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On lane assignment of connected automated vehicles: strategies to improve traffic flow at diverge and weave bottlenecks

Hari Hara Sharan Nagalur Subraveti [a,*], Anupam Srivastava [b], Soyoung Ahn [b], Victor L. Knoop [a], Bart van Arem [a]

[a] *Department of Transport and Planning, Delft University of Technology, Stevinweg 1, 2628CN Delft, the Netherlands*
[b] *Department of Civil and Environmental Engineering, University of Wisconsin – Madison, United States*

A R T I C L E I N F O

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A B S T R A C T

This paper presents a novel approach to improve traffic throughput near diverge and weave bottlenecks in mixed traffic with human-driven vehicles (HDVs) and connected automated vehicles (CAVs). This is done by the strategic assignment of CAVs across lanes. The main principle is to induce strategic and necessary lane changes (LCs) (by CAVs and HDVs) well upstream of the potential bottleneck, so that the traffic flow approaching the bottleneck is organized and exhibits fewer throughput-reducing LCs at the bottleneck. A hybrid approach is used to investigate the problem: macroscopic analytical approach to formulate lane assignment strategies, and numerical simulations to quantify the improvements in throughput for various scenarios. Several strategies are formulated considering various operational conditions for each bottleneck type. Furthermore, compensatory behaviour of HDVs in response to the flow/density imbalance created by the CAV lane assignment is explicitly accounted for in our framework. Evaluation by numerical simulations demonstrates significant benefits of the proposed method, even at low to moderate CAV penetration rates: they can lead to an increase of throughput by several percent, thereby decreasing delays significantly.

1. Introduction

Congestion at recurrent motorway bottlenecks such as diverges and weaves can lead to lower throughput and increased travel times and is a source of major discomfort for road users. Car-following (CF) (Tampere et al., 2005; Knoop et al., 2008; Chen et al., 2014) and lane-changing (LC) behaviour (Cassidy and Rudjanakanoknad, 2005; Laval and Daganzo, 2006; Lee and Cassidy, 2008) have been attributed as the two major reasons which affect the throughput of motorways. Given the vast literature on this topic, we will focus our attention on the LC related studies, the phenomenon addressed in this paper.

Various empirical studies suggest that disruptive LCs are a major source of traffic breakdown. The impact of LCs on the bottleneck throughput at diverges has been observed in Bertini and Malik (2004), Martínez et al. (2011) and Rudjanakanoknad (2012). High exit flow at the off-ramp can induce congestion across all the mainline lanes and significantly reduce the throughput, especially if the diverging traffic travels at a lower speed than the mainline traffic (Martínez et al., 2011; Rudjanakanoknad, 2012). The impacts of LCs at weaving sections were similarly observed in Lee and Cassidy (2008) and Marczak et al. (2014) where the high LC concentration lead

* Corresponding author.
E-mail address: h.h.s.nagalursubraveti@tudelft.nl (H.H.S. Nagalur Subraveti).

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to the onset of congestion and consequent reduction in bottleneck throughput. These studies indicate that it is desirable to control the LC manoeuvres upstream of recurrent bottlenecks to avoid congestion.

Laval and Daganzo (2006) conjectured that when a vehicle changes lane at a speed lower than that of the prevailing traffic on the target lane, it often creates a void due to the bounded acceleration of the vehicle. These voids, when created near the bottleneck, are not utilized by other vehicles and contribute to reduced bottleneck throughput. Various studies based on this insight tried relating LCs to reduced throughput at various motorway bottlenecks such as merges (Leclercq et al., 2011, 2016a,b), diverges (Marczak and Buisson, 2014) and weaves (Marczak et al., 2015). The majority of the previous studies assume that insertions or desertions in a lane occur at a fixed location and the effect of the spatial distribution of LCs on traffic flow are not taken into account. As notable exceptions, Leclercq et al. (2016a,b) and Chen and Ahn (2018) consider the impact of spatially distributed LCs. Particularly, the former formulated capacity drop in a multi-lane context, albeit only for merging sections. The latter analysed three different types of extended bottlenecks (merge, diverge, and weave) in a more comprehensive manner but was limited to single lane motorways.

Traditionally, strategies such as ramp metering (Papageorgiou and Kotsialos, 2002; Papageorgiou et al., 2003), variable speed limits (Lu and Shladover, 2014) and advisory systems (Park et al., 2011; Schakel and Van Arem, 2014) have been designed and implemented to improve the bottleneck throughput and alleviate congestion. However, the effectiveness of these systems relies on the drivers to comply with the information presented. Both Park et al. (2011) and Schakel and Van Arem (2014) pointed to the need of very high compliance of drivers to the advices to realize the full benefits of control. At lower compliance and penetration rates, even negative improvements were reported (Schakel and Van Arem, 2014). The emergence of connected and automated vehicle (CAV) technologies present opportunities to develop active traffic management strategies to improve traffic flow efficiency at motorway bottlenecks in a more systematic manner. CAV technologies, when sufficiently mature, reduce the need for driver intervention and provide with the possibility for more intricate and precise control enabled by high compliance. Thus, CAV technologies can be utilized to strategically influence LCs around bottlenecks and increase the throughput in mixed traffic with human-driven vehicles (HDVs) and CAVs.

Several notable studies exist in the literature that utilize CAV technologies to improve traffic operations. For instance, Sulejic et al. (2017) optimized the distribution of LC positions for weaving segments in order to increase the bottleneck throughput using a Connected-Intelligent Transport Systems (C-ITS) application. Tanaka et al. (2017) developed vehicle control algorithms to avoid conflicts in weaving sections in order to achieve throughput improvement. Tilg et al. (2018) studied how automated vehicle (AV) technology can be used to improve traffic operations at weaving sections by optimizing the LC positions of automated vehicles as well as considering reduced reaction time for AVs. This study was however restricted to single lane mainline with an additional auxiliary lane connecting the two ramps. These studies also do not take into consideration the effect of secondary merges at weaving sections which contribute to turbulence downstream of the bottleneck (Van Beinum et al., 2018a,b). Roncoli et al. (2015, 2017) developed optimization based traffic control strategies for efficient lane assignment of CAVs at motorway bottlenecks. Letter and Elefteriadou (2017) presented a merging algorithm aimed at maximizing the average travel speed by optimizing the trajectories of the merging vehicles. Khattak et al. (2020) developed a prototype CAV-enabled lane control signal (LCS) and provided a preliminary assessment of the potential improvement it offers over traditional gantry operated LCS. Amini et al. (2021) proposed a model to optimize the trajectories of CAVs at motorway weaving segments assuming a 100% market penetration rate. However, most of these studies assume 100% market penetration of CAVs. More importantly, none of these studies directly address the LC mechanism causing the throughput loss and propagation of disturbances. Chen et al. (2017), Ghiasi et al. (2017) and Amirgholy et al. (2020) examined the effects of reserved lanes for AVs in a multi-lane motorway setup and found that segregation of AVs and HDVs can lead to lower capacities especially at lower penetration rates. In a mixed traffic framework, HDVs are expected to show compensatory behaviour in response to traffic control of CAVs. This compensation may have an impact on the effectiveness of the system but has rarely been considered in the current literature.

The state-of-the-art in traffic flow theory present a major gap and an opportunity to develop concepts and matching traffic control strategies that can improve mixed traffic flow by directly addressing the LC mechanism causing the throughput loss at bottlenecks. To this end, this paper aims to formulate a control concept, utilizing CAV technologies, to mitigate throughput-reducing LCs (by HDVs and CAVs) at motorway bottlenecks. Specifically, at near-critical conditions (traffic is not yet congested, but approaching breakdown density), LCs near the bottleneck create irreversible traffic voids which remain unutilized thereby reducing the throughput of the section. Chen and Ahn (2018) suggested that LCs close to the downstream end of the bottleneck area are more likely to create persisting voids. These voids contribute most to throughput loss. LCs spatially spread out over a large distance and occurring further upstream of a bottleneck are less likely to create persisting voids which propagate downstream due to the re-utilization of voids: the probability that a LC contributes to throughput loss decreases and approaches zero as it occurs farther upstream of the bottleneck. The main principle of the proposed control strategy is to therefore move necessary LCs (by CAVs and HDVs) well upstream of the bottleneck, so that the traffic flow approaching the bottleneck is organized and exhibits fewer LCs at the bottleneck. This minimizes the creation of persistent voids and increases the probability of re-utilization of such voids when created, thus increasing throughput. Strategic lane assignment entails distributing CAVs by destination (at or downstream of the bottleneck), while also considering induced lane re-organization by HDVs to compensate for the CAV assignment. We develop macroscopic analytical formulation for various lane assignment strategies, considering different traffic operational conditions. The analytical formulation is based on sound physical principles and provides insights into the various parameters involved and their impact on throughput via the reduction of the number of LCs at the bottleneck. The throughput improvements due to the implementation of these strategies are then quantified through microscopic, vehicle-based numerical simulations. The results indicate that significant improvements in throughput can be realized (up to 7%), even at low to moderate CAV penetration. More specifically, the results show that (1) at diverge bottlenecks, maximum throughput can be achieved at low CAV penetration rates even for high exit flows; (2) lane assignment at weaves based on the predominant weaving flow leads to the best results; and (3) at high merging and diverging flow, lane assignment of exit flows is preferred over creating space for the merging flow.

The main contribution of this paper is three-fold: (1) it proposes a control concept that directly addresses the void creation mechanism by LCs, based on decades of scientific research in this realm, with the aim to increase bottleneck throughput; (2) it presents a rigorous analytical formulation of the method providing important insights and hence is theoretically grounded; and (3) it deals with multi-lane mixed traffic under arbitrary penetration rate of CAVs, incorporating HDV behaviour, which is rarely seen in the literature.

This paper is organized as follows: Section 2 presents the research problem and setup. Section 3 introduces the lane assignment strategies for various bottleneck types (diverges and weaves). Section 4 presents the framework and results of the numerical simulations used to quantify the impacts of lane assignment on the throughput. Conclusions are provided in Section 5 along with discussions on the limitations of this research and desirable future research.

2. Problem setup

In this paper, CAV lane assignment strategies are designed by leveraging knowledge of their approximate destination information (i.e. at or downstream of the bottleneck), thereby reducing LCs and their impact on throughput in the neighbourhood of the bottleneck. It is assumed that all CAVs are controllable and have full compliance with respect to the assigned target lanes. The control strategies discussed are limited strictly to lane assignment for CAVs, where destination information is exploited to assign CAVs approaching an exit closer to the exit lanes and those continuing through on the mainline further from the exit lane. Further, it is assumed that there are no communication latencies. In any case, short duration of communication latencies should not affect the effectiveness of the system much if the control time step is large enough. We consider three types of bottlenecks: diverge, merge, and weave bottlenecks, focusing on the diverge and weave bottlenecks where destination-based lane assignment is most useful.

The general principle that forms the baseline of the lane assignment strategies is to minimize LCs in the vicinity of the bottleneck as stated earlier. This is achieved through a combination of two objectives: pre-allocating exiting CAVs to the right lanes, and pre-allocating through vehicles to the left lanes in order to create space on right lane for merging and/or exiting vehicles. The number of CAVs that can be effectively assigned to each lane is determined by numerous factors including the destination demand for CAVs, expected merge volumes and exit proportions for all vehicles and lane capacities. Lane capacity forms the driving constraint in the strategies as shown in Section 3. While traffic dynamics nuances are not considered for the analytical design of control strategies (to keep the analysis tractable), they are incorporated into the simulation-based evaluations presented in Section 4.

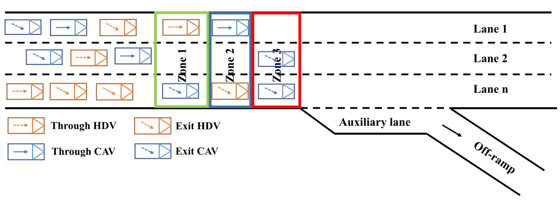
2.1. Diverges

Diverge bottlenecks present the first scenario of interest where destination-based lane assignment can be utilized to mitigate LCs close to the bottleneck, and thus improve throughput. A general setup of a diverge bottleneck is considered with n mainline lanes indexed 1 to n in ascending order from left to right such that left-most lane is lane 1. For brevity, we assume that the section has a single auxiliary (deceleration) lane, representing the majority of observed geometries at exit ramps, while noting that the framework can be easily extended to include other geometries. A schematic representation of the diverge section is shown in Fig. 1a.

