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Modelling of ultrasonic beam propagation from an array through transversely isotropic fibre reinforced composites using Multi Gaussian beams

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Abstract
Ultrasonic arrays are used for non-destructive evaluation of structures for aerospace and other applications. With the increase in the usage of fibre-reinforced composites in aerospace structures, this evaluation becomes complex due to the effects of attenuation and reflection from the layer boundaries in composite laminates. Hence there is considerable interest in developing beam models for accurate evaluation in such anisotropic media.
In anisotropic media, the phase velocity varies with direction of propagation of the ultrasonic beam. Hence, the slowness surface and its properties play an important role in the beam models which are based on the paraxial approximation. The beam from a single element transducer is well collimated. However the beam from individual array elements is not well collimated and may affect the beam propagation through the composite structure. In this paper, Multi modular Gaussian beam (MMGB) model based on the paraxial approximation is applied to study the propagation of beam from an ultrasonic array in transversely isotropic fibre-reinforced composites. The effect of the slowness surface properties on the beam diffraction and skew through the composite structure are studied along with the influence of array parameters on the beam propagation through the structure.
This work has demonstrated that the overall beam profile for quasi longitudinal beam from an ultrasonic array propagating in transversely isotropic fibre reinforced composites can be modelled as multi Gaussian beams. Simulation results are presented which illustrate the effects of? slowness properties on beam propagation in unidirectional CFRP in the symmetric planes.

Keywords: Laser ultrasound, time of flight (TOF), welding, aerospace, carbon fiber composite

1. Introduction

For many years phased array ultrasonics have been used in medical diagnostics. They are now being used more frequently in the field of NDT in the aerospace industry?. A phased array consists of multiple elements to which a time delay can be applied to steer and focus the sound beam generated from them. Beam steering and focusing in a single medium has been studied extensively by Wooh and his colleagues[1]. Beam models for phased array transducers are based on models developed for single element transducers. Models such as the Rayleigh Sommerfeld model have been used to predict the wave fields generated by the arrays. By far the most efficient and versatile model for single element transducers is the Multi Gaussian beam model [2]. This model uses the superposition of Gaussian beams to predict the wave field from a single element transducer. As the use of phased array increased the multi Gaussian beam model was expanded to predict the wave field from a phased array [3].

The necessity of nondestructively testing carbon fibre composites lead to the development of beam models to understand wave propagation in anisotropic media. The multi Gaussian beam model for a single element transducer was expanded to include the effects of the slowness surface and was used to simulate the wave field in a unidirectional gr/ep composite [4].The properties of the slowness surface play an important role in the beam models which are based on the paraxial approximation[5]. The curvatures of these slowness surfaces determine the rate at which the beam converges or diverges due to diffraction

In this paper we propose using the expanded multi Gaussian beam for phased array transducers combined with the inclusion of the slowness surface parameters to simulate the propagation of ultrasonic beam from an array into an anisotropic solid. Simulation results are presented for unidirectional CFRP when the quasi longitudinal wave propagates in the symmetry plane of the material. The beam is then steered and focused using the delay laws and the simulation results are presented.

2. Modular multi Gaussian beam model for Linear Phased Arrays

2.1 Gaussian Beam in Anisotropic media

In a solid, the velocity amplitude and the phase of Gaussian beam of type α can be described by the solution of the paraxial equation where y_3 axis is taken along the group velocity direction and the y_1-y_3 plane is taken as the plane of incidence. c_α and u_α are the magnitudes of the phase velocity and group velocity of the wave of type α.

$$v = \frac{V(0)}{\sqrt{\det[A^P + B^P M(0)]}} \hat{d} \exp\left[i\omega \left(\frac{y_3}{u_\alpha} + \frac{1}{2} Y^T M_{y_3}{}^{\alpha} Y \right) \right] \tag{1}$$

Where

$$M_{y_3}{}^{\alpha} = \left[C^P + D^P M(0) \right] \left[A^P + B^P M(0) \right]^{-1} \tag{2}$$

The propagation matrices in the solid are given by

$$A^P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C^P = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$D^P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$B^P = \frac{c_\alpha}{u_\alpha} \begin{bmatrix} \left(c_\alpha - 2C^\alpha\right) y_3 & -D^\alpha y_3 \\ -D^\alpha y_3 & \left(c_\alpha - 2E^\alpha\right) y_3 \end{bmatrix} \tag{3}$$

