Inherently Balanced Spherical Pantograph Mechanisms

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Abstract. This paper investigates the possibilities for designing shaking force balanced spherical pantograph mechanisms. Three different spherical designs are presented, which are a balanced general spherical pantograph, a balanced double spherical pantograph and a double S-shaped mechanism with surrounding 4R four-bar linkage. Also variations of the balanced designs are presented. As compared to the planar balanced pantograph, the same underlying system of principal vectors exists, of which the geometries can be visualized in the orthogonal planes. The feasible variations of link lengths for which force balance is maintained are discussed.

1 Introduction

Mechanisms used in applications with high accelerations or large moving masses, such as manipulators, combustion engines or land-based telescopes [1–3] can suffer from vibrations due to the generated shaking forces and shaking moments, which can be significant [4]. These can be fully eliminated by applying dynamic balancing which requires a specific distribution of the masses of the links [1].

Contrary to the balancing of planar and spatial mechanisms, of which a significant amount of research is known, the balancing of spherical mechanisms in specific has received limited attention. Gosselin [5] applied static balancing to spherical mechanisms, using springs to reduce the overall mass and inertia as compared to shaking force balancing using mass redistribution. Moore [6] created an algebraic method for force balancing of spherical four-bar mechanisms, using complex variables and Laurent polynomial factorisation. Borugadda [7] applied a counterweight and adjustable kinematic parameters with real time control to achieve force balance and partial moment balancing of a spherical mechanism. Partial force and moment balancing was also achieved by Gill et al. [8] using optimisation of the mass distribution. Most solutions for dynamic balancing of spherical mechanisms are based on the placement of additional mass and inertia, resulting in more complex systems with often larger power requirements or reduced performance [1]. Inherent balancing, on the contrary, is known for resulting in balanced solutions with relatively low mass, inertia and complexity, which however has not yet been explored for spherical mechanisms.

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The goal of this paper is to investigate a new approach for the design of shaking force balanced spherical mechanisms using inherently balanced spherical pantograph mechanisms. First, the transformation of a force balanced planar pantograph into a force balanced spherical pantograph is shown and subsequently four force balanced spherical variations are presented.

2 Balanced Spherical Pantograph

First the planar force balanced pantograph will be explained in Sect. 2.1, which is used in Sect. 2.2 as the basis for a spherical force balanced pantograph.

2.1 Planar Pantograph

The planar pantograph linkage shown in Fig. 1 is an inherently balanced planar geometry, which functions as the basis for a multitude of inherently balanced designs [1,9]. The basic geometry is a parallelogram SP_1AP_2 with two sets of parallel and equally long links of lengths a_1 and a_2, which are connected using revolute joints.

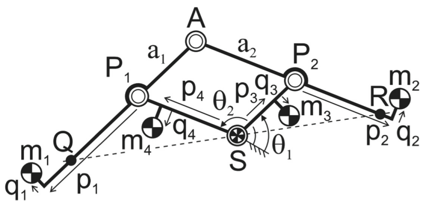

Fig. 1. Planar shaking force balanced pantograph consisting of four moving elements with a common center of mass located in base point S for any motion of the linkage [10].

The positions of link masses m_i are described by distances p_i along the link and q_i normal to the link as illustrated. For specific conditions, the balance conditions, the common Center of Mass (CoM) is stationary in point S, invariable to the movement of the linkage. The balance conditions of a planar pantograph are based on the principal vectors [1], which was shown to also apply for spatial pantographs [10]. Additionally, a principle of mirrored motion is visible within this geometry, with similarity point Q moving similarly and oppositely to similarity point R, resulting in zero net reaction forces in point S. Other planar geometries also show this principle when balanced [9].

2.2 Spherical Pantograph

Figure 2a shows the spherical version of the balanced planar pantograph which is obtained when the links are curved with equal radii in alternating directions for which the joints move along the surfaces of multiple spheres. The curvatures cannot be in the same direction for a force balanced design since a 2-fold radial symmetry is required to have the common CoM in the base pivot in S for any pose of the linkage. Due to the alternating curves of the links, joint A can no longer exist and both links are disconnected with their extremities in A_1 and A_2 as illustrated. These points then move towards and away from one another by the links SP_1 and SP_2 rotating in opposite directions, resulting in mirrored spherical trajectories.

