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COMPUTATIONAL MODELLING OF TRAFFIC FLOWS ON CONTROLLED ROADS

Alexey B. Kiselev, AnastasiaV. Kokoreva, Valeriy F. Nikitin and Nickolay N. Smirnov

Mechanics and Mathematics Faculty of Moscow M.V. Lomonosov State University
Leninskie Gory, MSU, Main Bldg, Moscow 119992, Russia
e-mail: akis@mech.math.msu.su

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Abstract. *A model of transient single-lane traffic flows is proposed, taking into account the basic components used to control road traffic (traffic lights and "sleeping policemen"), which is radically different from those traditionally considered in continuum mechanics. The model takes into account the main property of traffic flows, namely, the property of self-organisation, and enables the conditions required to ensure maximum carring capacity to be describned correctly both qualitatively and quantitatively, as well as the occurrence and evolution of "travelling traffic jams" on road, as well as the effect of road traffic control units. Unlike the first mathematical models, which describe traffic flows[1-6], and the corresponding research, generalized in the monograph[7], a model of fraffic flows was proposed in[8, 9] which contains not only a continuity equation but also a differential equation of the motion, and takes into account the limits on speed acceleration of the traffic flow, the technical charactristics of the vehicles and the response of a driver to a change in the road conditions. According to this model, the problem of traffic flow has no direct hydrodynamic analogy. Below, developing this model, we take into account additional road conditions, namely, the different forward visibility distances for a driver, and the presence on the road of traffic lights and "sleeping policemen".*

1 THE MODEL OF THE TRAFFIC FLOW ALONG AN ARTERIAL ROAD

Consider the unidirectional flow of vehicles along a sngle-lane road. An intersection with other roads and the presence of traffi lights will be taken into account by appropriate boundary conditions. We will introduce an Euler system of coordinates x along the arterial road in the direction of the traffic flow and the time t.

We will define the mean flow density $r(x,t)$ as the ratio of area of the traffic lane, occupied by the vehicles to the whole section of the traffic lane considered

$$r = \frac{S_{tr}}{S} = \frac{hnl}{hL} = \frac{nl}{L}$$

where h is the width of the trafic lane, L is the length of the controlled sectio o the road, l is the mean length of the vehicle, and n is the number of vehicles in the controlled section. Thus, the flow density inroduced is a dimensionless quantity, which varies from zero to unity.

We wil introduce the flow velocity $v(x,t)$, which can vary from zero to $v^0_{\max}$ - the maximum allowed speed on the arterial road outside the systems for controlling traffic. It follows from the definitions that the maximum density $r = 1$ corresponds to the situation when the vehicles are practically up against one another (“bumper to bumper”). In this case it is natural to take $v = 0$, i.e. there is a “traffic jam” on the road.

By calling the quantity

$$m = \int_0^L r dx$$

the “mass”, concentrated n a section of length L, we can white the change n mass on th arterial road. For a continuous flow of vehicles we will have the followin equation of continuity

$$\partial r / \partial t + \partial (rv) / \partial x = 0 \qquad (1.1)$$

We will write the equation of the dynamics of traffic flow, more exactly, the equation of the change in the mode of motion. Th mode of motion of vehicles on the road is defined by the following main factors: the response of a driver to a change in the road conditions and the actions which he takes, the response of the traffic to the driver’s action, and the technical characteristics of the vehicles. In developing the model of traffic dynamics we made the following main assumptions.

1. In view of the fact that it is the average traffic that is being described, and not the motion of each vehicle separately, the model operates with the average characteristics o the vehicles, and ignores any individual differences in power, inertia, brakn distances, etc.

2. It is assumed that,on average, te response of all drivers to a change in the road conditions is adequate, namely,it s assumed that, on seeing a re trafic light or a speed limitation sign, for eample, that there is a “sleeping policeman” ahead, or a pile-up of vehicles in front, thedriver

shows down to a complete stop or to a permissible speed, and does not cntinue to accelerate andsubsequently have to use emergency braking.

