# Cross-sectional stability of tidal inlets using a process-based model

Application of a process-based model to compute the equilibrium cross-sectional areas of the Frisian inlet

J.J. van den Berg



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Application of a process-based model to compute the equilibrium cross-sectional areas of the Frisian inlet

by

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#### Summary

In this thesis the equilibrium cross-sectional areas of a tidal inlet are investigated. The most common used methods to study cross-sectional stability are the Escoffier method (Escoffier, 1940) and the Modified Escoffier method (Van de Kreeke, 2004). The Escoffier methods are mainly empirical and lack physical justification. Therefore, in this thesis a process-based model, as proposed by Hinwood et al. (2012), is used to show that in the long-term the inlet cross-sectional area approaches a stable equilibrium or closes. Additionally, the influence of higher harmonics on the equilibrium cross-sectional areas is studied. The stable equilibrium cross-sectional area of the Frisian inlet in the Dutch Wadden Sea is computed and the results are compared to earlier findings.

In the process-based model the inlet is schematized as two cells, the tidal basin and tidal inlet. The inlet cross-section is rectangular with a width B. The process-based model consists of a set of four process-based equations; two hydrodynamic and two morphodynamic equations. The hydrodynamic equations consist of a one-dimensional equation of motion and a one-dimensional continuity equation. The morphodynamic equations consist of a simple equation relating the sediment concentration in the inlet to the velocity and a sediment balance equation. The sediment concentrations in basin and ocean are known quantities,  $C_b$  and  $C_o$  respectively. The equations of the process-based model are written in a non-dimensional form and solved using an explicit finite-difference scheme. To investigate the existence of equilibrium cross-sectional areas, the equations are solved for a large number of initial inlet depths (sixty). Calculations are extended over a large number of tidal cycles (thousand) with a computational time in the order of seconds. As a test, the process-based model is applied to a representative inlet and the results for the velocity and water level are compared to the semi-analytical Keulegan solution (Keulegan, 1951). In addition, the results for the velocity, water level and inlet depth are compared to the results of the numerical scheme used in Hinwood et al. (2012). The results show good agreement.

Applying the process-based model to a representative inlet, the existence of a stable and unstable equilibrium cross-sectional area is shown. Referring to Figure 1, for a cross-sectional area smaller than the unstable equilibrium, the inlet closes. For a cross-sectional area larger than the unstable equilibrium, the cross-sectional area approaches the stable equilibrium. The foregoing conclusions are in agreement with the assumptions underlying the empirical Escoffier stability analysis. It is shown that, when at equilibrium, the volume of sand deposited in the inlet during flood equals the volume of sand eroded from inlet during ebb. When at equilibrium, there is a net transport of sand; depending on the relative magnitude of  $C_o$  and  $C_b$ , the net transport is away or into the basin. In addition to the process-based model, the Escoffier and Modified Escoffier diagrams are constructed for the representative inlet and the Frisian inlet. The calculated stable and unstable equilibrium cross-sectional areas are compared with those of the process-based model.



Figure 1: Change in cross-sectional area A(t) for 60 different initial inlet cross-sectional areas.

By applying a Fourier analysis to the velocity and basin water level calculated with the process-based model, it is shown that a mean basin water level and velocity, as well as a first, second and third harmonic are present. When the variation of depth with tidal stage is neglected, the mean basin water level and velocity, as well as the second harmonic, are negligible. The third harmonic is still present. The second harmonic can potentially play a role in sand transport and thus in cross-sectional stability of a tidal inlet (Van de Kreeke and Robaczewska, 1993). To determine the effect of the second harmonic on the cross-sectional stability, the process-based model was applied to the representative inlet without including depth variations with tidal stage. The inlet still shows a stable and unstable equilibrium, which shows that the absence of a second harmonic not necessarily precludes the presence of a stable or unstable equilibrium cross-sectional area. Additionally, it is shown that the magnitude of the stable equilibrium cross-sectional area is not sensitive to whether or not the effect of tidal stage is included. On the contrary, the unstable equilibrium cross-sectional area is sensitive to inclusion of the effect of tidal stage.

In 1969 the basin surface area of the Frisian inlet was reduced by 30%. The inlet cross-sectional area after basin reduction was 24.500  $m^2$ . The stable and unstable equilibrium cross-sectional areas of the Frisian inlet after basin reduction are computed with the process-based model and the Modified Escoffier method. For the process-based model in order to obtain reliable results, the sediment parameters, including  $C_o$ ,  $C_b$  and m, are determined using the approximately 40 year long time series of the inlet cross-sectional areas after basin reduction. The stable equilibrium cross-sectional areas calculated with the Modified Escoffier stability method and the process-based model are 14.700  $m^2$  and 17,100  $m^2$  respectively. Referring to Figure 7.5, it seems reasonable to assume that at the time this report was written, December 2018, the inlet has reached a stable equilibrium.

# Contents

1	Intr	roduction	1
	1.1	Background	1
	1.2	Objectives	1
	1.3	Methods to attain objectives	2
<b>2</b>	Pro	cess-based model	3
	2.1	Tidal inlet basin system schematization	3
	2.2	Model equations	4
		2.2.1 Hydrodynamic equations	4
		2.2.2 Morphodynamic equations	5
		2.2.3 Non-dimensional form of the equations	6
3	Esc	offier stability analysis	8
	3.1	Escoffier diagram	8
	3.2	Modified Escoffier diagram	9
4	Арр	plication to a representative tidal inlet basin system	11
	4.1	Tidal inlet basin schematization and parameter values	11
	4.2	Inlet velocity, basin water level and higher harmonics	11
	4.3	Sediment concentration	13
	4.4	Stable and unstable equilibrium	14
	4.5	Escoffier diagram	16
	4.6	Modified Escoffier diagram	17
<b>5</b>	$\mathbf{The}$	e effect of depth variations with tidal stage on the stability of tidal inlets	18
	5.1	Higher harmonics neglecting depth variations with tidal stage	18
	5.2	Stable and unstable equilibriums	19
	5.3	Escoffier diagram	19

	5.4	Modified Escoffier diagram	20
6	Sen	sitivity analysis	<b>22</b>
	6.1	Sediment concentration in the ocean and basin: $C_o$ and $C_b$	22
	6.2	Empirical sediment parameter m	23
7	Fris	ian inlet	<b>25</b>
	7.1	Observations	25
	7.2	Schematization of Frisian inlet and parameter values	27
	7.3	Determination of the value of the friction factor	27
	7.4	Determination of morphodynamic parameters	28
	7.5	Stable and unstable equilibrium	29
	7.6	Modified Escoffier diagram	29
8	Dis	cussion, conclusions and recommendations	31
	8.1	Discussion	31
	8.2	Conclusions	32
	8.3	Recommendations	33
Bi	bliog	graphy	34
$\mathbf{A}_{j}$	ppen	dix A Numerical solution to the governing equations	35
	A.1	Numerical solution	35
	A.2	Comparison process-based model and HydSed	36

# List of Figures

1	Change in cross-sectional area $A(t)$ for 60 different initial inlet cross-sectional areas	
2.1	Schematization of a tidal inlet basin system top view (source: Hinwood et al. (2012))	3
2.2	Schematization of a tidal inlet basin system side view (source: Hinwood et al. (2012))	4
3.1	Escoffier diagram with the stable (C) and unstable (B) equilibrium cross-sectional area of the inlet.	8
3.2	Modified Escoffier diagram (source: Van de Kreeke (2004))	10
4.1	Ocean water level, basin water level and inlet velocity for two tidal cycles	12
4.2	Sediment concentration and velocity. During $\operatorname{ebb}$ the velocity is positive, during flood negative	13
4.3	Inlet depth $h(t)$ , for different initial inlet depths $h_0$ . Stable and unstable equilibrium depth	14
4.4	Depth variation during a tidal cycle when the inlet is in stable equilibrium.	15
4.5	Sediment volumes entering and leaving the inlet during ebb and flood. The net transport over a tidal cycle in the inlet is zero, the inlet is in equilibrium.	15
4.6	Escoffier diagram for the representative inlet. The closure curve includes the effect of depth variation with tidal stage. The values of the stable and unstable equilibriums are calculated with the process-based model and are listed in Table 4.5.	16
4.7	Modified Escoffier diagram. Stable and unstable equilibriums are computed with the process- based model and listed in Table 4.5.	17
5.1	Non dimensional inlet depth changes $h(t)$ for different initial inlet depths, when neglecting the effect of tidal stage.	19
5.2	Escoffier diagram; closure curves $\hat{u}(A)$ with and without the effect of tidal stage. Green and pink dots indicate stable and unstable equilibriums.	20
5.3	Modified Escoffier diagram with and without the effect of tidal stage. The closure curves are presented in terms of the velocity $\hat{U}$ and cross-sectional area A	20
6.1	Inlet depth $h^*(t^*)$ for five different values of $C_b$	22
6.2	Inlet depth $h^*(t^*)$ for five different values of $C_o$	23
6.3	Inlet depth $h^*(t^*)$ for five different values of $m$	24
7.1	Location of the Frisian inlet within the Wadden Sea. The area of interest is within the black square. Source: Van de Kreeke and Hibma (2005)	25

7.2	Measurements of the Frisian inlet, performed by Rijkswaterstaat in 2000. The green area indicates the inlet. The darkness of the red indicates the depth qualitatively; increased darkness indicates increasing depth. The yellow and white areas indicate elevations above sea level	26
7.3	Water level in the basin and ocean and inlet velocity	28
7.4	Comparison of measured (dots) and computed (solid line) cross-sectional areas. $\ldots$	28
7.5	Inlet depth $h(t)$ without the effect of tidal stage, for different initial inlet depths $h_0$ . Stable and unstable equilibrium depth.	29
7.6	Modified Escoffier diagram for the Frisian inlet. The closure curve is calculated without the effect of tidal stage.	30
A.1	Velocity and basin water level computed by the process-based model and HydSed	37
A.2	Depth $h^*(t^*)$ as computed by the process-based model and HydSed	37

