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THE COLLEGE OF AERONAUTICS
CRANFIELD

MEASUREMENT OF THE LATERAL STABILITY DERIVATIVES
OF A MORANE-SAULNIER M.S.760 'PARIS' AIRCRAFT G-APRU

by

W. G. Bradley

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THE COLLEGE OF AERONAUTICS
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Measurement of the lateral stability derivatives
of a Morane-Saulnier M.S. 760 'Paris' aircraft G-APRU

by

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SUMMARY

Flight tests were carried out on the College of Aeronautics M.S. 760 'Paris' aircraft G-APRU, in order to measure certain of the lateral derivatives. The derivatives were obtained by 'time-vector' analysis of the 'Dutch roll' oscillation: the sideslip-dependent derivatives were also measured by asymmetric and sideslip flight tests.

The Dutch rolls were carried out with the aircraft in two different inertia configurations, and the differences encountered in certain of the derivatives for the two conditions are attributable to the changes in wing dihedral and wing twist caused by the weight of fuel in the wing-tip tanks in the high inertia condition.

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LIST OF SYMBOLS

Axes and Variables

O, x, y, z	System of stability or wind axes
a_y	Lateral acceleration
v	Sideslip velocity
V	True airspeed
α	Body incidence
$\beta \left(= \frac{v}{V} \text{ radians} \right)$	True sideslip angle
ϕ	Bank angle
ξ	Aileron angle
ζ	Rudder angle
ψ	Yaw angle

Subscripts

B	Refers to aircraft body datum
G	Refers to gyro body datum
i	Refers to indicated or recorded value.

A dot above a quantity, such as $\dot{p}$, denotes differentiation with respect to time.

A bar above a quantity, such as $\bar{p}$, denotes a time vector.

Dutch Roll Parameters

T	Dutch roll oscillation period
$\omega = \frac{2\pi}{T}$	Damped circular frequency of Dutch roll oscillation.
t_D	Oscillation damping time; time for oscillation to damp to $\frac{1}{e}$ of original amplitude.
$\epsilon_D = \tan^{-1} \left(\frac{1}{\omega t_D} \right)$	damping angle
$\omega_0 \left(= \frac{\omega}{\cos \epsilon_D} \right)$	Undamped circular frequency of the Dutch roll oscillation
ϵ_{pr}	Phase angle by which rate of roll lags rate of yaw
$\epsilon_{\beta r}$	Phase angle by which sideslip angle lags rate of yaw
ϵ_{ayr}	Phase angle by which lateral acceleration lags rate of yaw

General Data

ASIR/IAS	Air Speed Indicator Reading/Indicated Air Speed (kts.) (Instrument error of A.S.I. used is zero.)
W	Aircraft weight (lbs.)
$m \left(= \frac{W}{g} \right)$	Aircraft mass (slugs)

b	Wing span (ft.)
S	Gross wing area (ft. ²)
ρ	Air density (slugs/ft. ³)
$\hat{t} \left(= \frac{m}{\rho S V} \right)$	Unit of aerodynamic time (seconds)
$\tau \left(= \frac{t}{\hat{t}} \right)$	Non-dimensional time (airsecs)
$\mu_2 \left(= \frac{m}{\rho S \frac{b}{2}} \right)$	Aircraft relative density

Aerodynamic Derivatives (Non-dimensional)

l_p	Rolling moment due to rate of roll
l_r	Rolling moment due to rate of yaw
l_v	Rolling moment due to sideslip
l_ξ	Rolling moment due to aileron
l_ζ	Rolling moment due to rudder
n_p	Yawing moment due to rate of roll
n_r	Yawing moment due to rate of yaw
n_v	Yawing moment due to sideslip
n_ξ	Yawing moment due to aileron
n_ζ	Yawing moment due to rudder
y_p	Side force due to rate of roll
y_r	Side force due to rate of yaw
y_v	Side force due to sideslip
y_ξ	Side force due to aileron
y_ζ	Side force due to rudder

1. Introduction

This report describes work carried out at the College of Aeronautics, Cranfield, to extract lateral aerodynamic derivatives from full scale flight test data, using the M.S.760 'Paris' aircraft.

The importance of aerodynamic derivatives measured from flight tests is considerable. They are of great use as a feedback into methods of estimation of derivatives, and as a check on wind tunnel values.

Derivatives measured in flight are also desirable to enable a better representation of the aircraft to be made for stability and control work, for the extrapolation of handling characteristics, and for the synthesis of transfer functions for use in auto-pilot and autostabiliser design.

Of the several methods of measuring aerodynamic derivatives in flight described in Reference 10, only three techniques have emerged as prominent:-

(i) The Frequency Response method (references 11 and 12), in which the transient response of the aircraft is analysed in the frequency plane can, in theory, yield all the lateral stability derivatives. Experience to date, however, has shown that the instrumentation requirements for this technique are extremely stringent. Several other features of the technique are by no means finalised and work is still proceeding to try to make the technique reliable.

(ii) The Equations of Motion technique (ref. 12) in theory enables all the lateral derivatives to be extracted. The technique consists of reading the flight record at frequent intervals and inserting the values of the parameters into the equations of motion. Several sets of simultaneous equations are then solved for the derivatives, the final values being statistically averaged. This technique has the disadvantage of a very high order of accuracy being required for the instrumentation, and requires the measurement of angular acceleration. The Equations of Motion technique is however quite workable.

(iii) The Time Vector technique (ref. 7) consists of analysis of the Dutch roll mode only, by transferring the aircraft response into the time domain, and transposing the equations of motion into vector polygons, which are then solved to give the stability derivatives. The fundamental limitation of this technique is that it is possible to extract only two derivatives from each degree of freedom.

The Time Vector technique was adopted for the current investigation mainly because some knowledge of its application existed at the College of Aeronautics. Other factors influencing the choice were that its instrumentation requirements were less exacting than for the other two methods, and that although the analysis work load is not less than with the other methods, it does enable the analyser to obtain a clear understanding of the system mechanics and the relationships between the derivatives. It was considered that the measurement of the sideslip-dependent derivatives ℓ_v , n_v and y_v by asymmetric and sideslip tests would compensate for the inability of the technique to yield all the lateral derivatives.

2. Synopsis of tests

Before the Time Vector technique could be applied to the analysis of damped Dutch rolls, a number of other tests were necessary.

Tests were carried out on the aircraft instrumentation system to obtain instrument dynamic response characteristics. The method adopted utilised a compound pendulum.

Airborne tests were carried out on the aircraft's sideslip vane to measure the effect of fuselage interference using the Flat Turn technique. This work is described in reference 5.

Considerable effort was expended in the measurement of the aircraft's moments of inertia, in pitch, roll and yaw, together with the product of inertia. The inertia measurements are described in reference 3.

Further airborne tests were carried out to measure aileron rolling power and the sideslip dependent derivatives l_v , n_v and y_v . The derivatives measured in this way were compared with the values obtained from Dutch roll tests.

Dutch roll tests were carried out at a constant altitude of 10,000 feet, covering the level speed range of the aircraft. These manoeuvres were carried out with the aircraft in two different inertia conditions, achieved by flying with the wing tip tanks first empty and then full of fuel.

3. Equipment

3.1 The Aircraft

The aircraft used in the tests was a Morane-Saulnier M.S.760 'Paris'. The 'Paris' is a four-seat, twin turbojet, all-metal monoplane. The aircraft is shown in figure 1 and principal data are given in table 1.

The aircraft was powered by two Turbomeca Marboré II turbojets situated in the fuselage and each delivering 883 lb. static thrust at sea level. The usable level speed range of the aircraft was from 100 to 295 kts. IAS at 10,000 feet.

Flying controls are operated manually by rods; the undercarriage, flaps, dive brakes and variable-incidence tailplane are operated electrically.

A principal feature of the 'Paris' is the wing-tip fuel tanks, each holding 50 gallons of fuel. These tanks have been modified so that the aircraft can fly with any desired quantity of fuel in either tank. This provides a simple means for using an asymmetric fuel loading to determine aileron power and for varying the aircraft's inertia distribution. In order to avoid fuel sloshing effects, Dutch rolls were carried out with the tip tanks either empty or completely full. The addition of full tip fuel increases the roll inertia by a factor of 2.7 and the yaw inertia by 1.7.

3.2 Instrumentation

The instrumentation requirements for the flat turn, sideslip and Dutch roll tests were basically the same, and consisted of the following quantities:-

Sideslip Angle
Rate of Roll
Rate of Yaw

Lateral Acceleration
Bank Angle
Heading Angle
Aileron Angle
Rudder Angle.

The various combinations of these quantities desired for each test were recorded on a Hussenot-Beaudoin A.13 photographic trace recorder. The paper width was 89 mm. and nominal paper speed $\frac{3}{4}$ inch per second.

The exact system used for recording each quantity will now be described in detail.

3.2.1. Sideslip angle

The angle of sideslip was measured by a simple balanced wind vane, mounted on a probe 2 ft. in front of the aircraft nose. Vane position was measured by an Elliott type W.121 inductive pick-off. The 400 c/s carrier frequency supplied a low pass filter/demodulator unit together with an R. - C. damping network which fed a S.F.I.M. E.301 galvanometer. Range normally used was $\pm 12^\circ$.

3.2.2. Rates of roll and yaw

Two C.I.D. single-axis rate gyros were used for these tests, having ranges of 10 and 20 degrees per second, and natural frequencies of 10 and 15 cycles per second. The pick-offs used were A.C. inductive types directly driving S.F.I.M. E.12 ratimeters. These gyros were oil-damped and as such are known to be temperature sensitive; this effect was minimised by installing the gyros in the aircraft cockpit which was maintained at approximately ground temperature. These gyros are also known to be extremely sensitive to supply frequency. To combat this effect strict monitoring of aircraft supply frequency was necessary to keep it as close as possible to that used during ground calibration.

3.2.3. Lateral acceleration

Lateral acceleration was measured by a S.F.I.M. type J.21 $\pm 0.6g$ accelerometer. This accelerometer is of the spring-mass type and incorporates a D.C. potentiometric pick-off directly driving a S.F.I.M. E.301 galvanometer.

3.2.4. Bank Angle

For the asymmetric and sideslip tests a S.F.I.M. type J.32 pendulum was mounted in the recorder itself. The natural frequency of the pendulum was 4 cycles per second, and the calibrated range $\pm 20^\circ$.

The flat turn tests demanded a more sensitive instrument. For this purpose a Smith's 2-axis free gyro was used. This instrument has an A.C. inductive pick-off and a calibrated range of $\pm 12^\circ$. The gyro has a recaging accuracy of $\pm \frac{1}{4}^\circ$.

3.2.5. Heading angle

Heading angle was measured by a simple ex-Luftwaffe directional gyro, modified by the addition of a multi-mirror system to the gimbal axis, and fitted directly to the recorder. Calibrated range was $\pm 30^\circ$.

3.2.6. Aileron and rudder angles

The control angles were measured by Penny and Giles linear potentiometers, type LP2-S. The potentiometers were fixed to the airframe and measured the movement of the control rods. The recording elements were S.F.I.M. E.512 ratimeters.

3.2.7. Dynamic calibration

The recording systems of sideslip, roll and yaw rates and lateral acceleration were dynamically calibrated to determine their phase angle frequency response. The calibrations were carried out using a compound pendulum (see fig. 2). The calibrations were carried out using the aircraft's recording system so that the phase lags shown in figs. 3, 4, 5, and 6 are absolute, that is, they include the recording elements. Extreme care had to be taken to determine phase angles of such small magnitude, overall accuracy being somewhat better than $\pm 0.2^\circ$. For the rate gyro calibration the peak to peak method was used and for the displacement quantities a zero crossing technique was adopted.

4. Preliminary tests

The following measurements were required for detailed analysis of the Dutch roll tests:-

- (a) The horizontal and vertical coordinates of the aircraft's centre of gravity in relevant fuel conditions.
- (b) The aircraft's moments of inertia in roll and yaw, together with the product of inertia, in relevant fuel conditions. For the sake of completeness, the pitching moment of inertia was also measured.
- (c) The airborne calibration of the aircraft's sideslip vane.

The determination of the desired quantities in (a) and (b), using the methods described in references 1 and 2, are described by the author in reference 3. The airborne calibration of the sideslip vane by the method of reference 4 is described by the author in reference 5.

4.1 Asymmetric tests

4.1.1. Theory

The wing-tip fuel tanks on the 'Paris', aircraft provide a ready means by which an asymmetric flight condition can be imposed on the aircraft. If the aircraft

is flown under an asymmetric condition in roll, then the out-of-balance rolling moment needed to maintain the wings level must be provided by the ailerons. This then provides a simple means by which the aileron rolling power derivative l_{ξ} can be determined.

If the aircraft is flown with an asymmetric rolling moment, applied by an asymmetric fuel condition, then the rolling moments due to aileron, rudder, sideslip and asymmetric rolling moment must be in equilibrium

$$\beta l_v + \xi l_{\xi} + \zeta l_{\zeta} + l_1 = 0 \quad (1)$$

The last term in equation (1) is defined as

$$l_1 = \frac{L_1}{\frac{1}{2} \rho V^2 S b / 2}$$

where L_1 is the asymmetric rolling moment.

Assuming that the aircraft is flown at zero sideslip, then equation (1) reduces to

$$\xi_{\beta=0} l_{\xi} + \zeta_{\beta=0} l_{\zeta} + l_1 = 0 \quad (2)$$

where $\xi_{\beta=0}$ and $\zeta_{\beta=0}$ are the aileron and rudder angles to achieve zero sideslip.

Dividing equation (2) by $\xi_{\beta=0}$ and re-arranging

$$l_{\xi} = - \left\{ \frac{l_1}{\xi_{\beta=0}} + l_{\zeta} \left(\frac{\zeta}{\xi} \right)_{\beta=0} \right\} \quad (3)$$

The slopes $l_1/\xi_{\beta=0}$ and $(\zeta/\xi)_{\beta=0}$ can readily be drawn by cross-plotting from sideslip results, carried out with varying values of L_1 . The rudder rolling power l_{ζ} can be estimated or a wind tunnel value used.

