Correlation singularities in partially coherent electromagnetic beams

Shreyas B. Raghunathan,[1] Hugo F. Schouten,[2] and Taco D. Visser[1,2,*]

[1]Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, Delft 2628 CD, The Netherlands

[2]Department of Physics and Astronomy, and Institute for Lasers, Life and Biophotonics, VU University, Amsterdam 1081 HV, The Netherlands

*Corresponding author: T.D.Visser@tudelft.nl

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We demonstrate that coherence vortices, singularities of the correlation function, generally occur in partially coherent electromagnetic beams. In successive cross sections of Gaussian Schell-model beams, their locus is found to be a closed string. These coherence singularities have implications for both interference experiments and correlation of intensity fluctuation measurements performed with such beams.

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The subject of singular optics [1,2] is the structure of wave fields in the vicinity of optical vortices and polarization singularities. Most studies deal with monochromatic, and hence fully coherent, light. Many wave fields that are encountered in practice, however, are partially coherent. Examples are the fields generated by multimode lasers and fields that have traveled through a random medium such as the atmosphere. The statistical properties of these fields are described by correlation functions, such as the *spectral degree of coherence* [3,4]. A few years ago it was pointed out that these correlation functions can also exhibit singular behavior [5]. Such correlation singularities, or "coherence vortices," occur at pairs of points at which the fields are completely uncorrelated. Coherence vortices have since been found in optical beams [6], in focused fields [7], and in fields produced by Mie scattering [8]. These studies are all limited to scalar fields. Although the concept of a spectral degree of coherence has been generalized to electromagnetic beams [9], the possible existence of *electromagnetic* coherence singularities in practical physical systems has not yet been examined. In this Letter we show that these singularities occur quite generally in a wide class of electromagnetic beams, namely those of the Gaussian Schell-model type. We describe their evolution in successive cross sections of these beams, and their physical implications.

The state of coherence and polarization of a random beam that propagates along the z-axis is characterized by the *electric cross-spectral density matrix* [9]

$$\mathbf{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\begin{pmatrix} W_{xx}(\mathbf{r}_1,\mathbf{r}_2,\omega) & W_{xy}(\mathbf{r}_1,\mathbf{r}_2,\omega) \\ W_{yx}(\mathbf{r}_1,\mathbf{r}_2,\omega) & W_{yy}(\mathbf{r}_1,\mathbf{r}_2,\omega)\end{pmatrix}, \quad (1)$$

where

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle E_i^*(\mathbf{r}_1,\omega)E_j(\mathbf{r}_2,\omega)\rangle, \qquad (i,j=x,y). \quad (2)$$

Here $E_i(\mathbf{r},\omega)$ is a Cartesian component of the electric field at a point $\mathbf{r}$ at frequency ω, of a typical realization of the statistical ensemble representing the beam. The spectral degree of coherence $\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)$ of the field is defined as

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{\mathrm{Tr}\mathbf{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{[\mathrm{Tr}\mathbf{W}(\mathbf{r}_1,\mathbf{r}_1,\omega)\mathrm{Tr}\mathbf{W}(\mathbf{r}_2,\mathbf{r}_2,\omega)]^{1/2}}, \quad (3)$$

where Tr denotes the trace. A correlation singularity occurs at pairs of points for which $\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)=0$. (From here on the ω-dependence of the various quantities is suppressed.) The physical meaning of correlation singularities is twofold. First, when the fields at two points $\mathbf{r}_1$ and $\mathbf{r}_2$ are combined in Young's experiment, the visibility of the ensuing interference fringes depends on the value of $\eta(\mathbf{r}_1,\mathbf{r}_2)$ [9, Section 9.2]. At a singularity, where $\eta(\mathbf{r}_1,\mathbf{r}_2)=0$, the fringe visibility will be zero. Second, in Hanbury Brown–Twiss experiments, one determines the correlation of intensity fluctuations at two points [10]. These correlations depend on the so-called *degree of cross polarization* [11]. It is easily seen that correlation singularities coincide with a divergence of the degree of cross polarization. The consequences of this are discussed by Hassinen *et al.* [12]. In view of these effects and because of the practical importance of partially coherent beams, it is therefore of interest to ask whether they contain coherence vortices.