# List of Tables

4.1	Parameter values for the representative inlet	11
4.2	Maximum water level and velocity using the Keulegan solution and process-based model. $\ldots$	12
4.3	Coefficients of basin water level and inlet velocity harmonics.	13
4.4	Mean basin water level, mean velocity and amplitudes of higher harmonics for the Keulegan solution and the process-based model.	13
4.5	Stable and unstable equilibrium cross-sectional areas for the representative inlet calculated using the process-based model including depth variations with tidal stage. Corresponding tidal prisms, velocity amplitudes $\hat{U}$ and maximum velocities $\hat{u}$ .	14
5.1	Mean basin water level, mean velocity and amplitudes of higher harmonics for the process-based model, with and without including effects of tidal stage. Ebb direction is positive.	18
5.2	Stable and unstable equilibrium cross-sectional areas of the representative inlet calculated using the process-based model excluding depth variations with tidal stage. Corresponding tidal prisms, velocity amplitudes $\hat{U}$ and maximum velocities $\hat{u}$	19
6.1	The equilibrium depth is computed for varying values of $C_o$ and $C_b$ , but the sum of the two parameters remains constant. The computed equilibrium depths are identical in all three cases.	23
7.1	Cross-sectional area Frisian inlet from 1970 till 2012.	26
7.2	Parameter values used in the process-based model for the Frisian inlet	27
7.3	Measured (Van de Kreeke and Dunsbergen, 2000) and computed tidal prism, basin water level and maximum velocity for the Frisian Inlet using the process-based model including depth variations with tidal stage.	27
7.4	Stable and unstable equilibrium cross-sectional areas of the Frisian Inlet and corresponding max- imum velocities and tidal prisms calculated using the process-based model without depth varia- tions with tidal stage.	29
7.5	Stable and unstable equilibrium cross–sectional areas and velocities as computed with the Mod- ified Escoffier diagram, as well as the corresponding tidal prisms	30
A.1	Comparison amplitudes of the higher harmonics for Keulegan, the process-based model and HydSed.	36

# List of Symbols

$a_o$ Fourier mean velocity $[m/s]$ or water level $[m]$ $\hat{u}_{1-3}, \hat{b}_{1-3}$ Fourier amplitudes velocity $[m/s]$ or water level basin $A$ Cross-sectional area inlet $[m^2]$ $A_b$ Surface area basin $[m^2]$ $a_o$ Tidal amplitude $[m]$ $B$ Width inlet $[m]$ $C$ Empirical constant $[-]$ $C_l$ Empirical constant $[-]$ $C_l$ Empirical constant $[-]$ $C$ Sediment concentration $[-]$ $C^*$ Sediment concentration in the ocean $[-]$ $C_b$ Sediment concentration in the basin $[-]$ $f$ Bottom friction factor $[-]$ $g$ Gravitational acceleration term $[m^2/s]$ $h$ Inlet depth $[-]$ $h^*$ Inlet depth $[m]$ HydSedNumerical model used in Hinwood et al. (2012) $K$ Entrance/exit loss coefficient $[-]$	[ <i>m</i> ]
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HydSed Numerical model used in Hinwood et al. $(2012)$	
K Entrance/exit loss coefficient [-]	
K1 Keulegan repletion coefficient [-]	
L Length inlet $[m]$	
<i>m</i> Empirical sediment transport constant [-]	
$P$ Tidal prism $[m^3]$	
q Empirical constant [-]	
Q River discharge [-]	
$Q^*$ River discharge $[m^3/s]$	
t Time [-]	
$t^*$ Time $[s]$	
T Period M2 tide $[s]$	
u Inlet velocity [-]	
$\hat{u}$ Velocity amplitude $[m/s]$	
$u^*$ Inlet velocity $[m/s]$	
$u_{cr}$ Threshold scour velocity [-]	
$u_{cr}^*$ Threshold scour velocity $[m/s]$	
$u_0$ Mean velocity $[m/s]$	
$u_{1-3}$ Amplitude higher harmonics velocity $[m/s]$	
u Equilibrium volocity [m/c]	
$u_{eq}$ Equiportum velocity $[m/s]$	
$\hat{U}$ Velocity amplitude (Eq. 3.5) $[m/s]$	
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## Chapter 1

## Introduction

## 1.1 Background

In the context of the thesis, a tidal inlet is defined as a relatively short and narrow channel between two barrier islands. The inlet constitutes the connection between the ocean and a tidal basin. A pronounced feature on the ocean side of the inlet is the ebb delta. Sand is transported towards the inlet by the long-shore current. Part of this is carried into the inlet by the flood tidal current. The remaining part continues its path along the coast. When the inlet is at equilibrium, the volume of sand deposited in the inlet during flood is eroded by the ebb currents.

As a result of the long-shore sand transport inlets have a tendency to change position, referred to as location stability. In addition to the change in position, the exchange of sand in the inlet can lead to considerable variations in width and depth and thus cross-sectional area. This is referred to as cross-sectional stability. Tidal inlets serve as access to ports and marinas. Depth and width variations could result in the inlet becoming too narrow and too shallow for navigation, leading to substantial maintenance dredging. Being able to predict these changes is important in managing the inlets.

This thesis deals with cross-sectional stability. The method presently used to address cross-sectional stability is the stability analysis presented by Escoffier (1940) and an extension thereof in Van de Kreeke (2004). Escoffiers method is largely empirical. The main assumption is that the inlet, including the cross-sectional area, approaches a morphological equilibrium. The existence of a morphological equilibrium had been suggested earlier by a study of O'Brien (1931). A state of the art review on cross-sectional stability of tidal inlets is presented in Van de Kreeke and Brouwer (2017).

Because the Escoffier stability analysis is empirical and lacks physical justification in this study a process-based model is used to address cross-sectional stability. The results are compared with those of the Escoffier analysis. The process-based model uses a set of equations proposed by Hinwood et al. (2012).

## 1.2 Objectives

The overall objective of the study is to investigate the cross-sectional equilibriums of a tidal inlet using a process-based model and compare the results with the Escoffier Stability analysis.

Specific research objectives are:

- To develop a numerical scheme that solves the system of process-based equations within a relatively short computational time
- To investigate, whether a long-term equilibrium cross-sectional area exists
- To calculate the equilibrium cross-sectional areas for a representative inlet and construct the Escoffier and Modified Escoffier diagrams

- To investigate the effect of including variation in depth with tidal stage on the equilibrium cross-sectional areas
- To calculate the equilibrium cross-sectional areas of the Frisian Inlet after basin reduction and construct the Escoffier and Modified Escoffier diagrams

## 1.3 Methods to attain objectives

In this section is explained how the objectives described in the previous section are attained.

In Chapter 2 the process-based model for the computation of the equilibrium cross-sectional areas of a tidal inlet is presented. It is explained how the inlet is schematized and how the set of process-based equations are solved numerically. The numerical solution is presented in Appendix A and the results of the numerical solution for both the hydrodynamic and morphodynamic equations are compared to the results of Hinwoods numerical solution (Hinwood et al., 2012).

In Chapter 3 background information on the Escoffier and modified Escoffier diagram is presented.

In Chapter 4 the process-based model is applied to a representative inlet. Section 4.1 gives the inlet-basin schematization and parameter values. For validation, the results of the model are compared to the results of the semi-analytical Keulegan solution in Section 4.2. In Section 4.4 it is shown that the representative tidal inlet approaches a long-term equilibrium. In Sections 4.5 and 4.6 the Escoffier and Modified Escoffier diagrams are constructed. The closure curves are calculated using the numerical solution to the hydrodynamic equations. The unstable and stable equilibrium cross-sectional areas calculated with the process-based model are presented in the Escoffier and modified Escoffier diagrams.

In Chapter 5 the influence of including depths variations with tidal stage in the hydrodynamic equations on the equilibrium cross-sectional areas is investigated. Using the process-based model, the stable and unstable equilibrium cross-sectional areas are computed, with and without the effect of tidal stage. Additionally, the Escoffier and Modified Escoffier diagrams are constructed neglecting the effect of tidal stage and are compared to the same diagrams including the effect of tidal stage in Sections 5.3 and 5.4.

In Chapter 6, it is shown how the equilibrium cross-sectional area depends on the morphological parameters  $C_o$ ,  $C_b$  and m and an indication is given how these parameters affect the stable and unstable equilibriums.

In Chapter 7 the stable and unstable equilibrium cross-sectional areas of the Frisian inlet, after basin reduction, are calculated. The value of the friction factor f is determined by comparing calculated and observed water levels and velocities. Values of the morphological parameters are determined by comparing calculated and observed inlet cross-sectional areas for the period 1970-2012.

In Chapter 8 the results of the process-based model are discussed and the conclusions are presented. Suggestions are made for topics which require further investigation.

## Chapter 2

## **Process-based model**

In this chapter a process-based model is presented. The model uses the equations presented in Hinwood et al. (2012). The schematization of the tidal inlet and basin is presented in Section 2.1. The model equations are presented in Section 2.2 and its numerical solution is presented in Appendix A. The numerical solution to the set of process-based equations is used for the computations in Chapters 4-7.

## 2.1 Tidal inlet basin system schematization

The area of interest is schematized as two cells: the tidal basin and inlet (Figure 2.1). The river discharge into the basin is indicated as  $Q^*$ . The tidal elevation  $(y_o^*)$ , the water level in the basin  $(y^*)$  and the inlet depth  $(h^*)$  are relative to the mean water level (MWL) in the ocean (Figure 2.2).



Figure 2.1: Schematization of a tidal inlet basin system top view (source: Hinwood et al. (2012)).



Figure 2.2: Schematization of a tidal inlet basin system side view (source: Hinwood et al. (2012)).

## 2.2 Model equations

The system of equations used in this thesis to compute the equilibrium cross-sectional area was earlier presented in Hinwood et al. (2012), Hinwood and McLean (2014) and Hinwood and McLean (2018). The system of equations consists of a hydrodynamic component and a sediment component. The equations constitute a strongly simplified set of process-based relations, which enables to compute the equilibrium state within a very short computation time (in the order of seconds). Since the equations are process-based, they could contribute to an improved understanding of whether or not equilibrium cross-sectional areas should expected to be present. The model equations are discussed in more detail in Sections 2.2.1 and 2.2.2.

Using the process-based model, the equilibrium state is computed by running the model for 1000 tidal cycles for approximately 60 different values of the inlet depth. The number of tidal cycles is chosen large enough to ensure the change in inlet depth is negligible at the end of the run. For a representative inlet, the results are discussed in Chapter 4. The computation time for the representative inlet is in the order of seconds. In the process-based model, the option for a river discharge in the tidal basin is included. The process-based model consists of two sets of equations: the hydrodynamic and morphodynamic equations.

