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ABSTRACT

The technology of integer ambiguity resolution enabled precise point positioning (also referred to as PPP-AR) has been proven capable of providing comparable accuracy, efficiency and productivity to long-baseline Real-Time Kinematic positioning (RTK) during the last decade. Commercial PPP-AR services have been provided by different institutions and companies and have been widely used in geodetic missions. However, the usage and research of the PPP-AR mostly concentrated on nonaviation applications, e.g., vehicle navigation, surveying and mapping and monitoring crustal motions. Few of them focused on fixing the ambiguities during an aircraft flight. In this contribution, we implemented the PPP-AR technique for the first time in an airplane flight test to investigate how much the fixed ambiguities could contribute to airplane positioning solutions in the challenging circumstances, including high velocity and severe maneuver. We first looked into the influences of the tropospheric delay on the positioning and ambiguity solutions since the height of the airplane may dramatically change within a narrow time span, and thus a proper constraint of this parameter was crucial for the computation of the tropospheric effects. Then how to fix the ambiguities successfully and reliably in the

challenging circumstances was discussed. Finally, the airplane data was processed in 15 s and 1 s interval with ambiguity float and fixed solution under different configurations to illustrate in which condition and to what extent the fixed ambiguities can improve the airplane positioning accuracy.

Keywords: GNSS; PPP; PPP-AR; Integer ambiguity resolution; Airplane navigation

INTRODUCTION

Global Navigation Satellite System (GNSS) has provided an unprecedented high accuracy, flexibility and tremendous contribution to navigation, timing and scientific issues related to precise positioning on Earth's surface (Teunissen and Kleusberg 2012). As one of its important applications, precise point positioning (PPP) uses undifferenced pseudo-range and carrier phase observations along with precise satellite orbit and clock products for standalone kinematic and static positioning (Zumberge et al. 1997; Kouba and Héroux 2001). Nowadays PPP has become an essential tool for providing position information to personal navigation (Wu et al. 2019; Psychas et al. 2019), vehicle and machinery control (Prabha et al. 2014), location-based monitoring (Richter et al. 2016), maritime operations (Ma et al. 2017) and cooperative mobility (Severi et al. 2018).

Among others, aircraft navigation by means of PPP has been widely studied, and the number of aircraft including airplanes and unmanned aerial vehicles (UAV) equipped with GNSS receiver chipset is increasing. Monico et al. (2019) implemented real-time PPP methodology in two airplane flight tests and found that the accuracy of 30 cm for the horizontal and 50 cm for the vertical component can be achieved in the use of GPS real-time orbit and clock products as compared to the relative positioning solutions. Teunissen et al. (2011) proposed a new algorithm for GNSS attitude determination and analyzed its performance in different platforms, including ground, maritime and airplane. The flight test results showed that the aircraft attitudes obtained from PPP compared very well with the precise relative attitude determination results, and the differences mostly contained within 0.2°.

Dorn et al. (2015) applied the PPP concept to the remotely piloted aircraft system (PARS) to obtain the position and velocity. However, their results were based on a driving-simulating-flying test which was much easier to verify the positioning solutions. And PPP was proved to achieve decimeter-level accuracy in the experiment. Roberts et al. (2005) investigated the synergies that exist between GNSS and vision for fixed-wing UAV navigation and control applications. The simulation test results presented that the root mean square (RMS) errors of roll, pitch and yaw angle are approximate 0.5^{o}. Imparato (2016) monitored the integrity of the navigation systems on an aircraft by exploiting the redundancy of the GNSS signals as collected at the receiver.

Although GNSS has been widely implemented in aircraft navigation, one of the bottlenecks is that the carrier-phase cannot contribute to the positioning solutions in the sense of fast and high-precision PPP parameter estimation because the ambiguities are not able to preserve their integer nature due to the presence of the satellite and receiver phase biases (Teunissen 1998a); and thus the standard PPP cannot perform integer ambiguity resolution. During the last decade, several methods which enable PPP to achieve integer ambiguity resolutions have been proposed and formulated (Ge et al. 2008; Teunissen et al. 2010; Geng et al. 2011; Li et al. 2013), and this integer ambiguity resolution enabled PPP is referred to as PPP-AR. These PPP-AR methods differing in the used model and applied corrections, as well as their connections, were reviewed by Teunissen and Khodabandeh (2015).

Generally, two types of combination methodology for pseudo-range and carrier phase are mainly used in PPP-AR: ionosphere-free combination and uncombined observable (Odijk 2003; Odijk et al. 2016). Each combination has its own advantages in data processing and will give exactly the same solution when rigorously solved. In this study we prefer using the uncombined observable because it is flexible for further model strengthening, i.e., strengthened by the external ionospheric pseudo observable. Besides, the advantages of the uncombined observable also include the simplest observation variance-covariance matrix, and all parameters are available for scientific research. Teunissen et al. (2010) proposed an uncombined PPP-AR model by means of reparametrizing the undifferenced GNSS observation equations so as to eliminate the rank defects. And the results indicated that PPP-AR works very much like network RTK if precise ionospheric corrections are made available to the user.

Since then, Zhang et al. (2011) extended the usage of the undifferenced and uncombined PPP-AR to a sparse ground network since the ionospheric effects were considered in the functional model. Odijk et al. (2014) focused on the single-frequency PPP-AR application and proved that single-frequency PPP integer ambiguity resolution is feasible in less than 10 min when applying the ionosphere corrections in a small network. Nadarajah et al. (2018) provided numerical insights into the role taken by the multi-GNSS integration in delivering fast and high-precision positioning solutions using uncombined PPP-AR model.

Except for the scientific research, companies also provide commercial products such as satellite phase biases and ionospheric corrections along with satellite orbit and clock products to users to help them fix the integer ambiguities. For instance, Trimble RTX service offers flexible subscription options in order to meet user's requirements from meter to centimeter level (Chen et al. 2011; Alkan 2019). Fugro G2+ provides clients with the additional hardware biases that are computed using global reference stations to enhance positioning services with integer ambiguity resolved PPP for two GNSSs (GPS and GLONASS) and G4 provides the ambiguity float solutions for four GNSSs (GPS, GLONASS, Galileo and BDS) (Liu et al. 2015; Tegedor et al. 2016).

