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On the levitation force in horizontal core-annular flow with a large viscosity ratio and small density ratio

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A numerical study has been made of horizontal core-annular flow: the flow of a high-viscosity liquid core surrounded by a low-viscosity liquid annular layer through a horizontal pipe. Special attention is paid to the question how the buoyancy force on the core, caused by a density difference between the core and the annular layer, is counterbalanced. The volume-of-fluid method is used to calculate the velocities and pressures in the two liquids. At the start of the calculation the core is in a concentric position. Thereafter the core starts to rise under the influence of buoyancy until it reaches an eccentric equilibrium position where the buoyancy force is counterbalanced by hydrodynamic forces generated by the movement of a wave at the core-annular interface with respect to the pipe wall. At high Reynolds number of the flow in the annular layer core levitation is due to inertial forces, whereas at low Reynolds number viscous (lubrication) forces are responsible for levitation. We carried out two types of calculation. In the first we assume the interface to be smooth (without wave) at the start of the calculation and study how the wave develops during the rising period of the core. In the second a wave is already present at the start of the calculation. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4793701]

I. INTRODUCTION

We study the flow of a high-viscosity liquid surrounded by a low viscosity liquid through a horizontal pipe. This core-annular flow is very interesting from a practical and scientific point of view. Much attention has been paid in the literature to core-annular flow. Joseph and Renardy[1] have written a book about it. There are several review articles, see for instance Oliemans and Ooms[2] and Joseph *et al.*[3] Most publications deal with the development of waves at the interface between the high-viscosity liquid and the low-viscosity one, see Ooms,[4] Bai *et al.*,[5,6] Renardy,[7] Li and Renardy,[8] Kouris and Tsamopoulos,[9] and Ko *et al.*[10] These studies deal with vertical core-annular flow (the core has a concentric position in the pipe). In that case the buoyancy force on the core, due to a density difference between the two liquids, is in the axial direction of the pipe. Papageorgiou *et al.*,[11] Wei and Rumschitzki[12] and others have developed lubrication models for axisymmetric core-annular flow. They derive nonlinear evolution equations for the interface between the two liquids. These equations include a coupling between core and annular film dynamics thus enabling a study of its effect on the nonlinear evolution of the interface.

It is also important to pay attention to core-annular flow through a horizontal pipe. When the densities of the two liquids are different, gravity will push the core off-center in that case. Experimental results suggest that under normal conditions a steady eccentric core-annular flow (rather than a stratified flow) is achieved. Relatively little attention has been given to the explanation of the levitation mechanism. Ooms and Beckers,[13] Ooms *et al.*,[14] and Oliemans and Ooms[2] proposed a mechanism based on hydrodynamic lubrication theory. They showed that levitation could not take place without a hydrodynamic lifting action due to the waves present at the oil-water interface. In their theoretical work they assumed that the core viscosity is infinitely large. So any deformation of the interface was neglected and the core moved as a rigid body at a certain speed with respect to the pipe wall. The shape of the waves was given as empirical input. They were assumed to be sawtooth waves that were like an array of slipper bearings and pushed off the core from the wall by lubrication forces. In their case the core would be sucked to the pipe wall if the velocity was reversed.

However it was pointed out by Bai *et al.*,[6] that (at finite oil viscosity) the sawtooth waves are unstable since the pressure is highest just where the gap between the core and the pipe wall is smallest. So the wave must steepen where it is gentle and become smooth where it is sharp, and levitation of the core due to lubrication forces is no longer possible. To get a levitation force from this kind of wave inertial forces are needed according to Bai *et al.* They studied by direct simulation the wave development for a concentric vertical core-annular flow (taking into account inertial forces) under the assumption that the densities of the two liquids are equal. Moreover, they assumed that the viscosity of the core is so large that it moves as a rigid solid which may nevertheless be deformed by pressure forces in the annulus (by applying the Young-Laplace equation).

Bai *et al.* tried to draw some conclusions about the levitation force on the core in case of an eccentric horizontal core-annular flow. They considered what might happen if the core moved to a slightly eccentric position owing to a small difference in density. The pressure distribution in the liquid in the narrow part of the annulus would intensify and the pressure in the wide part of the annulus would relax according to their predicted variation of pressure with the distance between the core and the pipe wall for the concentric case. In that case a more positive pressure would be generated in the narrow part of the annulus which would levitate the core. This calculation was actually carried out later by Ooms *et al.*[15] for an eccentric core-annular flow with a rigid core and a wave shape as calculated by Bai *et al.*

Li and Renardy[8] improved the wave simulation of Bai *et al.* by applying a volume-of-fluid scheme (VOF) and by allowing for different densities for the two liquids. They assume axisymmetric vertical flow. Their initial condition is seeded with an eigenmode of largest growth rate as found from stability calculations. They state that the wave shape as calculated by Bai *et al.* is too rounded and smooth compared to the experiments, which show an almost symmetrical form of the crest. The crest is only slightly sharper on the front and less sharp on the back. These details are successfully reproduced by Li and Renardy.

The aim of our research is to continue with the VOF-calculations by Li and Renardy, but then for horizontal flow with a density difference between the liquids. We will start with a concentric core and then calculate the upward movement of the core due to buoyancy and see whether the core will stay free from the wall at a certain eccentricity due to hydrodynamic forces. In our calculations we will study two different cases: in one case we start with a smooth interface between the core and the annulus and in the other we start with an interface on which already a wave of finite amplitude is present. The viscosity ratio is assumed to be large and the density ratio small. Moreover, calculations will be done at high and low Reynolds number of the flow in the annular layer.

II. NUMERICAL SCHEME

The incompressible Navier-Stokes equations plus mass conservation equation have been solved, with a density ρ and viscosity μ which are determined in each cell by the volume fraction of oil and water present, i.e., if the volume fraction of oil is C, and the density of oil is ρ_1 and the density of water ρ_2, the density in a cell is $C\rho_1 + (1 - C)\rho_2$ and similar for μ. The equations for both phases have been solved using the Finite Volume method, with application of the VOF method (see Li[16]). So an additional advection equation for the oil fraction C is solved. The equations solved are as follows:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left\{\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\} + \rho g_i$$

$$\frac{\partial u_j}{\partial x_j} = 0$$

$$\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} = 0\,.$$

Here g_i is the gravity vector. We used the package OpenFOAM version 1.6 (see Ref. 17) for the simulations. Use was made of the interFoam solver, with backward Euler in time, limited linear for the advection terms of the velocity components, and van Leer for the advection of the scalar. Interface compression (Rusche[18]) has been used to get a sharp interface. The pressure-velocity coupling was done using the PISO scheme. The following linear solvers have been used: Preconditioned Conjugate Gradient for the pressure, and Preconditioned Bi-Conjugate Gradient for velocity components. For stability reasons the Courant number was kept below 0.02 for all simulations. Although this Courant number is quite low, Courant numbers of this size are found necessary to obtain a very thin interface with the current method, as reported by Gopala and van Wachem.[19]

The grids were equidistant and structured for the axisymmetric cases and block-structured for the 3D cases. For the axisymmetric cases we used 80×80, 128×128, and 256×256 grid points. The final runs were made on a 128×128 grid. The 3D grids used had 110, 80, and 60 grid points in radial, circumferential, and axial flow direction, a total of approximately 500.000 grid points. In axial direction we impose the periodic boundary condition. At the pipe wall we prescribe the no-slip condition.

Test runs have been made for the axisymmetric cases to ascertain that the grids were fine enough and that the numerical schemes have not added too much numerical diffusion. We have made one run with a grid which was twice as fine in both coordinate directions, and we repeated the same run with the linear (non-limited) scheme. The results for the final wave form of the oil-water interface were graphically almost indistinguishable after transport over a small, non-relevant distance which indicated some difference in phase error.