#### 2.2.1 Hydrodynamic equations

In this section the hydrodynamic equations (Equation 2.1 and Equation 2.2) are presented and discussed. The equations pertain to a schematization of the tidal inlet (Figure 2.1).

The equations consist of an equation of motion for the inlet (Equation 2.1) and a one-dimensional continuity equation (Equation 2.2). Taking the ebb direction as positive, the equations are:

$$\frac{L}{g}\frac{du^*}{dt^*} + \left(\frac{K}{2g} + \frac{fL}{g(h^* + y_m^*)}\right)u^* \mid u^* \mid = y^* - y_o^*$$
(2.1)

$$A_b \frac{dy^*}{dt^*} = -u^* B(h^* + y_m^*) + Q^*$$
(2.2)

$$y_m^* = \frac{y_o^* + y^*}{2} \tag{2.3}$$

in which  $A_b$   $(m^2)$  is the surface area of the tidal basin,  $y^*$  (m) is the water level in the basin with respect to Mean Water Level (MWL) in the ocean,  $y_o^*$  (m) is the water level in the ocean,  $y_m^*$  (m) is the water level in the inlet,  $t^*$  (s) is time, B (m) is the width of the inlet,  $u^*$  (m/s) is the cross-sectional averaged velocity in the inlet,  $h^*$  (m) is the depth of the inlet measured with respect to MWL,  $Q^*$   $(m^3/s)$  is the river discharge, f (-) is the friction factor, K (-) is the loss coefficient, g  $(m^2/s)$  is the gravitational acceleration and L (m) is the length of the inlet. The velocity  $u^*$  and the depth  $h^*$  are assumed to be uniform over the length of the inlet.

Equation 2.1 contains a local acceleration term, bottom friction for the inlet, entrance losses and a term for variation of the water depth during the tidal stage. Density gradients are neglected. The water level in the

basin is assumed to be the same everywhere, also referred to as the "pumping mode". This is a reasonable approach for basins that are deep and have horizontal dimensions that are small compared to the length of the tidal wave. Equations 2.1 and 2.2 contain non-linear terms that generate higher harmonics. The quadratic velocity term  $(u^* \mid u^* \mid)$  is responsible for generating a third harmonic and the tidal variation of water depth within tidal stage  $(y_m^* + h^*)$  is responsible for generating a second harmonic. In particular the second harmonic in the velocity can be important for sediment transport (Van de Kreeke and Robaczewska, 1993).

#### 2.2.2 Morphodynamic equations

In this section the morphodynamic equations are presented (Equation 2.4 and Equation 2.5). Following Hinwood et al. (2012) the sediment concentration in the inlet is taken as:

$$C^* = m((\frac{u^*}{u_c^*})^2 - 1)$$
for  $|u^*| < u_c^* : C^* = 0$ 

$$L \frac{dh^*}{dt^*} = (h^* + y_m^*)u^* \Delta C^*$$
(2.5)

with

$$\Delta C^* = C_o - C^* \text{ for flood} \tag{2.6}$$

$$\Delta C^* = C^* - C_b \text{ for ebb} \tag{2.7}$$

in which  $C^*$  (-) is the sediment (volume) concentration at the downstream end of the inlet, m (-) is an empirical constant,  $u_c^*$  (m/s) the threshold velocity for both pick up and deposition,  $C_o$  (-) is the sediment concentration in the ocean,  $y^*$  (m) is the water level in the basin,  $u^*$  (m/s) is the velocity in the inlet,  $h^*$  (m) is the depth of the inlet and L (m) is the length of the inlet. For the process-based model Equations 2.4, 2.5, 2.6 and 2.7 are used to compute the morphological changes in the inlet.

For flow velocities lower than the threshold velocity  $u_c^*$ ,  $C^*$  becomes zero and the sediment is deposited on the bottom of the inlet. For flow velocities larger than  $u_c^*$  sand is picked up. The concentration  $C^*$  represents an equilibrium concentration, the maximum sediment concentration possible for a specific flow. When a water column enters the inlet, the sediment concentration in this water column is not equal to the equilibrium concentration instantly. The required distance for the sediment concentration to adapt to the equilibrium concentration is called the adaptation length. According to Galappatti and Vreugdenhil (1985) the adaptation length can be approximated as the distance a water column travels, in the time that is required for a sediment particle to travel from the top of the water column to the bottom:

$$\frac{h^*u^*}{w_s} \tag{2.8}$$

in which  $w_s$  is the fall velocity for the sediment. When the length of the inlet is larger than the adaptation length, the sediment concentration at the downstream end of the inlet is equal to the equilibrium concentration. For the inlets described in this thesis this condition is satisfied. For example, for the representative inlet described in Chapter 4, the maximum velocity is 1 m/s, the depth is 2.5 m and the fall velocity is 0.01 m/s, resulting in an adaptation length of 250 m. Since the length of the representative inlet is 500 m, the condition is satisfied.

The righthand side of Equation 2.5 with  $\Delta C^*$  given by Equation 2.6 represents the volume of sand that during flood enters the inlet at the ocean side and leaves the inlet at the basin side. It follows that when imposing the condition that during flood there is deposition, the volume of sand entering from the ocean side has to be larger than the volume of sand leaving the inlet at the basin side. This implies that  $C_o$  has to be sufficiently large, i.e.  $C_o >> C$ . Similarly, with Equation 2.7, the righthand side of Equation 2.5 represents the volume of sand that during ebb enters the inlet from the basin side and leaves the inlet at the ocean side. Imposing the condition that during ebb there is erosion, the volume of sand leaving the inlet at the ocean side has to be larger than the volume of sand entering at the basin side. It follows that  $C_b$  has to be sufficiently small, i.e.  $C_b << C$ . In summary, imposing the conditions that during flood there is deposition and during ebb there is erosion requires that  $C_o >> C_b$ .

When the inlet is in equilibrium, the deposition of sediment during flood is equal to the erosion during ebb. The velocity and therefore also the sediment concentration in the inlet  $(C^*)$  is more or less equal for both ebb and flood, which also means the volume of sediment leaving the inlet is equal for ebb and flood. However, since  $C_o > C_b$ , the volume of sediment entering the tidal basin during flood is larger than the volume of sediment leaving the tidal basin during flood is larger than the volume of sediment leaving the tidal basin during flood is larger than the volume of sediment leaving the tidal basin is filled up with sediment over time. This is shown for a representative inlet in Section 4.4. Sand that enters the inlet from the ocean and the basin side is assumed to be evenly deposited over the inlet.

#### 2.2.3 Non-dimensional form of the equations

To reduce the number of coefficients the equations are made dimensionless. The non-dimensional equations result in a reduction from 11 coefficients  $(A_b, L, B, a_0, T, k, g, m, f, C_b$  and  $C_o)$  in the dimensional equations to 6 coefficients  $(a_1, a_2, a_3, a_4, a_5 \text{ and } a_6)$  in the non-dimensional equations. The results are easily converted back to the dimensional form if one desires to do so. Introducing the scales suggested by Hinwood et al. (2012), the non-dimensional parameters are:

$$u = u^* \frac{TB}{4A_b}, h = \frac{h^*}{a_0}, y = \frac{y^*}{a_0}, y_o = \frac{y^*_o}{a_0}, Q = Q^* \frac{T}{A_b a_o}, C = \frac{C^*}{C_o}, t = \frac{t^*}{T}$$

in which T is the period of the tide and  $a_o$  is the amplitude of the tide. The factors as described above are substituted into equation 2.1 to 2.5, resulting in the following non-dimensional hydrodynamic equations:

$$\frac{du}{dt} = -\frac{a_1 u \mid u \mid}{(h+y_m)} - a_2(y_o - y) - a_3 u \mid u \mid$$
(2.9)

$$\frac{dy}{dt} = -4(h+y_m)u + Q \tag{2.10}$$

in which:

$$a_1 = \frac{A_b f}{2Ba_o}, a_2 = \frac{T^2 B a_o g}{4A_b L}, a_3 = \frac{2A_b K}{BL}$$

The non-dimensional form of the morphodynamic equations is:

$$C = a_4((\frac{u}{u_c})^2 - 1) \tag{2.11}$$

for 
$$|u| < u : C = 0$$

$$\frac{dh}{dt} = a_5(h+y_m)u\Delta C \tag{2.12}$$

with

$$\Delta C = 1 - C \text{ for flood} \tag{2.13}$$

$$\Delta C = C - a_6 \text{ for ebb} \tag{2.14}$$

in which:

$$a_4 = \frac{m}{C_o}, a_5 = \frac{4C_o A_b}{BL}, a_6 = \frac{C_b}{C_o}$$

The equations 2.9-2.12 are solved numerically as shown in Appendix A.

## Chapter 3

## Escoffier stability analysis

Presently the most widely used stability analysis is the one suggested by Escoffier (1940). A major component of his analysis is what has been referred to as the Escoffier Diagram, which is discussed in Section 3.1. The Modified Escoffier method is presented in Section 3.2, it eliminates some of the assumptions in the original Escoffier model. The closure curves of the Escoffier and Modified Escoffier diagrams are constructed with the process-based model in Chapters 4, 5 and 7.

## 3.1 Escoffier diagram

The Escoffier diagram consists of a closure curve  $\hat{u}(A)$  and an equilibrium velocity curve  $\hat{u}_{eq}$ , shown in Figure 3.1. Escoffier assumed that when at equilibrium the deposition in the inlet during flood is equal to the erosion during ebb. When at equilibrium, and assuming a sinusoidal velocity, the velocity amplitude  $\hat{u}$  equals the equilibrium velocity  $\hat{u}_{eq}$ . Escoffier took the equilibrium velocity to be independent of the inlet's cross-sectional area and equal to 0.9 m/s.

The intersections of the closure curve and equilibrium velocity curve represent a stable and an unstable equilibrium, shown as respectively points point B and C in Figure 3.1. When the cross-sectional area is between A and B, the inlet closes. When the cross-sectional area is in between B and C, the inlet cross-sectional area increases until it reaches an equilibrium at point C. When the inlet is in between C and D, the inlet cross-sectional area decreases until the cross-sectional area reaches point C.

Escoffier based the closure curve on a simplified equation of motion in which inertia and effects of tidal stage were neglected (Brown, 1928). In this thesis, the closure curve in the Escoffier diagram is calculated using a numerical solution of the one-dimensional hydrodynamic equations (Equations 2.9 and 2.10). These equations include inertia, exit entrance losses as well as the effect of tidal stage.



