Investigation into behaviour of coupled shear walls by means of continuous method

By

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Preface

This report is the final piece of the Master of Science thesis titled "Investigation into behaviour of coupled shear walls subjected to lateral forces by means of continuous method".

This thesis is part of the Civil engineering- Concrete structural engineering MSc program at the faculty of Civil Engineering and Geosciences of Delft University of Technology. This thesis work has been assessed and supported by the graduation committee, which consists of the following members:

- Dr.ir.C.R.Braam – Structural and building engineering, Delft University of Technology
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Abstract

In this thesis the static behaviour of coupled shear walls subjected to different types of lateral loading is investigated. The response of the coupled walls supported on three different types of foundations is studied by applying continuous and discrete methods.

In the first part of this thesis, the behaviour of coupled shear walls has been studied according to the continuous method by using a mathematical programme, Maple. Further, depending on the foundation types, some equations and curves have been presented which can be used to determine the internal forces, stresses and deflection of the shear walls and coupling beams.

In the second part, a research has been done on the behaviour of coupled shear walls according to a discrete method by using a frame analysis programme, MatrixFrame. Furthermore some comparisons have been made between the results obtained from the continuous method and the discrete method to determine the accuracy of the derived equations from the continuous method on the basis of some examples.

Moreover, the effect of the dimension change, stiffness and dimension ratios on accuracy of the results obtained by the continuous method is studied by representing some examples.

Finally the derived equations from the continuous method have been used to write a simplified programme which can be used in practical works. This work is provided as an excel sheet from which the internal forces and lateral deflection of the shear walls and connecting beams can be simply achieved with a very good accuracy to be used in the first phase of design.

Acknowledgement

It is a pleasure to thank the many people who made this thesis possible.

I would like to express my deep gratitude to my supervisor, Doctor Rene Braam, for his invaluable guidance, useful comments and remarks and engagement during this thesis. I also would like to thank doctor Braam for introducing me to the topic.

I am greatly indebted to Professor Hordijk and engineer Van Keulen, member of my master thesis committee, for their many constructive suggestions and their interest in this work.

The most special thanks go to my husband. Shahab, you gave me your unconditional support and love through all this long process.

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$2d_1$	Is the width of wall 1	[m]
$2d_2$	Is the width of wall 2	[m]
α	Is the stiffness factor which depends on width of opening, moment of inertia of walls, height of floor and centre to centre distance of the walls	[1/m]
α_f	Is the stiffness factor of foundation which depends on width of opening, moment of area of lintel, height of floor, moment of area of foundation beams and centre to centre distance of the walls	[1/m]
A_1	Is the cross sectional area of wall 1	[m^2]
A_2	Is the cross sectional area of wall 2	[m^2]
A	Is the Total cross sectional area of wall 1 and 2	[m^2]
A_b	Is the cross sectional area of the connecting beams	[m^2]
A_{f1}	Is the cross sectional area of foundation beam under wall 1	[m^2]
A_{f2}	Is the cross sectional area of foundation beam under wall 2	[m^2]
A_s	Is steel reinforcement area	[m^2]
A_{sw}	Is the area of shear reinforcement	[mm^2]
b	Is the span of opening	[m]
$c.g$	Centre of gravity	
E	Is the modulus of elasticity of the walls	[kN/m^2]
E_{cb}	Is the modulus of elasticity of the lintels	[kN/m^2]
$E_{s,b}$	Is the modulus of elasticity of the foundation beam	[kN/m^2]
F_1	Is the axial force factor in the walls	
F_2	Is the shear flow factor in the connecting beams	
F_3	Is the deflection factor of the walls	
F_4	Is the axial force factor in the connecting beams	
f_{yd}	Is the yield strength of steel reinforcement	[N/mm^2]
f_{ck}	Is the characteristic compression strength of concrete	[N/mm^2]

f_{cd}	Is the design value of the conrete compression force in the direction of the longitudinal memebr axis	[N/mm^2]
G_b	Is the shear modulus of the connecting beams	[kN/m^2]
h	Is the height of story	[m]
$h_{s,b}$	Is the height of foundation beam	[m]
H	Is the Total height if the coupled walls structure	[m]
I_1	Is the moment of inertial of wall 1	[m^4]
I_2	Is the moment of inertia of wall 2	[m^4]
i_t	Is the Total moment of area of wall 1 and 2	[m^4]
I_b	Is the moment of area of connecting beams	[m^4]
I_e	Is the equivalent moment of area of connecting beams which contains the shear effect	[m^4]
I_{f1}	Is the moment of area of foundation beam under wall 1	[m^4]
I_{f2}	Is the moment of area of foundation beam under wall 2	[m^4]
$I_{s,b}$	Is the moment of area of the stiffened beam at base	[m^4]
k	Is the stiffness factor which depends on cross sectional area of walls, second moment of area of walls and centre to centre distance of the shear walls	
k_f	Is the stiffness factor of foundation which depends on cross sectional area of foundation beams, second moment of area of foundation beams and centre to centre distance of the shear walls	
K_1	Is the percentage of moment carrying by individual action of walls	
K_2	Is the percentage of moment carrying by composite action of walls	
k_{v1}	Is the vertical stiffness of the foundation under wall 1	[kN/m]
k_{v2}	Is the vertical stiffness of the foundation under wall 2	[kN/m]
k_v	Is the equivalent vertical stiffness of the foundation	[kN/m]
$k_{\theta 1}$	Is the rotational stiffness of the foundation under wall 1	[kN.m]
$k_{\theta 2}$	Is the rotational stiffness of the foundation under wall 1	[kN.m]
k_θ	Is the equivalent rotational stiffness of the foundation	[kN.m]

l	Is the centre to centre distance of wall 1 and 2	[m]
λ	Is the modulus of elasticity of sub grade	[kN/m^3]
λ_r	Is a factor depends on rotational stiffness of the foundation, second moment of area of lintels, span op opening and height of story	[1/m]
λ_v	Is a factor depends on vertical stiffness of the foundation, second moment of area of lintels, span op opening and height of story	[1/m]
$M_1(z)$	Is the bending moment in wall 1 as a function of height	[kN.m]
$M_2(z)$	Is the bending moment in wall 2 as a function of height	[kN.m]
M_{10}	Is the bending moment in wall 1 at base	[kN.m]
M_{20}	Is the bending moment in wall 2 at base	[kN.m]
M_0	Is the bending moment in the walls due to axial force in the lintels	[kN.m]
M_{Ed}	Is the design moment	[kN.m]
$m(z)$	Is the external bending moment applied on walls as a function of the height due to external loads w, p and P	[kN.m/m]
μ_f	Is summation of λ_r and λ_v	
$N(z)$	Is the axial load in the walls as a function of height	[kN]
$n(z)$	Is the axial load in the connecting medium per unit height	[kN/m]
v	Is poisson ration of concrete	
P	Is the point load at the top of the walls	[kN]
p	Is the triangularly distributed load on the walls due to earthquake	[kN/m]
ψ	Is a factor depending on modulus of elasticity of lintels and foundation beam, second moment of area of lintels and foundation beam and height of story	
ζ^2	Is the young's 'modulus ratio of lintel to shear wall	
$q(z)$	Is the shear force in the connecting mediums per unit height	[kN/m]
Q_0	Is the shear force in foundation beam	[kN]
ρ	Is the reinforcement ratio	
S_1	Is shear force in wall 1	[kN]

S_2	Is shear force in wall 2	[kN]
s	Is the centre to centre distance of shear reinforcement	[mm]
σ	Is the stress	[kN/m^2]
V_{Ed}	Is the design shear force	[kN]
w	Is the uniform distributed load due to wind	[kN/m]
t_{beam}	Is the thickness of the connecting beams	[m]
t_{wall}	Is the thickness of the shear walls	[m]
$t_{s,b}$	Is the thickness of foundation beam	[m]
$x(z)$	Is the lateral deflection of the coupled walls system due to horizontal forces	[m]
z	Height coordinate	[m]
δ_v	Is the relative displacement at base due to elastic foundation	[m]
δ_θ	Is the rotational displacement at base due to elastic foundation	

Introduction

The main objective of this work is to review the previous works done on behaviour of coupled shear walls and gathering all information about analysing coupled walls based on the continuous method. Further, it is important to restudy the continuous method and prove the validity of the works done before by means of some comparing examples with the discrete method. The other important objective of this thesis is to extend the analysing of coupled shear walls according to the continuous method for different types of foundations. Finally the results will be used to develop a quick and simple programme to determine the behaviour of the coupled shear walls subjected to horizontal forces with an acceptable accuracy which can be used in the practical work.