Although PPP-AR has been widely implemented in scientific research and industrial applications, few publications focus on fixing the ambiguities on aircraft due to the challenging circumstances, including high velocity and severe maneuver. In this contribution, we implement the PPP-AR technique for the first time in an airplane flight test to investigate to what extend the fixed ambiguities could contribute to airplane positioning solutions. Besides, several key issues related to the positioning and ambiguity solutions are also discussed. We first investigate the influence of the tropospheric delays on PPP-AR estimations because a tight constraint is always given to this parameter for nonaviation applications based on the stable troposphere behavior on the ground. However, this is not the case for the aircrafts whose altitude may dramatically change within a narrow time span. Therefore, a proper constraint needs to be considered for the tropospheric delay.

Then different values of the success rate criterion of integer ambiguity resolution (Teunissen 2000) are assessed; as it is well known that both strength of underlying model and accuracy of float ambiguities are crucial factors for successful and reliable ambiguity fixing in real applications (Li et al. 2014), and the success rate represents the model strength to some extent. Therefore, we believe that a higher success rate criterion would be helpful to obtain the correct fixed ambiguities because wrongly resolved integer ambiguities may result in unacceptably large position errors (Verhagen et al. 2013). However, the higher success rate also means longer waiting time to get the first integer ambiguity solution.

Finally, the airplane data is processed in 15 s and 1 s interval with ambiguity float and fixed solutions under different configurations to illustrate in which condition and to what extent the fixed ambiguities can improve the airplane positioning accuracy. The reason for the 15 s interval data processing is that as mentioned before, the satellite phase bias corrections are needed for integer ambiguity resolution, and we generate these corrections as well as the satellite clock corrections through a GNSS network in the interval of 15 s. This also means that 1 s corrections need to be interpolated to meet the requirement of the data processing. Thus, the performance of the interpolated corrections is another focus of this contribution.

This article is organized as follows. An undifferenced and uncombined PPP-AR model at both network and user side is provided in the next section, as well as a brief description of the theory of integer ambiguity resolution. In the third section, a GNSS network with 20 reference stations is selected, and the airplane data is processed in ambiguity float and fixed solutions, respectively. The key issues mentioned above are also discussed in this section. The final section gives conclusions and remarks of this research.

PPP-AR THEORY AND INTEGER AMBIGUITY RESOLUTION

PPP-AR needs a GNSS network to process the data of a group of receivers to obtain various corrections such as satellite phase biases and clock offsets. The linearized undifferenced uncombined GNSS observation equations read as (Teunissen et al. 2010):

$$\begin{aligned} E\{\Delta\phi_{r,j}^{s}\} &= g_r^{s\mathrm{T}}\Delta x_r + m_r^s\tau_r - \mu_j\iota_r^s + dt_r - dt^s + \delta_{r,j} - \delta_{,j}^s + \lambda_j a_{r,j}^s \\ E\{\Delta p_{r,j}^{s}\} &= g_r^{s\mathrm{T}}\Delta x_r + m_r^s\tau_r + \mu_j\iota_r^s + dt_r - dt^s + d_{r,j} - d_{,j}^s \end{aligned} \qquad (1)$$

where $E\{\cdot\}$ is the expectation operator; $\Delta\phi_{r,j}^s$ and $\Delta p_{r,j}^s$ are the so-called observed-minus-computed phase and code observations on frequency j from satellite s to receiver r, in meters; g_r^s the line-of-sight unit vector from the satellite to the receiver; Δx the increment of the receiver position; τ_r the zenith tropospheric delay and m_r^s its corresponding mapping function which introduces an elevation-dependent scaling factor for each satellite; ι_r^s the slant ionospheric delay on the first frequency and having μ_j as the coefficient; dt_r and dt^s are the receiver and satellite clock offsets, respectively; note that they are common to both phase and code observation. $\delta_{r,j}$ and $\delta_{,j}^s$ are the receiver and satellite phase biases, in meters; $d_{r,j}$ and $d_{,j}^s$ are the receiver and satellite code biases; λ_j the wavelength and $a_{r,j}^s$ the integer ambiguity, in cycles.

However, the system of observation equations based on Eq. 1 is rank-deficient. To make it a full rank model, the S-system theory is applied to constrain a set of parameters as the S-basis. Examples of the applicability of this theory to PPP-AR can be found in (Odijk et al. 2017; Ma et al. 2020), and the constraint set we used to eliminate the rank deficiency is given by

$$\begin{cases} Pivot\ receiver\ clock:\ dt_p \\ Receiver\ and\ satellite\ code\ biases:\ d_{r,j}\ and\ d^s_{,j},\ \ r = 1, \dots, n,\ \ j = 1,2 \\ Pivot\ receiver\ phase\ biases:\ \delta_{p,j},\ \ j = 1,2 \\ Pivot\ receiver\ ambiguities:\ a^s_{p,j},\ \ s = 1, \dots, m,\ \ j = 1,2 \\ Pivot\ satellite\ ambiguities:\ a^p_{r,j},\ \ r = 2, \dots, n,\ \ j = 1,2 \end{cases} \tag{2}$$

It is worth mentioning that the choice of constraints is not unique, and after resolving the rank-deficient problem, some of the parameters, e.g., satellite clock offsets and phase biases do not represent their original parameters anymore. Instead, the estimable parameters are established by the combination of the original parameters and the constraints. Note that both receiver and satellite code biases are selected as the S-basis, indicating that these parameters will be absent in the rephrased observation equations. After reparametrizing Eq. 1 by means of the constraints of Eq. 2, the full rank observation equations can be constructed as:

$$\begin{gathered} E\{\Delta\phi^s_{r,j}\} = g^{s\,\mathrm{T}}_r \Delta x_r + m^s_r \tau_r - \mu_j \tilde{\iota}^s_r + d\tilde{t}_r - d\tilde{t}^s + \tilde{\delta}_{r,j} - \tilde{\delta}^s_{,j} + \lambda_j \tilde{a}^s_{r,j} \\ E\{\Delta p^s_{r,j}\} = g^{s\,\mathrm{T}}_r \Delta x_r + m^s_r \tau_r + \mu_j \tilde{\iota}^s_r + d\tilde{t}_r - d\tilde{t}^s \end{gathered} \tag{3}$$

The arguments $\tilde{\iota}^s_r$, $d\tilde{t}_r$, $d\tilde{t}^s$, $\tilde{\delta}_{r,j}$, $\tilde{\delta}^s_{,j}$ and $\tilde{a}^s_{r,j}$ refer to the same parameter as in Eq. 1, but their interpretation is different, as they are lumped with the constraints of Eq. 2. For instance, the ambiguity term $\tilde{a}^s_{r,j}$ is actually a double differenced ambiguity $\tilde{a}^s_{r,j} = a^s_{r,j} - a^s_{p,j} - a^p_{r,j} + a^p_{p,j}$ in which the superscript and subscript p denote the pivot satellite and receiver, respectively. It is worth noting that the temporal constraints would change the interpretation of these estimable parameters because additional rank deficiencies may occur in the absence of dynamic models. One can refer to Odijk et al. (2016) for more information about the details of solving the rank deficiency problem.

