

# DYNAMIC EQUILIBRIUM ASSIGNMENT CONVERGENCE BY EN-ROUTE FLOW SMOOTHING

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## Abstract

An essential feature in many dynamic traffic assignment (DTA) models used for planning purposes is to compute the (dynamic) equilibrium assignment, where travellers follow user-optimal routes, leading to minimal experienced route travel times. To compute these time-varying route flows in the equilibrium assignment, an iterative procedure is typically required, in which usually either the route flows or the route costs are averaged over successive iterations in order for the assignment to converge. To speed up this convergence, several methods have been proposed and tested. This paper proposes a new method using en-route route flow smoothing to efficiently derive the dynamic equilibrium assignment. At the same time, the newly proposed method aims at solving potential problems due to grid-locks. When grid-locks (are about to) occur, and consequently travel times increase on these road sections, travellers are rerouted (i.e., take a detour) thereby resolving the grid-lock conditions. These higher travel costs – due to detours and penalties for deviating from their initial route – will lead to different pre-trip route choice decisions, such that in the end an equilibrium still can be determined (where the equilibrium situation does not have grid-lock or circular routes). The method is described and tested on the Sioux Falls network. The application section shows that en-route smoothing indeed resolves grid-locks and speeds up the convergence rate. In the application, when applying en-route smoothing, approximately half the number of iterations is needed to find an assignment yielding an equal duality gap. Some explanations are given for this, and suggestions are made to further investigate how the method can be improved.

## 1 Background

Dynamic traffic assignment (DTA) models focus on estimating time-varying network conditions by describing route choice behaviour of travellers on an infrastructure network and the way in which the traffic dynamically flows over the network. We refer to [1] and [2] for an overview of DTA approaches, including a discussion of remaining challenges in DTA research and applications. With respect to the solution approaches, two classes of DTA models are generally distinguished: analytical and simulation-based. Analytical models are directly solved by using well-known optimization techniques, for instance, using mathematical programming or control theory approaches, or by formulating the model as a variational

inequality problem. Examples are models proposed by [3]-[5]. These models are usually limited to relatively small infrastructure networks, since they use solution procedures that do not take advantage of the specific characteristics of the transportation problems. On the other hand, simulation-based models are specifically designed for transportation problems and can handle larger and more realistic networks. These simulation-based DTA models are nowadays widely available and can define the problem either on a microscopic level (e.g., PARAMICS, AIMSUN2), a mesoscopic level (e.g., DYNASMART, INTEGRATION), or on a macroscopic level (e.g., INDY, MARPLE).

Route choice models as part of simulation-based DTA models typically describe optimizing travel behaviour (i.e., minimization of generalized travel costs). In these models, travellers are typically assumed to choose their route from origin to destination at the time of departure and not to switch routes while travelling. This relates to the non-equilibrium pre-trip assignment since, clearly, the chosen route may not be the fastest or most attractive route when the network conditions deviate from the predicted conditions. The difference in network conditions would yield an incentive to change to a different route if the traveller was aware of the prevailing conditions. Instead of allowing en-route route changes, typically an iterative procedure is used that allows travellers to choose a different route in the next iteration, based on actually experienced route travel times and costs. Repeating this process leads to a user-equilibrium assignment in which no traveler can unilaterally switch routes and be better off (Wardrop's equilibrium law). A comparative analysis of convergence methods for the dynamic equilibrium assignment is given by [6]. That this iterative convergence procedure tends to be time-consuming explains the body of research on efficient convergence methods.

Apart from the need for fast convergence, DTA models within an iterative equilibrium framework are prone to common problems with grid-locks. Typically, DTA models merely propagate travellers over the given routes and they cannot deviate from this route during the dynamic network loading process. In models using queuing and spillback, this may lead to grid-locks. Such grid-locks cause significant problems in the model, since the propagation process halts and travel times cannot be computed and no equilibrium can be determined. This problem arises mainly with intermediate route flows that have not converged yet to a user-equilibrium state and therefore some routes in a specific iteration have a too high flow rate. In practice, grid-locks may occur but are generally resolved by travellers turning around or taking detours. This is typical en-route route choice behaviour, which is not modelled in DTA models that usually have only pre-trip route choice.