Tall buildings are subjected to lateral forces due to wind and earthquake. The bending moment in the building caused by the lateral forces increases with the square of height. Therefore the effect of lateral forces will become progressively more important as the building height increases. The lateral stability of the buildings has to be provided by the vertical structural elements such as shear walls or rigid frames. Applying reinforced concrete shear walls is one of the most economical methods.

Mostly in the high-rise buildings a few rows of openings are required for windows and/or doorways. The effects of these openings on the stress distribution in the walls can be neglected if the openings are small. But in the case of larger openings, the wall could not work as one vertical element but as two or more coupled shear walls, depending on the number of rows of openings. Coupled shear walls will increase the total lateral stability of the building. Therefore, it is very important to obtain a simple method which can describe the behaviour of coupled shear walls under the action of lateral loadings.

There are different methods which can be used for analysis of the coupled shear walls. The choice depends on the degree of accuracy and the structural layout. The most general methods are the continuous method and the discrete method, both considering the linear elastic analysis. The continuous method is a simplified method which gives a series of charts for a rapid evaluation of the stress, axial force, bending moment and maximum deflection of coupled shear walls subjected to a lateral load. The discrete method is a computer based analysis in which the shear walls are treated as a framed structure.

The continuous method has been developed by Hubert Beck in 1962. After Beck, the continuous method has been studied and developed by many researchers such as Rosman, Coull and Choudhury. But there are still some limitations for the continuous method which previous works do not cover such as, shear walls on elastic foundation, the effect of crack formation on behaviour of the coupled walls and comparison with numerical programs. More details about previous works will be given in chapter 1.

The aim of this thesis is to consider the behaviour of coupled shear walls in high-rise building according to the continuous method by covering the limitation of previous works. There are seven main objectives in this thesis.

1. In the first chapter of this graduation work the literature will be reviewed to figure out the limitation of previous works on the continuous method
2. In the second chapter the behaviour of the coupled shear walls with one row of opening supported on rigid foundation will be reconsidered based on continuous method. Further the related equations and curves will be given for the walls subjected to a uniform distributed load, a

triangularly distributed load or a point load at the top of walls. Note that this is a restudy of previous works.

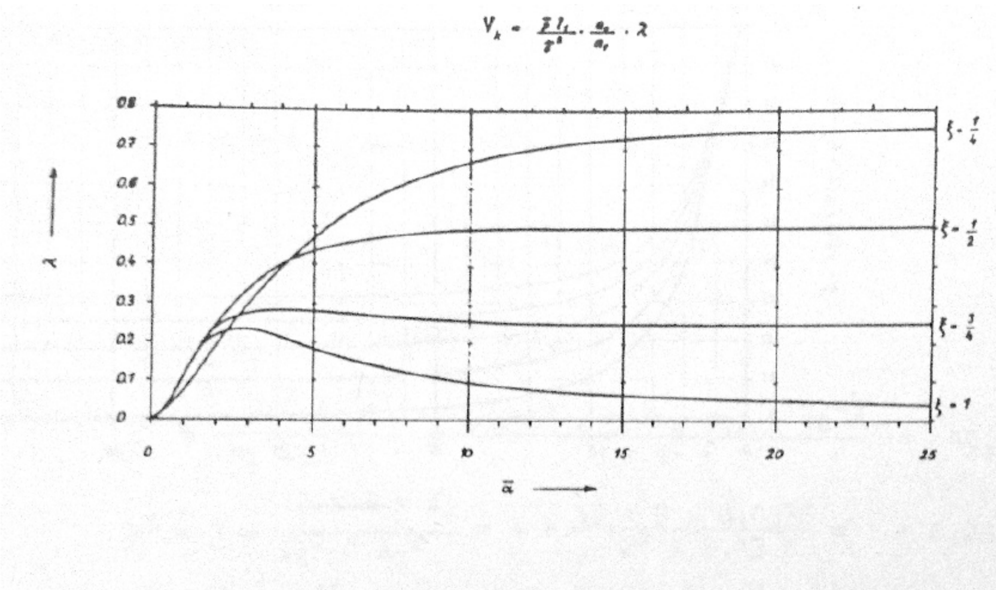
3. In the third chapter the behaviour of the coupled walls system with one row of opening supported by two individual elastic foundations will be reconsidered based on the previous studies.
4. The forth chapter will deal with investigating the behaviour of the coupled walls system with one row of openings on elastic foundation stiffened by a foundation beam at the base.
5. In the fifth chapter a comparison will be made between the results of the continuous method and the discrete method to determine the accuracy of the results obtained from the continuous method. Therefore the results of the models which have been obtained in the first two sections will be compared with a discrete method by using a linear elastic frame analysis computer program.
6. In the sixth section, the effect of connecting beams' stiffness variations on behaviour of the walls system will be figured out.
7. The effect of dimension variations on the coupled walls action will be studied in the seventh chapter.
8. Finally a conclusion will be drawn regarding to the accuracy and applicability of the continuous method for analysing the coupled shear walls.

1 literature review

Many studies have been done on the plane coupled shear walls subjected to lateral loading. Following describes some of the most important studies.

- Hubert Beck [2] in 1962 developed the simplified continuous method. A sample of chart for determination of the shear force in the connecting beams which is given by Beck is illustrated in Figure 1.1 where, ξ is the height ratio and λ and $\bar{\alpha}$ are stiffness parameters.

- Rosman [14], [18] completed the work of Beck and created some charts to determine the maximum shear force and the location of the maximum shear force, axial force and the maximum deflection on top of the walls depends on the stiffness of the walls. According to these graphs the axial forces, shear and bending could be determined on some certain points in the wall namely, at ¼, ½, and ¾ of the height and on the top of the walls. Determining the forces at the other points in the walls could only be done by using interpolation between the specified points.



(Beck, August 1962)

Figure 1.1 : A sample of curve given by Beck to determine the shear force in ¼ , ½ / ¾ and top of the walls

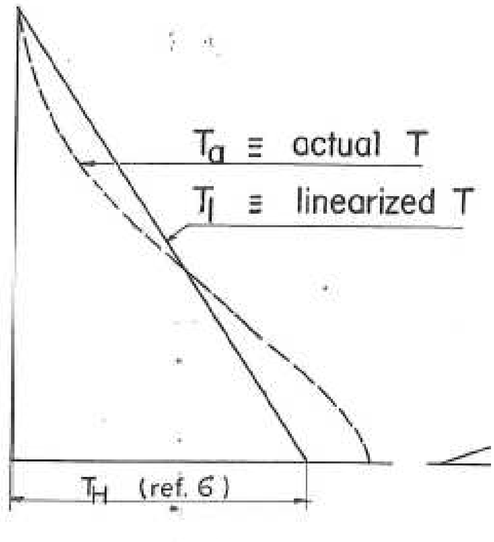
Note that in the above graph, V is the shear force in the connecting beams.

- Later in 1967 Coull and Choudhury [3], [4] attempted to complete the work of Rosman and Beck by introducing a simple and effective approach of analysis of coupled shear walls by the continuous method. The charts given by the Coull and Choudhury are more useful than the previous ones since, using these graphs is easier and they are applicable to determine the forces at each level of the walls. As well as the previous work the bending moment, shear and the deflections at each point can be found based on the height ratio and the stiffness parameter. Moreover, Coull and Choudhury considered not only the walls subjected to uniformly distributed load but also walls subjected to the triangularly distributed load and point load. In another study in 1972, Coull has considered the effect of the elastic foundation on the coupled shear walls for the case where either a settlement or a rotation of the foundation occurs.

- In 1972 TSO and Chan [6] improved the work done by Coull for coupled shear walls on elastic foundation when both vertical settlement and rotation occur.

- Blaauwendraad attempted to analyse the coupled shear walls according to the discrete method and by using frame analysis computer program. Further, he provided a good approximated method for the frame analysis programmes which do not have the possibility to define an element with infinite stiffness.

- Joseph Schwaighofer in 1967 studied the verification of the simplified approach of Rosman for the case of walls with more than one row of openings. Since the application of the theory of Rosman is very lengthy for the shear walls with more rows of openings, he introduced a simplified method for these kinds of shear walls. In his theory he assumed a linear shear distribution for the connecting beams along the height of the shear walls. (See Figure 1.2) This assumption results in considerable saving of time in the analysis. According to Schwaighofer's research, it appears that this linearization results in a very good engineering approximation and it can be accepted.



(Schawaighofer, November 1967)

Figure 1.2 : Actual and linearized T function (function of the shear force in the connecting beams)

- There are many other researches who have been studied the behaviour of the coupled shear walls according to the continuous method and the discrete method. However, it is beyond the scope of this thesis to mention the names of all researchers.

As shown above, previous researches considered the coupled shear walls with one and two rows of opening supported by rigid foundation. Furthermore, coupled walls supported on two individual elastic foundations have also been studied before. In this paper, previous studies will be revised in the first part. In the second part, the continuous method will be extended for analysis of coupled shear walls supported on elastic foundation with a stiffened beam at base for walls with one row of openings.