The satellite clock offsets and satellite phase delays estimated from Eq. **3** are provided to the user side, and the satellite orbits are available by an external provider, e.g., International GNSS Service (IGS). After applying these corrections and the same constraints as the network, the full rank PPP-AR user model reads:

$$\begin{aligned} E\{\Delta\phi_{u,j}^{s} + d\tilde{t}^{s} + \tilde{\delta}_{,j}^{s}\} \\ = g_u^{s^T}\Delta x_u + m_u^s\tau_u - \mu_j\tilde{\iota}_u^s + d\tilde{t}_u + \tilde{\delta}_{u,j} + \lambda_j\tilde{a}_{u,j}^s \\ E\{\Delta p_{u,j}^{s} + d\tilde{t}^{s}\} = g_u^{s^T}\Delta x_u + m_u^s\tau_u + \mu_j\tilde{\iota}_u^s + d\tilde{t}_u \end{aligned} \quad (4)$$

One can see that the satellite and receiver phase biases have been separated from the ambiguities so that they are possible to be fixed into integer values. Since we have obtained the observation equations in which the ambiguity can preserve the integer nature, in the following, we will fix the float ambiguities to integer values. To facilitate the interpretation, either network or user positioning equations can be written in the compact formula

$$E\{y\} = Aa + Bb, \quad D\{y\} = Q_{yy} \quad (5)$$

where y represents the vector with phase and code observables; a and A are the ambiguity parameters and the corresponding design matrix, while b and B are the baseline parameters and design matrix which include all other parameters except for the ambiguities. $D\{\cdot\}$ denotes the mathematical dispersion operation, and Q_{yy} refers to the variance matrix of the observation.

By applying an estimator, e.g., least-squares or Kalman filter (Verhagen and Teunissen 2017), the float solutions of ambiguity $\hat{a}$ and position components $\hat{b}$ can be obtained, as well as their individual variance matrix $Q_{\hat{a}\hat{a}}$ and $Q_{\hat{b}\hat{b}}$, and the covariance matrix $Q_{\hat{a}\hat{b}}$. Then the LAMBDA method (Teunissen 1993, 1995) is used to fix the ambiguities because of its efficiency and optimality. The first step of the LAMBDA is to transform the highly correlated ambiguities to a new set of decorrelated ambiguities by a transformation matrix Z^T (Teunissen et al. 1997):

$$\hat{z} = Z^T \hat{a}, \quad Q_{\hat{z}\hat{z}} = Z^T Q_{\hat{a}\hat{a}} Z \tag{6}$$

To preserve the integer nature of the ambiguities, the transformation matrix Z^T needs to be integer and volume preserving. Then the second step is to search the integer values of the float ambiguity in the space (Teunissen 1996):

$$(\hat{z} - z) Q_{\hat{z}\hat{z}} (\hat{z} - z) \leq \chi^2 \tag{7}$$

where $z \in Z^n$, and χ^2 defines a certain searching space instead of the whole integers in Z^n. The optimal integer estimator is the integer least-squares which has the maximum success rate of fixing ambiguities (Teunissen 1998b). Therefore, with the integer least-squares solution $\check{z}$, the ambiguities before decorrelation can be computed from the back transformation $\check{a} = Z^{-T} \check{z}$. The final step is to provide the ambiguity fixed baseline solution $\check{b}$ by adjusting the float solution $\hat{b}$

$$\begin{aligned} \check{b} &= \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a}) \\ Q_{\check{b}\check{b}} &= Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} Q_{\hat{a}\hat{b}} \end{aligned} \tag{8}$$

It is obvious that $Q_{\check{b}\check{b}} \leq Q_{\hat{b}\hat{b}}$, which means the ambiguity fixed baseline estimations are more precise than those of the ambiguity float. However, it is worth noting that the formula of $Q_{\check{b}\check{b}}$ is not rigorous since the fixed ambiguity $\check{a}$ is considered as a deterministic parameter here, and hence only acceptable if the success rate is very close to 1. For more information of taking into account the stochastic property of $\check{a}$ and how it influences the variance of $\check{b}$, one can refer to Teunissen (1998c).

Both strength of underlying model and accuracy of float ambiguities are two crucial factors for successful and reliable ambiguity fixing, and the integer ambiguity resolution success rate plays an important role in measuring the model strength. It has been demonstrated that the bootstrapped probability of obtaining the correct integer ambiguity vector is the lower bound of the integer least-squares estimator (Teunissen 2000), which reads as

$$P(\check{a} = a) = \prod_{i=1}^{n}(2\Phi\left(\frac{1}{2\sigma_{\hat{a}_{i|I}}}\right) - 1) \quad (9)$$

where Φ is the standard normal cumulative probability distribution and $\sigma_{i|I}$ is the standard deviation of ambiguity i, conditioned on all previous ambiguities, indicated by I.

Eq. **9** indicates the strength of the float ambiguities, and only in case the success rate is close to one, the ambiguities can be fixed reliably. Partial integer ambiguity resolution is implemented in the data processing, which means only a subset of ambiguities is fixed to integer values such that a user-defined success rate criterion is met, rather than fixing all ambiguities (Verhagen et al. 2011; Hou et al. 2016). This is because it might require a long time until reliable full ambiguity resolution is achieved, and the accuracy of the baseline parameters have been improved significantly after some of the ambiguities getting fixed (Teunissen and Verhagen 2009).

AIRPLANE DATA TESTS AND ANALYZES

As mentioned above, a GNSS network is needed in PPP-AR procedure for providing the satellite phase bias and clock corrections to users. Fig. **1** shows the network used in this study which contains 20 stations of the Brazilian Active Control Network (Fortes et al. 2009). The flight test was carried out in 1-Sept-2009 equipped with a NovAtel dual-frequency GPS receiver (Monico et al., 2019). The sample rate of the flight data is 2 Hz, and the duration is approximate 3 hours. The horizontal trajectory of the airplane can also be seen in Fig. **1**.