4.1.2 Method of asymmetric tests

Each tip tank on the aircraft has a capacity of 50 Imperial gallons. Asymmetric sideslips were flown with out-of-balance fuel loads, to both port and starboard, of 0, $12\frac{1}{2}$, 25, $37\frac{1}{2}$ and 50 gallons. The straight sideslips were carried out at 110, 160 and 220 kts. ASIR at 10,000 ft. The reason for carrying out straight sideslips was that the zero sideslip condition can best be determined by interpolation. The sideslip vane angle readings were corrected to true sideslip angle by the calibration obtained from the Flat Turn tests.

4.1.3. Results of asymmetric tests

Typical sideslip results are shown in figs. 7 and 8 for the flight condition of 160 kts. ASIR at 10,000 ft. The resulting plots $l_1/\xi_{\beta=0}$ and $(\zeta/\xi)_{\beta=0}$ for the three test speeds are shown in figs. 9, 10, and 11. The noticeable feature of the latter curves is the tendency for $(\zeta/\xi)_{\beta=0}$ to rapidly diminish with increase in speed. The assumed wind tunnel value of l_{ζ} was

$$l_{\zeta} = +0.013 - 0.008 C_L$$

The accuracy on the assumed value of ℓ_ξ is $\pm 40\%$. This large inaccuracy did not materially affect the asymmetric results because the contribution of the ℓ_ξ term in equation (3) is only 1.5% at the lowest airspeed. The values of ℓ_ξ calculated from equation (3) are given in Table 2. The scatter evident on the curves of ℓ_1 versus $\xi_{\beta=0}$ is equivalent to a $\pm 2\%$ error in $\frac{\ell_1}{\xi_{\beta=0}}$. This implies a probable error of from 3 to 4% in the measured values of ℓ_ξ .

4.2 Sideslip tests

A series of sideslip tests were carried out to enable the sideslip-dependent derivatives ℓ_v , n_v and y_v to be determined.

Considering the aircraft to be sideslipped at constant airspeed and altitude, the derivatives can be expressed as

$$y_v = - \left\{ \frac{1}{2} C_L \frac{d\phi}{d\beta} + y_\xi \frac{d\xi}{d\beta} + y_\xi \frac{d\xi}{d\beta} \right\} \quad (4)$$

$$\ell_v = - \left\{ \ell_\xi \frac{d\xi}{d\beta} + \ell_\xi \frac{d\xi}{d\beta} \right\} \quad (5)$$

$$n_v = - \left\{ n_\xi \frac{d\xi}{d\beta} + n_\xi \frac{d\xi}{d\beta} \right\} \quad (6)$$

The slopes $d\xi/d\beta$, $d\xi/d\beta$ and $d\phi/d\beta$ can be obtained from plots resulting from straight sideslips.

4.2.1 Method of sideslip tests

Straight sideslips were carried out at 110, 160, 220 and 295 kts. A.S.I.R. at 10,000 ft. and at 110, 160, and 220 kts. A.S.I.R. at 22,000 ft. Indicated sideslip angle, rudder and aileron angles, and bank angle were recorded in each condition.

4.2.2. Results of sideslip tests

The slopes $\frac{d\xi}{d\beta}$, $\frac{d\xi}{d\beta}$, $\frac{d\phi}{d\beta}$ are shown plotted against C_L in figs. 12, 13 and 14. It can be seen that there is a slight but discernable difference between the tests carried out at 10,000 and 22,000 ft.

The above slopes were then used to compute the sideslip-dependent derivatives ℓ_v , n_v and y_v from equations (4), (5) and (6). Of the control derivatives used in these computations, y_ξ and n_ξ were assumed to be negligible, n_ξ and ℓ_ξ were wind tunnel values, and ℓ_ξ was determined from asymmetric tests.

5. Dutch rolls and their analysis using time vectors.

5.1 Equations of motion

The aircraft's lateral equations of motion are expressed in Duncan's notation (ref. 6) as follows:-

5.4 Analysis of Dutch Rolls

A typical trace recording of a Dutch roll is shown in figure 15. The recordings were analysed on Benson-Lehner OSCAR trace reading equipment.

The times of occurrence and amplitudes of each peak were read off for each variable. The resulting peak times were then used to compute the 'raw' phase angles relative to rate of yaw, using a digital averaging method. The peak amplitudes were then filtered graphically to remove the spiral mode divergence and off-set zeros. Re-plotting the amplitudes against time on logarithmic graph paper yielded the damping parameters and the amplitudes of the recorded variables, after the instrument phase angle characteristics had been taken into account (equivalent to a time lag on decaying parameters).

The instrument phase lag characteristics were then applied to the relative phase angles. The ensuing amplitudes and phase angles were corrected for instrument alignment, sideslip vane calibration and position, and lateral accelerometer location. The preceding corrections are all explained in detail in Appendix II. The final values of the amplitudes and phase angles, referred to wind axes, were then in a suitable form to be used in the solution of the vector polygons.

5.5 Analysis of the vector polygons

The vector polygons consist of five vectors, two of which are known and three unknown. This means that one of the three unknown vectors must be assumed in order to solve the polygons, shown in figures 16 and 17.

It is usual, and indeed desirable, to assume the derivative having the smallest vector length. Thus it is ℓ_r which is assumed in the rolling moment polygon and n_r for the yawing moment polygon.

The assumed values were calculated from R.Ae.S. Data sheets and were

$$\ell_r = +0.015 + 0.209 C_L$$

$$n_r = -0.158 - 0.024 C_L$$

No side force diagrams were analysed; a sketch of a typical one is shown in fig. 18. It can be seen that it is practically impossible to determine y_p and y_r . Values of y_v were calculated from equation (14). The values of $\left| \frac{ay}{\beta} \right|$ were those measured from the Dutch roll oscillation.

6. Results

6.1 Low inertia condition

6.1.1 Rolling moment polygons

Solution of the rolling moment polygons for an estimated value of ℓ_r produced

produced the values of ℓ_v shown in fig. 19, which includes values of ℓ_v determined from wind tunnel tests, and values calculated from sideslip tests, as explained in section 4.

It can be seen that agreement with the values derived from sideslip tests is excellent, whilst there is only fair agreement between the Dutch roll and wind tunnel values, the values of ℓ_v measured in flight tests being approximately 0.02 lower than the wind tunnel results over the C_L range. The same C_L dependency is shown in both cases. The difference between the wind tunnel and flight test values is most probably due to scale effect and/or a power effect.

The values of ℓ_p determined from an estimated value of ℓ_r are shown in fig. 20, together with an estimated value derived from R.Ae.S. data sheets. The estimated value of -0.460 is a compromise value between -0.440, assuming the tip tanks to be absent, and -0.470, allowing for the tip tanks effect on aspect ratio by assuming that they are equivalent to end plates of height equal to tip tank diameter.

The values of ℓ_p calculated from vector analysis are in very good agreement with the estimated value, except at the higher speeds where the flight test values are seen to increase numerically to approximately -0.5. The reason for this apparent increase in roll damping at high speed is not easy to find. One possible explanation is that the assumed value of ℓ_r is too small at the low C_L values; this implies that the wing contribution to ℓ_r is larger than the estimation allows for.

The measured values of ℓ_v and ℓ_p shown in figures 19 and 20 are subject to quite considerable scatter - of the order of 15 - 20% in both cases. This scatter is caused by the conditioning of the vector diagram. It can be seen from figure 16 that the low inertia rolling moment polygon is rather long in the $\bar{p}$ and $\bar{\beta}$ directions making the determination of ℓ_p and ℓ_v very sensitive to the phase angle between the $\bar{p}$ and $\bar{\beta}$ vectors. The only means of reducing this scatter from a given set of flight results is to calculate ℓ_p and ℓ_r from an assumed ℓ_v , or ℓ_v and ℓ_r from an assumed ℓ_p .

6.1.2 Yawing moment polygons

The yawing moment polygons were solved for an assumed value of n_r . The resulting values of n_v and n_p are shown plotted against C_L in figs. 21 and 22.

The values of n_v derived from vector analysis are seen to be at variance with the values derived from straight sideslips and wind tunnel tests. The difference between the values obtained from wind tunnel tests and straight sideslips is obviously due to the value of n_ξ used in equation (6). The value of n_ξ used to determine n_v was itself derived from wind tunnel tests.

The values of n_v from vector analysis are seen to be 50% lower than the wind tunnel values at $C_L = 0.1$ and 10% lower at $C_L = 0.8$. The increase in the flight test values of n_v with C_L is quite surprising and indicates that the contribution of the wing to n_v is quite large.

The yawing moment polygon is well-conditioned for the determination of n_v from n_r ; this is shown by the low scatter distribution on the measured values (fig. 21). The scatter on n_v is approximately ± 0.006 throughout the speed range.

The measured values of the cross-derivative n_p , shown in figure 22, are quite different from the values estimated using ref. 9, and also from those quoted by the aircraft's manufacturers, the source of which is unknown. The flight test values of n_p show little or no dependency on lift coefficient. The scatter distribution on the n_p versus C_L plot is approximately ± 0.04 throughout the C_L range.

6.1.3 Side force equation

Fig. 23 shows the values of y_v determined from equation (14) plotted against C_L . Also shown are the values determined from straight sideslips (equation 4), and wind tunnel tests. It can be seen that the two sets of flight results are in very close agreement with each other, but differ markedly from the wind tunnel values, the differences being zero at $C_L = 0.20$ and 0.08 at $C_L = 1.0$. The flight test values of y_v exhibit a much greater variation with C_L than do the wind tunnel values; this implies a greater degree of wing contribution than the wind tunnel values indicate.

6.2 High inertia condition

Dutch rolls were also analysed for the tip tanks full condition, in which the rolling inertia was increased by 270% and the yawing inertia by 170%. These tests were carried out to compare derivatives in the two inertia conditions and to determine the precise effect of inertia distribution on the conditioning and shape of the vector polygons.

6.2.1 Rolling moment polygons

Solution of the vector polygons for the high inertia case for the derivatives ℓ_p and ℓ_v yields the derivatives shown in figures 24 and 25.

The high inertia values of ℓ_p are seen to be in reasonable agreement with the low inertia values. The tendency for the high inertia values of ℓ_p to increase numerically with decrease in C_L is again evident as in the low inertia case. This tendency is most probably due to the assumed value of ℓ_r (the same value as for the low inertia case) not being strictly applicable to the high inertia case, where the addition of wing tip fuel results in some wing deflection which would alter ℓ_r .

The high inertia values of ℓ_v shown in fig. 25 are in agreement with the low inertia values up to a C_L of approximately 0.6, above which they tend to diverge, and at a C_L of 0.9 the high inertia values of ℓ_v are approximately 0.02 numerically larger than the low inertia values. The reason for this divergence is probably that the high inertia ℓ_v is affected by wing distortion.

6.2.2 Yawing moment polygons.

The values of n_v and n_p obtained by solution of the high inertia polygons, for the same values of n_r as used in the low inertia case, are shown in figs. 26 and 27.

The high inertia values of n_v shown in fig. 26 are in poor agreement with the low inertia values. The high inertia values are increased by approximately 0.02 throughout the speed range; the same variation with C_L is shown in both cases.

The high inertia values of n_p shown in fig. 27 are also at variance with the low inertia values, being approximately one half of the low inertia values over the speed range.

6.2.3 Side force equation

The results of application of equation (14) to the results of the high inertia condition are shown in fig. 28. The values of y_v obtained are seen to be in extremely close agreement with the values obtained from the low inertia case.

7. Assessment of the accuracy of the Dutch roll results

In view of the large number of variables and the large number of data points needed to obtain one derivative, a strict error analysis of the Time Vector technique was not carried out on the work described herein.

A simpler approach to accuracy assessment is to divide the errors inherent in the technique into two categories - Random and Systematic - and then to determine the effects of these errors individually.

7.1 Random errors

Random errors are caused by errors in the instrumentation system, trace reading and subsequent processing of the recorded data. The effect of random errors can be clearly seen in the scatter on the plots of the resulting derivatives. This effect can be amplified or attenuated by the conditioning of the vector polygons. For example, in the rolling moment polygon, far less scatter would result in the derivative $\dot{\ell}_p$ if it were determined for an assumed value of ℓ_v rather than the assumed value of ℓ_r . Some idea of the experimental errors, that is the random errors up to the stage of drawing the vector polygons, can be gained from figs. 29 to 34. These graphs illustrate the desirability of analysing three consecutive Dutch rolls at the same airspeed. One noticeable feature of figs. 29 and 30 is that the amplitude ratios measured in the high inertia case have far greater scatter than the low inertia case. This scatter is not as evident in the derivatives because of the different polygon conditioning pertaining to the two inertia conditions.

The following table gives typical parameter scatter values:-

Parameter	Typical Scatter Distribution	
	Low Inertia	High Inertia
$\dot{\psi} / \dot{\phi}$	2%	4%
$\beta / \dot{\psi}$	$1\frac{1}{2}\%$	4%
a_y / β	4%	4%
ϵ_{pr}	$1\frac{1}{2}^\circ$	$1\frac{1}{2}^\circ$
$\epsilon_{\beta r}$	$1\frac{1}{2}^\circ$	$1\frac{1}{2}^\circ$

7.2 Systematic errors

Systematic errors are caused by inaccuracies in the derivatives assumed for solution of the vector polygons, and errors involved in the measured moments of inertia.

As an illustration of this effect, systematic errors are evaluated for a Dutch roll at 157 kts. E.A.S. at 10,000 feet in the low inertia condition.

An increase of 50% in the product of inertia term i_E decreases the value of $-\ell_v$ by 6%, increases $-n_p$ by 8.5% and decreases n_v by 5%. The value of ℓ_p is virtually unaffected.