C^α, D^α and E^α are the slowness surface curvatures which are measured in the slowness coordinates. These are obtained by expanding the y_3 component of the slowness vector s_α using a Taylor series expansion. For an isotropic material $C^\alpha = D^\alpha = E^\alpha = 0$

2.2 Multi Gaussian beam model

By the superposition of 10-15 Gaussian beams, the wave field of the circular piston transducer with radius a can be modelled [6] as shown below

$$v = \sum_{n=1}^{10} \frac{A_n V(0)}{\sqrt{\det[A^G + B^G M(0)_n]}} \hat{d} \exp\left[i\omega \left(\frac{y_3}{u_\alpha} + \frac{1}{2} Y^T M^{\alpha}(y_3) Y \right) \right] \tag{4}$$

Where the initial phase term $M(0)_n = \frac{2iB_n}{\omega a^2} I$. The A_n and B_n terms are the Wen and Breazzle coefficients.

The global matrices are assembled as matrix products of all the propagation and transmission ABCD matrices through the different interfaces

$$\begin{bmatrix} A^G & B^G \\ C^G & D^G \end{bmatrix} = \begin{bmatrix} A^p_{M+1} & B^p_{M+1} \\ C^p_{M+1} & D^p_{M+1} \end{bmatrix} \begin{bmatrix} A^t_M & B^t_M \\ C^t_M & D^t_M \end{bmatrix} \begin{bmatrix} A^p_M & B^p_M \\ C^p_M & D^p_M \end{bmatrix} \cdots \begin{bmatrix} A^p_1 & B^p_1 \\ C^p_1 & D^p_1 \end{bmatrix} \quad (5)$$

2.3 *Multi Gaussian beam model for linear phased arrays*

The Equation (4) is for circular piston transducer. In the case of a rectangular transducer with dimensions a_1 and a_2 radiating into an isotropic solid the multi Gaussian beam model for wave type α is given below [3]

$$v_n^\alpha = \hat{d} \exp\left(i\omega \frac{y_3}{u_\alpha} \right) \cdot \sum_{m=1}^{10} \sum_{n=1}^{10} V(0) \frac{A_n}{\sqrt{1 + c_\alpha y_3 [M(0)_{mn}]_{11}}} \frac{A_m}{\sqrt{1 + c_\alpha y_3 [M(0)_{mn}]_{22}}} \exp\left[\frac{1}{2} Y^T M_{mn}{}^{\alpha}(y_3) Y \right] \quad (6)$$

In terms of the global A,B,C,D matrices Equation (3) can be written as

$$v_n^\alpha = \hat{d} \exp\left(i\omega \frac{y_3}{u_\alpha} \right) \sum_{m=1}^{10} \sum_{n=1}^{10} V(0) \frac{A_n A_m}{\det[A^G + B^G M(0)_{mn}]} \exp\left[\frac{1}{2} Y^T M_{mn}{}^{\alpha}(y_3) Y \right] \quad (7)$$

Where

$$\left[M(0)_{mn}\right]_{11} = \frac{2iB_m}{\omega a_1{}^2}$$

$$\left[M(0)_{mn}\right]_{22} = \frac{2iB_n}{\omega a_2{}^2}$$

$$M_{mn}{}^{\alpha}(y_3) = \left[C^G + D^G \left[M(0)\right]_{mn} \right] \left[A^G + B^G \left[M(0)\right]_{mn} \right]^{-1} \quad (8)$$

The normalized velocity field from a rectangular transducer is given by (7). The normalized velocity field from an array of transducers can be then given as below

$$v^{\alpha} = \sum_{1}^{n} v_n^{\alpha} \exp\left(i\omega t_n\right) \tag{9}$$

Where t_n is the time delay applied to the nth array element to focus and steer the beam, v_n^{α} is the normalized velocity field of a single element.