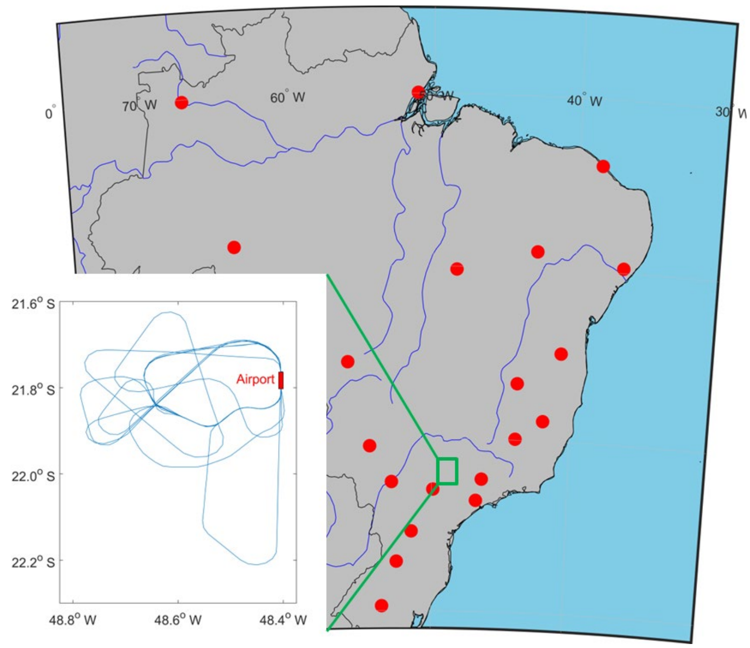

Fig. 1 GNSS network in which the reference stations is represented as the red point and location of the flight test in Sao Paulo State

A reference receiver was set at the airport to collect and store the GNSS data and was employed in the TOPCON-TOOLS commercial software (Gottsmann and del Potro 2008) to generate relative positioning solutions in a forward and backward filtering process with fixed double-differenced ambiguities. Since the baselines between the reference receiver and the airplane are always less than 50 km, these medium-distance relative positioning solutions are believed to be better than or at least comparable to PPP-AR ambiguity fixed positioning solutions, which makes it possible for the relative positioning positions to be regarded as the reference positions of the airplane for verifying the accuracy and the performance of the ambiguity float and fixed solutions of PPP-AR. However, one should keep in mind that the configurations adopted by the TOPCON-TOOLS may influence the performance of the PPP-AR solutions. For example, as the TOPCON-TOOLS implements a tight constraint for the tropospheric delay, the same tight troposphere constraint might be helpful for PPP-AR procedure to fit the TOPCON-TOOLS solutions.

Fig. **2** and Fig. **3** present the altitude and velocity of the airplane obtained by the TOPCON-TOOLS during the flight test, respectively. One can see that a series of maneuvers of the airplane, including sudden pushovers, accelerations and s-turns (as can be seen in Fig. **1**).

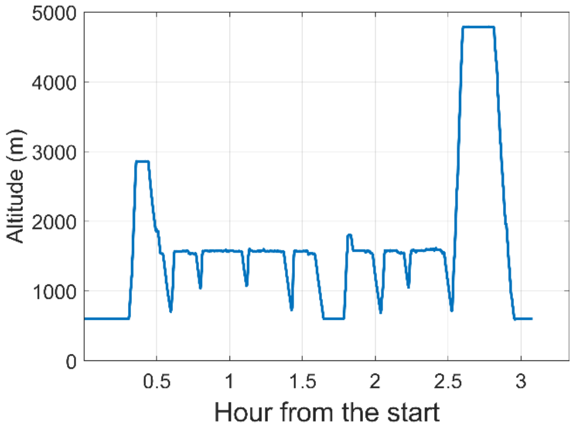

Fig. 2 Altitude of the airplane

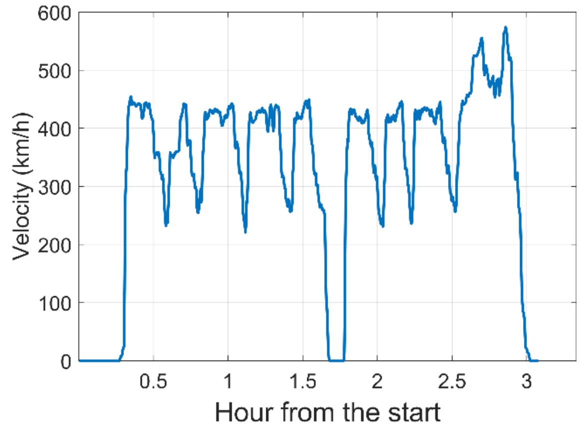

Fig. 3 Velocity of the airplane

The data processing strategy options are summarized in Table 1. The flight data are processed in the kinematic mode for which the coordinates are regarded as epoch-independent; meanwhile, the static mode is used for the network by considering the reference stations as stationary sites. GPS constellation with frequencies L1 and L2 is used because the receiver equipped on the airplane is a dual-frequency GPS receiver. The IGS final orbit product is applied in the experiment (Kouba and Héroux 2001) because this study focuses on the performance of the fixed ambiguity positioning solutions; thus, the error sources including the orbit error are to be eliminated as much as possible. A sample rate of 15 s interval is applied first since 15 s interval corrections, including the satellite clock and phase bias corrections for the flight data are generated from the chosen network.

Although signals are not likely to be affected by blockages or multipath, satellites at low elevation angles may suffer from unmodelled atmospheric delays. Besides, measurements at low elevation angles would not contribute much to the system since we applied elevation dependent weighting strategy. Those are the reasons why a 10^{o} elevation cutoff angle is chosen, which is the same as we usually use for nonaviation applications. The standard deviations of the phase and code observables are 0.005 m and 0.5 m, respectively. This is because, typically, the standard deviation of carrier-phase noise is less than 1 millimeter for a high carrier-to-noise-power-density ratio, and the code measurements are usually weighted at least 100 times lower than carrier-phase due to their high noises (Teunissen and Kleusberg, 2012).

The tropospheric hydrostatic delay is compensated by the Saastamoinen model (Saastamoinen 1972). The forward Kalman filter is implemented in the data processing, therefore, this procedure can be easily applied in the real-time case as long as the precise real-time orbits are provided. The receiver clock offsets and slant ionospheric delays are epoch-wise estimation parameters. And the receiver phase delays and ambiguity parameters are considered as constant according to their behavior in the data processing. Satellite clock offset and satellite phase delay are absent in the flight data processing as they are provided as the corrections from the network.

Since the purpose of this contribution is to assess the performances of the ambiguity fixed solutions during a challenging circumstance of an airborne receiver and to investigate how and to what extent the high velocity and severe maneuver may influence the integer ambiguity resolution, the ratio test is not implemented into the data processing. Because an unproperly selected ratio test procedure may reject some of the fixed ambiguities, no matter they are wrongly fixed or not. Interested readers are referred to Verhagen and Teunissen (2004) and Wang and Verhagen (2015) for more information on ratio test.

