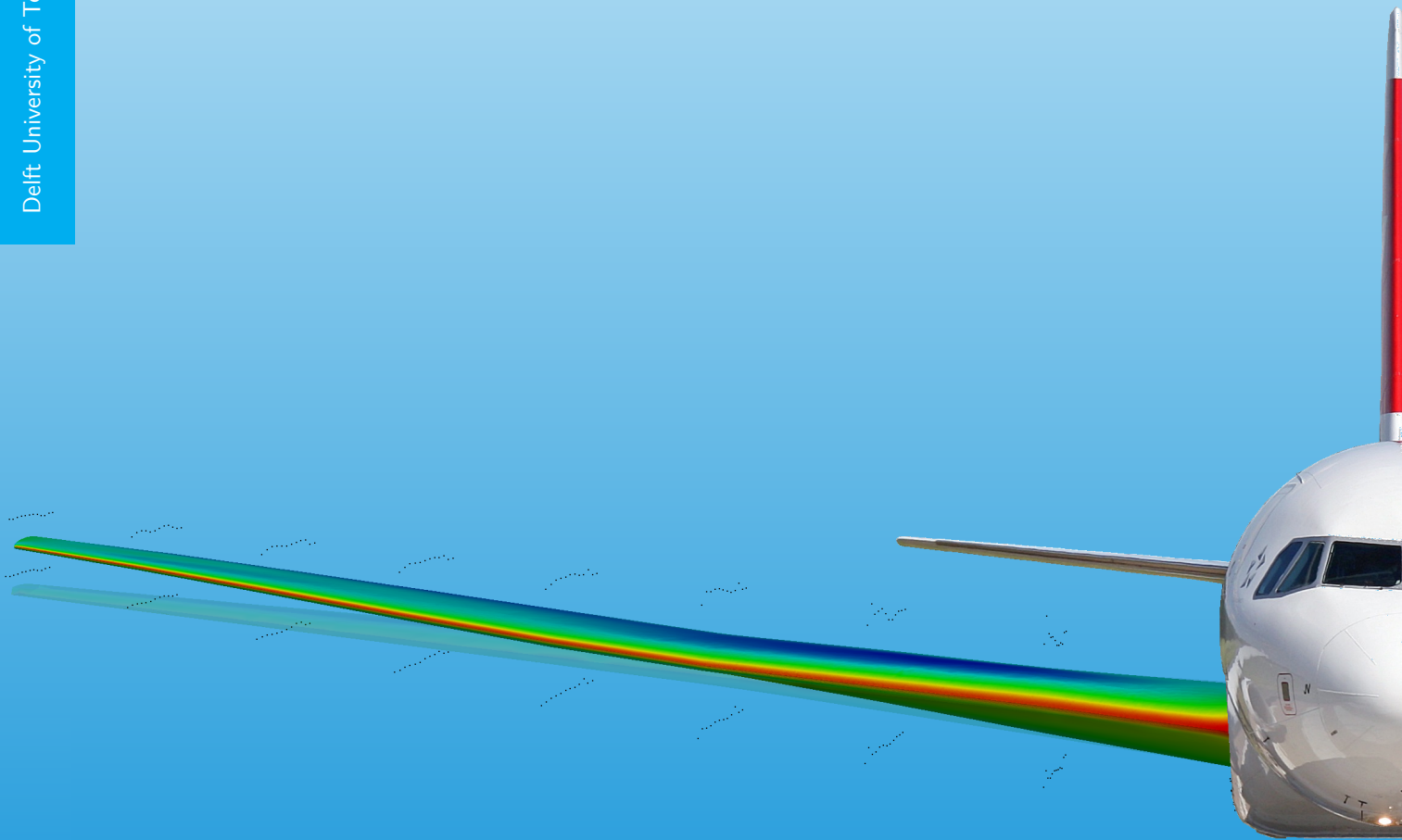


Wing aerostructural optimization using the Individual Discipline Feasible architecture

Jan E.K. Hoogervorst

Delft University of Technology



WING AEROSTRUCTURAL OPTIMIZATION USING THE INDIVIDUAL DISCIPLINE FEASIBLE ARCHITECTURE

by

Jan Eduard Kornelis Hoogervorst

in partial fulfillment of the requirements for the degree of

Master of Science
in Aerospace Engineering

at the Delft University of Technology.

Supervisor:	Dr. A. Elham
Thesis committee:	Dr. Ir. L. L. M. Veldhuis Dr. R. P. Dwight

Date:	10-02-2015
Thesis registration number:	074#16#MT#FPP
Student number:	1377256
Cover page image:	Edited. Credits: Björn, https://www.flickr.com/photos/24056116@N07/

An electronic version of this thesis is available at <http://repository.tudelft.nl/>.

SUMMARY

This thesis presents an effort to contribute to the minimization of fuel use of aircraft. The intention is to achieve this efficiently by using the Individual Discipline Feasible architecture for solving a gradient-based, aerostructural Multidisciplinary Design Optimization problem for a static aeroelastic wing. This wing is then optimized for minimal fuel consumption during the cruise phase.

The work in this paper is an effort continuing in the trend of using high-fidelity analyses for optimization of an aeroelastic wing. However this effort makes use of the Individual Discipline Feasible architecture, which decouples the aerodynamic and structural disciplines from each other. Using this approach the consistency of the system as a whole is maintained by the use of equality constraints on surrogate design variables. No coupled sensitivity information is required because of this decoupled system. This makes the system not only simpler, but also provides more freedom in software choice. Furthermore, the time to perform optimization is reduced with respect to the traditional Multidisciplinary Feasible architecture as the work of making the system consistent is removed from the computationally expensive individual disciplines and put it in the hands of the cheap optimization algorithm.

The aerodynamic and the structural disciplines hence independently calculate both the intermediate states of the system and the partial derivatives of these states with respect to the design vector. The performance module is dependant on the aerodynamic discipline and is therefore included in it. This module calculates the fuel weight.

SU2 is used within the aerodynamic discipline to deform the surface grid and the volume grid of the wing and its domain, calculate the flow properties and gain sensitivities of lift and drag with respect to surface perturbations of the wing. The software uses the 3D Free-Form Deformation parameterization technique to deform the surface grid. The code is originally meant to only deform the airfoil shapes of a wing, nevertheless it has been modified in order to also deform the wing according to the static aeroelastic deformation. The sensitivity analysis is performed by a continuous adjoint solver. It is shown however that the continuous adjoint method in SU2 does not capture the sensitivities of the trailing edge well due to assumptions of smoothness. This is why extra corrective factors are added to the false sensitivities. The results of the optimization verify the working of these corrective factors, except for the corrected sensitivities with respect to the planform design variables.

The Euler model is used for the flow analysis, due to its speed advantages. However, because viscosity is neglected in the Euler model, the viscous drag component and its sensitivity derivatives are estimated by a separate module.

For the structural discipline the FEMWET software is used, providing the static aeroelastic deformation and the aeroelastic axis of the wing. FEMWET uses equivalent panel thicknesses representing the wing box to calculate its deformation and failure modes. Its code is slightly modified for the purpose of this work, namely the Free-Form Deformation parameterization is included for the deformation of the airfoils.

Lastly, the weight of the wing structure is estimated by summing up the weight of the equivalent panels and empirical data.

The optimization design variables are selected to be the angle of attack, the exterior shape of the wing, being the airfoil and planform shapes, and the thicknesses of the equivalent panels representing the internal wing box. The problem is constraint by compression, tension, shear, buckling and fatigue failure modes. Moreover it is constraint by a minimum aileron effectiveness and a maximum wing loading. The aerodynamic analysis is performed under cruise conditions while the wing structure is also analyzed under the load cases of pull up, push over, gust load and a roll manoeuvre. The reference aircraft used throughout this report is a modern high-speed transport aircraft, the Airbus A320. The optimization algorithm chosen is The Sparse Nonlinear Optimizer, based on the Sequential Quadratic Programming optimization algorithm.

In this report the results of three aerostructural and one pure aerodynamic optimization problem are presented.

The aerodynamic optimization successfully showed the working of the aerodynamic discipline when controlled by the optimization algorithm and the ability to reduce induced and wave drag simultaneously.

The first aerostructural optimization was performed with a fixed planform. This optimization has resulted in a 7% reduction in the aircraft fuel weight. It is shown by this optimization that both the optimization tool and the corrective factors for the aerodynamic sensitivities work.

The second optimization included all planform design variables. It gave disappointing results. The fuel weight was reduced with 9%, however the equality constraint were violated too severe, making the solution invalid. It appeared that the correction factors did not work for the the sensitivities with respect to the planform design variables. To confirm this statement, the last optimization is performed with the sensitivities with respect to the planform design variables calculated by Finite Difference.

This last optimization resulted in a success. The optimizer managed to reduce the aircraft fuel weight with 11%, with an acceptable amount of constraint violation.

PREFACE

The present master thesis was accomplished within the faculty of Aerospace Engineering at the TU-Delft. It is an effort to contribute to the minimization of fuel use of aircraft by efficiently making use of high-fidelity analysis in the early stages of multidisciplinary wing design.

I would like to thank my supervisor Dr. Ali Elham for the support and input throughout this project. Furthermore I would like to thank the peers of room 6.01 for their company and interesting discussions on the topic of aircraft design. Last but not least I would like to thank my family, friends and especially my girlfriend for their support and interest in my work.

*Jan Hoogervorst
Delft, February 2016*

CONTENTS

Summary	iii
Preface	v
List of Figures	ix
List of Tables	xi
Nomenclature	xiii
1 Introduction and problem description	1
1.1 Wing aeroelasticity	2
1.2 Prior aerostructural wing optimization efforts	3
1.3 Research objective	4
1.4 Thesis outline	5
2 IDF architecture for wing multidisciplinary design optimization	7
2.1 Optimization architecture.	7
2.2 Aerostructural optimization problem formulation	9
2.2.1 XDSM	10
2.2.2 Objective function	12
2.2.3 Design variables	12
2.2.4 Constraints	13
2.3 Optimization algorithm.	13
2.3.1 Sequential Quadratic Programming	14
2.3.2 Optimality Conditions	15
2.4 Decoupled sensitivity analysis	15
2.4.1 Objective function sensitivity	16
2.4.2 Constraint sensitivity	16
3 Aerodynamic and performance analysis	19
3.1 Stanford University Unstructured	19
3.2 Free-Form Deformation parameterization	20
3.2.1 Implementation	20
3.3 Surface and volume grid	24
3.3.1 Grid deformation	24
3.4 Computational Fluid Dynamics.	25
3.4.1 Viscous drag estimation	25
3.4.2 Validation	26
3.5 Sensitivity analysis	26
3.5.1 Adjoint method	26
3.5.2 Sensitivity projection	27
3.5.3 Sensitivity verification	27
3.5.4 Sensitivity error origin	30
3.6 Fuel weight analysis.	31
3.7 Analysis structure	31
4 Structural analysis	35
4.1 EMWET/FEMWET	35
4.2 Weight estimation.	35
4.3 Static aeroelastic deformation analysis	36
4.3.1 Gird	36
4.3.2 Aerodynamic analysis	36

4.3.3	Finite Element Analysis	37
4.3.4	Aerostructural coupling	37
4.4	Failure modes and aileron effectiveness.	38
4.5	Validation and verification	38
4.5.1	Sensitivity verification	39
5	Test case application	41
5.1	Load cases	41
5.2	Initial surface and volume grid	41
5.2.1	Geometry	41
5.2.2	Initial surface and volume grid.	42
6	Results	45
6.1	Aerodynamic optimization	45
6.2	Fixed planform aerostructural optimization	46
6.3	Complete aerostructural optimization	53
6.3.1	Adjoint planform sensitivities	53
6.3.2	Finite Difference planform sensitivities	58
7	Conclusions and recommendations	65
7.1	Conclusions.	65
7.2	Recommendations	66
	Bibliography	67
	Appendix	71

LIST OF FIGURES

1.1	Wingbending of a Boeing 787 Dreamliner	2
1.2	Wingbending of a Boeing B-52 Stratofortress	2
1.3	Spanwise lift distribution difference between a Boeing B-47 Stratojet flexible and rigid wing	3
1.4	Chordwise pressure distribution difference between a Boeing B-52 Stratofortress flexible and rigid wing	3
2.1	XDSM of the MDF architecture	8
2.2	XDSM of the IDF architecture	9
2.3	XDSM of the present fuel weight optimization	11
3.1	Example of Free-Form Deformation: From a sphere to a blended-wing-body	20
3.2	Example of the usage of two FFD boxes to realize a translation in the z-direction in SU2	21
3.3	Example of an airfoil shape deformation using FFD control points	22
3.4	Example of a wing section transformation using FFD control points	22
3.5	Example of a wing section rotation using FFD control points	22
3.6	Definition of the planform design variables	23
3.7	Example of a wing section chord length change using FFD control points	24
3.8	Example of a volume grid deformation as a result of a surface deformation	25
3.9	FD sensitivity analysis for determination of Δx	28
3.10	Comparison between adjoint and FD sensitivities of lift with respect to the upper side FFD design variables at mid-span	28
3.11	Comparison between adjoint and FD sensitivities of drag with respect to the upper side FFD design variables at mid-span	29
3.12	An example of sectional deformation without TE	30
3.13	Aerodynamic discipline flowchart	32
4.1	Illustration of the wing box structure, represented by 4 equivalent panels	36
4.2	Example of the grid for the VLM and FEM analyses within FEMWET	37
4.3	Verification of implementation of the FFD method in FEMWET (below) using implementation of SU2 (above)	39
5.1	Reference aircraft Airbus A320 wing planform view in meters	42
5.2	Grid domain in meters	42
5.3	Planform view of the converged surface grid of the upper wing surface	43
6.1	Lift coefficient distribution comparison between initial and optimized wing of the aerodynamic optimization	46
6.2	Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing of the aerodynamic optimization	47
6.3	Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing of the aerodynamic optimization	47
6.4	Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing of the aerodynamic optimization	48
6.5	Lift coefficient distribution comparison between initial and optimized wing with fixed-planform	49
6.6	Comparison between the wing jig shape (blue) and the shape under 2.5g pull up load at sea level of the initial (grey) and optimized (red) wing with fixed-planform	50
6.7	Twist deformation distribution comparison between initial and optimized wing with fixed-planform of the jig shape (blue) and the 1g shape (red) wing with fixed-planform	50

6.8	Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing with fixed-planform	51
6.9	Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing with fixed-planform	51
6.10	Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing with fixed-planform	52
6.11	Planform view comparison between initial and optimized wing	54
6.12	Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing	55
6.13	Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing	56
6.14	Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing	57
6.15	Planform view comparison between initial and optimized wing	58
6.16	Lift coefficient distribution comparison between initial and optimized wing	59
6.17	Comparison between the wing jig shape (blue) and the shape under 2.5g pull up load of the initial (grey) and optimized (red) wing	59
6.18	Twist deformation distribution comparison between initial and optimized wing of the jig shape (blue) and the 1g shape (red) wing	60
6.19	Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing	61
6.20	Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing	62
6.21	Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing	63

LIST OF TABLES

2.1	Design vector	12
2.2	(In)equality constraint overview	14
3.1	Comparison between FD and adjoint sensitivities with respect to the P , α , U^* and EA^* design variables at mid-span	29
3.2	Comparison between FD and adjoint sensitivities with respect to the U^* design variables at mid-span excluding trailing edge	31
4.1	Comparison between FD and AD method of FEMWET sensitivity derivatives	39
5.1	Load cases for the Airbus A320 aircraft	41
5.2	Reference aircraft Airbus A320 wing geometry parameters	42
5.3	Grid maximum element sizes on surfaces for converged grid	43
6.1	Top-level aerodynamic optimization results	45
6.2	Top-level fixed-planform optimization results	49
6.3	Aerodynamic and structural fixed-planform optimization results	49
6.4	Top-level complete optimization results	53
6.5	Aerodynamic and structural complete optimization results	54
6.6	Top-level complete optimization results	58
6.7	Aerodynamic and structural complete optimization results	58

NOMENCLATURE

LATIN SYMBOLS

Variable	Description	Unit
$\mathcal{J}$	Cost-function	[–]
A	Aerodynamic residual	[–]
AIC	Aerodynamic Influence Coefficient matrix	[–]
b	Wing span	[m]
B	Bernstein polynomials	[–]
B	Byte	[–]
c	Airfoil chord length	[m]
c	Inequality constraint	[–]
c_{eq}	Equality constraint	[–]
c_j	Cruise specific fuel consumption	[$\frac{g}{N \cdot s}$]
C_D	Drag coefficient	[–]
C_{D_f}	Viscous drag coefficient	[–]
C_f	Flat-plate friction coefficient	[–]
$C_{f_{lam}}$	Laminar flat-plate friction coefficient	[–]
$C_{f_{turb}}$	Turbulent flat-plate friction coefficient	[–]
C_L	Lift coefficient	[–]
$C_{L_{cruise}}$	Cruise lift coefficient	[–]
C_p	Pressure coefficient	[–]
D	Drag force	[N]
$\vec{e}$	Unit vector of grid edge	[–]
EA	Vector of elastic axis positions	[m]
EA_{jig}	Vector of elastic axis positions of the jig-wing	[m]
f	Objective function	[–]
F	Failure modes	[–]
F	Force matrix	[N]
FFD	Vector of Free-From Deformation control point movements	[m]
g	Gravitational acceleration of the Earth	[$\frac{m}{s^2}$]
H	Height	[m]
H	Hessian matrix	[–]
Hz	Hertz	[–]
K	Stiffness matrix	[$\frac{N}{m}$]
K_{form}	Form-factor correction	[–]
L	Lagrangian	[–]
L	Lift force	[N]
L	Rolling moment	[Nm]
M	Mach number	[–]
M	Merit function	[–]
M_{ff}	Fuel-fraction	[–]
$\vec{n}$	Vector of the unit normal to a surface	[–]
n	Load factor	[g]
P	Vector of planform parameters	[–]
q	Dynamic pressure	[Pa]
R	Cruise Range	[m]
R	Residual	[–]
Re	Reynolds number	[–]

s	Slack variable	$[-]$
S	Structural residual	$[-]$
S	Surface	$[m^2]$
S_{ref}	Reference wing area	$[m^2]$
S_{wet}	Wetted wing area	$[m^2]$
SF	Safety Factor	$[-]$
t	Airfoil thicknesses	$[m]$
T	Vector of equivalent panel thicknesses	$[m]$
$\tilde{u}$	Vector of the displacement of a grid node	$[m]$
U	Vector of aeroelastic deformation	$[-]$
V	Aerodynamic right hand side vector	$[\frac{m}{s}]$
V_{inf}	Undisturbed airspeed	$[\frac{m}{s}]$
W	Weight residual	$[-]$
W_{fuel}	Fuel weight	$[kg]$
W_{MTO}	Maximum take-off weight	$[kg]$
W_{rest}	Maximum take-off weight minus wing and fuel weight	$[kg]$
W_{wbox}	Equivalent panel wing box structural weight	$[kg]$
W_{wing}	Wing structural weight	$[kg]$
WL	Wing loading	$[\frac{kg}{m^2}]$
WL_{init}	Initial wing loading	$[\frac{kg}{m^2}]$
x	Displacement	$[m]$
X	Vector of design variables	$[-]$
Y	Vector of state variables	$[-]$

GREEK SYMBOLS

Variable	Description	Unit
α	Angle of attack	$[deg]$
Γ	Wing sweep angle	$[deg]$
Γ	Vortex strength	$[s^{-1}]$
δ	Aileron deflection angle	$[deg]$
ϵ	Airfoil twist angle	$[deg]$
η	Spanwise position	$[-]$
θ	Airfoil twist due to aeroelastic deformation	$[deg]$
λ	Equality constraint Lagrange multiplier	$[-]$
λ	Wing taper	$[-]$
μ	Inequality constraint Lagrange multiplier	$[-]$
ρ	Air density	$[\frac{kg}{m^3}]$
ρ	Penalty parameter	$[-]$
σ	Stress	$[Pa]$
τ	Shear stress	$[Pa]$
ϕ	Adjoint vector	$[-]$
ϕ	Rotation around x-axis due to aeroelastic deformation	$[deg]$
ψ	Rotation around z-axis due to aeroelastic deformation	$[deg]$

1

INTRODUCTION AND PROBLEM DESCRIPTION

At present, on the aviation market a need exists for lighter and more efficient aircraft than the ones dominating the airspace today. These new generation aircraft consume less fuel per passenger for a given mission, which does not only reduce costs of an airline operating the aircraft, but also reduces air pollutants like carbon dioxide (CO₂), nitrogen oxides (NO_x) and soot. It is shown by Ruijgrok and van Paassen [1] that CO₂ increases linearly and that NO_x and soot even increase exponentially with increasing fuel flow. While the reasoning behind reduction in airline operating costs is obvious, that has not always been the case for the reduction of air pollutants. It is only in the modern age that people are aware of the scale of the potential damage of these pollutants. An example of this is the growing concern that NO_x increases the greenhouse effect when dumped in the upper region of the troposphere, see Ruijgrok and van Paassen [1] for more details.

One example of the demand for light and efficient aircraft is the orders for the Airbus A350 XWB. The amount of firm orders is 777, by 41 worldwide costumers, with 15 aircraft already deliveredⁱ. This is while Airbus is struggling to break even on the total programme cost basis of the much larger and less efficient Airbus A380ⁱⁱ. The Airbus A350 presents a 25% decrease in CO₂ exhaust with respect to the previous generation of long-range aircraftⁱ. Apart from the new engines used for the Airbus A350, this reduction is mainly gained by reducing the weight of the aircraft by using composite materials and by improving the aerodynamics by designing the wing from scratch.ⁱ.

When weight of an aircraft is reduced the necessary lift, mainly generated by the wings, to keep the aircraft level is also reduced. This also reduces the drag generated by the aircraft wings, as part of the total drag is dependent on the lift. Reduction of drag translates directly in a reduction of thrust required, reducing fuel use. If on top of this the wing is aerodynamically optimized the fuel use is even further reduced. Beside the above named reduction in operating costs and air pollutants, this reduction in fuel use can result in several advantages with respect to the performance of the aircraft, see McLean [2]:

- Increased range
- Increased payload capacity
- Decreased structural weight, which can decrease the fuel use even further
- Decreased of take-off field length
- Decreased take-off noise

The present thesis is an effort to contribute to this reduction of fuel use by optimizing both the internal wing box structure and external wing shape of a modern high-speed transport aircraft for minimal necessary fuel

ⁱA350 XWB by Airbus. <http://www.a350xwb.com/>, Last accessed: Jan 2016

ⁱⁱTelegraph, "Is Airbus's A380 a 'superjumbo' with a future or an aerospace white elephant?" by Alan Tovey. <http://www.telegraph.co.uk/>, Last accessed: Okt 2015

weight while maintaining its range specification. The novelty of this work is the use of the Individual Discipline Feasible (IDF) architecture to decouple the disciplines within the aerostructural optimization. This is further explained in section 1.3 Research objective and chapter 2.

1.1 WING AEROELASTICITY

When designing a wing one needs to keep in mind that a wing is flexible, meaning that the wing bends and twists when there are forces acting on it. The manufactured, unloaded shape of the wing is called the jig shape or 0-g shape. What happens in flight is that the wing structure is strained until an equilibrium of elastic and aerodynamic forces. This static aeroelastic equilibrium for the cruise condition is called the 1g-shape. In figure 1.1 and 1.2 several static aeroelastic equilibria are visible. As becomes clear from the figures, the wing bending can be quite high in cases of high aspect ratio, like the Boeing B-52 Stratofortress, or when flexible material like Carbon Fiber Reinforced Plastic (CFRP) is used, in the case of the Boeing 787 Dreamliner. When discussing flexible wings two deformation types are most pronounced: the up or downwards bending of the wing, the z-direction, and twisting of the wing around the lateral axis of the aircraft, the y-axis.



Figure 1.1: Wingbending of a Boeing 787 Dreamliner¹.

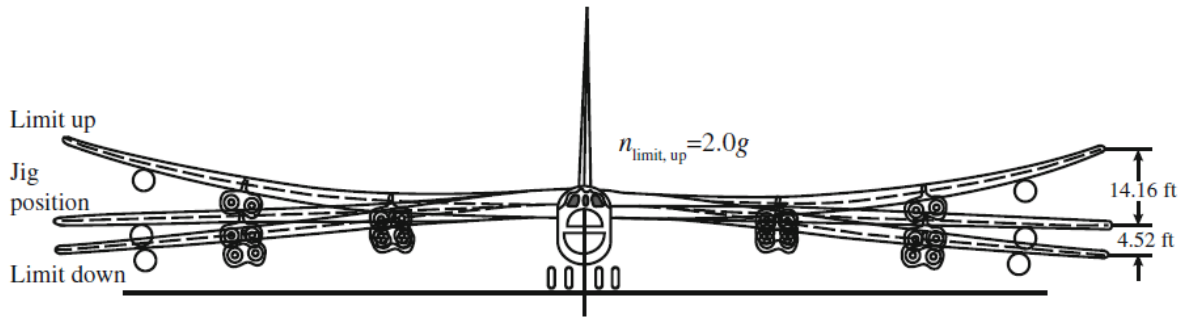


Figure 1.2: Wingbending of a Boeing B-52 Stratofortress[3].

The importance of taking wing bending and twist into account becomes apparent when comparing the spanwise lift distribution and chordwise pressure distributions of both a rigid wing and a flexible wing. Due to the twist the effective angle of attack of a wingspan section changes directly, influencing the pressure distribution and hence the local lift. Furthermore, when the wing is aft-swept, the effective angle of attack is also changed by pure vertical wing bending, without any twist. This is because the bending axis is at an approximately right angle to the elastic axis of the wing, not to the direction of the incoming airflow, see Vos and Farokhi [3] for more details.

¹C. Raezer. <https://www.flickr.com/photos/arlingtonpics/>, Last accessed: September 2015

In figure 1.3 the comparison between a rigid and a flexible Boeing B-47 Stratojet wing in spanwise lift distribution is plotted. It is visible that in this case the flexibility makes the distribution deviate further from the optimal elliptic distribution, increasing induced drag.

Figure 1.4 gives the chordwise pressure distributions for a rigid and a flexible Boeing B-52 wing at several spanwise positions. In these plots it is visible that the shockwave at the outboard sections of the wing is reduced in strength, reducing wave drag. It has to be noted that these are not general effects of wing flexibility, just examples of what the effect can be. In case of the Boeing B-47, the wing was designed without taking flexibility into account. However for the Boeing B-52 the wing bending was used to reduce drag at it high-speed cruise condition, see Vos and Farokhi [3].

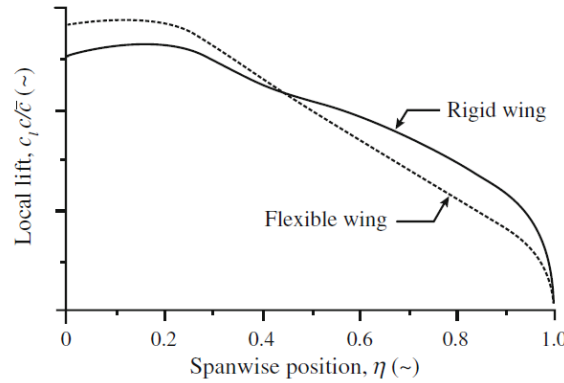


Figure 1.3: Spanwise lift distribution difference between a Boeing B-47 Stratojet flexible and rigid wing[3]

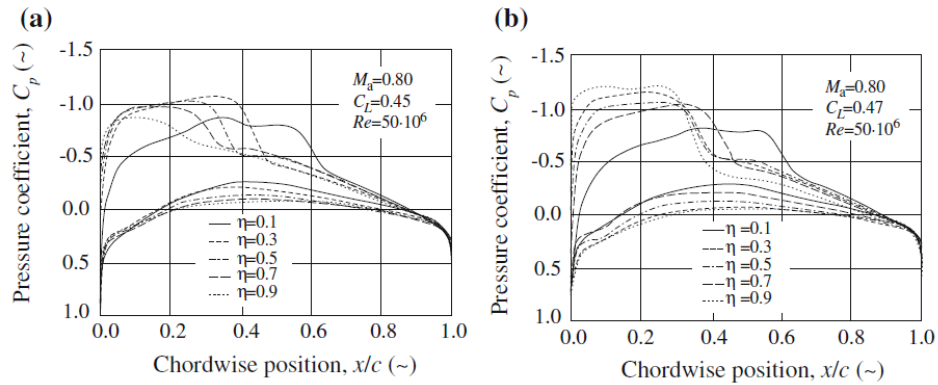


Figure 1.4: Chordwise pressure distribution difference between a Boeing B-52 Stratofortress flexible and rigid wing[3] (a) Flexible wing (b) Rigid jig-wing

In this project a flexible wing will be optimized for its cruise condition, hence for the 1-g shape of the wing. Apart from the effect on the lift and pressure distributions, one more effect which is a consequence of aeroelasticity will be taken into account: Reduced aileron effectiveness. The effect of dynamic aeroelasticity is not taken into account in this work due to time constraints.

1.2 PRIOR AEROSTRUCTURAL WING OPTIMIZATION EFFORTS

When the wing structure and aerodynamic shape is to be optimized for minimal fuel weight one arrives at a the field of Multidisciplinary Design Optimization (MDO), with at least two disciplines: The aerodynamic and the structural discipline. Within such an optimization problem the aerostructural system of various disciplines has to be consistent, meaning that the values of the data flowing to one discipline has to be in agreement with the data flowing back. Furthermore, the wing used for analysis has to be flexible and therefore the computational grid is to be deformed automatically. These issues and more have been addressed in the past. Below a short summary of these efforts is given.

The first efforts of a MDO of an aeroelastic wing are found to be in the 70s. An example of these is performed by Hafta [4], who sequentially solved the coupled aerostructural system. This system existed of a lifting line aerodynamic model and a simple structural Finite Element Analysis (FEA). This simple model for combined aerodynamic and structural optimization was also used by McGeer [5]. Afterwards, Grossman et al. [6], [7] found that when a wing is sequentially optimized the disciplines individually do not take into account the effect of changes to the other discipline. Therefore that way the optimization will not result in the optimal point in the design space. Wakayma and Kroo [8] showed that this optimum is strongly dependent by compressibility drag, aeroelasticity and structural design conditions.