Increasing the value of the estimated cross-derivative ℓ_r by 25% results in reductions of 9% and 10% in $-\ell_p$ and $-\ell_v$ respectively, whilst the same error in the estimation of n_r gives rise to a 15% decrease in n_v and a 32% increase in $-n_p$.

The effect of increasing the non-dimensional moment of inertia i_A by 3%, is a 4% increase in both $-\ell_p$ and $-\ell_v$. A 3% increase in i_C results in a 4% increase in n_v but practically no change in n_p .

The results quoted above refer only to the particular flight condition considered, but the magnitudes of the systematic errors are of the same order for all flight conditions, including the high inertia case.

Systematic errors are really a function of the conditioning of the vector polygons, and for any given aircraft configuration there is virtually nothing that can be done to reduce them.

8. Discussion

8.1 General

The results obtained from the vector analysis of Dutch rolls for the low inertia case are quite satisfactory.

The rolling moment derivative ℓ_p derived from vector analysis is in very good agreement with estimated values. The derivative ℓ_v is in excellent agreement with values determined from asymmetric and sideslip tests, but is approximately 0.02 lower than the wind tunnel value throughout the speed range.

The yawing moment derivative n_v is in reasonable agreement with wind tunnel results, but does show a greater variation with lift coefficient than do both wind tunnel and sideslip results. This difference could possibly be a power effect caused by the jet efflux affecting the fin effectiveness; this would have a C_L dependence and would not be present in wind tunnel tests.

The measured values of n_p seem quite satisfactory but do not agree at all well with estimated values, and are indeed of opposite sign to manufacturer's figures. The manufacturer's figures should, however, be viewed with some doubt.

The values of y_v measured from Dutch roll tests are in excellent agreement with sideslip values. The flight test values show far more variation with C_L than

wind tunnel values; again this is probably due to a power effect similar to that affecting n_v .

The results obtained from the high inertia configuration are less satisfactory than those from the low inertia condition. The rolling derivatives are, on the whole, in quite good agreement with the values in the low inertia condition. The changes in ℓ_p and ℓ_v at low speed are most probably due to a combination of wing distortion altering the high inertia value of ℓ_v and also the value of ℓ_r used in the calculation not being strictly applicable for the high inertia case.

The derivatives obtained from the yawing moment polygons for the two inertia conditions do not agree at all well, the values of n_v extracted from the high inertia case being approximately 0.02 larger than those obtained from the low inertia and the values of n_p being approximately one half of the low inertia values.

The values of the side force derivative y_v measured in the high inertia condition are in excellent agreement with low inertia values.

8.2 The yawing moment polygons

The discrepancy between the yawing moment derivatives obtained from the two inertia conditions tested is undoubtedly caused by the distortion of the airframe arising from the difference in inertia/mass distribution.

A simple ground test in which the wing tip deflection was measured as fuel was added to the tip tank showed that the wing tip was deflecting downwards and also twisting nose down.

The addition of tip fuel therefore changes the wing dihedral and twist which in turn alter some, or all, of the yawing derivatives. Why the yawing moment derivatives should be more affected than the rolling moment derivatives is not at all clear.

Assuming that the derivative n_r varies with wing deformation and n_v does not, it was decided to re-solve the high inertia polygons for the low inertia value of n_v . This assumption resulted in values of n_r 50% greater than the assumed low inertia values, and values of n_p 100% greater than their low inertia counterparts.

Alternatively, if the wind tunnel value of n_v were assumed for both inertia cases, then this has the effect of bringing the low and high inertia values of n_r and n_p together at lift coefficients below 0.4, above which they tend to diverge. This effect is similar to that observed in the rolling moment results.

8.3 Future work

Future work is necessary in order to fully investigate the effect of change of wing twist and dihedral on the lateral stability derivatives.

More Dutch roll tests should be carried out to fill in the gaps in the lower end of the speed range in both the low and high inertia conditions.

The control derivative ℓ_g should be measured at more frequent speed intervals

by carrying out more asymmetric tests. The derivative n_{ξ} should also be measured in flight possibly by means of the rudder impulse method.

The values of ℓ_{ξ} and n_{ξ} so determined could then be used in conjunction with straight sideslip tests to yield values of ℓ_v and n_v measured in both inertia conditions.

These values of ℓ_v and n_v could be used as the assumed derivatives in the solution of the rolling and yawing moment vector polygons. By this means it would be possible to determine ℓ_r , ℓ_p , n_r and n_p , with quite reasonable accuracy.

In conjunction with the derivative measurements, ground and possibly airborne tests would be necessary to accurately determine the wing deformation caused by tip tank fuel.

It should then prove possible to relate the changes in wing shape to the changes in the measured derivatives.

9. Conclusions

1. Asymmetric and sideslip tests were carried out to measure the derivatives ℓ_{ξ} , ℓ_v , n_v and y_v .
2. Vector analysis of Dutch rolls yielded the derivatives ℓ_v , ℓ_p , n_v , n_p and y_v . The rolling moment and side force derivatives so determined are in good agreement with the other full scale results (and estimated values in the case of ℓ_p). Agreement with wind tunnel results is only fair. The yawing derivatives are in reasonable agreement with wind tunnel values.
3. Dutch roll tests carried out with the aircraft in a high inertia condition, i.e. with tip tanks full, have yielded values of ℓ_v and ℓ_p in quite good agreement with values obtained from tests carried out without tip fuel. There is some slight but explicable divergence between the two sets of values.
4. Simple tests have shown that the aircraft wing deforms in both dihedral and twist with the addition of wing-tip fuel. Flight results indicate that this deformation affects the yawing moment derivatives more than the rolling moment derivatives.
5. Further work is needed to precisely determine the extent to which a known wing deformation affects the aerodynamic derivatives.

Addendum

After this report was completed, two possible causes of the discrepancy between the derivatives measured in the two inertia conditions have come to the author's notice.

The first possible explanation is that the 'additional mass correction', which has to be applied to the measured derivatives to allow for the movement of entrapped air, is dependent only on the aircraft shape. This correction has been ignored in this report and had not the tests been carried out in two different inertia conditions the point would have remained undetected. If this correction were applied, then because it is a constant the percentage effect on the two inertia cases would be different. Work would also be necessary to vary this correction for dynamic manoeuvres at altitude.

The second explanation concerns the possible presence of acceleration derivatives, which would have to be allowed for in the solution of the vector diagrams.

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APPENDIX I

The 'Time Vector' method for the analysis of a damped sinusoidal oscillation

Consider a simple spring/mass oscillatory system of mass m and spring constant c .

The equation of motion of such a system for a deflection x is

$$m\ddot{x} + cx = 0 \quad (\text{A.1})$$

a solution of which is

$$x = x_0 \sin [\omega_0 t + \phi] \quad (\text{A.2})$$

where $\omega_0 = \sqrt{\frac{c}{m}}$ = circular frequency and x_0 and ϕ constants dependent on boundary conditions.

Differentiate equation (A.2) with respect to time:-

$$\begin{aligned} \dot{x} &= \omega_0 x_0 \cos [\omega_0 t + \phi] \\ &= \omega_0 x_0 \sin \left[\omega_0 t + \frac{\pi}{2} + \phi \right] \end{aligned} \quad (\text{A.3})$$

The variation of x with time may be represented by the projection of a time vector $\bar{x}$ of constant length x_0 rotating at constant angular velocity ω_0 and starting from an initial angular position ϕ . The variation of $\dot{x}$ with time may be represented by a time vector $\bar{\dot{x}}$ of constant length $\omega_0 x_0$ and angular velocity ω_0 but starting from an initial angle of $\frac{\pi}{2} + \phi$, that is to say 90° phase advanced on $\bar{x}$. This is shown in Fig. 35a. The two vectors are time invariant with respect to each other.

If the spring mass system now has velocity damping of constant k added to it, the equation of motion becomes

$$m\ddot{x} + k\dot{x} + cx = 0$$

a solution of which is

$$x = x_0 e^{\frac{-k}{2m} t} \sin [\omega t + \phi] \quad (\text{A.4})$$

$$\text{where } \omega = \sqrt{\frac{c}{m} - \left(\frac{k}{2m}\right)^2} \quad (\text{A.5})$$

$\frac{k}{2m}$ is usually equated to $\frac{1}{t_0}$, where t_0 is called the 'damping time' and is the time taken for the oscillation to decay to $\frac{1}{e}$ of its original amplitude.

Hence equation (A.5) becomes

$$\omega = \sqrt{\omega_0^2 - \left(\frac{1}{t_0}\right)^2} \quad (\text{A.6})$$

The 'damping angle' is defined as ϵ_0 , where

$$\epsilon_0 = \tan^{-1} \left(\frac{1}{\omega t_0} \right)$$

Therefore equation (A.6) becomes

$$\omega = \omega_0 \cos \epsilon_0$$

and
$$\frac{1}{t_0} = \omega_0 \sin \epsilon_0.$$

Differentiating equation (A.4) with respect to time

$$\begin{aligned} \dot{x} &= x_0 e^{-\frac{t}{t_0}} \left\{ -\frac{1}{t_0} \sin [\omega t + \phi] + \omega \cos [\omega t + \phi] \right\} \\ &= x_0 \omega_0 e^{-\frac{t}{t_0}} \left\{ -\sin \epsilon_0 \sin [\omega t + \phi] + \cos \epsilon_0 \cos [\omega t + \phi] \right\} \\ &= x_0 \omega_0 e^{-\frac{t}{t_0}} \cos [\omega t + \phi + \epsilon_0] \\ \therefore \dot{x} &= x_0 \omega_0 e^{-\frac{t}{t_0}} \sin \left[\omega t + \phi + \epsilon_0 + \frac{\pi}{2} \right]. \end{aligned}$$

Similarly it can be shown that

$$\ddot{x} = x_0 \omega_0^2 e^{-\frac{t}{t_0}} \sin [\omega t + \phi + 2\epsilon_0 + \pi] \quad (\text{A.7})$$

Thus in the case of the damped oscillation x and $\dot{x}$ can be represented by $\bar{x}$ and $\bar{\dot{x}}$ rotating at constant angular velocity ω . The amplitudes of $\bar{x}$ and $\bar{\dot{x}}$ at zero time are x_0 and $\omega_0 x_0$, the amplitudes decrease with time at a rate of $e^{-\frac{t}{t_0}}$. The phase of $\bar{\dot{x}}$ is $\frac{\pi}{2} + \epsilon_0$ in advance of $\bar{x}$, where ϵ_0 is the 'damping angle'. Similarly the vector representation of $\ddot{x}$, namely $\bar{\ddot{x}}$, would have a zero time length of $x_0 \omega_0^2$ and have a phase angle of $\pi + 2\epsilon_0$ in advance of $\bar{x}$. The arrangement of these vectors is shown in fig. 35b.

Since the 'time vectors' are invariant with time with respect to each other, they can be added and subtracted in exactly the same manner as 'ordinary' vectors.

The solution of an aircraft's equations of motion can readily be carried out by means of time vectors. It is necessary to know the relative amplitudes and phase angles of the basic variables from analysis of a Dutch Roll. Differentials or integrals of variables can then be easily obtained in the manner outlined above.

APPENDIX II

Corrections to recorded data

1. Rates of roll and yaw

Since the rate gyros used to measure the aircraft's rates of roll and yaw are fixed in the airframe, they do not measure angular velocities about the wind axes for a given flight condition. This means that data obtained from the rate gyros must be transformed from gyro axes to wind axes. The system of axes used is shown in fig. 36.

The angle between the x-axis of the gyro axes and that of the wind axes is given by

$$x = \alpha + \epsilon \quad (\text{A.8})$$

where α is the trimmed body datum incidence and ϵ is the angle between the rate gyro datum and aircraft body datum.

Defining the time vectors representing the rates of roll and yaw about the gyro axes as $\bar{p}_g$ and $\bar{r}_g$, the transformation to wind axes is given by

$$\bar{r} = \bar{r}_g \cos x - \bar{p}_g \sin x \quad (\text{A.9})$$

$$\bar{p} = \bar{p}_g \cos x + \bar{r}_g \sin x \quad (\text{A.10})$$

The transformation is given by the addition of two vectors and is shown in fig. 37.

2. Lateral acceleration

Since it is not always possible to position a lateral accelerometer at the aircraft's centre of gravity, it is necessary to correct the reading of the lateral accelerometer for the effects of rates of yaw and roll. Fig. 38 shows a system of body axes with origin at the aircraft's centre of gravity 0. The lateral accelerometer position at C has coordinates of (x_1, y_1, z_1) with respect to the body axes. If ay_1 is the reading of the lateral accelerometer and ay the acceleration of the centre of gravity 0, (both positive to starboard), then

$$ay = ay_1 - x_1 \dot{r}_g + z_1 \dot{p}_g + y_1 (p_g^2 + r_g^2) \quad (\text{A.11})$$

It can be assumed that provided ϵ is small then $p_g = p_b$ and $r_g = r_b$. The last term in equation (A.11) is negligible since it represents the squares of small quantities. Equation (A.11) is then reduced to

$$ay = ay_1 - x_1 \dot{r}_g + z_1 \dot{p}_g \quad (\text{A.12})$$

The amplitude of the vectors $\bar{r}_g$ and $\bar{p}_g$ can easily be determined if the vectors $\bar{r}_b$ and $\bar{p}_b$ are known. The correction can then be applied vectorially as shown in fig. 39.

3. Sideslip angle

Sideslip angle was measured by a wind-vane 16.5 ft. in front of the aircraft's centre of gravity. Because of its position relative to the aircraft's c.g. the vane measured the indicated sideslip angle $\bar{\beta}_i$ and a component due to the aircraft's rate of yaw $\Delta\beta$.

This can be expressed as

$$\bar{\beta} = \bar{\beta}_i - \Delta\beta \quad (\text{A.13})$$

where $\bar{\beta}_i$ is $\bar{\beta}_i$ corrected for the fuselage effect by the flat turn calibration,

$$\text{and } \Delta\beta = \frac{\bar{r} \cdot \ell_x}{V}$$

where $\bar{r}$ = rate of yaw about wind axes

ℓ_x = distance of sideslip vane in front of aircraft c.g.