Table 1 Summary of the strategy of data processing for the network and airplane

Parameter	Strategy and value	
	Network	Airplane
Positioning mode	Static	Kinematic
Constellation	GPS	GPS
Frequency	L1 and L2	L1 and L2
Satellite orbits	IGS	IGS
Interval	15 s	15 s and 1 s
Elevation cutoff angle	10^{o}	10^{o}

Weighting strategy	Elevation dependent	Elevation dependent
Standard deviation (STD) of phase/code observable	0.005 m/0.5 m	0.005 m/0.5 m
Zenith hydrostatic delay	Saastamoinen model	Saastamoinen model
Slant ionospheric delay	Epoch by epoch	Epoch by epoch
Kalman filter	Forward	Forward
Receiver clock offset	Epoch by epoch	Epoch by epoch
Satellite clock offset	Epoch by epoch	/
Receiver phase delay	Constant	Constant
Satellite phase delay	Constant	/
Ambiguity	Constant	Constant
Integer ambiguity resolution	Partial	Partial

Influence of the tropospheric delay on the flight data processing

Table 2 shows the RMSs of the ambiguity float positioning solutions with different choices for the tropospheric delay process noises as compared to the reference positions obtained by the relative positioning of TOPCON-TOOLS commercial software. Note that the RMSs are calculated from 0.5 h to the end, during which the positioning solutions should have been converged. One can see that the positioning errors are increasing with the enlarged tropospheric delay process noise, and it becomes especially obvious for the vertical direction. This is because the tropospheric delay and the up component are highly correlated, and therefore the residuals of this type of delay due to the imperfect stochastic model would be mostly lumped into the vertical position.

Although the term *positioning errors* is used here and afterwards to assess the performances of PPP-AR with different troposphere process noises, it is actually the *displacements* between the PPP-AR solutions and the reference positions for a rigorous description because the reference might also be affected by GNSS error sources. We only use the *positioning errors* as an idiom for easy understanding.

Table 2 RMSs of the ambiguity float positioning solutions in different tropospheric delay process noise

Values (m^2/s)	Ambiguity float solutions (cm)			
	East	North	Horizontal	Up
0.0001	4.51	2.65	5.23	8.79
0.001	4.51	2.65	5.23	8.79
0.01	4.55	2.65	5.26	8.88

0.1	4.55	2.93	5.41	11.28

Fig. **4** presents the estimates of the tropospheric wet delay in different process noises. It can be seen that the wet delay values are not distinguishable for the tight constraints, i.e., 0.0001, 0.001 and $0.01\ m^2/s$, and the positioning errors of these three cases are very similar, as shown in Table 2. On the contrary, the loose constraint, $0.1\ m^2s$, naturally gives a less stable time series of the wet delay estimation. Big displacements of the troposphere estimates between the loose and tight constraints can be seen in two periods, one is around 1 h, and the other one is from 2.5 to 3 h. Correspondingly, large positioning errors of the up component have appeared in Fig. **5** in the same periods, which means that the loose constraint cannot represent the tropospheric wet delay very well and the impacts of the imperfect modelling are reflected in the vertical positioning errors. Therefore, we use a tight constraint $0.0001\ m^2/s$ in the following data processing.

Table 2 and Fig. **4** indicate that the tropospheric delay parameter needs to be tightly constrained in aviation applications. However, this might be due to the fact that the reference positions from TOPCON-TOOLS are obtained by a tight troposphere constraint, and thus a small troposphere process noise only fits the reference positions well. Even though it is difficult to claim that whether or not a tight troposphere constraint is better than a loose one due to the restriction of lacking airplane's true positions, we can conclude that the horizontal performance of PPP-AR is comparable with the medium-baseline RTK as the maximum distance between the TOPCON-TOOLS reference receiver and the airplane is 50 km. The RMS of the ambiguity float solutions is 5.23 cm with the tight troposphere constraint, and this value is further reduced to 2.52 cm with the ambiguity fixed solutions, as can be seen in Table 3.

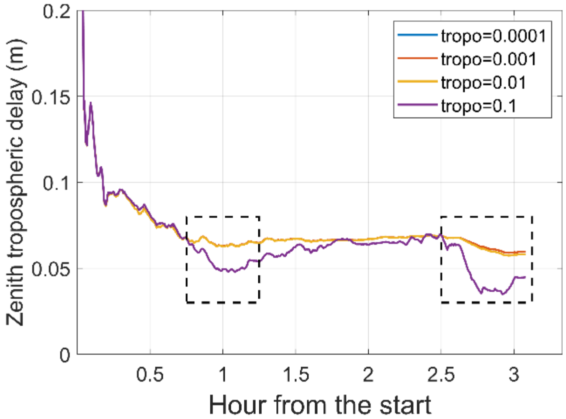

Fig. 4 Zenith tropospheric wet delay estimations by different process noises (the unit of the process noise is m^2/s)

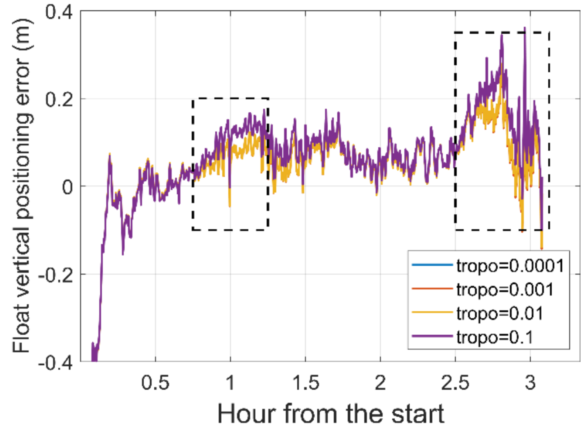

Fig. 5 Ambiguity float vertical positioning errors by different process noises (the unit of the process noise is m^2/s)

Influence of the integer ambiguity resolution success rate on the ambiguity fixed positioning solution

The success rate of Eq. 9 with the multiplication of the conditioned standard deviations of ambiguity is an aspect of the underlying model strength which is an essential factor for successful integer ambiguity resolution. Since we applied the partial ambiguity resolution, it means that at some epochs, only a subset of the ambiguity vector can be fixed rather than all ambiguities. In fact, as can be seen in Fig. **6**, the full ambiguity resolution cannot be achieved for most of the processing period because, on the one hand, a relatively long time is needed since the start of the data processing for the ambiguities to become precise enough to get fixed; and on the other hand, when new satellites rise above the cutoff angle, their ambiguities cannot be fixed immediately, which will cause the failure of full ambiguity resolution.