Figure 3.1: Escoffier diagram with the stable (C) and unstable (B) equilibrium cross-sectional area of the inlet.

The Escoffier stability model is useful for the determination of the stability of a tidal inlet. It also provides information on the required minimum cross-sectional area for the inlet to remain open. This can be helpful when the inlet is decreased in cross-section, for example after a storm. If a storm reduces the cross-sectional area below the value of point B in Figure 3.1, dredging should be performed immediately to minimize the amount of work. Additionally, the model can be used for studying the effects on for example deepening the inlet. An assumption made by Escoffier in formulating the model was that the sand entering is assumed to be distributed evenly over the inlet.

#### **3.2** Modified Escoffier diagram

The main reasons for developing a modified Escoffier diagram are Escoffier's assumptions that the inlet velocity is sinusoidal and the equilibrium velocity is 0.9 m/s (Van de Kreeke, 2004). These conditions are seldom satisfied. In the modified Escoffier diagram the equilibrium velocity is defined using the cross-sectional area tidal prism relationship, which can be written in the power form:

$$A = C * P^q \tag{3.1}$$

or as the linear equation as proposed by O'Brien (1969):

$$A = C_l * P \tag{3.2}$$

in which A is the cross-sectional area and P is the tidal prism. C,  $C_l$  and q are constants which have to be determined from observations. For a sinusoidal velocity, the tidal prism can be defined as:

$$P = \int_{0}^{\frac{1}{2}T} Audt = \int_{0}^{\frac{1}{2}T} A\hat{u}\sin\frac{2\pi}{T}tdt = \frac{TA\hat{u}}{\pi}$$
(3.3)

in which T is the period of the tide and u is the velocity. Rewriting Equation 3.3 results in the following expression for the maximum velocity:

$$\hat{u} = \frac{\pi P}{AT} \tag{3.4}$$

In analogy with Equation 3.4 a velocity amplitude is defined:

$$\hat{U} = \frac{\pi P}{AT} \tag{3.5}$$

It is noted that only for a sinusoidal velocity, the velocity  $\hat{U}$  equals the velocity amplitude  $\hat{u}$ . Eliminating P between Equations 3.1 and 3.5 results in an expression for the equilibrium velocity:

$$\hat{U}_{eq1} = \frac{\pi A^{\frac{1}{q}-1}}{TC^{\frac{1}{q}}} \tag{3.6}$$

Eliminating P between Equations 3.2 and 3.5 results in the following expression for the equilibrium velocity:

$$\hat{U}_{eq2} = \frac{\pi}{C_l T} \tag{3.7}$$

In Figure 3.2 the general shapes of the closure curve  $\hat{U}(A)$  and the equilibrium velocity curve  $\hat{U}_{eq1}(A)$  are presented. The intersections of the two curves represent the equilibrium cross-sectional areas. When using Equation 3.6, the closure curve has a similar shape as the closure curve in the Escoffier diagram, but the

equilibrium velocity is no longer independent of the cross-sectional area. The right-most intersection is the stable equilibrium and the left most intersection is the unstable equilibrium. When using Equation 3.7, the equilibrium velocity is independent of A, as is the case in the Escoffier diagram in Figure 3.1. Figure 3.2 is referred to as the modified Escoffier diagram.



Figure 3.2: Modified Escoffier diagram (source: Van de Kreeke (2004)).

## Chapter 4

## Application to a representative tidal inlet basin system

In this chapter the process-based model is applied to a representative tidal inlet basin system. In Section 4.1 the schematization of the representative inlet basin system and its parameters are presented. In Section 4.2 the numerical solution to the hydrodynamic equations is compared with the solutions from Keulegan (1951) and Hinwood et al. (2012). In Section 4.3 the calculated sediment concentration is discussed. In Section 4.4 the existence of equilibrium cross-sectional areas is shown. In Sections 4.5 and 4.6 the Escoffier and Modified Escoffier diagrams are presented for the representative inlet basin system.

#### 4.1 Tidal inlet basin schematization and parameter values

The process-based model is applied to a typical (representative) tidal inlet basin system as schematized in Figure 2.1 and with parameter values presented in Table 4.1. The inlet cross-sectional area is rectangular with width B and depth  $h^*$ . The system is forced with a sinusoidal ocean tide. Calculations are carried out with a time step  $\Delta t = 0.001$ . Values for  $C_o$ ,  $C_b$ , and m are taken from Hinwood et al. (2012).

Name	Value	Description
T (s)	44712	Period M2 tide
h (m)	2.5	Depth inlet
$A_b(m)$	$2.45^{*}10^{6}$	Surface area basin
$a_0 ({ m m})$	0.6	Tidal amplitude
B (m)	50	Width inlet
f (-)	0.025	Bottom friction factor
$g (m/s^2)$	9.81	Gravitational acceleration
K (-)	3	Entrance/exit loss coefficient
L (m)	500	Length inlet
$C_o$ (-)	$3*10^{-4}$	Sediment concentration in the ocean
$C_b$ (-)	$5*10^{-}5$	Sediment concentration in the basin
m (-)	$1.35^{*}10^{-}5$	Empirical sediment transport constant
$u_{cr} (m/s)$	0.3	Threshold scour velocity

Table 4.1: Parameter values for the representative inlet

### 4.2 Inlet velocity, basin water level and higher harmonics

To test the numerical solution presented in Appendix A, inlet velocity, basin water level and higher harmonics are calculated for the representative inlet basin system and the results are compared with the solutions from Keulegan (1951) in Table 4.2 and Hinwood et al. (2012) in Appendix A.



Figure 4.1: Ocean water level, basin water level and inlet velocity for two tidal cycles.

Ocean tide, basin water level and inlet velocity are presented together in Figure 4.1. The basin water level lags the ocean water level. The basin water level amplitude is slightly smaller than the ocean water level amplitude. In agreement with Equation 2.9, the velocity is approximately zero when the ocean tide and basin water level intersect. At that time it follows from Equation 2.10 that the basin water level is maximum. Whereas the water level curve is relatively smooth, the velocity curve during both ebb and flood shows a distinct break. This break is also present in the computations of the velocity with the numerical scheme used in Hinwood et al. (2012), as shown in Appendix A, Figure A.1.

The maximum basin water level and inlet velocity are compared with the Keulegan solution (Keulegan, 1951). The parameter values in Table 4.1 lead to a Keulegan repletion coefficient of K1 = 1. The maximum basin water level and inlet velocity in the Keulegan solution are given by equation 6.44 and equation 6.45 respectively in Van de Kreeke and Brouwer (2017). The results are presented in Table 4.2 and show reasonable agreement. The slight differences could in part be the result of neglecting inertia and effects of variations in depth with tidal stage in the Keulegan solution, while they are included in the numerical solution.

	Keulegan	Process-based model
Maximum basin water level (m)	0.51	0.56
Maximum inlet velocity $(m/s)$	1.20	1.24

Table 4.2: Maximum water level and velocity using the Keulegan solution and process-based model.

As shown in Van de Kreeke and Robaczewska (1993) higher harmonics in the velocity can play an important role in tidal sediment transport. To determine the presence of the higher harmonics a Fourier analysis was performed of velocity and basin water level. The Fourier series is written as:

$$y^* = a_0 + \hat{a}_1 \cos(\omega t) + \hat{b}_1 \sin(\omega t) + \hat{a}_2 \cos(2\omega t) + \hat{b}_2 \sin(2\omega t) + \hat{a}_3 \cos(3\omega t) + \hat{b}_3 \sin(3\omega t)$$
(4.1)

Values of the coefficients a and b for the first, second and third harmonic of velocity and basin water level are listed in Table 4.3. The mean velocity and basin water level set-up together with the amplitudes of the first, second and third harmonics are presented in Table 4.4. Except for the third harmonic, amplitudes decrease with increasing frequency.

The results of the process-based model are compared with those of the Keulegan solution (Equation 6.40 and 6.43 in Van de Kreeke and Brouwer (2017)), presented in the last column of Table 4.4. Amplitudes of the first and third harmonics from the numerical solution and the Keulegan solution show good agreement. As a result of neglecting depth variations with tidal stage, in the Keulegan solution amplitudes of the second harmonic, as well as the mean velocity and basin water level set up are zero.

	$y^*$ (m)	$u^* (m/s)$
$\hat{a}_1$	-0.2756	-1.1315
$\hat{b}_1$	0.4125	-0.7732
$\hat{a}_2$	-0.0320	-0.1014
$\hat{b}_2$	-0.0043	-0.0788
$\hat{a}_3$	0.0206	-0.0066
$\hat{b}_3$	-0.003	0.1743

Table 4.3: Coefficients of basin water level and inlet velocity harmonics.

	Amplitude	Amplitude
	process-based model	Keulegan
$u_0 (m/s)$	0.0471	
$\hat{u}_1 \ (\mathrm{m/s})$	1.37	1.38
$\hat{u}_2 \ (\mathrm{m/s})$	0.13	
$\hat{u}_3 (\mathrm{m/s})$	0.17	0.17
$y_0 (m)$	0.01305	
$\hat{y}_1$ (m)	0.50	0.48
$\hat{y}_2$ (m)	0.0323	
$\hat{y}_{3}$ (m)	0.021	0.021

Table 4.4: Mean basin water level, mean velocity and amplitudes of higher harmonics for the Keulegan solution and the process-based model.

Overall the numerical solution of the hydrodynamic part of the process-based model appears to give the right results.

## 4.3 Sediment concentration

The sediment concentration together with velocity in the inlet is shown in Figure 4.2. The sediment concentration is maximum, when the absolute value of the velocity is maximum. The sediment concentration is zero when the velocity is below the critical velocity for sediment suspension of 0.3 m/s.



Figure 4.2: Sediment concentration and velocity. During ebb the velocity is positive, during flood negative.

## 4.4 Stable and unstable equilibrium

Using equations 2.9 - 2.12, the presence of equilibrium depths is investigated. Calculations with approximately 25 values of the non-dimensional initial depth  $h_0$ , varying from 1.6 to 6, are carried out to determine which  $h_0$  values leads to closure of the inlet and which  $h_0$  values leads to a stable equilibrium. The results are shown in Figure 4.3. Because of the difficulty in calculating physically realistic velocities for small values of h, as explained in Section 4.5, calculations are limited to values of  $h_0$  larger than 1.6, corresponding with a dimensional depth  $h^* > 1$ m.