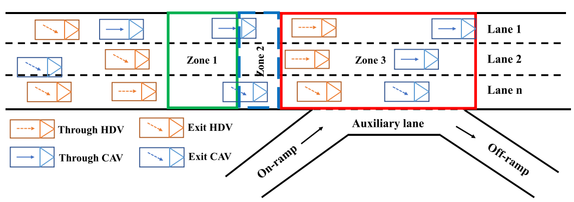
For a diverge section as presented above, high exit proportions across lanes can lead to increased LC activity immediately upstream of the bottleneck as vehicles perform (multiple) right-LCs to reach the exit lane. LC vehicles have to find gaps in the through traffic, while also gradually reducing speed to match the exit lane speed. The conflicting flows (through and exit) as well as the speed reductions involved may lead to onset of congestion and can adversely affect the throughput.

The LC conflicts at such a diverge bottleneck may be minimized through strategic destination-based target lane assignment for CAVs upstream of the section, such that vehicles with furthest destinations (through vehicles) are assigned to left lanes, and vehicles with nearer destinations (diverge exit) are assigned to right lanes. This forms the basis of our investigations later in the paper.

We start with dividing the roadway leading up to the exit location into three zones. Zone 3, closest to the exit is termed the ‘Capacity Impact’ (CI) zone with any LCs occurring in this zone creating irreversible traffic voids and reducing the throughput of the section. Without any lane assignment, it is assumed that vehicles change lanes in this zone to exit at the off-ramp. The primary control objective for the lane assignment strategy therefore involves reducing the number of LCs in the CI zone. Zones 2 and 1 are both upstream of the CI zone and are far enough upstream that LCs within these zones do not have a direct impact on the bottleneck throughput making them non-impact zones. Zone 1 (further upstream than Zone 2) is termed the ‘Control’ zone. All CAV lane assignments and corresponding



(a) Diverge section schematic



(b) Weave section schematic

Fig 1. Schematic representation of the bottlenecks.

CAV LCs occur within Zone 1. Since any LCs initiated due to the control strategy may cause an imbalance in the lane flow distribution, HDVs may re-adjust their lanes (discretionary LCs) to (partially) balance lane flows. These discretionary LCs occur in Zone 2, the 'Compensatory' zone immediately downstream of the control zone (but still upstream of CI). While Zone 2 would always extend somewhat downstream of Zone 1, it is possible for these zones to overlap. The lengths of each zone depend on various factors such as the number of LCs within the zone, LC intensity (number of LCs per distance), prevailing speeds etc.

2.2. Merges

The focus of this paper is to develop CAV lane assignment strategies to improve throughput (minimize throughput reduction) at a bottleneck. Merge bottlenecks are believed to be formed typically due to a queue build up on the rightmost mainline lane upstream of the merge, caused by the merging of oncoming vehicles from the ramp (Cassidy and Rudjanakanoknad, 2005). This queue can often spill over laterally to the left lanes as well, further disrupting the throughput. The root cause of the disruption in such scenarios is the merging LCs performed by the merging vehicles into the rightmost mainline lane as well as inward lanes (through secondary merging). A lane assignment strategy that would mitigate the disruptions caused by merging vehicles would involve creating voids on the rightmost lane upstream of the merge. While carefully designed CAV control can efficiently improve throughput at merge bottlenecks, lane assignment based strategy like the one of interest to this work is trivial for merge bottlenecks and is not studied separately. Instead, the principal of creating space for merging vehicles through pre-emptive lane assignment of CAVs is explored as a part of the more complex weave bottleneck scenario. If desired, the reader may consider a specific application of the weave strategies presented, with exit volume set to 0, in order to simulate a merge scenario.

2.3. Weaves

Weaving sections are the third bottleneck type that are of interest to us. A weave section differs from a diverge section due of the proximity of the merge and diverge discontinuities to each other, resulting in a combination of left LCs (due merging vehicles) and right LCs (due to exiting vehicles) woven within the same spatial region. It is important to note that a weave bottleneck is caused due to the combined impact of the merging and diverging (weaving) LCs, and a weave section where the merge behaviour alone leads to the bottleneck formation, a merge bottleneck, is different from a weave bottleneck. The weaving movement of vehicles leads to unique impacts on throughput and inspires control strategies unique from the isolated diverge/merge scenarios.

For brevity, we consider weaves sections with a merge and a diverge connected through a single auxiliary lane, and with an equal number of mainline through lanes upstream and downstream of the weave section, once again noting that the framework and insights presented can be easily expanded to more generic schematics. The mainline lanes are labelled similarly to diverge sections with lane 1 being the lane furthest from the on/off ramps and lane n being the mainline lane immediately adjacent to the auxiliary lane (Fig. 1b).

Since the weave section is a bottleneck created due to the combined effect of merging and diverging vehicles and their LCs, the area starting from upstream of the on-ramp gore to downstream of the off-ramp gore is considered as the 'Capacity Impact' zone or Zone 3. Like for the diverge scenario, any LC within the CI zone is considered to have a direct detrimental impact on the throughput. In addition to the region between the ramps, the CI zone extends upstream of the weave section. The 'Control' zone, Zone 1 is once again further upstream of the CI zone and is where any CAV target lane assignments are executed.

3. Lane assignment strategies

Formalizing the control concept in Section 2, this section presents the analytical formulation of lane assignment strategies for diverge and weave bottlenecks and the flows and number of LCs resulting from these strategies. The analytical formulations described in this section provide an important foundation to form any lane assignment strategy. Depending upon the CAV penetration rate, exit rate, lane-wise flows etc., feasible CAV lane assignment strategies can be identified by using the analytical formulations given in this section. The strategies for diverge bottlenecks are introduced first and based on the insights obtained, strategies for weaving sections are introduced. The strategies aim to better organize the traffic flow approaching the bottleneck such that the number of disruptive LCs near the bottleneck are minimized. The strategies are prefixed with D for diverge sections and W for weaving sections.

3.1. Diverges

Let n be the number of lanes on the mainline. The mainline traffic on each lane is described by the kinematic wave theory (Lighthill and Whitham, 1955; Richards 1956) and a triangular fundamental diagram with free-flow speed u, wave speed w and capacity C/n. The capacity of the mainline is then given by C. This capacity refers to the maximum possible outflow under ordinary conditions (without any LCs). The mainline flow is equal to λC where $\lambda(\leq 1)$ is the ratio of flow to capacity. $\lambda = 1$ implies that the mainline is flowing at capacity. We consider conditions where the mainline flow is less than the capacity ($\lambda < 1$) but high enough for LCs to instigate congestion and reduction in throughput. In order to maintain tractability, certain assumptions are made to reduce the number of parameters. Instead of considering lane-specific penetration rates for CAVs (p_i) and flow to capacity ratios (λ_i), we assume that the CAVs are evenly distributed across lanes with a mean penetration rate of p and similar flows on all the lanes ($\lambda C/n$). Nevertheless, the principle of CAV lane assignment remains the same. From here on, the strategies are formulated based on the reduced parameter set. If p is the penetration rate of the CAVs in lane i and λ is the ratio of flow to capacity in lane i (where i is the lane index), then the CAV flow in each lane is $p\lambda C/n$ and the HDV flow will be $(1-p)\lambda C/n$. The proportion of vehicles exiting through the off-ramp and the lane-wise proportions (with respect to lane flow) are denoted by α and α_i respectively. It should be noted that the movement from lane n (the farthest lane from the median) to the off-ramp is not considered as a LC since it does not result in an insertion to a new stream.

Ideally, it is preferred that the flow approaching the bottleneck is organized in such a way that the vehicles taking the exit are already in the right-most lane (lane n) and the through flow is in the inner lanes so that the LC conflicts between these two traffic streams near the bottleneck are minimized. CAVs can be used to organize the flow by assigning them to lanes according to their destinations with respect to the off-ramp much upstream of the bottleneck. This minimizes the number of LCs in the capacity impact zone (Zone 3). Lane assignment of CAVs depends upon various factors such as the penetration rate of CAVs, the exit flow and the flow in mainline.

Based on the lane flow distributions and capacity constraints of each lane, possible control strategies for CAV lane assignment in the control zone (Zone 1) are identified and the traffic flow and LCs resulting from the implementation of these strategies are formulated analytically. There are three possible scenarios for CAV lane assignment.

All exit CAVs (intending to exit at the off-ramp) can be assigned to lane n and all through CAVs initially in lane n (intending to exit further downstream of the bottleneck) can be assigned to the left lanes.

This requires that there is enough space (by space, we refer to spare capacity) in lane n to accommodate all exit CAVs in the left lanes (1 to $n-1$) per time interval.

$$\frac{p\lambda C}{n}(\alpha_1+\alpha_2+\cdots+\alpha_{n-1}) \le \frac{C}{n}(1-\lambda)+(1-\alpha_n)\frac{p\lambda C}{n} \tag{1}$$

The term on the left represents the number of exit CAVs in lanes 1 to $n-1$. The term on the right indicates the space available in lane n to accommodate the exit CAVs from the left after the through CAVs from this lane are assigned to the left lanes. Likewise, this also requires that there is enough space in the left lanes to accommodate all through CAVs from lane n.

$$(1-\alpha_n)\frac{p\lambda C}{n} \le \frac{C}{n}(1-\lambda)(n-1)+\frac{p\lambda C}{n}(\alpha_1+\alpha_2+\cdots+\alpha_{n-1}) \tag{2}$$

Combining Eqs. (1) and (2) results in the following condition:

$$\max\left[0,\frac{p\lambda-(1-\lambda)(n-1)}{np\lambda}\right] \le \alpha \le \min\left[1,\frac{1-\lambda+p\lambda}{np\lambda}\right] \tag{3}$$

which indicates that all the CAVs can be assigned based on their destinations respective to the bottleneck location. The min and max operators are applied as the exit proportions can only vary between 0 and 1.

There is also the possibility that all exit CAVs are assigned to lane n but only some through CAVs in lane n are assigned to the left lanes due to capacity constraints in the left lanes. This scenario usually occurs when λ is high (near capacity). This implies that the inequality in Eq. (2) is reversed leading to α less than the lower bound in Eq. (3).

$$\alpha < \max\left[0,\frac{p\lambda-(1-\lambda)(n-1)}{np\lambda}\right] \tag{4}$$

It is also possible that only some exit CAVs can be assigned to lane n due to capacity constraint on this lane and corresponding through CAVs from lane n are assigned to the left lanes. This scenario can usually occur when α is very high and greater than the upper bound in Eq. (3).

$$\alpha > \min\left[1,\frac{1-\lambda+p\lambda}{np\lambda}\right] \tag{5}$$

Below, we formulate three CAV lane assignment strategies for diverge sections based on the conditions relating exit proportions to the capacity constraints of lanes given by Eq. (3).

3.1.1. D-Strategy 1 (DS1):

In this strategy, all CAVs can be successfully assigned to lanes based on their destination with respect to the exit. This strategy is implemented when Eq. (3) is satisfied. The allocation of through CAVs in lane n to lanes 1 to$n-1$ is determined proportional to the lane-specific exit proportions (α_i) such that a higher CAV volume is assigned to a lane with a higher exit proportion. This ratio is given by

$$\alpha_i/(n\alpha-\alpha_n) \tag{6}$$

However, it is possible that the flow allocation according to this ratio cannot be accommodated in certain lanes due to capacity constraints, even though there is enough combined space across lanes 1 to $n-1$. This is represented by the equation:

$$(1-\alpha_n)\frac{p\lambda C}{n}*\frac{\alpha_i}{n\alpha-\alpha_n} \le \frac{C}{n}-\left[(1-p)\frac{\lambda C}{n}+(1-\alpha_i)\frac{p\lambda C}{n}\right] \tag{7.1}$$

Simplifying Eq. (7.1) results in the condition:

$$\alpha_i \geq \left|\frac{(n\alpha - \alpha_n)(1-\lambda)}{p\lambda(1-n\alpha)}\right| \tag{7.2}$$

In such a case, through CAVs in lane n are allocated in the left lanes according to the space available in each lane. This would result in a higher volume of through CAVs from lane n being assigned to a left lane with more space available. The ratio of space available in a particular left lane to the overall space available in all the left lanes is given by:

$$S_i = \frac{1-\lambda+\alpha_i p\lambda}{(1-\lambda)(n-1)+p\lambda(n\alpha-\alpha_n)} \tag{8}$$

Thus, two possible cases of this strategy are possible, denoted by: (1) DS1-C1 - when the through CAVs from lane n are assigned according to the ratio given by Eqs. (6) and (2) DS2-C2 – when assigned by Eq. (8).