As can be seen in Fig. **6**, the number of the fixed ambiguities with a lower success rate is always larger than those with a higher success rate at the beginning of data processing when full ambiguity resolution has not been achieved. Besides, full ambiguity resolution with a lower success rate can be achieved faster than higher success rates. However, the risk of the low success rate is that ambiguities may not be fixed correctly due to the imprecise model, and wrongly resolved ambiguities may result in unacceptably large position errors (Verhagen et al. 2013). Therefore, it is a trade-off decision for choosing the success rate so that the application scenarios and circumstance need to be carefully considered.

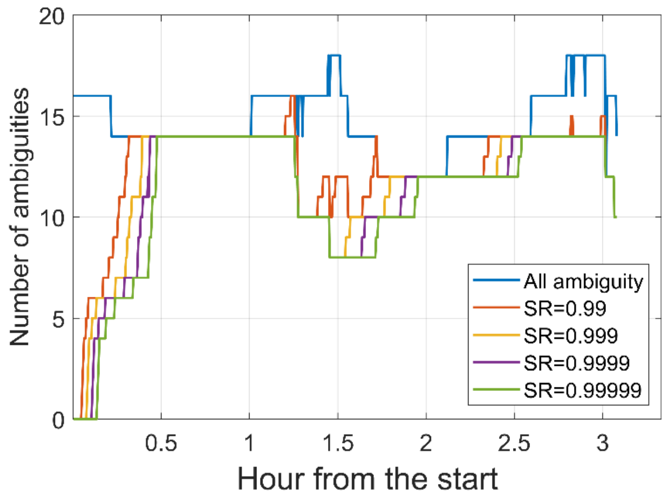

Fig. 6 Number of fixed ambiguities with different success rate criteria for partial ambiguity resolution as compared to all ambiguities. The blue line indicates the number of float ambiguities at each epoch, and the rest of the lines indicate the number of fixed ambiguities in different success rate criteria

Here we present the ambiguity float and fixed positioning solutions under the success rate criterions of 0.99 and 0.99999 in Fig. **7** and Fig. **8**, respectively. One can see that in both figures, the ambiguity fixed solutions have a short convergence time as compared to that of the ambiguity float solutions. This is because the horizontal component is highly correlated with ambiguities; therefore, the east and north component can be improved significantly after most of the ambiguities are correctly fixed. Because of the inclined angle of the constellation, it is reasonable to see a significant improvement in the east component because the satellite-receiver geometry on the east-west direction is not as good as that of the north-south due to the trajectory of the satellites; thus the ambiguity float solution of the east component is worse than the north component. However, fixing ambiguity can compensate for the unfavorable geometry in the east-west direction and thus leads to an equal level of accuracy in the east and north component.

For certain epochs at the beginning of data processing for both positioning solutions, the ambiguity fixed solution is close to the float solution because the contribution of ambiguity fixing is not obvious when not too many ambiguities get fixed. Once most integer ambiguities are resolved, i.e., at 0.3 h with the success rate criterion of 0.99 and at 0.5 h with the success rate criterion of 0.99999, which can be seen in Fig. **6**, the ambiguity fixed solutions experience a large improvement with the errors being at the centimeter-level, compared to a long convergence time of the ambiguity float positioning errors.

One can also see that the first ambiguity fixed solution with the success rate criterion of 0.99999 appears later than that with the success rate criterion with the 0.99 because it needs more time for the positioning model to become such strong. However, the positioning solutions with the 0.99 criterion seem to suffer from the wrong fixing ambiguities at around 0.25 h of Fig. **7** because all positioning components have the unexpectedly increased errors at the same period.

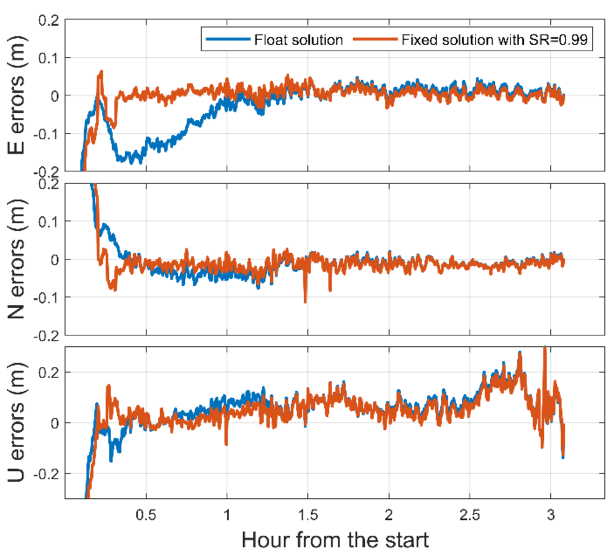

Fig. 7 Ambiguity float and fixed positioning errors with the success rate criterion of $\mathbf{0.99}$

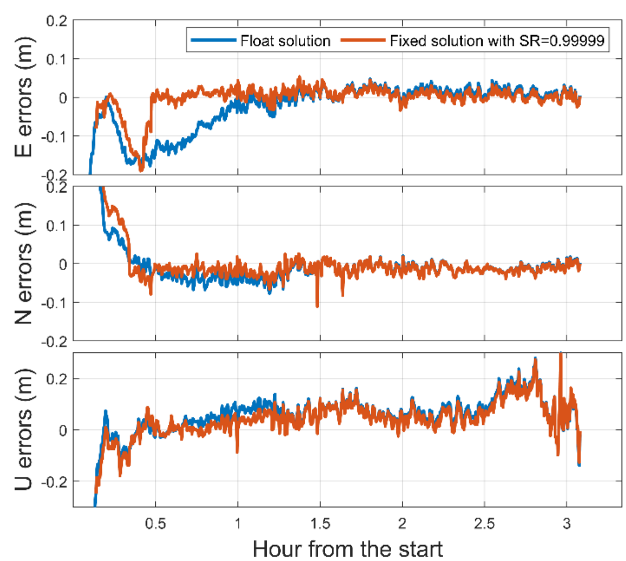

Fig. 8 Ambiguity float and fixed positioning errors with the success rate criterion of $\mathbf{0.99999}$

As can be seen in Fig. **7** and Fig. **8**, the ambiguity fixed and float solutions are likewise similar after a long convergence time, e.g., 1.5 hour. Because the advantage of the fixed ambiguities would not be obvious when the model has been strong enough. Therefore, the reduced RMSs of the fixed solutions presented in Table 3 are mainly due to the period 0.5 to 1.5 h since the RMSs are calculated from 0.5 h to the end, during which the positioning solutions should have been converged. Besides, the fixed solutions with the two success rate criterions are also almost the same after most ambiguities are fixed.