III. RESULTS

A. Upward vertical core-annular flow with buoyancy effect

To check our calculations we started with a comparison of our results with those presented by Li and Renardy for an upward vertical core-annular flow with buoyancy effect. Like them we assumed that at the start of the axisymmetric calculation a concentric core-annular flow with a wave of very small amplitude ($A = 0.000001$ m) is present at the interface. The radii of the core and the pipe are, respectively, given by $R_1 = 0.00372$ m and $R_2 = 0.00476$ m. The wavelength α is chosen (as by Li and Renardy) to be equal to the fastest growing wavelength found from linear stability calculations $\alpha = 0.0116$ m. The dynamic viscosities of the liquids in the core and annulus are, respectively, $\mu_1 = 0.601$ kg/ms and $\mu_2 = 0.001$ kg/ms. The densities are $\rho_1 = 905\,\mathrm{kg/m^3}$ and $\rho_2 = 995$ kg/ms. The interfacial tension is $T = 8.54 \times 10^{-3}\,\mathrm{kg/s^2}$. The pressure gradient in the axial direction is $dp/dz = f = -9172\,\mathrm{kg/m^2\,s^2}$. The velocity of the core liquid at the centreline of the pipe is then $V_0 = 0.166$ m/s (for fully developed smooth core-annular flow). This gives for the Reynolds number of the core liquid $Re_1 = \rho_1 V_0 R_1/\mu_1 = 0.93$ and the Reynolds number of the annular liquid $Re_2 = \rho_2 V_0 (R_2 - R_1)/\mu_2 = 171$. The other dimensionless parameters, as used by Li and Renardy, are $m = \mu_2/\mu_1 = 0.00166$, $a = R_2/R_1 = 1.28$, $\zeta = \rho_2/\rho_1 = 1.1$, $J = T R_1 \rho_1/\mu_1^2 = 0.0795$, and $K = (f + \rho_1 g)/(f + \rho_2 g) = -0.4552$. The result is shown in Figure 1 for $t = 0.0$, 0.4, 0.8, 1.2, 1.6, and 2.0 s. In the calculation the reference system is chosen in such a way that at the start of the calculation the core is at rest and the pipe wall is moving downward. This is done to follow more easily the development of the wave at the interface. However with the wave development the force on the core increases and it starts to move downward. Moreover, the wave moves with respect to the core, although the viscosity ratio between core liquid and annular liquid is rather large. These are the reasons why the wave changes position in the figure. We compared our results with those of Li and Renardy for wave shape, final amplitude and wave speed. Beyond the linear regime the core annular flow evolves into a flow with "bamboo" waves of constant amplitude at the interface. For the wave speed c (made dimensionless with the centreline velocity) we found is $c/V_0 = 0.85$ and for the maximum amplitude (made dimensionless by the core radius) $A_{max}/R_1 = 0.18$. All in good agreement with the results of Li and Renardy.

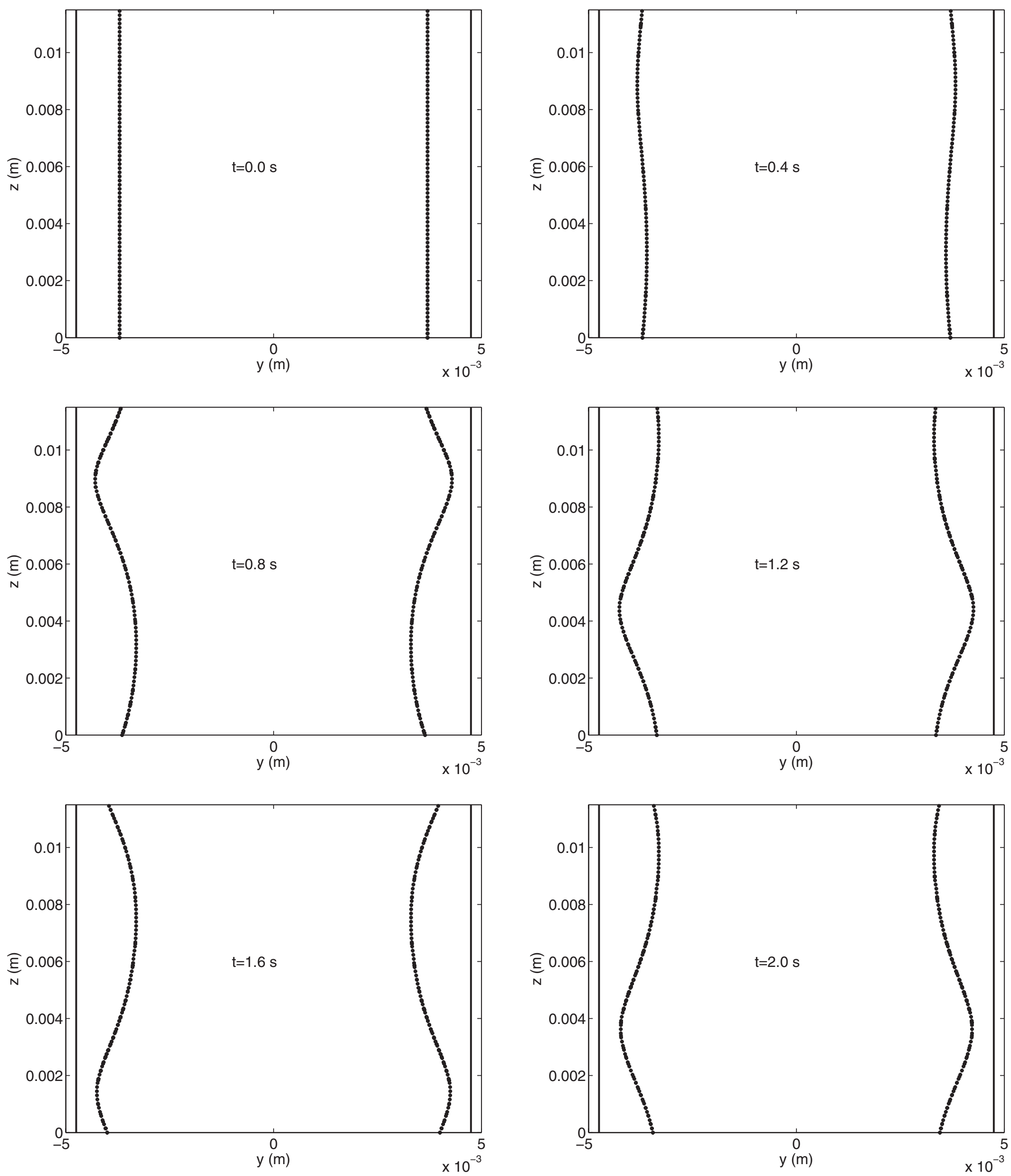

FIG. 1. Cross-section of vertical core-annular flow with buoyancy effect. The straight vertical lines in this figure represent the pipe wall.

B. Core-annular flow without buoyancy effect

Before studying the influence of buoyancy on horizontal core-annular flow we wanted to investigate whether a wave develops at the interface when there is a density difference between core liquid and annular liquid, but gravity is not taken into account ($g = 0$). The parameters are chosen to be the same as in the foregoing case of an upward vertical core-annular flow with buoyancy effect, except the pressure gradient is different. As the liquids are no longer pulled down by gravity, the pressure gradient can be chosen significantly smaller to achieve a net flow in the axial direction of the pipe. It is chosen to be $dp/dz = f = -150\,\mathrm{kg/m^2\,s^2}$. The velocity of the core liquid at the centreline of the pipe is then $V_0 = 0.330$ m/s. In the calculation the reference system is again chosen in such a way that at the start of the calculation the core is at rest and the pipe wall moving downward