V = true airspeed.

The transformation of $\bar{\beta}_i$ to $\bar{\beta}$ is shown vectorially in fig. 40.

TABLE 1

M.S. 760 'Paris' Aircraft
Principal dimensions and data

Wing

Span (to tip tank centre line)	31.8 ft.
Gross Area	198 sq.ft.
Aspect Ratio	5.1
Standard mean chord	6.23 ft.
Taper Ratio	0.71
Leading edge sweepback	$5\frac{1}{2}^{\circ}$
Dihedral	8°

Ailerons

Total area aft of hinge	14.3 sq.ft.
Mean chord aft of hinge	1.36 ft.
Stick/aileron gearing	0.44 rad/ft.
Aileron travel	16° up, 8.5° down.

Tailplane and Elevator

Combined gross area	31.85 sq.ft.
Elevator area aft of hinge	9.78 ft.
Elevator mean chord	1.22 ft.
Stick/elevator gearing	0.498 rad/ft.
Tailplane travel	$\pm 2.5^{\circ}$
Elevator travel, tailplane set at $+1.5^{\circ}$	19° up, 6° down

Fin and Rudder

Combined gross area	18.4 sq.ft.
Rudder area aft of hinge	5.9 sq.ft.
Rudder mean chord aft of hinge	1.92 ft.
Pedal/rudder gearing	0.97 rad/ft.
Rudder travel	$\pm 20^{\circ}$

Centre of gravity and inertia characteristics are given in reference 3.

TABLE 2

Values of rolling power calculated from asymmetric tests

Equivalent Airspeed (Kts.)	Lift Coefficient C_L	Rolling Moment Due to Rudder l_ζ	Aileron Rolling Power l_ξ
107	0.905	+0.006	-0.185
157	0.415	+0.010	-0.199
214	0.223	+0.011	-0.187



FIG. 1. THE MORANE-SAULNIER M.S. 760 'PARIS'

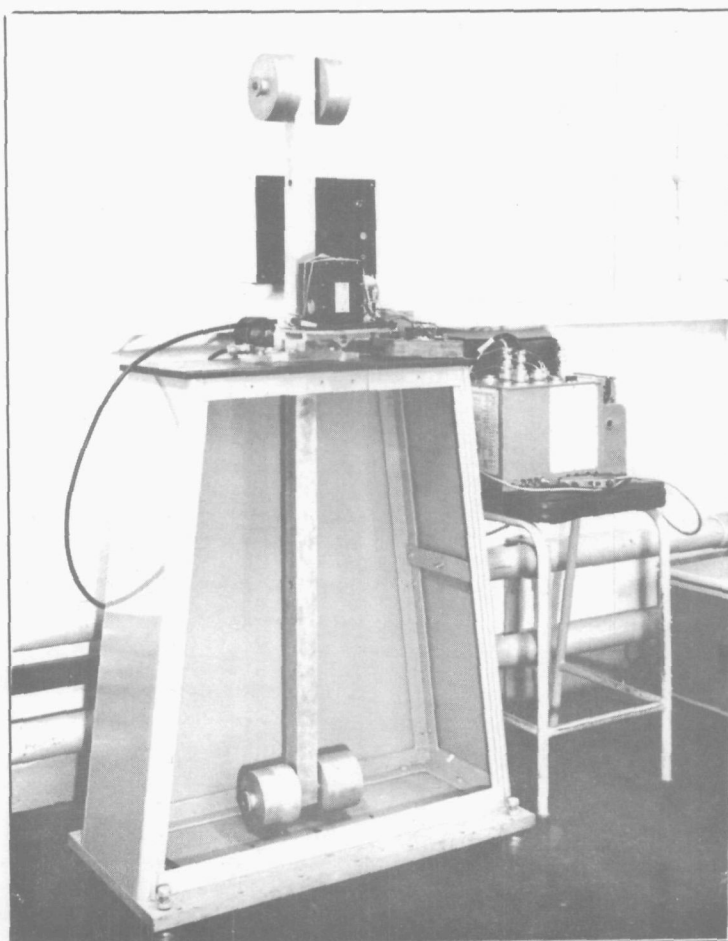


FIG. 2. THE COMPOUND PENDULUM

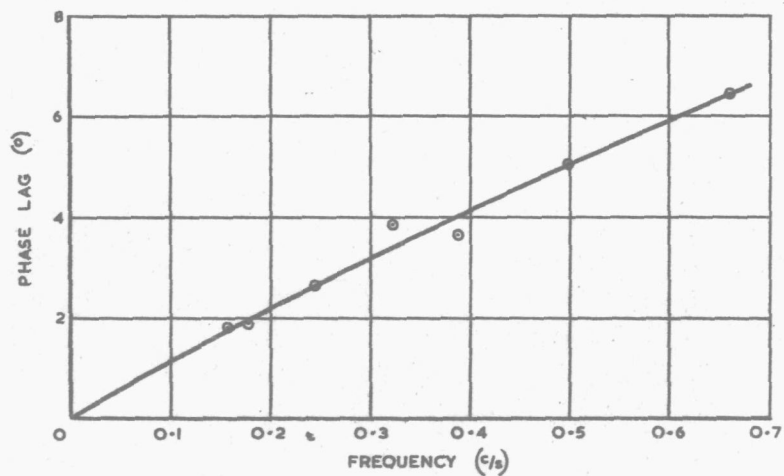


FIG.3. INSTRUMENT DYNAMIC CALIBRATIONS : ROLL RATE GYRO

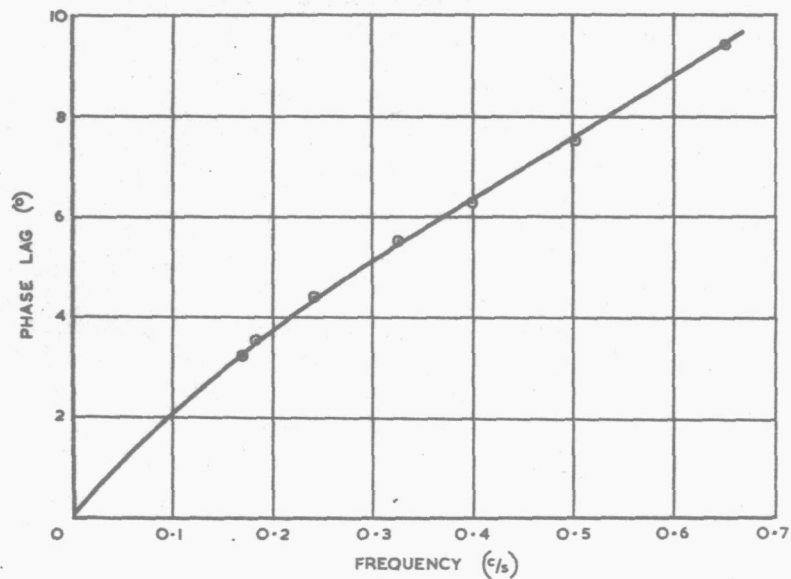


FIG.4. INSTRUMENT DYNAMIC CALIBRATIONS : YAW RATE GYRO.

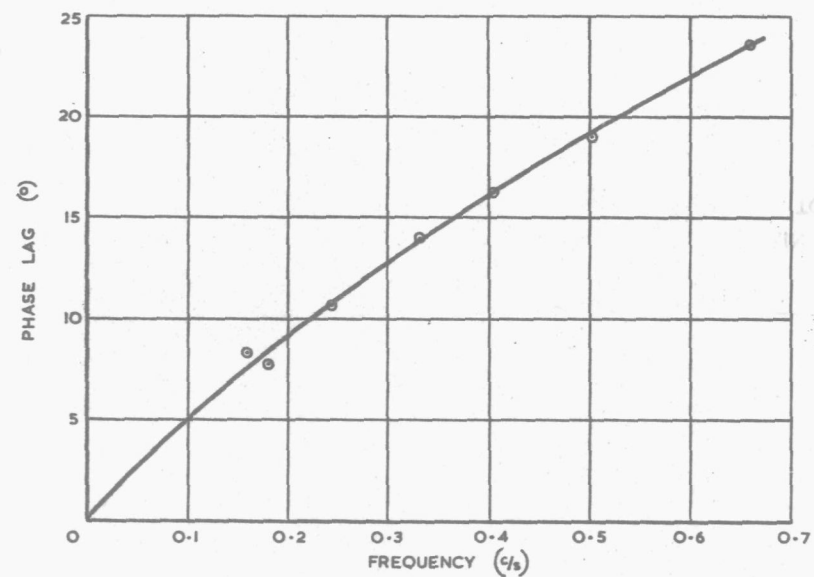


FIG.5. INSTRUMENT DYNAMIC CALIBRATIONS : SIDESLIP VANE.

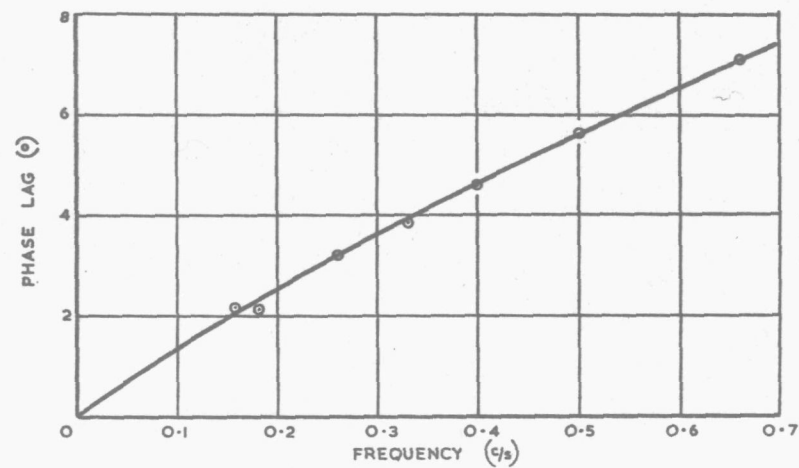


FIG.6. INSTRUMENT DYNAMIC CALIBRATIONS : LATERAL ACCELERATION.

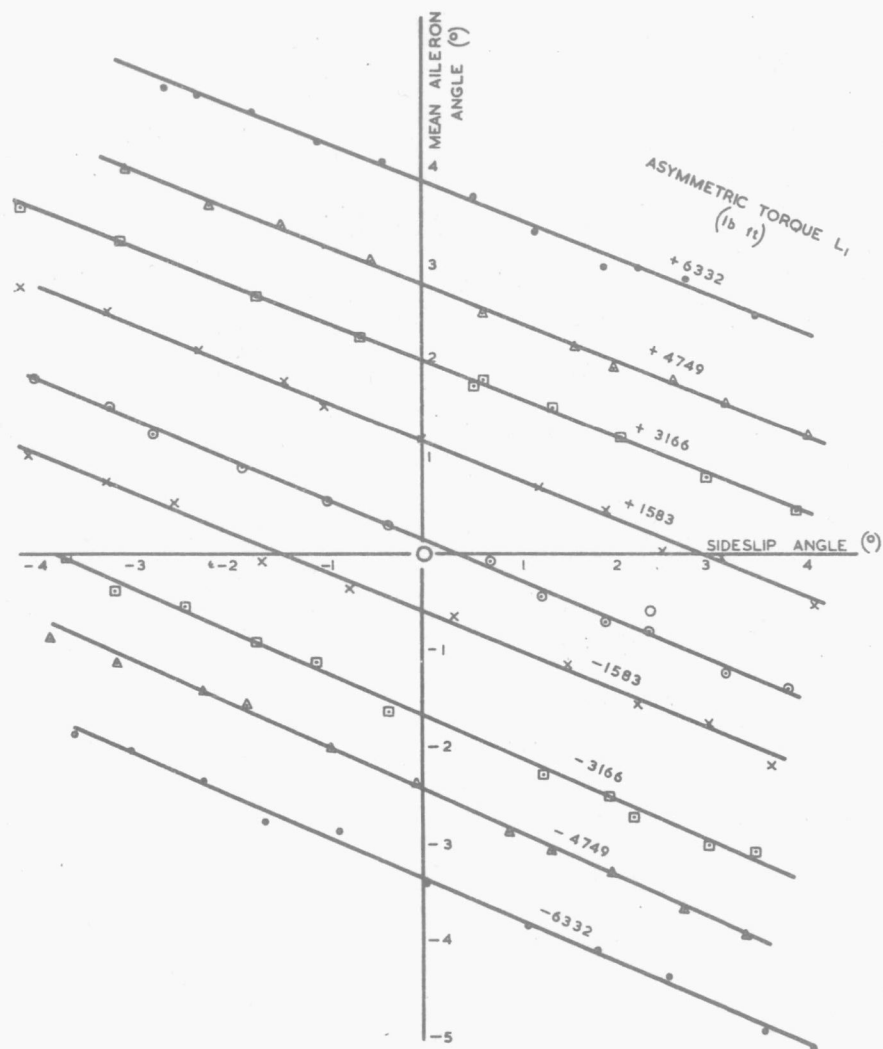


FIG. 7. TYPICAL ASYMMETRIC RESULTS: MEAN AILERON ANGLE AGAINST SIDESLIP ANGLE FOR VARYING VALUES OF ASYMMETRIC TORQUE.