Borland et al. [9], Chattopadhyay and Pagaldipti [10] and Baker and Giesing [11] started using high-fidelity methods within the MDO problem, but were only able to afford a limited amount of design variables. In continuation, Manning [12] demonstrates large-scale design using industry-standard analyses and Barcelos and Maute [13] present the effects of viscous drag inclusion in high-fidelity aerodynamic analysis.

Nowadays, the use of high-fidelity analyses within an aerostructural MDO is facilitated by the ever increasing computational power available. These high-fidelity analyses are especially valuable for wing-root and wing-tip optimization, as these areas are ruled by interference and 3D phenomena. Furthermore, for the analysis of unconventional design concepts high-fidelity analyses are of high value, as no empirical data exists. Brezillon et al. [14] presents methods for wing shape, planform and aero-acoustic MDO. The paper states that grid-deformation techniques are to be used instead of regenerating the grid for every design iteration, for simplification of the wing deformation process. Furthermore the paper confirms that the non-linear interactions between the disciplines are only captured by high-fidelity analyses.

Even though the advantages of high-fidelity aerodynamic analysis is clear, recent examples of aerostructural optimization efforts continue using panel methods, like Liem et al. [15] and Kennedy and Martins [16], [17]. The latest examples of higher fidelity aerostructural MDO's are given by Kenway et al. [18], [19], [20] and Liem et al. [21].

The reason why methods based on the panel method, full-potential or quasi-3D analyses, see Mariens et al. [22], are still used is because using higher fidelity CFD analyses within a MDO can be extremely time-consuming. This is because in the prior aerostructural MDO efforts the aerodynamic and structural disciplines have been coupled directly, meaning that consistency of the system has to be reached before moving to the next optimization iteration. This method for MDO is not optimal, see chapter 2 for more details.

1.3 RESEARCH OBJECTIVE

The work in this report is an effort to make more efficient use of high-fidelity analyses for the optimization of an aeroelastic wing, hereby reducing the computational effort. This is achieved by using the IDF MDO architecture in stead of using the prior used Multidisciplinary Feasible (MDF) architecture. The IDF architecture decouples the aerodynamic from the structural discipline. By decoupling the disciplines the work of making the system consistent is removed from the computationally expensive individual disciplines and put it in the hands of the cheap optimization algorithm. This is why the architecture is able to maintain consistency of the multidisciplinary system while searching for an optimum in the design space. Besides this advantage, the IDF architecture implies less demands on the sensitivity analyses of the disciplines. This gives more freedom for choosing software for the analyses. The details of this architecture are explained in chapter 2.

The complete research objective of the present thesis can now be formulated. It is presented below:

To contribute to the minimization of fuel use of aircraft, by using the Individual Discipline Feasible architecture for solving a gradient-based aerostructural Multidisciplinary Design Optimization problem and by optimizing a static aeroelastic wing for minimal fuel consumption during the cruise phase.

In order to achieve this objective, several sub-goals have been set:

- Set-up and formulate the aerostructural optimization problem using the IDF architecture.
- Gather and adapt the available software and methods so they can be used within the optimization.
- Verify the sensitivities gained from the software.
- Run the optimization for minimal fuel weight of the test case aircraft.

The last item is part of the test case application performed in this report. For this test case the reference aircraft used is a modern high-speed transport aircraft, the Airbus A320. This model is chosen because of its wide usage by airlines and the availability of its geometrical, aerodynamic and structural data.

Next to the above objective of this report, the work forms a basis for further high-fidelity optimization of for example a wing tip extension or a complete aircraft.

1.4 THESIS OUTLINE

This report is structured in the following way: Chapter 2 elaborates on the IDF implementation in the optimization problem and describes the optimization formulation. Afterwards in chapter 3 and 4 the aerodynamic and structural disciplines are unfolded and explained. Chapter 5 provides the details and the set-up of the reference aircraft model. Finally chapter 6 presents the results gained after optimization and 7 provides the conclusions and recommendations of this master thesis project.

2

IDF ARCHITECTURE FOR WING MULTIDISCIPLINARY DESIGN OPTIMIZATION

Optimization of a wing is truly a multidisciplinary task, as the physics of a wing of an aircraft depend on both aerodynamic and structural parameters. To illustrate this, an example of the interconnection is as follows: The shape of the wing defines the spanwise lift and moment distribution. The wing cannot fail under this loading, hence the structure is a function of the wing loading. However, the wing box structure defines the wing deformation due to the wing bending of the flexible wing. This means that the wing loading is also dependent on the wing box structure. Much like this interdependency, there exists an interdependency between necessary fuel weight needed for an aircraft's range and the wing structural weight.

These are examples of a system with multidisciplinary interdependencies and there are several ways to assure its consistency. The area of research of optimization of these systems is called MDO, as becomes clear from the definition given by Alexandrov [23]: "[MDO is] an area of research concerned with developing systematic approaches to the design of complex engineering artifacts and systems governed by interacting physical phenomena".

This chapter focuses and elaborates first of all on the IDF architecture within this MDO problem. Afterwards the optimization problem is formulated and the objective function, the design variables and the constraints are given. Then the optimization algorithm used in this study will be explained. Finally it is shown how the sensitivity analysis is performed in a strictly decoupled way.

2.1 OPTIMIZATION ARCHITECTURE

The optimization architecture has a great influence in how efficient the sometimes expensive discipline computations are used. This is because it controls the interactions between disciplines. This hence has an effect on how often these disciplines are called to perform their analyses.

The publication of Martins and Lambe [24] describe four single-level, or monolithic, architectures: The MDF, IDF, Simultaneous Analysis and Design (SAND) and All-At-Once (AAO) architecture. Multi-level, or distributed, algorithms are not of interest for this work, as no sub-level optimizations are performed inside the main optimization. The MDF and IDF architectures will be visualized using the eXtended Design Structure Matrix (XDSM), which shows not only the data dependency but is also extended with the flow and numbering of the process, see Lambe and Martins [25].

The MDF architecture works as follows: Within each optimization iteration it loops all separate disciplines until consistency is reached for all state variables. The earlier aerostructural wing optimization effort as discussed in chapter 1 incorporate this architecture. Many of these efforts include the Newton method to update and steer the state variables to convergence. Figure 2.1 provides the XDSM of the following general optimization problem:

$$\begin{aligned} \min \quad & f(X, Y(X, Y)) \\ \text{s.t.} \quad & c_0(X, Y(X, Y)) \leq 0 \\ & c_i(X_0, X_i, Y_i(X_0, X_i, Y_{j \neq i})) \leq 0 \end{aligned} \tag{2.1}$$

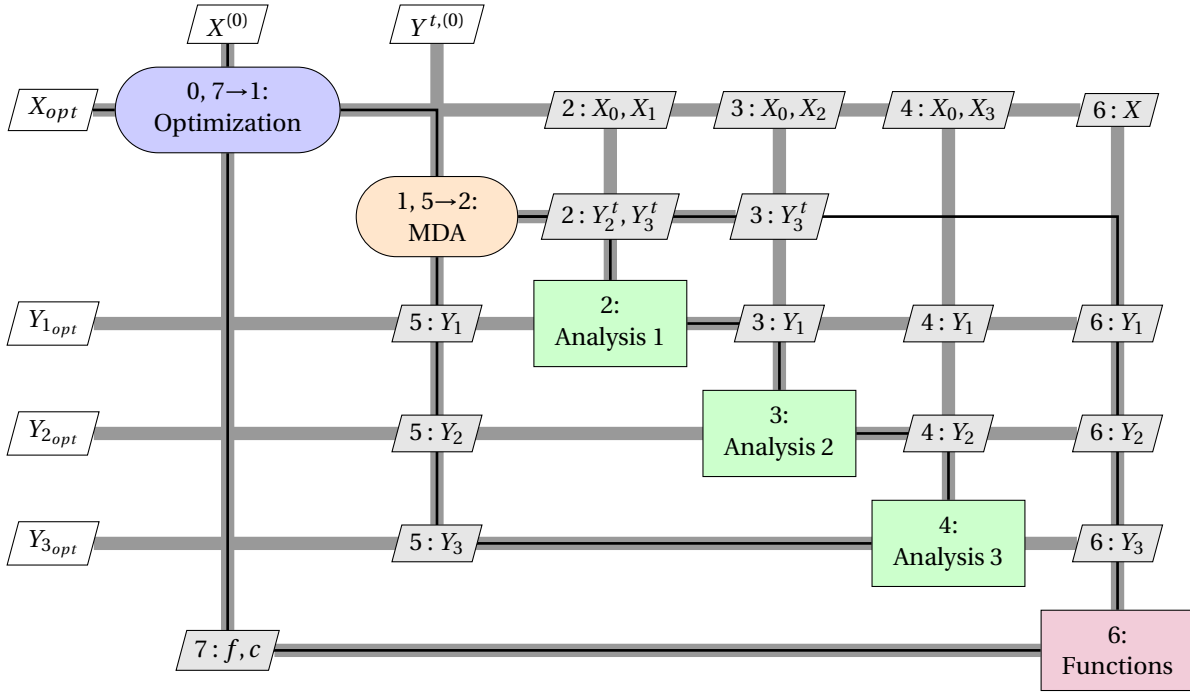


Figure 2.1: XDSM[25] of the MDF architecture

In the above figure and equation X represents the design vector, Y the state variables, f the objective function and c the constraints. Superscript t means that it is a copy of the variable and subscripts i and j give the number of the disciplinary analysis in question.

Advantages of this architecture are that the optimization formulation is as compact as possible; only the design variables, objective function and constraint values and their derivatives are needed. Furthermore, the consistency of the system is guaranteed at each optimization iteration. This means that the optimization will provide a valid output of the multidisciplinary system even when it is stopped halfway.

However, one disadvantage exists when using the MDF architecture. The internal Multidisciplinary Analysis (MDA) loop needs to result in a consistent system before the next optimization iteration can be performed. This means that the disciplines and their sensitivity calculations are called multiple times before each step in the direction of an optimum. Especially when computationally expensive disciplines are used, the optimization itself becomes very computationally expensive as well.

The second architecture, the IDF architecture, is different due to the removal of the MDA loop within the optimization. In stead of this loop, consistency is realized by imposing equality constraints on the interdisciplinary state variables. These state variables are constraint to surrogate variables, which are included in the design vector. Equation 2.2 provides the general formulation for the optimization problem to be solved using the IDF architecture.

$$\begin{aligned}
 \min \quad & f(X, Y(X, Y^*)) \\
 \text{s.t.} \quad & c_0(X, Y(X, Y^*)) \leq 0 \\
 & c_i(X_0, X_i, Y_i(X_0, X_i, Y_{j \neq i}^*)) \leq 0 \\
 & c_{eq_i} = Y_i^* - Y_i(X_0, X_i, Y_{j \neq i}^*) = 0
 \end{aligned} \tag{2.2}$$

Where the superscript $*$ flags the surrogate design variables and c_{eq} are the equality constraints. Figure 2.2 provides the XDSM for the IDF architecture.

The drawback of this architecture is that the amount of design variables and constraints increase. If this increase is too large the efficiency of the architecture will be compromised.

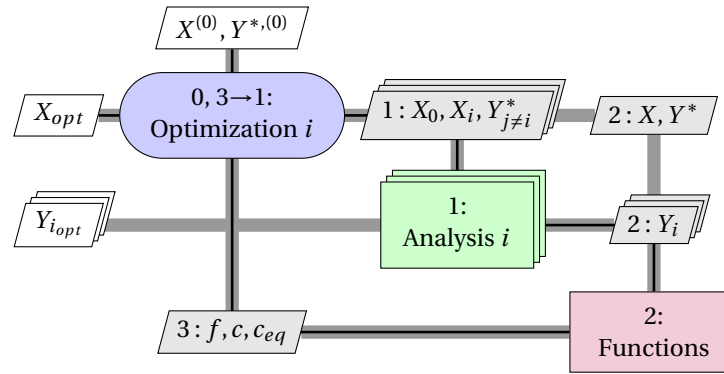


Figure 2.2: XDSM[25] of the IDF architecture

Nevertheless, the removal of the MDA loop results in two large advantages. First of all, according to Cramer et al. [26] the IDF architecture results in overall less computational costs than the MDF architecture, when the increase in design variables is kept low. By removing the MDA loop the responsibility of consistency of the system is removed from the disciplines themselves. Instead, it is placed in the hands of the optimization algorithm. This means that the optimizer can steer the design variables to an optimum and to multidisciplinary consistency simultaneously. Furthermore, it facilitates the ability to run the different decoupled analyses in parallel.

Another large advantage is that the calculation of design sensitivities is much more simple. For the MDF architecture the sensitivities of one discipline need to include the effect of all other disciplines having an effect on it. These sensitivities can efficiently be calculated using the coupled adjoint method, see for example Kenway and Martins [20]. However, this method requires the calculation of the sensitivities of the residual of one discipline with respect to the state variables of another. For the IDF architecture this information is not needed as the disciplines are completely decoupled. All inputs of a discipline are included in the design vector and the effect of one discipline to another is captured through the equality constraints.

The last two architectures, the SAND and AAO architectures, include the residuals of the individual disciplines and constrain them to 0. The difference between the two is that for the SAND architecture the consistency constraints as used for the IDF architecture are removed. The job of consistency is therefore fully carried by the residual constraints. The advantage of both architectures is that they are computationally the fastest of all, see Cramer et al. [26]. However, they are difficult or impossible to implement for black-box components. This is because for these architectures analysis the values of the residuals and their sensitivities are needed. The software used for the optimization in this report does not provide this information and are in essence black-boxes.

From the above overview between the available architectures one can conclude that for this optimization where external software is used only the MDF and IDF architectures are valid options. Between these two architectures IDF has a great advantage due to its efficiency and simpler sensitivity calculations. In the section below the two architectures will be compared using the formulation of the problem at hand.

2.2 AEROSTRUCTURAL OPTIMIZATION PROBLEM FORMULATION

Now that the MDF and IDF architecture have been explained, one can go along and formulate the aerostuctural optimization problem for both architectures.

The goal of this study is to minimize the fuel weight W_{fuel} used for the defined range of the reference aircraft by changing the planform shape P , the airfoil shapes FFD , the wing box structure thicknesses T and the angle of attack α of a flexible wing under constraints governed by structural failure modes F , aileron effectiveness L_δ , wing loading WL and the cruise lift coefficient $C_{L_{cruise}}$. Equation 2.3 provides the formulation of this

problem, in MDF form.

$$\begin{aligned}
& \min W_{fuel}(X) \\
& X = [FFD \ P \ T \ \alpha] \\
& s.t. \ F_i \leq 0 \\
& \quad 1 - L_\delta / L_{\delta_0} \leq 0 \\
& \quad WL / WL_{init} - 1 \leq 0 \\
& \quad C_L / C_{L_{cruise}} - 1 = 0
\end{aligned} \tag{2.3}$$

In order to make this formulation in IDF form, one needs to add surrogate design variables and equality constraints, as mentioned in the section above. Besides the original design variables, the aerodynamic analysis requires the aeroelastic deformation U and the elastic axis EA variables. The fuel weight analysis and structural analyses require W_{fuel} and the maximum take-off weight W_{MTO} besides the original design variables. This results in the formulation of the present aerostructural optimization problem in IDF form as given in equation 2.4.

$$\begin{aligned}
& \min W_{fuel}^*(X) \\
& X = [FFD \ P \ T \ \alpha \ U^* \ EA^* \ W_{fuel}^* \ W_{MTO}^*] \\
& s.t. \ F_i \leq 0 \\
& \quad 1 - L_\delta / L_{\delta_0} \leq 0 \\
& \quad WL / WL_{init} - 1 \leq 0 \\
& \quad U / U^* - 1 = 0 \\
& \quad EA / EA^* - 1 = 0 \\
& \quad C_L / C_{L_{cruise}} - 1 = 0 \\
& \quad W_{fuel} / W_{fuel}^* - 1 = 0 \\
& \quad W_{MTO} / W_{MTO}^* - 1 = 0
\end{aligned} \tag{2.4}$$

In the section above the advantages for using IDF were given. It was stated that the disadvantage of this architecture was that its efficiency could be compromised when the amount of surrogate design variables is too large. In this case, the efficiency will not be compromised. This is because for this optimization the amount of additional design variables is well under the amount of actual design variables.

However, the largest reason for choosing the IDF architecture for this aerostructural optimization problem is the simplicity of the sensitivity calculation. For high-fidelity aerodynamic analysis it saves a lot of work to use an off-the-shelf CFD solver, instead of creating one yourself. These CFD solvers usually do not provide the sensitivities of the aerodynamic residual with respect to the structural state variables. This is also the case for the CFD solver used in this project, see chapter 3. This means that when IDF is used for aerostructural optimization the requirements on the software for the disciplinary analyses are much lower, giving more freedom of choice and enhances the ability to treat the disciplines as interchangeable black-boxes.

It is for these reasons that IDF has been chosen as the architecture of this aerostructural optimization problem. Below, the frameworks provided by this architecture is filled in and presented using the XDSM diagram. Further in this section the objective function, design variables and constraints of the current optimization will be elaborated on.

2.2.1 XDSM

Figure 2.3 gives the XDSM of the IDF architecture for the optimization in this report. In this figure the design variables are stated above. The decoupled disciplines are the aerodynamic and the structural discipline. Because the Fuel weight analysis is dependent on the aerodynamic analysis, it is included in the aerodynamic discipline. The design vector includes all the inputs of the two disciplines, realizing the decoupled structure.

The aerodynamic analysis provides the C_L value, as well as the C_D variable which is fed to the fuel weight analysis. The elaboration on the method of the analysis is given at the end of chapter 3.

The structural discipline calculates the U , EA , F and L_δ values. See chapter 4 for further details on the method of calculation. In that chapter the calculation of the values of W_{MTO} and WL are also given. In the present XDSM they are part of the Functions block.

These disciplinary outputs are constraint to their corresponding surrogate variables. However, the outputs C_L , F , L_δ and WL are constraint using the original constraints of the optimization problem, as explained in section 2.2.4 Constraints.

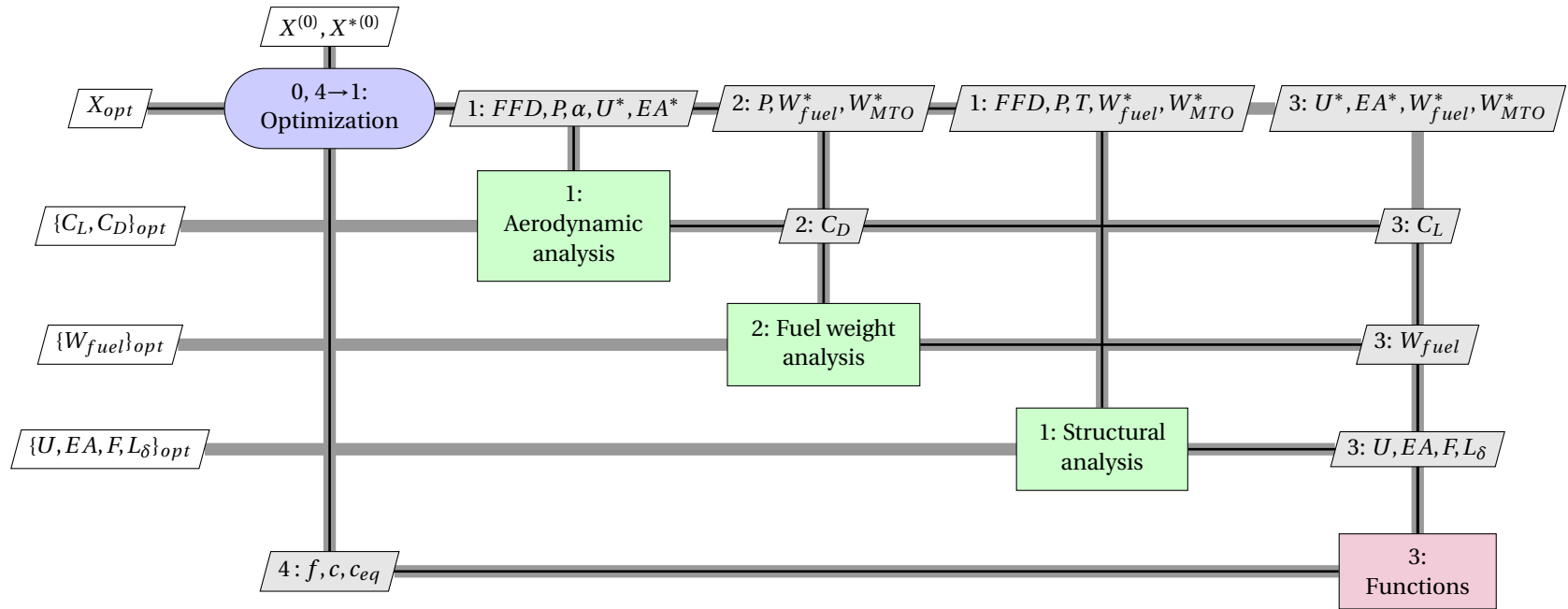


Figure 2.3: XDSM[25] of the present fuel weight optimization

For simplicity, the sensitivities are not presented in this overview. A complete overview and explanation on their calculation is given in section 2.4 Decoupled sensitivity analysis.

2.2.2 OBJECTIVE FUNCTION

The objective function for this optimization problem is the reduction of fuel weight needed for the range of the reference aircraft, as stated and argued in chapter 1. See chapter 3 for the explanation of how this fuel weight is determined. The used method dictates that the total fuel weight is dependant on the lift over drag ratio L/D of the aircraft and its maximum take-off weight W_{MTO} . The planform and airfoil shape design variables have a direct influence on the first parameter, while the wing box structure design variables have a direct influence on the latter. However, as was illustrated in the beginning of this chapter, the influences of the design variables also cross-over through indirect dependencies.

In this report the fuel weight W_{fuel} itself is not used as objective function. In stead, the surrogate design variable W_{fuel}^* is used. This is possible as these two parameters are equated through an equality constraint.

2.2.3 DESIGN VARIABLES

The design variables of the optimization in this study can be divided into three parts. Firstly the design variables which determine the exterior shape of the wing, namely the planform and airfoil shapes. Secondly the design variables which determine the internal structure, namely the wing box panel thicknesses. The last design variable is the angle of attack.

The wing exterior shape is defined by 9 2D airfoils. For a wing shape optimization performed by Palacios et al. [27] 5 sections are used. For this optimization extra sections are added, for example at the kink position. This position is needed because the planform change is realized by translation and scaling of the same 2D sections. More about the implementation of the wing shape design variables is given in chapter 3.

The planform is defined using the following 8 planform parameters: Root chord length c_r , taper from root to kink λ_1 , taper from kink to tip λ_2 , wing span from root to kink b_1 , wing span from kink to tip b_2 , leading edge sweep angle from root to tip Γ , kink twist angle ϵ_1 and tip twist angle ϵ_2 . Using one constant leading edge sweep angle is not considered an underestimation, as recent passenger aircraft also have a wing design with a constant leading edge sweep angle. The design variables for the airfoil shapes are defined as the displacement of the Free-Form Deformation (FFD) box control points. This optimization uses 9 control points per airfoil side. Also the concept of the FFD box and its control points is explained in chapter 3.

The internal structure design variables are defined as 40 equivalent panel thicknesses. This number comes from the fact that the wing box is divided into 10 elements and that each of these elements has an upper, lower, front and rear equivalent panel. This same number of sections representing the wing box is used and validated for the same reference aircraft in the publication of Elham and van Tooren [28]. More details on the equivalent panel thickness design variables can be found in chapter 4.

The last design variable, the angle of attack, is included to facilitate the change in lift generation necessary to maintain level flight in cruise.

Table 2.1 provides the complete overview of the design vector.

Table 2.1: Design vector

Parameter	Design Variable	number
FFD	Free-Form Deformation control points	162
P	Planform parameters	8
T	Equivalent panel thickness	40
α	Angle of attack	1
U^*	Deformation (4 DOF: y, z, ϕ, θ)	32
EA^*	Elastic Axis	24
W_{fuel}^*	Fuel weight	1
W_{MTO}^*	Maximum take-off weight	1
	Total	269

It is visible that beside the 211 design variables 58 more surrogate variables are present at the lower side of the table, denoted with the superscript *. These surrogate variables exist of the deformation in 4 Degrees Of Freedom (DOF) of the aeroelastic wing, defined at 8 of the 9 airfoils as the root is not deformed, the elastic axis position of the 1-g shape of the wing at the same 8 airfoils, the fuel weight and the maximum take-off weight. The reasoning behind taking 4 DOF for the deformation of the wing in stead of 6 is because the translation in x direction and the rotation ψ , around the z axis, are negligible in comparison to the other 4 DOF. The used DOF are: Translation in y and z direction and rotation around the x and y axis, ϕ and θ , respectively.

2.2.4 CONSTRAINTS

Two types of constraints are used in this optimization routine: equality and inequality constraints. First of all, the equality constraints are used to make sure the surrogate design variables are equal to the outputs of the disciplines, like explained in section 2.1 Optimization architecture. Of these equality constraints the only exception is the constraint on the lift coefficient C_L , as this constraint does not constrain surrogate design variables. To make sure enough lift is maintained the C_L is constraint to the lift coefficient needed to maintain horizontal flight in cruise $C_{L_{cruise}}$.

In equation 2.5 the calculation of $C_{L_{cruise}}$ is given. This equation is extracted from the method by Torenbeek [29] for mid-cruise weight calculation.

$$C_{L_{cruise}} = \frac{9.80665 \sqrt{W_{MTO}(W_{MTO} - W_{fuel})}}{0.5 \rho V_{inf}^2 S_{ref}} \quad (2.5)$$

Where W_{MTO} is one of the outputs of the structural analysis and W_{fuel} is the output of the fuel weight analysis, they are hence not the surrogate design variables. S_{ref} is the reference wing surface area, ρ the density of air and V_{inf} the undisturbed airspeed.

Most of the inequality constraints are present to make sure the wing box structure does not fail under static load or fatigue. Moreover a constraint is set so that the value of L_δ , being the roll moment L with respect to the aileron deflection δ , is equal or higher than the value of the reference aircraft. Lastly a constraint is present to limit the wing loading WL at take-off conditions, which should be equal or lower than the value of the reference aircraft. This is done to meet take-off and landing requirements and is also used in the optimization by Elham and van Tooren [28]. More on these inequality constraints can be found in chapter 4.

A total overview of the equality and inequality constraints can be found in table 2.2. The calculation of the failure modes F can be found in equations 4.7 to 4.9.

2.3 OPTIMIZATION ALGORITHM

The optimizer running the MDO architecture decides which next point in the design space is evaluated, taking into account the value of the objective function, the violation of the (in-)equality constraints and their sensitivities with respect to the design vector. A great variety of optimization algorithms exist, ranging from for example Genetic Programming or Particle Swarm Optimization which can find a global optimum to Sequential Linear Programming or Sequential Quadratic Programming (SQP), which are able to find a local optimum, see Langelaar and van Keulen [30] for a greater overview. The aerodynamic discipline of this optimization is computationally expensive, which is why it is preferred that the amount of discipline evaluations is as low as possible while still achieving a satisfactory optimum.

It was found by Mariens et al. [22] that for wing shape optimizations in the conceptual design phase the SQP is superior over the global exploration algorithm Local Optima Smoothing (LOS). LOS is an example of a global algorithm which uses random exploration of the design space to find the region of attraction. In the cited study SQP is found to be superior because the LOS algorithm required 22 times more computational time while the reduction in objective function was less then 0.5% higher. This proves that a local optimum can be sufficient for a wing shape optimization as the one in the present thesis. Moreover Langelaar and van Keulen [30] states that SQP is generally seen as the best general-purpose method for constraint optimization problems. These are the reasons for the choice of the SQP algorithm for the present optimization problem.

There are multiple implementations of the SQP algorithm available for MATLAB. The Sparse Nonlinear Optimizer (SNOPT) is well-suited for smooth, nonlinear problems with a high number of constraints and vari-

Table 2.2: (In)equality constraint overview

Type	Constraint	Load Case	Implementation	Number
Inequality	Compression upper panel	1	$c = F_{yield}$	64
	Buckling upper panel	1	$c = F_{buckling}$	64
	Tension lower panel	1	$c = F_{yield}$	64
	Shear front panel	1	$c = F_{yield}$	32
	Buckling front panel	1	$c = F_{shear buckling}$	32
	Shear rear panel	1	$c = F_{yield}$	32
	Buckling rear panel	1	$c = F_{shear buckling}$	32
	Compression upper panel	2	$c = F_{yield}$	64
	Buckling upper panel	2	$c = F_{buckling}$	64
	Tension lower panel	2	$c = F_{yield}$	64
	Shear front panel	2	$c = F_{yield}$	32
	Buckling front panel	2	$c = F_{shear buckling}$	32
	Shear rear panel	2	$c = F_{yield}$	32
	Buckling rear panel	2	$c = F_{shear buckling}$	32
	Tension upper panel	3	$c = F_{yield}$	64
	Compression lower panel	3	$c = F_{yield}$	64
	Buckling lower panel	3	$c = F_{buckling}$	64
	Shear front panel	3	$c = F_{yield}$	32
	Buckling front panel	3	$c = F_{shear buckling}$	32
	Shear rear panel	3	$c = F_{yield}$	32
	Buckling rear panel	3	$c = F_{shear buckling}$	32
	Fatigue lower panel	4	$c = F_{yield_{LP}} + 0.58$	64
	Aileron effectiveness	5	$c = 1 - L_{\delta} / L_{\delta_0}$	1
	Wing Loading	-	$c = WL / WL_{init} - 1$	1
Equality	C_L	-	$c_{eq} = C_L / C_{L_{cruise}} - 1$	1
	U	-	$c_{eq} = U / U^* - 1$	32
	EA	-	$c_{eq} = EA / EA^* - 1$	24
	W_{fuel}	-	$c_{eq} = W_{fuel} / W_{fuel}^* - 1$	1
	W_{MTO}	-	$c_{eq} = W_{MTO} / W_{MTO}^* - 1$	1
Total				1085

ables, see Gill et al. [31]. This is why it is chosen over other, less powerful, MATLAB based optimization functions like fmincon or SQPlab. The working of the SQP algorithm used by SNOPT is shortly explained in the section below.