2 Walls on rigid foundation:

One of the important methods to analyse coupled shear walls is the continuous method. The Continuous method is an approximated method which has been used since 1962. In this method the horizontal connecting beams will be replaced by equivalent continuous mediums over the height of the building which connects the coupled walls as has been illustrated in Figure 2.1. Furthermore, it is assumed that:

- Coupled shear walls exhibit flexural behaviour.
- Coupling beams carry axial forces, shear forces and bending moment
- The axial deformation of the coupling beam can be neglected
- The effect of gravity load on the coupling beam is not taken into account.
- The horizontal displacement at each point in wall 1 is equal to the horizontal displacement at the corresponding point of wall 2 because of the presence of the coupling beams.
- The curvature of the two walls are same at any level
- The point of contra flexure is positioned at the mid-pint of the clear span of the beam
- The dimensions of the walls and connecting beams will not change through the height of the structure

In this section, the walls supported on a rigid foundation under an uniform distributed load will be considered to derive the governing differential equations. Furthermore, these equations will be used to consider the behaviour of the coupled walls under other types of loadings and also for the coupled walls supported on an elastic foundation. The design curve and the equation for walls subjected to point load and a triangularly distributed load have been illustrated in the appendix 2.

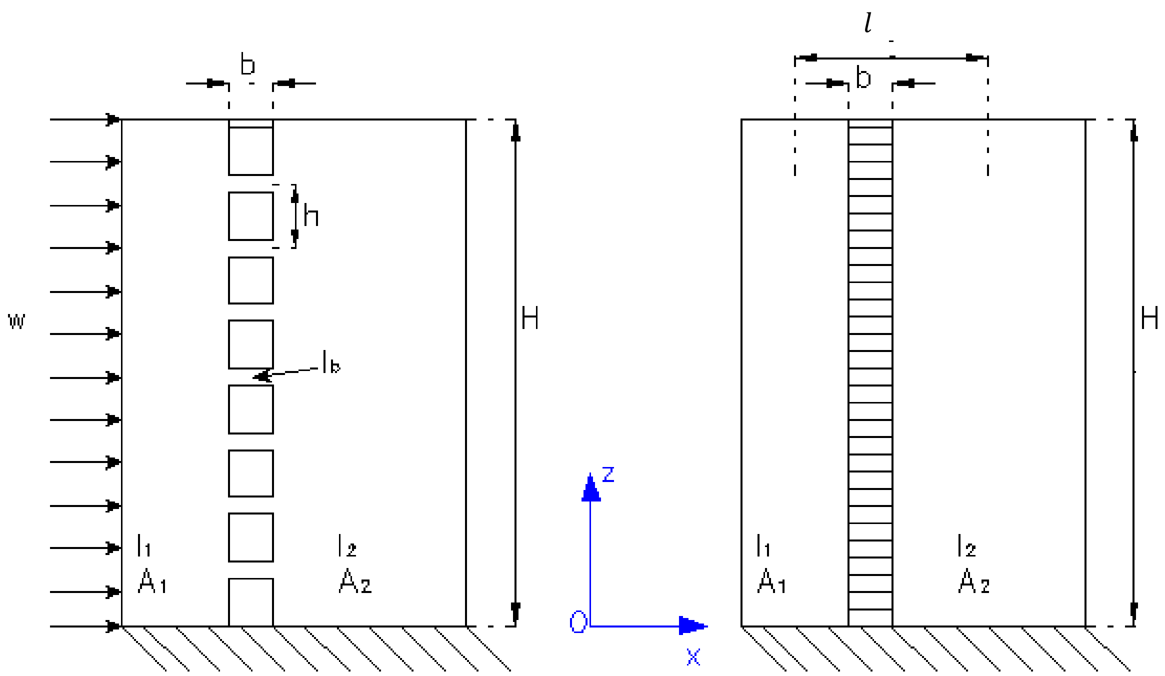


Figure 2.1 : Coupled shear walls according to continuous method model

Note that the approach and the notations which have been used in this section are based on the study of A.Coull [3]. The graphics and the curves have been made by using mathematical programme, Maple. The other figures have been drawn by using AutoCAD programme. Further, the Maple programme has been used to solve the obtained differential equations and simplify them.

Consider the coupled shear walls system shown in Figure 2.1. The connecting beams will be replaced by equivalent continuous mediums. To derive the equations related to internal forces in the beams and walls, the connecting medium will be cut along the vertical line of contraflexure. (SeeFigure 2.2). As has been mentioned in the assumptions, the point of contraflexure in the medium is identical to the mid-point of the clear span of each connecting beam. The applied forces on the shear walls and the connecting mediums will be as following:

- $n(z)$ is axial force in the connecting mediums (per unit height)
- $q(z)$ is the shear flow per unit height applied in the connecting mediums
- $M_1(z)$ is the bending moment in wall 1
- $M_2(z)$ is the bending moment in wall 2
- $N(z)$ is the axial force in walls namely, tension in wall 1 and compression in wall 2
- w is the uniform distributed load due to wind
- $m(z)$ is the external bending moment applied on walls system due to external load w

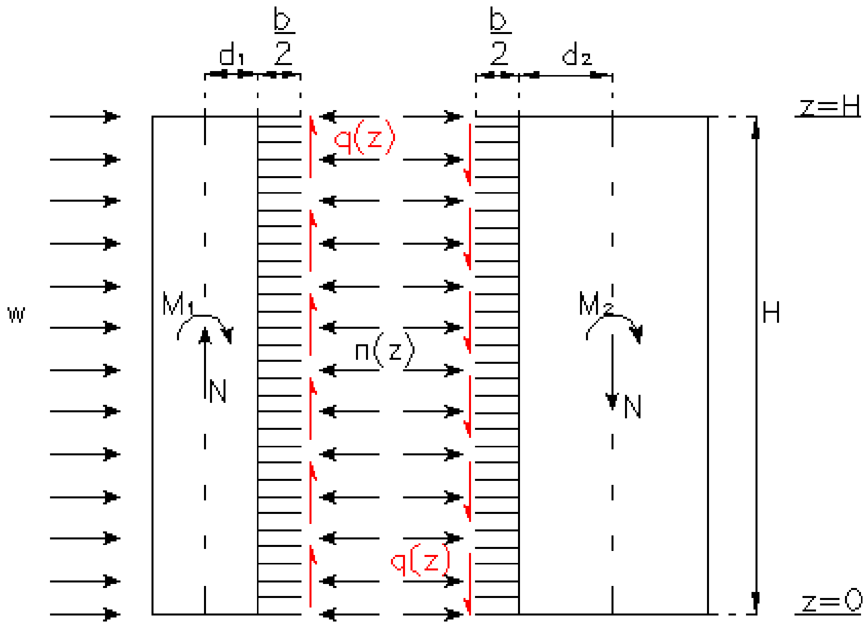


Figure 2.2 : Internal forces in coupled shear walls

Since the cantilevered laminas have been cut along the line of contraflexure some relative displacements as have been shown in Figure 2.3 will occur at the cut ends.