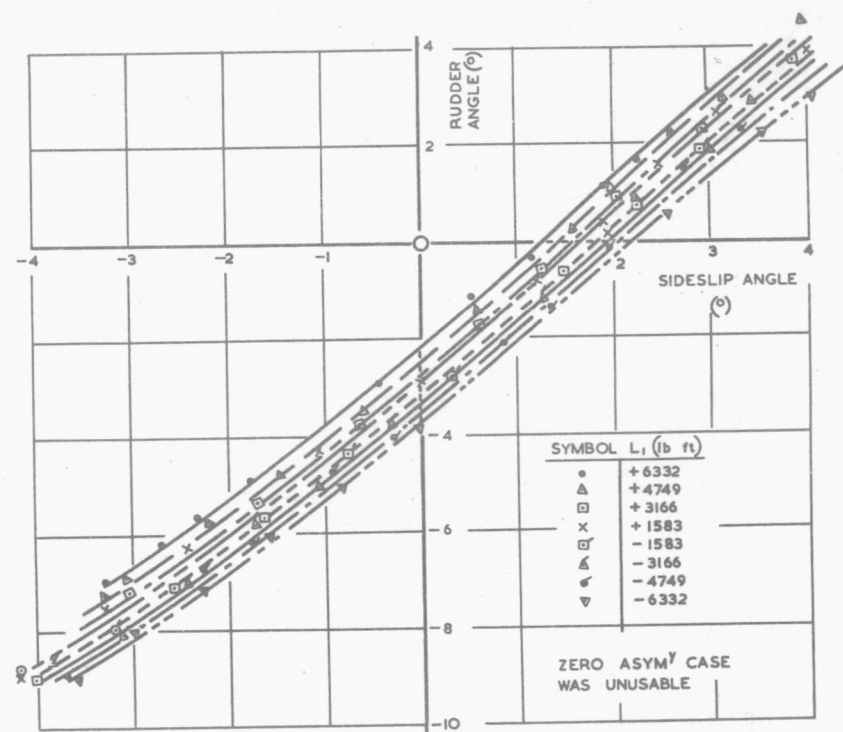


FIG. 8. TYPICAL ASYMMETRIC RESULTS: RUDDER ANGLE AGAINST SIDESLIP ANGLE FOR VARYING VALUES OF ASYMMETRIC TORQUE.

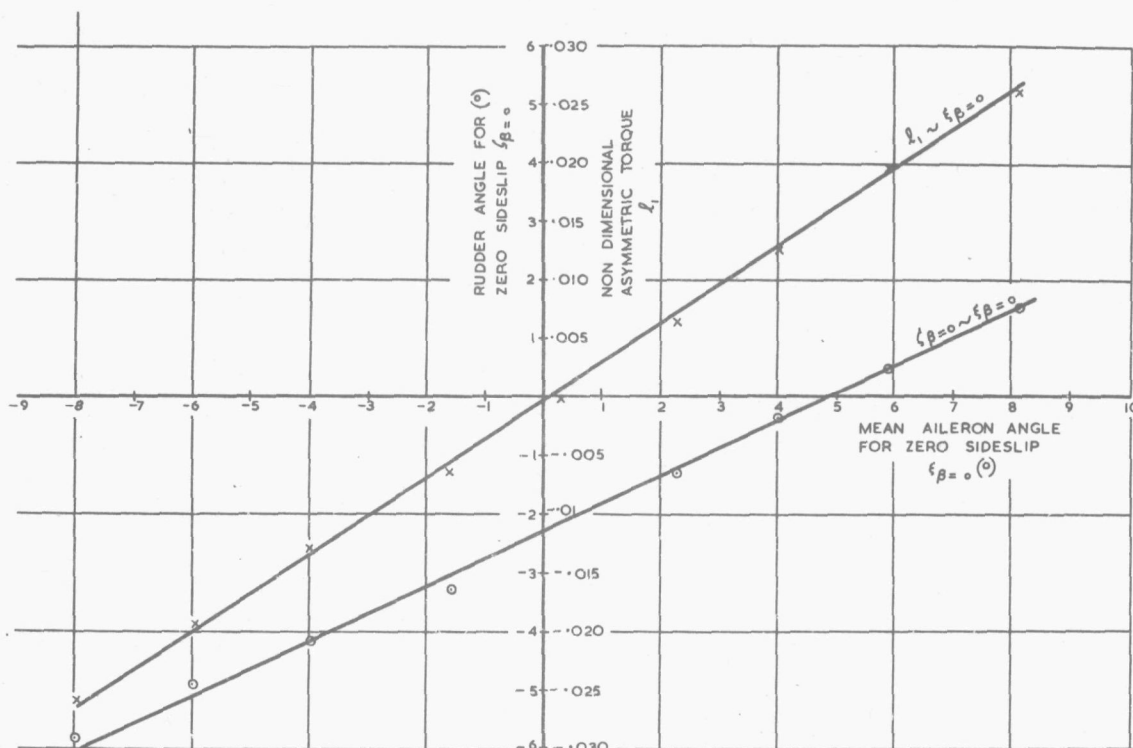


FIG.9. RUDDER ANGLE FOR ZERO SIDESLIP AND NON-DIMENSIONAL ASYMMETRIC TORQUE AGAINST MEAN AILERON ANGLE FOR ZERO SIDESLIP: 110 KTS. ASIR, 10,000 FT.

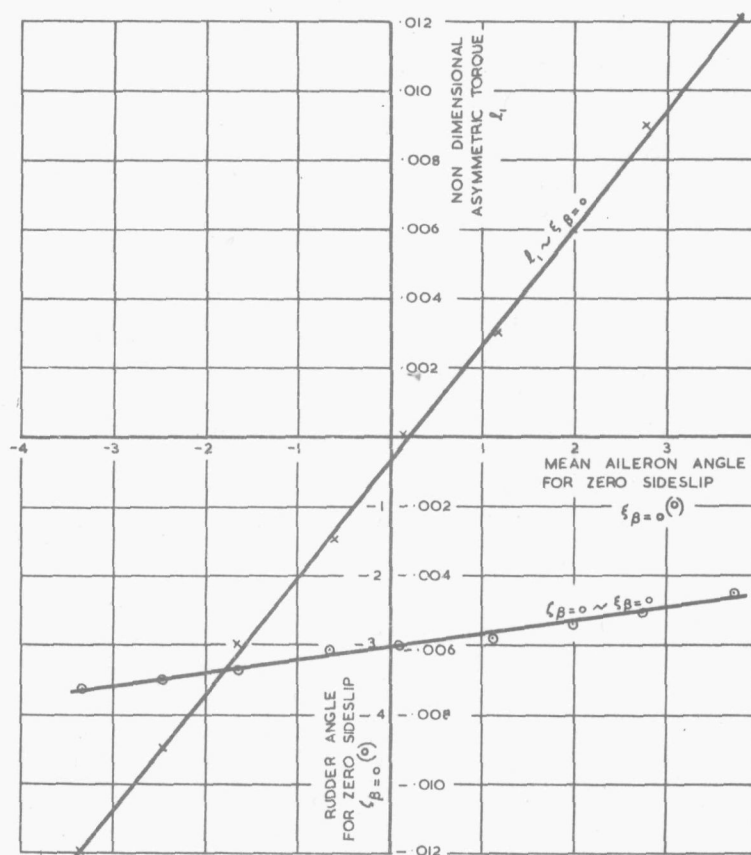


FIG.10. RUDDER ANGLE FOR ZERO SIDESLIP AND NON-DIMENSIONAL ASYMMETRIC TORQUE AGAINST MEAN AILERON ANGLE FOR ZERO SIDESLIP: 160 KTS. ASIR, 10,000 FT

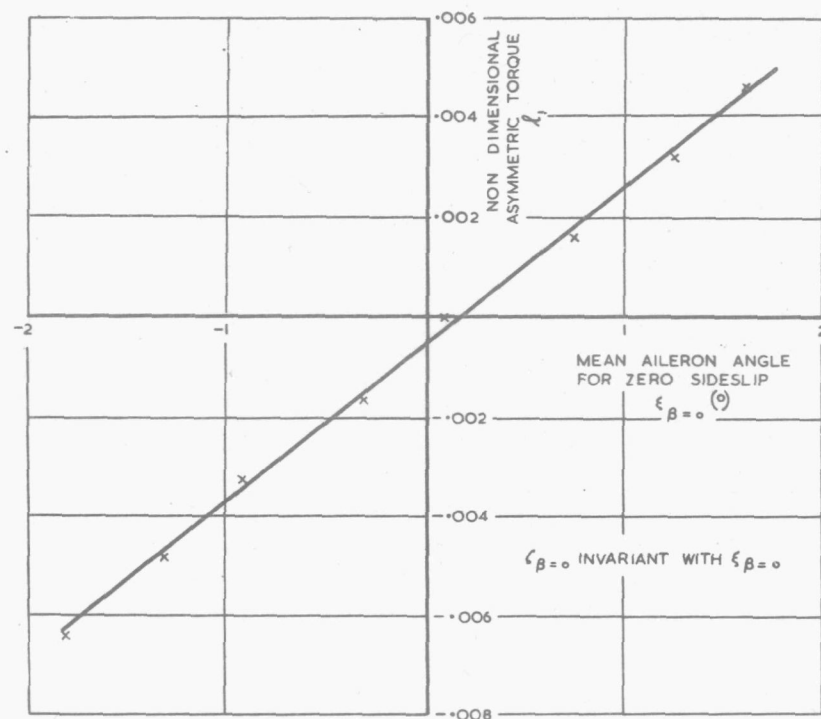


FIG.11. NON-DIMENSIONAL ASYMMETRIC TORQUE AGAINST MEAN AILERON ANGLE FOR ZERO SIDESLIP: 220 KTS. ASIR, 10,000 FT.

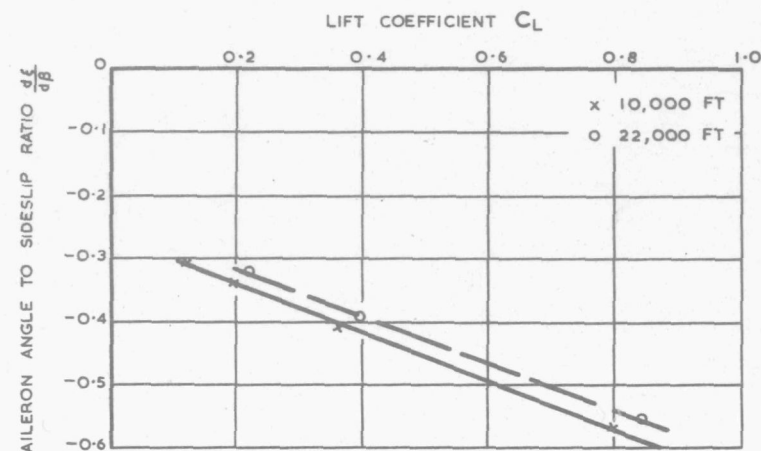


FIG.12. STRAIGHT SIDESLIPS:AILERON ANGLE TO SIDESLIP RATIO AGAINST LIFT COEFFICIENT.

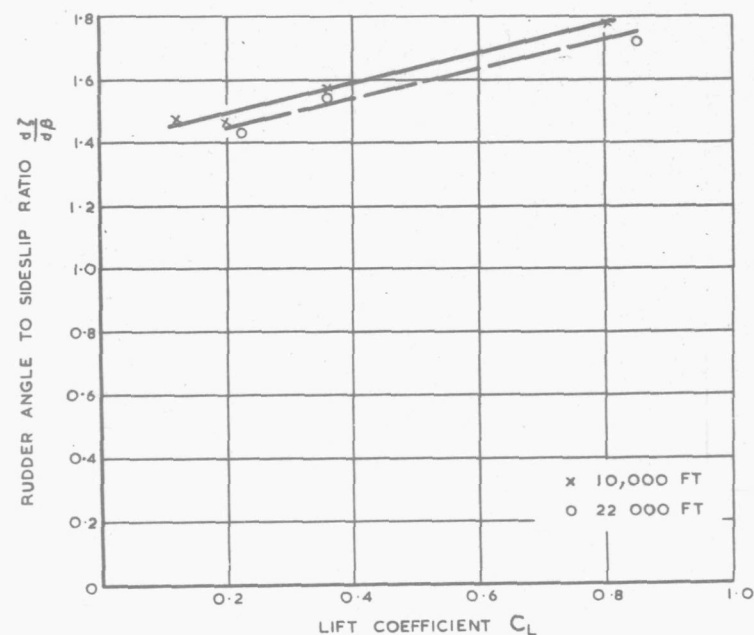


FIG.13. STRAIGHT SIDESLIPS:RUDDER ANGLE TO SIDESLIP RATIO AGAINST LIFT COEFFICIENT.

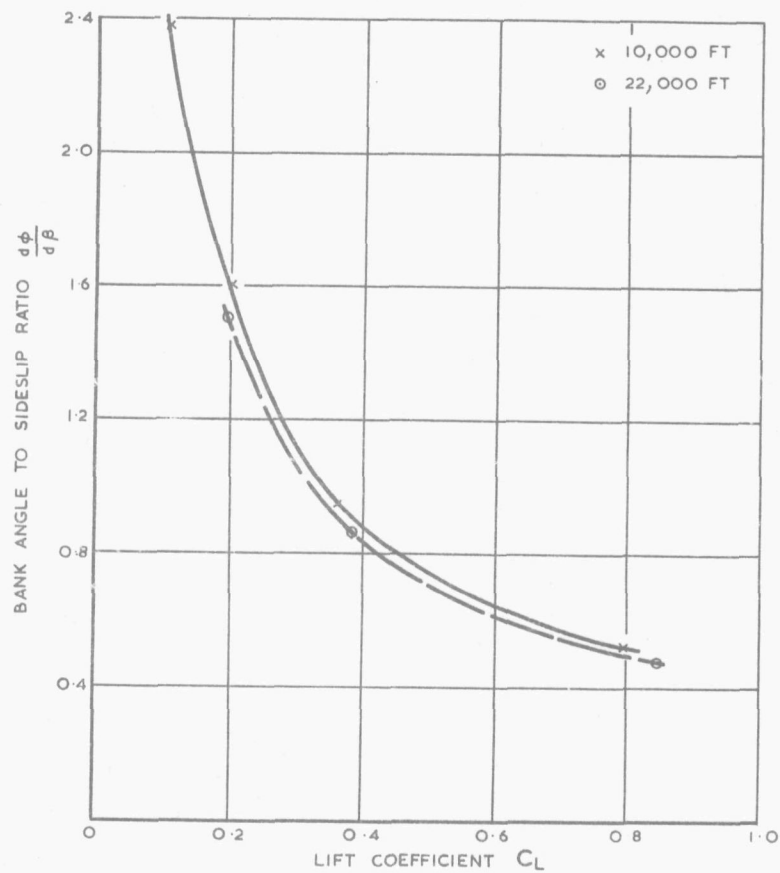


FIG.14. STRAIGHT SIDESLIPS: BANK ANGLE TO SIDESLIP RATIO AGAINST LIFT COEFFICIENT.

