is shown in C(2007b), leading to a new fracture mechanics theory and a new transformation of the Airy stress function, making exact solutions possible as applied for the derivations of the Wu mixed mode I-II fracture criterion and the derivations of the right fracture energies and the relation between mode I and II stress intensities and energy release rates. According to C(2007b), the matrix stresses can be expressed in orthotropic stresses as follows:

The stress in wood $\sigma_{x,or}$ is n_1 times the stress in the matrix σ_x due to the reinforcement in x-direction: $\sigma_{x,or} = (E_x / E) \cdot \sigma_x = n_1 \cdot \sigma_x$, while the reinforcement in y-direction is regarded to belong to the matrix, thus $\sigma_{y,or} = \sigma_y$ and $E_y = E$ of the matrix. For the shear stress, the multiplying factor is $n_6 = (2 + \nu_{xy} + \nu_{yx}) \cdot G_{xy} / E$. Thus E_x , E_y , G_{xy} , ν_{xy} and ν_{yx} are the orthotropic values of wood due to the reinforcements.

Eq.(a1) applies for a material with equal tension and compression strength. For unequal axial strengths, the failure condition e.g. in x-direction is: $(\sigma_x - X) \cdot (\sigma_x + X') = 0$, where X is the tensile strength and $-X'$, the compression strength, as given in Fig.1a.

This condition can be written like:

$$\left(\sigma_x - \frac{X - X'}{2} \right)^2 = \left(\frac{X + X'}{2} \right)^2 \quad \text{or:} \quad \sigma_x - p_x = \pm \bar{X} \quad (\text{a.2})$$

and the behavior is identical to that of a material with equal tension and compression strengths of $\bar{X}$ being pre-stressed by stress p_x .

This result follows from the applied linear transformation. Because Eq.(a.1) is a physical property, it should be independent of the chosen vector space and according to the additivity rule of linear mapping (linear transformation) is:

$f(\mathbf{x} + \mathbf{y}) = f(\mathbf{x}) + f(\mathbf{y})$, or in this case: $f(\boldsymbol{\sigma} - \mathbf{p}) = f(\boldsymbol{\sigma}) + f(-\mathbf{p})$ giving:

$$f(\boldsymbol{\sigma}) = f(\boldsymbol{\sigma} - \mathbf{p}) - f(-\mathbf{p}) \quad (\text{a.3})$$

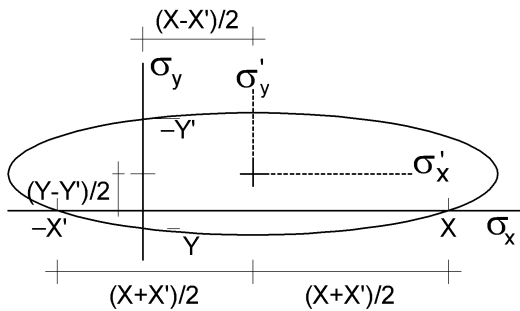


Figure 1a. – von Mises criterion for wood.

Substitution of $\sigma_x - p_x$, $\sigma_y - p_y$ and $\sigma_z - p_z$ for respectively σ_x , σ_y and σ_z in Eq.(1) gives:

$$\begin{aligned} & \left(\frac{\sigma_{x,or}}{n_1} - \frac{p_{x,or}}{n_1} - \sigma_{y,or} + p_y \right)^2 + \left(\sigma_{y,or} - p_y - \sigma_{z,or} + p_z \right)^2 + \left(\sigma_{z,or} - p_z - \frac{\sigma_{x,or}}{n_1} + \frac{p_{x,or}}{n_1} \right)^2 + \\ & + 6 \sum \tau_{ij}^2 / n_i = 2C \quad (= 6EW_d / (1 + \nu)) \quad \text{and after subtraction of: } f(-\mathbf{p}) \text{ this is:} \\ & \frac{\sigma_{x,or}^2}{n_1^2} + \sigma_{y,or}^2 + \sigma_{z,or}^2 - \frac{\sigma_{x,or}\sigma_{y,or}}{n_1} - \sigma_{z,or}\sigma_{y,or} - \frac{\sigma_{x,or}\sigma_{z,or}}{n_1} + \left(p_y + p_z - \frac{2p_{x,or}}{n_1} \right) \frac{\sigma_{x,or}}{n_1} + \end{aligned}$$

$$\begin{aligned}
 & + \left(p_y - 2p_z + \frac{p_{x,or}}{n_1} \right) \sigma_{z,or} + \left(p_z - 2p_y + \frac{p_{x,or}}{n_1} \right) \sigma_y + 3\sum \tau_{ij}^2 / n_i = -C_p + f(-\mathbf{p}) + C = \\
 & = C = 3EW_d / (1+\nu)
 \end{aligned} \tag{a.4}$$

with: $f(-\mathbf{p}) = p_{x,or}^2 / n_1^2 + p^2 + p_z^2 - pp_{x,or} / n_1 - pp_z - p_z p_{x,or} / n_1 = C_p$.

following from inserting $\sigma_x = p_x$, $\sigma_y = p_y = p$ and $\sigma_z = p_z$ in Eq.(a.1).