We now derive the changes in flow on each lane due to the CAV lane assignment and the number of LCs for each zone. The detailed formulation is shown for DS1-C1. The important metrics for DS1-C2 as well as other strategies can be formulated similarly based on the flow conservation principle.

Control zone (Zone 1):

Following the CAV lane assignment, the flow leaving the control zone and entering Zone 2 in lane n is given by:

$$\frac{\lambda C}{n} - (1-\alpha_n)\frac{p\lambda C}{n} + \frac{p\lambda C}{n}(\alpha_1+\alpha_2+\cdots+\alpha_{n-1}) \tag{9.1}$$

In Eq. (9.1), the first term is the flow entering the control zone, the second term is the through CAV flow assigned to the left lanes and the third term is the exit CAV flow in the left lanes assigned to lane n. Simplifying Eq. (9.1) results in:

$$\frac{\lambda C}{n}(1+n\alpha p-p) \tag{9.2}$$

The flow leaving control zone in the left lanes (1 to n −1) is:

$$\frac{\lambda C}{n} - \alpha_i\frac{p\lambda C}{n} + (1-\alpha_n)\frac{p\lambda C}{n} * \frac{\alpha_i}{n\alpha-\alpha_n} = \frac{\lambda C}{n}\left[1+\alpha_i p\left(\frac{1-n\alpha}{n\alpha-\alpha_n}\right)\right] \tag{10}$$

Then, the total number of LCs in the control zone due to the CAV assignment is the sum of the number of LCs from left to right and vice versa given by:

Number of LCs from left to right: $\frac{\alpha_1 p\lambda C}{n}(n-1) + \frac{\alpha_2 p\lambda C}{n}(n-2) + \cdots + \frac{\alpha_{n-1} p\lambda C}{n}(1)$

Number of LCs from right to left: $\frac{(1-\alpha_n)p\lambda C}{n} * \frac{\alpha_1}{n\alpha-\alpha_n}(n-1) + \cdots + \frac{(1-\alpha_n)p\lambda C}{n} * \frac{\alpha_{n-1}}{n\alpha-\alpha_n}(n-1)$

Combining these, the total number of LCs is given by:

$$\underbrace{\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i\right]}_{LCs\ from\ left\ to\ right} + \underbrace{\left(\frac{1-\alpha_n}{n\alpha-\alpha_n}\right)\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i\right]}_{LCs\ from\ right\ to\ left} \tag{11}$$

Table 1 gives the equations for the lane-specific flows leaving Zone 1 and the number of LCs in the control zone as a result of this strategy.

Compensatory zone (Zone 2):

Downstream of the control zone lies the compensatory zone (Zone 2). In this zone, HDVs may react to the imbalance in lane flows caused by the lane assignment in the control zone and perform LCs. We assume that the compensatory LCs are associated exclusively with HDVs. Driving rules, which can vary highly across regions, affect the way in which HDVs may compensate and rebalance in response to the CAV lane assignment. In this study, it is assumed that the HDVs compensate to return to the original lane flow distribution similar to the LC motivation considered in Shiomi et al. (2015). A new parameter β is introduced here which denotes the rate of compensation by the HDVs. In DS1, the flow leaving the control zone and entering the compensatory zone in lane n can either increase or decrease after the lane assignment depending on the number of exit CAVs and through CAVs. The flow will decrease if the number of exit CAVs is less than the number of through CAVs in lane n. Thus, it is assumed that to restore balanced lane flows, a fraction of exit HDVs from the left lanes will move to lane n. If the number of through CAVs in lane n is more than the total number of exit CAVs in the left lanes, the flow in lane $n$will decrease after the CAV assignment. This condition is given by Eq. (12.2):

Table 1
Metrics of Zone 1 for D-Strategy 1.

Condition	Flow in lane$n$	Flows on lanes 1:n −1	# of LCs to the right	# of LCs to the left
DS1 - C1	$\frac{\lambda C}{n}(1+n\alpha p-p)$	$\frac{\lambda C}{n}[1+\alpha_i p\left(\frac{1-n\alpha}{n\alpha-\alpha_n}\right)]$	$\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i\right]$	$\left(\frac{1-\alpha_n}{n\alpha-\alpha_n}\right)\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i\right]$
DS1 - C2	$\frac{\lambda C}{n}(1+n\alpha p-p)$	$\frac{\lambda C}{n}[1+p(S_i - S_i\alpha_n - \alpha_i)]$	$\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i\right]$	$(1-\alpha_n)\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}S_i - \sum_{i=1}^{n-1} iS_i\right]$

$$(1-\alpha_n)\frac{p\lambda C}{n} > \frac{p\lambda C}{n}(\alpha_1+\alpha_2+\cdots+\alpha_{n-1}) \tag{12.1}$$

$$\alpha \le (1/n) \tag{12.2}$$

If Eq. (12.2) is satisfied, flow in lane n decreases and the flow imbalance in lane n due to CAV lane assignment is given by:

$$\frac{\lambda C}{n} - \frac{\lambda C}{n}(1+n\alpha p-p) = \frac{p\lambda C}{n}(1-n\alpha) \tag{13}$$

The flow leaving the compensatory zone in lane n is hence given as:

$$\underbrace{\frac{\lambda C}{n}(1+n\alpha p-p)}_{Flow\ entering\ Zone\ 2} + \underbrace{\beta^{*}\frac{p\lambda C}{n}(1-n\alpha)}_{Unbalanced\ flow} \tag{14}$$

β varies between 0 and 1. $\beta = 0$ implies no compensation and $\beta = 1$ implies full compensation (i.e. lane-specific flows return to the initial state). However, if there are not enough exit HDVs in lanes 1 to $n-1$ to compensate for the flow imbalance in lane n, the assumed (required) level of compensation is not possible. This is given by the condition:

$$\underbrace{(1-p)\frac{\lambda C}{n}(n\alpha-\alpha_n)}_{Exit\ HDVs\ in\ left\ lanes} < \underbrace{\frac{p\lambda C}{n}(1-n\alpha)}_{Unbalanced\ flow} \tag{15}$$

In this case, the maximum possible rate of compensation is given by $\frac{(1-p)(n\alpha-\alpha_n)}{p(1-n\alpha)}$ and the range of β is then given by [0, $\frac{(1-p)(n\alpha-\alpha_n)}{p(1-n\alpha)}$]. In DS1-C2, the flow in lane n always decreases because if $\alpha > (1/n)$, Eq. (7.2) is never satisfied. The important metrics of the compensatory zone (Zone 2) for DS1 are then given in Table 2 and 3:

In the case where the flow in lane n decreases after CAV lane assignment (Eq. (12.2)), it is assumed that the compensating vehicles are only exit HDVs in the left lanes performing LCs to occupy the space created in lane n. However, this may not always be the case. There is a possibility that some through HDVs from the left lanes will also move to lane n. In order to account for this, two compensation rates β_1 and β_2 are introduced which denote the compensation rates of the exit HDVs and through HDVs respectively. $\beta_1(\beta_2)$ is the ratio of the compensating exit (through) HDV flow to the total compensating flow. They are related to the overall compensation rate β as follows:

$$\text{mean}(\beta_1,\beta_2) = \beta \tag{16}$$

After lane assignment of CAVs and compensation by HDVs, the number of LCs in the capacity impact zone (Zone 3) is computed based on the lane flows entering this region and their destinations. The number of LCs in the zone is determined by the number of exit HDVs remaining in lanes 1 to $n-1$ assuming no further compensation. The number of LCs in this region directly relates to the magnitude of throughput loss observed at the diverge bottleneck. The number of LCs in the capacity impact zone after CAV lane assignment in the control zone and compensation by HDVs in Zone 2 is the given in Table 4.

When β equals to its upper bound (the ideal case), the number of LCs in Zone 3 is equal to zero. This implies that all exit CAVs are assigned to lane n in Zone 1, and all exit HDVs move to lane n in Zone 2 through the compensatory behaviour and there are no exiting vehicles on other lanes except for lane n.

3.1.2. D-Strategy 2 (DS2)

In this strategy, all exit CAVs are assigned to lane n and only some through CAVs in lane n are assigned to the left lanes due to capacity constraints in the left lanes. This strategy is implemented when Eq. (4) is satisfied. This strategy is usually implemented when λ is high (near capacity).

In this scenario, the through CAVs on lane n are assigned to the left lanes based on the space available on the left lanes. Even after this assignment, there are still some through CAVs remaining in the right lane (lane n). The space left on the left lanes after the exit CAVs are removed is given by:

$$\frac{C}{n}(1-\lambda)(n-1) + \frac{p\lambda C}{n}(n\alpha-\alpha_n) \tag{17}$$

Table 2
Metrics for Zone 2 (DS1-C1).

Flow in lane$n$	Flow in lanes 1:$n-1$	# of LCs in this region
Flow in lane n increases (Eq. (12.2) not satisfied) and enough exit HDVs available to compensate (Eq. (15) satisfied)		
$\frac{\lambda C}{n}[1+(1-\beta)(n\alpha p-p)]$	$\frac{\lambda C}{n}\left[1+\frac{\alpha_i(p(1-n\alpha)+\beta(1-p)(1-\alpha_n))}{n\alpha-\alpha_n}\right]$	$\frac{\beta p\lambda C}{n^2\alpha-n\alpha_n}(n\alpha-1)(n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)$
Flow in lane n decreases (Eq. (12.2) satisfied) and enough exit HDVs available to compensate (Eq. (15) satisfied)		
$\frac{\lambda C}{n}[1+(1-\beta)(n\alpha p-p)]$	$\frac{\lambda C}{n}\left[1+\frac{\alpha_i p(1-n\alpha)(1-\beta)}{n\alpha-\alpha_n}\right]$	$\frac{\beta p\lambda C}{n^2\alpha-n\alpha_n}(1-n\alpha)(n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)$

Table 3
Metrics for Zone 2 (DS1-C2).

Flow in lane$n$	Flow in lanes 1:$n-1$	# of LCs in this region
Flow in lane n decreases (Eq. (12.2) satisfied) and enough exit HDVs available to compensate (Eq. (15) satisfied)		
$\frac{\lambda C}{n}[1+(1-\beta)(n\alpha p-p)]$	$\frac{\lambda C}{n}[1+p(S_i-S_i\alpha_n-\alpha_i)]-\frac{\alpha_i\beta p\lambda C}{n^2\alpha-n\alpha_n}(1-n\alpha)$	$\frac{\beta p\lambda C}{n^2\alpha-n\alpha_n}(1-n\alpha)(n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)$

Table 4
Number of LCs in Zone 3 (DS1).

	# of LCs in Zone 3
DS1-C1 (Eq. (12.2) not satisfied)	$(1-p)\frac{\lambda C}{n}\left[n\sum_{i=0}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)\right]$
DS1-C1 and DS1-C2 (Eq. (12.2) and (15) satisfied)	$\frac{\lambda C}{n}\left[1-p-\frac{\beta p(1-n\alpha)}{n\alpha-\alpha_n}\right]\left[n\sum_{i=0}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)\right]$

The flow leaving the control zone and the number of LCs resulting from the implementation of this strategy in the control zone are given in Table 5. These are derived in a similar way as Tables 1–4.

The flow imbalance due to the CAV lane assignment in lane n is given by $(n-1)(1-\lambda)\frac{C}{n}$. In this strategy, flow in lane n always decreases after CAV lane assignment. The number of exit HDVs in lanes 1 to $n-1$ required to compensate for the flow imbalance in lane n is given by the condition:

$$\underbrace{(1-p)\frac{\lambda C}{n}(n\alpha-\alpha_n)}_{\text{Exit HDVs in left lanes}}<\underbrace{(n-1)(1-\lambda)\frac{\beta C}{n}}_{\text{Unbalanced flow}} \tag{18}$$

The flows leaving the compensatory zone and LCs in this zone are shown in Table 6:

Finally, the number of LCs in the capacity impact zone is given by $\frac{C}{n}\left[\lambda-p\lambda-\frac{\beta(n-1)(1-\lambda)}{n\alpha-\alpha_n}\right]\left[n\sum_{i=0}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)\right]$. when $\beta\leq\frac{(\lambda-p\lambda)(n\alpha-\alpha_n)}{(n-1)(1-\lambda)}$. Otherwise, the number of LCs is equal to zero.