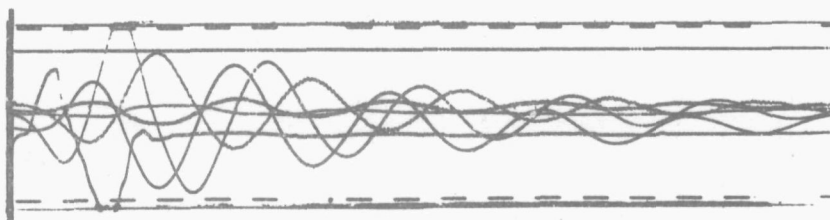


FIG. 15. TRACE RECORDING OF A TYPICAL DUTCH ROLL

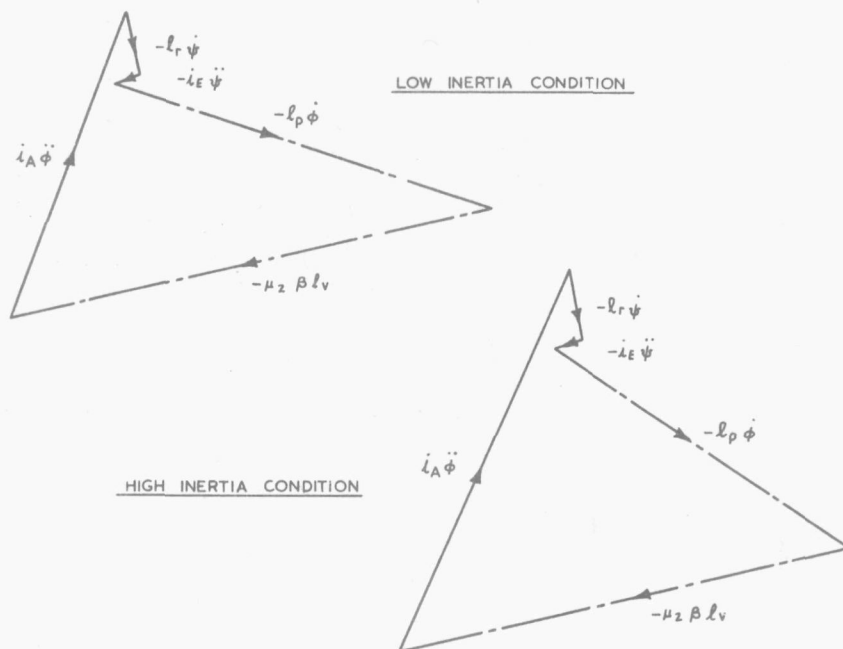


FIG.16. TYPICAL ROLLING MOMENT POLYGONS.

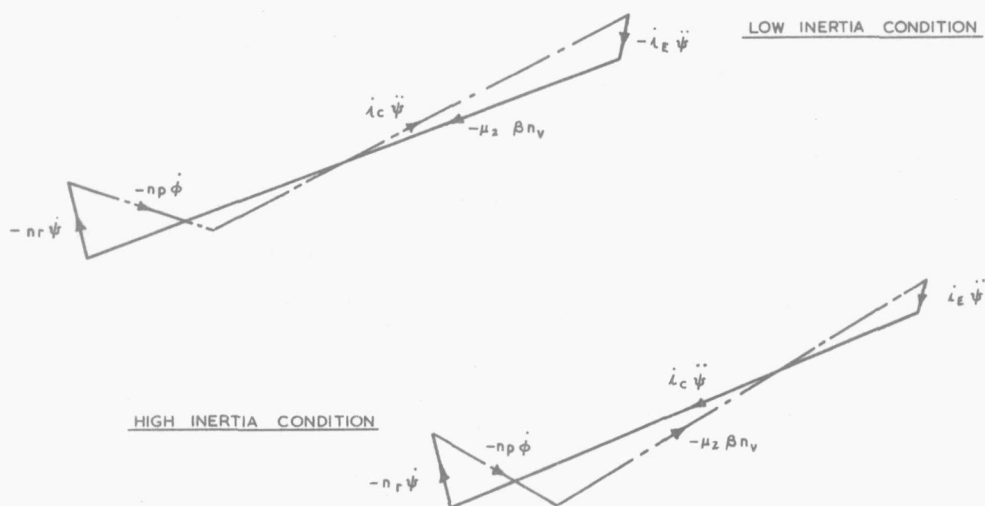


FIG.17. TYPICAL YAWING MOMENT POLYGONS

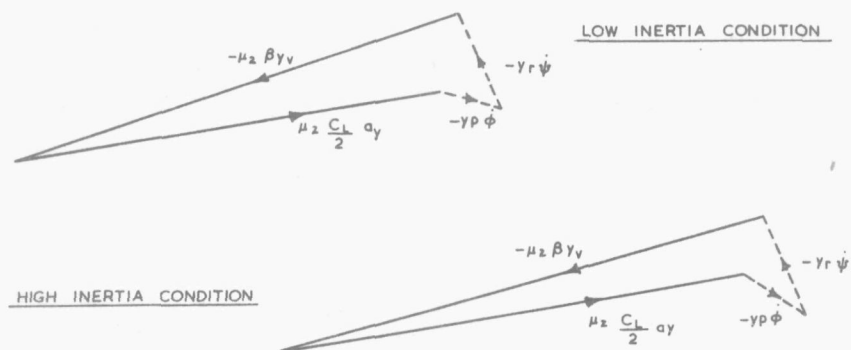


FIG.18. TYPICAL SIDE-FORCE POLYGONS.

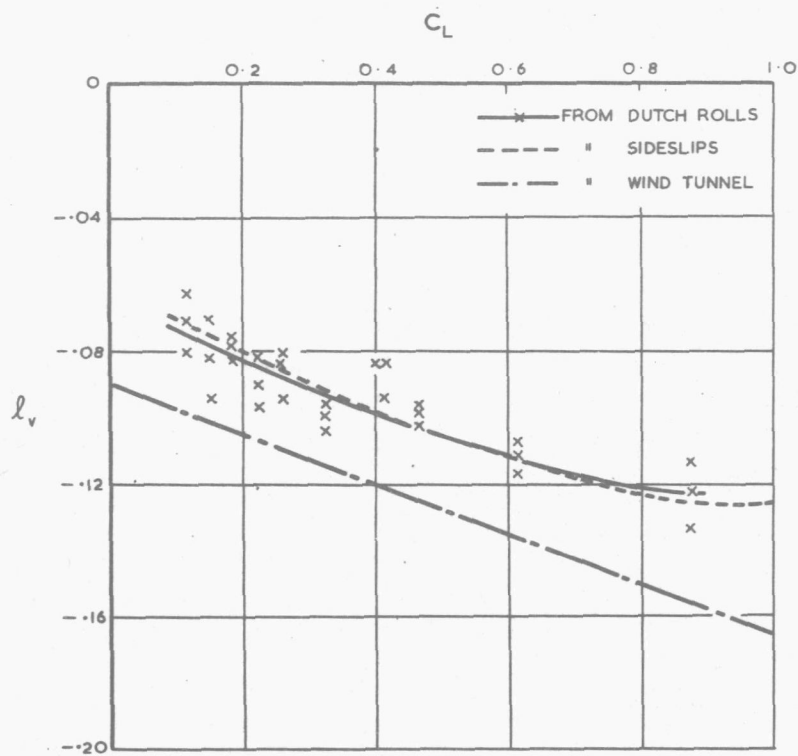


FIG.19. VARIATION OF THE ROLLING MOMENT DUE TO SIDESLIP DERIVATIVE l_v WITH LIFT COEFFICIENT: LOW INERTIA CONDITION.

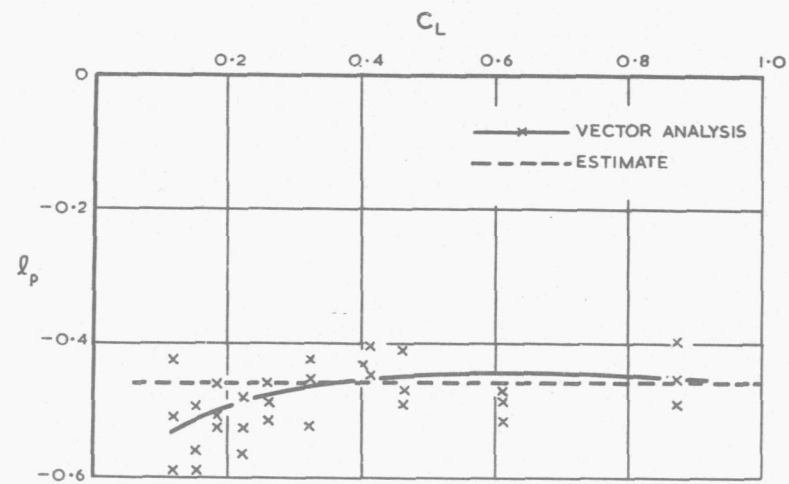


FIG.20. VARIATION OF THE ROLLING MOMENT DUE TO RATE OF ROLL DERIVATIVE l_p WITH LIFT COEFFICIENT: LOW INERTIA CONDITION

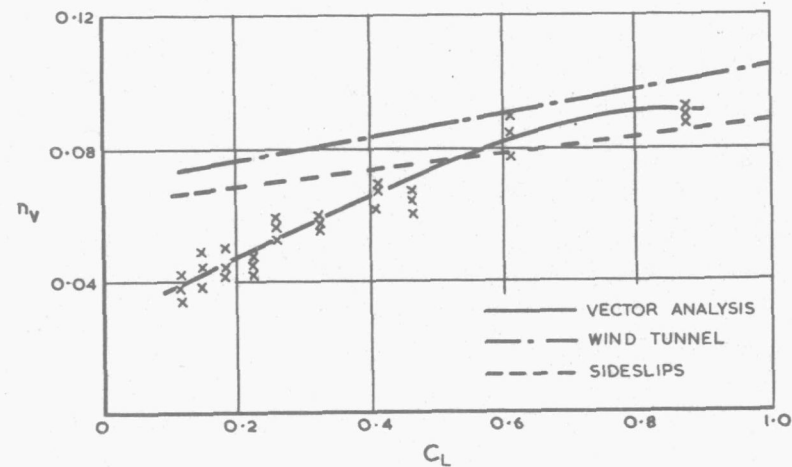


FIG.21. VARIATION OF THE YAWING MOMENT DUE TO SIDESLIP DERIVATIVE n_v WITH LIFT COEFFICIENT: LOW INERTIA CONDITION.

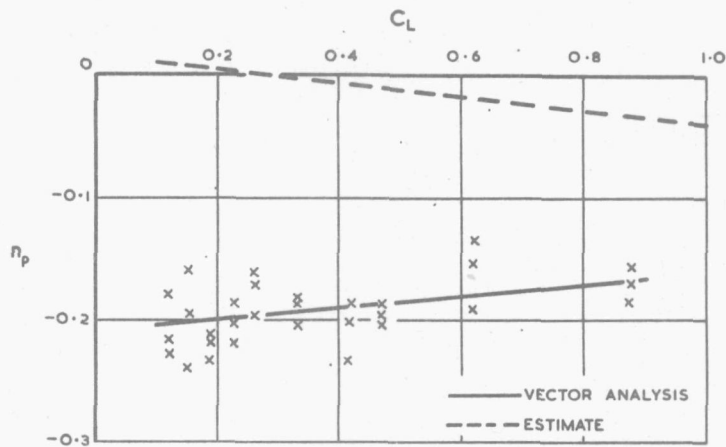


FIG.22. VARIATION OF THE YAWING MOMENT DUE TO RATE OF ROLL DERIVATIVE n_p WITH LIFT COEFFICIENT: LOW INERTIA CONDITION.

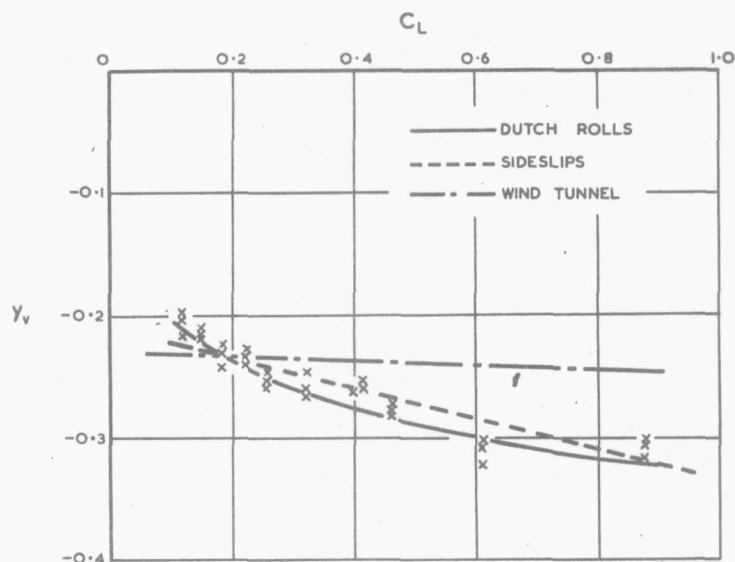


FIG.23. VARIATION OF THE SIDEFORCE DUE TO SIDESLIP DERIVATIVE y_v WITH LIFT COEFFICIENT: LOW INERTIA CONDITION.