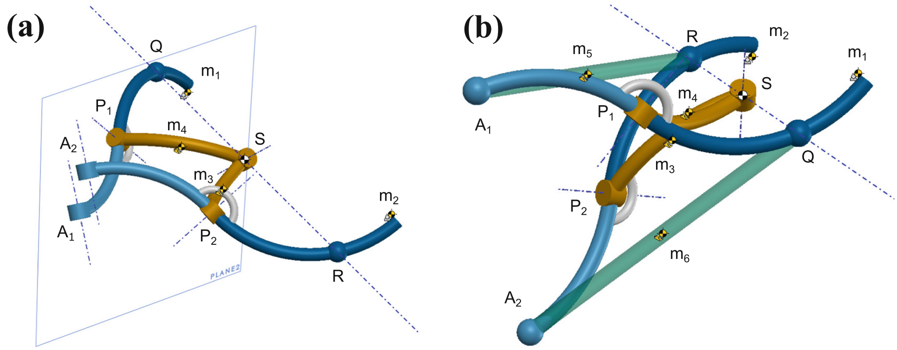

Fig. 2. a) Balanced spherical pantograph design with the similarity plane through S normal to the line through Q, S and R; b) Additional links with spherical joints are needed to constrain the pantograph properly and have the common CoM in base pivot S for all motions.

The force balance of the spherical pantograph can be evaluated by the projections of the linkage onto the orthogonal planes. Figure 3a shows the projection onto the plane through Q, R, and A, Fig. 3b shows the projection onto the similarity plane through S and normal to the line through Q, S and R, as shown in Fig. 2a, and Fig. 3c shows the projection onto the third orthogonal plane. The projection in Fig. 3a can be referred to as the in-plane projection, which shows a planar balanced pantograph as in Fig. 1. The projection in Fig. 3b can be referred to as the similarity projection, showing a mirrored geometry with respect to the vertical line through S and resulting in similar opposite movements of m_1 and m_3 with respect to m_2 and m_4 which is a known force balanced geometry. The projection shown in Fig. 3c consists of parallelograms, which therefore is also a balanced planar pantograph but with a different geometry as compared to Fig. 1. For each projection the mass parameters for force balance can be calculated with the equations of the planar balanced pantograph. Due to the limited space of the paper, this has not been elaborated here.

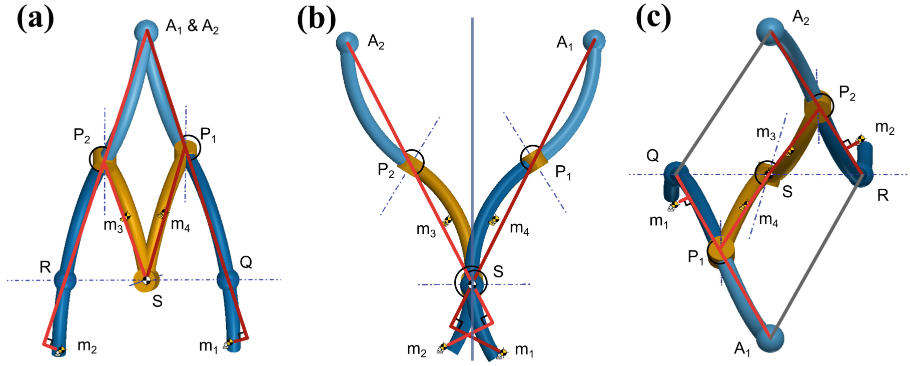

Fig. 3. Projections of the balanced spherical pantograph onto the 3 orthogonal planes: a) in-plane projection; b) Similarity projection; and c) Third projection.

The design in Fig. 3a is not properly constrained to move as a spherical pantograph since points A_1 and A_2 can move independently. These two points have to move within the similarity plane and also their rotational axes must remain parallel as illustrated with dashed lines in Fig. 2a. This can be accomplished by introducing two new equal links which connect points A_1 with R and A_2 with Q with ball joints as shown in Fig. 2b. The mass of these two links can be included for force balance by modeling their link mass with two equivalent masses located in the joints and combining the four equivalent masses with the mass of their respective connecting link. For balance of the spherical pantograph it is necessary that the links SP_1 and SP_2 have an equal length and that the links P_1A_1 and P_2A_2 have an equal length, while the length for each pair may be different. Also links QA_2 and RA_1 need to have an equal length for balance. The links can be curved with different radii when the link masses are scaled inversely with the radii.

3 Variations of the Balanced Spherical Pantograph

Variations of the inherently balanced spherical pantograph are presented here, with an alternative configuration of the spherical pantograph in Sect. 3.1. Section 3.2 and Sect. 3.3 show spherical versions of two planar balanced pantograph variations from [9], namely a double pantograph and a double S-shaped mechanism within a connecting 4R four-bar linkage.

