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SIMULATION OF THE DYNAMICS OF AN OLYMPIC ROWING BOAT

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Abstract. *A tool for the prediction of the performances of olympic rowing boats is presented and discussed. The equations of motion for the dynamical system composed by the boat, oars and oarsmen are obtained by means of suitable models for the rowers motion, oar forces and fluid-structure intercation forces. The proposed algorithm is implemented in a C++ code which has proved to produce consistent results for any crew configuration tested.*

1 Introduction

The periodic forces imposed at the oars and generated by the movement of the rowers induce on a rowing scull some *secondary motions* which cause an additional resistance to the main forward motion. This additional resistance may account for a significant part of total dissipated energy.

This aspect is often neglected in the common design studies, hence a more in-depth analysis could grant significant improvements in the boat efficiency.

A full dynamic model requires to simulate inertial forces produced by the rowers, the thrust forces at the oarlocks and the fluid-dynamic forces. It is a rather complex fluid-structure interaction problem which may be tackled with different approaches. For instance, by employing a full Navier-Stokes free surface model for the stationary motion of the boat, while computing the energy dissipation due to the secondary motions by means of a suitable potential problem.

The dynamics of rowing has been studied extensively in the last years, for instance by W.C. Atkinson, A. Dudhia, M. van Holst and L. Lazauskas. Yet, none of these studies considers the full dynamics of the boat. Often, the study is limited to the horizontal acceleration effects. We have taken instead the approach of computing the complete scull movement in the symmetry plane, including pitching and vertical acceleration. The computation of the inertial forces exerted by the rowers requires to build an adequate model for their movement. This has been one of the crucial aspects of our work.

The model has been used to analyze the performance of several scull and crew configurations. It can also be used to analyze the effect of different positions of the crew members and rowing styles.

2 Reference frames

We denote with $(\mathbf{O}; X, Y, Z)$ the global (inertial) reference frame, and with $\mathbf{e}_X$, $\mathbf{e}_Y$ and $\mathbf{e}_Z$ the corresponding versors. The X axis is directed horizontally and points towards the bow, being aligned with the mean velocity of the boat. The Z axis is directed vertically pointing upwards, while the undisturbed water free surface is located at $Z = h^0$, where h^0 is a constant value. Since only the motion in the (X, Z) plane is studied, all the considered forces lie in this plane.

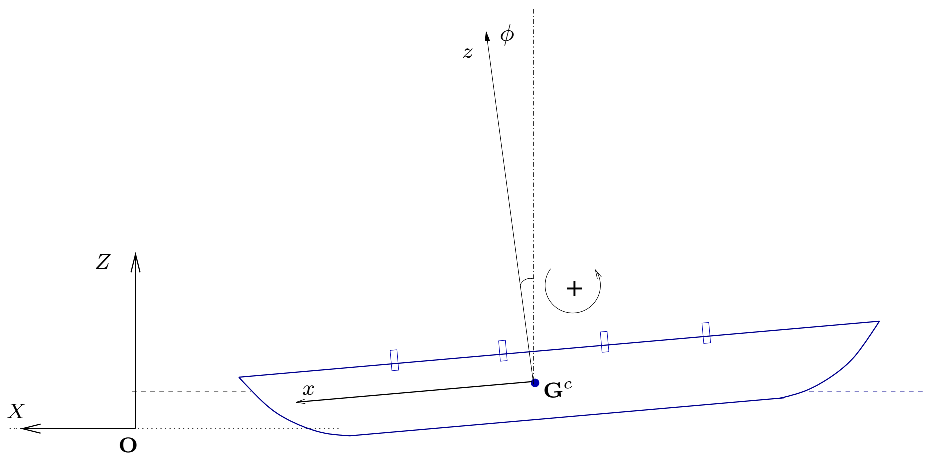

Figure 1: The global and relative reference frames

We also introduce a relative reference frame $(\mathbf{G}_c; x, y, z)$, attached to the boat hull (supposed to be a rigid body) and centered in its baricenter $\mathbf{G}_c$. The x, y, z axes versors in this frame of reference will be $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ (see Fig 1).