3. It is assumed that all drivers obey that taffic rules, in particular, they do not exceed the maximum speed permitted on the road, and maintain a safe distance between the vehicles, depending on the speed.

The equation of the change in speed can then be written in the form

$$\frac{dv}{dt} = a; \qquad a = \max\{-a^-, \min\{a^+, a'\}\}$$

$$a' = s_0 a_r + (1 - s_0)\int_0^Y w(y) a_r(t, x + y)dy + \frac{V(r) - v}{t}, \qquad a_r = -\frac{k^2}{r}\frac{\partial r}{\partial x} \tag{1.2}$$

Here a is the accelerationof the traffic flow, a^+ is the maximum possible acceleration, a^- is the emergency braking deceleration, and the quantities a^+ and a^- are positive and ar defined by the technical characteristics of each vehicle. The parameter $k > 0$ is, as has been shown previously[8, 9], the propagation velocity of small perturbations ("the velocity of sound") in traffic flow. The parameter t has the meaning of the delay time due to the finitrness of the speed of the driver's reaction to a change in the road conditions and the technical characteristics of his vehicle. The parameter corresponds to tendency of the driver to maintain a speed corresponding to the maximum safe speed $V(r)$ for the flow density r [8, 9]

$$V(r) = \{-k \ln r, v < v^0_{\max}; v^0_{\max}, v \geq v^0_{\max}\}$$

The speed $V(r)$ is determined from the condition for the car speed v to depend on the flow density of r for the conditions of a smple wave, while occurs when the flow starts to spread out from the point where $r_0 = 1$ and $v = 0$, takn into account the limitation on the maximum permissible speed $(v \leq v^0_{\max})$. The value of the parameter t may be different, depending on whether it is necessary to decelerate or accelerate in oder to reach the maximum safe speed $V(r)$, namely

$$t = \{t^+, V(r) < v; t^- V(r) \geq v\}$$

The remaining parameters in formulae (1.2) have the following meaning: $Y = in\{Y_0, L - x\}$ is the characteristic visibility along the flow, which depends on the weather conditions, $w(y)$ is the "weight" of the state of flow in front of the vehicle for takin a decision on whether to change the type of driving, which can be defined, foe example, as follows:

$$w(y) = \frac{w_0(y)}{\int_0^Y w_0(y)dy}, \qquad w_0 = \{1, 0 \leq y \leq Y_0; 0, y < 0, y > Y_0\}$$

and s_0 is a dimensionless parameter $(0 \le s_0 \le 1)$, characterizing the "weight" of the local situation compared with the situatio at a certain distance in frony of the vehicle.

Hence, in the expression for the acceleration o the traffic flow (1.2) the first term corresponds to the effect of local situation, the second term corresponds to the effect of the situation in front at a distance less than or equal to the characteristic visibility Y, while the third corresponds to the adjustment in the speed of the vehicle to the maximum safe one for the actual flow density r.

An estimate of the value of the propagation velocity of small perturbations k was made previously in[8, 9] starting from the following considerations. Suppose, starting the motion from a state of rest $(v = 0,\ r = 1)$ and accelerating to a velocity $v^0_{\max}$, the low reaches a maximum permissible density, r_*, which guarantees that the motion is safe.We mean by safe density that for which th distance between the vehicles is no less than the braking distance $X(v)$. Then,

$$r_* = (1 + X(v^0_{\max})/l)^{-1}, \qquad k = v^0_{\max} \ln(1 + X(v^0_{\max})/l)$$

When $v^0_{\max} = 80 km/h$ the braking istance of VAZ type car is 45 m, whih, for a mean length of the car (taking into account the minimum distance between stopped cars) $l = 5m$ gives a propagation velocity of weak perturbations $k = 35 km/h$. For such a value of $v^0_{\max}$ the maximum possible safe flow density is $r_* = 0.1$. The maximm accelerations for cars of this class are $a^+ = 1.63 m/s^2$ and $a^- = 5.5 m/s^2$.

The "velocity of sound" k, estimated in this way, agrees well with experimental data[3, 4].