According to Eq. (3), coherence vortices occur in a transverse plane z when both

$$|W_{xx}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)|=|W_{yy}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)|, \quad (4)$$

$$\mathrm{Arg}[W_{xx}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)]-\mathrm{Arg}[W_{yy}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)]=\pi \pmod{2\pi}. \quad (5)$$

For fixed $\boldsymbol{\rho}_1$ and z, the points $\boldsymbol{\rho}_2$ that satisfy condition (4) generally form a line. The same holds true for the solutions of Eq. (5). We therefore expect the simultaneous solutions, i.e., the coherence vortices, to be isolated points in the two-dimensional $\boldsymbol{\rho}_2$-plane. Note that when the fields at the two points that form an electromagnetic coherence singularity are combined in Young's experiment, the local modulations of $|E_x|^2$ and $|E_y|^2$ on the observation screen have equal magnitude and opposite sign, resulting in zero visibility of the total spectral density.

As we will show, such correlation singularities generically occur in Gaussian Schell-model beams [9], a wide

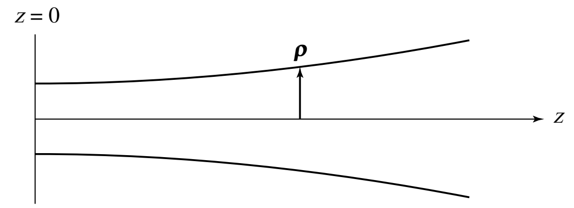

Fig. 1. Illustrating the notation. The vector $\boldsymbol{\rho} = (x, y)$ indicates a transverse position.

class of partially coherent electromagnetic beams that includes the lowest-order Gaussian laser mode. For these beams, the elements of the cross-spectral density matrix in the source plane $z = 0$ (see Fig. 1) read

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z = 0) = \sqrt{S_i(\boldsymbol{\rho}_1) S_j(\boldsymbol{\rho}_2)} \mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1), \tag{6}$$

with the spectral densities $S_i(\boldsymbol{\rho}) = W_{ii}(\boldsymbol{\rho}, \boldsymbol{\rho})$ and the degree of correlation $\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)$ both Gaussian functions; i.e.,

$$S_i(\boldsymbol{\rho}) = A_i^2 \exp(-\boldsymbol{\rho}^2 / 2\sigma_i^2), \tag{7}$$

$$\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1) = B_{ij} \exp[-(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2 / 2\delta_{ij}^2]. \tag{8}$$

The parameters A_i, B_{ij}, σ_i, and δ_{ij} are independent of position, but may depend on the frequency ω. In addition, they have to satisfy certain constraints to ensure that the field is beamlike [9]. As the beam propagates to a plane $z > 0$, and if we take $\sigma_x = \sigma_y = \sigma$, the matrix elements become ([9], where the one but last minus sign of Eq. (10) on p. 184 should be a plus sign)

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \frac{A_i A_j B_{ij}}{\Delta_{ij}^2(z)} \exp\left[-\frac{(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)^2}{8\sigma^2 \Delta_{ij}^2(z)}\right] \times \exp\left[-\frac{(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2}{2\Omega_{ij}^2 \Delta_{ij}^2(z)}\right] \exp\left[\frac{ik(\boldsymbol{\rho}_2^2 - \boldsymbol{\rho}_1^2)}{2R_{ij}(z)}\right], \tag{9}$$

where

$$\Delta_{ij}^2(z) = 1 + (z / k\sigma\Omega_{ij})^2, \tag{10}$$

$$\frac{1}{\Omega_{ij}^2} = \frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}, \tag{11}$$