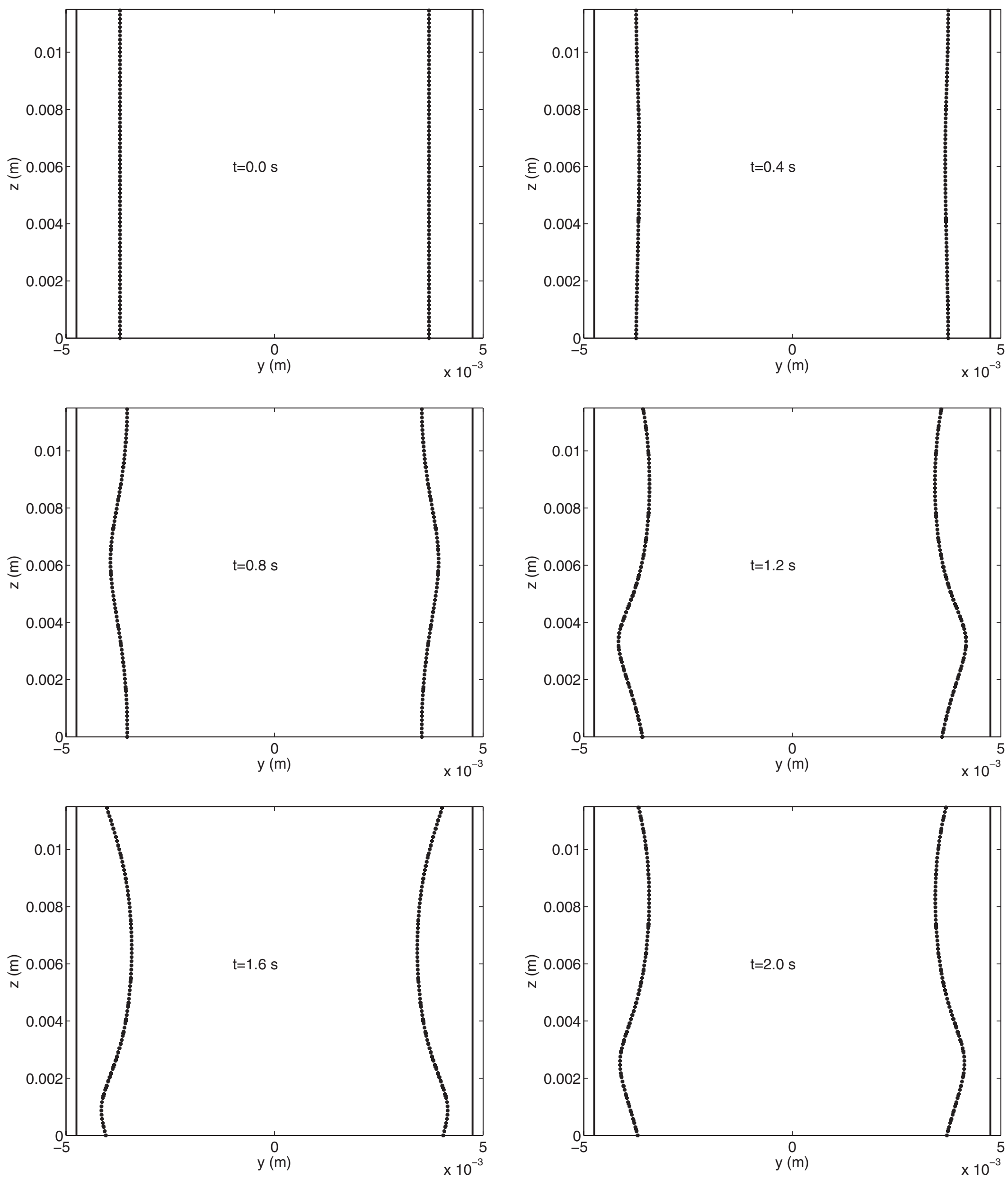

FIG. 2. Core-annular flow without buoyancy effect.

with a velocity of 0.330 m/s. The result is given in Figure 2 for $t = 0.0$, 0.4 , 0.8 , 1.2 , 1.6, and 2.0 s. Again a wave develops as function of time. After a certain period the shape and amplitude do not change anymore. For this case both axisymmetric calculations and 3D calculations were carried out. The results were in agreement. For all the coming cases 3D calculations were performed. We will use the result of this case as initial condition for some of the coming calculations.

C. Horizontal core-annular flow with buoyancy effect

1. Density difference is 20 kg/m^3

Next we considered the flow that we really wanted to investigate: horizontal core-annular flow with a large viscosity ratio and small density ratio, taking into account gravity. We keep all

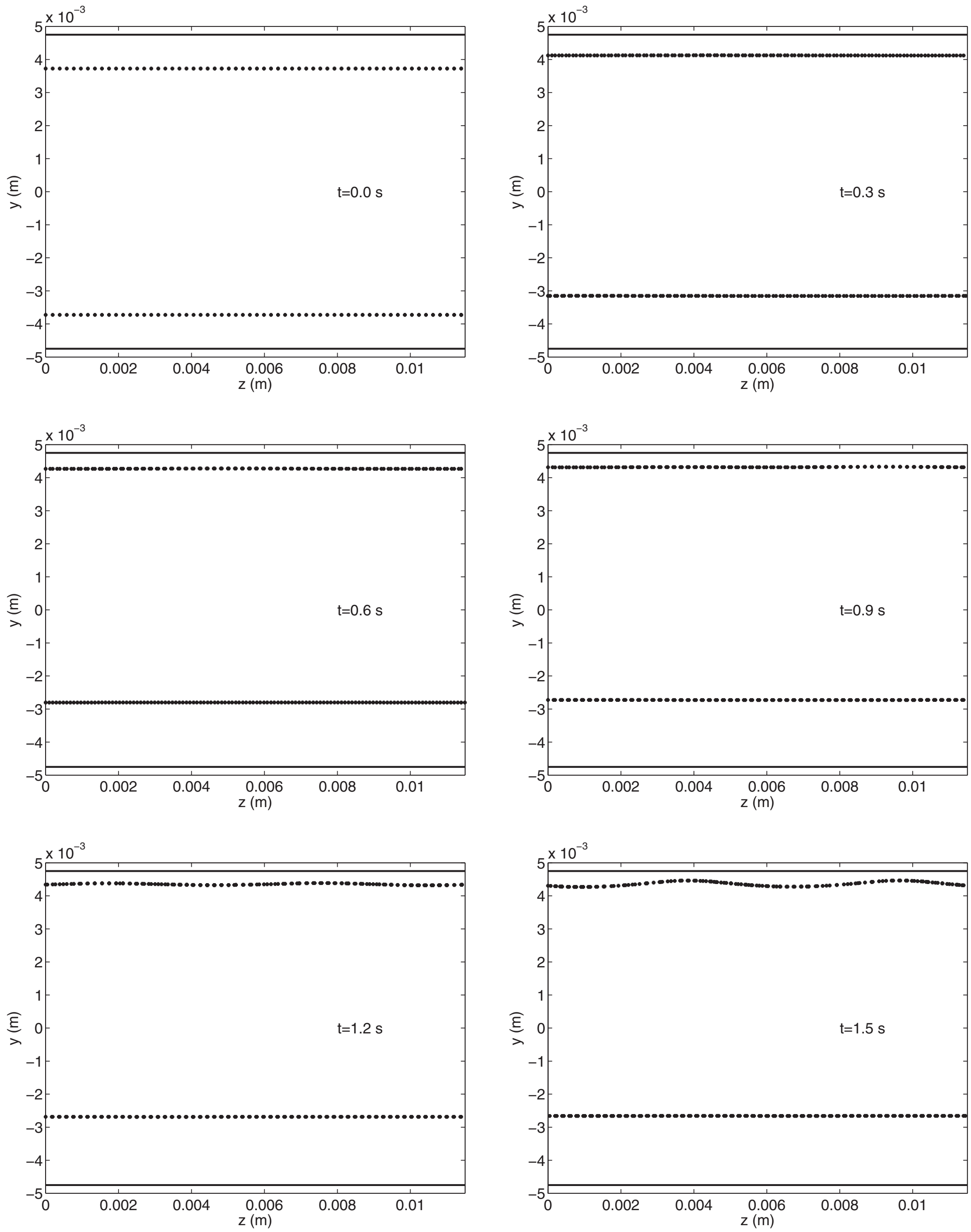

FIG. 3. Horizontal core-annular flow with an initial smooth interface and with buoyancy effect.

the parameters the same as in the foregoing case, except that the density of the core liquid is $\rho_1 = 980\,\mathrm{kg/m^3}$, the density of the annular liquid $\rho_2 = 1000\,\mathrm{kg/m^3}$ and gravity is now switched on in the direction perpendicular to the pipe axis. At the start the reference system is chosen in such a way that the core is at rest and the pipe wall moving to the left with a velocity of 0.330 m/s. The result is shown in Figure 3 for $t = 0.0$, 0.3 , 0.6 , 0.9 , 1.2, and 1.5 s. As can be seen the core moves upward under the influence of buoyancy. However when it arrives close to the pipe wall a wave develops. The wavelength is this time half the length of the wave in the foregoing cases. The core remains free from the pipe wall. The explanation of the levitation force will be given in the following.