For both focusing and steering the beam the time delay to be applied t_n is given below

$$t_n = \frac{F}{c_{\alpha}} \left\{ \left[1 + \left(\frac{\bar{N} d}{F} \right)^2 + \frac{2\bar{N} d}{F} \sin\theta_s \right]^{1/2} - \left[1 + \left(\frac{\left(n - \bar{N} \right) d}{F} \right)^2 + \frac{2\left(n - \bar{N} \right) d}{F} \sin\theta_s \right]^{1/2} \right\}$$

d is the centre to centre spacing between adjacent elements, F is the focal length, N is the number of elements, $n = 0,1\ldots.N\text{-}1$, $\bar{N} = (N-1)/2$ and θ_s is the steering angle

2.4 Multi Gaussian beam model for linear phased arrays for anisotropic materials

By using the curvature values in of the slowness surface of an anisotropic material to define the propagation matrices for a unidirectional CFRP in equation (3) and combining this with Equation (7) and Equation (9) we can model the wave field of an ultrasonic array through unidirectional CFRP

3. Calculation of radiation beam fields

In this section we present some results simulated by the Multi Gaussian beam model (which one did you use? From the previous section?). The unidirectional CFRP composite considered is transversely isotropic with its properties given as $C_{11} = C_{11} = 15, C_{12} = 7.7, C_{23} = C_{13} = 3.4, C_{33} = 87, C_{44} = C_{55} = 7.8, C_{66} = 3.65$ GPa and $\rho = 1.595$ gm/cm^3 .

The major parameters of the phased array ultrasonic transducer in this study are as follows: Centre frequency is 5 M Hz, element width is 0.8 mm, number of elements is 32 and element spacing is 0.2 mm as shown in Figure 1. The beam profile is computed up to 400 mm in the solid (in which direction?)

Figures 2 (a) and (b) show the calculated radiation beam fields from the phased array into unidirectional CFRP focused at 71.6 mm and unfocused respectively. The beam spread is higher in the unidirectional CFRP owing to the anisotropic nature and different bulk wave velocities as compared to what? I have not seen any baseline results to compare with here?.

Next the beam was steered at angles of 10° and 20° . Figures 3 (a) and (b) show the calculated radiation beam fields with a steering angle of 10° focused at 71.6 mm and unfocused respectively. Figures 3(c) and (d) show the calculated radiation beam fields with a steering angle of 20° focused at 71.6 mm and unfocused respectively. Owing to the anisotropic nature of the unidirectional CFRP, beam skew and beam divergence due to diffraction are observed when the beam is steered, hence the beam is steered (actively or as a result of the anisotropy) at higher angles than desired.

Another observation is the presence of grating lobes. The main lobeweakens and widens with an increase in the steering anglewhereas the grating lobe strengthens progressively.

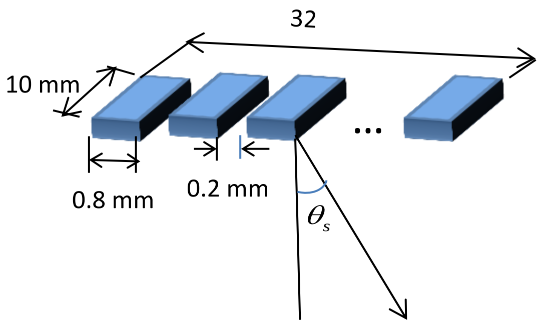

Fig. 1 Diagram of the linear phased array transducer

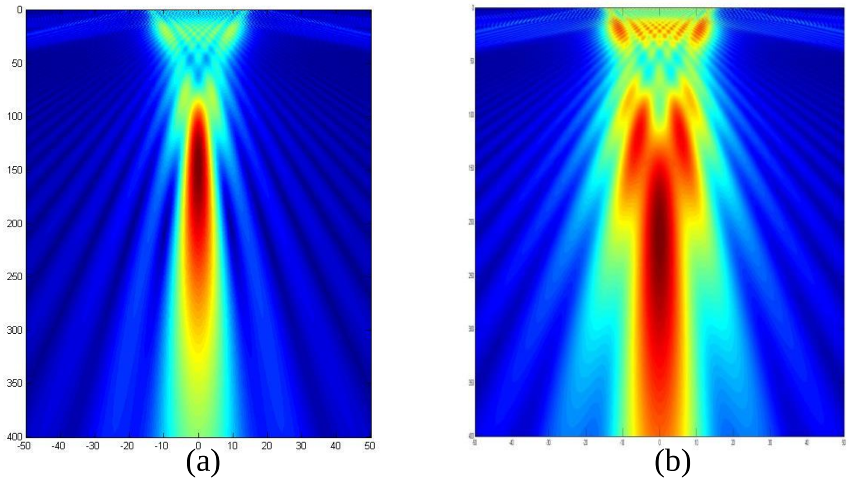

Fig. 2 Calculated radiation beam fields focused at 71.6 mm (a) and unfocused (b)