As stated, the optimization algorithm decides the direction of search and the next point in the design space. It continues its search until the stopping criterion is reached or the optimality conditions are met. The stopping criterion used in this project is 50 major iterations of SNOPT. The optimality conditions are also explained separately below.

2.3.1 SEQUENTIAL QUADRATIC PROGRAMMING

The SQP algorithm uses quadratic sub-optimization problems minimizing the quadratically approximated objective function, subjected to the linearised original constraints. The objective function f_{k+1} and constraints c_{k+1} and $c_{eq,k+1}$ are approximated using Newton's method, where $k+1$ stands for the updated location in the design space. This results in a the quadratic optimization sub-problem as given in equation 2.6

$$\begin{aligned}
 \min_{\Delta x} \quad & f + \left\{ \frac{df}{dx} \right\}^T \Delta x + \frac{1}{2} \Delta x^T H \Delta x \\
 \text{s.t.} \quad & c + \frac{dc}{dx} \Delta x \geq 0 \\
 & c_{eq} + \frac{dc_{eq}}{dx} \Delta x = 0
 \end{aligned} \tag{2.6}$$

Where Δx the search direction towards an optimum and H the approximation of the Hessian, which is gained using the quasi-Newton approach Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm. This quadratic

sub-problem is solved using an active-set algorithm, see Langelaar and van Keulen [30] and Gill et al. [31] for more information on these algorithms.

After the search direction Δx is gained, a line search is performed to find a sufficient decrease of the Merit function M . The calculation of this function is given in equation 2.7.

$$M = f - \mu^T (c - s) + \lambda^T (c_{eq} - s) + \frac{1}{2} \sum_{i=1}^m \rho_{c_i} (c_i - s_i)^2 + \frac{1}{2} \sum_{j=1}^n \rho_{c_{eq_j}} (c_{eq_j} - s_j)^2 \quad (2.7)$$

Where μ and λ are the Lagrange multipliers, s the slack variables and ρ the penalty parameters, see Gill et al. [31] for further details.

2.3.2 OPTIMALITY CONDITIONS

In order to check if a candidate for an optimum is a real local optimum the Karush-Kuhn-Tucker (KKT) conditions are one of the conditions that need to be satisfied. They are given by equations 2.8 to 2.10. They are the necessary conditions for a local optimum, the sufficient condition is given further below.

$$\frac{\partial L}{\partial X} = \frac{\partial f}{\partial X} + \sum \bar{\mu} \frac{\partial \bar{c}}{\partial X} + \sum \bar{\lambda} \frac{\partial \bar{c}_{eq}}{\partial X} = 0^T \quad (2.8)$$

$$c \leq 0, c_{eq} = 0 \quad (2.9)$$

$$\lambda \neq 0, \mu \geq 0, \mu c = 0 \quad (2.10)$$

In these equations L is the Lagrangian as given in equation 2.11.

$$L(X, \mu) = f(X) + \mu^T c(X) + \lambda^T c_{eq}(X) \quad (2.11)$$

In short, equation 2.8 gives the optimality condition, stating that a stationary point in the design space is reached. This condition provides the Lagrange multipliers. The feasibility condition in equation 2.9 checks if the point in the design space is actually feasible. The last condition is the complementarity condition in equation 2.10 which checks if the Lagrange multipliers do not help in satisfying the optimality condition. Lastly, if all the KKT conditions are satisfied, the sufficient conditions need to be satisfied in order to verify that the constraint is not only a stationary point put also an optimum. This is done by checking if the objective and the feasible domain are locally convex. The condition is given in equation 2.12.

$$(\partial X)^T \frac{\partial^2 L}{\partial X^2} \partial X > 0 \quad (2.12)$$

2.4 DECOUPLED SENSITIVITY ANALYSIS

As visible in the equation 2.6 in the prior section, the SQP algorithm needs sensitivity information of the objective function and the constraints with respect to the design vector. However, as the disciplines within the IDF MDO architecture are fully decoupled, the sensitivities also need to be decoupled. This means sensitivities of the objective function and intermediate states of the optimization with respect to the design variables and surrogate design variables are required. This section will expand the objective function and constraint sensitivities with respect to the design variables until a fully decoupled system is obtained. For extra clarity, the design vector X is given again by equation 2.13

$$X = \begin{bmatrix} FFD & P & T & \alpha & U^* & EA^* & W_{fuel}^* & W_{MTO}^* \end{bmatrix} \quad (2.13)$$

All sensitivities of the equality, non-equality and objective function are normalized by multiplying with the normalization vector X_0 . This normalization vector consist of the starting values of all (surrogate) design variables. The value for normalization of the FFD design variables is taken to be 0.1, as stated in section 2.2.3 Design Variables. Furthermore, the addition of 1 to the FFD control point design variables has no influence on the sensitivities, as it only an addition.

2.4.1 OBJECTIVE FUNCTION SENSITIVITY

The sensitivity of the objective function of the present optimization with respect to the design variables is given by equation 2.14:

$$\frac{dW_{fuel}^*}{dX} \quad (2.14)$$

The surrogate design variable W_{fuel}^* is also present inside the design vector. Hence its sensitivity with respect to X is a vector of zeros except for the sensitivity with respect to itself, which is 1.

2.4.2 CONSTRAINT SENSITIVITY

The sensitivities of the equality constraints with respect to the design variables all have the same form. They are given by equations 2.15 to 2.19. Of which only the first one is fully worked out.

$$\frac{dceq_U}{dX} = \frac{\partial ceq_U}{\partial X} + \frac{\partial ceq_U}{\partial U} \frac{dU}{dX} + \frac{\partial ceq_U}{\partial U^*} \frac{dU^*}{dX} = \frac{dU}{dX} \frac{1}{U^*} - \frac{dU^*}{dX} \frac{1}{U^{*2}} U \quad (2.15)$$

$$\frac{dceq_{EA}}{dX} = \frac{dEA}{dX} \frac{1}{EA^*} - \frac{dEA^*}{dX} \frac{1}{EA^{*2}} EA \quad (2.16)$$

$$\frac{dceq_{W_{fuel}}}{dX} = \frac{dW_{fuel}}{dX} \frac{1}{W_{fuel}^*} - \frac{dW_{fuel}^*}{dX} \frac{1}{W_{fuel}^{*2}} W_{fuel} \quad (2.17)$$

$$\frac{dceq_{W_{MTO}}}{dX} = \frac{dW_{MTO}}{dX} \frac{1}{W_{MTO}^*} - \frac{dW_{MTO}^*}{dX} \frac{1}{W_{MTO}^{*2}} W_{MTO} \quad (2.18)$$

$$\frac{dceq_{C_L}}{dX} = \frac{dC_L}{dx} \frac{1}{C_{L_{cruise}}} - \frac{dC_{L_{cruise}}}{dx} \frac{1}{C_{L_{cruise}}^2} C_L \quad (2.19)$$

Within these expanded sensitivities there are still many unknown sensitivities. The way of calculating each one of them is given below.

First of all, the sensitivities $\frac{dU^*}{dX}$, $\frac{dEA^*}{dX}$, $\frac{dW_{fuel}^*}{dX}$ and $\frac{dW_{MTO}^*}{dX}$ are sparse matrices with a diagonal line of ones for the sensitivity of a surrogate design variable to itself, being 1. The sensitivity $\frac{dEA}{dX}$ is a summation of $\frac{dU}{dX}$ and $\frac{dEA_{jig}}{dX}$, because the output of the structural discipline is the elastic axis of the jig wing and EA^* is defined as the deformed elastic axis. Both sensitivities $\frac{dU}{dX}$ and $\frac{dEA_{jig}}{dX}$ are calculated by the structural discipline using Automatic Differentiation (AD). The sensitivities $\frac{dW_{fuel}}{dX}$ and $\frac{dW_{MTO}}{dX}$ are expanded as shown in equations 2.20 and 2.21.

$$\frac{dW_{fuel}}{dX} = \frac{\partial W_{fuel}}{\partial X} + \frac{\partial W_{fuel}}{\partial C_D} \frac{dC_D}{dX} \quad (2.20)$$

$$\frac{dW_{MTO}}{dX} = \frac{\partial W_{MTO}}{\partial X} + \frac{\partial W_{MTO}}{\partial W_{wing}} \frac{dW_{wing}}{dX} + \frac{\partial W_{MTO}}{\partial W_{fuel}} \frac{dW_{fuel}}{dX} \quad (2.21)$$

Where W_{wing} is the wing structural weight. All unknown partial derivatives of the above equations and $\frac{dW_{wing}}{dX}$ are calculated using AD. The sensitivity $\frac{dC_{L_{cruise}}}{dx}$ is expanded as given in equation 2.22.

$$\frac{dC_{L_{cruise}}}{dX} = \frac{\partial C_{L_{cruise}}}{\partial MTOW} \frac{dMTOW}{dX} + \frac{\partial C_{L_{cruise}}}{\partial W_{fuel}} \frac{dW_{fuel}}{dX} \quad (2.22)$$

Where $\frac{\partial C_{L_{cruise}}}{\partial W_{fuel}}$ and $\frac{\partial C_{L_{cruise}}}{\partial W_{MTO}}$ are given in equations 2.23 and 2.24.

$$\frac{\partial C_{L_{cruise}}}{\partial W_{fuel}} = \frac{9.80665}{0.5\rho V_{inf}^2 S_{ref}} \frac{-W_{MTO}}{2\sqrt{W_{MTO}(W_{MTO} - W_{fuel})}} \quad (2.23)$$

$$\frac{\partial C_{L_{cruise}}}{\partial W_{MTO}} = \frac{9.80665}{0.5\rho V_{inf}^2 S_{ref}} \frac{2W_{MTO} - W_{fuel}}{2\sqrt{W_{MTO}(W_{MTO} - W_{fuel})}} \quad (2.24)$$

The last unknown sensitivities are the $\frac{dC_L}{dX}$ and $\frac{dC_D}{dX}$, which are given by equations 2.25 and 2.26 respectively.

$$\frac{dC_L}{dX} = \frac{\partial C_L}{\partial X} + \frac{\partial C_L}{\partial S_{ref}} \frac{dS_{ref}}{dX} \quad (2.25)$$

$$\frac{dC_D}{dX} = \frac{\partial C_D}{\partial X} + \frac{\partial C_D}{\partial S_{ref}} \frac{dS_{ref}}{dX} \quad (2.26)$$

Of the partial derivatives $\frac{\partial C_L}{\partial X}$ and $\frac{\partial C_D}{\partial X}$ the sensitivities with respect to T , W_{fuel}^* and W_{MTO}^* are 0. The rest is calculated using the aerodynamic solver implemented in the aerodynamic discipline. $\frac{\partial C_L}{\partial S_{ref}}$ and $\frac{\partial C_D}{\partial S_{ref}}$ is equal to $-\frac{C_L}{S_{ref}}$ and $-\frac{C_D}{S_{ref}}$, respectively, and $\frac{dS_{ref}}{dX}$ is calculated using AD.

Most of the sensitivities of the inequality constraints only depend on the structural discipline and are therefore calculated by the structural discipline, using AD. This is also the case for the aileron effectiveness, of which the sensitivity is given by equation 2.27

$$\frac{dC_{L\delta}}{dX} = -\frac{dL_\delta}{dX} \frac{1}{L_{\delta_0}} \quad (2.27)$$

Where $\frac{dL_\delta}{dX}$ is again calculated by the structural discipline using AD.

The sensitivity of the constraint on wing loading is given by equation 2.28.

$$\frac{dc_{WL}}{dX} = \frac{dWL}{dX} \frac{1}{WL_{init}} = \left(\frac{\partial WL}{\partial X} + \frac{\partial WL}{\partial W_{MTO}} \frac{dW_{MTO}}{dX} \right) \frac{1}{WL_{init}} \quad (2.28)$$

of which $\frac{\partial WL}{\partial X}$ is provided using AD and $\frac{\partial WL}{\partial W_{MTO}}$ is given by $\frac{1}{S_{ref}}$.

3

AERODYNAMIC AND PERFORMANCE ANALYSIS

The present chapter will elaborate on two separate analyses: The aerodynamic and the performance analyses.

The aerodynamic analysis is responsible for calculating the drag and lift forces of the wing. The analysis uses the same distinct steps as described by Samareh [32] for aerodynamic shape optimization (ASO):

- Geometry parameterization.
- Surface grid generation.
- Volume grid generation, regeneration, or deformation.
- Computational fluid dynamics (CFD) function and sensitivity analyses.

This chapter elaborates on all these steps and the used tool, being Stanford University Unstructured (SU2).

The performance analysis is responsible for calculation of fuel weight necessary for the range specification of the reference aircraft. This module receives the wing drag directly from the aerodynamic calculations. The details of the performance module calculation is shown in section 3.6 Fuel weight analysis.

Furthermore, at the end of this chapter an overview of the analysis within the complete discipline is given.

3.1 STANFORD UNIVERSITY UNSTRUCTURED

SU2 is an open-source computational analysis and design software collection which has been released in January 2012 by Palacios et al. [27] at Stanford University. This tool is chosen to perform the aerodynamic analysis for the present optimization problem because of its abilities and its availability. Moreover, the TU-Delft has grown to be one of the most active contributors to the code. This thesis can therefore also be seen as a competence check for further usage of SU2.

The software collection of SU2 is able to perform all the named steps in the introduction of this chapter and more using different modules downloadable as a complete package. The first three steps are performed by a volume mesh deformation script, controlled by design variables deforming the surface grid. This surface grid deformation is performed using the FFD parameterization. For the last step, the flow solver of SU2 is able to solve the governing equations of laminar Navier-Stokes, Reynolds-averaged Navier-Stokes (RANS), Euler and/or Full Potential equations for unstructured grids. The sensitivity analysis is performed by a continuous adjoint solver, which calculates gradients efficiently of a potentially very large number of design variables. This collection of abilities make SU2 especially suitable for performing aerodynamic analysis for an optimization problem.

In the next sections, the modules of SU2 and the application to the present optimization problem are explained in more depth.

3.2 FREE-FORM DEFORMATION PARAMETERIZATION

As stated, one of the tasks of the aerodynamic discipline is to change the wing shape and planform before the flow analysis is performed. However, it would be very inefficient to have all x, y and z coordinates of all grid points of the wing shape as design variables. This is why an efficient parametrization is needed. Samareh [33] provides an overview of eight available geometry parameterization techniques, of which the Bezier curves, the analytical shape functions and FFD stand out as compact and hence efficient approaches. The technique used by SU2 to parametrize and deform the 3D wing is the FFD parametrization. This will hence be the approach used for both the aerodynamic as the structural discipline to deform the airfoil shape. For the aerodynamic discipline the FFD approach will also be used for planform changes.

FFD is a parametrization technique which parametrizes deformation instead of the geometry itself. It has its origins in computer graphics where it is used for the deformation of solid geometric models, see Sederberg and Parry [34]. It uses a so-called FFD-box which encloses an initial geometry which is to be deformed. On the edges of this FFD-box control points are defined, which are mapped onto the geometry through a trivariate tensor product Bernstein polynomial. These control points can be used as design variables to change the geometry in a free-from manner. The Bernstein polynomials localize the deformation and ensure continuity. In figure 3.1 an example is given of the freedom of the user of FFD. In this figure a sphere is morphed into a blended wingbody.

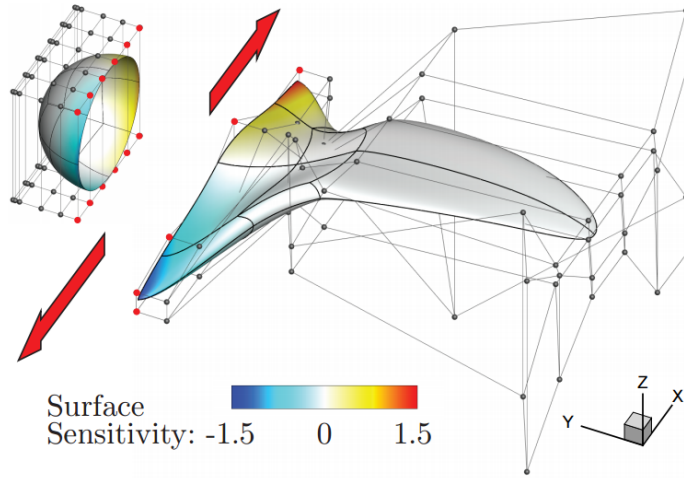


Figure 3.1: Example of Free-Form Deformation: From a sphere to a blended-wing-body[35]

Due to this large deformation freedom with only relatively few design variables FFD is a very efficient parametrization technique which is increasingly being used in aerospace design by for example Nielsen and Anderson [36], Anderson et al. [37], Samareh [32] and Palacios et al. [27]. For example, Samareh [38] uses the FFD not only to change the airfoil shape in the deformation to aerodynamic shape design variables such as thickness, camber, twist, shear, and planform.

3.2.1 IMPLEMENTATION

The FFD parametrization has already been implemented in SU2 as given in equation 3.1 [27].

$$x(u, v, w) = \sum_{i=0}^{l-1} \sum_{j=0}^{m-1} \sum_{k=0}^{n-1} FFD_{i,j,k} B_i^l(u) B_j^m(v) B_k^n(w) \quad (3.1)$$

Where x is the deformation of a point on the geometry and $u, v, w \in [0, 1]$ are the coordinates of that point. $FFD_{i,j,k}$ is the control point movement vector in all directions. All coordinates and movements are normalized w.r.t. the FFD box. i, j, k are the indices of the control points and l, m, n are the number of control points. B are the Bernstein polynomials. They are calculated as given by equation 3.2.

$$B_v^n(t) = \frac{n!}{v!(n-v)!} t^v (1-t)^{n-v} \quad (3.2)$$

As this parametrization is 3-dimensional, any sectional deformation will have an effect, however small, on all other sections of the wing. This is not desirable, as span-wise sections are isolated within the structural discipline, see chapter 4. This discrepancy is solved by using several FFD-boxes to cover the wing surface as the deformation within one FFD box does not influence the geometry in another box. These boxes are placed in between the spanwise sections. When a section is deformed by a design variable the two adjacent FFD-boxes will realize this deformation together. An example of how sectional deformation is realized by two separate FFD boxes is given in figure 3.2, where the mid-span section is translated in the z-direction.

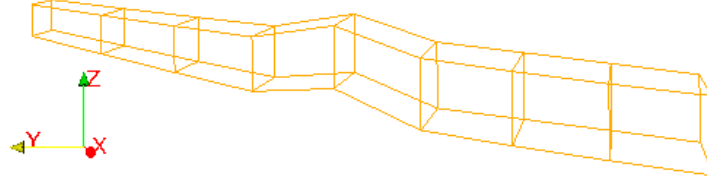


Figure 3.2: Example of the usage of two FFD boxes to realize a translation in the z-direction in SU2

The algorithm as explained above is also included within the structural discipline for deformation of the airfoils. For this purpose the 2D version of equation 3.1 is used.

Deforming airfoils is however not the only application of the FFD parametrization. Also the aeroelastic deformation and planform of the wing is parametrized using the FFD method. In order to realize deformations as for example sectional translation, twist or planform changes the FFD control points are moved as a group, not individually. Using groups of FFD control points for aeroelastic or planform deformation was however not yet implemented in SU2. This extension to the code was created by the author and can be found in the appendix. This piece of C++ code is inserted in the file `grid_movement_structure.cpp` of the downloadable source code of SU2. Links to the new design variables have been inserted throughout the SU2 source code. In the same piece of code a part is created to calculate the sensitivity with respect to EA^* . This calculation is performed in two steps. First a reverse implementation of the rotational sectional deformation using the grouped FFD control points is performed, after which the same sectional deformation is applied again but with a slightly changed position of EA^* . In section 3.5.2 Sensitivity projection it is further explained how a sectional deformation is projected onto the design variables to gain the right sensitivity. The amount with which EA^* is changed is taken to be the same as for all other sensitivity calculations within SU2: $0.001[m]$, again see section 3.5.2 Sensitivity projection.

All applications of the FFD parametrization method are further explained in the sections below.

AIRFOIL SHAPE

Airfoil shape deformation is realized by 9 2D airfoil sections. The shape deformation of each section is parametrized by the movement of 18 FFD control points distributed over the upper and lower side of each FFD-box. These control points are constraint to motion along the z-axis. Figure 3.3 gives an example of an airfoil shape deformation using the FFD control points. It can be seen that as the control points of the FFD-box move the airfoil shape changes accordingly. The airfoil plotted is the normalized root-airfoil of the reference wing.

It has to be stated that no investigation is performed towards the effect of adding more or reducing the amount of 2D airfoil sections or FFD control points per section. This is because at this moment, SU2 has a limit of 10 FFD boxes. Furthermore, the amount of FFD control points per section was taken to be similar as in the wing shape optimization as performed by [39]. In this publication 16 FFD control points were used per section.

The amount of FFD control points in figure 3.3 is different than the amount of control points used as design variables, 24 in stead of 18. The two leading edge control points and the last four trailing edge control points are not included in the design vector and will hence not move to deform the airfoil shape. The reason for the decision to keep the corner control points at the leading edge and trailing edge constant is to keep the design variables separated. In theory, the FFD control points could make the airfoil increase its incidence angle in stead of deforming the airfoil. This could be realized by moving all control points in the front half of

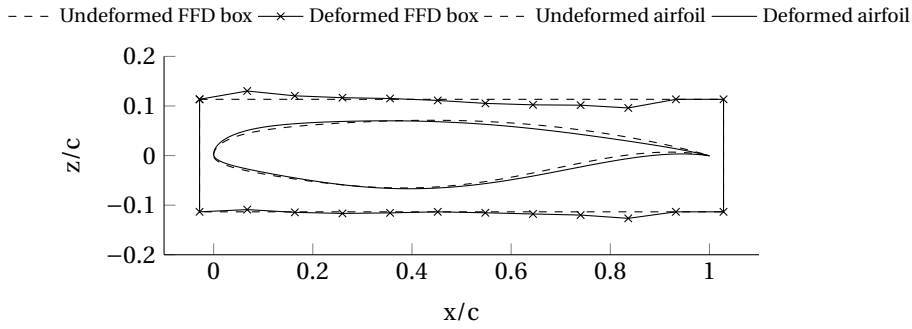


Figure 3.3: Example of an airfoil shape deformation using FFD control points

the airfoil upwards and all the point at the rear half of the airfoil downwards. However, this incidence angle is separately controlled by the planform design variables. To leave no doubt about separation of the different types of design variables, the corner FFD control points are held constant.

On top of this, the one-to-last couple of trailing edge control points are also held constant. The reason for this decision is given later in this chapter, in section 3.5 Sensitivity analysis.

WING AEROELASTIC DEFORMATION

Wing aeroelastic deformation knows 4 degrees of freedom, namely translation in the y and z direction and rotations ϕ and θ . These degrees of freedom are per spanwise section, together making up the total wing aeroelastic deformation. The translation in x direction and rotation ψ are neglected, as explained in chapter 2.

Translation of a section is done by displacing all sectional control points with the same displacement. This way the shape of the airfoil is not changed. An example of this is given in figure 3.4, where a section is translated in the z direction with 0.025.

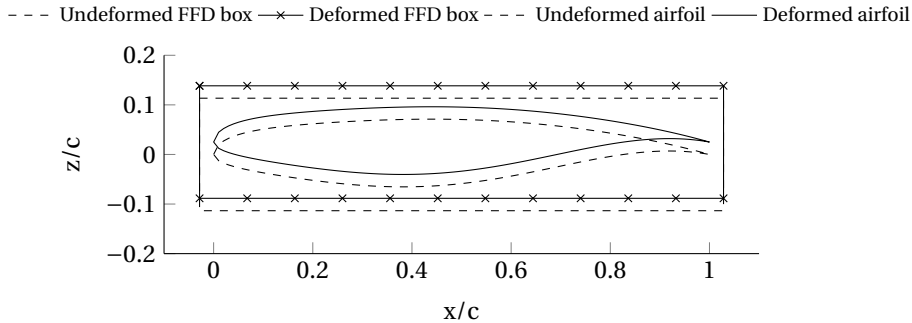


Figure 3.4: Example of a wing section transformation using FFD control points

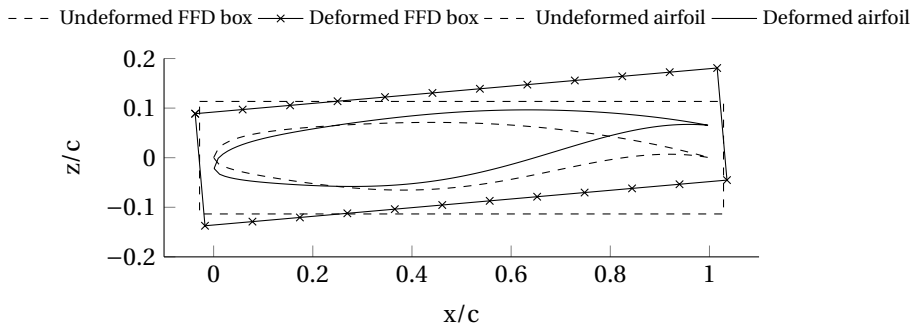


Figure 3.5: Example of a wing section rotation using FFD control points

The two rotational degrees of freedom need an axis around which the section is rotated. This is the elastic axis. The deformation is done by rotating all sectional control points with the same amount around the elastic axis. Again, the shape is not changed with this deformation. Figure 3.5 illustrates an example where the airfoil is rotated around the y axis with -5° . The elastic axis is in this example equal to the quarter chord point.

PLANFORM

The planform design variables are listed in chapter 2. For extra clarity they are plotted in figure 3.6.

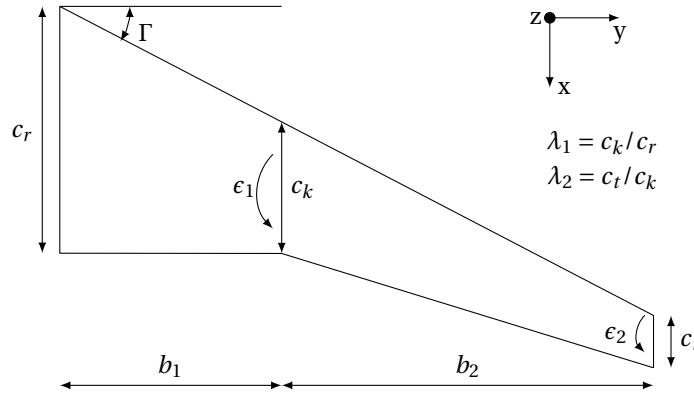


Figure 3.6: Definition of the planform design variables

Where the subscripts r , k and t stand for root, kink and tip, respectively.

For parametrization these design variables are transformed in the following parameters per spanwise section: x, y and z location of the leading edge, twist and chord length. Four of these parameters are implemented in the same way as for wing aeroelastic deformation, see the section above. For twist the quarter chord position is taken as point around which the section is rotated. The last parameter to be implemented is the chord length. This time, the FFD implementation is to be performed in reverse, because the geometry is known, but the accompanying location of the FFD control points is not. In order to obtain this control point deformation with the geometry deformation as an input equation 3.1 has to be manipulated. Writing this equation in its 2D and matrix form it can be written as shown in equation 3.3.

$$\begin{bmatrix} B_0^{l-1}(u_0)B_0^{m-1}(v_0) & \dots & B_{l-1}^{l-1}(u_0)B_0^{m-1}(v_0) & \dots & B_{l-1}^{l-1}(u_0)B_{m-1}^{m-1}(v_0) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ B_0^{l-1}(u_g)B_0^{m-1}(v_g) & \dots & B_{l-1}^{l-1}(u_g)B_0^{m-1}(v_g) & \dots & B_{l-1}^{l-1}(u_g)B_{m-1}^{m-1}(v_g) \end{bmatrix} \begin{bmatrix} FFD_{0,0} \\ \vdots \\ FFD_{l-1,0} \\ \vdots \\ FFD_{l-1,m-1} \end{bmatrix} = \begin{bmatrix} x_{0,0} \\ \vdots \\ x_{g,g} \end{bmatrix} \quad (3.3)$$

Where the first matrix on the left hand side is the matrix made by the Bernstein polynomials, the second matrix the movement of the FFD control points and the matrix on the right hand side of the equation the deformation of the geometry itself. The last matrix is gained by simply stretching and thickening the airfoil to the required chord length. The equation implies that the control point deformation is given by the inverse of the Bernstein polynomial matrix times the geometry deformation. However, the resulting system is overdetermined. The solution is found by using the normal equations. The solution is given in equation 3.4.

$$FFD = (B^T B)^{-1} B^T x \quad (3.4)$$

Where B is the matrix filled with Bernstein Polynomials, FFD is the vector of FFD displacements and x the vector of geometry displacements, see equation 3.3.

The result of this inversion can be seen in figure 3.7. In the figure the chord is extended by 0.2. It is found that the percentage increase of the chord is equal to the percentage increase of the FFD box. However, the leading edge of the airfoil need to remain at its original position. This is why the whole FFD-box is moved slightly to the left. This last point makes the use of the inverse FFD implementation as shown in equation 3.4 necessary.