In this contribution, we propose a new method to compute the dynamic equilibrium which aims at both speeding up convergence and resolving the problem of grid-locks. The proposed method makes use of en-route flow smoothing to spread the route flows during the execution of the dynamic network loading model. This en-route route flow smoothing procedure is explained in the next section. Characteristics of the procedure are investigated on the Sioux Falls network in the application section thereafter. The convergence efficiency (measured here by the duality gap) of the en-route smoothing procedure is compared to that of classical convergence approaches. In Section 4, we draw some conclusions on the convergence procedure and the presented results from the application, and make recommendations for further research.

## 2 En-route flow smoothing method

The proposed en-route smoothing method is based on the hybrid route choice model described by [7] which combines travellers' pre-trip and en-route route decisions. In short, this hybrid route choice model allows for en-route route decisions in the sense that every intersection provides travellers the possibility to decide (with some penalty) to deviate from their pre-trip chosen route when route costs on an alternative route are smaller. The model formulation is described in the ensuing of this section.

Consider a road network  $G = (N, A)$ , where  $N$  is the set of network nodes and  $A$  is the set of network links (arcs). Let the modelling time horizon be given by set  $T$ . Furthermore, a set of origin nodes  $R \subset N$  and destination nodes  $S \subset N$  are given. Travel demand for a given time period  $K \subset T$  is given for each origin-destination (OD) pair  $(r, s) \in RS$  in terms of vehicles per hour and is denoted for each departure time instant  $k$  by  $D^{rs}(k)$ . The origin and destination nodes are assumed to be connected with so-called connector links to the network, which are included in set  $A$ .

First, suppose that within a certain iteration  $i$  (in the iterative equilibrium framework) all travellers have been prescribed a certain route  $p$ . Let  $P^{rs}(k)$  be the set of routes that are relevant for OD pair  $(r, s)$  at departure time  $k$ . The total travel demand for OD pair  $(r, s)$  which is given by  $D^{rs}(k)$  is distributed according to prescribed route rates  $\chi_p^{rs,(i)}(k)$  over the routes  $p \in P^{rs}(k)$  where evidently  $\sum_{p \in P^{rs}(k)} \chi_p^{rs,(i)}(k) = 1$ . The route flows  $f_p^{rs,(i)}(k)$  are then computed as

$$f_p^{rs,(i)}(k) = \chi_p^{rs,(i)}(k) D^{rs}(k). \quad (1)$$

These route flows are model input for the underlying dynamic network loading (DNL) model which simulates the traffic flows over the network and yields actual experienced (route) travel times. In this work, we applied the analytical DNL procedure including dynamic queuing and spillback, explained in [8].

To ensure convergence, we use the method of successive averages (MSA) in which the route traffic flows are averaged over successive iterations. Thus, the prescribed route rates in iteration  $i$ , used in Equation (1) to determine the route flows, are computed as

$$\chi_p^{rs,(i)}(k) = \begin{cases} \hat{\chi}_p^{rs,(i)}(k) & \text{if } i = 1 \\ \chi_p^{rs,(i-1)}(k) + \frac{1}{i} (\hat{\chi}_p^{rs,(i)}(k) - \chi_p^{rs,(i-1)}(k)) & \text{otherwise} \end{cases}, \quad (2)$$

where  $\chi_p^{rs,(i-1)}(k)$  are the route flow rates for the routes  $p \in P^{rs}(k)$  for all OD-pairs  $(r, s) \in RS$  at departure time  $k$  in the previous iteration  $(i-1)$ , while  $\hat{\chi}_p^{rs,(i)}(k)$  are the intermediate route flow rates for the current iteration  $i$ , computed as

$$\hat{\chi}_p^{rs,(i)}(k) = \begin{cases} \exp(-\alpha^{(i)} \tilde{\tau}_p^{rs}(k)) / \sum_{p' \in P^{rs}(k)} \exp(-\alpha^{(i)} \tilde{\tau}_{p'}^{rs}(k)) & \text{if } i = 1 \\ \exp(-\alpha^{(i)} \tau_p^{rs,(i-1)}(k)) / \sum_{p' \in P^{rs}(k)} \exp(-\alpha^{(i)} \tau_{p'}^{rs,(i-1)}(k)) & \text{otherwise} \end{cases} \quad (3)$$