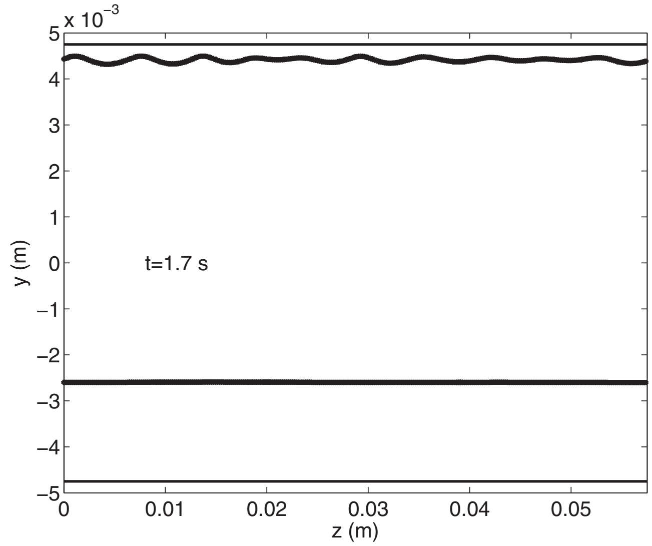

FIG. 4. Horizontal core-annular flow with an initial smooth interface and with buoyancy effect, for a pipe which is five times longer (0.0580 m) than the one used for Figure 3.

We repeated this calculation, but for a pipe which is five times longer (0.0580 m). The result (see Figure 4) is the same as in the preceding case. A wave develops at the interface and the core rises to an eccentric, equilibrium position. The wavelength is the same as in Figure 3. The wave shape and amplitude vary as function of time.

In the following calculation we started with a wave of finite amplitude at the interface. To that purpose we choose the wave shape as found for the case of core-annular flow without buoyancy effect (see Figure 2). All other parameters are the same as in the foregoing case with an initial smooth interface and pipe length of 0.0116 m. The result is shown in Figure 5. In this figure we show the results between t = 0.0 s and t = 1.0 s. Thereafter the wave shape did not change noticeably anymore. The amplitude remains finite. The crest is only slightly sharper on the front than on the back. Again the core rises until it reaches an eccentric equilibrium position. The eccentricity is smaller than in the preceding case (for which a smooth initial interface was used). The wavelength does not change during the upward flow and is twice as large as for the case with an initial smooth interface. We continued the calculation much longer. The total upward momentum of the core is given in Figure 6. It is clear that after about t = 2.5 s the stable position has been reached. The 3D structure of the wavy interface (corresponding to the stable position) and the associated distribution of the reduced pressure (pressure without gravity contribution) are given in Figure 7. This will be discussed in more detail.

We have also computed the axial velocity distribution in the annular layer at the top of the pipe (see Figure 8 for velocity distributions at six axial positions at $t = 0.8$ s). In the chosen reference system the pipe wall is at rest. The wave shape and position at $t = 0.8$ s can be seen in Figure 5. The straight vertical part of the distributions is the velocity of the core liquid which has a viscosity much larger than the viscosity of the annular liquid. At this condition the wave moves relative to the core with a velocity of about 0.04 m/s in the $-z$ direction. So when we subtract in Figure 8 about 0.21 m/s from the axial velocity we choose a reference system according to which the wave is at rest. In this reference system we find a velocity distribution with negative and positive parts. It is negative close to the pipe wall, which is moving in the $-z$ direction, and positive close to the interface. So in that reference system there is a recirculating flow in the trough of the wave. As mentioned already by Bai *et al.*,[6] this eddy (as they call the recirculating flow) plays an important role in the development of the pressure distribution in the annular layer. More details are given by Ooms *et al.*[15] Also contour plots and profiles for the axial velocity in a total cross section of the pipe when the stable position

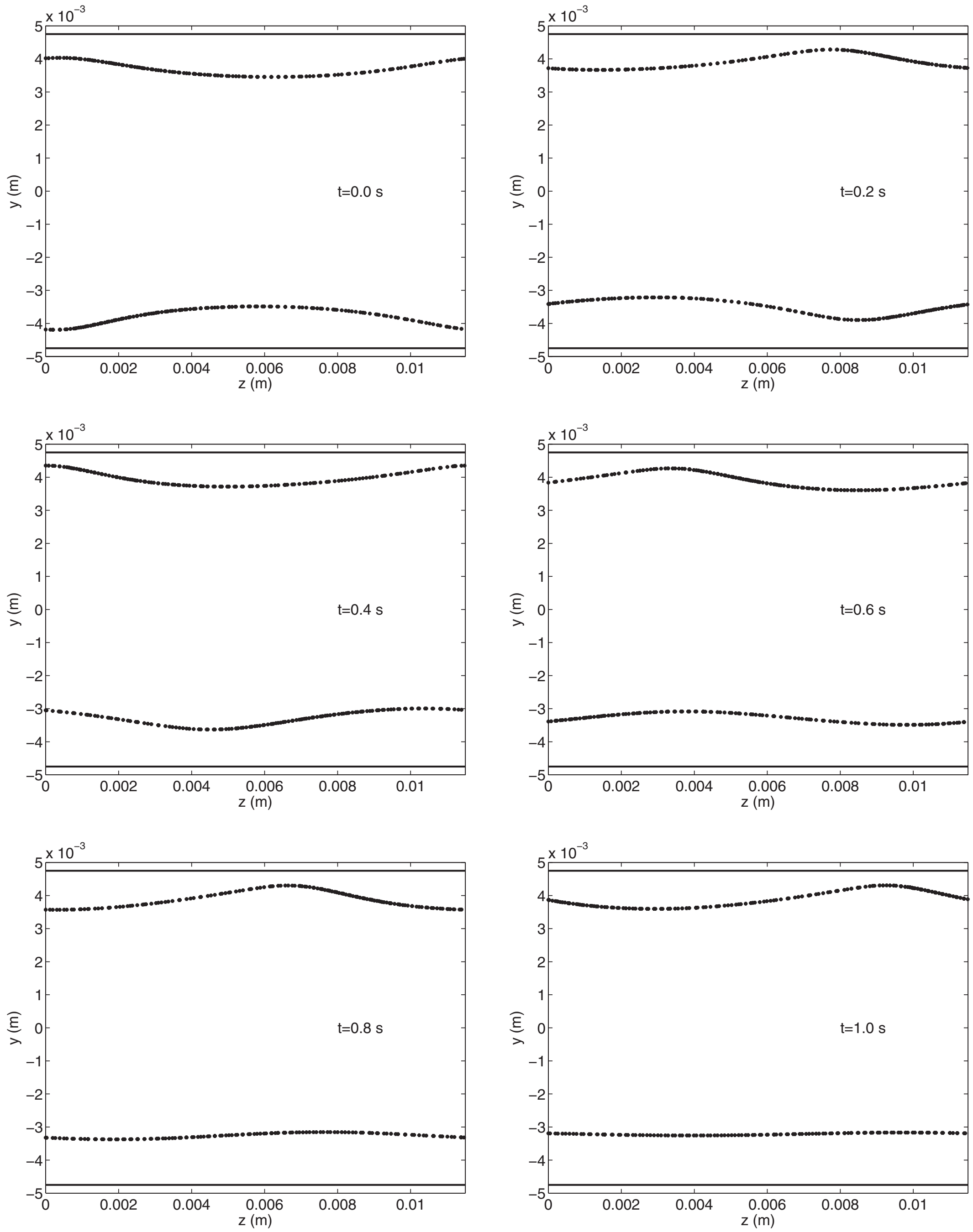

FIG. 5. Horizontal core-annular flow with an initial wave of finite amplitude at the interface and with buoyancy effect.

has been reached, are shown in Figure 9 for a reference system according to which the pipe wall is at rest. As can be seen again, there is a recirculation region in the trough of the wave.