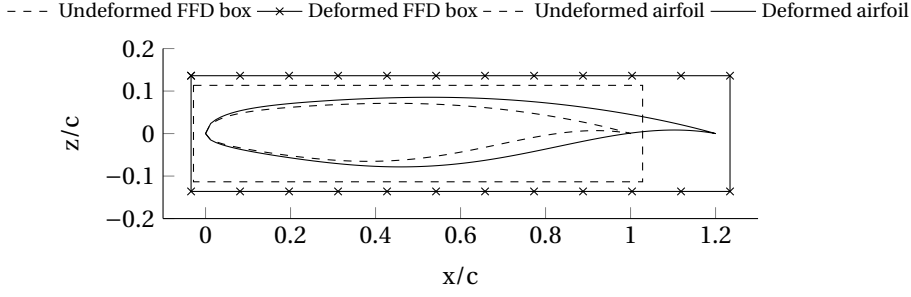


Figure 3.7: Example of a wing section chord length change using FFD control points

3.3 SURFACE AND VOLUME GRID

For any numerical aerodynamic analysis the grid can be seen as the most important part, as its quality directly influences the quality of the simulation. In general, grids can be divided in two types: Structured and unstructured grids. Structured grid cells are quadrilaterals in 2D and hexahedra in 3D, while unstructured grids typically use triangles in 2D and tetrahedra in 3D. Mixed or hybrid grid approaches are unstructured but also consist of quadrilaterals in 2D and hexahedra and tetrahedra in 3D to properly simulate the boundary layer, see Rizzi [40]. SU2 is however only able to solve the flow for unstructured grids. Moreover, as the Euler CFD model is used no mixed grid is needed for boundary layer simulation. The reasoning behind the CFD model choice is given in section 3.4 Computational Fluid Dynamics.

The unstructured grid method produces a random collection of elements, filling the area. This can be performed almost completely automatically. The unstructured methods are fast and enable the solution of very large and detailed problems in a relative short period of time, enhancing flexibility. However, the user lacks of control when setting up the mesh. For more detail on mesh characteristics see Rizzi [40]. ANSYS ICEM is used to create the initial unstructured grid used for this optimization, see the ANSYS ICEM CFD User Manual[41] for details on the software.

The initial wing geometry, surface and volume grid used for the test case application will be given in chapter 5. Note that this is only the initial definition. During the course of the optimization the grid and thereby also the geometry of the wing will be deformed. This deformation process is explained in the subsection below.

3.3.1 GRID DEFORMATION

Earlier in this chapter the implementation of the FFD parametrization to deform the geometry surface is explained. One of the large advantages of SU2 is that it also deforms the volume mesh to conform the deformed surface. This releases the user from a lot of work as meshing can be a time-consuming job.

The responsive volume deformation within SU2 is solved iteratively. Within each step the volume grid is deformed based on the classical spring method. In this spring method a stiffness is applied to the connections between two corners of a volume element. This effectively means that every edge of a volume element acts as a spring. The displacement of a corner $\vec{u}_i$ is therefore calculated using the displacements of its neighbours $\vec{u}_j$ using equation 3.5, as published by Palacios et al. [27].

$$\left(\sum_{j \in N(i)} K_{ij} \vec{e}_{ij} \vec{e}_{ij}^T \right) \vec{u}_i = \sum_{j \in N(i)} K_{ij} \vec{e}_{ij} \vec{e}_{ij}^T \vec{u}_j \quad (3.5)$$

Where $N(i)$ is the set of neighbouring corners, $\vec{e}_{ij}$ the unit vector in the direction of the edge connecting the two corners and K_{ij} is the stiffness matrix. The stiffness of the edges are a function of the distance to a wall. By making the volumes which are closest to the wall the stiffest it is less likely that the module creates negative volumes. These negative volumes occur if the edges of a volume switch positions, which happens when the deformation is large and the stiffness is too low. These large deformations occur for the deformation according to the chord length changes, see for the root chord length and taper design variables. As it could not be prevented that the grid deformation module creates negative volumes, these design variables have been bounded. Only lower bounds were found to be necessary. The values of these bounds can be found in chapter

6.

Beside the variables of taper, this method is able to handle all deformations of the grid which are encountered during this optimization. An example of a surface deformation with the accompanying volume grid deformation is illustrated in figure 3.8.

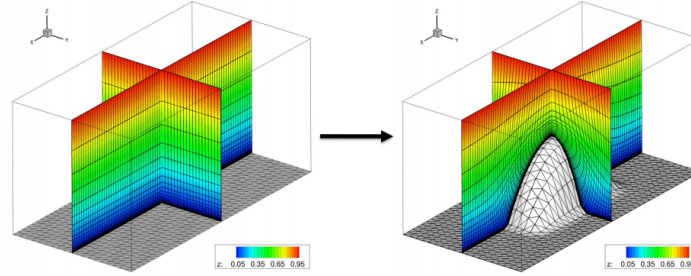


Figure 3.8: Example of a volume grid deformation as a result of a surface deformation[42]

3.4 COMPUTATIONAL FLUID DYNAMICS

In the introduction of this thesis it is already stated that high-fidelity CFD analyses are especially valuable for wing-tip and root optimization and for unconventional design concepts. This is the case in the present optimization as the shape and planform of the wing tip and root are included in the design variables. Furthermore, this thesis forms a basis for other optimization effort, like for example the wing tip or unconventional design concepts.

The RANS CFD model would capture all main physical phenomena of the geometry at hand accurately, as presented by Lyu et al. [43]. The RANS model simplifies the Navier Stokes equations by decomposing them into time-averaged and fluctuating quantities. RANS uses turbulence models to gain the turbulent stresses, being the so-called Reynolds stresses. Because this way the turbulence is not modelled directly by the Navier-Stokes equations the grid can be coarser than for more fundamental methods like the Large Eddy Simulation (LES) and the Direct Numerical Simulation (DNS). This makes RANS method usable for larger scale CFD applications, like aircraft, see Rizzi [40] for more details. A computationally cheaper option is the Euler model. The Euler equations are gained by neglecting viscosity completely within the Navier-Stokes equations. This simplifies the equations greatly, resulting in reduction of computational time but also in accuracy. Lyu et al. [43] performed a wing shape optimization using both the Euler model and the RANS model. It presented that using the Euler model the total drag coefficient at the optimum point was only 3.7 count higher then when using the RANS model. While the same amount of iterations was used, the Euler model saved considerable computational time. It used 26% of the computational time of the RANS solution. This is why the Euler model is used for the aerodynamic and sensitivity analyses in this thesis.

Because viscosity is neglected in the Euler model, the viscous drag component and its sensitivity derivatives are not included. This is why this component has to be calculated by a separate module. This viscous drag estimation module is explained below.

3.4.1 VISCOUS DRAG ESTIMATION

For this optimization the viscous drag contribution cannot be assumed to be constant because the wing surface area will change during the routine. This naturally has a large effect on the viscous drag component. The airfoil shape also has an effect on the viscous drag, but significantly less. The viscous drag estimation used in this report is based on the viscous drag estimation published by Raymer [44]. This estimation uses the flat-plate turbulent skin friction corrected using form factor corrections to include the airfoil thickness effect, see equation 3.6.

$$C_{D_f} = \frac{\sum(K_{form_s} C_{f_s} S_{wet_s})}{S_{ref}} \quad (3.6)$$

Where C_{D_f} is the viscous drag component and S_{wet} the wetted surface area, approximated using equation 3.7. The subscript s indicates that the parameters are taken per spanwise wing section. K_{form} is the form-

factor correction calculated using equation 3.8 and C_f is the flat-plate skin friction coefficient.

$$S_{wet} = S_{ref} (1.977 + 0.52(t/c)) \quad (3.7)$$

$$K_{form} = \left(1 + \frac{0.6}{(x/c)_m} \left(\frac{t}{c}\right) + 100 \left(\frac{t}{c}\right)^4\right) (1.34M^{0.18} (\cos \Gamma_m)^{0.28}) \quad (3.8)$$

In the above equations t/c is the thickness-to-chord ratio, $(x/c)_m$ is the chordwise location of the airfoil maximum thickness point, M the Mach number, Γ_m the sweep of the maximum thickness line.

According to the method of Raymer [44], C_f can be calculated as a weighted average between the laminar flat-plate skin friction coefficient $C_{f_{lam}}$ and the turbulent flat-plate skin friction coefficient $C_{f_{turb}}$. The approach of reversed engineering is used to find the weighting factors. The friction drag is aimed at a value of 0.0049 for the wing of the reference aircraft, taken from Elham and van Tooren [28]. This method found that 29% of laminar flow over the wing is to be used to gain the same friction drag as used in the reference stated. However, in the reference a full turbulent flow over the wing is used. This means that the viscous drag estimation is overpredicted by the present method for the given reference aircraft. However, as a form of correction for this overprediction the unrealistic portion of laminar flow of 29% maintained in use. In chapter 7 a recommendation is added for a better calculation of the boundary layer and the resulting friction drag. The coefficients $C_{f_{lam}}$ and $C_{f_{turb}}$ are calculated using equation 3.9 and 3.10 respectively.

$$C_{f_{lam}} = \frac{1.328}{\sqrt{Re}} \quad (3.9)$$

$$C_{f_{turb}} = \frac{0.455}{(\log_{10} Re)^{2.58} (1 + 0.144M^2)^{0.65}} \quad (3.10)$$

In the above equations Re is the Reynolds number.

3.4.2 VALIDATION

Validation of the SU2 CFD code is provided by Palacios et al. [27] and the viscous drag model has been corrected to provide the friction drag values as found in literature. Furthermore, a grid and domain convergence study is performed in section 5.2 Surface and volume grid to make sure the grid and domain has no or little influence on the flow solution. Lastly, the improvements resulting from the optimization are not compared to other models than the initial model. Because of these reasons the CFD analysis is not validated within this project.

Aside from this, the sensitivities provided by SU2 are verified. This, together with the explanation on how these sensitivities are acquired, is explained in the next section.

3.5 SENSITIVITY ANALYSIS

One of the reasons for using SU2 as a CFD solver is the availability of design sensitivities. When these sensitivities are accurate they can speed up a optimization significantly as they provide the direction towards an optimum. In addition to this, the computational time of the whole process can be greatly reduced when these sensitivities are calculated in an efficient way. A method for efficient calculation of the sensitivities is the adjoint method. The section below will explain this further. Moreover, this section will elaborate on how SU2 obtains the sensitivities with respect to the design vector using the surface sensitivities provided by the adjoint method. Finally the obtained sensitivities will be verified using Finite Differencing (FD).

3.5.1 ADJOINT METHOD

It is known that the adjoint method is a very efficient method for calculation of sensitivities of a cost-function with respect to a large number of design variables. This is because the calculations only have to be performed once per cost-function, instead of once per design variable. The basics of the adjoint approach is explained below.

The derivative to be calculated is given by equation 3.11.

$$\frac{d\mathcal{J}}{dX} = \frac{\partial \mathcal{J}}{\partial X} + \frac{\partial \mathcal{J}}{\partial Y} \frac{dY}{dX} \quad (3.11)$$

In this equation $\mathcal{J}$ is a cost-function, X is the design vector and Y the vector of state-variables. Of the above equation $\frac{dY}{dX}$ is difficult to calculate as its calculation would require one converged flow solution per design variable. In order to solve this in a more efficient way another sensitivity equation is used, being the sensitivity with regard to the residual R . The equation for this sensitivity is given in equation 3.12.

$$\frac{dR}{dX} = \frac{\partial R}{\partial X} + \frac{\partial R}{\partial Y} \frac{dY}{dX} = 0 \quad (3.12)$$

Substituting equation 3.12 into equation 3.11 results in equation 3.13.

$$\frac{d\mathcal{J}}{dX} = \frac{\partial \mathcal{J}}{\partial X} - \frac{\partial \mathcal{J}}{\partial Y} \left(\frac{\partial R}{\partial Y} \right)^{-1} \frac{\partial R}{\partial X} \quad (3.13)$$

The adjoint vector ϕ is given as in equation 3.14, which is called the adjoint equation.

$$\frac{\partial R}{\partial Y} \phi = \frac{\partial \mathcal{J}}{\partial Y} \quad (3.14)$$

The partial derivatives in equation 3.13 are easier to be calculated then the full derivative $\frac{dY}{dX}$. Using the adjoint method the computational effort of calculating the sensitivities is comparable to the computational effort of one converged flow solution. The output of the adjoint method as implemented in SU2 are the sensitivity of C_L and C_D with respect to all grid nodes on the surface of the wing.

Adjoint methods can be divided into continuous and discrete adjoint methods. The continuous adjoint method is implemented in SU2, whereas the discrete is not yet implemented at the date of writing. The difference between the two methods is that for the continuous method the adjoint method is applied at the level of the governing equations and afterwards discretized. Using the discrete method the governing equations are discretized before the adjoint equation is derived. For a derivation of both methods see Palacios et al. [27].

3.5.2 SENSITIVITY PROJECTION

Using the adjoint method described above, SU2 is able to calculate the sensitivity of C_L and C_D with respect to a perturbation of each surface grid node of the wing surface in the direction normal to that surface. Having each node on the surface as a design variable is however not optimal, as stated before. This is why a projection method is implemented in SU2. This method projects the surface sensitivity onto a certain design variable, in this case the FFD control points. The projection is performed using the approximation given in equation 3.15, as published by Economou et al. [45].

$$\frac{\partial \mathcal{J}}{\partial FFD} = \int_S \left\{ \frac{\partial \mathcal{J}}{\partial S} \right\} \frac{\partial S}{\partial FFD} ds \approx \sum_{i \in N(S)} \left\{ \frac{\partial \mathcal{J}}{\partial S} \right\}_i \frac{\vec{n}_i \cdot \Delta \vec{x}_i}{\Delta FFD} \Delta S_i \quad (3.15)$$

Where $\mathcal{J}$ is the cost-function, in this case being C_L or C_D and FFD the design variable. S is the affected surface, $N(S)$ is the amount of nodes on S , $\left\{ \frac{\partial \mathcal{J}}{\partial S} \right\}_i$ the surface sensitivity on node i as calculated by the adjoint method, $\vec{n}_i$ is the unit normal to the surface at i , $\Delta \vec{x}_i$ is the change of node i in Cartesian coordinates after a perturbation of the design variable, and ΔS_i is the area surrounding node i . The geometrical values of the above equation are gained by deforming the FFD coordinates with a small amount. During this project 0.001[m] or 0.001[deg] is taken as perturbation. It was found that an increase or decrease in this value did not have an effect on the sensitivity.

Within SU2 the projection onto FFD control points individually was already implemented. However for this project also the projection on the other (surrogate) design variables is necessary, like the aeroelastic deformation or planform changes. For this purpose the same extension to the SU2 code as described in section 3.2 Free-Form Deformation parameterization was used. This is possible because the values of $\Delta \vec{x}_i$ in equation 3.15 are in essence a grid deformation as discussed in section 3.3 Surface and volume grid.

3.5.3 SENSITIVITY VERIFICATION

Having the correct sensitivity values is important for a smooth optimization process. If these values are not accurate they will have an adverse effect on the optimization process, especially near a minimum, according

to Nielsen and Anderson [36]. This is why it is of high importance to verify the sensitivity derivatives gained by SU2. This section will show this verification.

The sensitivity derivatives gained by the disciplines will be compared with the same derivatives approximated using Finite Differencing (FD). FD is a straight-forward method for derivative comparison, but it is prone to truncation or linearization errors. This is why first an analysis has to be performed to determine the value with which the design variable of interest is changed, being Δx . This analysis has been performed for two sensitivities: The sensitivity of C_L with respect to a vertical translation of the mid-span section in meters and the sensitivity of C_L with respect to the rotation θ of the mid-span section in degrees, see figure 3.9 for the results. The value of 0.0001 m is selected as the Δx value for the FD analysis of the aerodynamic sensitivities with respect to the translations and the value of 0.1 deg is chosen for the sensitivities with respect to rotations. For these values the sensitivities are not affected by truncation nor linearisation errors.

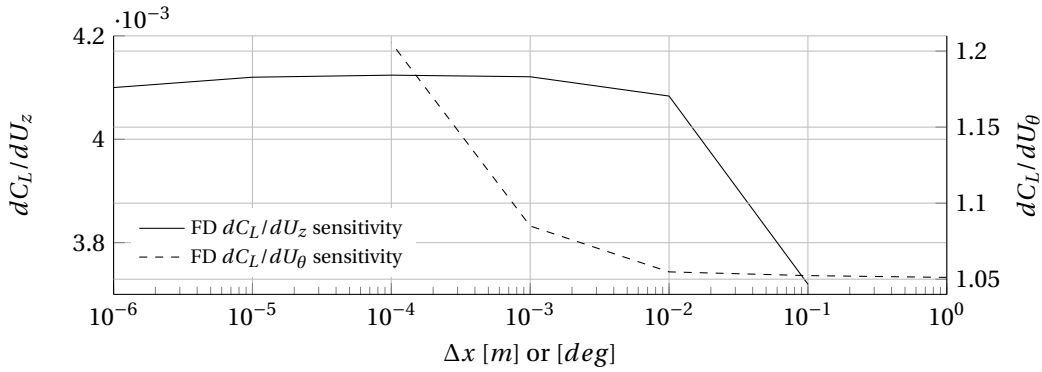


Figure 3.9: FD sensitivity analysis for determination of Δx

In order to perform the sensitivity verification for all aerodynamic sensitivities with respect to 269 design variables one would need to run 270 CFD analyses. In order to save effort only a selection of the sensitivities is compared for the present verification. The design sensitivities of C_L and C_D with respect to the upper airfoil shape design variables, planform variables, angle of attack, aeroelastic deformation surrogate variables and elastic axis position surrogate variables at the mid-span section are verified in this section.

Figures 3.10 and 3.11 and table 3.1 show the comparison between the adjoint and FD method for the sensitivity derivatives provided by SU2.

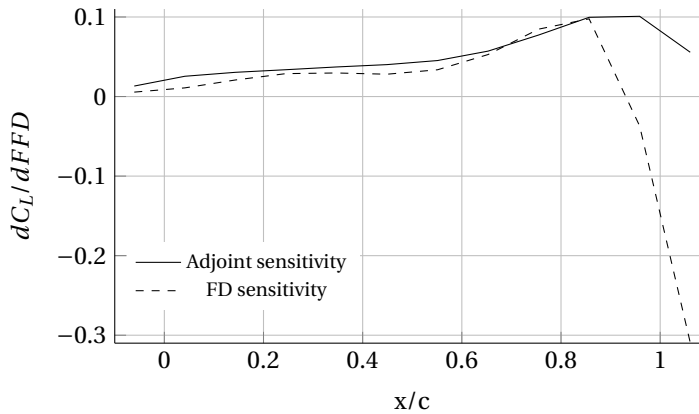


Figure 3.10: Comparison between adjoint and FD sensitivities of lift with respect to the upper side FFD design variables at mid-span

In the figures the sensitivity is plotted against the chordwise positions of the FFD control points. The definition of the planform design variables as used in the table is given in figure 3.6. The subscripts y and z stand for transformations in the y - and z -direction, respectively. The subscripts ϕ and θ stand for rotations around the x -axis and the y -axis, respectively.

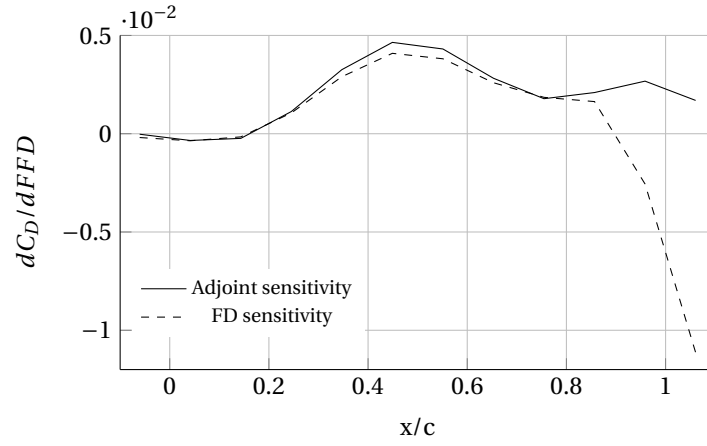


Figure 3.11: Comparison between adjoint and FD sensitivities of drag with respect to the upper side FFD design variables at mid-span

Table 3.1: Comparison between FD and adjoint sensitivities with respect to the P , α , U^* and EA^* design variables at mid-span

Sensitivity	Adjoint	Finite Difference	Difference ratio
dC_L/dP_{c_r}	0.204800	0.049411	0.241264
dC_L/dP_{λ_1}	1.262100	0.548132	0.434302
dC_L/dP_{λ_2}	0.136430	0.073995	0.542368
dC_L/dP_{b_1}	-0.663952	0.073718	-0.111029
dC_L/dP_{b_2}	-0.200557	0.022879	-0.114077
dC_L/dP_{Γ}	-1.267325	-0.091545	0.072235
dC_L/dP_{e_1}	-7.086881	4.367452	-0.616273
dC_L/dP_{e_2}	-0.895321	1.390726	-1.553327
dC_L/dU_y^*	-0.128709	0.002426	-0.018849
dC_L/dU_z^*	1.092200	0.004124	0.003776
dC_L/dU_{ϕ}^*	0.015439	0.003278	0.212328
dC_L/dU_{θ}^*	-0.965147	1.052352	-1.090354
dC_L/dEA_x^*	-0.014966	-0.000058	0.003876
dC_L/dEA_y^*	-0.037037	-0.000141	0.003807
dC_L/dEA_z^*	-0.003868	0.000131	-0.033864
$dC_L/d\alpha$	0.131180	0.138230	1.053743
dC_D/dP_{C_r}	0.014292	0.002727	0.190784
dC_D/dP_{λ_1}	0.072277	0.022210	0.307283
dC_D/dP_{λ_2}	0.006936	0.003262	0.470311
dC_D/dP_{b_1}	-0.024199	0.002855	-0.117979
dC_D/dP_{b_2}	-0.005488	0.000216	-0.039442
dC_D/dP_{Γ}	0.025027	-0.018208	-0.727534
dC_D/dP_{e_1}	-0.343528	0.209228	-0.609057
dC_D/dP_{e_2}	-0.032725	0.050448	-1.541598
dC_D/dU_y^*	-0.004491	0.000203	-0.045207
dC_D/dU_z^*	0.039328	0.000591	0.015027
dC_D/dU_{ϕ}^*	0.000655	0.000172	0.262741
dC_D/dU_{θ}^*	-0.033723	0.042245	-1.252705
dC_D/dEA_x^*	-0.000539	-0.000008	0.014845
dC_D/dEA_y^*	-0.001334	-0.000020	0.014996
dC_D/dEA_z^*	-0.000138	0.000003	-0.021804
$dC_D/d\alpha$	0.006917	0.006974	1.008241

It is observed that most sensitivities with respect to the FFD design variables are in good agreement. This

is except for the last two control points near the trailing edge. Also the sensitivity with respect to α are correct. However, the sensitivities with respect to all other design variables are completely off. Some of them have the wrong sign and the difference ratios variate between 0.0034 and 96.1765, absolutely spoken. This is an issue which has to be solved for the optimization algorithm to find a good optimum. In the section below the origin of this problem is explained and a temporary solution is proposed.

It has to be stated that all the sensitivities calculated in this section using the adjoint method neglect the sensitivities in proximity of the trailing edge. The reason for this is also given in the section below.

3.5.4 SENSITIVITY ERROR ORIGIN

The difference in sensitivity derivatives when compared to FD is too severe to neglect, see the previous section. To gain an understanding of the origin of this error further investigation is needed. In this section this investigation and its results are presented. After this a temporary solution is proposed.

The first striking observation is that the derivatives of lift and drag with respect to the individual FFD control points are most of the time in agreement with the FD results, except for the derivatives near the trailing edge. In figures 3.10 and 3.11 it becomes clear that the nearer the trailing edge of the airfoil, the greater the error in the derivative value. The last two data points are even of the wrong sign. This observation resulted in the hypothesis that the error present at the sensitivity derivatives with respect to P , U^* and EA^* is caused by a large error at the trailing edge of the surface sensitivity. Meaning that the error originates from the continuous adjoint module in SU2.

The above hypothesis can be tested by excluding the trailing edge from the comparison completely. This effectively means that a new FFD box is formed around the wing, excluding the trailing edge. This is illustrated in figure 3.12.

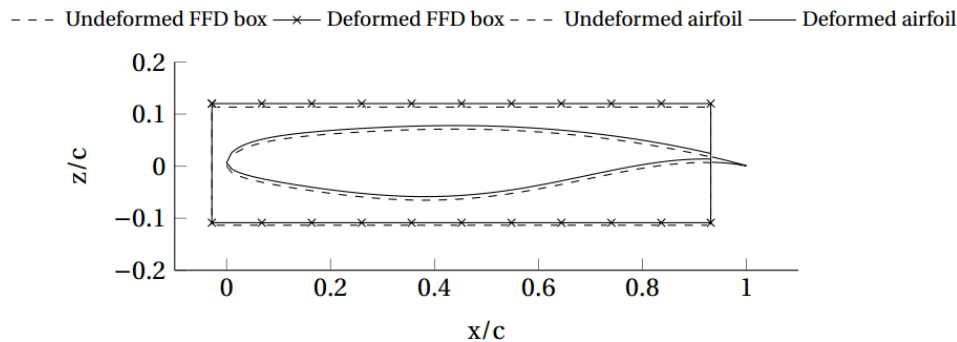


Figure 3.12: An example of sectional deformation without TE

When this new FFD box is deformed, the surface sensitivities of the trailing edge are not included in the projection onto the (surrogate) design variables. Moreover, when the box is deformed for FD analysis, the trailing edge will not deform or move. Nevertheless, a discontinuity is not present because the displaced nodes on the surface grid remain connected to the non-displaced nodes of the trailing edge surface, keeping the surface intact. The comparison of the sensitivity derivatives without trailing edge are given in table 3.2. The same mid-span section is used for this comparison.

It can be observed that the sensitivities are now in much better agreement. This fact confirms the hypothesis of that the trailing edge surface sensitivities are the origin of the error in the previous section.

This error origin was confirmed by the developers of the SU2 code. They state that the surface sensitivities near the trailing edge are inaccurate due to the assumption within the derivation of the continuous adjoint equations that the surface is smooth, see Palacios et al. [39] and CFD-Online¹.

They have not quantified the error, however they do present a solution, which is to discard the trailing edge

¹CFD-Online Forum. <http://www.cfd-online.com/Forums/su2/157134-trailing-edge-sensitivity-anlysis.html>, Last accessed: Sept 2015

Table 3.2: Comparison between FD and adjoint sensitivities with respect to the U^* design variables at mid-span excluding trailing edge

Sensitivity	Adjoint	Finite Difference	Difference ratio
dC_L/dU_y^*	-0.160506	-0.129634	0.807660
dC_L/dU_z^*	0.992069	0.945899	0.953460
dC_L/dU_ϕ^*	0.009432	0.007202	0.763582
dC_L/dU_θ^*	-0.757714	-1.044903	1.379021
dC_D/dU_y^*	-0.006387	-0.015811	2.475318
dC_D/dU_z^*	0.123954	0.107068	0.863773
dC_D/dU_ϕ^*	0.001128	0.001335	1.184011
dC_D/dU_θ^*	-0.019825	-0.063084	3.182011

surface sensitivities completely. This discarding of the trailing edge sensitivities is used for the adjoint calculations given in figures 3.10 and 3.11 and table 3.1, because it resulted in a slight improvement of the values. This solution however does not improve the sensitivity derivatives enough, as can be seen by the comparison with the FD values. This comparison has quantified the error and has shown that the sensitivities at the trailing edge cannot simply be removed.

A real solution to the problem could in theory be to change from the continuous adjoint method to the discrete adjoint method. This way no assumption has to be made with regard to the smoothness of the surface. However, the discrete adjoint method has not yet fully been implemented in the SU2 code. In the near future this method will be available.

Because of the limited time frame of this project, a temporary solution is implemented. First of all, the 4 control points closest to the trailing edge, 2 at the top and 2 at the bottom, are held constant. This way the false sensitivities with respect to the airfoils shape are removed. Second of all, the sensitivity derivatives with respect to the planform variables, aeroelastic deformation surrogate variables and elastic axis position surrogate variables are multiplied with corrective factors. The difference ratios given in table 3.1 are used as the corrective factors. This means that the corrective factors gained by comparing the mid-span section are used for all span wise sections. The author is aware that this is a very crude solution, however it showed to give satisfactory results, see chapter 6. In chapter 7 recommendations will be added to improve the sensitivities for further optimization efforts based on this research.

3.6 FUEL WEIGHT ANALYSIS

The performance analysis exists of the fuel weight analysis. As can be seen in figure 2.3 this analysis receives C_D directly from the aerodynamic analysis and is therefore not a decoupled discipline.

The fuel weight is gained using the fuel-fraction method described by Roskam [46] and given by equations 3.16 and 3.17.

$$W_{fuel} = 1.05 * (1 - M_{ff}) W_{MTO} \quad (3.16)$$

$$R = \frac{V_{inf}}{c_j} \frac{L}{D} \ln(M_{ff_{cruise}}) \quad (3.17)$$

Where in the first equation the factor 1.05 makes sure that there is 5% of fuel in reserve and where M_{ff} is the overall mission fuel-fraction. The fuel-fraction values for all mission elements not being cruise are taken as a constant and the fuel-fraction for cruise $M_{ff_{cruise}}$ is calculated using the lower equation, the Brequet's range equation. In this equation R is the cruise range and c_j the specific fuel consumption at cruise. The lift L is taken to be equal to the mid-cruise weight of the aircraft, as given in chapter 2. The drag D is gained by summing the wing drag with the rest drag of the reference aircraft, being the drag of the engines, pylons, fuselage and tail section. This rest drag is gained through Obert [47].

3.7 ANALYSIS STRUCTURE

At this point it is clear what the aerodynamic discipline consists of and what the capabilities of SU2 include. The missing part is the overall structure in which all these discipline fractions are placed. This structure is given by the flowchart in figure 3.13.