With these assumptions, the pitch angle ϕ is the angle between $\mathbf{e}_X$ and $\mathbf{e}_x$, and is positive when the bow lowers. Once we have introduced the rotation matrix

$$\mathcal{R}(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}, \tag{1}$$

we can write the coordinate transformation law for a generic point P as

$$\begin{bmatrix} R_X^P \\ R_Y^P \\ R_Z^P \end{bmatrix} = \mathcal{R}^T(\phi) \begin{bmatrix} r_x^P \\ r_y^P \\ r_z^P \end{bmatrix} + \begin{bmatrix} G_X^c \\ G_Y^c \\ G_Z^c, \end{bmatrix} \tag{2}$$

where positions in the global system are denoted by capital letters.

Transformations between velocity and acceleration vectors in the two reference frames assume the form

$$\mathbf{V}^P = \dot{\mathbf{P}} = \mathbf{v}^P + \dot{\mathbf{G}}^C + \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{G}^C), \tag{3}$$

$$\mathbf{A}^P = \ddot{\mathbf{P}} = \mathbf{a}^P + \ddot{\mathbf{G}}^C + \dot{\boldsymbol{\omega}} \times (\mathbf{P} - \mathbf{G}^C) + \boldsymbol{\omega} \times \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{G}^C) + 2\boldsymbol{\omega} \times \mathbf{v}^P, \tag{4}$$

being $\boldsymbol{\omega} = \dot{\phi}\,\mathbf{e}_Y$ the angular velocity vector. Here, the dot symbol denotes time derivatives.

3 Dynamic system and governing equations

We assume now that the motion of the rowers in the relative reference frame is assigned, namely $\mathbf{g}^{v^i} = \mathbf{g}^{v^i}(t) = (g_x^{v^i}(t), g_z^{v^i}(t))$ is the motion law for the baricenter of the i-th rower. It can be recast in the absolute reference frame by means of transformation (2). We can finally write the motion equations for a system composed by scull, oars, and oarsmen as

$$\begin{aligned} M\ddot{\mathbf{G}}^c + \left(\mathcal{O}(\phi)\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\right)\ddot{\phi} = & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (5a) \\ -2\dot{\phi}\mathcal{O}(\phi)\sum_{i=1}^n M^{v^i}\dot{\mathbf{g}}^{v^i} - \dot{\phi}^2\mathcal{R}^T(\phi)\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i} - \mathcal{R}^T(\phi)\sum_{i=1}^n M^{v^i}\ddot{\mathbf{g}}^{v^i} + & \\ + \mathcal{R}^T(\phi)\sum_{i=1}^n \mathbf{f}^{r^i}(t) + M\mathbf{g} + \mathbf{F}^a, & \end{aligned}$$

$$\begin{aligned} \left(\mathcal{R}^T(\phi)\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\right) \times \ddot{\mathbf{G}}^c + \left(I_y^c + \sum_{i=1}^n M^{v^i}||\dot{\mathbf{g}}^{v^i}||^2\right)\ddot{\phi} = & \qquad\qquad\qquad\qquad\qquad (5b) \\ -2\dot{\phi}\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\cdot\dot{\mathbf{g}} - \sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\times\ddot{\mathbf{g}} + \sum_{i=1}^n M^{v^i}(G_X^{v^i} - G_X^c)g + M^a. & \end{aligned}$$

Here g is the module of the gravity acceleration ($9.81\ m/s^2$), M^{v^i} is the mass of the i-th rower, I_y^c is the moment of inertia of the boat around the y axis, while the matrix $\mathcal{O}(\phi)$ is defined as

$$\mathcal{O}(\phi) = \frac{d}{d\phi}\mathcal{R}^T(\phi) = \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & 0 & 0 \\ -\cos\phi & 0 & -\sin\phi \end{bmatrix}. \quad (6)$$

We now have a system of three second order ordinary differential equations in the time variable, in which $\mathbf{u} = [G^c_X(t), G^c_Z(t), \phi(t)]$ is the unknown vector, its components being the position of the scull center of gravity $\mathbf{G}^c(t)$ and the pitch angle.