Here,  $\tilde{\tau}_p^{rs}(k)$  is the free flow travel time on route  $p$  from  $r$  to  $s$  departing at  $k$ , and  $\tau_p^{rs,(i-1)}(k)$  is the actual experienced travel time in the previous iteration ( $i-1$ ). Hence, the prescribed route flows are, for the first iteration, based on free flow travel times, while in next iterations these are determined by the actual experienced (route) travel times in the previous iteration. The scale parameter  $\alpha^{(i)}$  in the logit model in Equation (3) determines the pre-trip smoothing in route flows. Note that lower values for  $\alpha^{(i)}$  lead to a more uniformly distributed OD travel demand over the relevant routes  $p \in P^{rs}(k)$ . In case of computing the deterministic dynamic equilibrium assignment, the scale parameter needs to be set sufficiently high.

The actual experienced route travel times used in Equation (3) can be computed as a dynamic sum of consecutive link travel times along the route,

$$\tau_p^{rs,(i)}(k) = \int_0^T \sum_{a \in A} \delta_{ap}^{rs}(k,t) \theta_a^{(i-1)}(t), \quad (4)$$

where  $\delta_{ap}^{rs}(k,t)$  is the dynamic link-route incidence indicator that equals 1 if flow on route  $p$  departing at  $k$  reaches link  $a$  at time instant  $t$ , and zero otherwise, and  $\theta_a^{(i-1)}(t)$  is the link travel time for vehicles entering link  $a$  at time instant  $t$ , in the previous iteration ( $i-1$ ).

Now, even though these routes  $p$  have been prescribed to travellers, in our en-route smoothing procedure, they may switch routes en-route. If current traffic conditions are such that travellers are better off by deviating to another route, they might do so. In the following, travellers with the same prescribed route  $p$  can be seen as belonging to the same class of travellers. Hence, the formulation in this section is actually a multiclass formulation where each class is a distinct route. Let  $q \in Q^{ns}(t)$ , where  $Q^{ns}(t)$  denotes the set of all alternative routes  $q$  from intersection node  $n$  to the destination  $s$  at time instant  $t$ . The fraction of travellers of class  $p$  (i.e., having route  $p$  as pre-trip prescribed route) following route  $q$  is given by the probability that route  $q$  has minimal generalized route costs,

$$\hat{\chi}_{pq}^{ns,(i)}(t) = \Pr\left(c_{pq}^{ns,(i)}(t) \leq c_{pz}^{ns,(i)}(t), \quad \forall z \in Q^{ns}(t)\right), \quad (5)$$

Here  $\hat{\chi}_{pq}^{ns,(i)}(t)$  is the fraction of class  $p$  travellers following route  $q$  at time instant  $t$ , based on the generalized route costs  $c_{pq}^{ns,(i)}(t)$ . These costs,  $c_{pq}^{ns,(i)}(t)$ , are the costs of following route  $q$  (which may (partially) overlap with route  $p$ ) while having pre-trip prescribed route  $p$ . These generalized route costs are computed as

$$c_{pq}^{ns,(i)}(t) = \theta_q^{ns,(i)}(t) + \ell_{pq} \omega^{(i)}, \quad (6)$$