The explanation of the levitation force can be understood from the reduced pressure distribution in the annular layer at the top and the bottom of the pipe (see Figure 10). As can be seen the size of the variations in the pressure distribution remains about the same at the top of the pipe, whereas at the bottom the variations disappear with increasing eccentricity. The reduced pressure at the top can be larger or smaller than at the bottom, leading to downward and upward forces on the core. However, on average the downward force at the front of the wave crest is larger than the upward

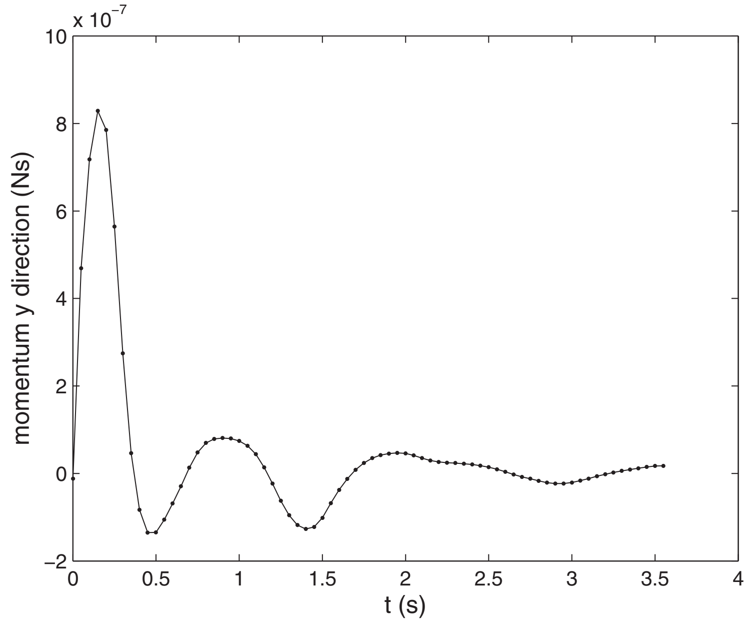

FIG. 6. Total upward momentum of the core as function of time. (The time step in the calculation is very much smaller than the time between two consecutive points shown in the figure.)

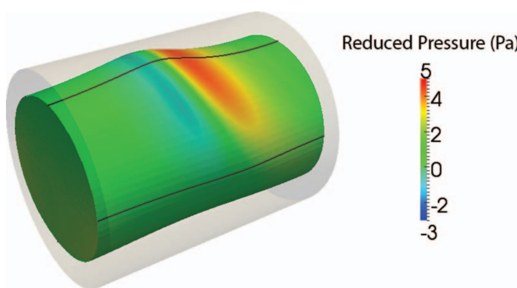

FIG. 7. 3D structure of the wavy interface (corresponding to the stable position) and the associated reduced pressure distribution.The pipe wall is shown as a transparent image.

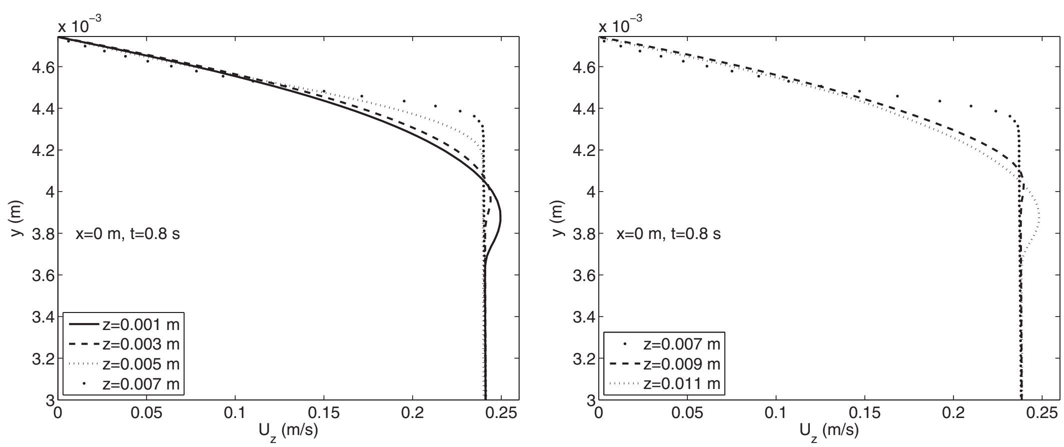

FIG. 8. Axial velocity distribution in the annular layer at the top of the pipe for horizontal core-annular flow with an initial wave of finite amplitude at the interface and with buoyancy effect.

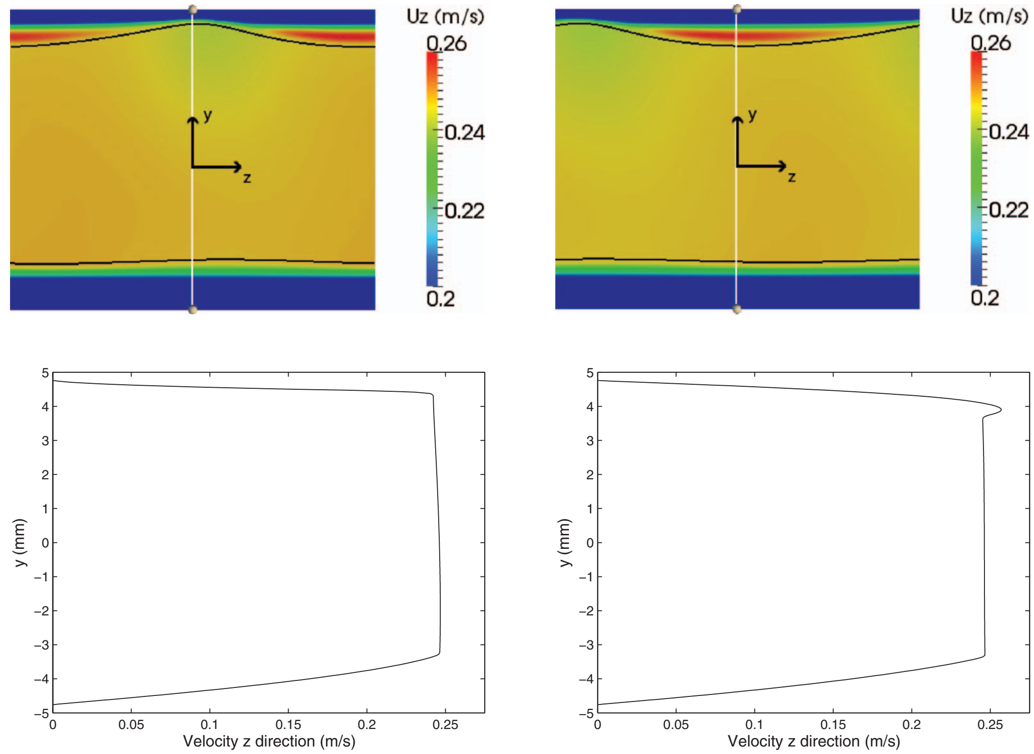

FIG. 9. Contour plot for axial velocity at t = 2.30 s (top, left) and t = 2.35 s (top, right). Axial velocity profile in cross section through the wave top (bottom, left) and velocity profile in a cross section through the trough of the wave (bottom, right).

force at the back. So there is a net downward force that counterbalances the upward buoyancy force when the core is in its equilibrium position.