To close the problem, however, besides providing a motion law for the baricenter of each rower, we must determine the values of the traction forces applied on each of the oars —namely $\mathbf{f}^{r^i}(t)$— and the forces and moments acting on the hull, due to its interaction with the surrounding water.

4 Rowers motion law

The rowers baricenter is assumed to move on a path shaped as an ellipse in the XZ plane (see Fig. 2), inclined of an angle $\sigma = 7/75$ with respect to the x axis.

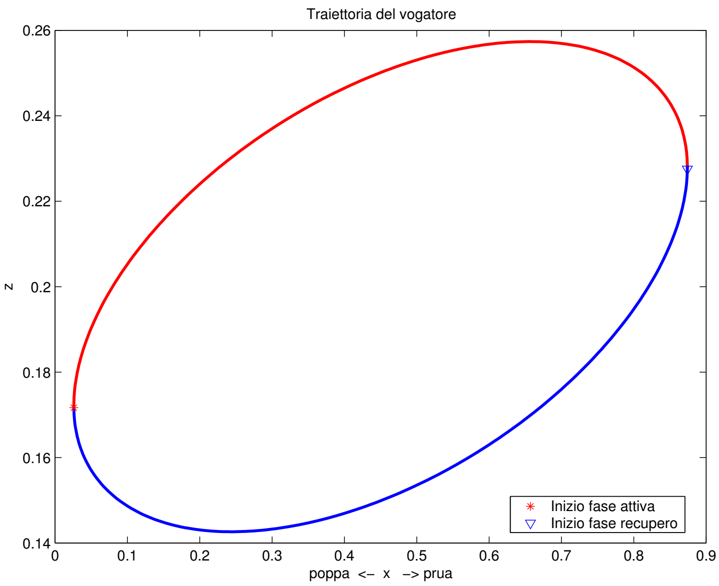

Figure 2: The elliptic rower baricenter path in the XZ plane

The equations describing the path are

$$\begin{aligned} \tilde{g}^v_x(t) &= \tilde{x}_0 + a_x\cos(\theta(t))\cos(\sigma) - a_z\sin(\theta(t))\sin(\sigma) \\ \tilde{g}^v_x(t) &= \tilde{z}_0 + a_x\cos(\theta(t))\sin(\sigma) + a_z\sin(\theta(t))\cos(\sigma) \end{aligned} \quad (7)$$

Here$\tilde{x}_0 = (L^c + L^g)/2$, $\tilde{z}_0 = 4L^g$ and $a_x = (L^c + L^g)/2$, $a_z = L^g$. L^c is the rowers seat excursion, L^g is the distance between the rowers seat and their baricenter. The motion law, velocity and accelerations are readily computed from this equation.

Alternatively we may use a trajectory inferred from telemetry measurements. Work is ongoing in this direction.

5 Oars traction forces

These forces can be computed by analyzing the dynamics of the oar itself. Assuming a rigid oar having negligible mass, we can write, composing the linear and angular momentum conservation law around the oarlock, that

$$\mathbf{f}^{r^i}(t) = \frac{r_b}{L_r}\mathbf{f}^{s^i}(t), \tag{8}$$

being L_r the total lenght of the oar, and r_b the distance between the rower's hands and the oarlock. The oarlock forces $\mathbf{f}^{s^i}(t)$ can be measured by placing suitable sensors in the oarlock, and are therefore assigned. For the stroke period we use the analytical law (Fig. 3), which well fits experimental data.

$$f_x^{s^i} = F_x^{max}\sin^2(\frac{\pi t}{\tau_1}) \qquad f_z^{s^i} = F_z^{max}\sin^2(\frac{\pi t}{\tau_1}), \tag{9}$$