where  $\theta_q^{ns,(i)}(t)$  is the travel time on route  $q$  from  $n$  to  $s$ , and  $(\ell_{pq} \omega^{(i)})$  is the minimum improvement that is required for travellers to be rerouted. This minimum improvement depends on the cost term  $\omega^{(i)}$  and the route deviation proportion  $\ell_{pq} \in [0,1]$ . The cost term,  $\omega^{(i)}$ , states that the new route  $q$  should be at least  $\omega^{(i)}$  faster for travellers to be rerouted to this route. The route deviation proportion is the relative length of route  $q$  which does not coincide with the pre-trip prescribed route  $p$ . Consequently, we assume that the more route  $q$  deviates from the pre-trip route  $p$ , the larger the minimum improvement needs to be in order to switch routes. Note that in case route  $q$  fully overlaps with (the remainder of) route  $p$ , then  $\ell_{pq} = 0$ , and  $c_{pq}^{ns,(i)}(t) = \theta_q^{ns,(i)}(t)$ . On the other hand, if route  $q$  deviates from route  $p$ , then  $\ell_{pq} > 0$ , and route  $q$  should be at least  $(\ell_{pq} \omega^{(i)})$  faster in order for travellers to be rerouted to this route.

In this work, we use the instantaneous travel time to determine en-route route switching, or here called en-route flow smoothing. Since the pre-trip route fractions are based on actual experienced travel times (in the previous iteration), while the en-route flow smoothing is based on instantaneous travel times, the equilibrium state for the pre-trip assignment is not equal to the equilibrium state for the assignment with both pre-trip prescribed routes and en-route flow smoothing. However, the Wardrop user-equilibrium can still be reached by fading out the en-route flow smoothing over the subsequent iterations. Therefore, the cost term  $\omega^{(i)}$  in Equation (6) is iteration-dependent. More specifically, the minimum improvement for en-route rerouting increases as the duality gap decreases, such that en-route route smoothing is less likely with higher convergence.

The instantaneous route travel times  $\theta_q^{ns,(i)}(t)$  can be computed as

$$\theta_q^{ns,(i)}(t) = \sum_{a \in A} \left[ \delta_{aq}^{ns}(t) (\theta_a^{(i)}(t) + \varepsilon_a) \right], \quad (7)$$

where  $\delta_{aq}^{ns}(t)$  is the static link-route incidence indicator (since instantaneous travel times are considered here) that equals 1 if link  $a$  belongs to route  $q$ , and zero otherwise, and the instantaneous link travel times  $\theta_a^{(i)}(t)$  are computed by the DNL model. The error term  $\varepsilon_a \sim N(0, \sigma_a^2)$ , with  $\sigma_a^2 > 0$ , results in a spread of traffic flow among the (instantaneous) fastest routes.

Equations (5)-(7) provide no closed-form expression to determine the class-specific en-route reroute fractions, which necessitates solving these by means of simulation. The assumption of independent and identically Normal distributed link error terms leads to the Probit assignment model [9]-[10]. To limit the required number of independent consecutive draws (to replicate the error distribution), low discrepancy sequences are used. In this work, the Modified Latin Hypercube Sampling (MLHS) method is applied [11].

In sum, the proposed en-route smoothing procedure allows for en-route rerouting when alternative routes provide a minimum improvement (in instantaneous route travel time). This minimally required improvement increases over successive iterations, thus fading out the effect, such that in the end the proposed procedure converges to a (pre-trip) dynamic equilibrium state similar to the dynamic equilibrium assignment computed by classical convergence procedures. The procedure has two advantages. First of all, when grid-locks (are about to) occur, and consequently travel times increase on these road sections, travellers are able to deviate to a different route (i.e., to take a detour) thereby resolving the grid-lock conditions. The DTA model is not halted and travel times can be computed. Note that these higher travel costs (due to detours and penalties for deviating from their initial route) will lead to different pre-trip route choice decisions, such that in the end an equilibrium still can be determined (where the equilibrium situation does not have grid-lock or circular routes). Second of all, it can be reasoned that convergence is faster by starting (within the hybrid route choice model) with en-route flow smoothing and progressively moving towards pre-trip route decisions. Namely, procedures based on pre-trip routing converge over successive iterations, while the proposed en-route smoothing procedure converges both over successive iterations and within a single iteration (during simulation). Both these advantageous characteristics are tested and discussed in the next section.