As mentioned in the Introduction according to Bai *et al.* inertial forces are needed to get levitation. To check this statement we carried out the same calculation as given in Figure 5, but this time without inertial terms in the equations of motion. The result is shown in Figure 11. Indeed inertial forces are essential for core levitation. Without inertial terms the core continues to rise and finally touches the pipe wall. It is important to point out, that this statement holds for high Reynolds numbers for the flow in the annular layer. At low Reynolds numbers this conclusion is not valid, as we will show in Sec. III C 3.

2. Density difference is 40 kg/m³

We performed also a calculation for a horizontal core-annular flow similar to the one shown in Figure 5, but with a larger density difference of $\rho_2 - \rho_1 = 40\,\mathrm{kg/m^3}$, a larger pressure drop $dp/dz = f = -450\,\mathrm{kg/m^2\,s^2}$ and an initial amplitude of 0.0001 m. The other parameters remain the same. The core velocity at initial (concentric) position is now about 1 m/s. The result is given in Figure 12. Again the core rises in the tube until an equilibrium position is reached. However, this time the wave shape continues to change. Several wavelengths are possible.

3. Low Reynolds number

It is interesting to study horizontal core-annular flow with buoyancy effect also at low Reynolds number of the flow in the annular layer. Bai *et al.*[6] state in their paper that at low Reynolds number stationary core-annular flow is not possible as the vertical force on the core is then upward instead of downward. The pressure forces in the annular layer are associated with the form of the waves that they generate. According to them waves also develop at low Reynolds number, but the pressure

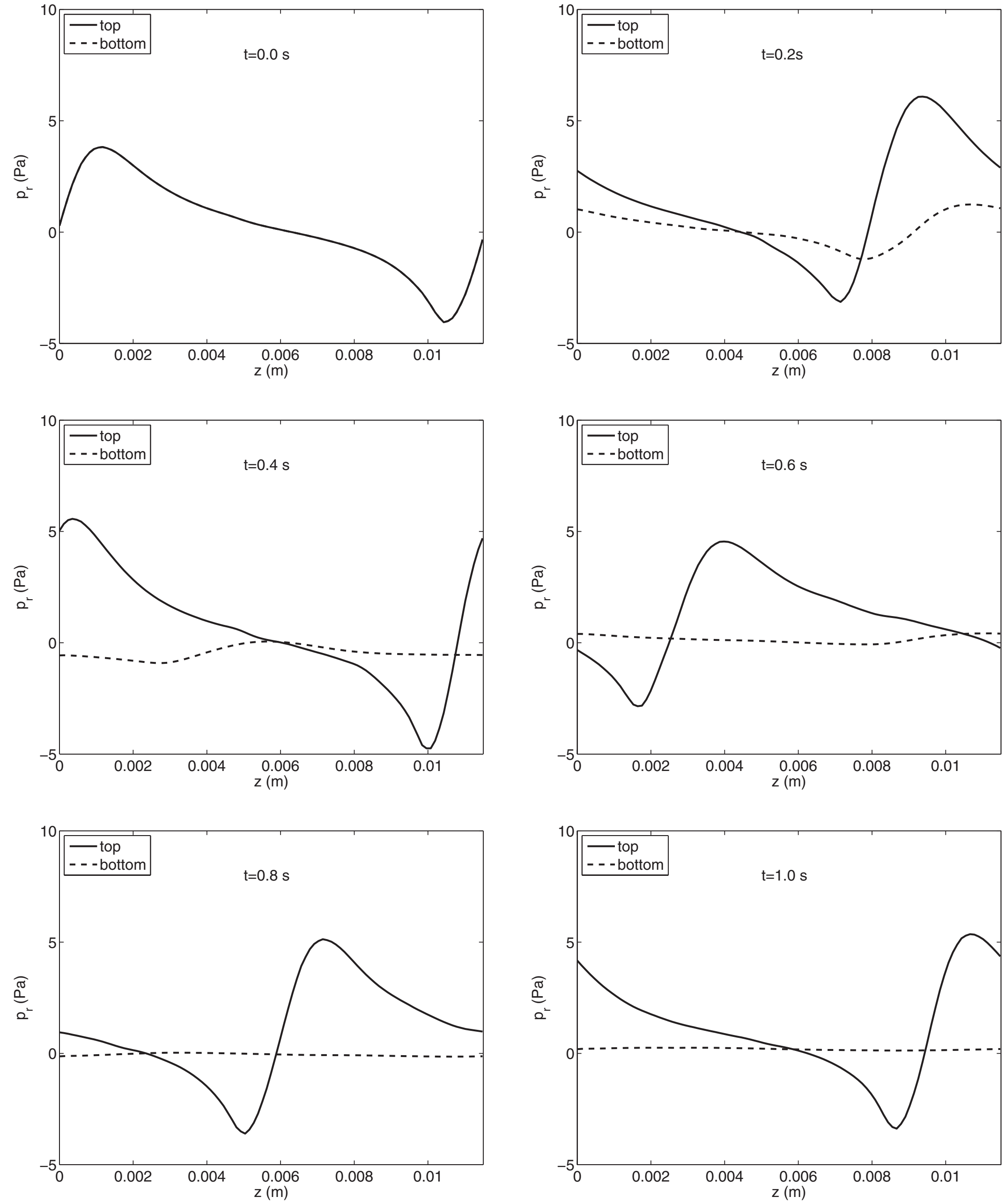

FIG. 10. Reduced pressure distribution for horizontal core-annular flow with an initial wave of finite amplitude at the interface and with buoyancy effect. At t = 0.0 s the reduced pressure distribution at the top and bottom coincide.

forces associated with these waves are negative like those on the reversed slipper bearing which pull the slipper to the wall. Only when the Reynolds number is higher than a certain threshold value, high positive pressures are generated, that cause core levitation.

As mentioned earlier Li and Renardy[8] improved the wave simulation of Bai *et al.* by applying a VOF scheme and by allowing for different densities for the two liquids. They state that the wave shape as calculated by Bai *et al.* is too rounded and smooth compared to experiments, which show an almost symmetrical form of the crest. The crest is only slightly sharper at the front and less sharp at the back. These details are successfully reproduced by Li and Renardy. Moreover, it is important to realize that Bai *et al.* studied the wave development for a concentric vertical core-annular flow