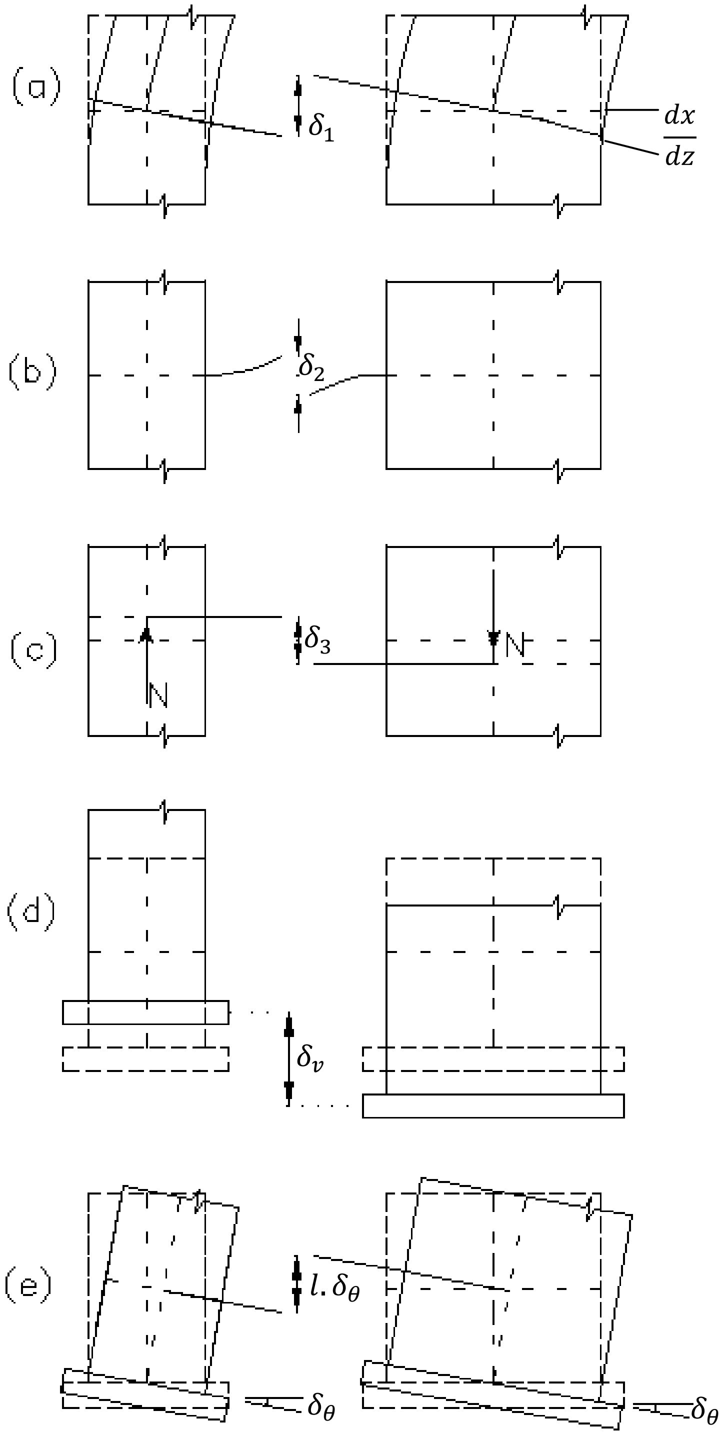


Figure 2.3 : Relative displacements of the laminas

1) Rotation of the wall cross section due to bending. (Figure 2.3 a). Due to the applied bending moment on the walls, the cross sections will rotate. It is worth to mention that there are two bending moments which act on the walls, first the bending moment due to external load and second the bending moment due to the shear force in the connecting laminas.

$$\delta_1 = \left(\frac{b}{2} + d_1\right).\frac{dx}{dz} + \left(\frac{b}{2} + d_2\right).\frac{dx}{dz} = l.\frac{dx}{dz} \tag{2.1}$$

2) Deformation of the connecting mediums due to shear flow. (Figure 2.3 b). By considering a small part of the connecting medium with depth of dz and the flexural rigidity of EI_e the deformation of the laminas due to shear flow can be derived[1]. For more details see appendix 1.

$$\delta_2 = -\text{q}.\frac{b^3 h}{12EI_e} \tag{2.2}$$

In which E is the modulus elasticity of the beams and EI_e is the equivalent flexural rigidity of the connecting beams. Note that I_e includes the effect of shearing and bending deformation of the beams and can be expressed as:

$$I_e = \frac{I_b}{1 + r} \tag{2.3}$$

$$r = \frac{12EI_b}{b^2 GA_b}.\lambda \tag{2.4}$$

3) Deformation of the walls due to axial force (Figure 2.3 c). As can be seen, due to the deflection of the connecting beams, the applied load will introduce a tensile force in wall 1 and a compressive force in wall 2. Note that in this the modulus of elasticity for both beams and walls is assumed to be equal which defines by E.

$$\delta_3 = -\frac{1}{E}.\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^z N(z)dz \tag{2.5}$$

4) Vertical and rotational relative displacements, at the base which will occur in the case of elastic foundation and it depend on the modulus of sub-grade reaction (Figure 2.3 d & e). As has been mentioned before, in the first part of this thesis, the walls on rigid foundation will be considered which means that, relative displacement (δ_v) and rotation (δ_θ) at base will be zero.

As is known, in reality there is no relative displacement at the point of contraflexure in the connecting mediums. If it is assumed that the summation of the relative displacements is equal to δ, then it can be concluded that δ is equal to zero.

$$\delta = \delta_1 + \delta_2 + \delta_3 = 0 \tag{2.6}$$

[1] The verification of the equivalent flexural rigidity of the beam and the deflection of laminas are given in appendix 1 section 1.1

$$\delta = l\left(\frac{d}{dz}x(z)\right) + \frac{1}{12}\frac{\left(\frac{d}{dz}N(z)\right)b^3h}{EI_e} - \frac{\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left(\int_0^z N(z)dz\right)}{E} = 0 \qquad (2.7)$$

2.1 Axial force in walls

Considering the vertical force equilibrium in the walls (Figure 2.4) , it can be concluded that the axial force in the walls at each level is equal to the integral of the shear flow in the connecting medium from that particular level to the top.

$$\left(\int_z^H q(z)dz\right) = N(z) \qquad (2.8)$$

By differentiating from the above equation, shear flow in the mediums can be obtained.

$$q = -\frac{dN}{dz} \qquad (2.9)$$

The bending moments in each wall is a superposition of a) free bending due to external applied load and b) reverse bending due to shear and axial force in the connecting beams. Considering the moment equilibrium at the centre line of the walls in the Figure 2.4, the moment curvature relationship of the walls can be written as:

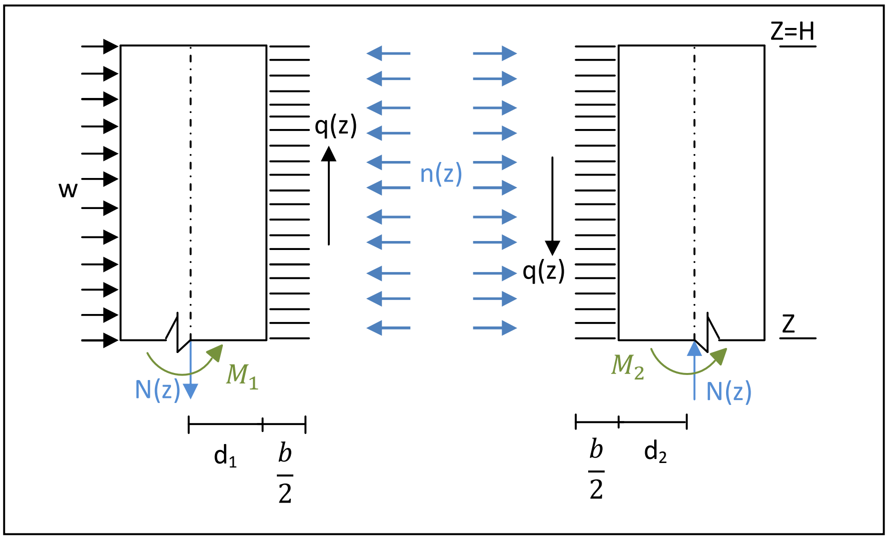


Figure 2.4 : Axial force in the walls

$$M_1 = m(z) - \left(\frac{b}{2} + d_1\right).\int_z^H q(z)dz - M_0 = EI_1.\frac{d}{dz}\left(\frac{dx}{dz}\right) \quad (2.10)$$

$$M_2 = -\left(\frac{b}{2} + d_1\right).\int_z^H q(z)dz + M_0 = EI_2.\frac{d}{dz}\left(\frac{dx}{dz}\right) \quad (2.11)$$

$$M_1 + M_2 = m(z) - l\left(\int_z^H q(z)dz\right) = (EI_1 + EI_2)\left(\frac{d^2}{dz^2}x(z)\right) \quad (2.12)$$

Where M_0 is the moment caused by the axial force, $n(z)$, in the connecting beams. $m(z)$ is the applied external moment which is equal to $\frac{w(H-z)^2}{2}$ in case if uniform distributed load. Substituting equation (2.8) into above equation gives the moment curvature relation equal to:

$$M_1 + M_2 = m(z) - l.N(z) = (EI_1 + EI_2)\left(\frac{d^2}{dz^2}x(z)\right) \quad (2.13)$$

This can be written as:

$$\left(\frac{d^2}{dz^2}x(z)\right) = \frac{m(z) - lN(z)}{(EI_1 + EI_2)} \quad (2.14)$$

By differentiating equation (2.7) and substituting equation (2.14) into it, the following equation can be derived:

$$\frac{d\delta}{dz} = l\left(\frac{d^2}{dz^2}x(z)\right) + \frac{1}{12EI_e}\left(\frac{d^2}{dz^2}N(z)\right)bh^3 - \left(\frac{1}{A_1} + \frac{1}{A_2}\right)\frac{N(z)}{E} = 0 \quad (2.15)$$

$$l\left(\frac{m(z) - lN(z)}{E(I_1 + I_2)}\right).\frac{12EI_e}{bh^3} + \left(\frac{d^2}{dz^2}N(z)\right) - \left(\frac{A_1 + A_2}{A_1.A_2}\right)\frac{N(z)}{E}.\frac{12EI_e}{bh^3} = 0 \quad (2.16)$$