### 3 Application

To investigate the characteristics of the proposed en-route smoothing procedure, the following application is chosen. The considered network is the Sioux Falls network, shown in Figure 1. The network layout is taken from [12], and originally consists of 76 network links, and 24 nodes. To make the network suitable for dynamic assignment, the (original) origins/destinations are offset from the network nodes, thus creating an additional 48 connector links and 24 origins/destinations. Network characteristics (speed, capacity, number of lanes, etc.) are approximated using satellite images of the real network provided by Google Maps. The static travel demand [12] is distributed over time according to the departure time profile given in Table 1.

**Table 1** Departure time profile for dynamic travel demand

From [h]	Till [h]	Fraction
00:00	00:40	0.10
00:40	01:20	0.15
01:20	02:00	0.25
02:00	02:40	0.25
02:40	03:20	0.15
03:20	04:00	0.10

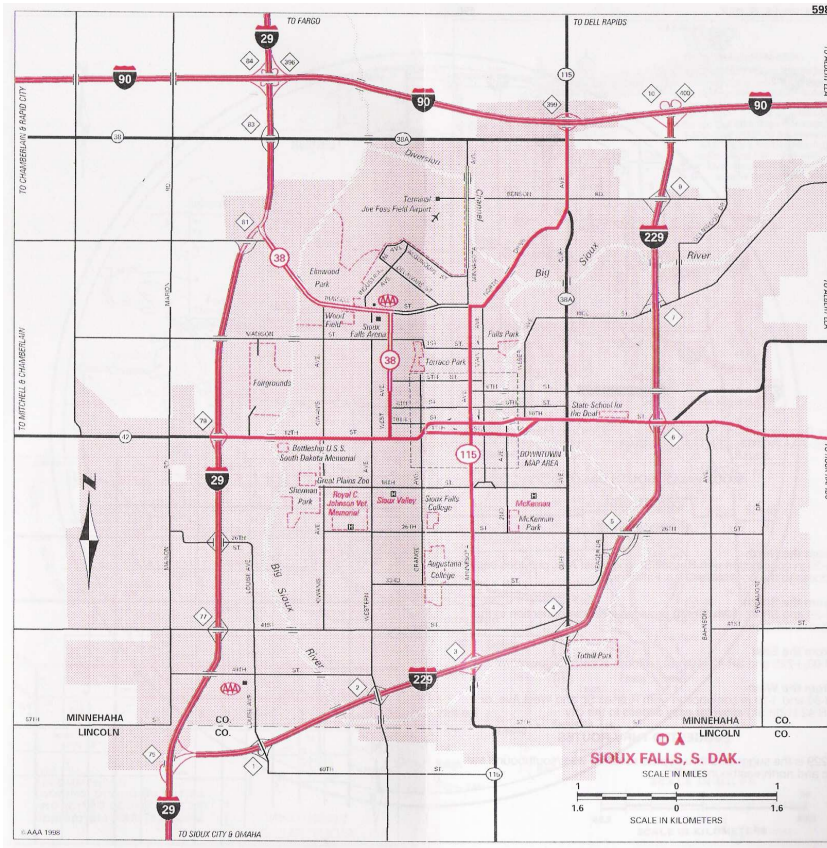


Figure 1 Sioux Falls network (source: [12])

### 3.1 Experimental setup

In the remainder of this section, we compare the proposed en-route smoothing procedure with alternative convergence procedures. The tested procedures are:

- no smoothing:** the scale parameter  $\alpha^{(i)}$  in the logit model in Equation (3) is set sufficiently high, such that the intermediate route flow rates on the fastest route  $p^*$  approximate 1,  $\hat{\chi}_p^{rs,(i)}(k) \approx 1$ , while these rates approximate zero on all other (slower) routes. The minimum improvement for travellers to be rerouted,  $\omega^{(i)}$ , is set sufficiently high, such that en-route rerouting/smoothing does not occur, and consequently pre-trip prescribed routes are followed from origin to destination.
- pre-trip smoothing:** intermediate route flow rates are smoothed over available routes  $p \in P^{rs}(k)$ , where routes with lower travel times (in the previous iteration) receive a higher share of traffic flow. To this end, the scale parameter  $\alpha^{(i)}$  in the logit model in Equation (3) is set to a sufficiently low value. However, to ensure reaching the equilibrium state, pre-trip smoothing decreases over successive iterations, meaning that the scale parameter  $\alpha^{(i)}$  increases with an increasing iteration. Here, we set  $\alpha^{(i)} := 2 \cdot i$ . Note that letting  $\alpha^{(i)}$  increase in this way is made based on some test runs, but was not extensively evaluated. Other ways of decreasing the pre-trip smoothing may lead to different results.