Here

$$0 \le t \le \tau_1 \rightarrow \begin{cases} \theta(t) = -\pi(\frac{t}{\tau_1}+1) \\ \dot{\theta} = -\frac{\pi}{\tau_1} \\ \ddot{\theta} = \left(\frac{\pi}{\tau_1}\right)^2 \end{cases} \qquad \tau_1 < t \le T \rightarrow \begin{cases} \theta(t) = -\pi(\frac{t-\tau_1+\tau_2}{\tau_2}+1) \\ \dot{\theta} = -\frac{\pi}{\tau_2} \\ \ddot{\theta} = \left(\frac{\pi}{\tau_2}\right)^2 \end{cases}, \tag{10}$$

where the stroke period function and the recovery period function are respectively

$$\tau_1 = 0.00015625(r-24)^2 - 0.008125(r-24) + 0.8 \qquad \tau_2 = (60-\tau_1 r)/r, \tag{11}$$

r being the cadence.

6 Forces due to the hull interaction with water

The hydrostatic and hydrodynamic forces and moments are decomposed in the following way

$$\begin{aligned} \mathbf{F}^a &= S^a\mathbf{e}_Z - R^a\mathbf{e}_X + \mathbf{D}^a, \\ M^a &= M_S^a + M_D^a. \end{aligned} \tag{12}$$

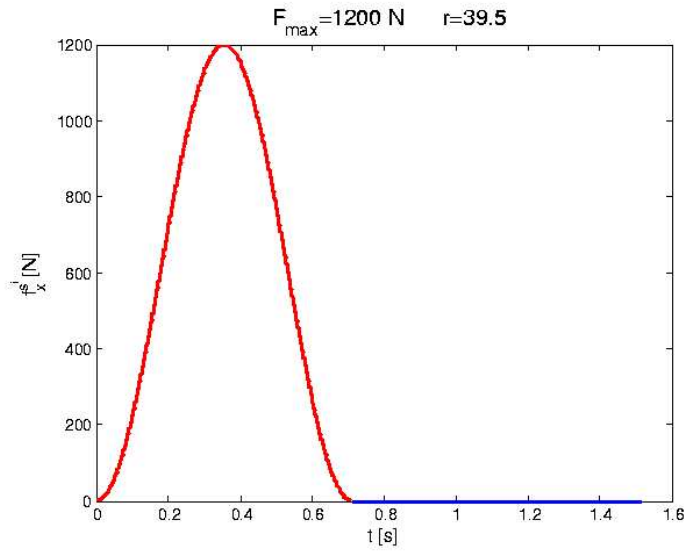

Figure 3: The oarlock force during a period

Here S^a and M_S^a are the hydrostatic lift and moment respectively, and depend on the instantaneous position of the hull. The drag due to the primary motion R^a is computed by means of the empirical formula

$$R^a = \frac{1}{2}\rho S_{ref} C_{dX} (\dot{G}_X^c)^2, \tag{13}$$

being S_{ref} a reference surface (again depending on the instantaneous position of the boat), and C_{dX} a drag coefficient. The latter is computed for each boat by performing a Navier–Stokes simulation of the stationary motion.

Finally, the forces and moments due to the secondary motions of the boat, namely $\mathbf{D}^a$ and M_D^a, are computed by solving the following elliptic partial differential problems (see [2]) for the complex velocity potential ψ_α

$$\begin{cases} \Delta\psi_\alpha = 0 & \text{on } \Omega \\ \dfrac{\partial\psi_\alpha}{\partial z} - \dfrac{\omega^2}{g}\psi_\alpha = 0 & \text{on } \Gamma_{fs} \\ \dfrac{\partial\psi_\alpha}{\partial n} - i\dfrac{\omega^2}{g}\psi_\alpha = 0 & \text{on } \Gamma_\infty \\ \dfrac{\partial\psi_\alpha}{\partial n} = 0 & \text{on } \Gamma_b \\ \dfrac{\partial\psi_\alpha}{\partial n} = N_\alpha & \text{on } \Gamma_c \end{cases} \qquad \alpha = 1, 2, 3. \tag{14}$$

By a physical point of view, we solve three problems where a periodic motion (of frequency $f = \omega/2\pi$) is imposed to the boat surface, in the direction of each of the three degrees of freedom considered. The non-homogeneous Neumann conditions applied on the boat surface for each problem are therefore the components of the generalized normal vector $N = [n_x, n_z, zn_x - xn_z]$.