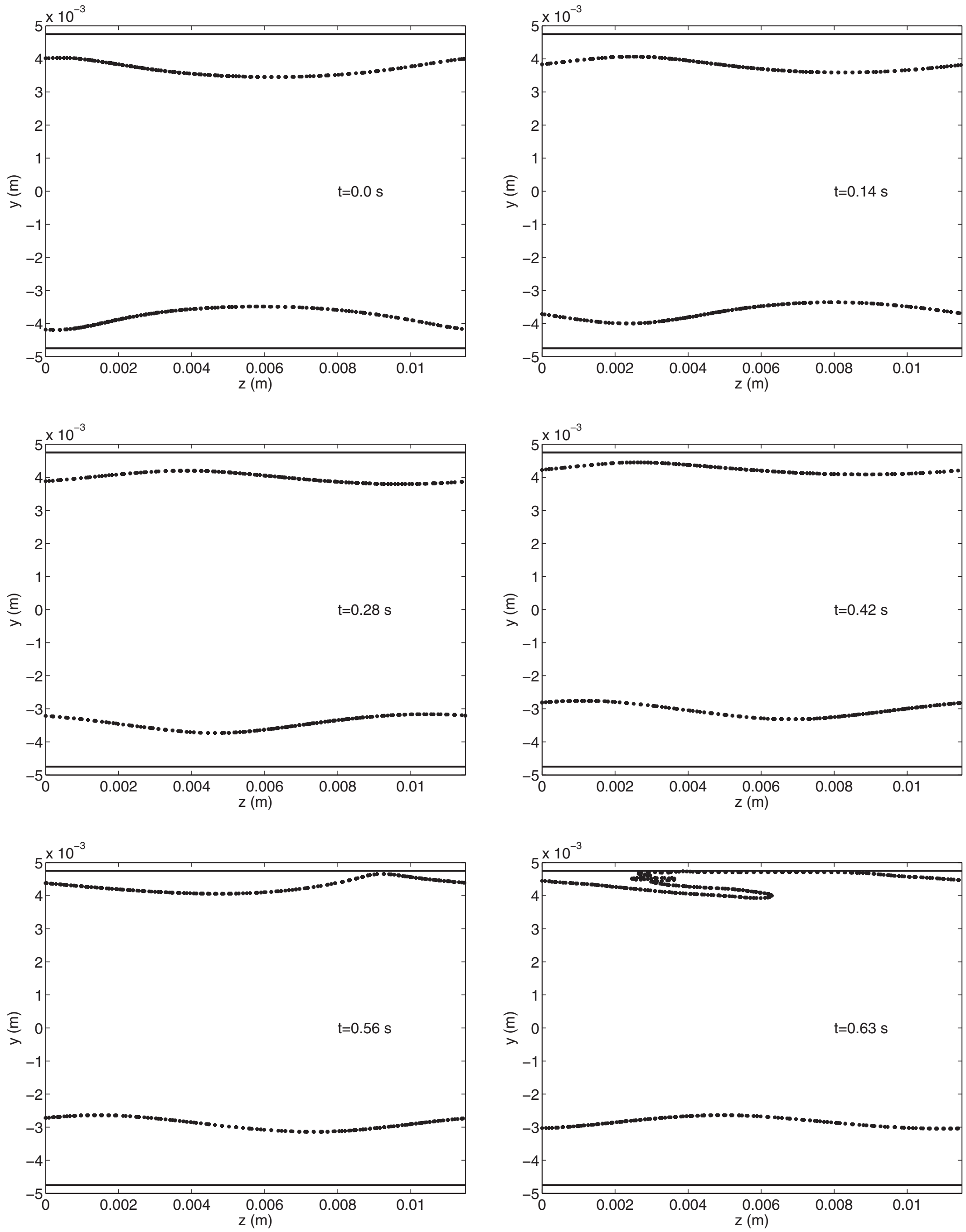

FIG. 11. Horizontal core-annular flow with an initial wave of finite amplitude at the interface and with buoyancy effect, as calculated without inertial terms in the equations of motion.

under the condition that the viscosity of the core is so large, that it moves as a rigid solid which may nevertheless be deformed by pressure forces in the annulus (by applying the Young-Laplace equation).

Like Li and Renardy we apply the VOF method and satisfy the full dynamic interface conditions (continuity of velocities and of normal and tangential stresses). This may lead to different results at low Reynolds number than as found by Bai *et al.* For that reason we studied by our method horizontal core-annular flow with buoyancy effect also at low Reynolds number. This was done by increasing

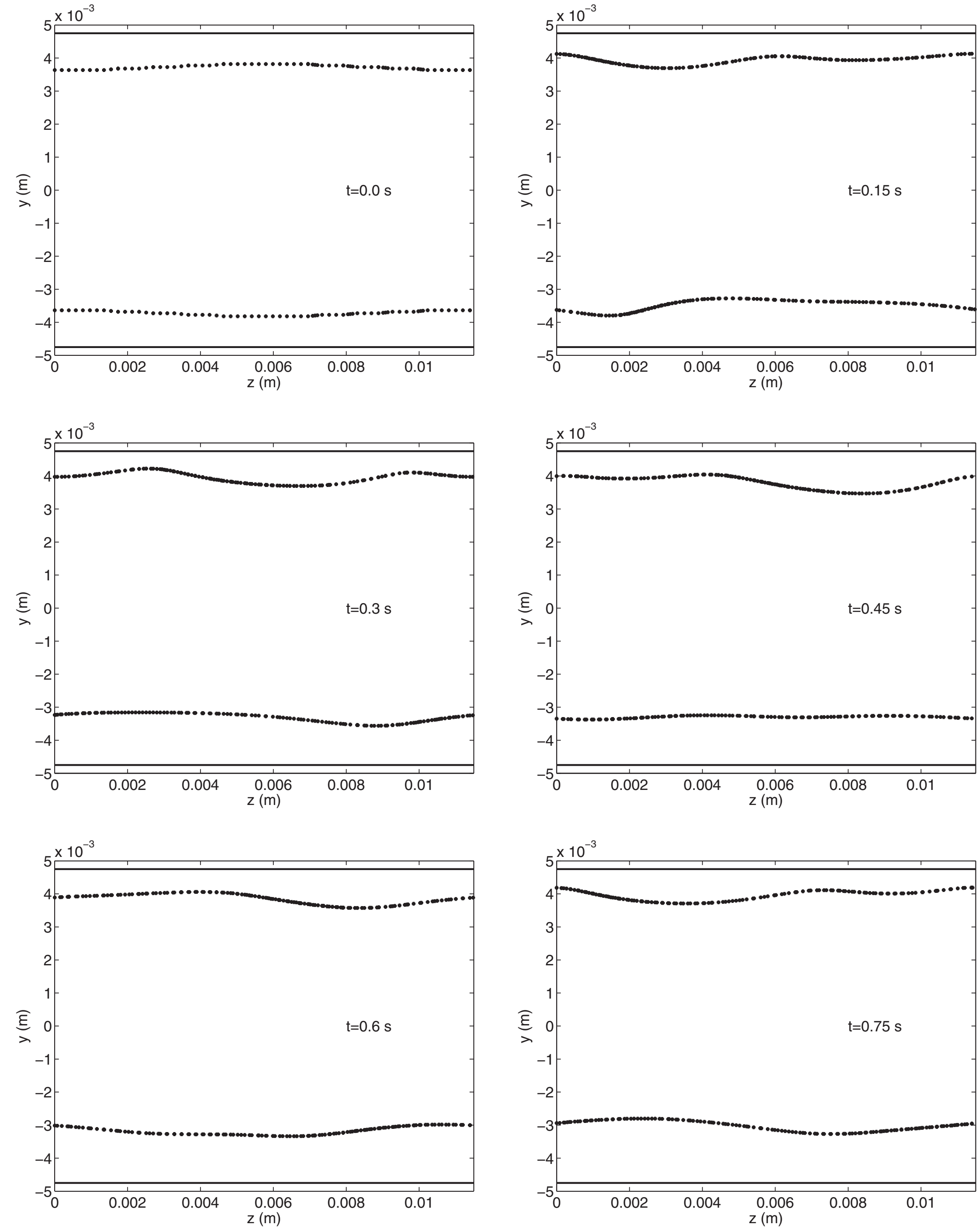

FIG. 12. Horizontal core-annular flow as in Figure 5, but with larger density difference $\rho_2 - \rho_1 = 40\,\mathrm{kg/m^3}$, a larger pressure drop $dp/dz = f = -450\,\mathrm{kg/m^2\,s^2}$ and initial amplitude of 0.0001 m.

the viscosity of the annular liquid by a factor of 40 ($\mu_2 = 0.04\,\mathrm{kg/ms}$). The Reynolds number of the annular layer is then $Re_2 = 8.5$. The core viscosity was chosen to be ($\mu_2 = 2.4\,\mathrm{kg/ms}$). So the ratio of the viscosities is still significant (60). All other parameters were kept the same, apart from the density difference between the core liquid and annular liquid, that was increased in steps from $\rho_2 - \rho_1 = 20\,\mathrm{kg/m^3}$ to $\rho_2 - \rho_1 = 150\,\mathrm{kg/m^3}$. The calculations were done with and without inertial terms. We found that for all cases the core-annular flow becomes eccentric and stable. For the cases without inertial terms levitation is due to viscous (lubrication) forces that create a pressure built-up