3.1.3. D-Strategy 3 (DS3)

In this strategy, only some exit CAVs can be assigned to lane n due to capacity constraint in this lane and the corresponding through CAVs from lane n are assigned to the left lanes. This strategy is implemented when Eq. (5) is satisfied. This scenario is common when α is very high.

Table 7 indicates the flow leaving the control zone and the number of LCs resulting from the CAV lane assignment.

In this strategy, the flow in lane n always increases after the CAV lane assignment due to the high exit flow entering lane n. Hence, it is assumed that certain through HDVs in lane n will move to the left lanes to restore the lane balance. The flow imbalance is caused by the lane assignment is equal to $\frac{C}{n}(1-\lambda)$. The important metrics for the compensatory zone is shown in Table 8.

Since no exit HDVs were involved in the compensation, the number of LCs occurring in the capacity impact zone would be equal to the number of LCs performed by the exit HDVs remaining in the left lanes which is given by

$$(1-p)\frac{\lambda C}{n}\left[n\sum_{i=0}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)\right] \tag{19}$$

3.2. Weaves

Based on the insights obtained from diverge sections, CAV lane assignment strategies for weaving sections are formulated. In weaving sections, both merging and diverging LCs can occur near the bottleneck. Thus, in addition to the LC conflicts between through and exit vehicles upstream of the on-ramp, there are LC conflicts between the weaving and non-weaving traffic which negatively affects the throughput of the bottleneck. At weaving sections, the flow approaching the weave segment can be better organized to minimize conflicts between through and exit traffic (similar to diverge sections). However, this may reduce the space available in the shoulder lane for the incoming ramp demand. Thus, another possibility would be to create space in the shoulder lane by moving

Table 5
Metrics for Zone 1 (DS2).

Flow in lane$n$	Flow in lanes 1:$n-1$	# of LCs to the right	# of LCs to the left
$\frac{C}{n}(1-n(1-\lambda))$	$\frac{C}{n}$	$\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i\right]$	$\frac{C}{2}(1-\lambda)(n-1)+\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i\right]$

Table 6
Metrics for Zone 2 (DS2).

	Flow in lane n	Flow in lanes 1:$n-1$	# of LCs in this region
Eq. (18) is satisfied	$\frac{C}{n}[1+n\lambda-n+\beta(n-1)(1-\lambda)]$	$\frac{C}{n}\left[1-\frac{\alpha_i\beta(n-1)(1-\lambda)}{n\alpha-\alpha_n}\right]$	$\frac{\beta C}{n^2\alpha-n\alpha_n}(n-1)(1-\lambda)(n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)$

Table 7
Metrics for Zone 1 (DS3).

Flow in lane n	Flows in lanes 1:$n-1$	# of LCs to the right	# of LCs to the left
$\frac{C}{n}$	$\frac{C}{n}\left[\lambda-\alpha_i\left(\frac{1-\lambda}{n\alpha-\alpha_n}\right)\right]$	$\frac{C}{n^2\alpha-n\alpha_n}(1-\lambda+p\lambda-\alpha_n p\lambda)\left[n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i\right]$	$\left(\frac{1-\alpha_n}{n\alpha-\alpha_n}\right)\frac{p\lambda C}{n}\left[n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i\right]$

Table 8
Metrics for Zone 2 (DS3).

	Flow in lane n	Flow in lanes 1:$n-1$	# of LCs in this region
$\beta\le\frac{(\lambda-p\lambda)(1-\alpha_n)}{(1-\lambda)}$ i.e. Enough HDVs available for compensation	$\frac{C}{n}[1-\beta(1-\lambda)]$	$\frac{C}{n}\left[\lambda-\frac{\alpha_i}{n\alpha-\alpha_n}(1+\beta)(1-\lambda)\right]$	$\frac{\beta C}{n^2\alpha-n\alpha_n}(1-\lambda)(n\sum_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i)$

vehicles to the inner lanes to accommodate the merge flow and shifting the LCs of exit CAVs downstream of the on-ramp in the weaving segment. These two possible CAV lane assignment strategies are analysed in this paper and the resulting LCs from these strategies are formulated as follows:

The capacity impact zone in weaving sections is divided into two parts – the first part is the area upstream of the on-ramp (denoted as CIZ-1) and the second part is the area between the two ramps plus the area extending downstream of the off-ramp (denoted as CIZ-2): see Fig. 2 for the sketch.

Let q_r be the demand entering from the on-ramp. The minimum number of LCs in CIZ-2 is equal to the total merging and diverging flow. There are four possible scenarios to evaluate at weaving sections with respect to the merge and diverge flow. They are: (1) low merge and low diverge flow (2) low merge and high diverge flow (3) high merge and low diverge flow and (4) high merge and high diverge flow. Depending upon the scenario, specific CAV lane assignment strategies described below can be implemented.

3.2.1. W-Strategy 1 (WS1)

This strategy is based on the principle of diverging sections and is preferred when the diverge flow at the weaving section is high. In this strategy, all exit CAVs are assigned to lane n and through CAVs in lane n are moved to the left lanes to create space for the exit CAVs. The primary objective is to organize the exit flow and minimizing LC conflicts in CIZ-1 without taking into consideration the incoming ramp demand to preserve simplicity in control.

In this strategy, the flow entering CIZ-1 will be similar to the diverging sections and the number of LCs in the first half of the capacity impact zone upstream of the on-ramp will also be similar. The formulation for flows and number of LCs in each zone (control zone, compensatory zone and first part of capacity impact zone - CIZ-1) will hence be similar to those for the diverging sections.

In CIZ-2, the weaving LCs occur with the exit vehicles moving from the mainline to the auxiliary to take the off-ramp while vehicles from the on-ramp enter into the mainline via the auxiliary lane. The number of diverging LCs from the mainline is given by $\alpha\lambda C$. Van Beinum et al. (2018a,b) highlighted the contribution of secondary merges, additional LCs by the merging vehicles to inner lanes, to the turbulence observed at weaving sections. In that empirical study, approximately 40% of the merging flow performed secondary merges for a short weaving segment with high flow ($\lambda\approx0.8$). The effect of secondary merges (which has been neglected in existing studies) is considered while computing the number of LCs in CIZ-2 where secondary merges occur. The merging LCs from the auxiliary lane to mainline, including the secondary merges, is given by:

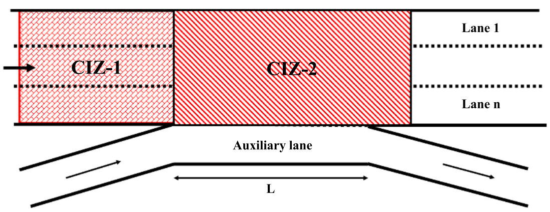


Fig 2. Capacity impact zone (CIZ) for weaving sections.

$$q_r(1+\sum_{i=n-1}^{1} SM_i) \tag{20}$$

In the above equation, SM_i represents the percent of total merging flow that will perform secondary merges to the inner lane i. If SM_{n-1} is 40%, then 40% of the merging flow will change lane from lane n to lane $n-1$. The value of i may be bounded as the number of LCs that can be performed by the merging vehicles depends upon the length of the weaving segment. There might still be secondary LCs occurring far downstream of the weave; however, these do not contribute to the loss in throughput at the bottleneck.

3.2.2. W-Strategy 2 (WS2)

The primary objective of this strategy is to create enough space for entering ramp demand in lane n by moving through CAVs in lane n to the left lanes. The LCs of exit CAVs in the left lanes are shifted downstream of the on-ramp in this strategy. This strategy is usually preferred when the merge flow at weaving sections is moderately high.

Van Beinum et al. (2018a,b) reported that a high proportion (almost 90%) of exit vehicles are in the right lane before the start of the on-ramp. Without any CAV lane assignment, we assume that all the exit HDVs from the left lanes will move to lane n in order to take the off-ramp in the upstream capacity impact zone (CIZ-1). Hence there are no LCs by exit HDVs in CIZ-2. The minimum flow in lane n in CIZ-1 is hence given by:

$$\underbrace{\frac{\lambda C}{n}+(n\alpha-\alpha_n)(1-p)\frac{\lambda C}{n}}_{Exit\ \ HDVs}=\frac{\lambda C}{n}(1+n\alpha-\alpha_n-n\alpha p+\alpha_n p) \tag{21}$$

The space available (or remaining) for the ramp demand q_r to merge into in lane n is given by:

$$\max\left\{0,\frac{C}{n}[1-\lambda(1+n\alpha-\alpha_n-n\alpha p+\alpha_n p)]\right\} \tag{22}$$

The merging flow q_r can either be greater than or less than the space left in lane n given by Eq. (22). The two possible scenarios and the strategies related to these two scenarios are discussed as follows:

3.2.2.1. WS2-C1.

If the ramp demand q_r is less than the space available given by Eq. (22), then there is no need to create additional space. If the number of through CAVs in lane n are greater than the number of exit CAVs in the left lanes, then Eq. (12.2) is satisfied and all CAVs can be assigned based on their destinations. In this way, the CAVs can be arranged based on their destinations as well as create enough space for the incoming ramp demand. This will again lead to a scenario similar to the diverging sections.

However, if the number of through CAVs in lane n are less than the exit CAVs in the left lanes, then assigning CAVs based on their destinations might lead to the reduction in space available for the on-ramp demand due to the increase in flow in lane n. In such a scenario, the number of exit CAVs from the left lanes assigned to lane n will be less than or equal to the through CAVs in lane n ensuring that Eq. (12.2) is satisfied. This means that LCs of some exit CAVs in the left lanes are shifted downstream of the on-ramp.

If X_1 is the proportion of exit CAVs which can be assigned to lane n, then to ensure Eq. (12.2) is satisfied, the following criterion must be satisfied:

$$X_1\frac{p\lambda C}{n}(n\alpha-\alpha_n)\leq(1-\alpha_n)\frac{p\lambda C}{n} \tag{23.1}$$

$$X_1\leq\frac{1-\alpha_n}{n\alpha-\alpha_n} \tag{23.2}$$

The flows and number of LCs up to CIZ-1 can be calculated using the formulations given for diverge sections as long as Eq. (12.2) is satisfied. The number of LCs in CIZ-2 is given by:

$$\text{Diverging LCs from mainline to ramp} - \alpha\lambda C+(1-X_1)\frac{p\lambda C}{n}\left[n\sum_{i=0}^{n-1}\alpha_i-\sum_{i=1}^{n-1}i\alpha_i\right] \tag{24}$$

$$\text{Merging LCs from ramp to mainline (including the secondary merges)} - q_r\left(1+\sum_{i=n-1}^{1}SM_i\right) \tag{25}$$

3.2.2.2. WS2-C2.

If the incoming ramp demand $q_r$is greater than the space remaining in lane n, then there is a need to create additional space. This can be achieved by assigning some of the through CAVs in lane n to the left lanes while keeping the exit CAVs in the left lanes. The flow that is needed to be reassigned to create space is given by:

$$q_r-\frac{C}{n}[1-\lambda(1+n\alpha-\alpha_n-n\alpha p+\alpha_n p)] \tag{26}$$

If the number of through CAVs $(1-\alpha_n)p\lambda C/n$ in lane n is greater than the space required, necessary buffer can be created. However, space on the left lanes also needs to be taken into account to ensure they are able to accommodate the incoming through CAVs from lane n. This gives rise to the condition

$$p \leq \frac{(1-\lambda)(n-1) + (n\alpha - \alpha_n)(\lambda - p\lambda)}{\lambda(1-\alpha_n)} \tag{27}$$

When the above condition is satisfied, the left lanes are able to accommodate the incoming through CAVs from lane n and necessary space can be created to accommodate the on-ramp demand. In the earlier strategies, lane assignment of CAVs were performed in the control zone upstream of the capacity impact zone while taking into consideration the compensation by HDVs. However, in this case, if the CAV lane assignment is performed far upstream, any space created for the on-ramp demand might be occupied by the exit HDVs in the left lanes via compensation. This will reduce the benefits of the lane assignment. Hence, to avoid this situation and to realize the full benefits of this strategy, the CAV lane assignment in this strategy is performed in the first part of the capacity impact zone (CIZ-1). While this may cause disturbances on the left lanes, enough space might be created in lane n where the on-ramp demand enters the mainline. The trade-off between creating additional disturbances on the left lanes upstream of the on-ramp along with shifting the LCs of left exit CAVs downstream of the on-ramp to create space for the on-ramp demand will be analysed in the next section while quantifying the bottleneck throughput using microscopically driven numerical simulations.