Of interest for failure by flat crack propagation is the plane stress equation with

$\sigma_z = p_z = 0$; $\tau_{xz} = \tau_{yz} = 0$ and with $p_y = p_{y,or} = p$, giving for Eq.(a.4):

$$\frac{\sigma_{x,or}^2}{C' \cdot n_1^2} - \frac{\sigma_{x,or} \cdot \sigma_{y,or}}{C' \cdot n_1} + \frac{\sigma_{y,or}^2}{C'} + \frac{\sigma_{x,or}}{C' \cdot n_1} \left(p - 2 \frac{p_{x,or}}{n_1} \right) - \frac{\sigma_y}{C'} \left(2p - \frac{p_{x,or}}{n_1} \right) + 3 \frac{\tau_{or}^2}{C' \cdot n_6^2} = 1 \tag{a.5}$$

For $\sigma_{y,or} = \tau_{or} = 0$, Eq.(a.5) becomes:

$$\frac{\sigma_{x,or}^2}{C' \cdot n_1^2} + \frac{\sigma_{x,or}}{C' \cdot n_1} \left(p - 2 \frac{p_{x,or}}{n_1} \right) = 1$$

This is identical to $(\sigma_{x,or} - X)(\sigma_{x,or} + X') = 0$, or to: $\sigma_{x,or}^2 + (X' - X)\sigma_{x,or} - XX' = 0$,

showing that: $(pn_1 - 2p_{x,or}) = X' - X$, and $C' \cdot n_1^2 = XX'$

The same applies in the perpendicular y-direction for the uniaxial tension and compression strengths Y and Y' giving: $C' = YY'$ and $(p_{x,or} / n_1 - 2p) = Y' - Y$

This last result is to be expected because according to the molecular theory, the strength is proportional to the E-modulus and thus is $YY' = XX' / n_1^2$ and $X' - X = n_1(Y - Y')$. Then

also is: $p = p_{x,or} / n_1 = Y - Y' = (X - X') / n_1$.

Eq.(a.5) becomes:

$$\frac{\sigma_{x,or}^2}{C' \cdot n_1^2} - \frac{\sigma_{x,or} \cdot \sigma_{y,or}}{C' \cdot n_1} + \frac{\sigma_{y,or}^2}{C'} - p \frac{\sigma_{x,or}}{C' \cdot n_1} - p \frac{\sigma_{y,or}}{C'} + 3 \frac{\tau_{or}^2}{C' \cdot n_6^2} = 1 \quad \text{or:} \tag{a.6}$$

$$\frac{\sigma_{x,or}^2}{XX'} + \frac{\sigma_{x,or}}{X} - \frac{\sigma_{x,or}}{X'} - 2F_{12} \sigma_{x,or} \sigma_{y,or} + \frac{\sigma_{y,or}^2}{YY'} + \frac{\sigma_{y,or}}{Y} - \frac{\sigma_{y,or}}{Y'} + \frac{\tau_{or}^2}{S^2} = 1 \tag{a.7}$$

where S is the shear strength and: $2F_{12} = 1 / C' n_1 = 1 / \sqrt{XX'YY'}$ (a.8)

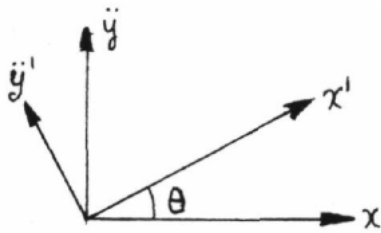
This value of F_{12} applies for the elastic state. At initial stress redistribution and micro-cracking of the matrix and F_{12} becomes lower reaching a near zero value at yield or failure initiation. This may indicate an early dissipation of the elastic distortional energy for formation of initial micro-cracks. This dissipation of distortional energy is according to the incompressibility condition and thus follows a minimum energy principle of yield. At the end of this stress redistribution, yield occurs according to Eq.(a.7) with $F_{12} = 0$. This last means an absence of coupling terms between the normal stresses. This only is possible when the reinforcement takes the whole normal loading and no shear, causing the matrix to fail by shear and the critical distortional energy principle thus reduces to the Tresca criterion. The necessary validity of the Tresca criterion is confirmed in [17] and [18], where the strongly increased (6-fold) compression strength under the load of locally loaded blocks and the increased embedding strength of dowels and nails, is explained by the construction of the equivalent slip line field in the specimen, using the Tresca criterion. The Tresca criterion satisfies the normality rule and thus inherently the theorems of limit analysis for matrix failure. The normality rule thus does not apply for hardening. This condition is shown to be replaced by the minimum work condition for dissipation

represented by the yield equation and the hardening state constants C_d and C_t of Eq.(17). Thus after initial yield, shear strength hardening is possible according to the mixed mode Wu equation and finally when the empty spaces in wood are pressed away, real hardening is possible by confined dilatation at locally compression loading of the isotropic matrix. This is discussed in Section D.