Hence, to describe the dynamics of traffic flow along a single-lane highway, from Eqs (1.1) and (1.2) we obtain a system of two-quasi-linear partial differential equations in divergent form

$$\frac{\partial r}{\partial t} + \frac{\partial (rv)}{\partial x} = 0, \qquad \frac{\partial (rv)}{\partial t} + \frac{\partial (rv^2)}{\partial x} = ra$$

The acceleration a is given by the last three formulae of (1.2).

We will formulate the boundary conditions at the ends of the part of the arterial road $0 \le x \le L$. Two versions of the boundary conditions are possible aththe beginning of the flow where $x = 0$:

1. when there ia no "jam", the flow density and the maximum safe speed for the given density is specified:

$$r(0,t) = r_0, \quad v(0,t) = V(r_0)$$

2. under conditions of a travelling or fixed "jam", adjoining the entrance part of the highway $x = 0$, we impose the condition that the density gradient is equal to zero, while the speed is equal to the maximum safe speed for the given density:

$$\frac{\partial r}{\partial x}|_{x=0} = 0, \qquad v(0,t) = V(r)$$

The presence or absence of a “travelling jam”, close to the left boundary of the calculated region $(x = 0)$, is determined after calculating the next time step according to the following criterion: if

$$\frac{\partial r}{\partial x}|_{x=0} > 0 \quad \text{and} \quad r > r_0$$

then there is a “travelling jam”.

At the exit of the flow, when $x = L$, we impose the “free exit” condition

$$\frac{\partial r}{\partial x} = 0, \qquad \frac{\partial v}{\partial x} = 0$$

We will take as the initial conditions the fact on a part of length x_0, measured from the entrance $(x = 0)$, the arterial road is occupied by a flow of vehicles of density r_0, moving at a speed $V(r_{0)}$, and when $x_0 < x \le L$ the road is free of vehicles $(r = 0, v = 0)$.

2 MODELS OF SYSTEMS FOR CONTROLLING ROAD TRAFFIC

We wll consider two versions of the traffic control, characteristic for city roads: traffic lights and so-called “sleeping policemen”.

Traffic lights. The main parameters of the operation of trafic lights are the duration of the signals: the green light t_g, the yellow light t_y and the red light t_r respectively. We propose the following algorithm to model the operation of trafic lights.

1. At the instant of switching from the green light to the yellow light we calculate the distance

$$x_r = (v^0_{\max})^2 /(2a_r)$$

where a_r is the regular braking decelation, which is less than the emergency braking value a^-. Vehicles which are a distance less than x_r from the traffic lights are unable to stop before the traffic lights with a standart braking deceleration of a_r, and hence they cross on the yellow light, which corresponds to the rules of traffic motion.

2. While the yellow light is operating we will assume that the maximum speed is

$$v^l_{\max}(x_l) = v^0_{\max} t_{ys} / t_y$$

where t_{ys} is the time which has elapsed since the yellow light showed. Then x_l is a point which is shifted towards the traffic light in accordance with the relation

$$x_l = L_1 - x_r t_{ys} / t_y$$

where L_1 is the coordinate of the point where the trafic control system is situeted (in this case, the traffic lights). As a resultat of this, at the instant when the red light shows vehicles stop at th frafic lights.

3. At the instant when the red light is switched to a green light the maximum permitted speed of crossing the traffic lights will be $v^0_{\max}$, as at the reaining points of the section of road considered.

"Sleeping policemen". The system of limiting the speed o trafic flow, usually called "sleeping policemen", is modelled by specifying that the maximum speed $v_{\max}$ at the point where the "sleeping policemen" is situated is considerably less than for the main part of the road - $v^o_{\max}$. In this paper we consider the case of two "sleeping policemen" at the distance d from the one to the another, which is the situation most often encountered in practice. The point where the first "sleeping policemen" is situated is $x = L_1$. Then the maximum permitted speed along the section o road considered $0 \le x \le L$ is specified as allows:

$$v_{\max} = \{v_p, x \in [L_1, L_1 + d]; v^0_{\max}, x \in [L_1, L_1 + d]\}$$

where $v_p < v^0_{\max}$, and the parameter v_p (the maximum crossin speed) of a "sleeping policemen" is one of the fundamental parameters of the model.