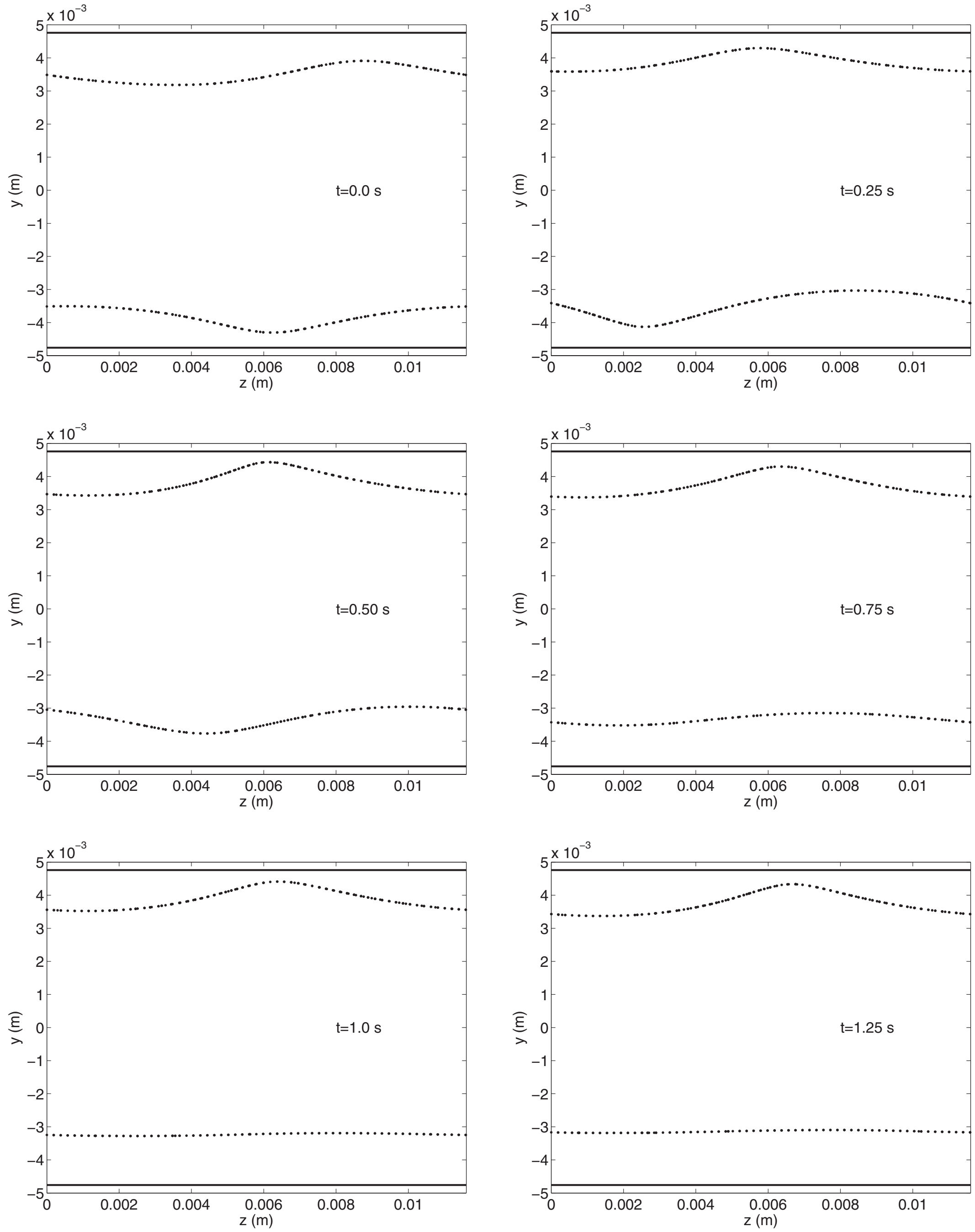

FIG. 13. Cross-section of horizontal core-annular flow with buoyancy effect at low Reynolds number, as calculated without inertial term in the equation of motion.

strong enough to counterbalance the upward buoyancy force. An example is given in Figure 13, which shows the flow development for the case without inertial terms and with a density difference $\rho_2 - \rho_1 = 150\,\text{kg/m}^3$. So, contrary to the statement of Bai *et al.*, we find that stable core-annular flow is possible at low Reynolds number of the flow in the annulus. We have continued this calculation to $t = 2.65$ s. In Figure 14 the reduced pressure distribution at the top and bottom of the pipe is shown at $t = 2.65$ s. The reduced pressure variations are considerable larger than as shown in Figure 10. The reason is that in this case the density difference is much larger.

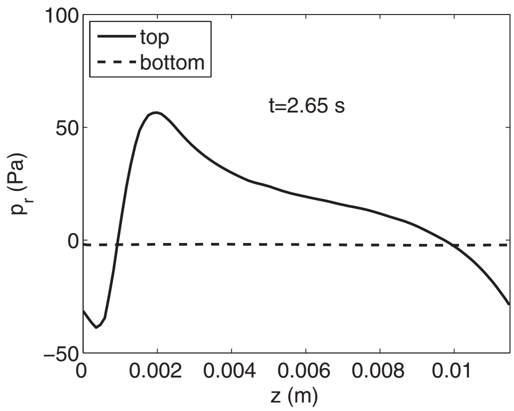

FIG. 14. Reduced pressure distribution for horizontal core-annular flow with buoyancy effect at low Reynolds number at $t = 2.65$ s, as calculated without inertial term in the equation of motion.

IV. CONCLUSION

Horizontal core-annular flow with a large viscosity ratio and small density ratio is a fascinating physical phenomenon. In the case that the density of the core liquid is smaller than the density of the annular liquid, one would expect that the core continues to rise until it touches and fouls the upper part of the pipe. However that does not happen as we know from experiments and now also from numerical simulations. Waves develop at the core-annular interface. When the core rises, the movement of these waves with respect to the pipe wall generates pressure variations in the annular layer that are larger at the top of the pipe than at the bottom. These pressure variations cause downward and upward forces on the core, but the downward forces are larger. When a certain eccentricity has been reached, the net downward force counterbalances the upward buoyancy force on the core. The core has then reached its equilibrium position. We have simulated this process by direct simulation of the flow in the core and the annulus. At high Reynolds numbers of the annular flow levitation is caused by inertial forces and at low Reynolds numbers levitation is due to viscous (lubrication) forces. To the best of our knowledge such simulations have not been done before.

From experiments it is known that stationary, eccentric horizontal core-annular flow with a large viscosity ratio and small density ratio is only possible within a certain region of parameter values (density difference, pressure gradient, viscosity ratio, percentage of annular liquid). More information about this point is given by Oliemans and Ooms.[2] In future work we want to study, whether we can numerically simulate and understand these limitations to core-annular flow. The purpose of this publication is to show that it is possible to simulate numerically horizontal core-annular flow and study the levitation mechanism at low and high Reynolds numbers.

Another point of further study is the influence of the chosen wavelength in the numerical study. For the vertical core-annular flow with buoyancy effect (that we discussed) we choose (like Li and Renardy) the wavelength to be equal to the length of the fastest growing disturbance found in linear stability calculations. We kept this wavelength for the other calculations. Perhaps this choice is not too bad, as we know from nonlinear stability calculations that a range of wavelengths is present in finite amplitude waves. Moreover from experiments it is known that the wavelength is about the same as the pipe diameter (like we assumed).

A point of concern in our simulations is that the bottom layer becomes practically smooth. This does not seem to be consistent with what has been observed in horizontal core-annular flow experiments (see, for example, Sotgia *et al.*,[20] where oil properties are similar to those used in the simulations). Contrary to the simulation results, in eccentric core-annular flow experiments the observed wave amplitude (and wavelength) is larger at the bottom.

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