Figure 4.3: Inlet depth h(t), for different initial inlet depths  $h_0$ . Stable and unstable equilibrium depth.

In Figure 4.3 can be seen that as time progresses the inlet either approaches a stable equilibrium or closes. The unstable equilibrium is the initial depth for which the inlet will just remain open or close. Referring to Figure 4.3, the unstable equilibrium is at a non-dimensional depth of 2.25. An initial depth lower than this value causes the inlet to close. When the initial depth is larger than this value the inlet remains open and approaches the stable equilibrium, which is a depth of 3.53. In summary, as time progresses inlets either approach a stable equilibrium or close. This agrees with the premise of the Escoffier stability analysis.

The cross-sectional area and maximum velocity for the stable and unstable equilibrium are presented in Table 4.5. The tidal prism and the velocity amplitude  $\hat{U}$  for which the inlet is in stable or unstable equilibrium are included in the same table. The tidal prism is computed by multiplying the basin tidal range with the surface area of the tidal basin. The equilibrium cross-sectional area is determined from  $A = Bh^*$ . The velocity amplitude  $\hat{U}$  is computed with Equation 3.5. It follows that the equilibrium velocity  $\hat{u}$  is considerably higher than the 0.9 m/s proposed by Escoffier (Section 3.1). Since the process-based model computes a non-sinusoidal velocity (Figure 4.1), the maximum velocity  $\hat{u}$  is not equal to the velocity amplitude  $\hat{U}$ , as explained in Section 3.2.

	Stable equilibrium	Unstable equilibrium
Equilibrium cross-sectional area $(m^2)$	106	67.5
Tidal prism $(m^3)$	$2.35^{*}10^{6}$	$1.57^*10^6$
Velocity amplitude $\hat{U}$ $(m/s)$	1.56	1.62
Maximum velocity $\hat{u}$ (m/s)	1.31	1.29

Table 4.5: Stable and unstable equilibrium cross-sectional areas for the representative inlet calculated using the process-based model including depth variations with tidal stage. Corresponding tidal prisms, velocity amplitudes  $\hat{U}$  and maximum velocities  $\hat{u}$ .

When the inlet is in stable equilibrium, the depth changes of the inlet during a tidal cycle are shown in Figure 4.4. The depth of the inlet at the beginning of ebb equals the depth at the end of flood, which means there is no net deposition or erosion of sediment during a tidal cycle. Furthermore, the difference in depth between the

beginning and the end of ebb and the beginning and the end of flood is 0.0184 (-). With L=500m and B=50m, this amounts to an erosion of the inlet of ((0.0184 x 0.6) x 50 x 500 =) 276 m<sup>3</sup> of sand during ebb and equals the same volume of deposition during flood. For comparison, typical values of longshore sand transport at tidal inlets vary between  $100.000m^3$  and  $1.000.000m^3$  per year or  $135m^3$  and  $1350m^3$  per tidal cycle (Van de Kreeke and Brouwer, 2017).



Figure 4.4: Depth variation during a tidal cycle when the inlet is in stable equilibrium.

The volumes of sediment entering and leaving the inlet during ebb and flood are computed using Equation 2.12. For the computation of the volume of sediment entering the inlet during flood,  $\Delta C$  is replaced by  $C_o$  in Equation 2.12. For the sediment entering the inlet during ebb  $\Delta C$  is replaced by  $C_b$ . For the sediment leaving the inlet for both ebb and flood  $\Delta C$  is replaced by C. The result is shown in Figure 4.5. It can be seen that the net transport in the inlet over a full tidal cycle is zero, which means the inlet is in equilibrium. The volume of sediment leaving the inlet during ebb is approximately the same as the volume of sediment leaving the inlet during flood. The volume of sediment entering the basin is larger than the volume of sediment leaving the basin, which means the basin is slowly filling with sediment. This is physically unrealistic and a shortcoming of the process-based model.



Figure 4.5: Sediment volumes entering and leaving the inlet during ebb and flood. The net transport over a tidal cycle in the inlet is zero, the inlet is in equilibrium.

## 4.5 Escoffier diagram

The Escoffier diagram for the representative inlet is given in Figure 4.6. The closure curve  $\hat{u}(A)$  is calculated using equations 2.9 and 2.10. Because the velocity is not sinusoidal (Figure 4.1),  $\hat{u}$  is taken as the maximum velocity. The maximum velocity  $\hat{u}$  is calculated at intervals  $\Delta h = 0.05$  for values of h between  $h^* = 1m$  and  $h^* = 3.5m$ . The cross-sectional area is determined from  $A = Bh^*$ .

The shape of the closure curve is the same as shown in Figure 3.1. The position of the stable and unstable equilibrium depths as calculated with the process–based model are obtained from Table 4.5 and have been added in Figure 4.6 . As shown in Walton Jr (2004), in calculating the closure curve, the assumption of a constant width as compared to the assumption of a geometrically similar cross-section makes little difference for the part of the closure curve to the right of the stable equilibrium. Values to the left of the stable equilibrium can deviate substantially.

At values of  $h^* < 1m$ , the closure curve showed an aberrant character and did not gradually approach zero. Therefore calculation of the closure curve was halted at  $h^* = 1$ . The aberrant character is attributed to approximating the water level in the inlet by the average of the basin water level and the ocean water level:

$$y_m = \frac{y_o + y}{2}$$

From Equation 2.9, with  $y \approx 0$ , to a first approximation:

$$u \mid u \mid = \frac{a_2}{a_1}(h + y_m)(-y_o)$$

It follows that with y and  $y_o$  having finite values, u does not approach zero when h goes to zero. As shown in Figure 5.2 in Section 5.3, the aberrant behaviour of the closure curve for small values of h is no longer present when neglecting the depth variation with tidal stage.

Escoffier took the equilibrium velocity curve to be parallel to the horizontal axis in the Escoffier diagram, with a value of 0.9 m/s. This is considerably lower than the value of approximately 1.3 m/s for  $\hat{u}$  in Figure 4.6.



Figure 4.6: Escoffier diagram for the representative inlet. The closure curve includes the effect of depth variation with tidal stage. The values of the stable and unstable equilibriums are calculated with the process-based model and are listed in Table 4.5.

## 4.6 Modified Escoffier diagram

The modified Escoffier diagram for the representative inlet is presented in Figure 4.7. The closure curve  $\hat{U}(A)$  is calculated using Equation 3.5. The tidal prism is computed by multiplying the basin tidal range with the surface area of the tidal basin. The cross-sectional area of the inlet is determined from  $A = Bh^*$ . Calculations were carried out for: 1.6 < h < 4.3. The shape of the closure curve in Figure 4.7 is similar to the shape of the closure curve in the modified Escoffier diagram in Figure 3.2. The inlet cross-sectional areas, tidal prisms and velocity amplitudes  $\hat{U}$  corresponding with the stable and unstable equilibrium are computed with the process-based model and presented in Table 4.5. Using these values the positions of the stable and unstable equilibriums are plotted in Figure 4.7. The minimum cross-sectional area in Figure 4.7 is  $50m^2$  which corresponds to a minimum depth of 1m. Smaller values lead to aberrant behaviour as explained in Section 4.5.



Figure 4.7: Modified Escoffier diagram. Stable and unstable equilibriums are computed with the process-based model and listed in Table 4.5.

When taking Equation 3.6 as the expression for the equilibrium velocity and using the values of the tidal prisms and cross-sectional areas of the stable and unstable equilibriums given in Table 4.5, q is 1.10 and C is  $9.69 \times 10^{-6}$ . These value are within the range found in the literature (Van de Kreeke and Brouwer, 2017).

## Chapter 5

## The effect of depth variations with tidal stage on the stability of tidal inlets

In this chapter the effect of depth variations with tidal stage on the stable and unstable equilibriums is investigated. When the depth variation with tidal stage is neglected, the process-based model solves Equations 2.9, 2.10 and 2.12 with  $y_m$  (Equation 2.3) equal to zero. The process-based model is applied to the representative inlet, for which the parameters are presented in Table 4.1. The computations made in this chapter are similar to the computations in Chapter 4, except that the depth variations with tidal stage are neglected. According to Van de Kreeke and Robaczewska (1993) the effect of depth variations with tidal stage and the resulting generation of even higher harmonics are important for sediment transport in the inlet and therefore might affect the equilibrium cross-sectional areas.

## 5.1 Higher harmonics neglecting depth variations with tidal stage

In a similar way as in Section 4.2 the mean velocity, basin water level set-up and amplitudes of the first, second and third harmonic are calculated with the process-based model, neglecting depth variations with tidal stage. The results, together with the results from Table 4.4 when including the effect of tidal stage, are presented in Table 5.1. When neglecting effects of tidal stage, the mean velocity and basin water level set-up are extremely small and also the amplitude of the second harmonic becomes significantly smaller. Amplitudes of the third harmonic remain approximately the same. Van de Kreeke and Robaczewska (1993) showed that the even higher harmonics play an important role in the transport of sediment in tidal inlets.

	Amplitude	Amplitude
	with tidal stage	without tidal stage
$u_0 (m/s)$	0.0471	0.0009
$\hat{u}_1 \ (m/s)$	1.37	1.37
$\hat{u}_2 \ (\mathrm{m/s})$	0.13	0.0015
$\hat{u}_3 ~({ m m/s})$	0.17	0.19
$y_0 (m)$	0.01305	-0.0005
$\hat{y}_1$ (m)	0.50	0.51
$\hat{y}_2$ (m)	0.0323	0.0004
$\hat{y}_3$ (m)	0.021	0.023

Table 5.1: Mean basin water level, mean velocity and amplitudes of higher harmonics for the process-based model, with and without including effects of tidal stage. Ebb direction is positive.

## 5.2 Stable and unstable equilibriums

To determine the effect of neglecting the tidal stage on the equilibrium depth and velocities, the changes in depth (h) for different values of the initial depth  $(h_0)$  are calculated. The results are presented in Figure 5.1. Referring to Figure 5.1, the unstable equilibrium is at a non-dimensional depth of 1.42 and the stable equilibrium is at a non-dimensional depth of 1.42 and the stable equilibrium is at a non-dimensional depth variations with tidal stage are included, it becomes clear that the stable equilibriums are almost equal. The unstable equilibriums, however, differ by approximately 50%. Contrary when including the effect of tidal stage, computations using values of a non-dimensional depth smaller than 1.6 result in realistic values.