The flows exiting the capacity impact zone (CIZ-1) along with # of LCs in this zone are given in Table 9:

The number of diverging LCs in the second part of the capacity impact zone for this case would be equal to the number of LCs performed by the exit CAVs in the left lanes which is equal to:

$$\frac{p\lambda C}{n}\left[n\sum_{i=0}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i)\right] \tag{28}$$

The number of merging LCs in the second half remain the same as in the previous scenarios.

Finally, if there are not enough through CAVs to create sufficient space for the ramp demand, we can either still send the available through CAVs in lane n to the left lanes to create as much space as possible for the on-ramp demand while taking into consideration the space available on the left lanes as well. In such a scenario, the number of through CAVs in lane n that can be assigned to the left lanes to create space for the on-ramp demand will be equal to:

$$min\left\{(1-\alpha_n)\frac{p\lambda C}{n}, q_r - \frac{C}{n}[1-\lambda(1+n\alpha-\alpha_n-n\alpha p+\alpha_n p)], \frac{C}{n}[(1-\lambda)(n-1)+(n\alpha-\alpha_n)(\lambda-p\lambda)]\right\} \tag{29}$$

In the above equation, the first term is the number of through CAVs available in lane n for lane assignment, the second term represents the space to be created in lane n for the on-ramp demand and the third term indicates the space available in the left lanes to accommodate the through CAVs from lane n.

3.3. Overview

In this section, the analytical formulation of CAV lane assignment strategies for diverge and weave bottlenecks and the flows and LCs resulting from these strategies are provided. The analytical formulations give important insights about the system behaviour with respect to the various parameters involved. It must be noted here that the flows and resultant LCs for the various strategies have been derived assuming a fixed value of C. Empirical studies (Cassidy and Bertini, 1999; Bertini and Leal, 2005) have shown that the capacity of a section diminishes after the onset of congestion, a phenomenon commonly referred to as capacity drop. The primary aim of the various strategies formulated here is to avoid or delay congestion. Nonetheless, these strategies are applicable in congested traffic, where formation of irreversible voids remains a major concern for throughput reduction. However, a different value of maximum flow (C) should be assumed in congested conditions. For simplicity, switching logic from capacity in ordinary conditions to a reduced maximum flow in congested conditions is not discussed in this paper.

4. Numerical simulations for throughput quantification

In the previous section, lane assignment strategies for CAVs were formulated with the objective to reduce the number of disruptive LCs near diverge and weave sections. In this section, we aim to quantify the improvements in throughput as a result of implementing these strategies. The throughput improvement depends on various parameters such as the penetration rate of CAVs, exit proportions, merge flows etc. Hence, the strategies are evaluated for a range of these parameters. We further examine the trajectory plots to illustrate the working mechanisms of the control concept. In the remainder of this section, we will first introduce the setup for the numerical simulations (Section 4.1). The results of the throughput analysis for diverging sections for the various strategies are first presented (Section 4.2) followed by the results for weaving sections (Section 4.3).

4.1. Simulation setup

In a multi-lane setup, traffic dynamics can get very complex with LC interactions over space and time and across multiple lanes. Considering this complex nature of traffic dynamics, it is difficult to formulate a closed-form analytical solution for throughput. As cited earlier, previous efforts on throughput formulation were restricted to single lane motorways or a single type of bottleneck. Most of these studies were also from a macroscopic perspective and detailed LC interactions from a microscopic perspective are not fully

Table 9
Metrics for CIZ-1 (WS2-C2).

	Flow in lane n	Flow in lanes $1{:}n-1$	# of LCs
Eq. (27) is satisfied	$\frac{C}{n} - q_r$	$\frac{\lambda C}{n} - (1-p)\frac{\alpha_i \lambda C}{n} + \left[\left(q_r - \frac{C}{n}[1 - \lambda(n\alpha - \alpha_n - n\alpha p + \alpha_n p)]\right) * \frac{\alpha_i}{n\alpha - \alpha_n}\right]$	$\left(\left(1-p\right)\frac{\lambda C}{n} + \frac{Eqn.(26)}{n\alpha - \alpha_n}\right)\left[n\sum_{i=0}^{n-1}\alpha_i - \sum_{i=1}^{n-1} i\alpha_i)\right]$

captured. In this light, we employ microscopically driven numerical simulations to quantify the throughput at the bottleneck for the formulated strategies. A similar approach was adopted in Chen and Ahn (2018) to quantify capacity drop at extended bottlenecks albeit for single lane motorways.

The LC mechanism proposed in Laval and Daganzo (2006) is used here wherein the LC vehicle acts as a moving bottleneck traveling at a lower speed creating a persisting void downstream. The free-flow speed, wave speed and capacity of a lane are denoted by u_i, w_i and C_i respectively with i being the lane index. The vehicles maintain their headway according to Newell's car-following model (Newell, 2002). Newell's model is used for its simplicity, its direct connection to triangular FD and effectiveness in reproducing the important traffic flow phenomena such as disturbance propagation. The treatment of inserting and exiting vehicles in a lane is performed as follows:

- A vehicle inserts and occupies the equilibrium position of a vehicle in the target lane. The vehicle in the target lane adjusts its position by temporarily stopping and gaining the desired spacing before following the inserting vehicle. (Of note, this assumption does not affect the throughput).
- Exiting vehicles are assumed to instantly reduce their speed to the speed on the target lane (if the speed on the target lane is lower than the speed on the current lane; otherwise they exit at the speed of their current lane) before exiting their current lane.
- Vehicles merging from an on-ramp are assumed to insert into the mainline at a lower speed (v_{mer}) and accelerate at a constant rate till they reach the prevailing speed on the mainline.
- Similarly, vehicles exiting to an off-ramp leave the mainline by instantly reducing their speed to an exit speed (v_{div}) before moving to the off-ramp.
- LC vehicles are always admitted into their target lane.
- The merging and exiting positions and times are uniformly distributed over the space–time domain.

The authors refer to Chen and Ahn (2018) for a detailed description of these assumptions. The LC process in this study is assumed to be instantaneous. In general, the process of LC takes several seconds (Olsen et al., 2002; Toledo and Zohar, 2007). LC durations are dependent upon various factors such as traffic density, driver type, surrounding vehicles etc. While this can be explicitly modelled (Schakel et al., 2012) to increase realism, we favour the simpler approach with the following logic. An instantaneous LC would overestimate throughput compared to the LCs with a particular duration. However, this is compensated by our conservative approach to consider speed adaptation during the LC process. It is assumed that the drivers fully adapt their speeds in the origin lane, whereas in normal conditions, relaxation is observed in the target lane (Treiber et al., 2000; Zheng et al., 2013). Therefore, these two simplifying assumptions have opposing effects. While they may not completely cancel out each other, the effects are in the same order of magnitude, and we expect reasonable accuracy.

For the throughput analysis, a 3-lane motorway is chosen. In the case of weaving sections, a single lane on-ramp and off-ramp are assumed to be connected via an auxiliary lane. The merging and diverging into and from the mainline occurs from and to the auxiliary lane respectively. To introduce heterogeneity, lane-specific FDs are considered in the simulations. One may consider both class and lane-specific FDs. However, given the lack of empirical evidence on which has a stronger influence, only lane-specific FDs are assumed to preserve the main focus of this paper which is the control strategy itself. The lane-specific FD parameters are given in Table 10. The merging speed from the on-ramp (v_{mer}) is taken as 60 km/h and the diverging speed to the off-ramp (v_{div}) is 60 km/h. The acceleration rate is fixed as 1 m/s^2. The simulation period is set to be 70 min.

4.2. Diverge sections

In this section, the impact of CAV lane assignment on the throughput at diverge bottlenecks using numerical simulations is presented. The layout of the diverge bottleneck is described first followed by analysis of the throughput under various CAV penetration rates and exit flows. In order to get a better understanding of the working mechanism of control, the trajectory plots are analysed with and without CAV lane assignment for a particular scenario. This is followed by a discussion on the lengths of the control and compensatory zone and the effect of compensation by HDVs on the throughput.

Table 10
Lane-specific FD parameters.

	Lane 1	Lane 2	Lane 3
Free-flow speed (km/h)	120	105	90
Wave speed (km/h)	24	21	18
Capacity (veh/h)	2000	2000	2000

A 3-lane motorway with a single lane off-ramp is considered with an auxiliary lane of length 300 m connecting the mainline to the off-ramp. Exiting vehicles slowdown in lane n to the exiting speed before exiting the mainline and entering the auxiliary lane. Van Beinum et al. (2018a,b) reported a change in the lane flow distribution at around 700 m upstream of the off-ramp with higher speeds observed upstream of 750 m location. This can be attributed to the LC activity with exiting vehicles in the left lanes moving to the right-most lane in order to take the exit. Hence, the length of the capacity impact zone was set to be 700 m to indicate the start of the LC activity of exit vehicles in the left lanes. It is assumed that the length of the control zone is not constrained by the presence of any intervening ramps. When the control zone (and compensatory zone) is sufficiently long, any reduction in the flow leaving these zones can be avoided. 20 simulation runs are performed for each scenario and the throughput is obtained by taking the average of the results of the 20 runs. The overall flow as well as the exit flow is assumed to be equally distributed across the lanes with λ being set to 0.9. Hence, the demand on each lane is 1800 veh/h. The strategies are examined for both low (300 veh/h) and high (750 veh/h) exit flows. The total throughput is given as the sum of the mainline and diverging flow and the mainline flow is measured 300 m downstream of the off-ramp.

CAV lane assignment strategies for diverge sections are based on the criterion relating exit proportions to the capacity constraints of lanes given by Eq. (3). Given the input parameters, Fig. 3 shows the criterion for a particular diverge strategy to be implemented. Exit rate (α) in the figure is defined as the ratio of the exit flow to the total flow. The solid lines represent the upper and lower bounds given by Eq. (3). When the exit rate lies between the two curves, the condition given by Eq. (3) is satisfied and DS1 is implemented wherein all the CAVs are assigned to lanes based on their respective destinations. If the exit rate is less than the lower bound, the condition given by Eq. (4) is satisfied and DS2 is implemented. Only some through CAVs in lane n can be moved to the left lanes (due to capacity constraints in the left lanes) and all the exit CAVs in the left lanes are assigned to lane n.

For low exit flows, when the CAV penetration rate is less than or equal to 20%, DS1 can be implemented and when the penetration rate is higher, DS2 can be implemented. When the exit flow is high, DS1 can be implemented for CAV penetration rates of lower than or equal to 30% and DS2 for higher penetration rates. Since α is always lower than $(1/n)$, the flow in lane n decreases and flow in the left lane increases after the lane assignment. Thus, to restore the original lane flow distribution, HDVs from the left lanes move to lane n exhibiting compensatory behaviour.

Fig. 4 illustrates the throughput for the diverging section under various penetration rates. The 0% penetration rate represents the no control case where no lane assignment is performed. The no control case is considered as the benchmark against which any performance gains resulting from the CAV lane assignment are compared.[1] When the exit flow is high, the throughput drops by 8.3% from the maximum possible outflow. In case of low exit flows, the throughput loss is 1.7%. From the figure, it can be seen that the full benefits of CAV lane assignment can be realized even at low to medium penetration rates (i.e. even from 10% onwards). Maximum throughput was achieved when the penetration rate was greater than 30% for high exit flows and 10% for low exit flows. The effect of compensation on the throughput is more pronounced when the exit flow is high. For low exit flows, compensation does not seem to play a major role with similar throughputs observed for 50 and 100% compensation rates. In the case of higher exit flows, higher compensation rate lead to a comparatively higher throughput than lower compensation rate. As flow in lane n decreases after lane assignment, the space created in lane n is occupied by exit HDVs in the left lanes. This leads to more exit vehicles reaching lane n farther upstream thus reducing the LC activity and conflicts near the off-ramp. When the length of the auxiliary lane was fixed to 0 m, (i.e. the off-ramp is directly connected to the mainline), the throughput dropped by an additional 1% in the high exit flow case and 0.3% in the low exit flow case without any CAV lane assignment. Hence it can be said that the length of the auxiliary lane does not seem not affect the throughput significantly for the chosen demand.