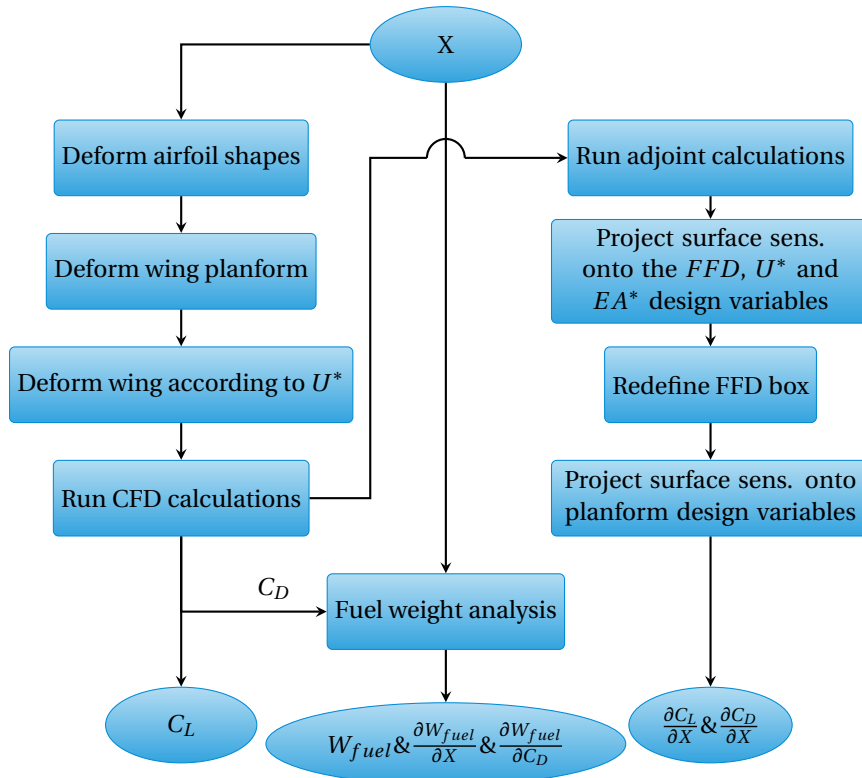


Figure 3.13: Aerodynamic discipline flowchart

It was already shown in figure 2.3 that when 'zoomed out', the discipline receives the design vector and produces C_L , W_{fuel} and the derivatives of both with respect to the design vector. The present flowchart provides the 'zoomed in' view. Below a step-by-step explanation of the flowchart is given, starting with the aerodynamic analysis.

First of all, the airfoil shape is deformed. It is important to state that the mesh at this point is untwisted. This is done because the structural discipline deforms the airfoil shapes with untwisted airfoils. The same is to be done in SU2. After this the wing planform is changed, of which again the airfoil changes are performed first, being the change in chord length. This is followed by the rest of the planform, being the translation of the kink and tip airfoil in x, y and z direction and twist. It is at this moment where also the original twist is added to the wing. The last step in deformation of the wing is the aeroelastic deformation. As stated before, the aeroelastic deformation is performed around the elastic axis of the wing.

After this deformation process the CFD flow analysis is performed, providing the values of C_L and C_D and the complete pressure and velocity distributions on the wing surface. The C_L is a direct output of the discipline and the C_D is fed to the fuel weight analysis. The pressure and velocity distributions are fed to the adjoint calculation module.

The adjoint calculation module uses CFD results to calculate the surface- and angle of attack-sensitivity of the wing. This is done twice, once for the sensitivities of C_L and once for C_D . The surface sensitivity is then yet to be projected to the FFD control points and other design variables to gain the requested sensitivity values. This is done in the next step.

Within this last step the projection onto the FFD , U^* or EA^* design variables is performed. Firstly, it has to be stated that the perturbation of the FFD control points to calculate the sensitivity has to be in the untwisted z direction with respect to the untwisted airfoil to comply with the deformation as performed in the structural discipline. This is why the perturbation direction on this twisted and aeroelastically deformed wing

is also rotated with the same amount as the twist and deformation.

After this the sensitivities with respect to P have to be projected. This actually requires the program to first make a new FFD box around the deformed wing, as the sections of effect are not the airfoils anymore, but the root, kink and tip section.

The fuel weight analysis module uses the design vector X and C_D , as stated. The output is the fuel weight W_f and its partial derivatives with respect to X and C_D . How these derivatives are used can be found in section 2.4 Decoupled sensitivity analysis.

4

STRUCTURAL ANALYSIS

In this chapter the method for calculating the outputs of the structural discipline is given. First of all, a short background will be given on the tools used within this discipline. After that, some details of the method of weight estimation is given. In the following section the aeroelastic deformation analysis is explained, including the Finite Element Analysis (FEA), the aerodynamic analysis and the coupling between the two. Lastly, the constraints on the thicknesses of the wingbox equivalent panels and aileron effectiveness are provided.

4.1 EMWET/FEMWET

The aeroelastic analysis tool 'Finite element based Elham Modified Weight Estimation Technique' (FEMWET), by Elham and van Tooren [48], is used for calculation of the aeroelastic deformation and elastic axis position within this discipline. It is developed in 2015 and was originally an extension to 'Elham Modified Weight Estimation Technique' (EMWET), by Elham et al. [49]. This extension was essential for a more precise estimation of the wing weight and a better distribution of this weight to cope with aeroelastic effects such as aileron effectiveness and, with a lower degree of precision, fatigue.

Within this combination of tools, EMWET is used as an initializer for sizing of the wing structure. FEMWET uses the equivalent panel thicknesses to provide the aeroelastic analysis. However, at this stage the equivalent panel thicknesses are taken as design variables. This results in that FEMWET can be used independently from EMWET after gaining the initial estimation. This is with one exception: When wing structural weight is required, the module for weight estimation of EMWET is used. More detail on this module is given below.

The advantages of FEMWET is that it is computationally cheap and does not need detailed data about the wing structure, which in this stage of design is not available. Furthermore it provides sensitivities of any function of interest with respect to the design variables. The methodology of the aerostructural analysis of this tool is also given below.

4.2 WEIGHT ESTIMATION

As explained above, the wing weight estimation as used in this study is equal to the method used within EMWET. This is because it provides a good correlation between the wing weight estimate and the actual wing weight. As shown by Elham et al. [49], the error remains within 3% for various aircraft from different manufacturers, size and category. It is a Quasi-analytical weight estimation method and can predict the structural weight with the same accuracy as FEA based methods but with a much shorter computational time, as also presented by Elham and van Tooren [50]. Apart from this, the module of EMWET is easily implementable in combination with FEMWET as it also uses the equivalent panels to represent the wing box. Within this method, the wing weight is divided in two parts : primary and secondary wing weight.

The primary wing weight consist of all the parts that are necessary to divert the forces acting on the wing. These are the four equivalent panels of the wing box, the ribs and non-optimum structure weight. The equivalent panels represent each side of the wing box, where the upper and lower equivalent panels represent the box upper and lower skin, stringers and spars caps. The vertical equivalent panels represent the spar webs

of the wing box. The weight of the equivalent panels is gained using its summed volume and the density of the used material. For a better understanding, figure 4.1 provides an illustration of the wing box structure, represented by the 4 equivalent panels.

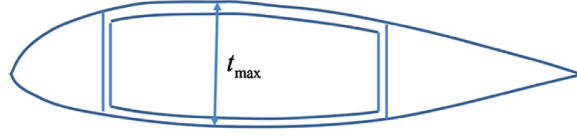


Figure 4.1: Illustration of the wing box structure, represented by 4 equivalent panels[50]

The non-optimum structure weight embodies the sheet taper, joints, cutouts, mountings, connections and attachments. Together with the weight of the ribs its weight is estimated by Elham and van Tooren [28] to be 1.5 times the weight of the wing box existing of only the equivalent panels, W_{wbox} .

The secondary wing weight includes the leading and trailing edge structure, leading and/or trailing edge high lift devices and control surfaces. Torenbeek [51] presents that the total secondary wing weight can be estimated by $15 \cdot S_{ref}$.

The above results in a weight for the complete wing structure W_{wing} given by equation 4.1.

$$W_{wing} = 1.5 \cdot W_{wbox} + 15 \cdot S_{ref} \quad (4.1)$$

With the wing weight and therefore W_{MTO} known, the wing loading WL can be calculated. See chapter 2 for its usage within the WL constraint. Equation 4.2 provides the calculation.

$$WL = \frac{W_{MTO}}{S_{ref}} \quad (4.2)$$

4.3 STATIC AEROELASTIC DEFORMATION ANALYSIS

In order to gain proper static aeroelastic deformation and design sensitivities of aeroelastic phenomena an aeroelastic deformation analysis is to be performed. As can be read above, FEMWET is the appropriate choice at an early design level, like in this study. FEMWET includes both a Vortex Lattice Method (VLM) as aerodynamic solver and a Finite Element Method (FEM) solver to obtain the static aeroelastic equilibrium of the wing. The details of the solvers are given below, followed by the coupling methodology.

4.3.1 GIRD

Firstly, the grids for the VLM and FEM within FEMWET have to be defined. As already stated in chapter 2, 9 airfoil sections are used as design variables for the present optimization problem. At these same positions the aeroelastic deformation and aeroelastic axis positions have to be defined. To omit the need for interpolation, the nodes for the FEA are placed at the same spanwise positions as the 9 airfoil sections. However, to increase the accuracy of the calculated deformation, an extra node is placed between all these airfoil sections. The deformations of these extra nodes are not further used within the optimization problem. The FEA node positions are also used to define the spanwise edges of the VLM mesh. An example of a grid used for the VLM and FEM analyses is given in figure 4.2.

4.3.2 AERODYNAMIC ANALYSIS

The aerodynamic analysis within FEMWET is based on the VLM as presented by Katz and Plotkin [52]. The governing equation to be solved is given in equation 4.3.

$$AIC \cdot \Gamma = V \quad (4.3)$$

Where AIC is the Aerodynamic Influence Coefficient matrix, Γ the vortex strength matrix and V the right hand side. Aerodynamic forces are calculated based on Γ . The AIC and V matrix are calculated based on the mesh used for the vortex lattice method. The Prandtl-Glauert compressibility correction is used to correct for compressibility effects and the boundary conditions are applied on the wing camber line to include the effects of airfoil shape. This airfoil shape is parametrized using the same FFD method as described in chapter

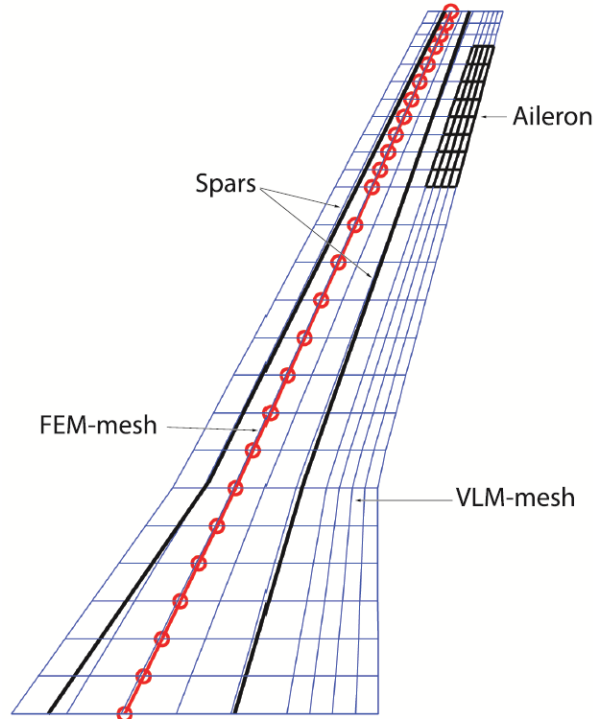


Figure 4.2: Example of the grid for the VLM and FEM analyses within FEMWET[28]

3. The initial airfoil shapes are given using Class Shape Transformation coefficients.

It has to be stated that this separate aerodynamic analysis is required and cannot be replaced with the high-fidelity CFD solver used in the aerodynamic discipline of this optimization problem. This is because SU2 is not able to provide sensitivities w.r.t. the moment and lift distribution.

4.3.3 FINITE ELEMENT ANALYSIS

The FEMWET tool uses a finite beam element analysis representing the wing. The elements of this beam are placed between the nodes which are placed at the elastic axis of the wing. It is assumed that the elastic axis is equal to the shear centre of the sections. These shear centres are calculated at each node with the method presented by Megson [53].

The second output of FEMWET is the deformation vector U . The structural governing equation is used to calculate this vector, see equation 4.4.

$$K \cdot U = F \quad (4.4)$$

Where K is the stiffness matrix and F the force matrix. These matrices are computed using the consistent shape functions for a 3-D 2-node Timoshenko beam element as presented by Luo [54], see Elham and van Tooren [48] for more details on the creation of these matrices. The solution of the coupled aerostructural problem of equation 4.4 is gained by the Newton coupling method, explained in the section below.

Once the nodal displacement is known, the stresses within the equivalent panels can be calculated. This is done using both the displacements and the geometry of the wing box equivalent panels. For this analysis each of these panels, situated between two nodes, are divided into smaller elements, 4 elements for the upper and lower panels and 2 for the front and rear panels. In section 4.4 Failure modes and aileron effectiveness it is explained how the yield and buckling stresses and the failure modes of these panel elements are calculated.

4.3.4 AEROSTRUCTURAL COUPLING

The aerodynamic and structural data are coupled. The Newton method is used by FEMWET to solve the system. Solving the system is necessary to guarantee its consistency, meaning that all residuals of the governing equations are 0. The system of governing equations is given in equation 4.5.

$$R = \begin{bmatrix} A(\Gamma, U, \alpha) \\ S(\Gamma, U, \alpha) \\ W(\Gamma, U, \alpha) \end{bmatrix} = \begin{bmatrix} AIC \cdot \Gamma - V \\ KU - F \\ L - n \cdot W_{MTO} \end{bmatrix} = 0 \quad (4.5)$$

Where A , S , and W stand for the residuals of the aerodynamic, structural and weight governing equations, respectively. The first two have already been introduced in the sections above, while the last one is introduced to maintain level flight by equating lift to the weight of the aircraft. Γ , U and α are the state variables of the same governing equations. Within this coupled system is the AIC matrix dependent on U , V dependent on U and α and both F and L are dependent on Γ .

The Newton method finds the update of the state variables using equation 4.6. This updating of the state variables is repeated until the norm of the update vector is less than 10^{-9} .

$$\begin{bmatrix} \frac{\partial A}{\partial \Gamma} & \frac{\partial A}{\partial U} & \frac{\partial A}{\partial \alpha} \\ \frac{\partial S}{\partial \Gamma} & \frac{\partial S}{\partial U} & \frac{\partial S}{\partial \alpha} \\ \frac{\partial W}{\partial \Gamma} & \frac{\partial W}{\partial U} & \frac{\partial W}{\partial \alpha} \end{bmatrix} \begin{bmatrix} \Delta \Gamma \\ \Delta U \\ \Delta \alpha \end{bmatrix} = - \begin{bmatrix} A(\Gamma, U, \alpha) \\ S(\Gamma, U, \alpha) \\ W(\Gamma, U, \alpha) \end{bmatrix} \quad (4.6)$$

As can be seen, for the Newton method coupled derivative information is required. These derivatives are gained using Automatic Differentiation and analytical techniques.

4.4 FAILURE MODES AND AILERON EFFECTIVENESS

The yield stresses are dependent on the used material, while the buckling stresses are computed by the FEM in the following way: The upper and lower panel buckling stress is calculated using the stiffened panel efficiency method, as presented by Niu [55]. The vertical panels are subjected to shear buckling, of which the stress is calculated using the same stiffened panel efficiency method. From these stresses the failure modes in equations 4.7 to 4.9 are defined.

$$F_{yield} = \frac{\sigma \cdot SF}{\sigma_{yield}} + \left(\frac{\tau \cdot SF}{\tau_{yield}} \right)^2 - 1 \quad (4.7)$$

$$F_{buckling} = \frac{\sigma \cdot SF}{\sigma_{buckling}} - 1 \quad (4.8)$$

$$F_{shear buckling} = \frac{\tau \cdot SF}{\tau_{buckling}} - 1 \quad (4.9)$$

This results in 10 constraints per beam element. Compression, tension and buckling for the upper and lower equivalent panel and shear load and shear buckling for each of the vertical panels. However, the amount of used constraints are less as some failure modes are not relevant, for example the tension failure mode at a panel under compression.

Fatigue is represented by a constraint on the stress on the lower equivalent panel: under a gust load of 1.3g the stress must be lower than 42% of the maximum allowable stress of the material. This estimation of fatigue is suggested by Hurlimann [56] and is used by Kenway and Martins [20] and Elham and van Tooren [48].

A total overview of the equality and inequality constraints can be found in table 2.2.

Aside from the structural and fatigue constraints governed by the failure modes, a constraint is added on the aileron effectiveness. It is presented by Elham and van Tooren [48] that aileron effectiveness is an active constraint, showing its importance for an aeroelastic aerostructural optimization. The constraint makes sure that the rolling moment due to a deflection of the aileron L_δ is equal or higher than the value of the reference aircraft. FEMWET provides this value and its sensitivity with respect to the design variables.

4.5 VALIDATION AND VERIFICATION

The validation of the output of the weight estimation module and FEMWET is published by Elham et al. [49] and Elham and van Tooren [28]. As the methodologies of these programs are not changed for this project, no additional validation needs to be performed.

However, one change to FEMWET is performed: The FFD parameterization is included for 2D airfoil shape

deformation. As SU2 already implemented this parameterization approach, the deformations can be compared. This can serve as verification of the FFD parameterization implementation in FEMWET.

Figure 4.3 shows the deformation as performed by SU2 in comparison with the deformation using the FFD approach in FEMWET. For both deformations 24 control points were used. Two control points were moved on the upper side and one on the lower. As can be seen, the shape deformations are a match.

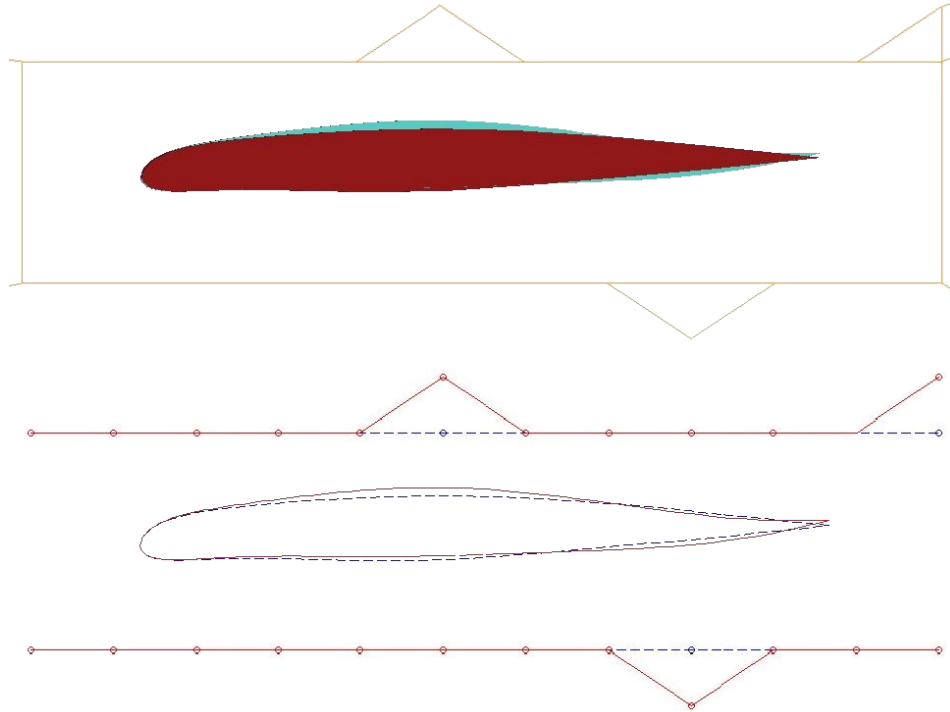


Figure 4.3: Verification of implementation of the FFD method in FEMWET (below) using implementation of SU2 (above)

4.5.1 SENSITIVITY VERIFICATION

For the same reasons named above, not all sensitivities need to be verified. This does not count for the sensitivities with respect to the *FFD* design variables, as they are newly implemented. The method of verification is equal as the one in chapter 3, section 3.5 Sensitivity analysis.

In table 4.1 the sensitivity derivatives calculated by FEMWET using AD are compared with the same sensitivities calculated using FD. The Δx is taken to be $1e-6$, the same value as used for verifying the sensitivities of the original FEMWET publication. The section used for verification is again at mid span, with the leading edge upper FFD control point as the design variable.

Table 4.1: Comparison between FD and AD method of FEMWET sensitivity derivatives

Sensitivity	Adjoint	Finite Difference
$dU_z/dFFD_{LE}$	0.00364	0.00364
$dEA_z/dFFD_{LE}$	0.00780	0.00780
$dF_{buckling\ upper}/dFFD_{LE}$	0.00673	0.00673
$dF_{shear\ front}/dFFD_{LE}$	-0.01602	-0.01602
$d\eta/dFFD_{LE}$	0.00347	0.00347

The verification of the structural analysis is considered a success as the sensitivities are a match.

5

TEST CASE APPLICATION

By now, the optimization problem, the disciplines and their content are explained. This chapter will apply this optimization tool to a test case. As stated in chapter 1, the reference aircraft used for this test case is a modern high-speed transport aircraft, the Airbus A320. The geometrical, aerodynamic and structural data is gained through the book of Obert [47].

5.1 LOAD CASES

The load cases of the reference aircraft used in this study can be found in table 5.1 and are estimated using the load diagram of an aircraft of the same class, presented by Dillinger et al. [57].

Table 5.1: Load cases for the Airbus A320 aircraft

Load case	type	M	q[Pa]	n[g]	H[m]
1	pull up	0.89	21200.0	2.5	7500
2	pull up	0.58	23500.0	2.5	0
3	push over	0.89	21200.0	-1.0	7500
4	gust	0.89	21200.0	1.3	7500
5	roll	0.83	29700.0	1.0	4000
6	cruise	0.82	10650.0	1.0	11000

The first three of these load cases are used for evaluation of the constraints on structural failure. The fourth is used for a constraint on fatigue of the wing structure. For the first four load cases the weight of the aircraft is taken to be the maximum take-off weight. The fifth load case in table 5.1 is used for the aileron effectiveness constraint. The last load case is used for the calculation of U and EA and for the aerodynamic discipline. For the last two load cases the aircraft mid-cruise weight is taken, as given in equation 5.1 as presented by Torenbeek [29].

$$W_{cruise} = \sqrt{W_{MTO}(W_{MTO} - W_{fuel})} \quad (5.1)$$

5.2 INITIAL SURFACE AND VOLUME GRID

In the present section the initial wing geometry, surface and volume grid is presented. This will only be the initial grid as throughout the optimization the grid is deformed as explained in section 3.3 Surface and volume grid.

5.2.1 GEOMETRY

The planform data of the reference aircraft is presented in table 5.2, which results in a planform-view of the wing as given in figure 5.1.

The original airfoils of this wing are defined at four locations: 0, 0.33, 0.66 and 1 span length. This data is interpolated to find the initial airfoil shapes at the airfoil sections used in this optimization.

Table 5.2: Reference aircraft Airbus A320 wing geometry parameters[47]

Position	x	y	z	Chord [m]	Twist at 1/4 chord [deg]
Root	0	0	0	7.0518	0
Kink	3.3006	6.3403	0.5547	3.7584	-2.50
Tip	8.8306	16.9635	1.4841	1.4958	-2.50

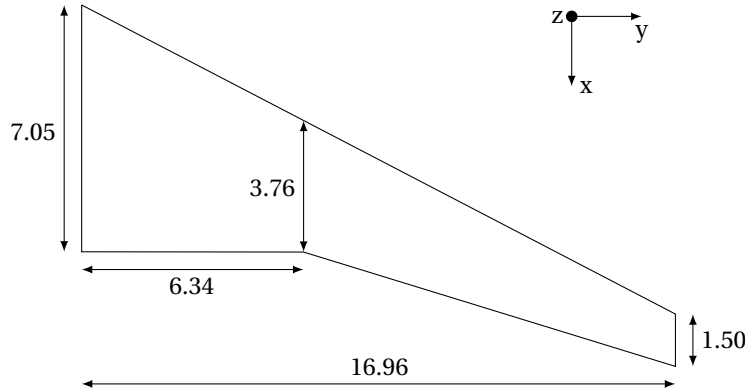


Figure 5.1: Reference aircraft Airbus A320 wing planform view in meters

5.2.2 INITIAL SURFACE AND VOLUME GRID

Before the grid can be created the domain size of the grid has to be determined. It is important that the edges of the domain do not interfere with the flow solution on the surface of the wing. To confirm this independence several domain sizes were investigated and tested. For this investigation the outlet position was handled separately, due to the presence of the wake of the wing. By visual inspection of the converged flow solution the approximate width and height of the domain and length in front of the wing was already found. This was done by making sure that the pressure disturbance due to the wing was relatively small in size at the inlet, upper, lower and side plane of the domain. This estimate was however just a starting point. Afterwards the domain was decreased in size with steps of approximately 10%. This was continued until the change in lift and drag coefficient became higher than 0.1%. After this, the outlet position was independently decreased until the same criterion was met. The result of this investigation is a domain of 250m wide, 650m deep and 600m high, see figure 5.2.

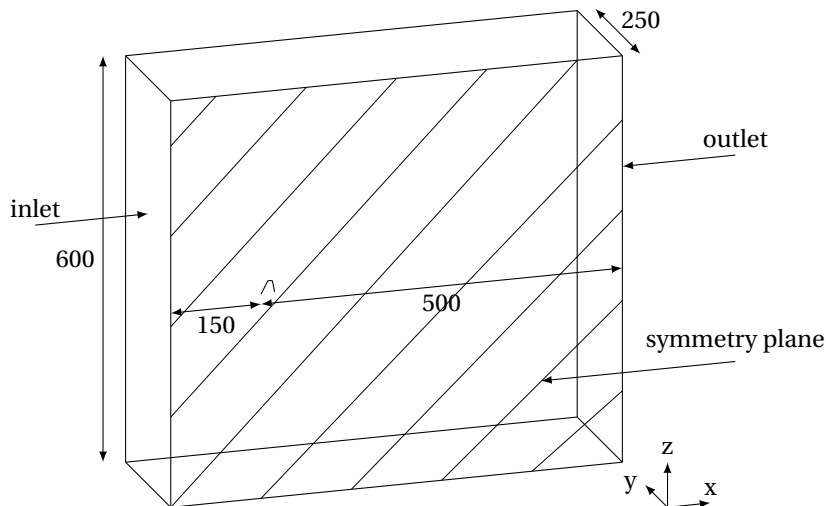


Figure 5.2: Grid domain in meters

Creating a grid is always a compromise between accuracy and computational speed, as in CFD the discretized

governing flow equations are solved for each of the cells in the volume grid. This is why a grid convergence study is performed. During this study a similar method is used as for determining the domain size as explained above. This time the maximum size of the elements on each surface is increased separately until the change in lift and drag coefficient become higher than 0.2%. The step-size of increase varied per surface, for the maximum element size the increase was 100%, for the upper and lower wing surfaces 0.05 and for the wing tip and trailing edge 0.01. This resulted in the maximum grid size values as given by table 5.3 and the surface grid as shown in figure 5.3. The total amount of elements in the converged grid is 1,134,344. This grid is used as the initial point of the optimization.

Table 5.3: Grid maximum element sizes on surfaces for converged grid

Surface	Maximum element size
Far-field	16 [m]
Wing upper surface	0.3 [m]
Wing lower surface	0.3 [m]
Wing tip surface	0.08 [m]
Wing trailing edge line	0.03 [m]

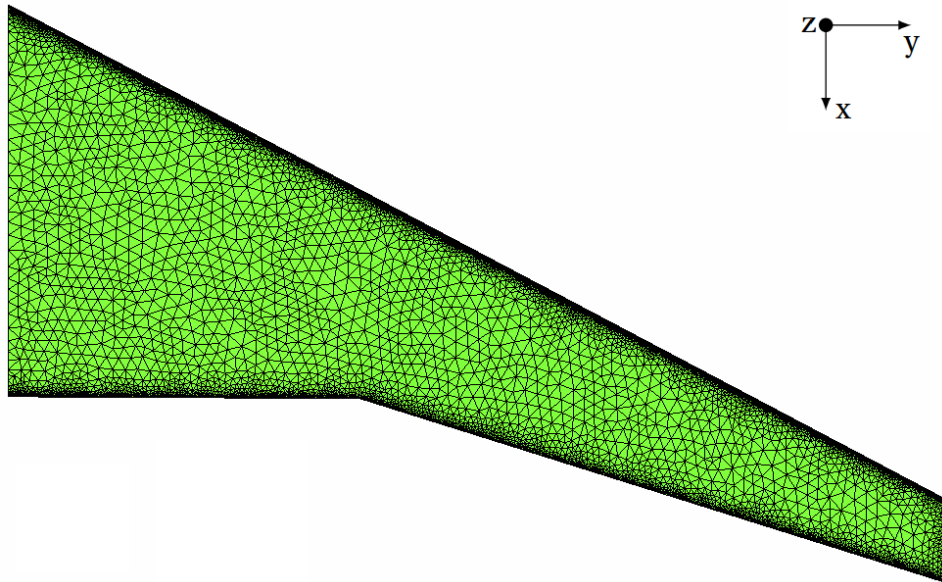


Figure 5.3: Planform view of the converged surface grid of the upper wing surface

It has to be noted that the created surface grid representing the wing is not twisted at all. The original twist given by the geometry of the reference aircraft is added at a later stage, see section 3.7 Analysis structure. This is done because in the structural discipline the shape of the airfoils is deformed using untwisted airfoils. To maintain coherence, the aerodynamic discipline also deforms the shape of the untwisted airfoils, after which the twist is added later using grid deformation.

6

RESULTS

After having described the decoupled disciplines and the optimization problem in which they operate, the results of this optimization can be shown and discussed. It is chosen to perform four optimizations: One pure aerodynamic optimization, one aerostructural optimization with a fixed planform, being the original planform of the reference aircraft, and two complete aerostructural optimizations including the planform design variables as described in chapter 2. Of the last three optimizations, it is expected that the third and fourth optimization problem will show a greater reduction in fuel weight, because of the increased degrees of freedom in the design space.

In the optimization code, the objective function and design variables of all these optimizations are normalized.

All results are gained using 8 3.50 GHz processors and 63 GB of RAM (Random Access Memory). Each single function evaluation has a computational cost of around 1 hour, for all optimization problems.