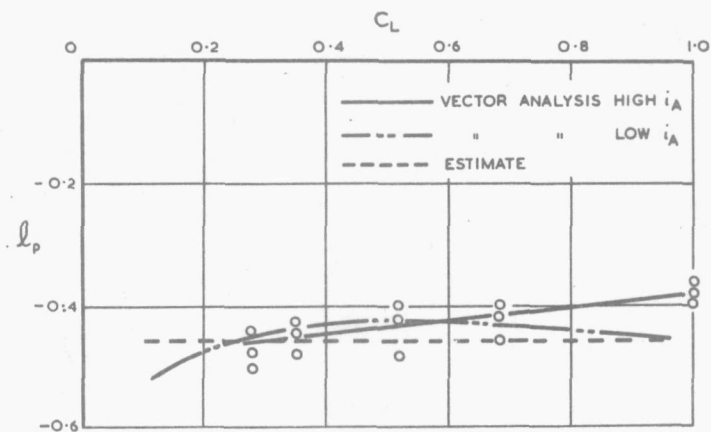


FIG.24. VARIATION OF THE ROLLING MOMENT DUE TO RATE OF ROLL DERIVATIVE l_p WITH LIFT COEFFICIENT: HIGH INERTIA CONDITION.

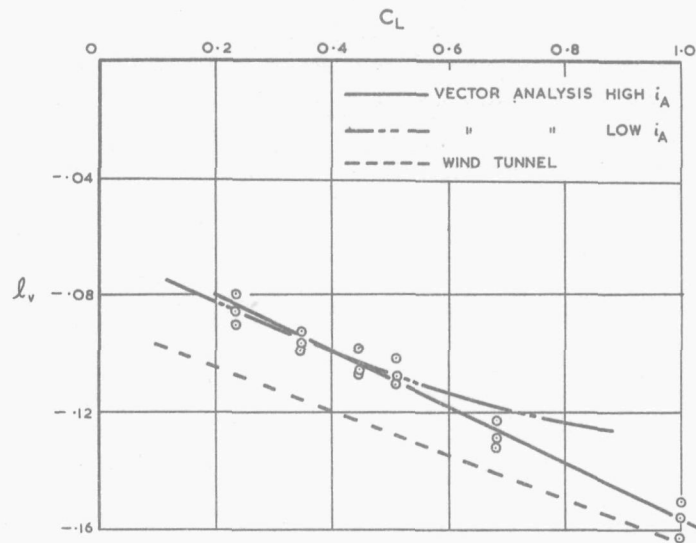


FIG.25. VARIATION OF THE ROLLING MOMENT DUE TO SIDESLIP DERIVATIVE l_v WITH LIFT COEFFICIENT: HIGH INERTIA CONDITION.

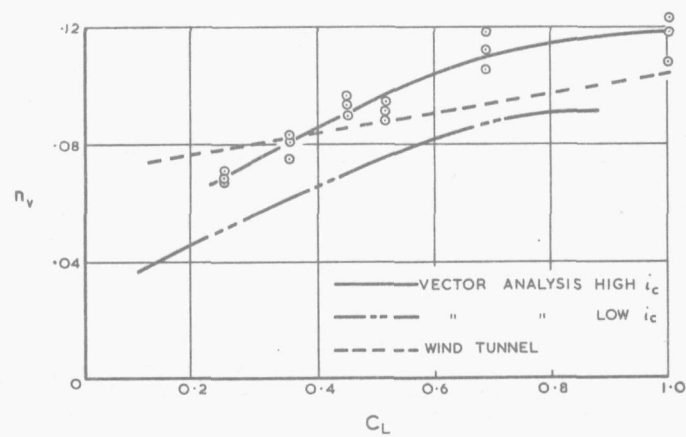


FIG.26. VARIATION OF THE YAWING MOMENT DUE TO SIDESLIP DERIVATIVE n_v WITH LIFT COEFFICIENT: HIGH INERTIA CONDITION.

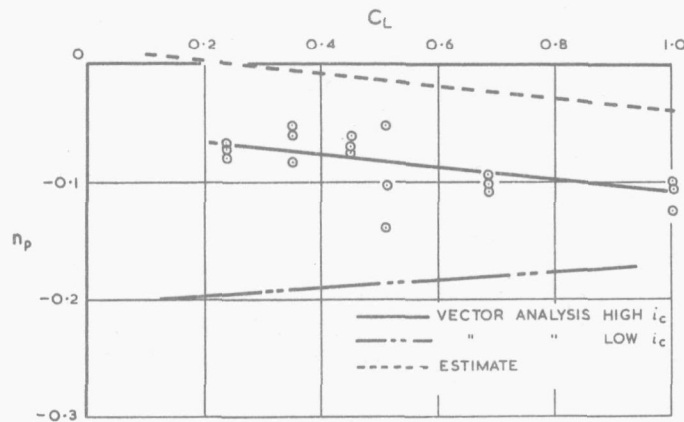


FIG.27. VARIATION OF THE YAWING MOMENT DUE TO RATE OF ROLL DERIVATIVE n_p WITH LIFT COEFFICIENT: HIGH INERTIA CONDITION.

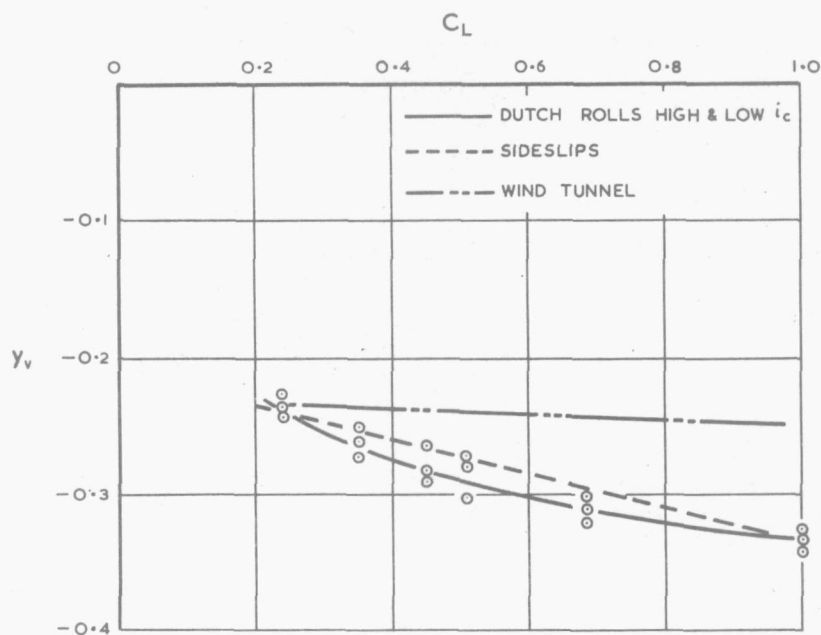


FIG.28 VARIATION OF THE SIDESLIP DERIVATIVE Y_v WITH LIFT COEFFICIENT: HIGH INERTIA CONDITION.

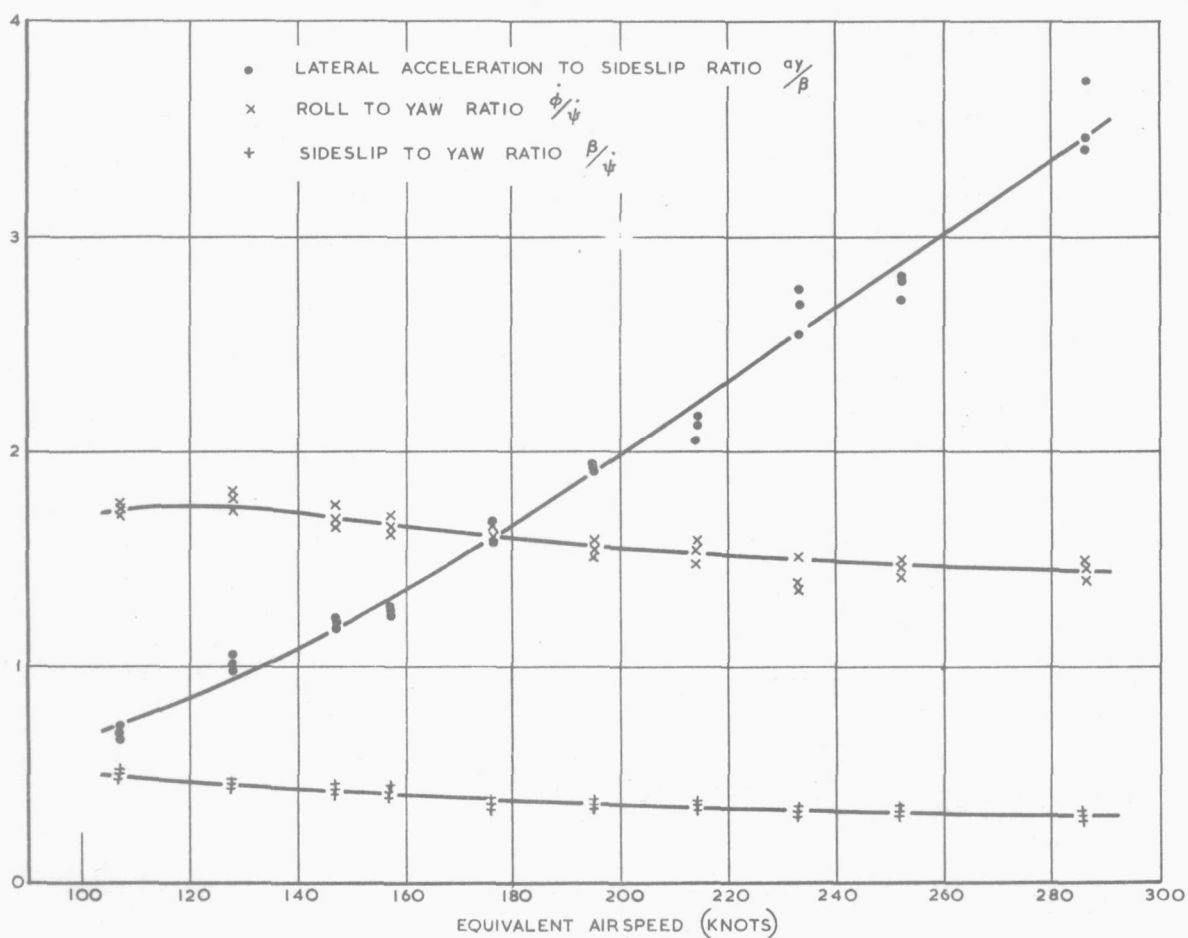


FIG.29. AMPLITUDE RATIOS DERIVED FROM THE DUTCH ROLL OSCILLATION: LOW INERTIA CONDITION.

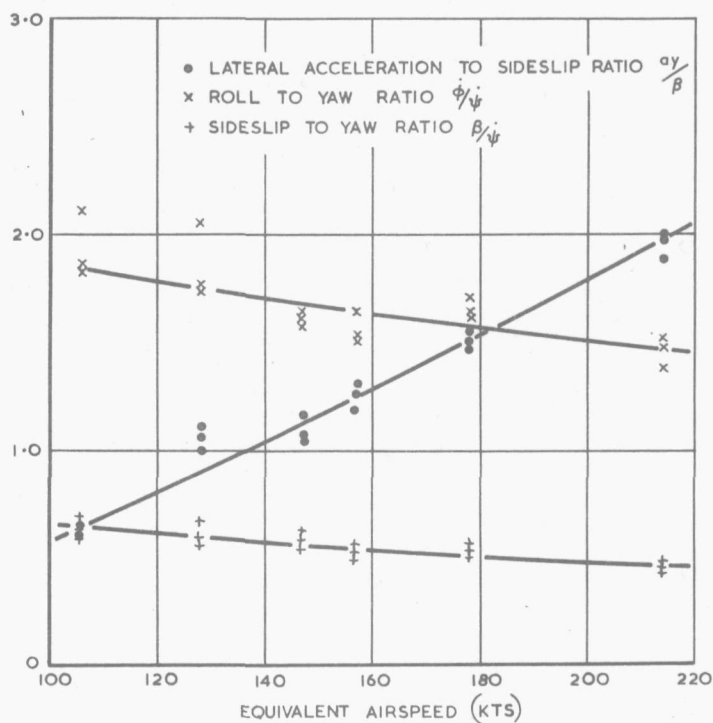


FIG.30. AMPLITUDE RATIOS DERIVED FROM THE DUTCH ROLL OSCILLATION: HIGH INERTIA CONDITION.