Simplifying the above equation and applying the following notations:

$$I_1 + I_2 = i_t \quad (2.17)$$

$$A_1 + A_2 = A \quad (2.18)$$

Gives:

$$\frac{d^2}{dz^2}N(z) - N(z).\left(\frac{12I_e l^2}{b^3.h.i_t}\right).\left(1 + \frac{Ai_t}{A_1.A_2.l^2}\right) = -\frac{m(z)l}{i_t}.\left(\frac{12I_e}{b^3.h}\right) \quad (2.19)$$

Assuming the following parameters and substituting them into equation (2.19) gives a second-order differential equations based on the normal force of the shear walls:

$$\left(\frac{12.I_e.l^2}{b^3.h.i_t}\right) = \alpha^2 \qquad (2.20)$$

$$\left(1 + \frac{A.i_t}{A_1.A_2.l^2}\right) = k^2 \qquad (2.21)$$

Note that, the stiffness factor ($k\alpha H$) depends only on dimension of the coupled walls system. Obviously the stiffness of the walls system will be increased by increasing the magnitude of k and α.

$$\frac{d^2}{dz^2}N(z) - k^2\alpha^2.N(z) = -\frac{\alpha^2}{l}.m(z) \qquad (2.22)$$

The normal force in the shear walls can be derived by solving the ordinary differential equation (ODE) and using the boundary conditions. Note that the calculations have been made in Maple programme. More details about solving and simplified equations can be found in appendix 1.

Boundary conditions:

The walls can be considered as cantilever beams which are restrained at the bottom. According to equation (2.8), the axial force at the top is equal to zero which gives the first boundary condition . The second boudary conditions can be obtaied by inserting z=0 in equation (2.7). It is assumed that, the walls are rigidly connected to the foundation wich leads to a zero slope at the base. Therfore the first term in the compatibility equation (2.7) will become zero. The third term in cmopatibilty equation is also zero since, the integration range is from zero to zero. Consequently two required boundry condtions to solve the ODE (2.22)will be as follow:

1. $At\ z = H$

$$N(H) = 0 \qquad (2.23)$$

2. $At\ z = 0$

$$\frac{d}{dz}N(z) = 0 \qquad (2.24)$$

As a result, the equation of normal force in the shear walls will be:

$$N_{dist}(z) = w.\frac{H^2}{k^2.l}.\left(\frac{1}{2}.\left(1-\frac{z}{H}\right)^2 + \frac{1}{(k\alpha H)^2}.\left(1 - \frac{\cosh(k\alpha z) + k\alpha H.sinh(-k\alpha z + k\alpha H)}{\cosh(k\alpha H)}\right)\right) \qquad (2.25)$$

The normal force in the walls can also be rewritten as follow:

$$N_{dist}(z) = F_1.\left(w.\frac{H^2}{k^2 l}\right) \tag{2.26}$$

Where (F_1) is the axial force factor in the walls due to the applied external load and can be defined as following:

$$F_1 = \frac{1}{2}.\left(1-\frac{z}{H}\right)^2 + \frac{1}{(k\alpha H)^2}.\left(1-\frac{\cosh(k\alpha z)+k\alpha H.sinh(-k\alpha z+k\alpha H)}{\cosh(k\alpha H)}\right) \tag{2.27}$$

The variation of the axial force in the walls due to uniformly distributed load has been drawn for different height ratio and stiffness factor in Figure 2.5

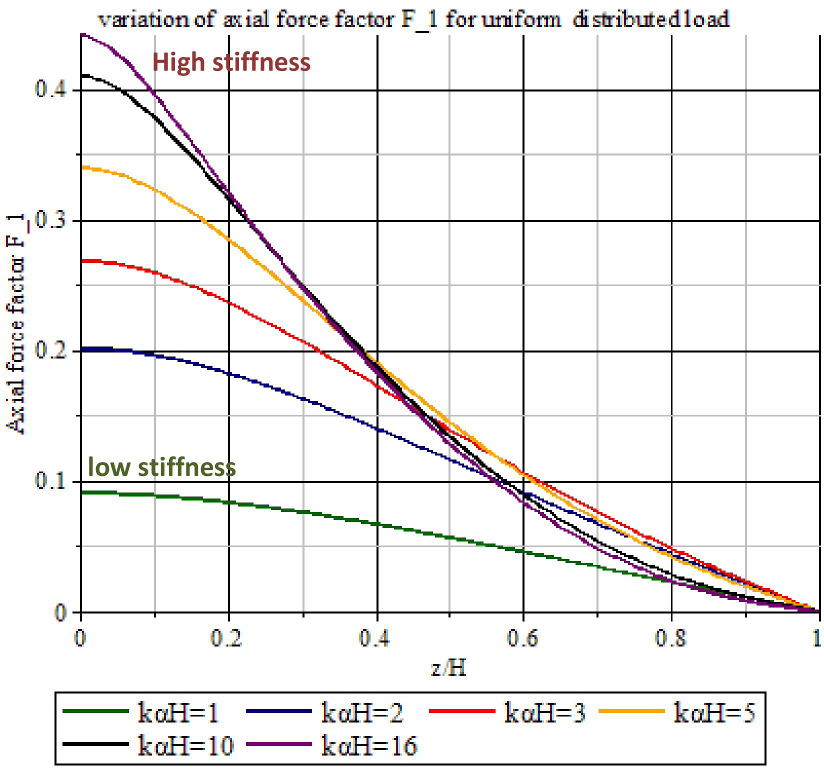


Figure 2.5: Variation of the axial force factor in the walls for uniform distributed load

Considering Figure 2.5, it can be seen that, the higher the stiffness of the connecting beams, the larger the amount of axial force in the walls will be for the same height ratio. According to equation (2.20) and (2.21)the stiffness factor $(k\alpha H)$ depends on the walls dimensions, connecting beam span, height of the connecting beams and the story height. $k\alpha H$ will be increased by decreasing the walls dimensions, decreasing the length of the connecting beams, increasing the story height and increasing the height of the connecting beams. Furthermore, as can be seen the axial force will be decreased along the height of the walls. The reason is that, the amount of normal force at each level is equal to the integration of the shear flow from that level to the top. At the lower level, the integration range is larger which leads into a lager axial force.

It is worth to mention that axial force in the walls is inversely proportional to the walls ‘bending moment. This means that, bending moment in the walls can be reduced by increasing the stiffness of the connecting beams which leads into increasing the axial force in the walls. Note that the equation of the bending moment will be derived later in section 0 and is given by equation (2.33).

2.2 Shear force in connecting beams

According to equation (2.9) by differentiating from the axial force in the wall, the shear flow in the connecting mediums can be determined:

$$q_{dist} = \frac{wH}{k^2 l} \cdot \left(\frac{1}{\mathrm{k\alpha H.}\cosh(k\alpha H)} \big(\sinh(k\alpha z) - k\alpha H.\cosh(k\alpha(H-z))\big) + k\alpha H.\cosh(k\alpha H) - k\alpha z.\cosh(k\alpha H) \right) \quad (2.28)$$

This equation can also be rewritten as a function of height ratio and stiffness ratio:

$$q_{dist}(z) = \frac{wH}{k^2 l} . F_2 \quad (2.29)$$

In which F_2 is the shear flow factor in the connecting mediums and can be given by the following equation:

$$F_2 = \frac{1}{\mathrm{k\alpha H.}\cosh(k\alpha H)} \big(\sinh(k\alpha z) - k\alpha H.\cosh(k\alpha(H-z))\big) + k\alpha H.\cosh(k\alpha H) - k\alpha z.\cosh(k\alpha H) \quad (2.30)$$

The variation of shear flow factor (F_2) versus the height ratio and the stiffness factor $k\alpha H$ has been illustrated in the Figure 2.6. As can be seen from the curves, the shear flow in the connecting beams is directly related to the stiffness parameter $K\alpha H$ which means that, increasing the stiffness leads to increasing shear flow in lamellas. Moreover, by increasing $K\alpha H$ the position of the maximum loaded beam will move downwards which is illustrated by the dashed line on the curve.