3.1 Alternative Configuration of the Spherical Pantograph

Figure 4 shows an interesting configuration of the spherical pantograph when links SP_2 and SP_1 of the spherical pantograph are made collinear, thereby forming a H shaped geometry. Balance is achieved due to QP_1 and RP_2 being parallel, causing points A_1 and A_2 as well as m_1 and $m_2$to move in opposite directions.

This required constraint for points A and m can be enforced by attaching rigid links (visible as grey lines) between A_1 and R as well as between A_2 and Q, connected with ball joints. These links form two opposing parallelograms which balance each other. The out of plane geometry is shown in Fig. 4b. The link lengths can be varied, however the geometry must remain symmetrical about S for force balance.

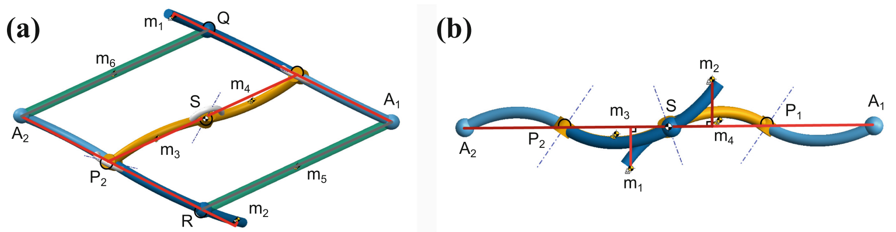

Fig. 4. a) Variation of the balanced spherical pantograph when links SP_2 and SP_1 are collinear; b) Side view.

3.2 Balanced Double Spherical Pantograph

Figure 5a shows the design of a balanced double spherical pantograph, which can be considered as a combination of two mirrored spherical parallelograms using solely revolute joints. The parallelograms are positioned such that the curvature of the links is mirrored in S. Link SP_1 is rigidly connected to link SP_4 and link SP_2 is rigidly connected to link SP_3, which makes both sides move synchronously with the joints of each side moving along the surface of a sphere. Each parallelogram can have links with different curvature and the conditions on the link lengths for each parallelogram are equal to the spherical pantograph in Fig. 2, with the links connecting in S having an equal length and the links distant from S having an equal length, while the length for each pair may be different, however both parallelograms must be proportionate for balance. Also here the projections of the mechanism onto the three orthogonal planes in Fig. 5b, 5c and 5d show planar balanced geometries from which the mass parameters can be derived as known.

An alternative version of this mechanism is obtained when the link curvatures are altered four times to create a wave like shape as shown in Fig. 6. This mechanism however requires ball joints in A_1 and A_2 to be movable and the links connecting in S being longer than the links distant from S.

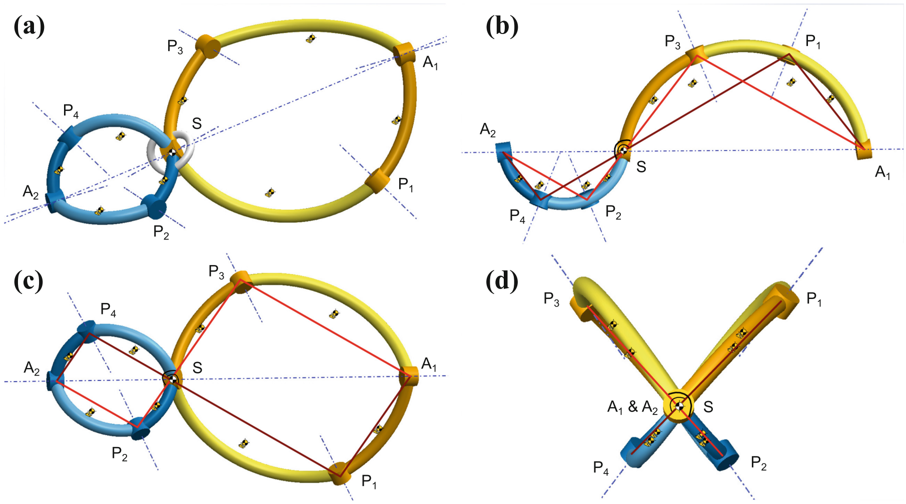

Fig. 5. a) Balanced double spherical pantograph with one side twice as large as the other; b) Third orthogonal projection; c) In-plane projection; d) Projection onto the similarity plane.

As for the planar pantograph it is also possible for force balance to shift links to another location as illustrated in Fig. 7a such that they remain parallel. As compared to Fig. 5a here links A_1P_1 and A_2P_4 have been shifted along their connecting links to $A_1^*P_1^*$ and $A_2^*P_4^*$, respectively. This results in the shorter link $P_1^*P_4^*$ through S.