Figure 5.1: Non dimensional inlet depth changes h(t) for different initial inlet depths, when neglecting the effect of tidal stage.

Values of the equilibrium cross-sectional areas and corresponding maximum velocities are presented in Table 5.2. Values of the tidal prism and velocity amplitude  $\hat{U}$  when the inlet is in stable or unstable equilibrium are included in the same table. The equilibrium velocity remains more or less constant.

	Stable equilibrium	Unstable equilibrium
	(without tidal stage)	(without tidal stage)
Equilibrium cross-sectional area $(m^2)$	110	42.5
Tidal prism $(m^3)$	$2.44^{*}10^{6}$	$0.94^{*}10^{6}$
Velocity amplitude $\hat{U}(m/s)$	1.56	1.56
Maximum velocity $\hat{u}$ $(m/s)$	1.32	1.3

Table 5.2: Stable and unstable equilibrium cross-sectional areas of the representative inlet calculated using the process-based model excluding depth variations with tidal stage. Corresponding tidal prisms, velocity amplitudes  $\hat{U}$  and maximum velocities  $\hat{u}$ .

#### 5.3 Escoffier diagram

The closure curve without the effect of depth variations with tidal stage, together with the positions of the stable and unstable equilibriums listed in Table 5.2, are presented in Figure 5.2. For comparison the closure curve when including effects of depth variations with tidal stage (Figure 4.6) is presented in Figure 5.2, as well as the corresponding stable and unstable equilibriums (Table 4.5). The aberrant behavior of the closure curve for small h is no longer present. As explained in Section 4.5 the aberrant behavior is caused by including the effect of tidal stage. Instead, when not including the effect of tidal stage, the closure curve approaches zero for small values of the inlet depth. The shape of the closure curve is similar to the shape of the closure curve in

Figure 3.1 (Escoffier, 1940). The closure curves with and without tidal stage are nearly equal to the right of the maximum, which is in agreement with Walton Jr (2004). To the left of the maximum the differences between the closure curves increase.



Figure 5.2: Escoffier diagram; closure curves  $\hat{u}(A)$  with and without the effect of tidal stage. Green and pink dots indicate stable and unstable equilibriums.

## 5.4 Modified Escoffier diagram

The modified Escoffier diagram without including depth variations with tidal stage is presented in Figure 5.3. The closure curve  $\hat{U}(A)$  is calculated in a similar way as described in Section 4.6. Calculations were carried out for: 0 < h < 4.3. The inlet cross-sectional areas, tidal prisms and velocity amplitudes  $\hat{U}$  corresponding with the stable and unstable equilibrium are presented in Table 5.2. For comparison the closure curve when including the effect of tidal stage (Figure 4.7) is added.



Figure 5.3: Modified Escoffier diagram with and without the effect of tidal stage. The closure curves are presented in terms of the velocity  $\hat{U}$  and cross-sectional area A.

When the equilibrium velocity is given by Equation 3.6, it follows from the data in Table 5.2 that when neglecting depth variations with tidal stage q is 0.997 and C  $4.65 \times 10^{-5}$ . These values differ substantially from the values when including depth variations with tidal stage in Section 4.6, for which q is 1.10 and C is  $9.69 \times 10^{-6}$ . However, they are still within the range of values reported in the literature (Van de Kreeke and Brouwer, 2017).

## Chapter 6

## Sensitivity analysis

In this chapter is shown how the stable equilibrium depth depends on the parameters  $C_o$ ,  $C_b$  and m. The sensitivity analysis shows how the change in parameters increases or decreases the stable equilibrium depth of the inlet. The initial values for  $C_o$ ,  $C_b$  and m, as well as the remaining parameters are chosen the same as for the representative inlet (Table 4.1). The results from this analysis are used for the calibration of the process-based model applied to the Frisian inlet in Chapter 7.

## 6.1 Sediment concentration in the ocean and basin: $C_o$ and $C_b$

Values of the sediment concentration  $C_o$  and  $C_b$  are not always available for a location. Therefore, it is useful to know what the effect of these parameters is on the stable and unstable equilibrium cross-sectional areas. Using the process-based model, in this section the equilibrium cross-sectional areas are computed for different values of  $C_o$  and  $C_b$ . The initial inlet depth is 2.5m for all cases. In each case one parameter is varied and all other variables are as listed in Table 4.1. The  $C_o$ ,  $C_b$  and m values listed in the table are those used by Hinwood et al. (2012).

In Figure 6.1 the equilibrium depth of the inlet is computed for five different values of  $C_b$ , keeping the other parameters constant. Increasing values of  $C_b$  lead to decreasing value of stable equilibrium. For values of  $C_b$  equal or larger than  $10^{-4}$  the inlet closes. The value of  $C_o$  is  $3 * 10^{-4}$  and m is  $1.35 * 10^{-5}$ .



Figure 6.1: Inlet depth  $h^*(t^*)$  for five different values of  $C_b$ .

In Figure 6.2 the equilibrium depth of the inlet is computed for five different values of  $C_o$ , keeping the other parameters constant. Increasing values of  $C_o$  lead to a decreasing value of stable equilibrium. For values of  $C_o$  equal or larger than  $4 * 10^{-4}$  the inlet closes. For  $C_b$  a value of  $5 * 10^{-5}$  is chosen and for m a value of  $1.35 * 10^{-5}$ .



Figure 6.2: Inlet depth  $h^*(t^*)$  for five different values of  $C_o$ .

The magnitude of  $C_b$  and  $C_o$  has a significant effect on the equilibrium depth, it can make the difference for the inlet to approach a stable equilibrium or close. It is noted that the value of the equilibrium depth is related to the sum of the two parameters  $(C_o + C_b)$ , rather than the individual magnitudes of these parameters. The equilibrium depth is computed for three cases, for which both  $C_o$  and  $C_b$  are varied, but the sum of the two parameters remains equal with a value of  $20 * 10^{-5}$ . The result is presented in Table 6.1. The equilibrium depth is identical for all three cases.

An explanation can be found looking at Equations 2.9-2.12. When the inlet is at equilibrium, the volume of sand deposited in the inlet during flood equals the volume of sand eroded from the inlet during ebb. From Equations 2.9-2.12 it follows that to a first approximation the the volume of sand entering during flood is  $B \int |u| C_o$  dt, where the integral is over the flood cycle. The volume of sand entering during ebb is  $B \int |u| h C_b$  dt, where the integral is over the ebb cycle. With |u| and h being approximately equal for ebb and flood it follows that the total volume of sand entering the inlet during flood and ebb is proportional to  $(C_o + C_b)$ . The same holds for the total volume of sand leaving the inlet when the inlet is at equilibrium.

$C_b$	$C_o$	Equilibrium depth (m)
$1 * 10^{-5}$	$19 * 10^{-5}$	3.511
$3 * 10^{-5}$	$17 * 10^{-5}$	3.511
$5*10^{-5}$	$15 * 10^{-5}$	3.511

Table 6.1: The equilibrium depth is computed for varying values of  $C_o$  and  $C_b$ , but the sum of the two parameters remains constant. The computed equilibrium depths are identical in all three cases.

#### 6.2 Empirical sediment parameter m

The empirical parameter m can only be obtained from observations. The magnitude of m has a large influence on the equilibrium depth, as can be seen in Figure 6.3. Five different values of m are chosen varying between  $1.2 * 10^{-5}$  to  $2 * 10^{-5}$ . For  $C_o$  a value of  $3 * 10^{-4}$  is chosen and for  $C_b$  a value of  $5 * 10^{-5}$ . All other parameters are as shown in Table 4.1. As time progresses, different equilibrium depths are reached. Increasing values of mlead to increasing values of stable equilibriums. For values of m equal or smaller than  $1.2 * 10^{-5}$  the inlet closes.



Figure 6.3: Inlet depth  $h^*(t^*)$  for five different values of m.

## Chapter 7

## Frisian inlet

The Frisian inlet is part of the Dutch Wadden Sea, a shallow sea shielded by several barrier islands. The inlet is located in between the two barrier islands Ameland and Schiermonnikoog (Figure 7.1). The inlet connected part of the Wadden Sea, including the Lauwersmeer, to the Northsea. In 1969 the Lauwersmeer was closed, reducing the tidal basin surface area by 30 % (Van de Kreeke, 1998). This had a significant effect on the morphology of the tidal inlet and led to reduction of the inlet cross-sectional area. The bathymetry of the tidal basin and inlet has been closely monitored by Rijkswaterstaat from 1970 till the present day. The change in cross-sectional area and the wealth of available data is what makes the Frisian inlet interesting for validation of the process-based model and the Escoffier stability analysis.



Figure 7.1: Location of the Frisian inlet within the Wadden Sea. The area of interest is within the black square. Source: Van de Kreeke and Hibma (2005).

## 7.1 Observations

In this section pertinent parameter values for the Frisian Inlet are presented. For the Frisian inlet the offshore mean annual significant wave height is 1.13 m and the mean tidal range is 2.25 m, with the M2 tide being the predominant tidal constituent (Van de Kreeke, 2004). The inflow of fresh water is negligible.

The information on basin water levels and inlet velocities is based on a measurement campaign carried out in 1992. The velocity was measured in the inlet at a depth of 1.5 m above the bottom, which resulted in an M2 velocity amplitude of 0.77 m/s (Van de Kreeke and Dunsbergen, 2000). The basin water level measurements carried out during the same campaign resulted in a basin water level amplitude of 1.14 m (Van de Kreeke and Dunsbergen, 2000). The basin area at high water and basin area at low water. Before reduction this results in a value of  $125 \times 10^6 m^2$  and after reduction  $90 \times 10^6 m^2$  (Van de Kreeke and Dunsbergen, 2000). In Van de Kreeke and Dunsbergen (2000) the tidal prism is reported as  $200 \times 10^6 m^3$ .

Information on sand concentrations in the inlet are reported in Van de Kreeke and Hibma (2005). Average sand concentrations in the inlet over the 14 day measurement period are 6.7 mg/L. High fluctuations in the measurements were present, with peaks up to 100 mg/L.