6.1 AERODYNAMIC OPTIMIZATION

The first optimization which is performed is a pure aerodynamic optimization. This is done to prove the functioning of the aerodynamic discipline when it is steered by the optimization algorithm. The U value of the initial wing is held constant and applied throughout the optimization. The objective of this optimization is to reduce C_D by using the FFD and α design variables only. C_{D_f} is held constant throughout the optimization. The problem is constraint by two constraints, being that the C_L value has to remain equal to the initial value and that the maximum thickness of each wing section has to be equal or greater than the initial value. The problem is formulated as given in equation 6.1.

$$\begin{aligned} \min \quad & C_D(X) \\ & X = [FFD \ \alpha] \\ \text{s.t.} \quad & C_L / C_{L_{init}} - 1 = 0 \\ & t_{max_{init}} - t_{max} \leq 0 \end{aligned} \tag{6.1}$$

The optimization results are shown in table 6.1

Table 6.1: Top-level aerodynamic optimization results

Parameter	Value
Objective function value	0.8382
Iterations	15
Function evaluations	131
Maximum constraint violation	1.5797e-4

The C_D value reduced with 16%, the constraints on t_{max} have not been violated and the constraint on C_L is violated with $1.5797e - 4$. However, this constraint violation translates to a violation of C_L of 0.02%, which is an acceptable amount. The total computational time of this optimization is 104 hours. Further details of the

results of this optimization are given below.

The airfoil shape changes not only reduced drag but also managed to reduce the α from 3.1584deg to 3.0022deg . The lift distribution comparison between the initial wing and the optimized wing is given in figure 6.1.

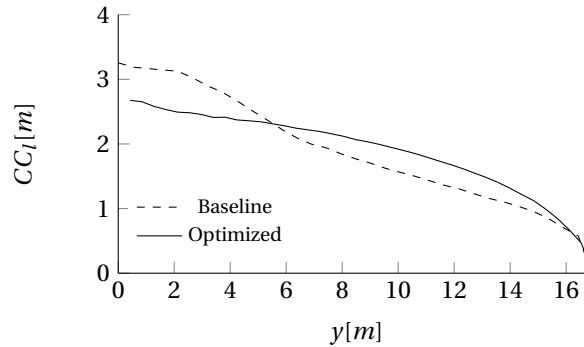


Figure 6.1: Lift coefficient distribution comparison between initial and optimized wing of the aerodynamic optimization

As can be seen, the optimization managed to make the lift distribution much more elliptical. The result of this is an induced drag reduction. The amount of this induced drag reduction can not be given, because SU2 is not able to divide the computed inviscid drag into induced and wave drag components.

The total drag reduction originated from a combined reduction of wave drag and induced drag. The reduction of wave drag can be investigated using the plots of the pressure coefficient and Mach number distributions, which are given in figures 6.2 and 6.3 respectively, and the airfoil shape changes and sectional pressure distributions, which are plotted in figure 6.4. From these plots it is found that especially at the inboard half of the wing the shock wave is removed, or at least reduced in strength. Around the mid-span section a shock wave remains near the trailing edge.

The change in spanwise load distribution as given above is also visible in these plots. Front loading is increased over the whole span, while the rear loading is reduced for the inboard sections of the wing.

The increased front loading together with the decreased rear loading of the inboard sections also result in that the pressure coefficient isobars run more parallel to the leading edge. This way more of the potential advantage of wing sweep is realized, as the root and tip effects are minimized, see Obert [47] for more explanation on straight isobars.

This described airfoil loading is a direct result of the airfoil shape changes. It is visible that for the inboard part of the wing the leading edge thickness increased and that the overall camber decreased. For the outboard airfoil sections the overall thickness and camber is increased.

With these results it is shown that the aerodynamic discipline reacts appropriate when it is controlled by the optimization algorithm and that the aerodynamic analysis is not only able to reduce induced drag and wave drag at the same time, but also to find an optimal balance between them.

6.2 FIXED PLANFORM AEROSTRUCTURAL OPTIMIZATION

For this optimization problem the planform is kept fixed throughout the optimization. This is done to show the potential and proof of functioning of the decoupled aerostructural system.

In order to realize keeping a fixed planform, three adaptations are made to the initial problem description. First of all, the P vector in the design variables is removed. Secondly, the wing loading constraint is removed as the wing surface will not change. It is expected that the wing loading will only decrease, as the W_{MTO} is expected to decrease. The third adaptation is that even though the airfoil shape changes, the friction drag is assumed to be constant. The last assumption is made because the wing surface has the greatest effect on the friction drag estimation and this parameter will not change. Equation 6.2 shows the formulation of this adapted optimization problem.

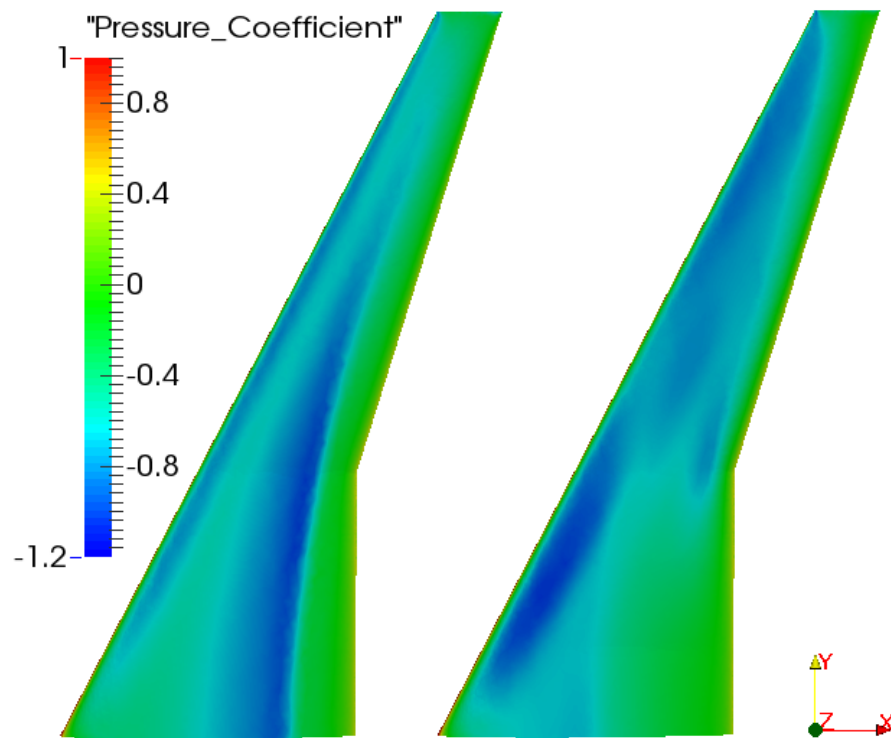


Figure 6.2: Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing of the aerodynamic optimization

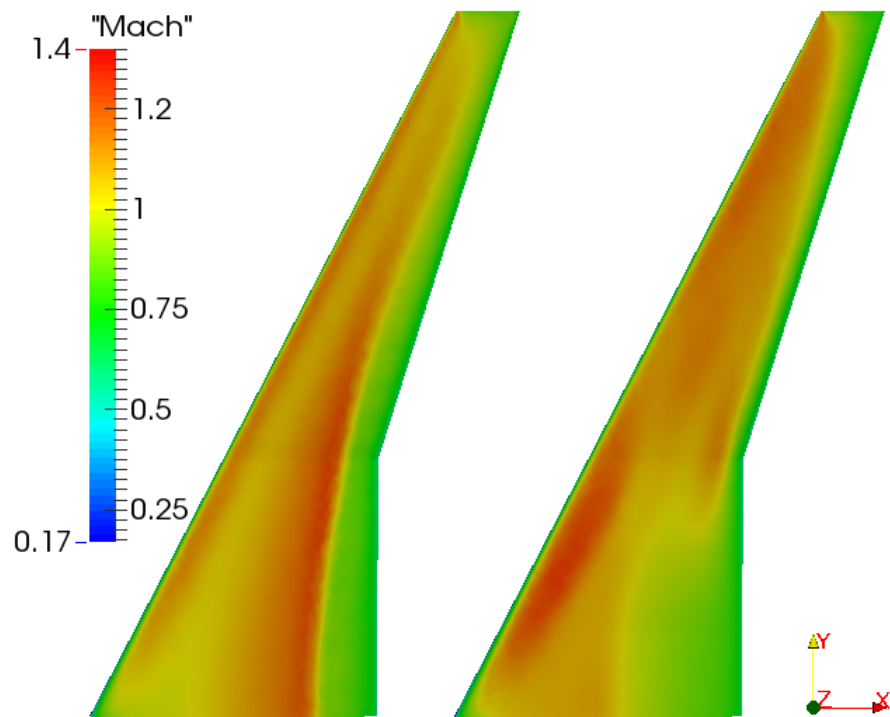


Figure 6.3: Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing of the aerodynamic optimization

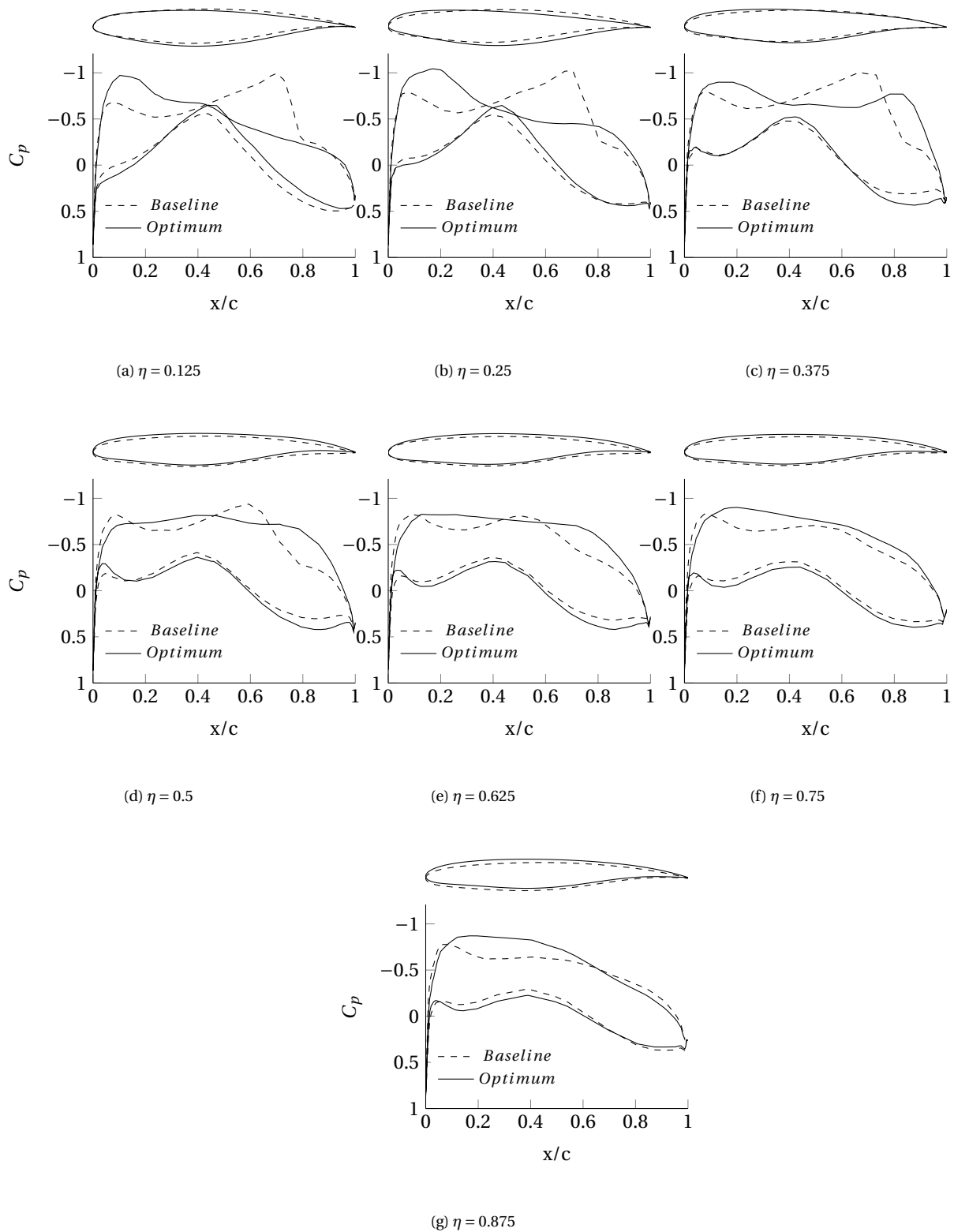


Figure 6.4: Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing of the aerodynamic optimization

$$\begin{aligned}
& \min W_{fuel}^*(X) \\
& X = [FFD \ T \ \alpha \ U^* \ EA^* \ W_{fuel}^* \ W_{MTO}^*] \\
& s.t. \ F_i \leq 0 \\
& \quad 1 - L_\delta / L_{\delta_0} \leq 0 \\
& \quad U/U^* - 1 = 0 \\
& \quad EA/EA^* - 1 = 0 \\
& \quad C_L/C_{L_{cruise}} - 1 = 0 \\
& \quad W_{fuel}/W_{fuel}^* - 1 = 0 \\
& \quad W_{MTO}/W_{MTO}^* - 1 = 0
\end{aligned} \tag{6.2}$$

Table 6.2 provides the basic results of the fixed planform optimization.

Table 6.2: Top-level fixed-planform optimization results

Parameter	Value
Objective function value	0.9333
Iterations	18
Function evaluations	90
Maximum constraint violation	1.3952e-5

On the used hardware the computational time for this optimization is around 90 hours. This is very little taken into account that the optimization including high-fidelity analysis has been performed on only 8 processors, while in earlier studies the amount of used processors is significantly higher and the total time to reach the optimum was similar, see for example Brezillon et al. [14].

From the result presented above it can be seen that 7% of fuel weight is saved with a negligible maximum constraint violation.

Table 6.3 gives the optimization results for the aerodynamic and structural properties of the aircraft.

Table 6.3: Aerodynamic and structural fixed-planform optimization results

	$\alpha[deg]$	C_L	C_D	C_{D_f}	$W_{MTO}[kg]$	$W_{fuel}[kg]$	$W_{wing}[kg]$
Initial	3.1584	0.5228	0.0195	0.0049	73,500	17,940	8,801
Optimized	2.5924	0.5209	0.0169	0.0049	72,571	16,743	9,068

It is observed that C_D is reduced with 13% while C_L is only slightly reduced. The reduction of C_L is a direct result of the reduction in W_{MTO} and W_{fuel} . The reduction of C_D is due to the improved shape of the airfoils which not only reduced wave drag, but also the induced drag through the improved spanwise lift distribution. This lift distribution of the initial and optimized wing is given in figure 6.5. The airfoil shape changes also increased the total lift at constant α . This last result is visible in the reduction of α , while the C_L remained almost constant. The total L/D increased with 15%, from 26.81 to 30.82.

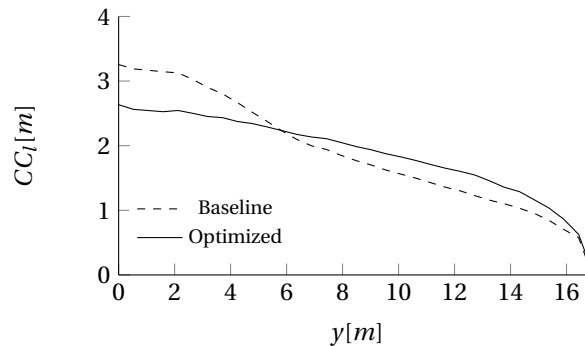


Figure 6.5: Lift coefficient distribution comparison between initial and optimized wing with fixed-planform

W_{MTO} is reduced with 1% and W_{wing} is increased with 3%. This is where the price is paid for the more elliptic

lift distribution. However, it seems worth it to pay that price, as the reduction of fuel use is stronger than the increase in structural wing weight. The fact that the outer wing sections produce more lift increases the wing bending. This is compensated by the heavier structure, but not completely. The vertical wing bending of the tip in the cruise condition increased with 15%, from 0.57m to 0.66m . This increase in wing flexibility is also visible in figure 6.6, which shows the jig shape and 2.5g-shape at sea level of the optimized wing and the initial wing. In figure 6.7 the twist distributions of the initial and optimized wing are plotted for their jig shape and the 1g-shape. Also here the increased wing flexibility becomes visible. The tip twist deformation at cruise condition increased with 37%, from -1.4° to -2.0° .

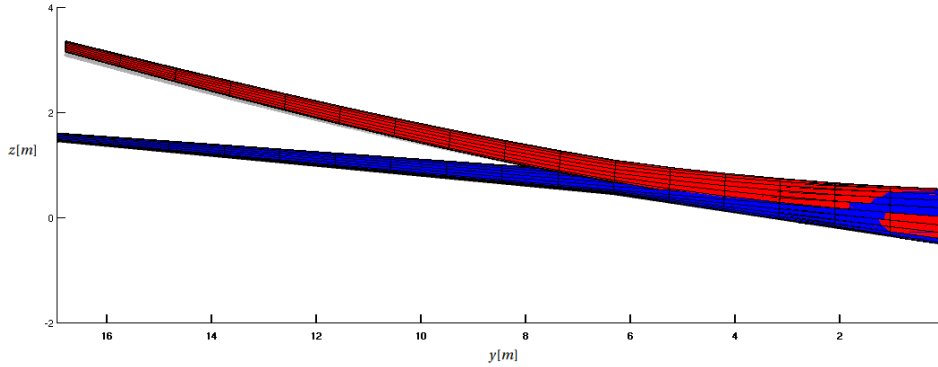


Figure 6.6: Comparison between the wing jig shape (blue) and the shape under 2.5g pull up load at sea level of the initial (grey) and optimized (red) wing with fixed-planform

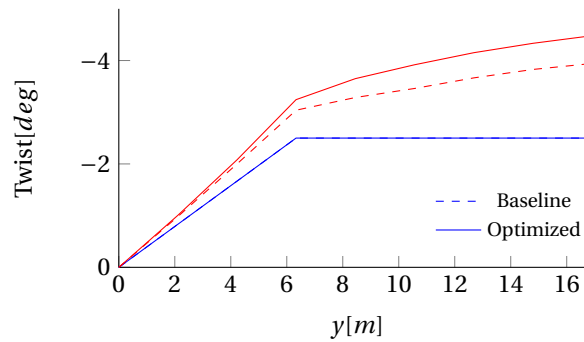


Figure 6.7: Twist deformation distribution comparison between initial and optimized wing with fixed-planform of the jig shape (blue) and the 1g shape (red) wing with fixed-planform

Furthermore, the comparison between the initial and optimal pressure coefficient and Mach number distribution on the upper surface of the wing are plotted in figures 6.8 and 6.9 respectively. From these figures a couple of things can be observed. Firstly, the shock wave is reduced in strength. For the reference wing a strong shock wave was present over almost the complete wing, whereas for the optimized wing this strong shock wave only appears at the outboard half of the wing. A logic explanation of this outboard shock wave is that the optimizer has to counter-act the increased negative twist. This way the outboard loading remains high and the elliptical lift distribution is maintained. The rest of the wing experiences weaker pressure jumps. The airfoil pressure distributions given in figure 6.10 show this in more detail.

Secondly, the Mach number and pressure coefficient plots also show that the optimization resulted in isobars which are more parallel to the leading edge. Just like as explained in the section above, this also results in reduced wave drag. One has to be careful for shock-induced boundary layer separation. Due to the larger aft loading at the tip-sections of the optimized wing the local Mach number has increased. Obert [47] states that shock-induced boundary separation occurs for local Mach numbers of 1.35 to 1.45. Figure 6.9 shows that these Mach numbers are however not reached at the outboard half of the wing. Also at the leading edge of the inboard part of the wing the local Mach number does not reach 1.35, the maximum Mach number is however close: 1.34. It is therefore recommended that more research is performed to make sure the boundary layer does not separate in this area.

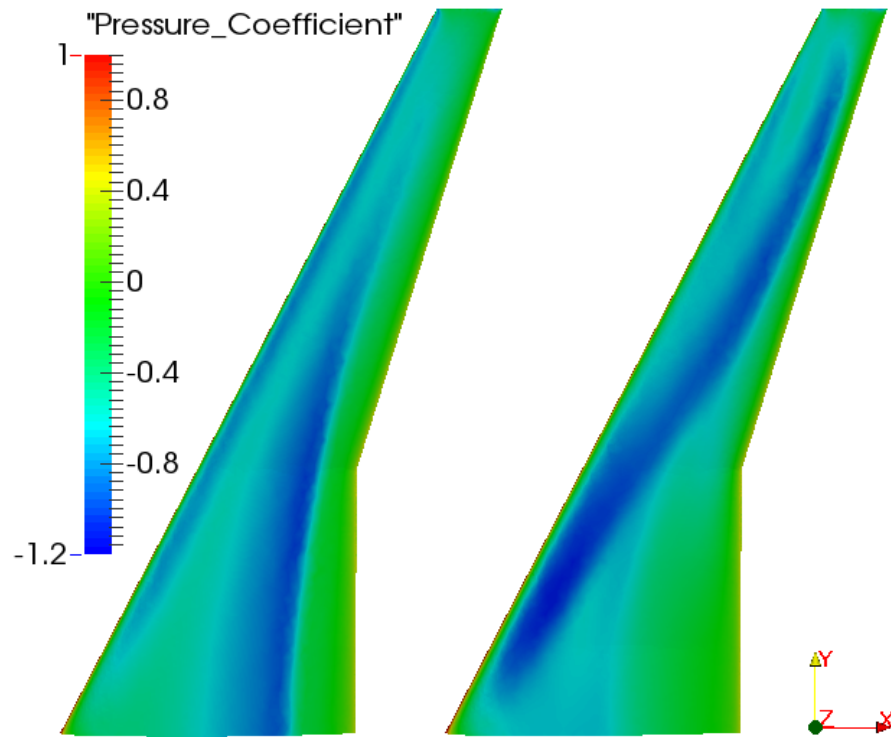


Figure 6.8: Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing with fixed-planform

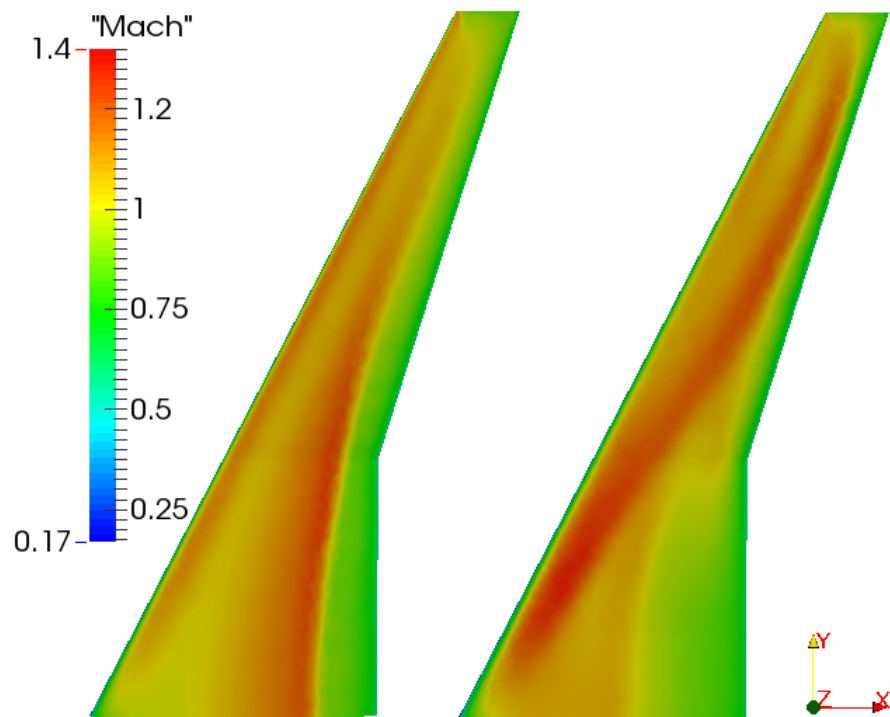


Figure 6.9: Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing with fixed-planform

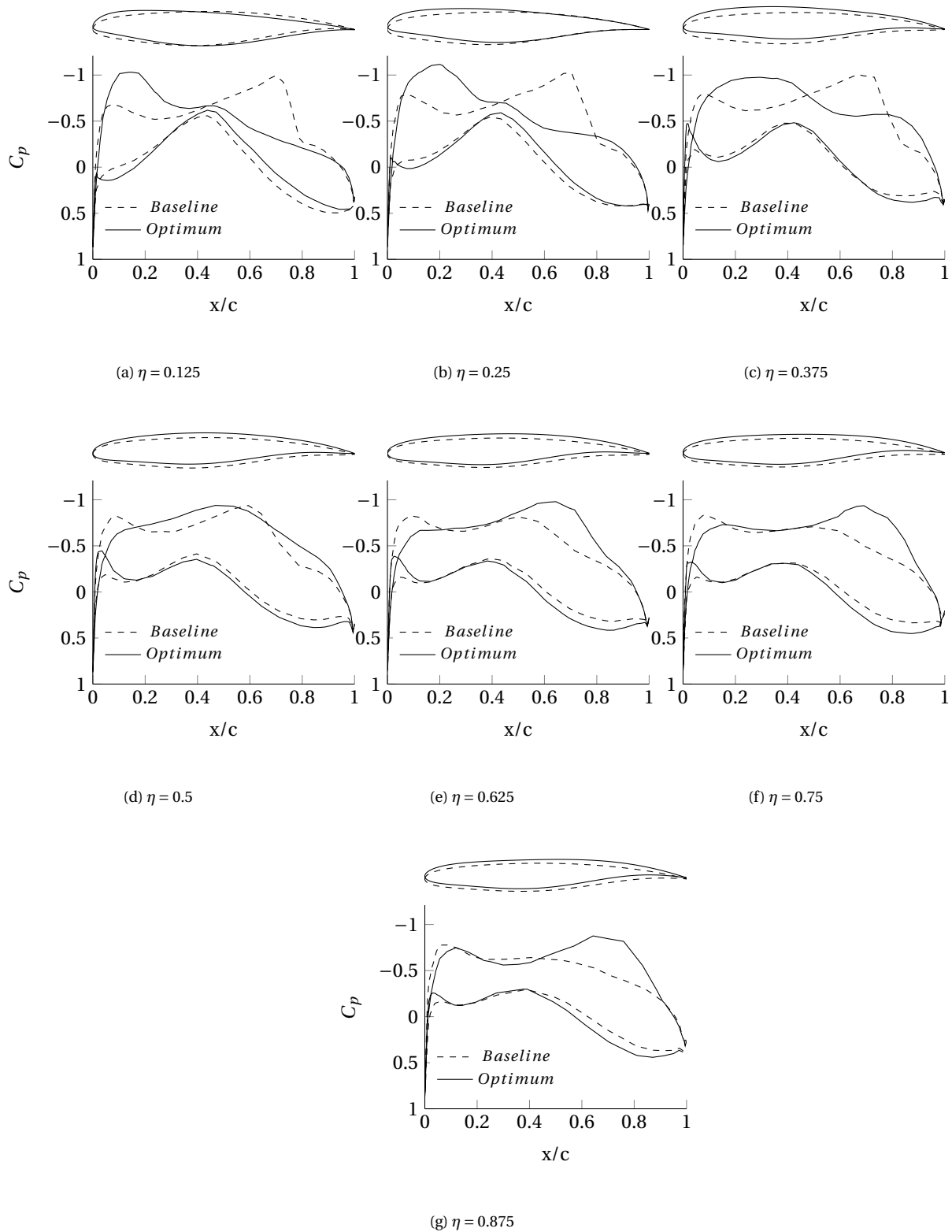


Figure 6.10: Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing with fixed-planform

Figure 6.10 provides the airfoil shapes accompanied by their pressure distributions. It is found that at the root of the wing the camber is decreased and that the leading edge of the airfoils are thickened, which is similar to the pure aerodynamic optimization. This is a desired result from a design point of view because the isobars are straightened and the section pitching moment coefficient is lowered.

In the middle sections the camber is increased, resulting in the more elliptical lift distribution.

Finally, at the tip sections aft loading is generated by thickening the trailing edge of the airfoils. This generates more lift at the outboard part of the wing.

From the presented results it can be deduced that the induced drag is reduced due to the improved lift distribution and that the wave drag is reduced due to the reduction of shock wave strength, especially at the inboard half of the wing, and isobars which are more parallel to the wing leading edge. These changes resulted in the inviscid drag coefficient to drop 18%, from a value of 0.0146 to 0.0120.

6.3 COMPLETE AEROSTRUCTURAL OPTIMIZATION

In the previous section it was shown that the fixed planform optimization resulted in a feasible wing design with a fuel reduction of 7%. Furthermore, the effects of design variables on the state variables and the objective function were confirmed. In this section the results of the full optimization, including the planform design variables, are presented. However, this section is divided in two parts: Firstly, an optimization is performed using the sensitivities of the aerodynamic discipline gained using the adjoint formulation as described in chapter 3. However, as described in the same chapter, these sensitivities with respect to the planform design variables had to be corrected using a simple correction factor.

The second part of this section shows the results of the same optimization problem but where the sensitivities with respect to the planform design variables are calculated using FD. Because this way of calculating the sensitivities is computationally much more expensive, the amount of planform design variables is reduced from 8 to 6.

6.3.1 ADJOINT PLANFORM SENSITIVITIES

This subsection provides the results of the optimization problem as described throughout this report. The normalized lower bounds the taper and root chord design variables are 0.99 on λ_1 , 0.85 on λ_2 and 0.9 on c_r . These bounds have been stretched until negative volumes appeared because of the deformation module in SU2.

Table 6.4 provides the basic results of this optimization.

Table 6.4: Top-level complete optimization results

Parameter	Value
Objective function value	0.8846
Iterations	1
Function evaluations	50
Maximum constraint violation	6.6888e-2

It can immediately be seen that the results are disappointing. The optimizer could not find a feasible optimum, which is visible through the severe constraint violation. The large amount of function evaluations without proceeding to the second iteration shows that the optimization routine tried to find the right search direction for an feasible better point in the design space, but failed to do so due to false sensitivities. It stopped prematurely while giving the message that it could not find a better point in the design space.