The shear force in each individual connecting beam can be calculated by integrating the shear flow over the height of each story. Calculating the area under the shear flow curve for a specified level gives the shear force in that beam.

$$Q_{distributed}(z) = \int_{z_i-\frac{h}{2}}^{z_i+\frac{h}{2}} q_{dist}(z)dz \tag{2.31}$$

$$Q_{distributed}(z) = \frac{wHh}{k^2 l} + \frac{2w.sinh(k\alpha z_i).sinh\left(\frac{1}{2}k\alpha h\right)}{k^4 l\alpha^2.cosh(k\alpha H)} + \frac{2w.Hsinh(k\alpha H).\sinh(k\alpha z_i).sinh\left(\frac{1}{2}k\alpha h\right)}{k^3 l\alpha.cosh(k\alpha H)} - \frac{2w.Hcosh(k\alpha z_i).sinh\left(\frac{1}{2}k\alpha h\right)}{k^3 l\alpha} - \frac{wz_i h}{k^2 l} \tag{2.32}$$

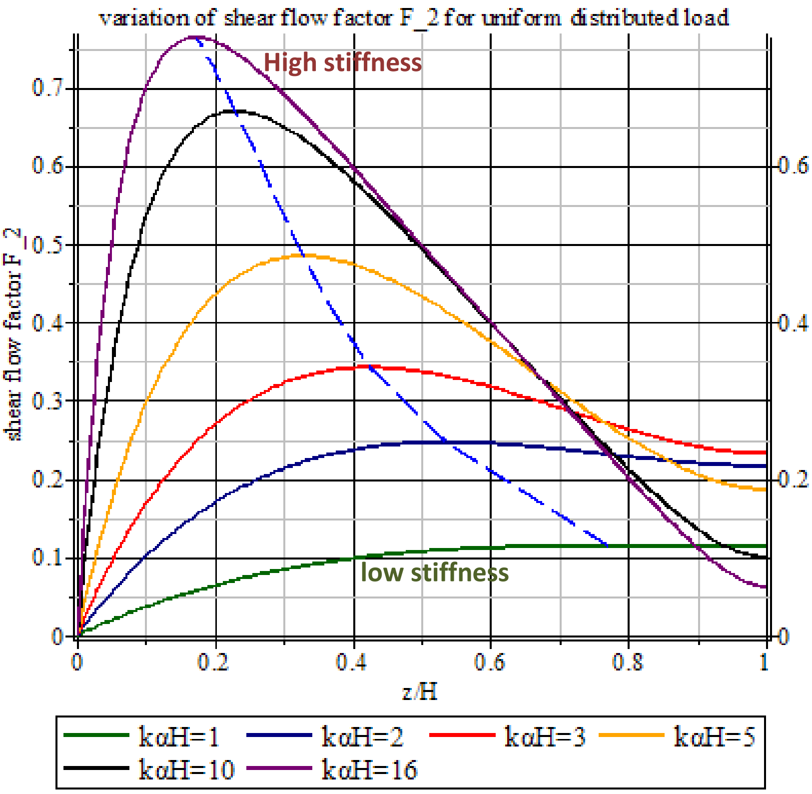


Figure 2.6 : Variation of shear flow in the beams for uniform distributed load.

2.3 Bending moment in the walls

By considering moment curvature relation of the walls and the assumption of identical deflection of the walls, it can be concluded that, bending moment in each wall is proportional to its flexural rigidity. So for the coupled shear walls subjected to uniform distributed load, the moment at each level of height (z) will be:

$$M_1 = \frac{I_1}{i_t} . \left(\frac{1}{2} wH^2 \left(1 - \frac{z}{H}\right)^2 - N(z). l \right) \qquad (2.33)$$

Where N(z) is the axial force in the walls and it has been derived earlier. According to the equation (2.26) the bending moment of the walls can be rewritten as follows:

$$M_1 = \frac{I_1}{2. i_t} . wH^2 \left(\left(1 - \frac{z}{H}\right)^2 - \frac{2}{k^2} . F_1 \right) \qquad (2.34)$$

$$M_2 = \frac{I_2}{2. i_t} . wH^2 \left(\left(1 - \frac{z}{H}\right)^2 - \frac{2}{k^2} . F_1 \right) \qquad (2.35)$$

As can be seen the axial force in the walls has an inverse effect on the amount of the walls' bending moment. The larger the axial force, the smaller the bending moment will be.

2.4 Deflection

The equation of the lateral deflection of the system can be determined by using the equation (2.7) and (2.14) and getting rid of the axial force. As a result, a fourth order differential equation according to the height of the structure will be found.

$$N(z) = \frac{m(z) - \left(\frac{d^2}{dz^2} x(z) \right) E i_t}{l} \qquad (2.36)$$

$$\frac{d^2}{dz^2} N(z) = \frac{\left(\frac{d^2}{dz^2} m(z) \right) - \left(\frac{d^4}{dz^4} x(z) \right) E i_t}{l} \qquad (2.37)$$

$$\frac{d\delta}{dz} = l \left(\frac{d^2}{dz^2} x(z) \right) + \frac{1}{12 E I_e} \left(\frac{d^2}{dz^2} N(z) \right) bh^3 - \left(\frac{1}{A_1} + \frac{1}{A_2} \right) \frac{N(z)}{E} = 0 \qquad (2.38)$$

Substituting the second derivative of the normal force from equation (2.36) into the first derivative of the compatibility equation (2.7) gives the equation of the deflection.

$$l\left(\frac{d^2}{dz^2}x(z)\right)+\frac{b^3h}{12EI_el}\left(\frac{d^2}{dz^2}m(z)-\frac{d^4}{dz^4}x(z).(EI_1+EI_2)\right) -\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\frac{1}{El}\left[m(z)-(EI_1+EI_2)\left(\frac{d^2}{dz^2}x(z)\right)\right]=0 \quad (2.39)$$

By substituting the equations of $k^2$and α^2, the ODE of deflection can be written as follows:

$$\left(\frac{d^4}{dz^4}x(z)\right)-\left(\frac{d^2}{dz^2}x(z)\right)k^2\alpha^2=\frac{1}{Ei_t}.\left(\frac{d^2}{dz^2}m(z)-m(z).(k^2-1)\alpha^2\right) \quad (2.40)$$

Where for walls subjected to uniform distributed load, external bending moment is equal to:

$$m(z)=\frac{w(H-z)^2}{2}$$

Boundary conditions

The governing boundary conditions at the top and bottom of the coupled walls on the rigid foundation are:

1. $At\ x=0$

$$x(z)=0 \quad (2.41)$$

$$\frac{dx}{dz}|_{z=0}=0 \quad (2.42)$$

2. $At\ x=H$

$$\frac{d^2}{dz^2}|_{z=H}=0 \quad (2.43)$$

$$\frac{d^3}{dz^3}x(z)-(k\alpha)^2.\frac{d}{dz}x(z)=\frac{1}{Ei_t}.\left(\frac{d}{dz}m(z)-\alpha^2(k^2-1).\int_0^H m(z)dz\right) \quad (2.44)$$

By determining the value of N and its first derivative $\frac{dN}{dz}$ according to the equation (2.36) and substituting them into the compatibility equation (2.7),the last boundary condition at z=H can be derived.

Now, the ordinary differential equation of the deflection can be solved by using these boundary conditions. Consequently the deflection of the walls due to uniform distributed load $x(z)_{dist}$ at each height level will be:

$$x(z)_{dist} = \frac{1}{24} \cdot \frac{1}{k^6 E.\, i_t \alpha^4 cosh\,(k\alpha H)} \cdot \big(w(6\alpha^4 k^6 H^2 z^2 .\, cosh(k\alpha H) - 4z^3 H k^6 \alpha^4 .\, cosh(k\alpha H)$$
$$+ z^4 k^6 \alpha^4 .\, cosh(k\alpha H) - 6\alpha^4 H^2 k^4 z^2 .\, cosh(k\alpha H) + 4\alpha^4 H z^3 k^4 .\, cosh(k\alpha H)$$
$$- \alpha^4 z^4 k^4 .\, cosh(k\alpha H) + 24 z H k^2 \alpha^2 .\, cosh(k\alpha H) - 12 z^2 k^2 \alpha^2 .\, cosh(k\alpha H)$$
$$+ 24 H k \alpha .\, sinh(k\alpha H - k\alpha z) - 24\, k\alpha H .\, sinh(k\alpha H) + 24 .\, cosh(k\alpha z) - 24)\big) \quad (2.45)$$

According to this equation the deflection depends on the height (z) and stiffness parameter α and k. Therefore it is not possible to draw a diagram which shows the variation of the deflection against the height ratio. It is known that the maximum deflection occurs at the top of the coupled walls. Therefore, the height level (z) in the above equation will be substituted equal to total height of the wall (H) which gives the maximum lateral deflection of the wall as a function of $K\alpha H$ and K. The variation of the deflection factor is illustrated in Figure 2.7.

$$x_{dist}(H) = \frac{wH^4}{8Ei_t} .\, F_3 \quad (2.46)$$

Where F_3 is the deflection factor of the walls system due to the uniform distributed load and can be given by following equation:

$$F_3 = -\frac{1}{k^2} + 1 - \frac{8.\, \sinh\,(k\alpha H)}{H^3 \alpha^3 k^5 .\, \cosh\,(k\alpha H)} - \frac{8}{H^4 \alpha^4 k^6 .\, \cosh\,(k\alpha H)} + \frac{4}{H^2 \alpha^2 k^4} + \frac{8}{H^4 \alpha^4 k^6} \quad (2.47)$$

Considering Figure 2.7, it can be seen that, as expected the larger stiffness of the walls leads to a smaller lateral deflection at the top. Furthermore, for the larger amount of k the deflection at the top is also larger. As can be seen in equation (2.20) and (2.21) the stiffness parameters k and α depend on the dimension of the walls and beams. Increasing the amount of k when the value of $k\alpha H$ remains constant means that, the value of α have been decreased. The value of α can be diminished by reducing the beam height which will have negative influence on coupling effectivity of the shear walls. As a result, the top deflection of the walls will be increased.