Fig. 5 shows the trajectories for a particular simulation run with and without CAV lane assignment in the capacity impact zone after a warm-up period. The capacity impact zone starts from the 1300 m location and is 700 m long (area between the green dotted lines in Fig. 5). The auxiliary lane starts downstream of the capacity impact zone from 2000 m. The chosen simulation runs have a throughput similar to the average throughput of all the simulation runs. In the figure, the blue solid lines represent the through vehicles in a lane, the red dotted lines represent the exit vehicles and the black dotted lines indicate the exiting vehicles coming from the left. The black asterisk denotes the exiting location of the vehicles originally in that lane while the magenta diamond indicates the exiting positions of the vehicles coming from the adjacent lanes. The control case shown in the figure corresponds to the 30% CAV penetration rate with high exit flow and 50% compensation.

We take a closer look at the lane-wise flows to understand the effect of the CAV lane assignment on the flows per lane. Table 11 gives the lane-wise throughput for the control and no control case. It can be seen that the throughput in lanes 1 and 2 increased and decreased in lane 3 when CAV lane assignment is performed. This is because through CAVs in lane 3 are assigned to lanes 1 and 2 in the control zone in order to create space for the left exit CAVs. All the exit CAVs in lanes 1 and 2 are moved to lane 3 in the control zone. As DS2 is implemented, flow in lane 3 decreases after CAV lane assignment. This results in the exit HDVs in the left lanes to move to lane 3 in the compensatory zone in order to restore the lane flow distribution. Any remaining exit HDVs in the left lanes will then change to lane 3 in the capacity impact zone. This is illustrated by the reduction in the number of voids in the capacity impact zone in lanes 1 and 2 in the control case. And since the flow is organized upstream, the LC conflicts between the through and exit vehicles in lane 3 near the bottleneck are reduced. This significantly reduces the disturbances in lane 3 which can be clearly observed in Fig. 5. In the no control case, vehicles coming in from the left create disturbances which propagate upstream creating congestion in this lane. While through

[1] The reason for choosing the no control case as the benchmark is that comparison with other possible traditional strategies such as ramp-metering or LC advisories are dependent upon the underlying models and control algorithms and seem unfair. And there are no other commonly accepted CAV control schemes for lane assignment.

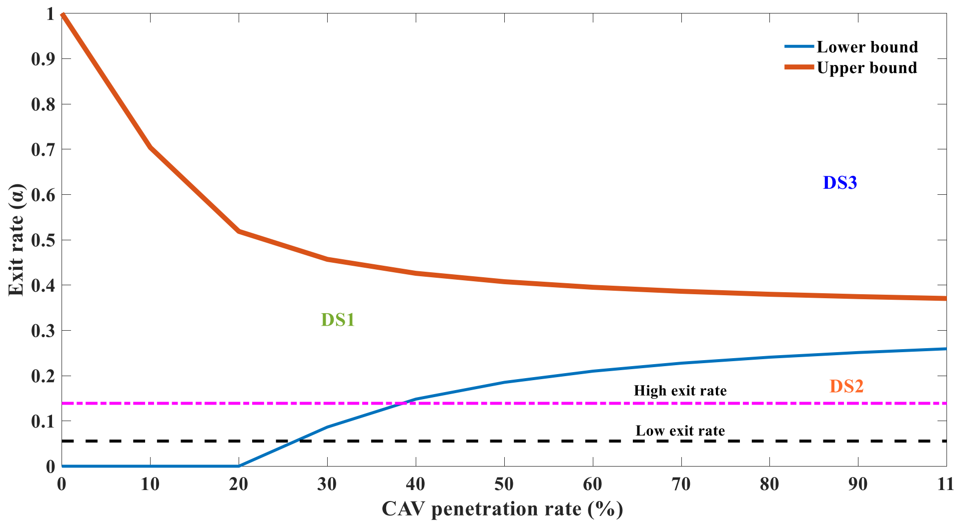


Fig 3. Condition for diverge strategy in low exit flow scenario.

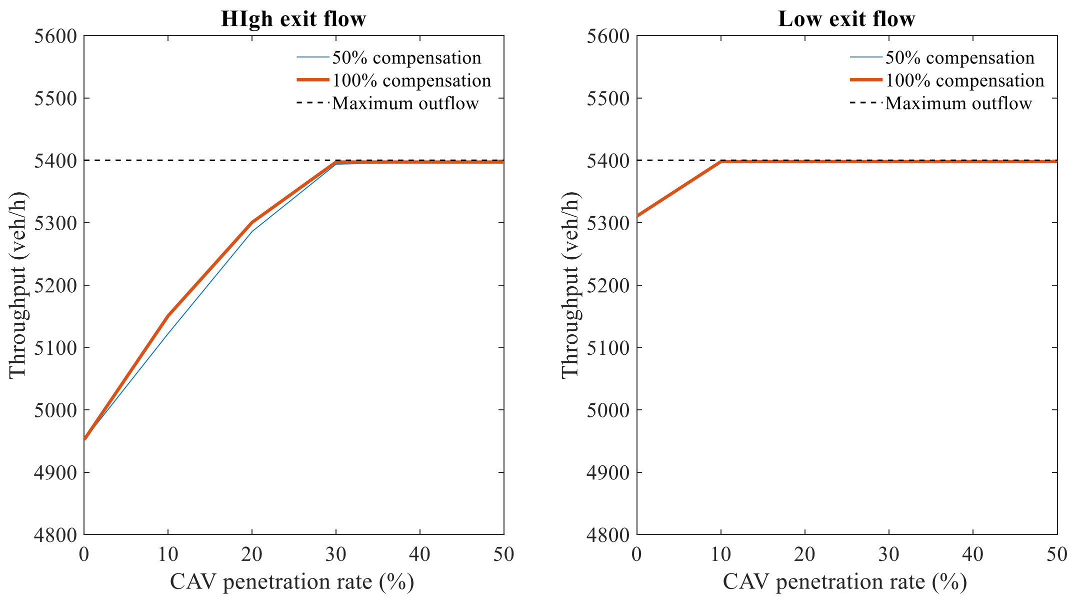


Fig 4. Throughput for diverging sections.

vehicles on lane 3 can move to the right lanes in the no control case to avoid congestion, this situation is unlikely. The left lanes are already at near-critical conditions and any LCs from the right can deteriorate the condition in these lanes and increase conflicts with exit vehicles coming from the left. Empirical analysis by Van Beinum et al. (2018a,b) also showed no changes in the fraction of flow on the left lane and only minor increase in the fraction of flow in the centre lane upstream of off-ramps. Hence, it can be said that with CAV lane assignment, the flow approaching the bottleneck is better organized which reduces the disturbances leading to a higher throughput. Fig. 6 shows the cumulative curves at the bottleneck. It can be seen that CAV lane assignment leads to higher throughput and lower delays when compared to the no control case.

The simulation results also showed that when the length of the control zone and compensatory zone were long enough, CAV lane assignment did not impact the throughput at the bottleneck. For example, with a CAV penetration rate of 10% and 100% compensation (in case of high exit flow), it was observed that when the length of the control zone plus compensatory zone was greater than 1000 m, lane assignment did not impact the throughput. This is because when the LC activity is spatially spread over a large distance, disturbances created by inserting vehicles can be absorbed by any voids resulting from exiting vehicles. In general, the lengths of the control and compensatory zone depend on various factors such as the number of LCs within these zones, LC intensity (number of LCs per distance) and prevailing speeds. However, if the lengths of these zones are chosen long enough (assuming the space is not

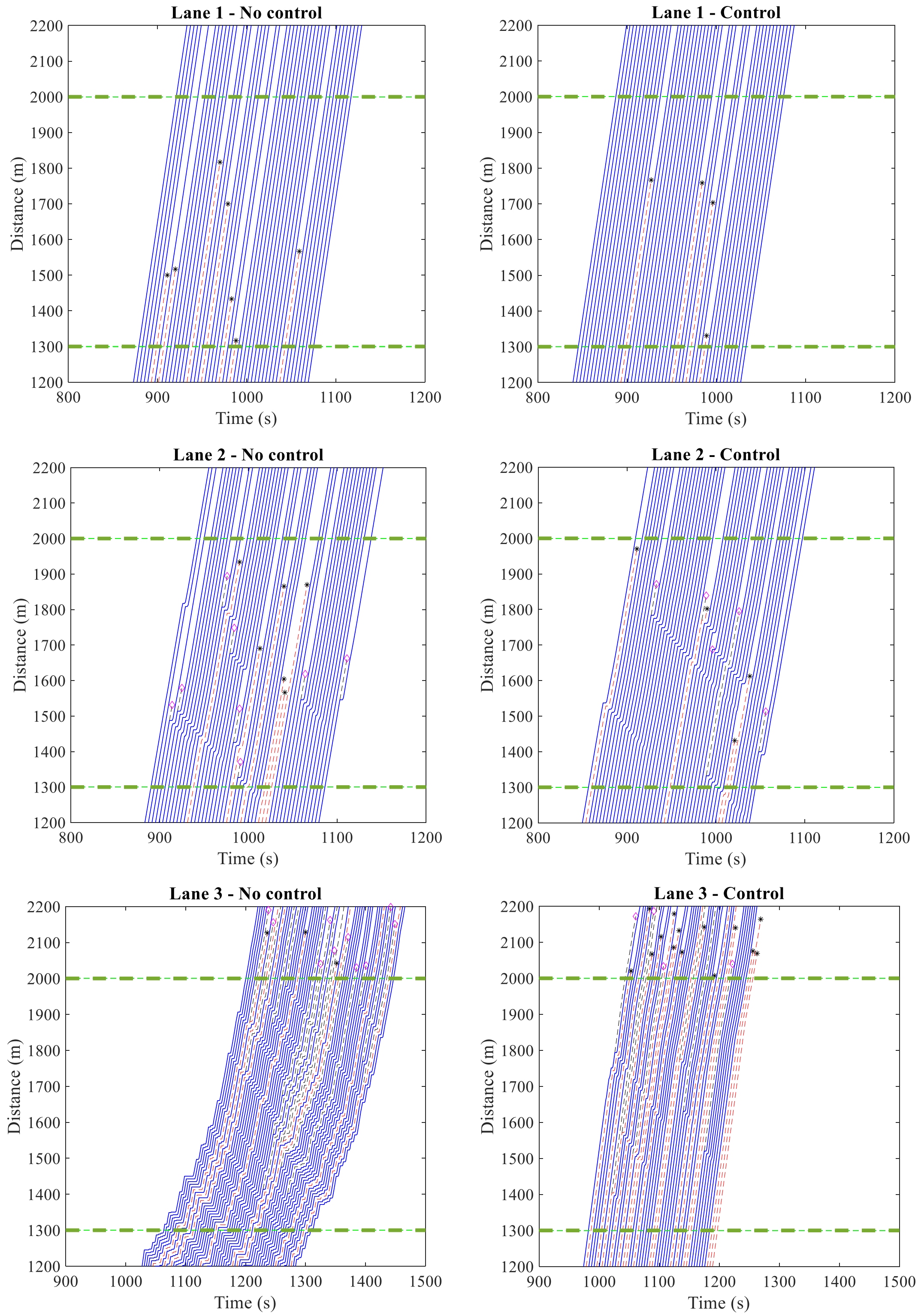


Fig 5. Trajectory plot for diverging sections.

constrained by any intervening ramps), then the CAV lane assignment will not create any additional congestion. It is possible that the LCs in the control zone can impact the throughput especially in free-flow conditions. But even in that situation, the total number of disruptive LCs in the system will reduce due to the spatial distribution of LCs and re-utilization of the voids. The benefit may be reduced

Table 11
Lane-wise throughput (veh/h).