The information on the cross-sectional area of the inlet consist of two parts. For the years 1970-1987 the cross-sectional areas were taken from Biegel (1991). The location of the cross-section is off Engelmansplaat. For the years 1987-2012 the cross-sectional areas were computed from the depth measurements made available by Rijkswaterstaat (Figure 7.2). In Figure 7.2 the green area marks the inlet. The length of the inlet is taken as 5000 m, which is also the average width of the adjacent barrier islands. The average cross-sectional areas over the length of the inlet for the period 1970-2012 are presented in Table 7.1. For a given year cross-sectional areas along the length of the inlet might differ by as much as 50%. The average width of the inlet was more or less constant throughout the years with a value of 3200m.



Figure 7.2: Measurements of the Frisian inlet, performed by Rijkswaterstaat in 2000. The green area indicates the inlet. The darkness of the red indicates the depth qualitatively; increased darkness indicates increasing depth. The yellow and white areas indicate elevations above sea level.

Year	Cross-sectional area $(m^2)$
1970	24500
1975	22900
1979	21900
1982	20800
1987	19830
1994	18929
1997	18988
2000	18796
2006	18943
2012	17830

Table 7.1: Cross-sectional area Frisian inlet from 1970 till 2012.

## 7.2 Schematization of Frisian inlet and parameter values

The Frisian inlet is schematized as shown in Figure 2.1. The parameter values are based on observations discussed in Section 7.1 and presented in Table 7.2. The values for the entrance/exit loss coefficient and critical velocity were taken the same as in Hinwood et al. (2012). For the value of the friction factor see Section 7.3. The dimensions of the inlet, L and B, are based on the measured bathymetry as shown in Figure 7.2. The inlet length is approximately equal to the width of the adjacent barrier islands.

Name	Value	Description
T (s)	44712	Period M2 tide
$A_{b1}(m)$	$125^{*}10^{6}$	Surface area basin before reduction
$A_{b2}(m)$	$90^{*}10^{6}$	Surface area basin after reduction
$a_0 (m)$	1.12	Tidal amplitude
B (m)	3200	Width inlet
f (-)	0.025	Bottom friction factor
$g (m/s^2)$	9.81	Gravitational acceleration
K (-)	3	Entrance/exit loss coefficient
L (m)	5000	Length inlet
$u_{cr} (m/s)$	0.3	Threshold scour velocity

Table 7.2: Parameter values used in the process-based model for the Frisian inlet.

## 7.3 Determination of the value of the friction factor

The value of the friction factor is determined by comparing observed and computed values for the velocity and basin water level. Using the parameter values in Table 7.2, the inlet velocity and basin water level are computed with the process-based model for values of the bottom friction factor (f) between 0.02 and 0.03. By comparing the computed velocity and basin water level to the measured values from Section 7.1, the bottom friction factor is found. Within the selected range, the bottom friction factor appeared to have little influence on the maximum velocity, maximum basin water level elevation and tidal prism. A value of 0.025 is selected, which is the same as reported in Hinwood and McLean (2015).

With f = 0.025, the computed velocity, basin water level and the ocean water level are plotted in Figure 7.3. The basin water level lags the ocean water level and its amplitude is slightly larger than the ocean water level amplitude. The maximum velocity, basin water level and tidal prism are presented in Table 7.3 and compared to the measurements by Van de Kreeke and Dunsbergen (2000). Since the measurements used by Van de Kreeke and Dunsbergen (2000). Since the measurements used by Van de Kreeke and Dunsbergen (2000) were performed in 1992, the hydrodynamic computations are carried with the corresponding inlet depth at that time. The inlet depth is found by dividing the average cross-sectional area from the years 1987 and 1994 from Table 7.1 by the width of the inlet, which results in a depth of 6 m. The tidal prism is computed by multiplying the basin area with the basin water level range. The values computed by the process-based model are close to the measured values.

	Measurements	Process-based model
	(Van de Kreeke and Dunsbergen, 2000)	
Tidal prism $(m^3)$	$200 * 10^6$	$214 * 10^{6}$
Basin water level amplitude $(m)$	1.14	1.19
Maximum velocity $\hat{u}$ $(m/s)$	0.77	0.72

Table 7.3: Measured (Van de Kreeke and Dunsbergen, 2000) and computed tidal prism, basin water level and maximum velocity for the Frisian Inlet using the process-based model including depth variations with tidal stage.



Figure 7.3: Water level in the basin and ocean and inlet velocity.

#### 7.4 Determination of morphodynamic parameters

The morphodynamic parameters  $C_o$ ,  $C_b$  and m are determined by matching computed and measured crosssectional areas. For the measured cross-sectional areas reference is made to Table 7.1. The initial inlet depth is found by dividing the cross-sectional area from 1970 by the width of the inlet. In Van de Kreeke (2004) a sand concentration of 6.7 mg/L was measured in the basin of the Frisian inlet. For a density of 1590  $kg/m^3$  for wet sand, a mass concentration of 6.7 mg/L corresponds with a volume concentration  $C_b$  of  $4.2 \times 10^{-6}$ , which is used as the initial value for  $C_b$ . In Wang et al. (1995) sediment concentrations in the order of  $O(10^{-5})$  are reported for the Frisian inlet, which is used as an initial value for  $C_o$ . According to Equation 2.11, with  $C^*$  $= 10^{-5}$ ,  $u^* = 1$  m/s and  $u_{cr}^* = 0.3$  m/s, this results in an initial m value of  $10^{-6}$ . The three morphological parameters are adjusted until a satisfying fit with the cross-sectional area measurements is found. This results in a  $C_o$  of  $1.3 \times 10^{-5}$ ,  $C_b$  of  $6.2 \times 10^{-6}$  and m of  $2.5 \times 10^{-6}$ , which lie within the same order of magnitude of earlier measurements and predictions by Van de Kreeke (2004) and Wang et al. (1995). A comparison between observed and calculated cross-sectional areas is presented in Figure 7.4. The standard deviation of the computed values is 400  $m^2$ .



Figure 7.4: Comparison of measured (dots) and computed (solid line) cross-sectional areas.

#### 7.5 Stable and unstable equilibrium

To determine the stable and unstable equilibrium cross-sectional areas of the Frisian inlet, the changes in depth (h) for different values of the initial depth  $(h_0)$  are calculated. The results are presented in Figure 7.5. Calculations with approximately 70 values of  $h_0$  varying from 0 to 8 are carried out to determine which  $h_0$  values lead to closure of the inlet and which  $h_0$  values lead to a stable equilibrium. Unfortunately, the unstable equilibrium depth is too small to be computed when including variations of depth with tidal stage; the computations for very small inlet depths lead to unrealistic values for the velocity, as explained in Section 4.5. Instead, to arrive at an estimate of the depth of the unstable equilibrium, the equations without the variations in depth with tidal stage are used, resulting in Figure 7.5. The value of the stable equilibrium differs little (approximately 3%) from that calculated when including depth variations with tidal stage in the equations. Values of the equilibrium cross-sectional areas are computed by multiplying the dimensional equilibrium depths with the width of the inlet and are presented with corresponding velocities in Table 7.4. In the same table the tidal prism for the stable and unstable equilibrium is presented, which is computed by multiplying the tidal range with the surface area of the basin.

	Stable equilibrium	Unstable equilibrium
Equilibrium cross-sectional area $(m^2)$	17.100	2.900
Tidal prism $(m^3)$	$217^*10^6$	$37.4^{*}10^{6}$
Maximum velocity $\hat{u}$ $(m/s)$	0.79	0.77

Table 7.4: Stable and unstable equilibrium cross-sectional areas of the Frisian Inlet and corresponding maximum velocities and tidal prisms calculated using the process-based model without depth variations with tidal stage.



Figure 7.5: Inlet depth h(t) without the effect of tidal stage, for different initial inlet depths  $h_0$ . Stable and unstable equilibrium depth.

## 7.6 Modified Escoffier diagram

In this section the modified Escoffier diagram without depth variation with tidal stage is constructed for the Frisian inlet and presented in Figure 7.6. The closure curve  $\hat{U}(A)$  is calculated with the process-based model using Equation 3.5. The tidal prism is computed by multiplying the basin tidal range with the surface area of the tidal basin. The equilibrium velocity curve is calculated using Equation 3.7. For the Dutch Wadden Sea  $C_l = 6.8 * 10^{-5}$  (Van de Kreeke, 1998). With T=44712s, this results in  $\hat{U}_{eq2} = 1.03$  m/s. The intersections of the closure curve and equilibrium velocity curve represent the stable and unstable equilibriums. Their values are listed in Table 7.5. Using a different schematization for the inlet basin system, in Van de Kreeke (2004) the Modified Escoffier diagram was constructed for the Frisian inlet. This resulted in a stable equilibrium

cross-sectional area of 15.500  $m^2$ , which is close to the 14.700  $m^2$  found in this study. Values of the unstable cross-sectional area were not presented in Van de Kreeke (2004).

	Stable equilibrium	Unstable equilibrium
Equilibrium cross-sectional area Modified Escoffier $(m^2)$	14.700	3800
Tidal prism $(m^3)$	$216 * 10^{6}$	$56.5 * 10^{6}$
Velocity amplitude $\hat{U}$ $(m/s)$	1.03	1.03

Table 7.5: Stable and unstable equilibrium cross–sectional areas and velocities as computed with the Modified Escoffier diagram, as well as the corresponding tidal prisms.



Figure 7.6: Modified Escoffier diagram for the Frisian inlet. The closure curve is calculated without the effect of tidal stage.

## Chapter 8

# Discussion, conclusions and recommendations

In this chapter the results of the study are discussed. In Section 8.1 the limitations and advantages are discussed for each of three different methods used to determine the equilibrium cross-sectional areas. In Section 8.2 the conclusions are summarized. In Section 8.3 recommendations are presented for additional research.

### 8.1 Discussion

The Escoffier method (Escoffier, 1940) is an empirical method based on the assumption of the existence of equilibrium cross-sectional areas. Other assumptions are a sinusoidal velocity and an equilibrium velocity of 0.9 m/s. A sinusoidal velocity does not include higher harmonics, which could be important for the transport of sediment in tidal inlets (Van de Kreeke and Robaczewska, 1993). In the Escoffier method the equilibrium velocity of 0.9 m/s is an empirical value and lacks physical justification. This and the assumptions of a sinusoidal velocity make this method difficult to apply to a real world inlet.