- **en-route smoothing:** route flow rates are updated during simulation (i.e., execution of the DNL model) based on the instantaneous travel times  $\theta_q^{rs,(i)}(t)$ , and a minimum improvement that is required in order to reroute  $\omega^{(i)}$ . The minimum improvement is set sufficiently low, such that travellers are rerouted to currently faster routes (based on instantaneous travel times), thus allowing en-route flow smoothing. To ensure reaching the equilibrium state, en-route smoothing decreases over successive iterations. To this end, the minimum required improvement  $\omega^{(i)}$  increases in subsequent iterations. Here, we set  $\omega^{(i)} := (i-1)/2$  minutes. Or, in words,  $\omega^{(i)} = 0$  in the first iteration, and  $\omega^{(i)}$  increases with half a minute in every subsequent iteration. Once again, letting  $\omega^{(i)}$  increase in this way is made based on some test runs, but was not extensively evaluated. A remark is made in the conclusions section on other ways of decreasing the en-route smoothing.
- **pre-trip and en-route smoothing:** the pre-trip smoothing (smoothing pre-trip intermediate route flow rates) and en-route smoothing (updating route flow rates during simulation) are both applied, each with the same settings as described above.

To compare the performance of these alternative convergence procedures, we use the duality gap as a measure of convergence rate. The duality gap,  $\pi^{(i)}$ , is given by

$$\pi^{(i)} = \frac{\int_0^T \sum_{(r,s) \in RS} \sum_{p \in P^{rs}(k)} [\tau_p^{rs,(i)}(k) f_p^{rs,(i)}(k)] dk - \int_0^T \sum_{(r,s) \in RS} [\tau_{p^*}^{rs,(i)}(k) D^{rs}(k)] dk}{\int_0^T \sum_{(r,s) \in RS} [\tau_{p^*}^{rs,(i)}(k) D^{rs}(k)] dk}. \quad (8)$$

In words, the duality gap computes the relative difference between the total experienced travel time (by all travellers) and the system travel time that would correspond with all travellers having the travel time belonging to the shortest route for their OD-pair, denoted by route  $p^*$ .