The forces due to secondary motions are finally computed by integrating the pressure obtained un the boat surface. It turns out that these forces present a component proportional to the acceleration vector $\ddot{\mathbf{u}}$ — giving rise to an added mass matrix $\mathcal{M}$— and a component proportional to the velocity vector $\dot{\mathbf{u}}$ — leading to a damping matrix $\mathcal{S}$. As for the ancular velocity ω, we have taken it correspondingly to the principal frequency of the rowers motion.

7 Numerical solution

Introducing all these quantities in equations (5) we get a system of the form

$$A(t, \mathbf{y}(t))\frac{d\mathbf{y}}{dt}(t) = \mathbf{B}(t, \mathbf{y}(t)), \quad t > 0 \tag{15}$$

where $\mathbf{y} = [\mathbf{G}^c_X(t), \mathbf{G}^c_Z(t), \phi(t), \dot{\mathbf{G}}^c_X(t), \dot{\mathbf{G}}^c_Z(t), \dot{\phi}(t)]$. Employing $\mathbf{y}$ instead of $\mathbf{u}$ leads to a first order ODE system, equivalent to the second order one given by (5). This allows the use of several numerical schemes developed for this kind of problems. In particular, we employed schemes included in GSL libraries [1], more precisely a time adaptive Runge–Kutta 45 scheme.

The algorithm here illustrated has been implemented in a C++ stand-alone program. Input data are

- a geometrical description of the boat (positions of rowers, oarlocks, footboards, etc.);
- a 3D grid representing the hull surface;
- a list of environmental parameters (gravity acceleration, water density, etc.);
- a description of rowers motion (cadence, oarlock forces, etc.);
- the added mass and damping matrixes;
- a list of parameters for the numerical solution (initial time step, initial time, final time, etc.).

8 Results

A typical solution for problem (15), in the case of a four oarsmen boat is depicted in Fig. 4. Each rower weights 85 kg and pushes with a force $F_x^{max} = 1200\ N$. The picture represents a time hystory plot for each component of the solution vector $\mathbf{y}$. The computed