$$R_{ij}(z) = [1 + (k\sigma\Omega_{ij} / z)^2] z. \tag{12}$$

We note that the matrix elements of Eq. (6) are real-valued and positive. Therefore, according to Eq. (5), there are no correlation singularities in the source plane. However, as we will now show, such singularities are created on propagation. In a cross section of the beam, we choose the point $\boldsymbol{\rho}_1$, and calculate for which points $\boldsymbol{\rho}_2$ both Eqs. (4) and (5) are satisfied. An example is shown in Fig. 2, in which the intersections of the curves, labeled ρ_A and ρ_B, indicate two simultaneous solutions. That these points are indeed coherence vortices is also evidenced by Fig. 3. At the two singular points, all phase

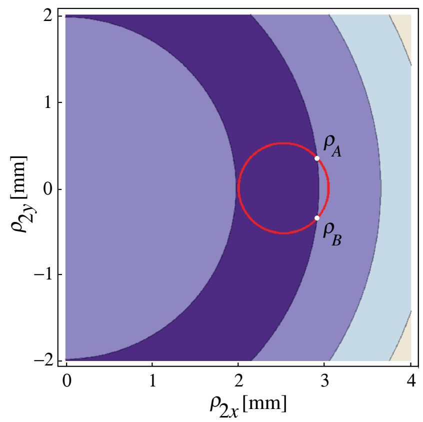

Fig. 2. (Color online) Locus of equal modulus of W_{xx} and W_{yy} (red curve), and the contours of $\mathrm{Arg}[W_{xx}] - \mathrm{Arg}[W_{yy}] = \pi$ (mod 2π). Their intersections, $\boldsymbol{\rho}_A$ and $\boldsymbol{\rho}_B$, are correlation singularities. In this example $A_x = 1$, $A_y = 3$, $\lambda = 632.8$ nm, $\sigma = 1$ mm, $\delta_{xx} = 0.2$ mm, $\delta_{yy} = 0.09$ mm, $z = 1.4$ m, and $\boldsymbol{\rho}_1 = (2.5, 0)$ mm.

contours coincide. It is seen that $\eta(\boldsymbol{\rho}_1, \boldsymbol{\rho}_A, z)$ and $\eta(\boldsymbol{\rho}_1, \boldsymbol{\rho}_B, z)$ have opposite *topological charge*, namely +1 and −1, respectively [1]. That the singularities formed by the pairs $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_A, z)$ and $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_B, z)$ lie well within the region of appreciable intensity is shown in Fig. 4, in which the normalized spectral density of the beam is plotted, together with the three points $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_A$, and $\boldsymbol{\rho}_B$. It is to be noted that for scalar Gaussian Schell-model beams [3, Eqs. (5.6)–(91)], such singularities do not exist.

When the cross-sectional plane z is taken close to the source plane and is then gradually moved away, there first are no coherence singularities, until the pair $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_A, z)$ and $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_B, z)$ is created. This observation explains the opposite topological charge of the two coherence singularities, because, just as for "ordinary" phase singularities, topological charge is conserved in the creation process [13]. When the plane z is taken further away from the source, the opposite takes place: the points $\boldsymbol{\rho}_A$ and $\boldsymbol{\rho}_B$ move closer together until they eventually annihilate. This is connected to the fact that as $z \to \infty$ condition (5) can no longer be satisfied.

The evolution of the pair of singularities $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_A, z)$ and $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_B, z)$ along the direction of propagation is shown in

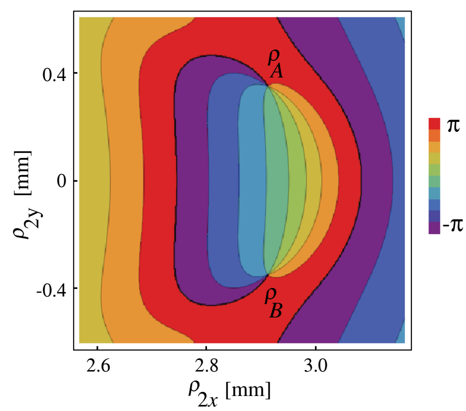

Fig. 3. (Color online) Color-coded phase plot of the degree of coherence $\eta(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$ in the plane $z = 1.4$ m. The singularities at $\boldsymbol{\rho}_A$ and $\boldsymbol{\rho}_B$ have opposite topological charge.