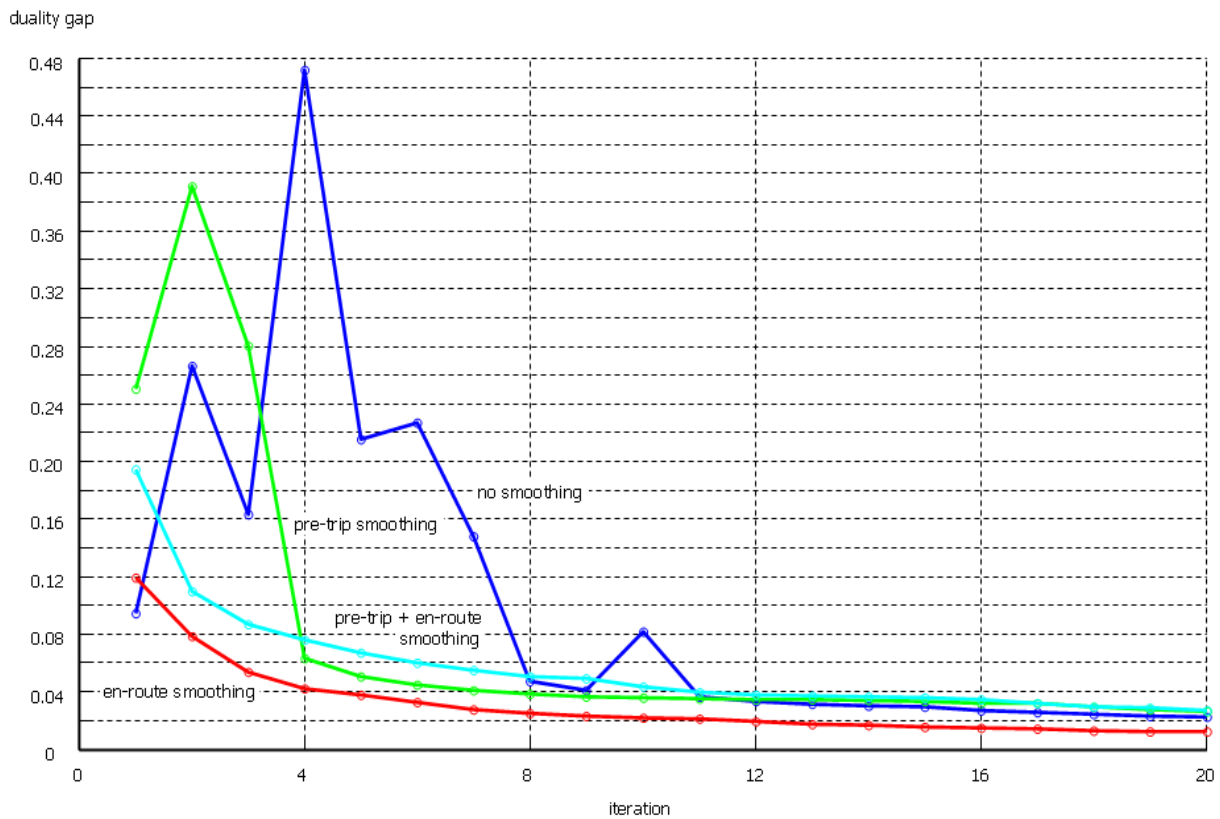
The results are presented and discussed next.

### 3.2 Numerical results

Before presenting the convergence results, we wish to remark here that initially problems with grid-lock occurred within early iterations in case of pre-trip smoothing. A possible explanation that in particular the pre-trip smoothing procedure was found to be prone to grid-lock problems is the following. Within early iterations while applying the pre-trip smoothing procedure, relatively large traffic flows are assigned to a number of the prevailing fastest routes. The number of used routes is higher than in case of no pre-trip smoothing (note that in the ‘no smoothing’ setting the number of used routes per OD-pair never exceeds the iteration number, thus being small in early iterations when grid-lock is most likely to occur). A higher number of routes being used, basically leads to a larger probability that route flows cross in such a way that grid-lock may occur. At the same time, once grid-lock occurs, the pre-trip smoothing procedure does not allow for rerouting. Hence, the grid-lock cannot be resolved.



The grid-lock problems in the pre-trip smoothing settings were solved by ensuring a minimum traffic flow (even when the links downstream of the node were fully occupied). Thereby, the propagation process could continue and the DTA model was capable of computing travel times, and new (intermediate) route flow rates. This minimum traffic flow was set as  $0.05/i$  of the upstream demand. Note that this ad hoc solution may solve the problem of grid-lock, yet yields underestimated route travel times since travel times are not corrected for severe delays due to grid-lock conditions. We wish to emphasize here that the proposed en-route smoothing procedure does yield correct route travel times, as the grid-lock conditions are avoided or solved in a coherent way by allowing travellers to take a detour. That this ad hoc solution in the pre-trip smoothing procedure leads to underestimated route travel times and hence evidently has a negative impact on the rate with which the assignment converges to the dynamic user-equilibrium is shown next.