A significant improvement of the ambiguity fixed east component has been demonstrated, and the north component can also be improved to some extent, even though the ambiguity float north component has been accurate already. However, it seems that a bias is lumped into the up component and the integer ambiguity resolution does not benefit the up component much as the horizontal component. This is because the model strength of the up component in GNSS is weaker than that of the horizontal component due to the design of the constellation, i.e., all visible satellites are ‘above’ the receiver. This situation is getting worse for PPP-AR because only one receiver is involved in the data processing. Meanwhile, it is acknowledged that the geometry of the relative positioning is better than single point positioning, which means that the reference positions that are obtained by relative positioning must be better or at least equal to PPP-AR. Therefore, the bias is due to the weak geometry of a single receiver. Besides, mismodelling tropospheric delay could also affect the solution of the up component because of the high correlation. Since the double-differenced measurements could to some extent remove tropospheric model errors, the undifferenced measurements must be influenced by these errors, leading to a worse up solution as compared to the double-differenced model.

Table 3 RMSs of the ambiguity float and fixed solutions with the success rate criterions $\mathbf{0.99}$ and $\mathbf{0.99999}$

Component	Ambiguity float solution (cm)	Ambiguity fixed solution with the success rate criterion 0.99	Ambiguity fixed solution with the success rate criterion 0.99999

		(cm)	(cm)
E	4.50	1.54	1.55
N	2.65	2.00	1.99
2D	5.22	2.52	2.52
U	8.88	7.89	7.84

Eq. 8 demonstrates that the precision of the baseline parameters could be improved once the ambiguities are getting fixed. Fig. **9** shows the standard deviations of the positioning components under the ambiguity float solution and fixed solution with the success rate criterion of 0.99. It can be seen that the largest improvement is presented in the east component, which also explains why the east component benefits the most from fixing ambiguities.

Both ambiguity float and fixed STD of the up component are worse than those of the horizontal component because of the design of GNSS, i.e., all satellites are above the receiver. Note that STDs of the up component start rising at 2.5 h and reach a peak value at around 3 h, indicating a bad geometry of the up component during this period. This could be one reason for the bad behavior of the up component within the same period in Fig. **7** and Fig. **8**. Another possible reason is the sudden rising attitude of the airplane from 2.5 h and the sudden dropping at 2.8 h (seen in Fig. **2**), which causes the residuals of the unmodelled tropospheric delay affecting the up component.

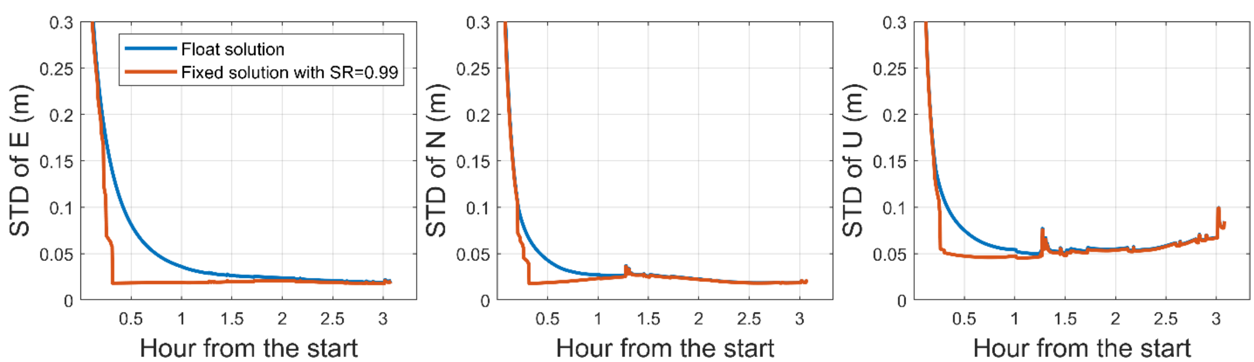

Fig. 9 Standard deviations of the positioning components

Process the flight data with **1 s** interval

One of the issues for processing the flight data with 1 s interval is that the required corrections need to be interpolated from the 15 s to1 s. We simply implemented the linear interpolation method because of its convenience and efficiency. Among these corrections, the interpolation of the satellite phase bias corrections may not be a problem because they can remain constant over a short time span. However, interpolated satellite clock would be meaningless because the variations of the clock offset are not easily captured and thus not accurately interpolated even during the 15 s period. Therefore, the standard deviations of the phase and code observable are enlarged from 0.005 and $0.5m$ to 0.01 and $1m$, respectively, since the biases of the inaccurate corrections would be lumped into the observables. Besides, the success rate applied in the 1 s interval data test is 0.99999 due to the potentially correlated observations for which the filter cannot handle with. Because the success rate criterion represents how much the precision of the ambiguities can achieve, and thus a higher criterion ensures the successful integer ambiguity resolution to some extent.

Fig. **10** shows the ambiguity float and fixed positioning solutions of 1 s interval data processing, in which one can see that the first ambiguity fixed solution appeared earlier than the fixed solutions in Fig. **8** which also applies the 0.99999 success rate. This is because the observations can be quickly accumulated by using a high sample rate, and thus the model strength is able to achieve the high success rate even having the standard deviations of the observables increased. However, fast integer ambiguity resolution does not mean that the ambiguities are fixed correctly. As can be seen in Fig. **10** that the fixed solution of E and N component before 0.25 h show large deviations as compared to the reference positions due to the correlation between ambiguities and horizontal component. This is also the reason for the less impacts on the ambiguity fixed up solution.

The accuracy of the up component for ambiguity float and ambiguity fixed solutions are decreased compared to 15 s data processing. This is because the linear interpolation cannot well represent the variations of the clock offsets. Therefore, the satellite clock interpolation and/or prediction need to be further investigated. One can refer to Wang et al. (2017) in which they proposed a dynamic satellite clock incorporated in Kalman filter to predict the clock corrections in a short latency. This model can also be used in interpolating the clock corrections.

Unfortunately, even we have known that wrong fixing ambiguities existed in the positioning solutions, we cannot identify which integer ambiguity is not correct. We even do not know which subset of the ambiguity vector is fixed in the partial ambiguity resolution since the original ambiguities have been transformed by Eq. 6 before fixing. Which also indicates that the ‘fixed’ ambiguities may not be integer values if full ambiguity resolution is not performed because the fixed ambiguities need to be computed from the back transformation.