3 RESULTS OF NUMERICAL CALCULATIONS

The above problems were solved numerically by the TVD method with second order of accuracy[10]. The number of nodes in the calculation grid was 201.

We used the following parameters in the calculations: $L = 1000$ m is the length of the calculated region, $x_0 = 100$ m is the length of the section occupied by the moving trafic at the initial instant of time $t = 0$, $L_1 = 500$ m is the point where the traffic control systems are situated (the traffic lights or the first "sleeping policemen"), $d = 50$ m is the distance between two "sleeping policemen", $r_0 = 0.1 \div 0.5$ is the traffic flow density at the entrance to the calculated region $x = 0$, $v^0_{\max} = 25$ m/s is the maximum speed on the main part of the road, $v_p = 3$ m/s is the maximum speed of crossing a "sleeping policemen", $k = 7.9$ m/s is the propagatio velocity of small perturbations in the traffic flow, $a^+ = 1.5$ m/s^2 is the maximum acceleration of the flow, $a^- = 5$ m/s^2 is the maximum (emergency) braking deceleration of the flow, $a_r = 1.5$ m/s^2 is the standart braking deceleration, $Y_0 = 100$ m is the characteristic forward visibility along the flow, $s_0 = 0.7$ is the "weight" of the local situation, $t^+ = 3.3$ s, $t^- = \infty$ is the time taken to adjust to a safe speed, and $t_g = 40 \div 300$ s, $t_y = 5$ s and $t_r = 30$ s are the durationsof the traffic-light signals.

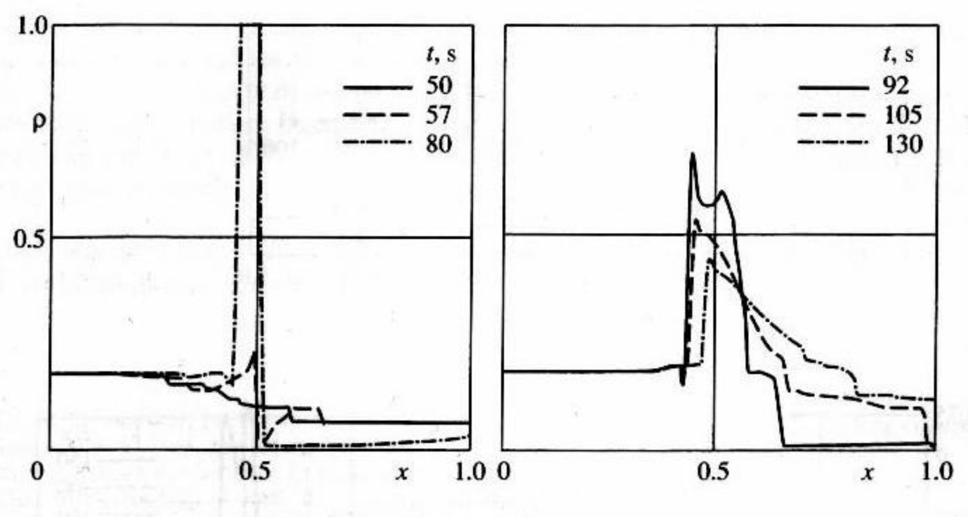

Figure 1

Nence, in the calculations we varied the density of the incoming flow r_0 (and, of course, its speed) and the duration t_g for which the green light operates.

The results of the calculations are presented in figures 1-4 and in the table.