**Figure 2** Duality gap for various smoothing procedures: no smoothing = blue graph, only pre-trip smoothing = green graph, only en-route smoothing = red graph, and both pre-trip and en-route smoothing = cyan graph

The convergence speed of the various procedures, as measured by the evolution of the duality gap (given by Equation (8)) over successive iterations, is plotted in Figure 2. Note that the computed duality gap in the first few iterations in the pre-trip smoothing case are incorrect in the sense that the actual experienced travel times are underestimated due to the ad hoc solution explained in the previous paragraph to solve the grid-lock problems. From the results in Figure 2, it can be seen that the pre-trip smoothing has no observable benefit over no smoothing (while having the previously discussed problems). This while in case of applying

the en-route smoothing procedure, approximately half of the iterations are needed to find an assignment with an equal duality gap. We should mention here that, in principal, the computation time of a single iteration while applying the en-route smoothing procedure increases, due to computing new route flow rates during simulation. However, two processes are at work here. On the one hand, computation time in early iterations increase by approximately 30-40 percent as route flow rates are relatively often updated. On the other hand, the computation time of later iterations is approximately equal or even lower. This is due to route flow rates being less often updated (as the minimum improvement to reroute increases), and less congested network conditions which leads to faster computation due to the way in which the DNL model is implemented. Thus the overall computation time of the various convergence procedures is comparable. Finally, it can be seen that combining the pre-trip and en-route smoothing procedures (where the ad hoc solution used in the pre-trip smoothing method is not required and thus route travel times and the duality gap are computed correctly) does not benefit the convergence. Apparently, these procedures are not compatible in the sense that, given the current implementation and application scenario, the advantages of pre-trip smoothing and those of en-route smoothing level each other out when applied simultaneously. The reason for this is somewhat unclear and requires further investigation. This however goes beyond the scope of this paper and is hence considered future research.

## 4 Conclusions

This contribution proposes a new procedure to compute the dynamic equilibrium assignment, which relies on en-route rerouting, here also called en-route flow smoothing. The proposed procedure aims at speeding up convergence within an iterative equilibrium framework (using, e.g., MSA), and at the same time solving common problems with grid-locks in a coherent way. The theory and mathematical formulation of the procedure are explained, and the characteristics regarding convergence speed and grid-locks are tested against other convergence procedures. Based on the presented results, the following conclusions can be drawn.

First of all, the en-route smoothing procedure allows the occurrence of grid-locks to be avoided or solved. In most DTA models, travellers are merely propagated over the given routes and they cannot deviate from this route during the loading process. In models using queuing and spillback, this may lead to grid-locks. Such grid-locks cause significant problems in the model (as also in the application presented in this paper), since the propagation process halts and travel times cannot be computed and no equilibrium can be determined. Our proposed en-route flow smoothing method allows for en-route rerouting, such that when grid-lock occurs (or is about to occur), travel times on these road sections increase, and travellers are rerouted (i.e., take a detour) thereby resolving the grid-lock conditions. These higher travel costs (due to detours and penalties for deviating from their initial route) will lead to different pre-trip route choice decisions, such that in the end an equilibrium still can be determined (where the equilibrium situation does not have grid-lock or circular routes).

Second of all, the en-route smoothing procedure enables route flow smoothing during simulation (i.e., during the execution of the DNL model) which allows for faster convergence to the dynamic

equilibrium assignment. In the presented application, when applying en-route smoothing, approximately half the number of iterations is needed to find an assignment yielding an equal duality gap. Herein, the en-route smoothing procedure helps in early iterations to compute route travel times which prove to be closer to the travel times under equilibrium conditions, thereby speeding up the convergence. Since the procedure is (currently) based on instantaneous route travel times, the en-route smoothing is faded out over subsequent iterations to ensure that the iterative assignment correctly converges to a (pre-trip) dynamic Wardrop user-equilibrium, similar to that computed by classical convergence procedures. This has been shown in the application section testing the proposed method against various other convergence methods on the Sioux Falls network.

Finally, based on the current research findings, further research is recommended on investigating the type of network conditions under which applying the en-route smoothing method has the intended benefits, and on testing how the en-route smoothing procedure works under various settings. A potential approach for the latter, is thought to be setting the minimum improvement for en-route rerouting as route specific as a function of the difference between the experienced travel time on this route (in the previous iteration) and the travel time on the fastest route (in the previous iteration) between the same origin and destination. This way, travellers following a route which proved to have a relative high travel time (in the previous iteration) are more prone to being rerouted, while travellers following a route with a travel time similar to that of the fastest route (in the previous iteration) are more likely to remain on their pre-trip prescribed route.

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