It seems a reduction of almost 12% in fuel weight is gained, however due to the violation in equality constraints, this fuel weight saving is actually 9%. On top of this there exist more unacceptable equality constraint violations. The largest equality constraint violation occurs at the C_L constraint: a difference of 6% exists between $C_{L_{cruise}}$ and the surrogate design variable C_L . This means that the C_L used for the aerodynamic analysis is actually too low, which worsens the results even further. The second largest equality constraint violation is at the W_{fuel} constraint, as already named. This violation is $3.0852e-2$, meaning an absolute difference of 489kg.

Table 6.5 gives the optimization results for the aerodynamic and structural properties of the aircraft.

The optimization stopped prematurely, however the departure from the starting point is a good one. Just like

Table 6.5: Aerodynamic and structural complete optimization results

	$\alpha[deg]$	C_L	C_D	C_{D_f}	$W_{MTO}[kg]$	$W_{fuel}[kg]$	$W_{wing}[kg]$
Initial	3.1584	0.5228	0.0195	0.0049	73,500	17,940	8,801
Optimized	1.8620	0.3972	0.0128	0.0048	73,732	16,355	10,612

the fixed planform optimization in the prior section the α , C_L , C_D and W_{fuel} decreased, W_{wing} increased and W_{MTO} remained close to its original value. Even though the C_D reduced because of an exaggerated decrease in C_L , the L/D is increased with 16%, from 26.81 to 31.03. This means that there has been an extra reduction in drag. Because this solution is infeasible only part of the details of this optimization are given, being the aerodynamic results.

The inviscid C_D reduction is again a combination of wave and induced drag reduction. Starting with the induced drag reduction: The initial and the optimized planform of the wing can be found in figure 6.11.

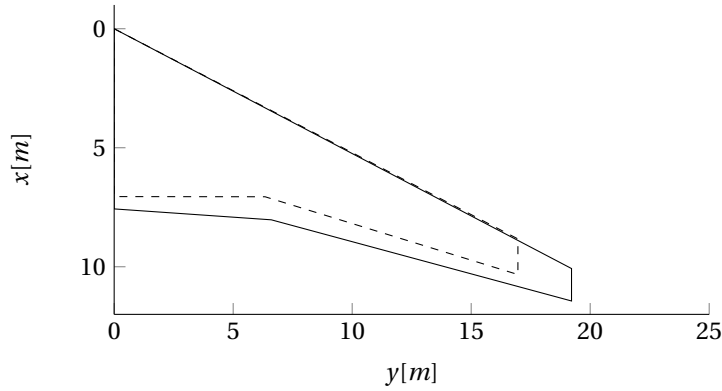


Figure 6.11: Planform view comparison between initial and optimized wing

It can be seen that the sweep angle is almost kept constant while the root chord and span length increased. This makes the whole wing surface increase with 25%, from $62.18m^2$ to $77.45m^2$. The root chord increase is not a preferred outcome, as it arrests the aspect ratio increase. The aspect ratio only increased from 9.25 to 9.54. This minor increase will create only little induced drag reduction.

Coming to the wave drag reduction, the figures 6.12 and 6.13 plot the pressure coefficient and the Mach number distribution for both the initial and the optimized wing. Figure 6.14 provides the airfoil shapes and accompanying pressure distributions. It can be observed that the distributions themselves did not change much, only that the peaks in pressure and Mach number are reduced in strength. This is a direct result of the reduced angle of attack. The reason behind this barely changed distribution is that the FFD control points did not move much. This can on its turn be explained with the fact that the normalized design sensitivities with respect to the planform are significantly higher than the ones with respect to the FFD design variables. The FFD design variables were not able to move yet as the optimizer stopped prematurely. The wave drag will hence be reduced, but not by a satisfactory amount.

The fault of the disappointing results lies in the crude correction on the inaccurate sensitivities given by SU2. The correction of the sensitivities with respect to U^* and EA^* proved to work as shown in the previous section, however the corrections are not valid any longer for the sensitivities with respect to P for a changing planform. This has been confirmed by performing the same optimization, but with the sensitivities with respect to the planform design variables calculated using FD. The results of that optimization run will be explained in the section below.

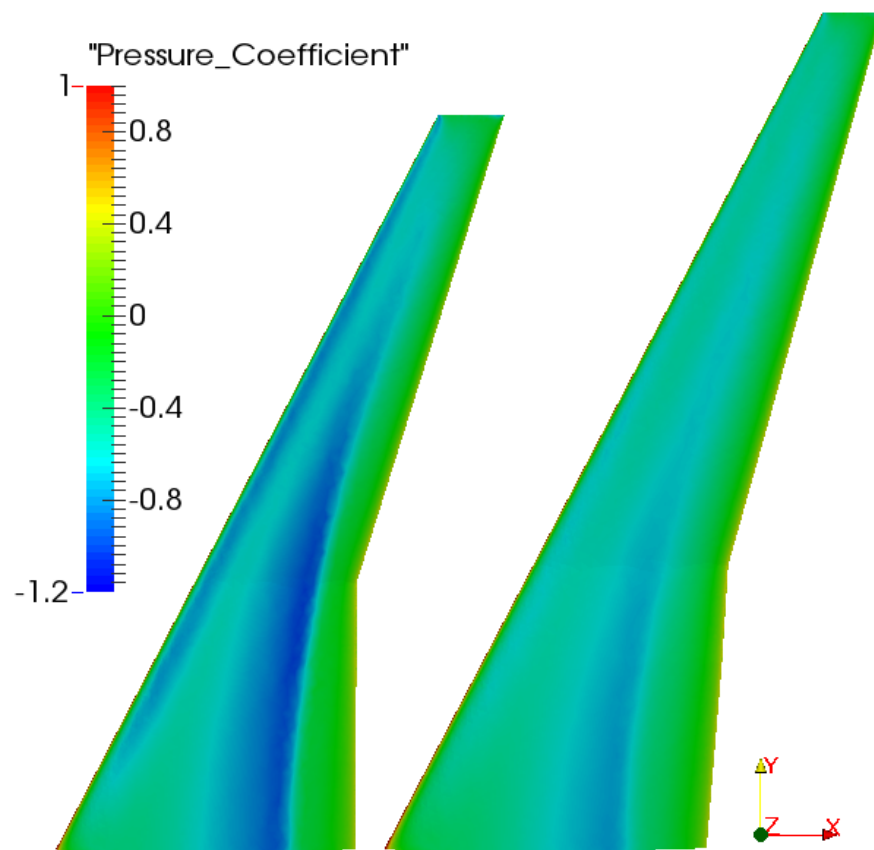


Figure 6.12: Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing

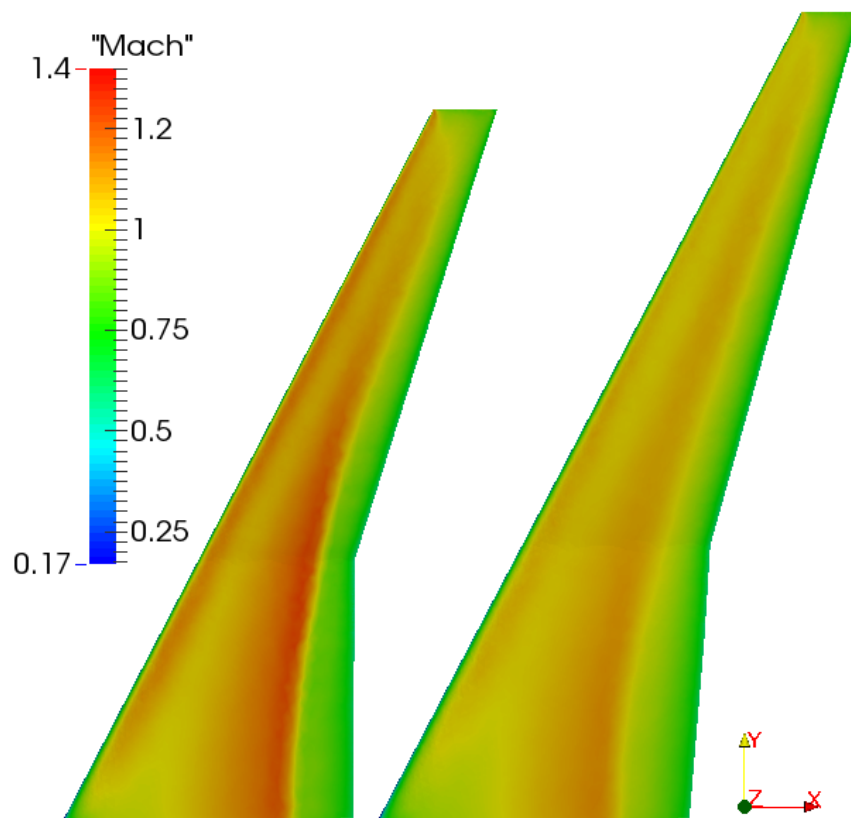


Figure 6.13: Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing

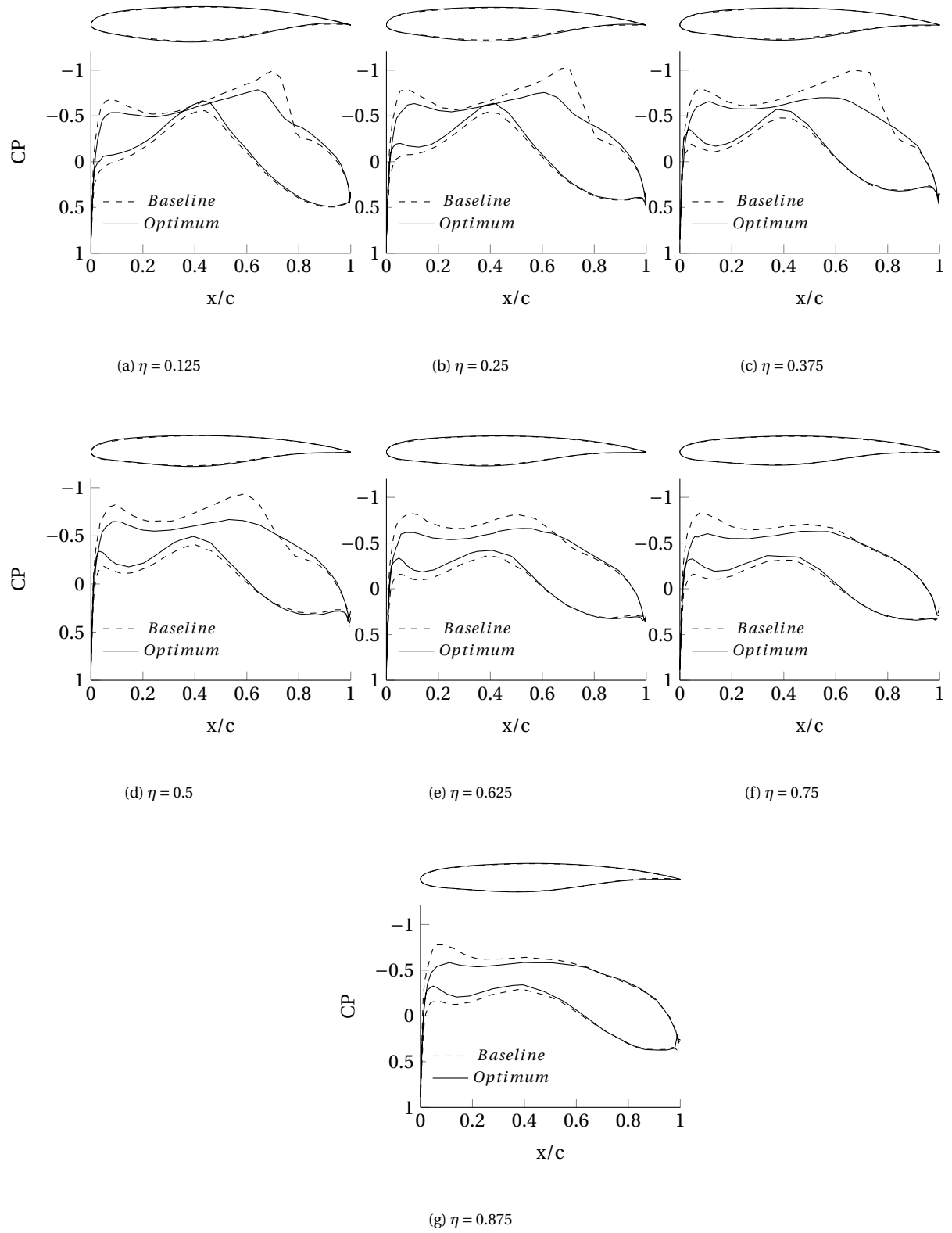


Figure 6.14: Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing

6.3.2 FINITE DIFFERENCE PLANFORM SENSITIVITIES

For the generation of these results the incorrect sensitivities with respect to the planform design variables as calculated using the adjoint formulation within SU2 are replaced with sensitivities calculated using FD. This is done in order to gain satisfactory results for the complete optimization problem and to prove that the fault lies in the false sensitivities.

This optimization run will cost significantly more time as the flow has to be solved for the calculation of each sensitivity with respect to P . This is why the original 8 planform design variables are reduced to 6, being the following: root chord length, taper ratio of the complete wing, wing span, leading edge sweep angle, kink twist angle and tip twist angle. The normalized lower bounds on taper is taken to be 0.85 and on the root chord length 0.9.

The top-level results given by table 6.6.

Table 6.6: Top-level complete optimization results

Parameter	Value
Objective function value	0.8873
Iterations	7
Function evaluations	51
Maximum constraint violation	4.6379e-5

Already looking at these results it can be seen that this optimization did produce satisfactory results. The fuel weight is reduced with more than 11% with a maximum equality constraint violation of 4.6379e-5. The maximum inequality constraint violation is found to be even less, 1.5855e-6. These values are low enough for the design to be called feasible.

For this optimization each function evaluation has the cost of 3 hours, meaning a total of 153 hours or 6.4 days.

Table 6.7: Aerodynamic and structural complete optimization results

	$\alpha[deg]$	C_L	C_D	C_{D_f}	$W_{MTO}[kg]$	$W_{fuel}[kg]$	$W_{wing}[kg]$
Initial	3.1584	0.5228	0.0195	0.0049	73,500	17,940	8,801
Optimized	1.4712	0.4793	0.0142	0.0050	70,948	15,914	8,270

Apart from the fuel reduction the optimization routine also managed to reduce the wing weight with 6%, see table 6.7. Both reductions together reduced the W_{MTO} with more than 3%.

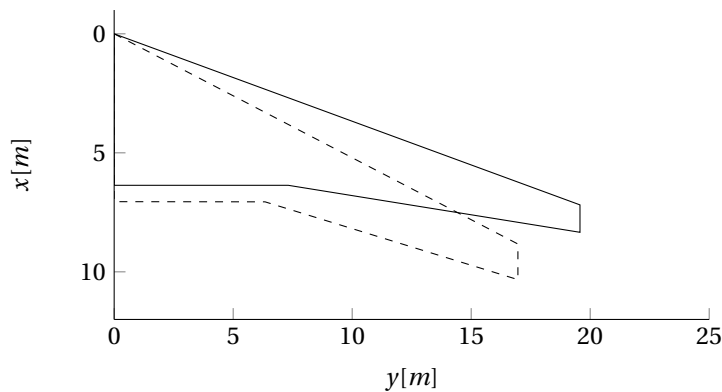


Figure 6.15: Planform view comparison between initial and optimized wing

The wing weight reduction is a result of the decreased sweep angle, from 27.5° to 20.2° . Although this is counter-acted by the increase in wing span, from $16.96m$ to $19.57m$ being an increase of 15%, a total wing weight reduction is observed. The optimized wing sweep angle and span can be observed from figure 6.15, which gives the planform view of the optimized wing in comparison with the initial wing. The aspect ratio

increased from 9.25 to 11.55 and the taper ratio of the wing reduced from 0.21 to 0.18. The wing surface is increased from 62.18m^2 to 66.32m^2 , being an increase of 7%. What is not visible from the above planform view is the kink and the tip jig-wing twist. The negative kink twist reduced from -2.5° to -1.8° and the negative tip twist reduced from -2.5° to -0.3° . Compared to the first coefficient, the change in the latter coefficient is large. This is one way how the optimizer realized extra lift on the outboard sections of the wing. The twist distribution after deformation and the change in lift distribution are further discussed below.

Because of the reduced wing and fuel weight the cruise C_L and α are also reduced. This reduces drag, however the L/D is increased with 26%, from 26.81 to 33.75. Hence the total drag reduction does not only come from the reduction in lift, but also from a combined reduction in induced and wave drag. The total drag coefficient is reduced from 0.0195 to 0.0142, while the friction drag is only increased marginally. The small increase in the friction drag component is due to the increase in wing wetted area. Helping with the induced drag reduction is the fact that the spanwise lift distribution has a more elliptical shape, just like the results shown in the sections above. See figure 6.16 for the comparison of lift distributions of this optimization run.

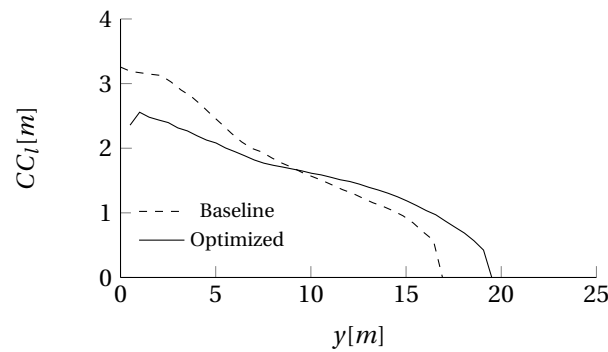


Figure 6.16: Lift coefficient distribution comparison between initial and optimized wing

It is remarkable that the wing weight is reduced, even though the loading near the wing tip is higher and the wing itself is longer. The effect of the reduction in sweep is hence observed to be stronger than the effect of the other two combined. Through the reduced wing weight the wing bending of the wing tip is larger: from 0.57m to 0.93m . In figure 6.17 the comparison between the jig shape and the 2.5g -shape at sea level between the initial and optimized wing is given.

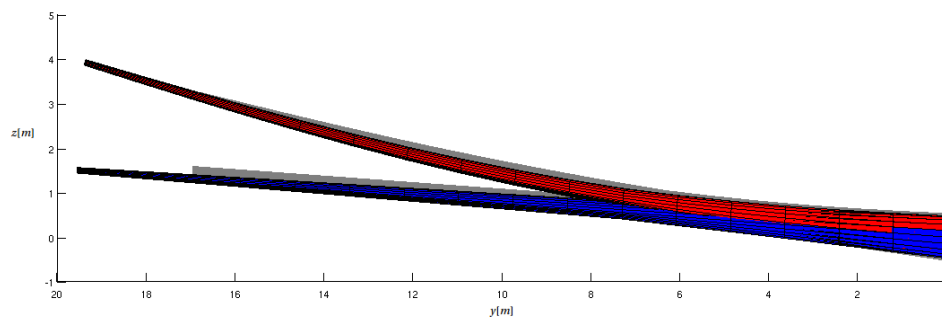


Figure 6.17: Comparison between the wing jig shape (blue) and the shape under 2.5g pull up load of the initial (grey) and optimized (red) wing

Figure 6.18 shows the twist distributions of the initial and optimized wing for their jig shape and the 1g -shape.

Even though the negative twist deformation at the tip increased from -1.4° to -1.6° , the total negative twist of the complete wing is reduced. As found in the figure and stated above, this is because the negative twist of the jig shape wing has been reduced, especially at the outboard half of the wing. An explanation of this that it is done to help the outboard sections creating lift.

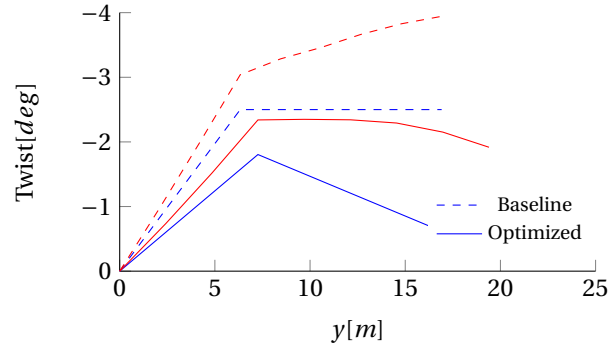


Figure 6.18: Twist deformation distribution comparison between initial and optimized wing of the jig shape (blue) and the 1g shape (red) wing

The reduction in wave drag is realized by improving the airfoil shapes so that the shock wave is reduced or even removed. This is counteracted by the reduction in sweep angle. However the trade-off has been made in favour of the reduction in wing weight. The resulting airfoil shapes and pressure distributions are further discussed below. In the figures 6.19 and 6.20 the pressure coefficient and Mach number distributions of the initial and optimized wing are given. It is found that the isobars are straightened, making the wing sweep more effective. This helps with the reduction in wave drag. From these images the earlier discussed increased loading on the outboard wing sections also becomes visible. The leading edge of these sections carry this increased loading.

Figure 6.21 presents the comparison of airfoil shapes at several spanwise positions. Furthermore, it gives the comparison of the accompanying pressure distributions. At the inboard sections the camber has been decreased slightly. At the outboard sections the airfoils are mostly made thinner. The lower surfaces are made less convex.

Even though the wing sweep angle increased it can be seen that at all positions the shock wave is almost completely removed, except for at the quarter span position. At this section the shock wave is weakened and moved forward. Moreover, what is striking from these results is the fact that the optimizer did not create rear loading at the outboard sections, like it did in the fixed planform optimization case. Instead, the increase in lift generation is realized by the less convex lower surfaces and a more convex upper surface leading edge. This is now possible because of the twist-tailoring of the jig wing.

Lastly, the aileron effectiveness is found to be an active constraint. This is expected from literature, see Elham and van Tooren [48]. However, the wing loading constraint is not active. It appears that the increase in span and thereby an increase in wing area are the preferred direction, instead of a decrease in surface area. This could be a result of the fact that a local optimum is found. The global optimum could in theory be a wing with a smaller surface area. For the reasons stated in chapter 2 a global optimization is outside the scope of this research.

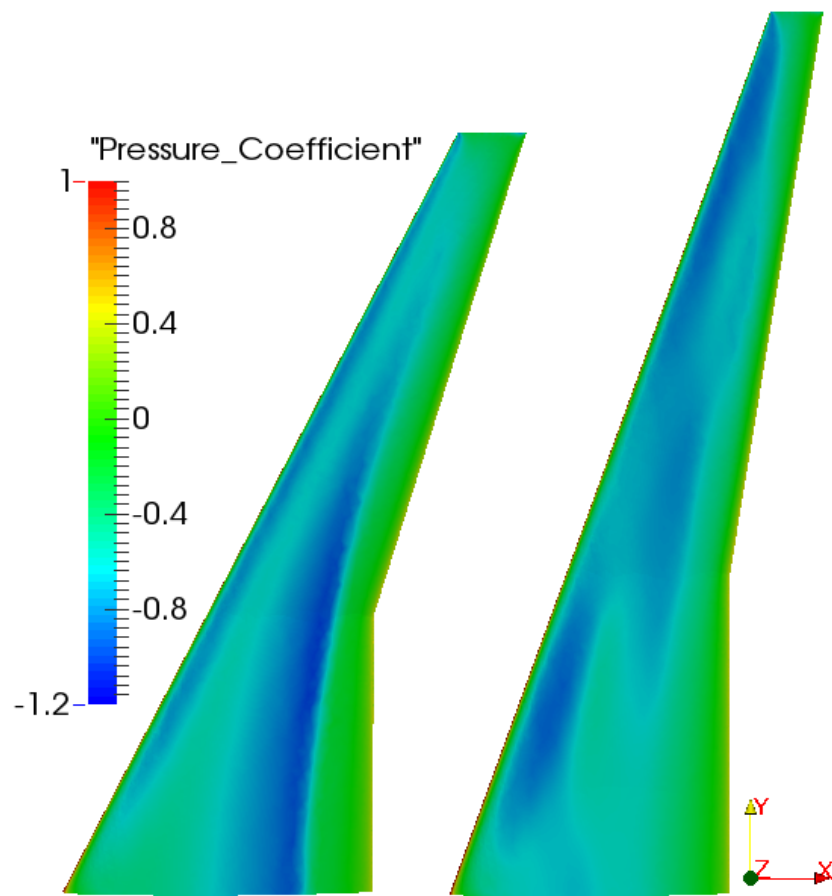


Figure 6.19: Upper wing pressure coefficient distribution comparison between initial (left) and optimized (right) wing

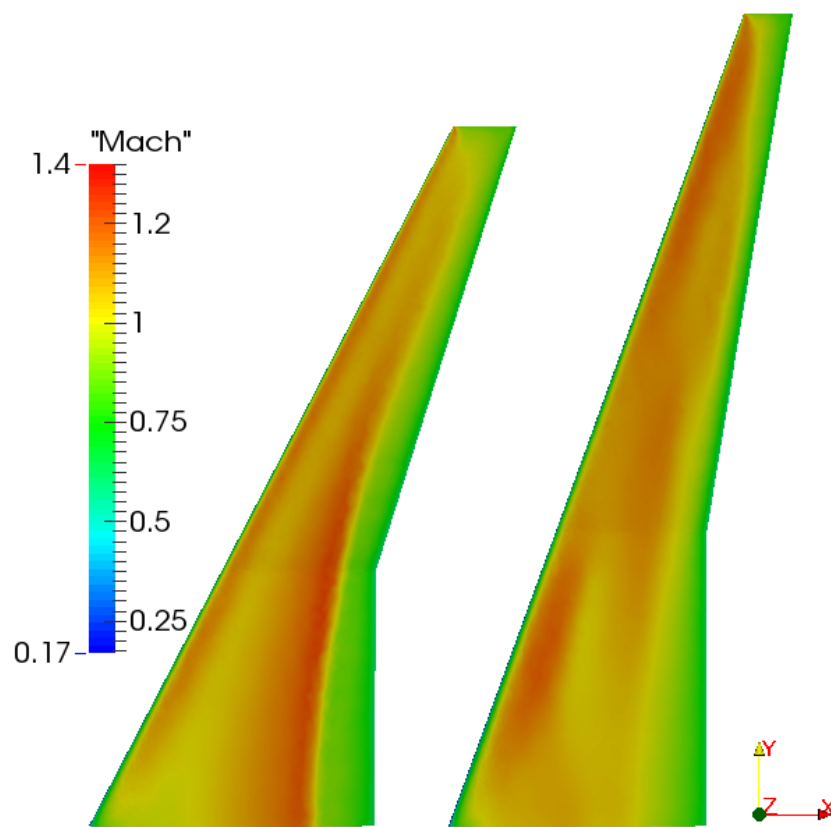


Figure 6.20: Upper wing Mach number distribution comparison between initial (left) and optimized (right) wing

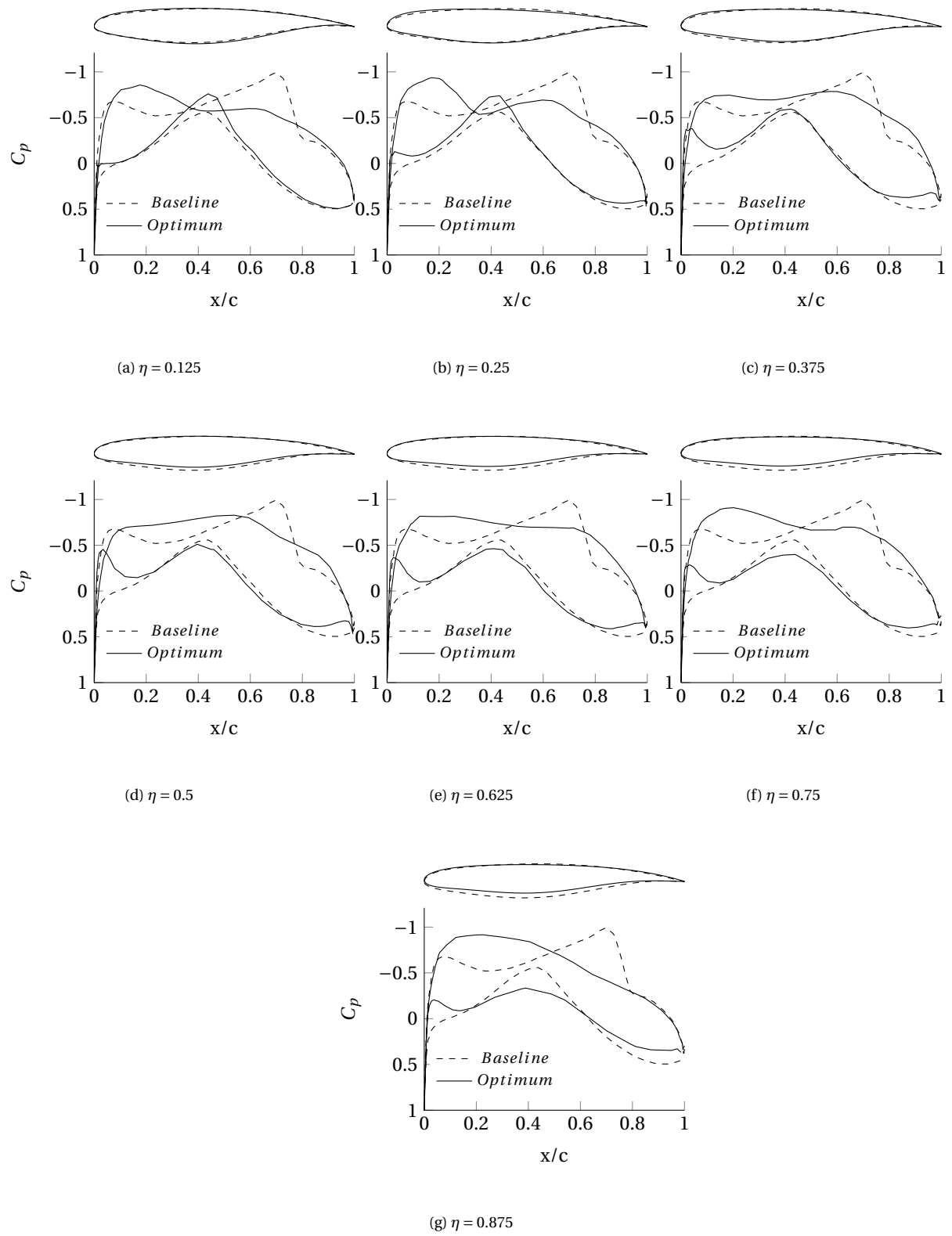


Figure 6.21: Spanwise airfoil shape and pressure coefficient distribution comparison between initial and optimized wing

CONCLUSIONS AND RECOMMENDATIONS

The final conclusions and recommendations for further research based on this research will be given in this last chapter of the present thesis.