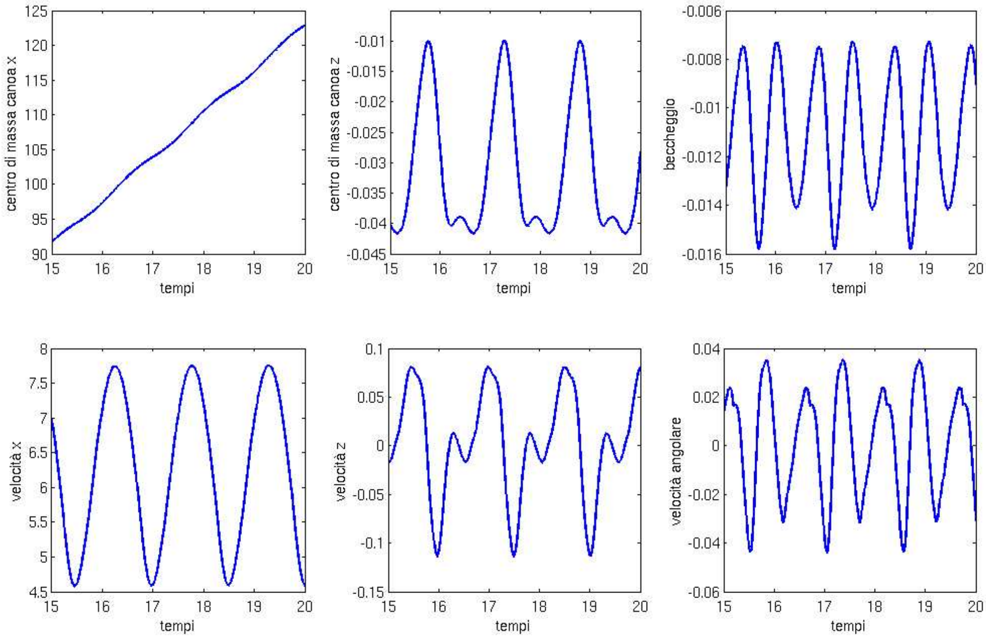

Figure 4: Solution for a four crew boat

average velocity in X direction (about $6m/s$) is compatible with the performances of actual olympic athletes.

Other solutions are shown in Fig. 5 for two different single rower bats. One athlete (green line) weights 106 kg, the other (red line) 85 kg, while they both push with $F_x^{max} = 1200\ N$. Here the average speeds are again compatible with real athletes ones. Moreover, we observe how the heavy rower's boat lowers more into the water, with respect to the light rower's one. This determines an increase of the wet surface, and therefore of the total drag, leading to a lower speed.

Capturing this physical mechanism, our model proves to be able to compute, for example, how much harder the heavier rower should push in order to compensate his weight disadvantage, and the additional energy needed to do it. This code therefore, is not only meant to help designers of new boats, but also meant to address trainers and athletes in their decisions and strategies.

9 Conclusions

The algorithm presented has proved to be robust, and produced physically correct results for any crew configuration tested so far. Still, improvements have to be made in several areas. In particular, the estimation of oar forces, fluid dynamic forces, and of inertia forces due to the rowers movement can be improved by analyzing experimental data coming by telemetry devices and accelerometers placed on rowing boats.

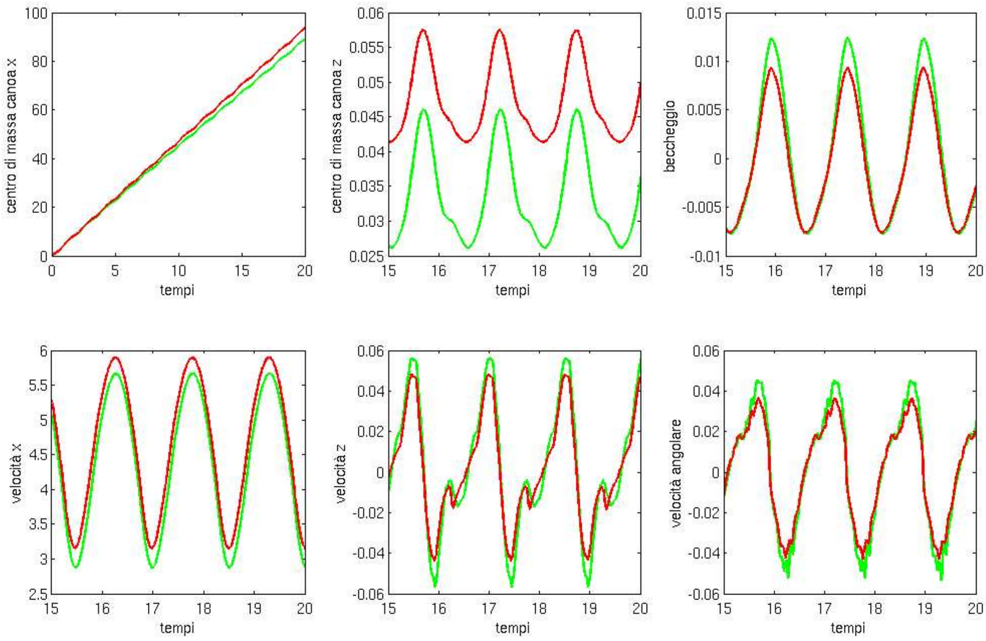

Figure 5: Comparison between singles pushed by heavy (green line) and light (red line) rowers

Besides, interfacing the dynamical system for the boat motion with a different fluid-dynamic model (based on the solution of Navier–Stokes equations with free surface) is ongoing, with the aim to further validate the model.

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