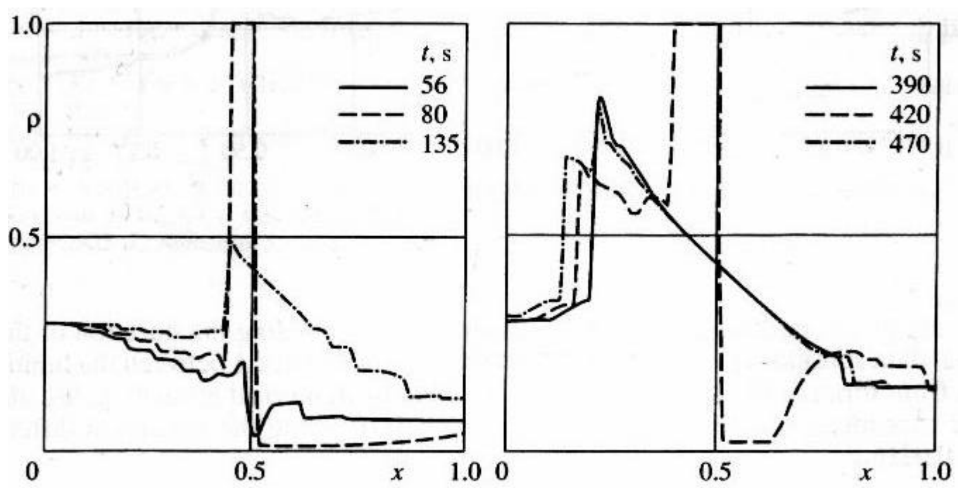

Figure 2

In figures 1 and 2 we show the distributions of the traffic flow density r with respect to the coordinate of the calculated region x at diferent instants of time, indicated on the figures, in case when the flow is controlled by the traffic lights. In this case the working time of the green light $t_g = 50$ s, while the initial traffic flow densities are $r_0 = 0.18$ (figure 1) and $r_0 = 0.3$ (figure 2). The time in figure 1 corresponds to the following operating cycles and signals of the traffic lights: 50 s – the first cycle, end of the green light, 57 s – the first cycle, the yellow light, 80 s – the first cycle, the end of the red light, 92 s – the second cycle, the green light, 105 s – the second cycle, the green light, 130 s – the second cycle, and the end of the red light. In figure 2: 56 s – the first cycle, the yellow light, 86 s – the fifth cycle, the end of the red light, 135 s – the second cycle, the end of the green light, 390 s – the fifth cycle, the of the green light, 426 s – the fifth cycle, the end of the red light, 476 s – the sixth cycle, the end of the green light. As can be seen from these graphs, when $r_0 = 0.18$ a "travelling jam" is not formed (figure 1), while in the case when $r_0 = 0.3$ a travelling jam is formed, which moves in the opposite direction to the trafficflow, the speed of the vehicles in which is reduced considerably.

The results of an investigation of how the limiting initial flow density r_0^*, for which a travellin jam is not formed, deponds on the duraton o the green light t_g, are presented in the table. The remaining initial parameters are fixed. The dependence of r_0^* on t_g is described quite well by the followin formula

$$r_0^* = a\ln(bt_g) \tag{3.1}$$

where a and b are parameters which depend on many factors, including the duration of the red signal t_r. For the initial data considered $a = 0.054$ and $b = 0.87$. Ther diference Δ between the limiting densities r_0^*, calculated rom formula (3.1), and the values obtained by numerical modelling, are also shown in the table. The root mean square deviation is equal to 0.0116, while the maximum difference in the densities $\Delta = 0.0216$.

t_g, s	40	60	80	100	150	200	250	300
r_0^*	0.18	0.21	0.23	0.23	0.23	0.29	0.31	0.31
$\Delta \times 10^3$	11.68	3.56	-0.89	11.16	-6.95	-11.41	-19.36	-9.52

Table

In figures 3 and 4 we show profiles of the traffic flow density r with respect to the coordinate of the calculated region x for the case when the flow is controlled by two "sleeping policemen" at different successive time intervals, indicated on the graphs, for an initial flow density $r_0 = 0.1$ (figure 3) and $r_0 = 0.3$ (figure 4).