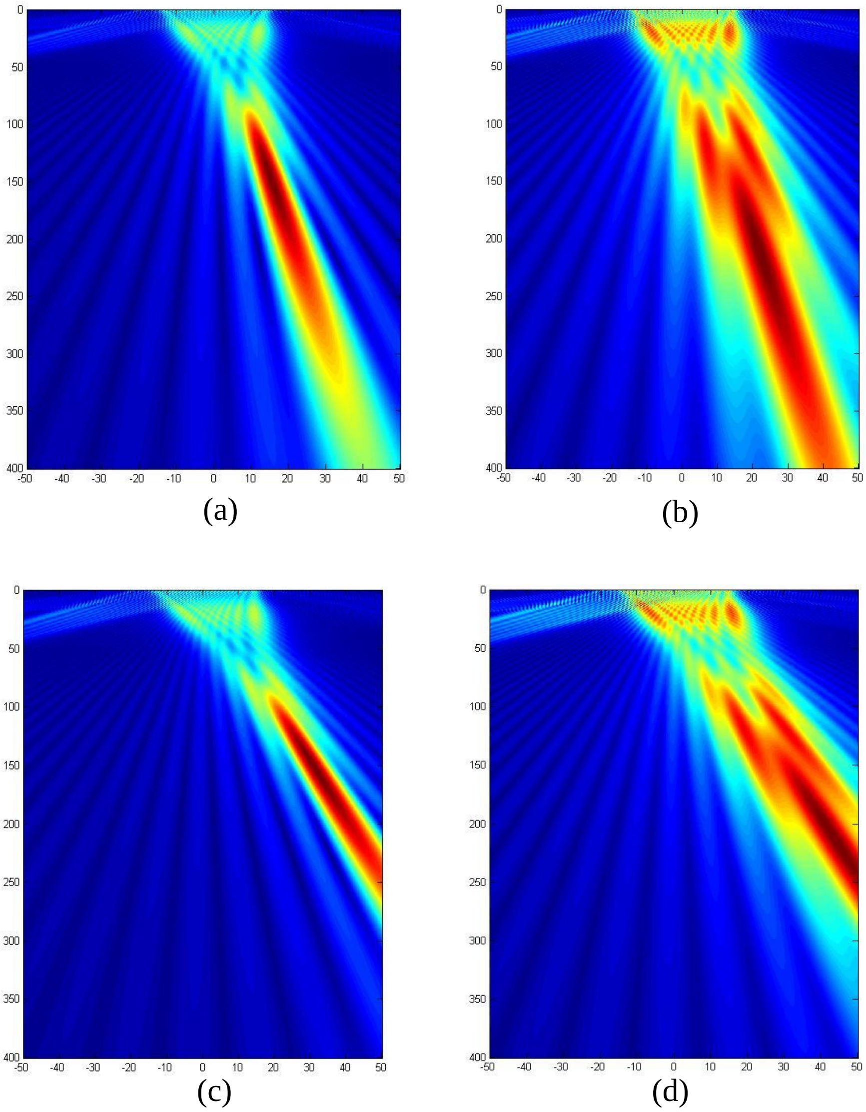

Fig. 3 Calculated radiation beam fields with focusing at 71.6 mm and unfocused steered at 10° ((a), (b)) and 20° ((c), (d))

4. Conclusions

In this paper, we have proposed using the expanded multi gaussian beam model using the values of the curvature of the slowness surfaces to calculate the radiated beam fields in anisotropic media. The simulation results presented for unidirectional CFRP show that the model is able to predict the wave field in anisotropic media. In the case of low steering angles the model calculates the radiated beam field highlighting the beam divergence and beam skew due to the anisotropic nature of the composite. Hence the model proposed in the paper can be used to predict the wave field in an anisotropic media for low steering angles The next step would be to verify the proposed model with Rayleigh Sommerfeld integral model.

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