In the Modified Escoffier method (Van de Kreeke, 2004) some of the limitations of the Escoffier method are no longer present. The Modified Escoffier method is based on the Escoffier method, but it does not assume a sinusoidal velocity and an equilibrium velocity of 0.9 m/s. In this method, the equilibrium velocity is based on the A-P relation (O'Brien, 1931), for which a physical basis is presented in Van de Kreeke (2004). A problem that remains is that for application to a real world inlet, the A-P relation for the inlet must be known.

The process-based model consists of a set of four equations and calculates the inlet depth as a function of time. Major assumptions are uniform flow in the inlet and a water level that is linear between the ocean water level and basin water level. For small values of the depth, the approximation of the water level leads to unrealistic velocities (Section 4.5).

Instead of assuming an equilibrium cross-sectional area, the process-based model is run until the change in depth over time is negligible and the inlet approaches a stable equilibrium. When the inlet is at a stable equilibrium, the condition that there is deposition during flood and erosion during ebb, leads to a net transport of sand to the basin (Sections 4.4 and 2.2.2). The basin acts as an infinitely large sink or source of sand. This is physically unrealistic.

Although the process-based model is based on physical principles, the morphodynamic equations contain the empirical parameters  $C_o$ ,  $C_b$  and m, which are not a priori known. As shown for the Frisian inlet (Chapter 7) these parameters require calibration. For application to a real world inlet this can be a problem, since measurements are not always available. More research on the morphological parameters is therefore desirable.

As shown by Van de Kreeke and Robaczewska (1993) the second harmonic of the velocity can be important for sediment transport in a tidal inlet. For the process-based model the effect of the second harmonic on the equilibrium cross-sectional area is investigated by excluding the depth variation with tidal stage in the computations of the equilibrium cross-sectional areas (Chapters 4 and 5). Whether or not including the effect of depth variations with tidal stage, the inlet approaches a stable equilibrium. This shows that the absence of a second harmonic not necessarily precludes the presence of a stable or unstable equilibrium cross-sectional area. More research is needed to evaluate the role of the second harmonic in determining the cross-sectional equilibrium.

Additionally, it is shown that in the Escoffier and Modified Escoffier method the part of the closure curve to the right of the maximum (and thus the stable equilibrium) is not sensitive to whether or not the effect of tidal stage is included. On the contrary, the part to the left of the maximum (and thus the unstable equilibrium) is sensitive to inclusion of the effect of tidal stage. The most likely explanation is that generation of higher harmonics for the relatively small cross-sectional areas to the left of the maximum is more pronounced than for the relatively large cross-sectional areas to the right of the maximum.

For the Frisian inlet, computations with the process-based model resulted in a stable equilibrium cross-sectional area of 17.100  $m^2$ , while computations with the Modified Escoffier resulted in a value of 14.700  $m^2$  (Sections 7.5 and 7.6). The difference in these two values can be explained by the fundamental difference in approach. The computation of the process-based model is based on the physical processes in the inlet and leads to an equilibrium. The computation of the Modified Escoffier method is based on the empirical A-P relation and assumes an equilibrium cross-sectional area. Additionally, the value for  $C_l$  of the equilibrium velocity (Equation 3.7) is based on a regression analysis for five inlets in the Wadden Sea. As shown in Van de Kreeke and Brouwer (2017), the equilibrium cross-sectional area of an individual inlet can deviate from the value computed with the regressions analysis. For the process-based model calibration of the morphodynamic parameters was necessary, which also introduces an uncertainty in the result.

## 8.2 Conclusions

In this section the conclusions of the study are presented. The conclusions address the objectives stated in Section 1.2.

The non-dimensional equations 2.9-2.12 that constitute the bases of the process-based model are solved using an explicit finite difference scheme. The finite difference form of the equations and the procedure used to solve these equations are presented in Appendix A.

Applying the process-based model to a representative inlet, it is shown that in the long-term the cross-sectional area of the inlet approaches a stable equilibrium. In addition an unstable equilibrium is identified (Section 4.4). For cross-sectional areas smaller than the unstable equilibrium the inlet closes. For inlets larger than the unstable equilibrium the inlet cross-sectional area goes to the stable equilibrium.

Using the process-based model the stable and unstable equilibrium cross-sectional areas for a representative inlet show good qualitative agreement with calculations carried out with the Escoffier and Modified Escoffier stability analysis (Section 4.5 and 4.6).

Using the results of the representative inlet it is concluded that excluding depth variations with tidal stage makes little difference on the magnitude of the stable cross-sectional area. Excluding depth variations with tidal stage considerably decreases the magnitude of the unstable cross-sectional area (Chapter 5).

After determination of the values of the friction factor and morphodynamic parameters, the process-based model predicts a value of 17.100  $m^2$  for the stable equilibrium cross-sectional area of the Frisian inlet (Section 7.5). The Modified Escoffier method, with the equilibrium velocity derived from the linear A-P relationship for the Dutch Wadden Sea, yields a value of 14,700  $m^2$  for the stable equilibrium (Section 7.6). The value of the unstable cross-sectional area, when neglecting depth variations with tidal stage in the computations, is 2900  $m^2$ .

Based on the results of the computations it is concluded that the Frisian inlet is close to its equilibrium state.

## 8.3 Recommendations

More research is needed for the following subjects:

- The effect of the second harmonic on the equilibrium cross-sectional areas of a tidal inlet.
- The values of the morphodynamic parameters, i.e.  $m, C_o$  and  $C_b$ , and their effect on the equilibrium cross-sectional areas of a tidal inlet.
- Replace the relatively simple morphodynamic Equations 2.4 and 2.5 with a set of more advanced morphodynamic equations.

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## Appendix A

# Numerical solution to the governing equations

In this Appendix the numerical solution used in the process-based model is presented in Section A.1. The results for the basin water level, velocity and inlet depth for the representative inlet (Chapter 4) are compared to the results of the numerical scheme used in Hinwood et al. (2012), also referred to as HydSed, in Section A.2.

#### A.1 Numerical solution

The numerical scheme used in the process-based model to solve Equations 2.9 to 2.12 is presented in this section. The equations with the unknowns velocity (u), basin water level (y), concentration (C) and inlet depth (h), are written in a finite difference form.

The differential Equations 2.9, 2.10 and 2.12 are solved using an explicit scheme (Zijlema, 2015). All the variables (u, y, C and h) are defined at the same time level i. The time derivatives are centered around  $i - \frac{1}{2}$ . The resulting finite difference equations corresponding to the differential equations 2.9 and 2.10 are:

$$\frac{u(i) - u(i-1)}{\Delta t} = -a_1 \frac{u(i) \mid u(i-1) \mid}{h(i-1) + y_m(i-\frac{1}{2})} - a_2(y_o(i-\frac{1}{2}) - y(i-\frac{1}{2})) - a_3u(i) \mid u(i-1) \mid$$
(A.1)

$$\frac{y(i) - y(i-1)}{\Delta t} = -4(h(i-1) + \frac{y_m(i-\frac{1}{2})}{2})u(i-1) + Q$$
(A.2)

with

$$y_m(i - \frac{1}{2}) = \frac{y(i) + y(i - 1) + y_o(i) + y_o(i - 1)}{2}$$

The resulting finite difference equations corresponding to the equations 2.11 and 2.12 are:

$$C(i) = a_4((\frac{u(i)}{u_{cr}})^2 - 1)$$
(A.3)

$$\frac{h(i) - h(i-1)}{\Delta t} = a_5(y_m(i-\frac{1}{2}) + h(i-\frac{1}{2}))u(i-\frac{1}{2})\Delta C(i)$$
(A.4)

with

$$\Delta C(i) = 1 - C(i) \text{ for flood}$$
(A.5)

$$\Delta C(i) = C(i) - a_6 \text{ for ebb} \tag{A.6}$$

The finite difference Equations A.1-A.4 are solved for y, u, C and h with the following initial conditions:

$$u(0) = 0, y(0) = a_o, C(0) = 0 \text{ and } h(0) = h_0$$

The ocean tide  $y_o$  is defined as:

$$y_o(i) = \sin\left(2\pi t(i)\right) \tag{A.7}$$

Starting with Equation A.2, y(1) is calculated. Subsequently u(1), C(1) and h(1) are calculated using Equations A.1, A.3 and A.4. Computations are carried out until the change in h is negligible and the inlet is in equilibrium.

The variables u, y and h are dependent on each other. Instead of solving these variables simultaneously, they are solved explicitly. For the computation of y(i) in Equation A.2, u and y should both be solved at the same time step(i). However, since u(i) is unknown, u(i-1) is used instead.

The accuracy of the explicit scheme was tested by comparing some of the results of the process-based model with the numerical scheme used in (Hinwood et al., 2012), from now on referred to as HydSed. HydSed uses the same equations as the process-based model, but a different numerical scheme. The results of the comparison are presented in the next section (Section A.2). No significant differences are found in the comparison and therefore the use of an explicit scheme seems justified.

#### A.2 Comparison process-based model and HydSed

For the representative inlet (Chapter 4), the results of the process-based model are compared to the results of the numerical scheme used in Hinwood et al. (2012), referred to as HydSed. The results of the velocities and the basin water levels from the process-based model and HydSed are shown in Figure A.1. In Figure A.2 the depth change in time and the stable equilibrium inlet depth for both models are shown. In Table A.1 the higher harmonics for both numerical models, as well as for Keulegan (Keulegan, 1951) are shown. It is clear that there are no significant differences present.

	amplitude process-based model	amplitude Keulegan	amplitude HydSed
$u_1  [\mathrm{m/s}]$	1.37	1.38	1.35
$u_2  [\mathrm{m/s}]$	0.13		0.13
$u_3  [\mathrm{m/s}]$	0.17	0.17	0.19
$u_4  [\mathrm{m/s}]$	0.062		0.062
$u_5  [\mathrm{m/s}]$	0.072		0.075
$y_1  [\mathrm{m}]$	0.50	0.48	0.49
$y_2  [\mathrm{m}]$	0.032		0.033
$y_3 [m]$	0.021	0.021	0.022
$y_4  [\mathrm{m}]$	0.0059		0.0067
$y_5  [\mathrm{m}]$	0.0051		0.0047

Table A.1: Comparison amplitudes of the higher harmonics for Keulegan, the process-based model and HydSed.



Figure A.1: Velocity and basin water level computed by the process-based model and HydSed.



Figure A.2: Depth  $h^*(t^*)$  as computed by the process-based model and HydSed.