The case when $r_0 = 0.1$ corresponds to ree motion of the trafic flow through the zone in which the flow is controlled by the "sleeping policemen", while when $r_0 = 0.3$ a travelling jam is formed, which moves in the opposite directio to the flow. It can be seen that when the flow of vehicles traverses the section with the "sleeping policemen", two sectons of increased density are formed (figures 3 and 4), which when $r_0 < 0.2$ does not impede the free passage of the flow through the obstacles. If $r_0 \geq 0.2$, there is a travelling jam before the "sleeping policemen", the occurrene of which leads to a situatio in which, over the course o time, the density r at the entrance to the calculated region $x = 0$ begins to exceed the initial density r_0 and the motion before the obstacle zone becomes very slow (figure 4).

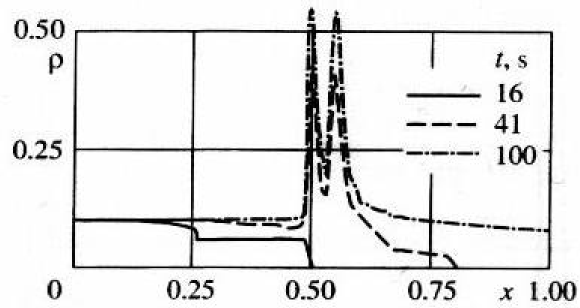

Figure 3

The case when $r_0 = 0.1$ corresponds to free motion of the trafic flow through the zone in which the flow is controlled by the "sleeping policemen", while when $r_0 = 0.3$ a travelling jam is formed, which moves in the opposite direction to the flow. It can be seen that the flow

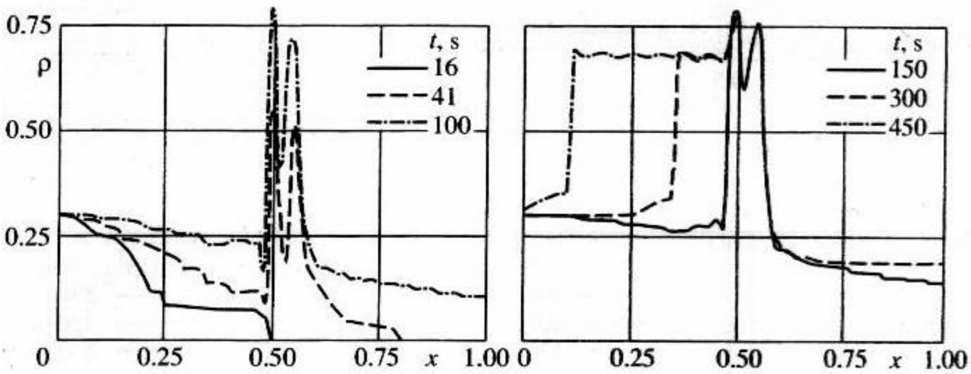

Figure 4

Of vehicles traverses the section with the "sleeping policemen", two sections of increased are formed (figures 3 and 4), which , when $r_0 < 0.2$ does not impede the free passage of the flow through the obstacles. If $r_0 \geq 0.2$, there is a travelling jam before "sleeping policemen", the occurrence of which leads to a situation in which, over the course of tim, the density r at the entrance to the calculated region $x = 0$ begins to exceed the initial density r_0 and the motion before the obstacle zone becomes very slow (figure 4).

4 CONCLSION

Our calculations enable us to conclude that, at low densities of the incomin traffic flow, "sleeping policemen" enable the speed to be controlled in the required way along the sections where they are installed, without interfering with the free motion of the traffic. However, when the density of the incoming traffic flow increases, they produce a "travelling jam", which moves in the opposite direction to the traffic flow, which, in the final analysis, leads to congestion on the road. Control of the traffic using lights one, by choosing the optimum mode of operation (the duration of the signals of different colour), to increase the througout considerably.

The model takes into account the main property of traffic flows, namely, self-organization, and enables the conditions required to ensure maximum throughput, the occurrence and evolution of "travelling jams" on roads, and the effect of the main components of traffic control, to be correctly described both qualitatively and quantitatively.

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