	Lane 1	Lane 2	Lane 3
No control	1549	1550	1196
Control	1782	1781	1079

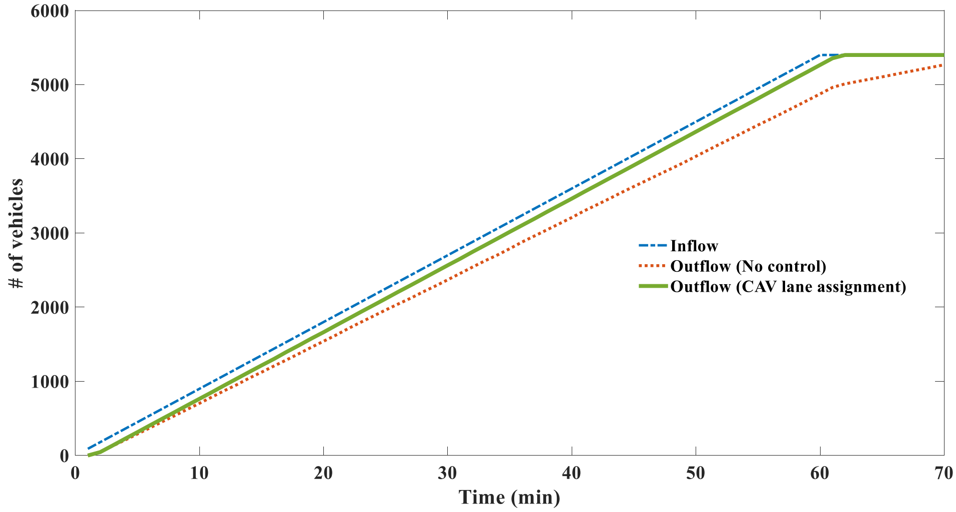


Fig 6. Cumulative curves at the diverge bottleneck.

but it would still be positive. It must be noted that the length of the control (and compensatory) zone will be site-specific and depend on the number of lanes, exit rates, traffic rules, etc. Thus, they should be tuned during initial deployment taking into account all these factors. It is also possible that enough space is not available upstream due to the presence of structural discontinuities. In such cases, the control zone can either be extended upstream of the discontinuity (in the case of another off-ramp upstream) or the lane assignment strategy can be complemented with another control strategy such as ramp metering (in the case of an on-ramp).

Since strategies DS1 and DS2 are implemented, flow in lane 3 reduces after CAV lane assignment. To restore the initial lane flow distribution and occupy the space created in lane 3, it is assumed that exit HDVs in the left lanes move to lane 3. However, as mentioned in the previous section, it is possible that through HDVs from lanes 1 and 2 may also move to lane 3 in order to compensate. Fig. 7 shows the impact of mixed compensation (both through and exit HDVs compensate) on the throughput for high exit flow with 10% CAV penetration rate.

For maximum benefits, it is preferred if the compensation is mostly performed by exit HDVs. In Fig. 7, the labels on the vertical axis

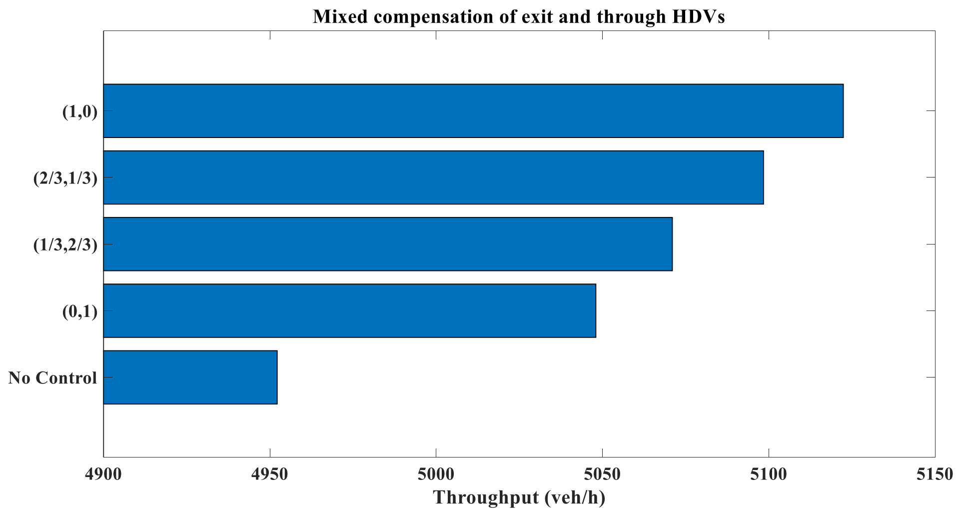


Fig 7. Throughput variation in the mixed compensation case.

represent the compensation rates of exit and through HDVs. The first coordinate in the parenthesis indicates β_1 (compensation rate of exit HDV) and the second coordinate represents β_2 (compensation rate of through HDV). The mean compensation rate β in this scenario is 50%. It can be seen that when the compensation is performed only by through HDVs, minor improvements in throughput is observed. However, if the compensation is performed only by exit HDVs, the throughput by 3.5% compared to no control.

4.3. Weave sections

We now analyse the impact of CAV lane assignment on the throughput at weave bottlenecks. The layout of the weave bottleneck is initially described followed by the analysis of the throughput under various CAV penetration rates and the four scenarios with respect to the merge and diverge flow (described in Section 3.2)

A 3-lane motorway is considered with an auxiliary lane of length 500 m connecting the merging segment (on-ramp) to the diverging segment (off-ramp). Merging vehicles from the on-ramp enter the mainline via the auxiliary lane at a speed of v_{mer} (60 km/h) and diverging vehicles from the mainline exit to the off-ramp at a speed of v_{div} (60 km/h). The demand and exit flows are assumed to be equally distributed across lanes with a flow to capacity ratio of 0.9. The merge and diverge flows were varied from low (300 veh/h) to high (750 veh/h). Various empirical studies (Lee and Cassidy, 2008; Marczak et al., 2014; Van Beinum et al., 2018a,b) reported that at weaving sections, majority of the weaving activity occurs in the first half of the auxiliary lane. Hence, in the simulations, the majority (90%) of the inserting and exiting positions are generated in the initial half of the auxiliary lane. As the main focus of this study is to evaluate the benefits of CAV lane assignment, the individual LC locations of the CAVs are not controlled in the weaving segment. The total throughput for a weaving section is given as the sum of the mainline and diverging flow and the mainline flow is measured 1000 m downstream of the off-ramp. Since the flow on the mainline is high (and the weaving segment is short), the percent of merging traffic that is assumed to perform secondary merges is assumed to be 40% (Van Beinum et al., 2018a,b). It was also mentioned in that study that the region of influence of secondary merges is around 400 m downstream of the weaving segment and thus the secondary merges are generated up to 400 m downstream of the weave.

The strategy WS1 attempts to organize the exiting traffic approaching the weaving segment such that exit vehicles are in lane 3 and LC conflicts between through and exit vehicles on the mainline are minimized. WS1 is similar to the strategies formulated for diverge sections. The insights from the results for diverging sections reveal that for the chosen exit flows (resulting in exit rates of 0.05 and 0.14; see Fig. 3), the CAV lane assignment leads to decrease in the flow of lane 3 resulting in exit HDVs moving to lane 3 from lanes 1 and 2 to compensate. In WS2, through CAVs from lane 3 are moved to the left to create space for the incoming on-ramp demand. The weave strategies WS1 and WS2 are evaluated for the 4 possible scenarios in the following order:

- low merge and low diverge flow
- low merge and high diverge flow
- high merge and low diverge flow
- high merge and high diverge flow

Fig. 8 illustrates the throughput for the weaving section under various penetration rates for these 4 scenarios. 100% compensation is assumed for all scenarios.

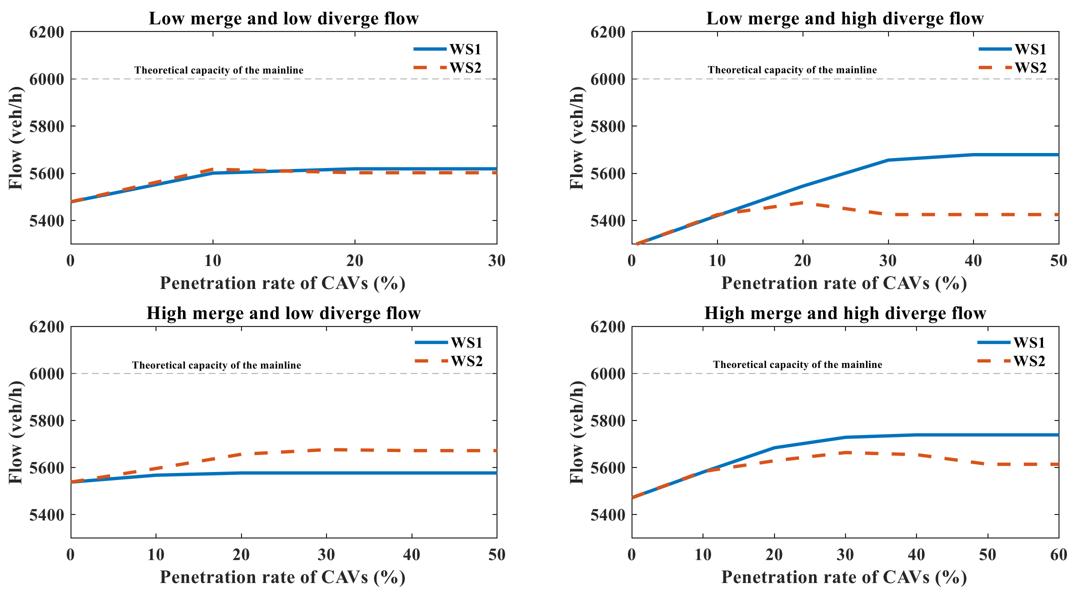


Fig 8. Throughput at weaving section.

In the low merge and diverge scenario, both WS1 and WS2 result in similar results under various CAV penetration rates with a maximum increase of 2.5% in throughput observed at the 10% penetration rate compared to the no control situation.

In the low merge and high diverge scenario, the performance of WS2 tails off from the 20% penetration rate onwards with an

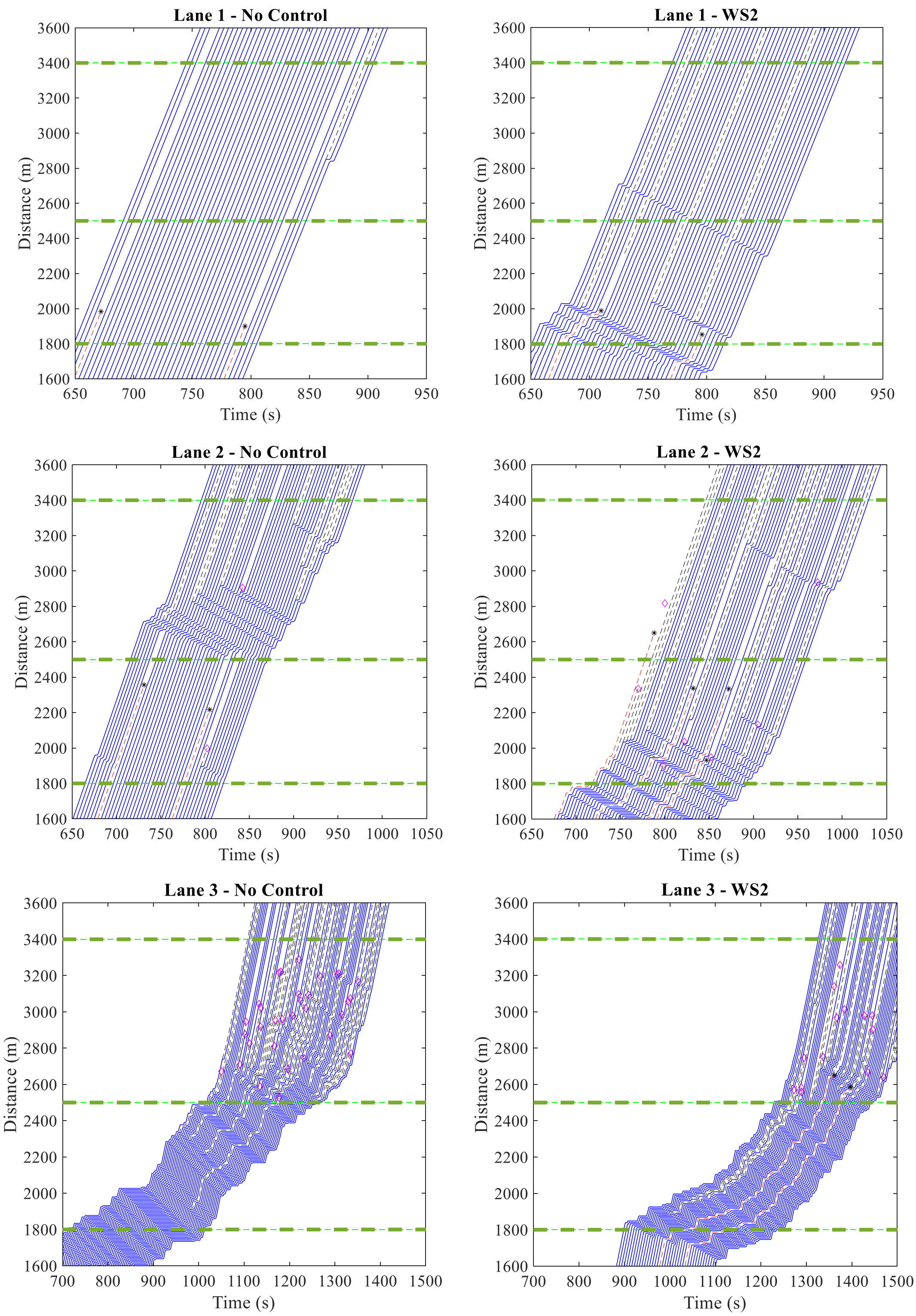


Fig 9. Trajectory plot for high merge-low diverge scenario.

increase of only 2.5% in the throughput compared to the no control case. However, WS1 yields high improvement in the throughput with an increase of 7.2% compared to the no control case. This is not surprising since WS1 is more geared toward high diverge flow conditions by organizing the flow with respect to the destination.