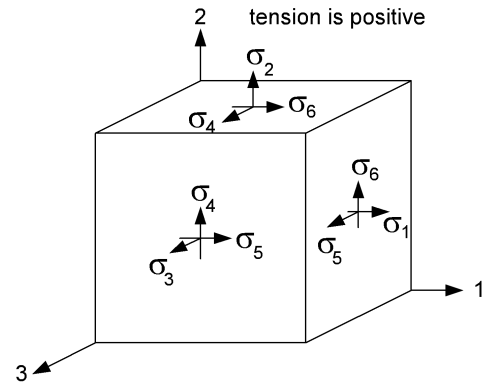
Appendix 2

Transformation of strength tensors:

F_{ij}



Positive rotation about the main 3-axis (z-axis)



Positive signs in right handed coordinate system

Sign convention for shear:

If an outward normal of a plane points to a positive direction, the plane is positive, and if on a positive plane the stress component acts in the positive coordinate direction, this component is positive.

In the x' , y' coordinates of figure above the strength tensors are:

$$F_i = \begin{Bmatrix} F'_1 \\ F'_2 \\ F'_3 \\ F'_4 \\ F'_5 \\ F'_6 \end{Bmatrix} ; F_{ij} = \begin{Bmatrix} F'_{11} & F'_{12} & F'_{13} & F'_{14} & F'_{15} & F'_{16} \\ & F'_{22} & F'_{23} & F'_{24} & F'_{15} & F'_{26} \\ & & F'_{33} & F'_{34} & F'_{35} & F'_{36} \\ & & & F'_{44} & F'_{45} & F'_{46} \\ & & & & F'_{55} & F'_{56} \\ & & & & & F'_{66} \end{Bmatrix}$$

symmetry

The principal strength components are:

$$F_i = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} ; F_{ij} = \begin{Bmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ & F_{22} & F_{23} & 0 & 0 & 0 \\ & & F_{33} & 0 & 0 & 0 \\ & & & F_{44} & 0 & 0 \\ & & & & F_{55} & 0 \\ & & & & & F_{66} \end{Bmatrix}$$

sym.

Section A, Failure criterion of wood and wood like polymers

Transformation about the 3- axis gives:

$$F_1' = \frac{F_1 + F_2}{2} + \frac{F_1 - F_2}{2} \cos(2\theta); \quad F_2' = \frac{F_1 + F_2}{2} - \frac{F_1 - F_2}{2} \sin(2\theta)$$

$$F_6' = -(F_1 - F_2) \sin(2\theta); \quad F_3' = F_3; \quad F_4' = F_5' = 0$$

F'_{ij}	invariant	$\cos 2\theta$	$\sin 2\theta$	$\cos 4\theta$	$\sin 4\theta$
F'_{11}	I_1	I_2	0	I_3	0
F'_{22}	I_1	$-I_2$	0	I_3	0
F'_{12}	I_4	0	0	$-I_3$	0
F'_{66}	$4I_5$	0	0	$-4I_3$	0
F'_{16}	0	0	$-I_2$	0	$-2I_3$
F'_{26}	0	0	$-I_2$	0	$+2I_3$
F'_{13}	I_6	I_7	0	0	0
F'_{23}	I_6	$-I_7$	0	0	0
F'_{36}	0	0	$-I_7$	0	0
F'_{44}	I_8	I_9	0	0	0
F'_{55}	I_8	$-I_9$	0	0	0
F'_{45}	0	0	I_9	0	0
F'_{33}	F_{33}	0	0	0	0

Read e.g. $F'_{11} = I_1 + I_2 \cos 2\theta + I_3 \cos 4\theta$

$$I_1 = (3F_{11} + 3F_{22} + 2F_{12} + 3F_{66})/8; \quad I_2 = (F_{11} - F_{22})/2; \quad I_3 = (F_{11} + F_{22} - 2F_{12} - F_{66})/8;$$

$$I_4 = (F_{11} + F_{22} + 6F_{12} - F_{66})/8; \quad I_5 = (F_{11} + F_{22} - 2F_{12} + F_{66})/8; \quad I_6 = (F_{13} + F_{23})/2;$$

$$I_7 = (F_{13} - F_{23})/2; \quad I_8 = (F_{44} + F_{55})/2; \quad I_9 = (F_{44} - F_{55})/2.$$