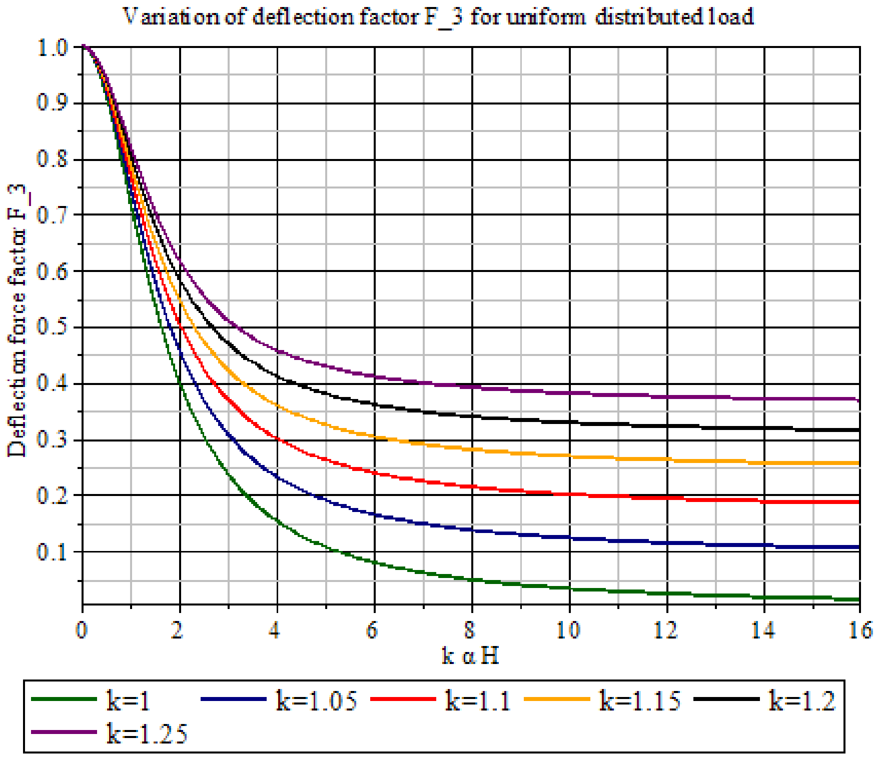


Figure 2.7 : Variation of top deflection factor for uniform distributed load

2.5 Walls' shear

The shear force of the walls can be derived by considering the moment equilibrium of a small part of the wall which is shown in Figure 2.8 .

$$\sum M = (M_1 + \delta M_1) - M_1 + (S_1 + \delta S_1).dz + \delta N.\left(\frac{b}{2} + d_1\right) = 0 \quad (2.48)$$

Considering the same conditions for the wall 2, the shear force in each wall will be:

$$S_1 = \left(\frac{I_1}{i_t}.l - \frac{b}{2} - d_1\right).\frac{dN}{dz} - \frac{I_1}{i_t}.\frac{d}{dz}m(z) \quad (2.49)$$

$$S_2 = \left(\frac{I_2}{i_t}.l - \frac{b}{2} - d_2\right).\frac{dN}{dz} - \frac{I_2}{i_t}.\frac{d}{dz}m(z) \quad (2.50)$$

In which m(z) is the external moment and for the uniform distributed load is equal to

$m(z) = \frac{1}{2}w(H-z)^2$ and $\frac{dN}{dz} = -q$

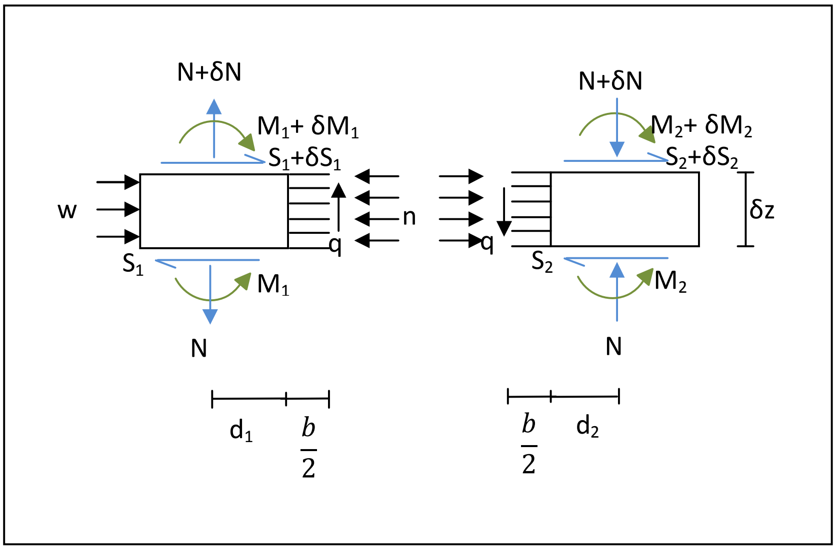


Figure 2.8 : Internal forces on small element of the coupled walls

Hence, the equation of the shear force of the walls can be rewritten as following:

$$S_1 = \frac{w.H.I_1}{i_t}\left(1-\frac{z}{H}\right) - \left(\frac{I_1}{i_t}l - \frac{b}{2} - d_1\right).q(z) \tag{2.51}$$

$$S_2 = \frac{w.H.I_2}{i_t}\left(1-\frac{z}{H}\right) - \left(\frac{I_2}{i_t}l - \frac{b}{2} - d_2\right).q(z) \tag{2.52}$$

In which q is defined by equation (2.28) and it is illustrated graphically in Figure 2.6. It is worth to mention that, if both walls have the same properties the shear force in the walls for a uniform distributed load is:

$$S_1 = S_2 = \frac{wH}{2}\left(1-\frac{z}{H}\right) \tag{2.53}$$

2.6 Axial force in the connecting beams

The axial force in the connecting beams can be derived by considering the horizontal force equilibrium of the small part of the walls which is shown in Figure 2.8.

Horizontal equilibrium in wall 1 (left part in Figure 2.8)

$$\sum F_H = n.dz - w.dz - S_1 - \delta S_1 + S_1 = 0 \tag{2.54}$$

Horizontal equilibrium in wall 2 (right part in Figure 2.8)

$$\sum F_H = n.dz + S_2 + \delta S_2 - S_2 = 0 \qquad (2.55)$$

$$n = \frac{1}{2}.\left(\frac{d}{dz}S_1 - \frac{d}{dz}S_2 + w\right) \qquad (2.56)$$

S_1 and S_2 have been defined in equations (2.51) and (2.52). As a result the equation of the normal force in the connecting beams can be derived.

$$n = \frac{wI_2}{i_t} - \frac{1}{k^3.l.\alpha.cosh(k\alpha H)}.\left(\left(\frac{I_1 l}{i_t} - \frac{b}{2} - d_1\right).w.\left(k\alpha.\cosh(k\alpha z) + \mathrm{k}^2\alpha^2\mathrm{H}.\sinh\left(k\alpha(H-z)\right) - \mathrm{k}\alpha.\cosh(k\alpha H)\right)\right) \qquad (2.57)$$

The normal force in the beams can also be rewritten as follows:

$$n = w\left[\frac{I_2}{i_t} - \frac{1}{k^2.l}\left(\frac{I_1}{i_t}.l - \frac{b}{2} - d_1\right).F_{4,distributed}\right] \qquad (2.58)$$

In which $F_{4,distributed}$ is the axial force factor in the connecting mediums due to uniform distributed load and can be given by the following equations:

$$F_{4,distributed} = \frac{\cosh(k\alpha z) + k\alpha H.\sinh\left(\mathrm{k}\alpha(\mathrm{H}-\mathrm{z})\right) - \cosh(k\alpha H)}{cosh(k\alpha H)} \qquad (2.59)$$

From Figure 2.9 can be seen that, the maximum variation of the axial load factor in the connecting beams occurs at the lower level of the walls. Considering equation (2.58), the beams at the lower level of the walls carry larger compression force. Further, it can be seen that, increasing the amount of $k\alpha H$ leads into higher axial force in the connecting beams.