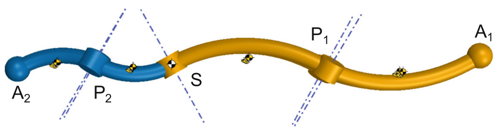

Fig. 6. Balanced double spherical pantograph with four times altering curvature and ball joints in A_1 and A_2.

3.3 Double S-Shaped Mechanism with Connecting 4R Four-Bar Linkage

As a complex example of a balanced spherical mechanism the double S-shaped mechanism with a surrounding 4R four-bar linkage is presented in Fig. 7b. This mechanism is derived from a planar solution of the grand 4R four-bar based linkage architecture which is an advanced combination of multiple balanced pantographs [9]. The linkage $A_1P_1SP_2A_2$, with solely revolute joints, can be seen as

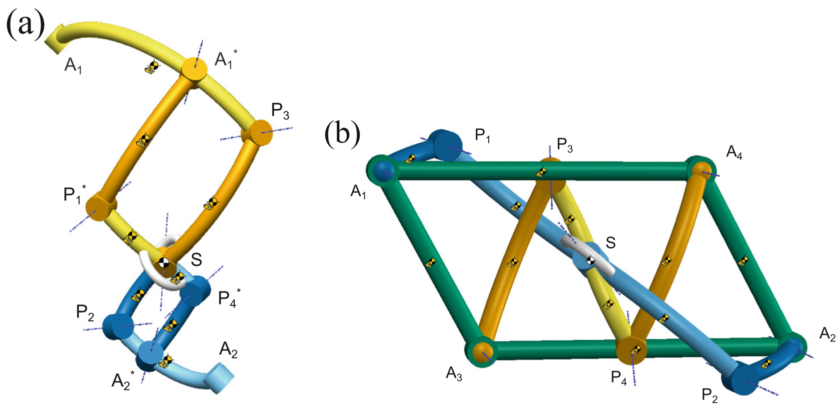

Fig. 7. a) Variation of the balanced spherical double pantograph with shifted links; b) Double S-shaped mechanism with external 4R four-bar linkage and common CoM in S.

a S-shaped geometry and linkage $A_3P_3SP_4A_4$, with solely revolute joints, can be seen as a second S-shaped geometry. Links P_1SP_2 and P_3SP_4 have a shared revolute joint in S with S being the center of each link. The surrounding 4R four-bar is a planar parallelogram linkage to which the S-geometries are connected with ball joints in the revolute joints A_i. The mechanism is balanced with the common CoM in joint S for any pose. Figure 8 shows two projections of the S-geometries of the mechanism.

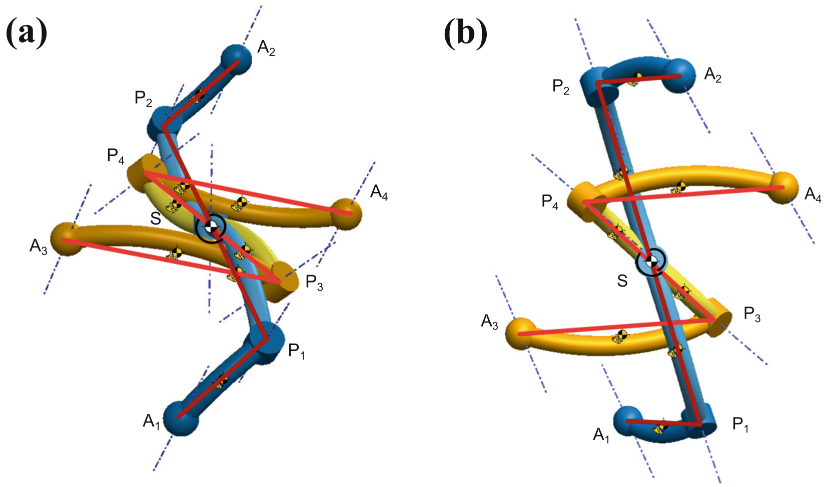

Fig. 8. Projections on two orthogonal planes of the two S-geometries.

4 Conclusion

A spherical shaking force balanced pantograph was presented, derived from the planar balanced pantograph. Based on this spherical pantograph, five variations of force balanced designs were shown, namely an alternative configuration of the spherical pantograph, three variations of a double spherical pantograph and a double S-shaped mechanism with external 4R four-bar linkage. The projections of the spherical mechanisms onto the orthogonal planes were shown to result into known planar geometries for force balance, mostly planar pantographs, which makes is possible to calculate the mass parameters for balance with the equations known for the planar case. The feasible variations of link lengths and link curvature radii for which force balance is possible have been discussed. With the presented spherical pantograph it is possible to design a wide variety of new inherently balanced spherical and non-spherical mechanisms, following the approach in this paper and in [1,9].

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