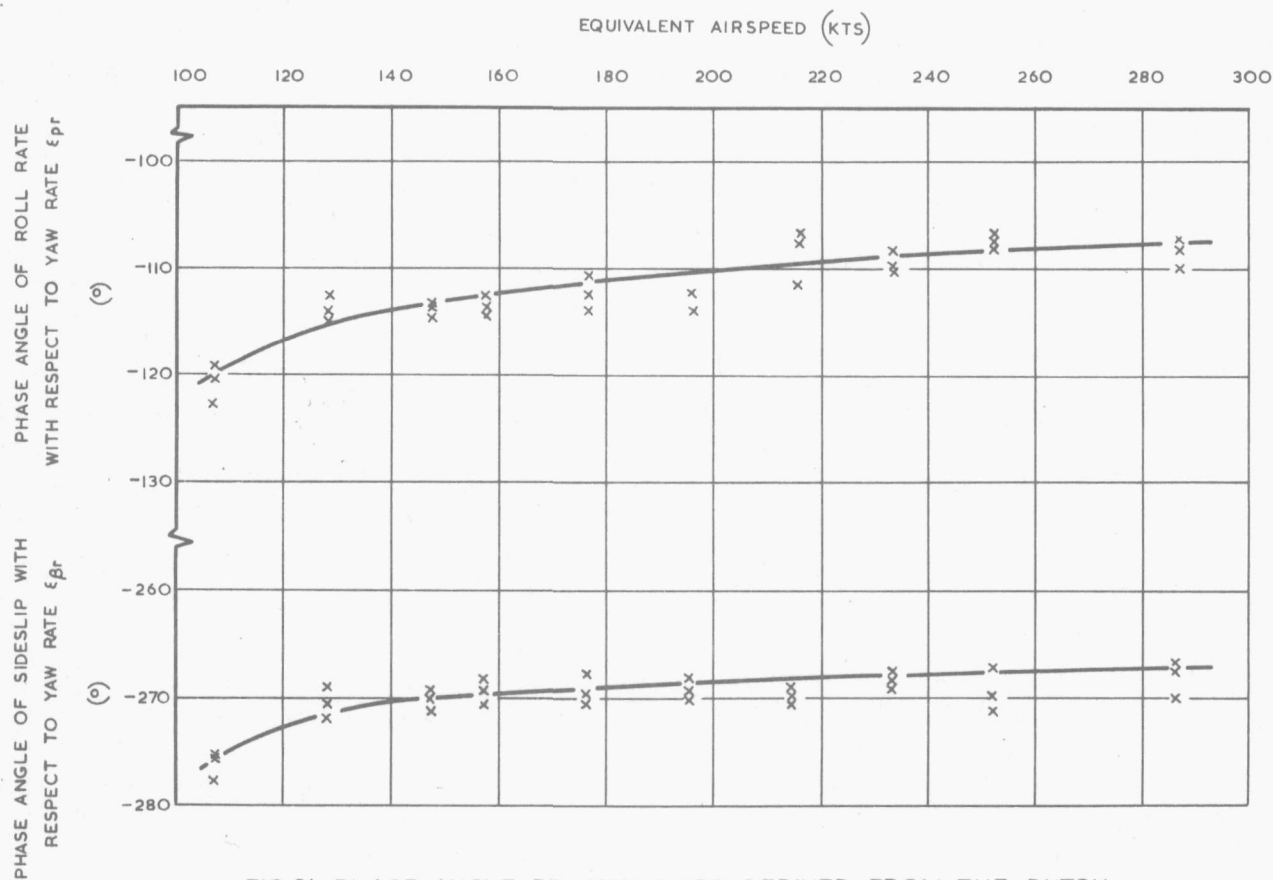


FIG.31. PHASE ANGLE RELATIONSHIPS DERIVED FROM THE DUTCH ROLL OSCILLATION: LOW INERTIA CONDITION.

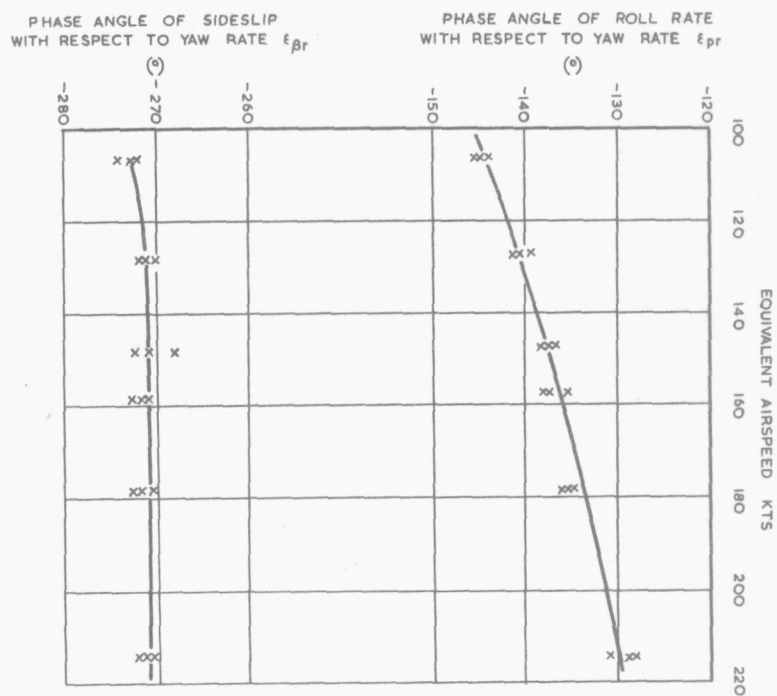


FIG.32. PHASE ANGLE RELATIONSHIPS DERIVED FROM THE DUTCH ROLL OSCILLATION: HIGH INERTIA CONDITION.

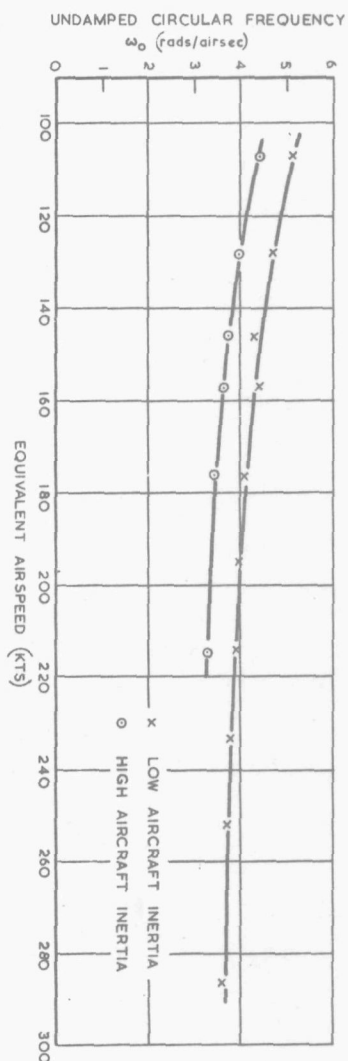


FIG.33. THE UNDAMPED CIRCULAR FREQUENCY OF THE DUTCH ROLL OSCILLATION.

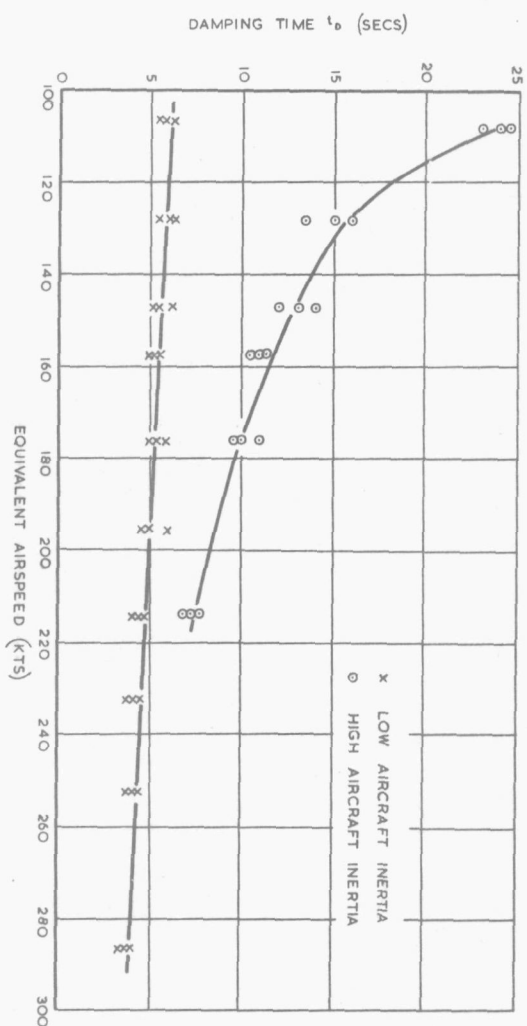


FIG.34. THE DAMPING TIME OF THE DUTCH ROLL OSCILLATION.

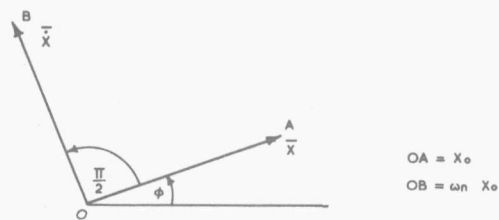


FIG. 35a. TIME VECTOR REPRESENTATION OF A SIMPLE SINUSOIDAL OSCILLATION WITHOUT VISCOUS DAMPING.

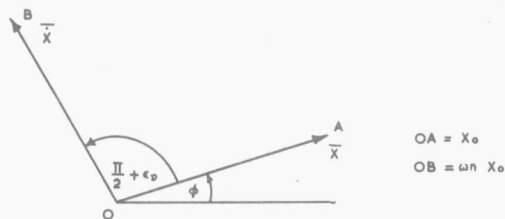


FIG. 35b. TIME VECTOR REPRESENTATION OF A SIMPLE SINUSOIDAL OSCILLATION WITH VISCOUS DAMPING.

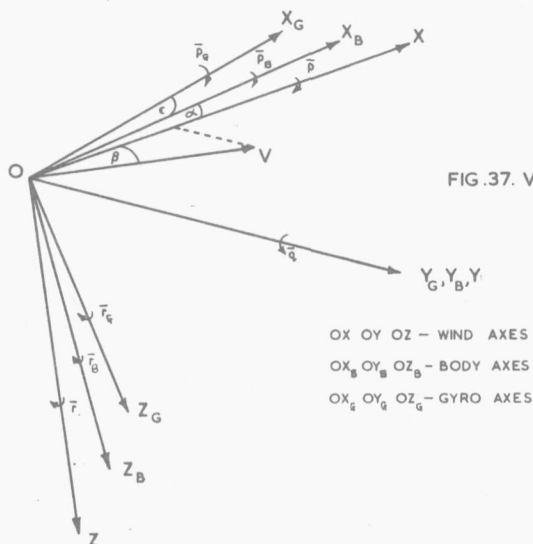


FIG. 36. AXES SYSTEMS USED FOR INSTRUMENTATION CORRECTIONS.

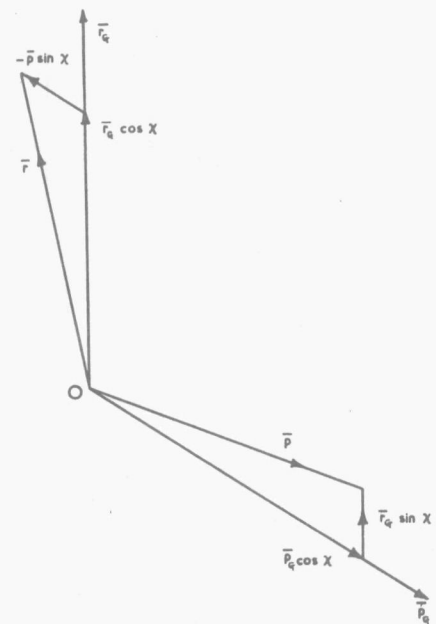
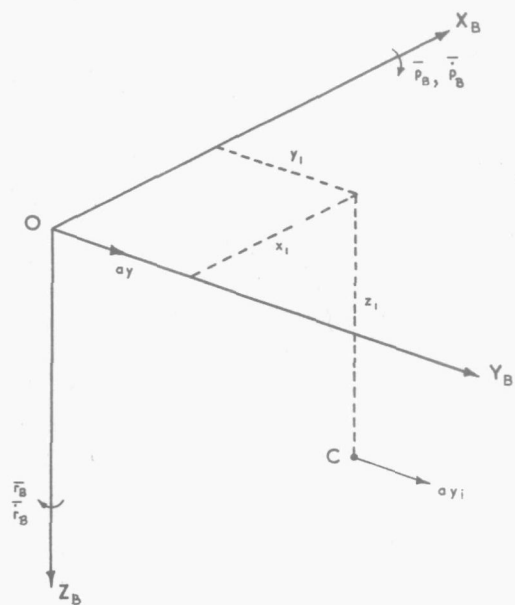


FIG. 37. VECTOR TRANSFORMATION OF RATES OF ROLL AND YAW FROM GYRO AXES TO WIND AXES.



$$a_y = a_{yi} - x_i \ddot{r}_B + z_i \dot{\theta}_B + y_i (\dot{\phi}_B^2 + \dot{\psi}_B^2)$$

FIG.38. BODY AXES SHOWING CORRECTION TO LATERAL ACCELEROMETER READING TO OBTAIN AIRCRAFT LATERAL ACCELERATION AT THE CENTRE OF GRAVITY.

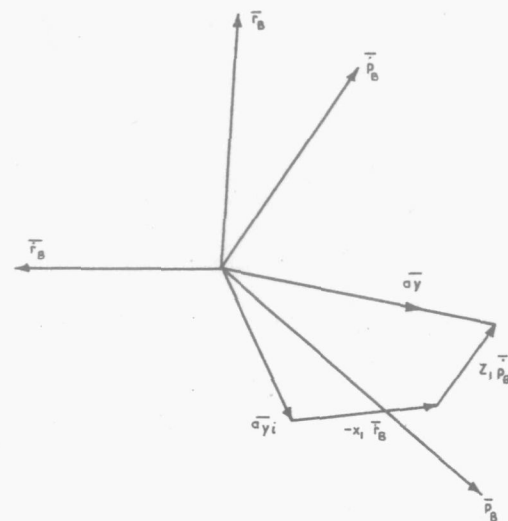
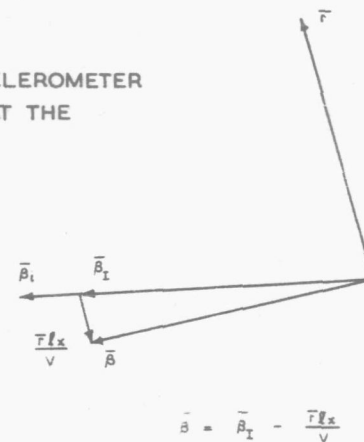


FIG.39. VECTOR TRANSFORMATION OF LATERAL ACCELEROMETER READING TO LATERAL ACCELERATION AT THE AIRCRAFT CENTRE OF GRAVITY.



$$\beta = \beta_I - \frac{\ddot{r}_B x}{V}$$

FIG.40. VECTOR TRANSFORMATION OF SIDESLIP VANE ANGLE TO SIDESLIP ANGLE.