7.1 CONCLUSIONS

In this study the IDF MDO architecture has been applied to solve a wing aerostructural optimization problem. The main advantage of IDF has been found to be that it does not require the coupled sensitivity analysis for gradient-based optimization. This provides the user with more freedom of software choice for the disciplinary analyses. On top of this, it reduces the overall computational costs with respect to the widely used MDF architecture. This increase in optimization efficiency comes from the fact that the IDF architecture is able to advance towards an optimum in the design space while realizing consistency of the multidisciplinary system simultaneously. This last argument has however not been proven in the present work.

The drawback of the IDF architecture is that if the increase in design variables due to this architecture is too large, its efficiency is compromised. It was however found that is not the case in this application.

Throughout the development of the tool it was found that the sensitivities with respect to the displacement of the trailing edge provided by SU2 were inaccurate. This was confirmed by its developers. This inaccuracy resulted in the need to introduce corrective factors to the sensitivities. These corrective factors proved to work for the aerodynamic sensitivities with respect to the aeroelastic deformation and elastic axis position, but not for the sensitivities with respect to the planform design variables.

The functioning of the tool has been tested on a test case: The wing of the Airbus A320. Before the aerostructural optimizations an aerodynamic optimization run is performed, to make sure all modules of SU2 works correctly when it is controlled by the optimization algorithm. This optimization is found to be a success as it resulted in the reduction of the total drag of 16%, through the reduction of both induced and wave drag.

Afterwards, three aerostructural wing optimizations are performed. All of these optimizations optimize the wing external shape and internal wing box structure for minimal fuel weight necessary for the range characteristics of the Airbus A320 reference aircraft. The first optimization kept the planform shape of the wing constant while for the last two optimizations this was also included in the design vector. The difference between the last two optimizations is the way of calculating the aerodynamic sensitivities with respect to the planform design variables. The second optimization used the continuous adjoint formulation for its calculation while the third used FD.

The fixed planform optimization has resulted in a 7% reduction in the aircraft fuel weight. The drag reduction was achieved by reducing the wave drag and induced drag of the wing. The wave drag was reduced in two ways. Firstly by making more use of the wing sweep by straightening the iso-bars on the wing and secondly by reducing the strength of the shock wave at the inboard half of the wing. The induced drag was reduced by making the spanwise lift distribution more elliptic. This last effect had the penalty of making the wing structure heavier. The optimization results show the optimum for this trade-off.

It is shown that the optimization approach performs well. With the relatively small computational power of

8 processors an optimum is found within 4 days. Moreover, it is proven that the corrective factors for the aerodynamic sensitivities with respect to the aeroelastic deformation and elastic axis position work. This is however not the desirable method of sensitivity calculation and a better optimum should be reached if these sensitivities were calculated in the correct way.

The complete optimization including the planform design variables provided disappointing results. Although the fuel weight was reduced with 9%, the equality constraints were violated beyond the acceptable. The optimization routine failed to simultaneously reduce the objective function and constraint violations and stopped prematurely. This is due to the inaccurate sensitivities given by SU2, even after correction. The reason why this optimization failed to converge while the fixed-planform optimization did converge is because the correction factors were not valid anymore for the sensitivities with respect to the planform shape. This was tested by performing the same optimization but with the sensitivities with respect to the planform design variables calculated with FD instead of the adjoint formulation. For this last optimization the amount of planform design variables was reduced from 8 to 6 to reduce the computational cost.

The last optimization resulted in a success. The optimizer managed to reduce the aircraft fuel weight with 11%. The constraint violations remained within the acceptable. At the found optimum, the constraint on aileron effectiveness is active, but the constraint on wing loading is not.

The wing friction drag is increased slightly, while the induced and wave drag are reduced. The induced drag reduction is a result of the increase in aspect ratio and improved spanwise lift distribution. The wave drag is reduced by removing the shock wave at almost all spanwise sections and by straightening the isobars over the wing. Both are managed through airfoil shape changes.

Even while the increased wingspan and the outbound wing loading have a negative effect on the aircraft wing loading, it is reduced. This is explained by the larger effect of the sweep angle decrease on the wing weight. The wave drag increase accompanied by this sweep angle decrease is more than compensated by the airfoil shape changes.

7.2 RECOMMENDATIONS

The present section presents recommendations for further research based on this thesis.

- It is highly recommended to improve sensitivity accuracy by using the discrete adjoint method, which will be implemented in SU2 in the near future. These new sensitivities should result in a successful verification and a converging optimization when planform design variables are included in the design vector.
- This research only estimated the viscous drag and neglected other boundary layer effects. It is recommended to include the boundary layer analysis in the optimization using RANS for a better estimation of the friction drag and shock-induced boundary layer separation.
- The airfoil shape design variables used are evenly spread over the upper and lower surface of the FFD-boxes. It is recommended to construct areas with a higher density of FFD control points, like for example the leading edge. This will improve the design freedom in areas with high design sensitivities.
- Throughout this study only static aeroelastic behaviour is taken into account. When dynamic aeroelasticity is included, a more realistic picture of the boundaries of the design space is created.
- It is furthermore recommended that a multi-point optimization for the complete mission of the aircraft is performed, as the wing shape of the optimized wing is only optimized for the cruise condition.
- The optimum found in this study is found using a gradient based, local optimization algorithm. It is recommended to perform a global optimization in order to find the optimal wing in the complete design space.

BIBLIOGRAPHY

- [1] G. Ruijgrok and D. van Paassen. *Elements of aircraft pollution*. Delft University Press, 2005.
- [2] D McLean. Wingtip devices: What they do and how they do it. *Boeing Performance and Flight Operations Engineering Conference*, 2005. Seattle, WA.
- [3] R Vos and S Farokhi. *Introduction to Transonic Aerodynamics*. Springer, 2015.
- [4] R T Hafta. Optimization of flexible wing structures subject to strength and induced drag constraints. *AIAA Journal*, 14(8):1106–1177, 1977.
- [5] T. McGeer. *Wing Design for Minimum Drag and Practical Constraints*. Ph.d. thesis, Stanford University, 1984.
- [6] B Grossman, G Strauch, W H Eppard, Z Gurdal, and R T Haftka. Integrated aerodynamic/structural design of a sailplane wing. *Journal of Aircraft*, 25(9):855–860, 1988.
- [7] B Grossman, R T Haftka, P J Kao, D M Polen, and M Rais-Rohani. Integrated aerodynamic-structural design of a transport wing. *Journal of Aircraft*, 27(12):1050–1056, 1990.
- [8] S. Wakayma and I. Kroo. Subsonic wing design using multidisciplinary optimization. In *Proceedings of the 5th Wrenn, AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, volume 2, pages 1358–1368, Sept 1994. Panama City Beach, FL.
- [9] C. J. Borland, J. R. Benton, P D. Frank, T.J. Kao, R. A. Mastro, and J. F M. Barthelemy. Multidisciplinary design optimization of a commercial aircraft wing – an exploratory study. In *Proceedings of the 5th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, volume 1, pages 505–519, Sept 1994. Panama City Beach, FL.
- [10] A. Chattopadhyay and N. Pagaldipti. A multidisciplinary optimization using semi-analytical sensitivity analysis procedure and multilevel decomposition. *Journal of Computers and Mathematics with Applications*, 29(7):55–66, 1995.
- [11] M. Baker and J. Giesing. A practical approach to mdo and its application to an hsct aircraft. In *1st AIAA Aircraft Engineering, Technology, and Operations Congress*, Sept 1995. Los Angeles, CA.
- [12] V. M. Manning. *Large-Scale Design of Supersonic Aircraft via Collaborative Optimization*. Ph.d. thesis, Stanford University, 1999.
- [13] M. Barcelos and K. Maute. Aeroelastic design optimization for laminar and turbulent flows. *Comput. Methods Appl. Mech. Engrg.* 197, pages 1813–1832, 2008.
- [14] J Brezillon, A Ronzheimer, D Haar, M Abu-Zurayk, K Lummer, W Kruger, and F J Nattere. Development and application of multi-disciplinary optimization capabilities based on high-fidelity methods. *AIAA Journal*, 2012.
- [15] R. P. Liem, C. A. Madery, and E. Lee. Aerostructural design optimization of a 100-passenger regional jet with surrogate-based mission analysis. In *AIAA Aviation conference*, Aug 2013. Los Angeles, CA.
- [16] G. J. Kennedy and J. R. R. A. Martins. A parallel aerostructural optimization framework for aircraft design studies. *Structure and Multidisciplinary Optimization.*, 50:1079–1101, 2014.
- [17] G. J. Kennedy and J. R. R. A. Martins. A comparison of metallic and composite aircraft wings using aerostructural design optimization. In *12th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference and 14th AIAA/ISSM 17-19*, Sept 2012. Indianapolis, IN.

- [18] G. K. W. Kenway, G. J. Kennedy, and J. R. R. A. Martins. Scalable parallel approach for high-fidelity steady-state aeroelastic analysis and adjoint derivative computations. *AIAA Journal*, 52, 2014.
- [19] G. K. W. Kenway, J. R. R. A. Martins, and G. J. Kennedy. Aerostructural optimization of the common research model configuration. In *AIAA Aviation conference*, Jun 2014. Atlanta, GA.
- [20] G. K. W. Kenway and J. R. R. A. Martins. Multipoint high-fidelity aerostructural optimization of a transport aircraft configuration. *Journal of Aircraft*, 51, 2014.
- [21] R P Liem, G K W Kenway, and J R R A Martins. Multimission aircraft fuel-burn minimization via multipoint aerostructural optimization. *AIAA Journal*, 53(1):104–122, 2015.
- [22] J Mariens, A Elham, and M van Tooren. Quasi-three-dimensional aerodynamic solver for multidisciplinary design and optimization of lifting surfaces. *Journal of Aircraft*, 51(2):547–558, 2014.
- [23] N. Alexandrov. Editorial - multidisciplinary design optimization. *Optimization and Engineering*, 6:5–7, 2005.
- [24] J. R. R. A. Martins and A. B. Lambe. Multidisciplinary design optimization: A survey of architectures. *AIAA Journal*, 51:2049–2075, 2013.
- [25] A. B. Lambe and J. R. R. A. Martins. Extensions to the design structure matrix for the description of multidisciplinary design, analysis, and optimization processes. *Structural and Multidisciplinary Optimization*, 46(2):273–284, 2012.
- [26] E. J. Cramer, J. E. Dennis Jr., P. D. Frank, R. M. Lewis, and G. R. Shubin. Problem formulation for multidisciplinary optimization. *SIAM Journal on Optimization*, 4(4):754–776, 1994.
- [27] F. Palacios, M. R. Colonna, A. C. Aranake, A. Campos, S. R. Copeland, T. D. Economom, A. K. Lonkar, T. W. Lukaczyk, T. W. R. Taylor, and J. J. Alonso. Stanford university unstructured (su^2): An open-source integrated computational environment for multi-physics simulation and design. *51st AIAA Aerospace Sciences Meeting*, January 2013.
- [28] A Elham and M J L van Tooren. Coupled adjoint aerostructural wing optimization using quasi-three-dimensional aerodynamic analysis. *16th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Jun. 2015. Dallas, TX.
- [29] E Torenbeek. *Advanced Aircraft Design, Conceptual Design, Analysis and Optimization of Subsonic Civil Airplanes*. John Wiley & Sons Ltd, West Sussex, 2013.
- [30] M. Langelaar and F. van Keulen. Lecture slides wb1440 - engineering optimization concepts and applications. TU Delft Faculty of Mechanical Engineering, 2013.
- [31] P. E. Gill, W. Murray, and M. A. Saunders. Snopt: An sqp algorithm for large-scale constrained optimization. *SIAM Journal on Optimization*, 12(4):979–1006, 2002.
- [32] J. A. Samareh. Aerodynamic shape optimization based on free-form deformation. *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, (2004-4630), July 2004. Albany, NY.
- [33] J. A. Samareh. Survey of shape parameterization techniques for high-fidelity multidisciplinary shape optimization. *AIAA Journal*, 39, 2001.
- [34] T. W. Sederberg and S. R. Parry. Free-form deformation of solid geometric models. *Computer Graphics*, 20(4):151–160, 1990.
- [35] H. Gagnon and D. W. Zingg. Two-level free-form deformation for high-fidelity aerodynamic shape optimization. *Proceedings of the 14th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference*, Sept. 2012. Indianapolis, IN.
- [36] E. J. Nielsen and W. K. Anderson. Aerodynamic design optimization on unstructured meshes using the navier-stokes equations. *7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization Proceedings*, pages 825–837, Sept. 1998.

- [37] G. R. Anderson, M. J. Aftosmis, and M. Nemec. Parametric deformation of discrete geometry for aerodynamic shape design. *60th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, Jan. 2012. Nashville, Tennessee.
- [38] J. A. Samareh. Multidisciplinary aerodynamic-structural shape optimization using deformation (massoud). *8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, (2000-4911), Sept. 2000. Long Beach, CA.
- [39] F. Palacios, T. D. Economon, A. D. Wendorff, and J. J. Alonso. Large-scale aircraft design using su2. *53rd AIAA Aerospace Sciences Meeting*, January 2015.
- [40] A. Rizzi. *Aerodynamic Design a Computational Approach*, KTH Course SD2611. KTH Department of Aeronautical & Vehicle Engineering, 2011.
- [41] *ANSYS ICEM CFD User Manual*. ANSYS, 14.5 edition, 2012.
- [42] T. Carrigan, T. Economon, F. Palacios, and T. Lukaczyk. Supersonic aircraft shape design powered by su2 and pointwise. *SU2 and Pointwise*, 2014.
- [43] Z. Lyu, G. K. W. Kenway, C. Paige, and J. R. R. A. Martins. Automatic differentiation adjoint of the reynolds-averaged navier–stokes equations with a turbulence model. *21st AIAA Computational Fluid Dynamics Conference*, July 2013. San Diego, CA.
- [44] D. P. Raymer. *Aircraft Design: A Conceptual Approach*. American Institute of Aeronautics and Astronautics, Inc., second edition, 1992.
- [45] T. D. Economon, F. Palacios, and J. J. Alonso. An unsteady continuous adjoint approach for aerodynamic design on dynamic meshes. *15th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Jun. 2014. Atlanta, GA.
- [46] J. Roskam. *Airplane Design*. DAR corporation, Lawrence, Kan, 1986.
- [47] E Obert. *Aerodynamic Design of Transport Aircraft*. Delft University Press, 2009.
- [48] A Elham and M J L van Tooren. Beyond quasi-analytical methods for preliminary structural sizing and weight estimation of lifting surfaces. *56th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Jan. 2015. Kissimmee, FL.
- [49] A. Elham, G. La Rocca, and M.J.L. van Tooren. Development and implementation of an advanced, design-sensitive method for wing weight estimation. *Aerospace Science and Technology*, 29:100–113, 2013.
- [50] A Elham and M J L van Tooren. Winglet multi-objective shape optimization. *Elsevier*, 2014.
- [51] E Torenbeek. Development and application of a comprehensive, designsensitive weight prediction method for wing structures of transport category aircraft. (Report LR-693), 1992. Delft University of Technology,.
- [52] J Katz and A Plotkin. *Low speed aerodynamics*. Cambridge University Press, 2001.
- [53] T H G Megson. *Aircraft Structures for Engineering Students*. Elsevier Aerospace Engineering Series, Oxford, 4th edition, 2007.
- [54] Y Luo. An efficient 3d timoshenko beam element with consistent shape functions. *Adv. Theor. Appl. Mech.*, 1(3):95–106, 2008.
- [55] M C Y Niu. *Airframe Stress Analysis and Sizing*. Conmilit Press Ltd., 1997.
- [56] F Hurlimann. Mass estimation of transport aircraft wing. PhD thesis, Swiss Federal Ins. of Technology Zurich, Zurich, 2010.
- [57] J K S Dillinger, T Klimmek, M M Abdalla, and Z Gurdal. Stiffness optimization of composite wings with aeroelastic constraints. *Journal of Aircraft*, 50(4):1159–1168., 2013.

APPENDIX

CODE FOR REALIZATION OF U^* AND P DEFORMATION AND EA^* SENSITIVITY CALCULATION

```
void CSurfaceMovement::SetFFD_U_DEF(CGeometry *geometry, CConfig *config,
CFreeFormDefBox *FFDBox, unsigned short iDV, bool ResetDef) {
```

```
    unsigned short iOrder, jOrder, kOrder;
    double x, y, z, movement[3];
    unsigned short index[3];
    string design_FFDBox;
```

```
    /*--- Set control points to its original value (even if the
        design variable is not in this box) ---*/
```

```
    if (ResetDef == true) FFDBox->SetOriginalControlPoints();
```

```
    design_FFDBox = config->GetFFDTag(iDV);
```

```
    if (design_FFDBox.compare(FFDBox->GetTag()) == 0) {
```

```
        /*--- xyz-coordinates of a point on the line of rotation. ---*/
```

```
        double u_x=config->GetParamDV(iDV, 2)*config->GetDV_Value(iDV);
        double u_y=config->GetParamDV(iDV, 3)*config->GetDV_Value(iDV);
        double u_z=config->GetParamDV(iDV, 4)*config->GetDV_Value(iDV);
        double cosPH=cos(-1*config->GetParamDV(iDV, 5)
            *config->GetDV_Value(iDV)*(PI_NUMBER/180.0));
        double sinPH=sin(-1*config->GetParamDV(iDV, 5)
            *config->GetDV_Value(iDV)*(PI_NUMBER/180.0));
        double cosT=cos(-1*config->GetParamDV(iDV, 6)
            *config->GetDV_Value(iDV)*(PI_NUMBER/180.0));
        double sinT=sin(-1*config->GetParamDV(iDV, 6)
            *config->GetDV_Value(iDV)*(PI_NUMBER/180.0));
        double cosPS=cos(-1*config->GetParamDV(iDV, 7)
            *config->GetDV_Value(iDV)*(PI_NUMBER/180.0));
        double sinPS=sin(-1*config->GetParamDV(iDV, 7)
            *config->GetDV_Value(iDV)*(PI_NUMBER/180.0));
        double elas_axis_x = config->GetParamDV(iDV, 8);
        double elas_axis_y = config->GetParamDV(iDV, 9);
        double elas_axis_z = config->GetParamDV(iDV, 10);
```

```
        /*--- The angle of rotation. ---*/
```

```
        /*double psi = config->GetDV_Value(iDV)*PI_NUMBER/180.0; */
```

```
        /*--- An intermediate value used in computations. ---*/
```

```

/*--- Change the value of the control point if move is true ---*/

for (iOrder = 0; iOrder < FFDBox->GetlOrder(); iOrder++)
    for (kOrder = 0; kOrder < FFDBox->GetnOrder(); kOrder++) {
        index[0] = iOrder; index[1] = int(config->GetParamDV(iDV, 1)); index[2] = kOrder;
        double *coord = FFDBox->GetCoordControlPoints(iOrder,
            int(config->GetParamDV(iDV, 1)), kOrder);
        x = coord[0]; y = coord[1]; z = coord[2];

        double elas_rel_x=(x+u_x)-elas_axis_x;
        double elas_rel_y=(y+u_y)-elas_axis_y;
        double elas_rel_z=(z+u_z)-elas_axis_z;

        double elas_rel_x_def_x=elas_rel_x;
        double elas_rel_y_def_x=cosPH*elas_rel_y + sinPH*elas_rel_z;
        double elas_rel_z_def_x=-sinPH*elas_rel_y + cosPH*elas_rel_z;

        double elas_rel_x_def_xy=cosT*elas_rel_x_def_x - sinT*elas_rel_z_def_x;
        double elas_rel_y_def_xy=elas_rel_y_def_x;
        double elas_rel_z_def_xy=sinT*elas_rel_x_def_x + cosT*elas_rel_z_def_x;

        double elas_rel_x_def=cosPS*elas_rel_x_def_xy + sinPS*elas_rel_y_def_xy;
        double elas_rel_y_def=-sinPS*elas_rel_x_def_xy + cosPS*elas_rel_y_def_xy;
        double elas_rel_z_def=elas_rel_z_def_xy;

        double x_def=elas_rel_x_def+elas_axis_x;
        double y_def=elas_rel_y_def+elas_axis_y;
        double z_def=elas_rel_z_def+elas_axis_z;

        movement[0] = x_def-x;

        movement[1] = y_def-y;

        movement[2] = z_def-z;

        FFDBox->SetControlPoints(index, movement);
    }
}

}

void CSurfaceMovement::SetFFD_EA_GRAD(CGeometry *geometry, CConfig *config,
CFreeFormDefBox *FFDBox, unsigned short iDV, bool ResetDef) {

    unsigned short iOrder, jOrder, kOrder;
    double x, y, z, movement[3];
    unsigned short index[3];
    string design_FFDBox;

    /*--- Set control points to its original value (even if the
    design variable is not in this box) ---*/

```

```

if (ResetDef == true) FFDBox->SetOriginalControlPoints();

design_FFDBox = config->GetFFDTag(iDV);

if (design_FFDBox.compare(FFDBox->GetTag()) == 0) {

    /*--- xyz-coordinates of a point on the line of rotation. ---*/

    double cosPH=cos(-1*config->GetParamDV(iDV, 2)*(PI_NUMBER/180.0));
    double sinPH=sin(-1*config->GetParamDV(iDV, 2)*(PI_NUMBER/180.0));
    double cosT=cos(-1*config->GetParamDV(iDV, 3)*(PI_NUMBER/180.0));
    double sinT=sin(-1*config->GetParamDV(iDV, 3)*(PI_NUMBER/180.0));
    double cosPS=cos(-1*config->GetParamDV(iDV, 4)*(PI_NUMBER/180.0));
    double sinPS=sin(-1*config->GetParamDV(iDV, 4)*(PI_NUMBER/180.0));
    double cos_PH=cos(config->GetParamDV(iDV, 2)*(PI_NUMBER/180.0));
    double sin_PH=sin(config->GetParamDV(iDV, 2)*(PI_NUMBER/180.0));
    double cos_T=cos(config->GetParamDV(iDV, 3)*(PI_NUMBER/180.0));
    double sin_T=sin(config->GetParamDV(iDV, 3)*(PI_NUMBER/180.0));
    double cos_PS=cos(config->GetParamDV(iDV, 4)*(PI_NUMBER/180.0));
    double sin_PS=sin(config->GetParamDV(iDV, 4)*(PI_NUMBER/180.0));
    double elas_axis_x = config->GetParamDV(iDV, 5);
    double elas_axis_y = config->GetParamDV(iDV, 6);
    double elas_axis_z = config->GetParamDV(iDV, 7);
    double elas_axis_x_grad = config->GetParamDV(iDV, 8);
    double elas_axis_y_grad = config->GetParamDV(iDV, 9);
    double elas_axis_z_grad = config->GetParamDV(iDV, 10);

    /*--- The angle of rotation. ---*/

    /*double psi = config->GetDV_Value(iDV)*PI_NUMBER/180.0; */

    /*--- An intermediate value used in computations. ---*/

    /*--- Change the value of the control point if move is true ---*/

    for (iOrder = 0; iOrder < FFDBox->GetlOrder(); iOrder++)
        for (kOrder = 0; kOrder < FFDBox->GetnOrder(); kOrder++) {
            index[0] = iOrder; index[1] = int(config->GetParamDV(iDV, 1)); index[2] = kOrder;
            double *coord = FFDBox->GetCoordControlPoints(iOrder,
                int(config->GetParamDV(iDV, 1)), kOrder);
            x = coord[0]; y = coord[1]; z = coord[2];

            /* Reverse rotation to get original coordinates*/
            double elas_rel_x_orig=x-elas_axis_x;
            double elas_rel_y_orig=y-elas_axis_y;
            double elas_rel_z_orig=z-elas_axis_z;

            double elas_rel_x_def_z_orig=cos_PS*elas_rel_x_orig + sin_PS*elas_rel_y_orig;
            double elas_rel_y_def_z_orig=-sin_PS*elas_rel_x_orig + cos_PS*elas_rel_y_orig;
            double elas_rel_z_def_z_orig=elas_rel_z_orig;

            double elas_rel_x_def_yz_orig=cos_T*elas_rel_x_def_z_orig
                - sin_T*elas_rel_z_def_z_orig;

```

```

double elas_rel_y_def_yz_orig=elas_rel_y_def_z_orig;
double elas_rel_z_def_yz_orig=sin_T*elas_rel_x_def_z_orig
    + cos_T*elas_rel_z_def_z_orig;

double elas_rel_x_def_orig=elas_rel_x_def_yz_orig;
double elas_rel_y_def_orig=cos_PH*elas_rel_y_def_yz_orig
    + sin_PH*elas_rel_z_def_yz_orig;
double elas_rel_z_def_orig=-sin_PH*elas_rel_y_def_yz_orig
    + cos_PH*elas_rel_z_def_yz_orig;

double x_orig=elas_rel_x_def_orig+elas_axis_x;
double y_orig=elas_rel_y_def_orig+elas_axis_y;
double z_orig=elas_rel_z_def_orig+elas_axis_z;

/* Do rotation to get new coordinates*/
double elas_rel_x=x_orig-(elas_axis_x+(elas_axis_x_grad*config->GetDV_Value(iDV)));
double elas_rel_y=y_orig-(elas_axis_y+(elas_axis_y_grad*config->GetDV_Value(iDV)));
double elas_rel_z=z_orig-(elas_axis_z+(elas_axis_z_grad*config->GetDV_Value(iDV)));

double elas_rel_x_def_x=elas_rel_x;
double elas_rel_y_def_x=cosPH*elas_rel_y + sinPH*elas_rel_z;
double elas_rel_z_def_x=-sinPH*elas_rel_y + cosPH*elas_rel_z;

double elas_rel_x_def_xy=cosT*elas_rel_x_def_x - sinT*elas_rel_z_def_x;
double elas_rel_y_def_xy=elas_rel_y_def_x;
double elas_rel_z_def_xy=sinT*elas_rel_x_def_x + cosT*elas_rel_z_def_x;

double elas_rel_x_def=cosPS*elas_rel_x_def_xy + sinPS*elas_rel_y_def_xy;
double elas_rel_y_def=-sinPS*elas_rel_x_def_xy + cosPS*elas_rel_y_def_xy;
double elas_rel_z_def=elas_rel_z_def_xy;

double x_def=elas_rel_x_def+(elas_axis_x+(elas_axis_x_grad*config->GetDV_Value(iDV)));
double y_def=elas_rel_y_def+(elas_axis_y+(elas_axis_y_grad*config->GetDV_Value(iDV)));
double z_def=elas_rel_z_def+(elas_axis_z+(elas_axis_z_grad*config->GetDV_Value(iDV)));

/* Movement is difference between old rotation and new*/
movement[0] = x_def-x;

movement[1] = y_def-y;

movement[2] = z_def-z;

FFDBox->SetControlPoints(index, movement);

}

}

}

void CSurfaceMovement::SetFFD_PLANFORM(CGGeometry *geometry, CConfig *config,
CFreeFormDefBox *FFDBox, unsigned short iDV, bool ResetDef) {

    unsigned short iOrder, jOrder, kOrder;
    double x, y, z, movement[3];
    unsigned short index[3];

```

```

string design_FFDBox;

/*--- Set control points to its original value (even if the
design variable is not in this box) ---*/

if (ResetDef == true) FFDBox->SetOriginalControlPoints();

design_FFDBox = config->GetFFDTag(iDV);

if (design_FFDBox.compare(FFDBox->GetTag()) == 0) {

    /*--- xyz-coordinates of a point on the line of rotation. ---*/

    double D_chord=config->GetParamDV(iDV, 2)*config->GetDV_Value(iDV);
    double l_chord=config->GetParamDV(iDV, 3);
    double D_FFD_x_0=config->GetParamDV(iDV, 4);
    double l_FFD_x=config->GetParamDV(iDV, 5);
    double l_FFD_z=config->GetParamDV(iDV, 6);

    double chord_ratio = (D_chord+l_chord)/l_chord;
    double l_FFD_x_new = l_FFD_x*chord_ratio;
    double l_FFD_z_new = l_FFD_z*chord_ratio;
    double FFD_x_mov = (l_FFD_x_new-l_FFD_x);
    double FFD_z_mov = (l_FFD_z_new-l_FFD_z)/2;

    /*--- The angle of rotation. ---*/

    /*double psi = config->GetDV_Value(iDV)*PI_NUMBER/180.0; */

    /*--- An intermediate value used in computations. ---*/

    /*--- Change the value of the control point if move is true ---*/
    for (iOrder = 0; iOrder < FFDBox->GetlOrder(); iOrder++)
        for (kOrder = 0; kOrder < FFDBox->GetnOrder(); kOrder++) {
            index[0] = iOrder; index[1] = int(config->GetParamDV(iDV, 1)); index[2] = kOrder;
            double *coord = FFDBox->GetCoordControlPoints(iOrder,
            int(config->GetParamDV(iDV, 1)), kOrder);
            x = coord[0]; y = coord[1]; z = coord[2];

            double movement_portion = (double) index[0]/(FFDBox->GetlOrder()-1);
            movement[0] = (movement_portion*FFD_x_mov)+D_FFD_x_0;

            movement[1] = 0;

            double k_Order = (double) kOrder;
            movement[2] = (FFD_z_mov*(k_Order-1))+(FFD_z_mov*k_Order);

            FFDBox->SetControlPoints(index, movement);

        }
    }
}

```