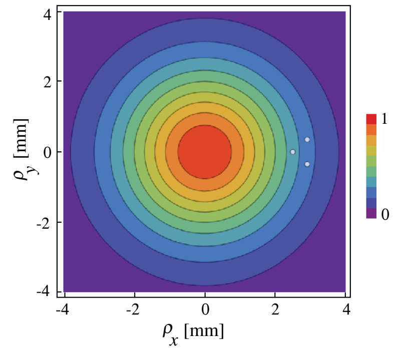

Fig. 4. (Color online) Normalized spectral density of the beam in the cross section $z = 1.4$ m. The points $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_A$, and $\boldsymbol{\rho}_B$ are indicated by the three white dots.

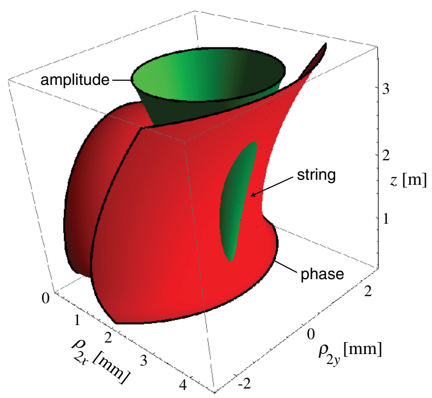

Fig. 5. (Color online) Intersection of a surface of equal amplitude (green) and a surface of opposite phase (red) constitutes a string of correlation singularities.

Fig. 5. The surface corresponding to Eq. (4) is depicted in green ("equal amplitude"), whereas the surfaces corresponding to Eq. (5) are depicted in red ("opposite phase"). It is seen that the singularities, i.e., the intersection of these surfaces, form a closed "string" or loop in the direction of the beam, with one half of the string formed by $\boldsymbol{\rho}_A$ and the other by $\boldsymbol{\rho}_B$, having opposite topological charge.

It follows from Eqs. (5) and (12) that the location of correlation singularities depends crucially on the parameters δ_{xx} and δ_{yy}, the transverse coherence length of the electric field components E_x and E_y, respectively. Indeed, if we increase δ_{yy} from 0.09 (as in all previous examples) to 0.12 mm, the string of singularities becomes markedly shorter, as is shown in Fig. 6. For a value near $\delta_{yy} = 0.13$ mm, the string disappears.

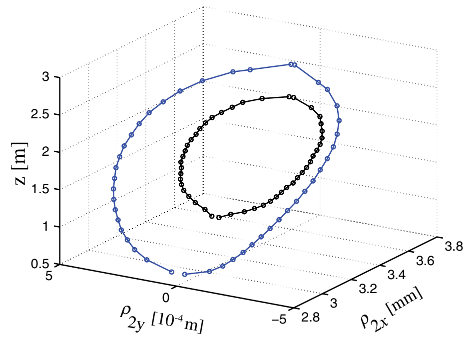

Fig. 6. (Color online) Two strings of correlation singularities in a partially coherent beam. The larger string (blue) is for $\delta_{yy} = 0.09$ mm, and the shorter string (black) is for the case $\delta_{yy} = 0.12$ mm.

In conclusion, we have demonstrated that a new type of correlation singularities, namely an electromagnetic coherence vortex, generically occurs in partially coherent beams of the Gaussian Schell-model type. In consecutive cross sections, the singularities form a closed loop. At the end points of the loop the singularities are created or annihilated pairwise. The presence of these singularities has profound consequences for interference experiments performed with partially coherent beams.

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