WS2 outperforms WS1 when the merge flow is high and the diverge flow is low. This is also not surprising since WS2 better accommodates the merge flow than WS1. To gain more insight, Fig. 9 displays the trajectories for lanes 2 and 3 when WS2 is implemented compared to the no control case. The CAV penetration rate is 30%. The CIZ-1 region lies between 1800 m and 2500 m while CIZ-2 lies between 2500 m and 3400 m (indicated by the green dotted lines in Fig. 9). Table 12 also shows the lane-wise throughput for this case. It can be seen from the figure that more disturbances are created in lane 2 in the CIZ-1 region which affects the throughput of lane 2. The disturbances in CIZ-2 from the secondary merges interact with the LCs of the through CAVs coming from lane 3 upstream which further affects the throughput of lane 2. However, assigning through CAVs from lane 3 to the left also creates space on lane 3 to accommodate the ramp demand which reduces the disturbances and severity of congestion in lane 3. The space created in the left lane helps in dissipating the disturbances created by the merging vehicles which helps in avoiding the congestion propagating too far upstream. This can be seen in Fig. 9 where more disturbances are observed in CIZ-2 in the no control case as compared to the scenario where WS2 was implemented. As mentioned earlier, the CAV lane assignment aims to avoid or delay congestion but the very physical mechanism (of formation and propagation of irreversible voids) is still applicable during congestion. Hence, the lane assignment can be continued in the control case even after the onset of congestion as seen in the trajectory plot of lane 3. The lane-wise flows given in Table 12 also show the reduction in the flow in lane 2 because of implementing WS2. While WS2 outperforms WS1 in this scenario, the observed improvements in throughput are not very high. This is because the number of CAVs which can be assigned to the left lanes is governed by three factors given in Eq. (29). When the CAV penetration rate is high, the space available on the left lanes reduces as most of the exit CAVs remain in these lanes reducing the space to accommodate the through CAVs from lane 3. Thus, although WS2 is better in high merge-low diverge scenarios, the increase in throughput is not very high.

Finally, in the high merge-high diverge scenario, WS1 performs better than WS2 with increasing CAV penetration rate. At lower penetration rate of 10%, both strategies yield similar improvements in throughput. However, with increasing CAV penetration rate, WS1 performs better than WS2 with a small decrease in the throughput from 50% onwards when WS2 is implemented. The results are somewhat unexpected but can be explained as follows: In WS2, exit CAVs in the left lanes are kept in their lanes and their LCs are shifted downstream of the on-ramp. This increases the number of LCs in the weaving segment as exit CAVs are changing lanes from the left while also interacting with the vehicles which have entered from the on-ramp.

More specifically, since the CAV lane assignment in WS2 are performed in CIZ-1, the LC activity in CIZ-1 is increased. And as the length of CIZ-1 is fixed higher number of LCs in this region will lead to decrease in the flow exiting this region. The LC activity in lane 2 particularly increases as there are insertions from the left (in terms of exit HDVs from lane 1), insertions from the right (through CAVs from lane 3) and desertions to the right and left (exit vehicles in lane 2 and inserting vehicles from lane 3 moving to lane 1). This causes the throughput of lane 2 to decrease with increasing CAV penetration rates. The exit CAVs in lanes 1 and 2 also start their LC activity downstream of the on-ramp which can lead to interactions with the ramp flow which has entered the mainline as part of the ramp flow also performs secondary merges. This further creates disturbances with higher penetration rates as more CAVs remain in the left lanes which increases the number of LCs in CIZ-2 as well. Hence, when both the merge and diverge flow are high, WS1 is preferred over WS2 especially when the CAV penetration rate is high.

Of further note, for high merge flow scenarios, the total merging flow is high and enough space cannot be created in lane 3 to accommodate the ramp demand. Fig. 10 shows the demand that cannot be accommodated in lane 3 under various penetration rates. The total demand on lane 3 is the sum of the flow in lane 3, exit vehicles coming from the left lanes and the ramp demand. For example, when the CAV penetration rate is 30% and the diverge flow is high, the demand on lane 3 is 1800 veh/h (flow in lane 3) + 750 veh/h (on-ramp demand) + 350 veh/h (exit HDVs coming from the left lanes) which equals to 2900 veh/h. The capacity of lane 3 is only 2000 veh/h. Hence there is a surplus demand of 900 veh/h. The through CAVs in lane 3 which can be assigned to the left lanes to create space to accommodate this demand equals 465 veh/h. Thus, there is a demand of 435 veh/h which cannot be satisfied even after CAV lane assignment. Note that the lane assignment in CIZ-1 can contribute to a reduction in the bottleneck throughput. Thus, in the case of high on-ramp demand, complementing the lane assignment with on-ramp metering may bring greater benefits. Such integrated strategy, however, needs a careful consideration of system-wide benefits including ramp delay, which is beyond the scope of this conceptual work. Nevertheless, the analytical formulation from this study provides an important foundation to form such a strategy.

5. Conclusions and discussions

This paper presented a novel traffic control approach of strategic CAV lane assignment to improve mixed traffic throughputs at diverge and weave bottlenecks. The main concept lies in organizing traffic flow well upstream of a bottleneck by strategically assigning CAVs across lanes, thereby minimizing the throughput-reducing LCs near the bottleneck. Compensatory behaviour of HDVs in response to CAV lane assignment was explicitly accounted for in the framework Taking a hybrid approach, CAV lane assignment strategies were formulated analytically based on the macroscopic flow conservation principle considering lane-wise demand, capacity constraints, CAV penetration rate and HDV compensation rate. The improvements in throughput due to these lane assignment strategies were then estimated using microscopically driven numerical simulations.

For diverge bottlenecks, three lane assignment strategies were proposed taking into consideration the prevailing lane flow distributions and capacity constraints of the individual lanes. Results from the numerical simulations revealed that at low CAV penetration rates, maximum throughput can be achieved by performing the CAV lane assignment even for high exit flows. The strategies which lead to the decrease in flow in the shoulder lane after lane assignment are highly beneficial as this causes the exit HDVs in the

Table 12
Lane-wise throughput in the high merge-low diverge scenario (veh/h).

	Lane 1	Lane 2	Lane 3
No control	1730	1959	1584
Control	1942	1899	1563

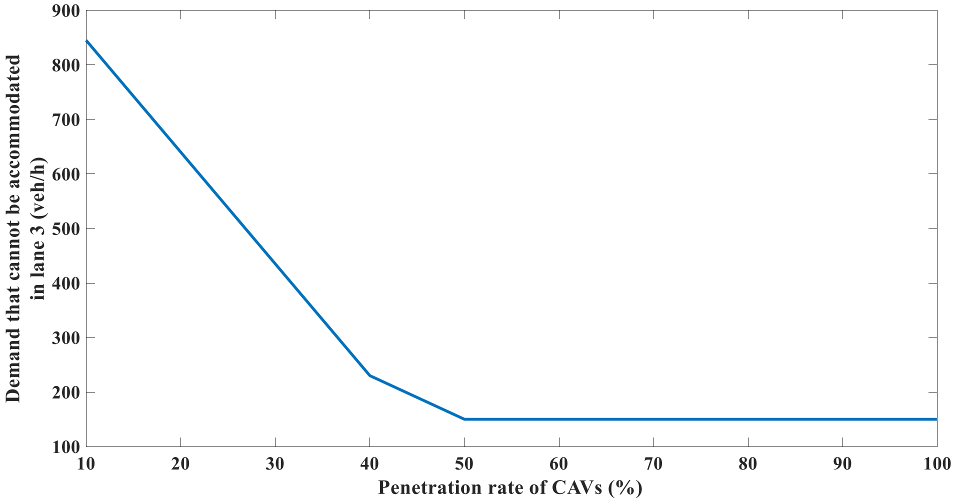


Fig 10. Demand that cannot be accommodated in high merge scenarios.

inner lanes to move to the shoulder lane to occupy the space created in that lane compensating for the flow imbalance. This reduces the LC conflicts between the through and exit traffic as the majority of the exit traffic is already in the shoulder lane. It was also determined that when the length of the area where the CAV lane assignment is performed is long enough, it does not contribute to loss in throughput.

For weave bottlenecks, two strategies with different and conflicting objectives were formulated. The first strategy was derived from the insights of diverge bottlenecks wherein the primary objective was to assign majority of the exit vehicles to the right-most lane far upstream which might reduce the space for the merging flow. The second strategy involved creating enough space in the right lane to accommodate the incoming merge demand which causes the exit vehicles to remain in the inner lanes and start their exit activity downstream of the on-ramp. The two strategies were evaluated for different combinations of merge and diverge flows. It was observed that when the merge and diverge flows were low, both strategies performed identically with minor improvements in throughput. In case of high merge and diverge flows, it was observed that at lower penetration rates (<20%), both strategies lead to similar improvements in throughput. But as the penetration rates of CAVs increased, the former strategy lead to better results with the improvements in the latter strategy falling off as the penetration rates increased. This was a result of the increase in the LC activity towards the left in the second strategy which occurred concurrently with the LC activity towards the right of exit HDVs. The results revealed that strategic lane assignment of CAVs in mixed traffic can lead to improvements in throughput even at low penetration rates. Earlier studies such as Chen et al. (2017) and Talebpour et al. (2017) pointed to negative benefits when CAVs and HDVs are segregated. Hence it can be said that designated infrastructure for AVs might not be preferable and with a good control strategy, performance gains can be obtained even in mixed traffic at low penetration rates. However, an efficient communication infrastructure is needed for the successful implementation of such proposed strategies.

Note that several simplifying assumptions were made for the purpose of developing the analytical framework and obtaining key insights. The overall mainline flow, exit flow and CAVs were assumed to be distributed evenly across the lanes. This assumption can be relaxed for more general formulations by introducing additional parameters to capture different CAV distributions across lanes. While the general formulation might yield different lane-specific improvements in throughput, the main insights do not change. The LC mechanism used in this study is based on the assumption that merging and diverging traffic have lower speeds than the mainline through traffic. The merging and diverging speeds were also assumed to be exogenous and determined externally. Furthermore, it was assumed that vehicles, including LC vehicles, follow Newell's simplified CF model and LC vehicles take the equilibrium position of a vehicle in the target lane. In reality, vehicles may exhibit anticipation and relaxation (Laval and Leclercq, 2008; Schakel et al., 2012; Zheng et al., 2013) which may reduce the void size affecting the estimation of throughput. We also assume vehicle homogeneity with vehicles exhibiting similar acceleration capabilities which affects the void size. High percentage of heavy vehicles (such as trucks) in the right lanes can also impact the performance of control by making it less desirable for exit HDVs to move to this lane in the compensatory zone. However, this can be accounted for in the framework by assuming lower compensation rates. And finally, limited by scope, we do not deal with the spatial LC distributions in the weave segment which are known to contribute to the throughput loss.

Future research is needed to relax these assumptions and thoroughly evaluate the impact of these parameters on the throughput of the bottleneck.

CRediT authorship contribution statement

Hari Hara Sharan Nagalur Subraveti: Conceptualization, Methodology, Investigation, Writing - original draft. **Anupam Srivastava:** Conceptualization, Supervision, Writing - review & editing. **Soyoung Ahn:** Conceptualization, Supervision, Writing - review & editing. **Victor L. Knoop:** Supervision, Writing - review & editing. **Bart Arem:** Supervision, Funding acquisition.

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