It is also worth noting that the successful integer ambiguity resolution starts from 0.25 h in the 1 s interval data processing, which is almost the same as Fig. 7, the 15 s interval. It indicates that other than the model strength, the geometry change is also a key factor for fixing ambiguity. Besides, both the ambiguity float and fixed up solutions of 1 s interval are worse than those of 15 s interval, which can also be seen in Table 4, the RMSs of the positioning solutions of the 1 s interval data processing. The statistics are again calculated from 0.5 h to the end. The worsening of the up component is due to the inaccurate satellite clock corrections.

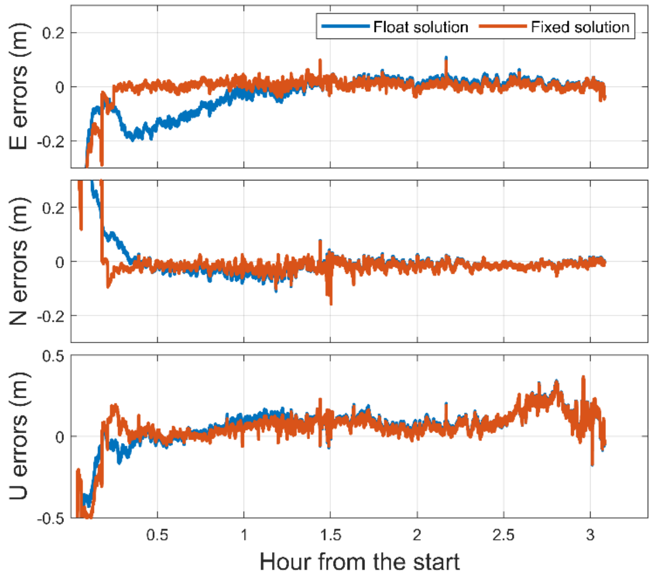

Fig. 10 Ambiguity float and fixed positioning solution of **1 s** interval data processing

Table 4 RMSs of the ambiguity float and fixed solutions in the data processing of **1 s** interval

Component	Ambiguity float solution (cm)	Ambiguity fixed solution (cm)
E	4.47	1.56
N	2.58	2.15
2D	5.16	2.65
U	11.14	10.24

SUMMARY

In this contribution, we implemented the PPP-AR concept in the aviation application since the technique of integer ambiguity resolution enabled PPP has been widely used in geodetic missions. The aviation applications may face the challenging circumstances including high velocity and severe maneuver, and therefore it is worthwhile to investigate if the integer ambiguity resolution is influenced by such circumstances. An undifferenced and uncombined positioning model which preserves the integer nature of the ambiguity was applied in network and user side. The satellite clock corrections as well as the satellite phase bias corrections were generated from the chosen GNSS network and provided to the flight data, and thus integer ambiguity resolution can be achieved on the receiver of airplane.

The flight data was collected from a 3 hours airplane experiment during which intense maneuvers were taken place, and the velocity reached more than 500 km/h. The performance of the PPP-AR was verified in these challenging circumstances with the data processing interval of 15 s and 1 s. And the reference positions are obtained from a relative positioning solution of the TOPCON-TOOLS commercial software with fixed double-differenced ambiguities.

The results show that the integer ambiguities can be correctly fixed when the model is strong enough, and the positioning accuracy is improved once most of the ambiguities get fixed, especially for the east component which is highly correlated with the ambiguities. Since the main purpose of fixing ambiguity is to improve the parameters' precision and thus reduce the convergence time, the improvement positioning behavior is mostly due to the period during when the ambiguity fixed solution has been converged, but the ambiguity float solution has not. For the 15 s data test, the accuracy of the horizontal component is improved from 5.22 cm with the ambiguity float to 2.52 cm with the ambiguity fixed solution. However, the improvement for the up component is not obvious, from 8.88 cm to 7.98 cm, because the up component is highly correlated with the tropospheric delays and receiver clock offsets.

Generally speaking, this PPP-AR procedure performs well in processing the flight data. Both ambiguity float and fixed solution are not significantly affected by the maneuvers of sudden pushovers, accelerations or s-turns. This is because, on the one hand, the positioning model takes into account almost all error sources of GNSS, including the slant ionospheric delay and satellite and receiver phase biases. The realistic model ensures the accuracy of the float ambiguities, and thus they can be fixed successfully. And on the other hand, the Kalman filter can handle the maneuvers very well because the transition matrix of the state updated equation of Kalman filter between consecutive epochs is obtained from the differential equations of the first-order linearized positioning model, and therefore the state transition matrix can well predict the position change between epochs.

Several other key issues are also discussed in this contribution. The first one is the influence of the tropospheric delay on the flight data. It is well known that the zenith wet delay should be considered as an unknown parameter and is sensitive to the altitude; therefore, a proper constraint is needed for the process noise of the wet delay as the height of the airplane may dramatically change within a narrow time span. Although the results indicate that a tight constraint of the tropospheric delay is still recommended for the airplane navigation, it could be due to the fact that the reference positions are obtained by a tight troposphere constraint, and thus a small troposphere process noise fits the reference well. However, we can still conclude that the horizontal performance of PPP-AR is comparable with the medium-baseline RTK as the maximum distance between the TOPCON-TOOLS reference receiver and the airplane is 50 km.

Secondly, different values of the integer ambiguity resolution success rate criterions are tested and discussed. The success rate is an aspect of the underlying model strength and relates to the number of fixed ambiguities. Since the main systematic errors are taken into account in the positioning model and the flight do not suffered by unexpected situations such as scintillation and weather events, a relatively low success rate criterion (0.99) already works well for the data processing. However, the value of the success rate criterion needs to be determined by the real conditions in different applications.

Finally, the flight data is processed in the interval of 1 s, which means that the 1 s corrections are interpolated by the 15 s corrections. The interpolation causes an accuracy degradation of the corrections, especially for the satellite clock because the clock offsets vary quickly even within a very short time span. As a consequence, the up component of the 1 s interval is worse than that of the 15 s interval. And the convergence times for the fixed solutions of the 1 s interval data processing are not shortened because the geometry change is also a key factor for ambiguity fixing.

DATA AVAILABILITY STATEMENT

Some data used during the study are available online in accordance with funder data retention policies. (Brazilian CORS network): http://geoftp.ibge.gov.br/informacoes_sobre_posicionamento_geodesico/rbmc/dados/

Some data used during the study were provided by a third party. (Flight test data). Direct request for these materials may be made to the provider (Embraer S.A.) as indicated in the Acknowledgments.

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