As well as the magnitude of axial force, the position of the minimum loaded beam depends also on the stiffness factor. The smaller the stiffness factor ($k\alpha H$), the higher the minimum loaded beam is positioned. Further it can be seen that, for the walls with higher stiffness factor the normal force in the beams changes significantly along the height of the walls though, for the walls with lower $k\alpha H$ this value will change slightly.

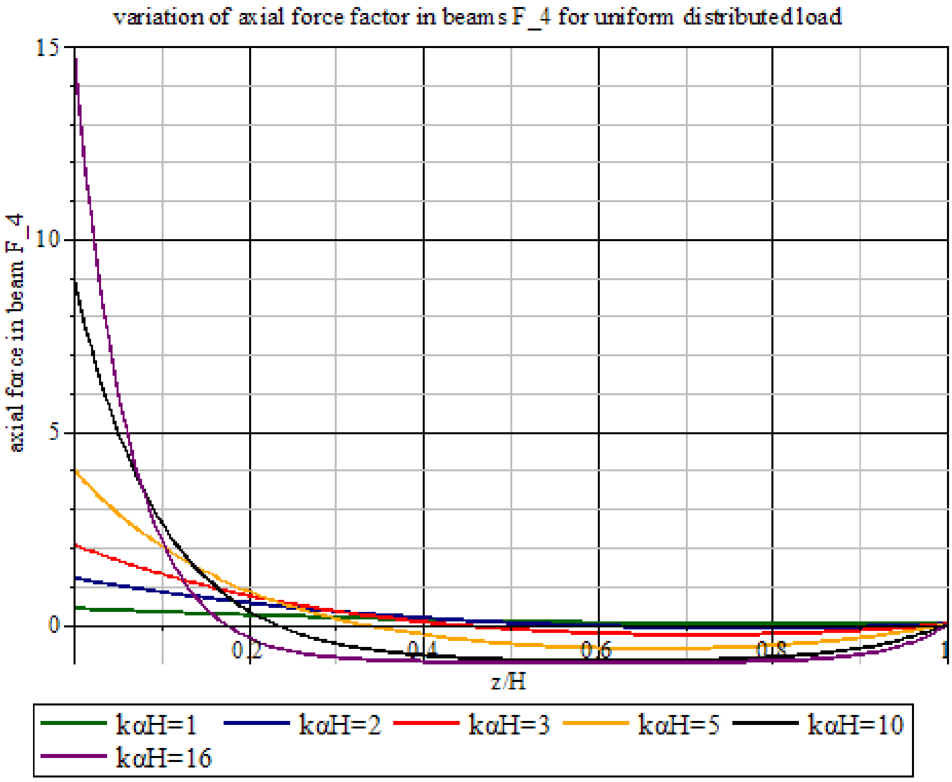


Figure 2.9 : Variation of axial force factor in the beams for uniform distributed load

2.7 Stress in the walls

Up to now the internal forces and the deflection of the coupled shear walls have been considered which are depending on the stiffness factor, $k\alpha H$, and the height ratio, $\frac{z}{H}$. Another important aspect about the coupled shear walls is the efficiency of the coupling which has a large impact on the design and optimization of the connection. Consider a pair of coupled shear walls. The stress distribution at any section due to the applied load is shown in Figure 2.10. The actual stress at each section is a superposition of the stress due to axial load on the wall and the stress due to acting bending moment in each wall.

$$\sigma_A = \frac{(m(z) - (N_{dist,load}).l)I_1.c_1}{i_t} + \frac{N_{dist,load}}{A_1} \tag{2.60}$$

$$\sigma_B = \frac{(m(z) - (N_{dist,load}).l)I_1.d_1}{i_t} + \frac{N_{dist,load}}{A_1} \tag{2.61}$$

In which $N_{dist,load}$ is the axial force in the walls. Further, i_t and A_1 are the summation of second moment of area of the walls 1 and 2 and cross sectional area of wall 1 respectively. Note that, the same expression holds for the stress in wall 2 at point C and D.

The actual stress distribution can also be derived by an alternative superposition of:

1. Assuming that the coupled walls act as a single composite cantilever system. In this case the neutral axis is situated at the centroidal axes of the wall element, as has been illustrated in Figure 2.10 c. If it is assumed that K_2 is the percentage of the bending moment which is resisted by the walls as a composite cantilever system, the total moment which will be carried by the walls is:

$$M_{composite} = m(z) * \frac{k_2}{100} \tag{2.62}$$

Consequently the stress distribution at each section will be:

$$\sigma_A = \frac{m(z)}{\left(i_t + \frac{A_1 A_2}{A}.l^2\right)} . \left(\frac{A_2.l}{A} + c_1\right) . \left(\frac{k_2}{100}\right) \tag{2.63}$$

$$\sigma_B = \frac{m(z)}{\left(i_t + \frac{A_1 A_2}{A}.l^2\right)} . \left(\frac{A_2.l}{A} - d_1\right) . \left(\frac{k_2}{100}\right) \tag{2.64}$$

2. Assuming that the coupled shear walls act as two completely independent cantilevers. In this case the neutral axis is situated at the centre of each wall. If K_1 is the percentage of the bending moment which is resisted by the two independent cantilever system the total moment carried by the walls will be:

$$M_{Individual} = m(z) * \frac{k_1}{100} \tag{2.65}$$

$$\sigma_A = \frac{M_1.c_1}{I_1} = \frac{m(z)c_1}{i_t} . \frac{(100 - k_2)}{100} \tag{2.66}$$

$$\sigma_B = -\frac{M_1.d_1}{I_1} = -\frac{m(z)d_1}{i_t} . \frac{(100 - k_2)}{100} \tag{2.67}$$

Substituting summation of equation (2.60), (2.63) equal to equation (2.66), the value of composite factor k_2 can be determined.

$$k_2 = \frac{200}{(k\alpha H)^2 . \left(1 - \frac{z}{H}\right)^2} . \left(\frac{-cosh(k\alpha z) - k\alpha H. sinh(k\alpha H - k\alpha z)}{cosh(k\alpha H)} + 1 + \frac{k^2\alpha^2}{2}(H - z)^2\right) \tag{2.68}$$

The variation of k_1 and k_2 as a function of height ratio and $k\alpha H$ has been illustrated in Figure 2.11. As can be seen, in the higher level of the walls the composite action factor is larger which can be concluded that, coupling of the walls is more effective for the tall buildings. Furthermore, it can be seen that the value of composite action will be increased by enlarging the stiffness factor, $k\alpha H$ till a certain point. Increasing the stiffness factor additionally, will reduce significantly the composite factor at the higher level of the walls. Moreover, increasing the stiffness parameter at the lower level does not change the magnitude of composite factor greatly.

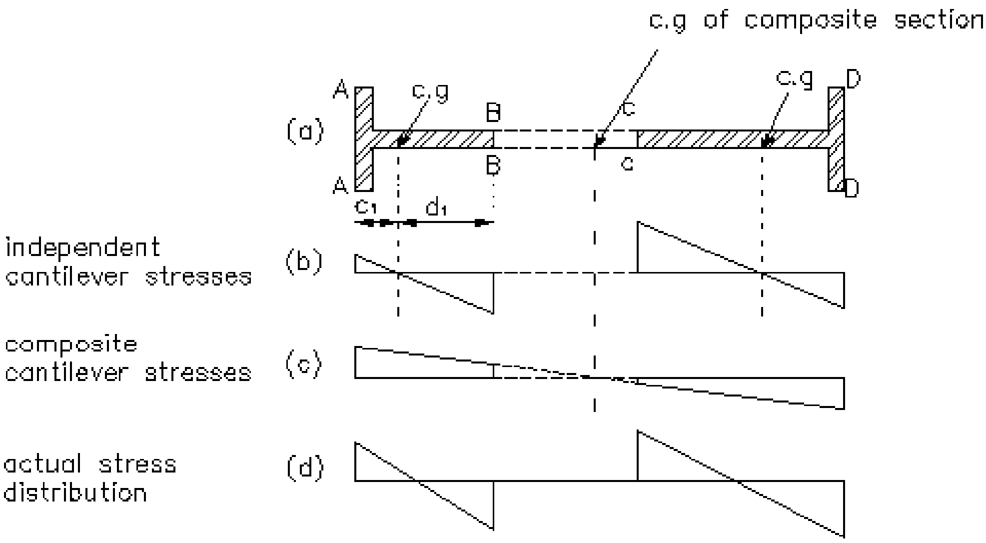


Figure 2.10 : Stress distribution due to composite and individual cantilever action

Note that, c.g is used as a notation for centre of gravity.

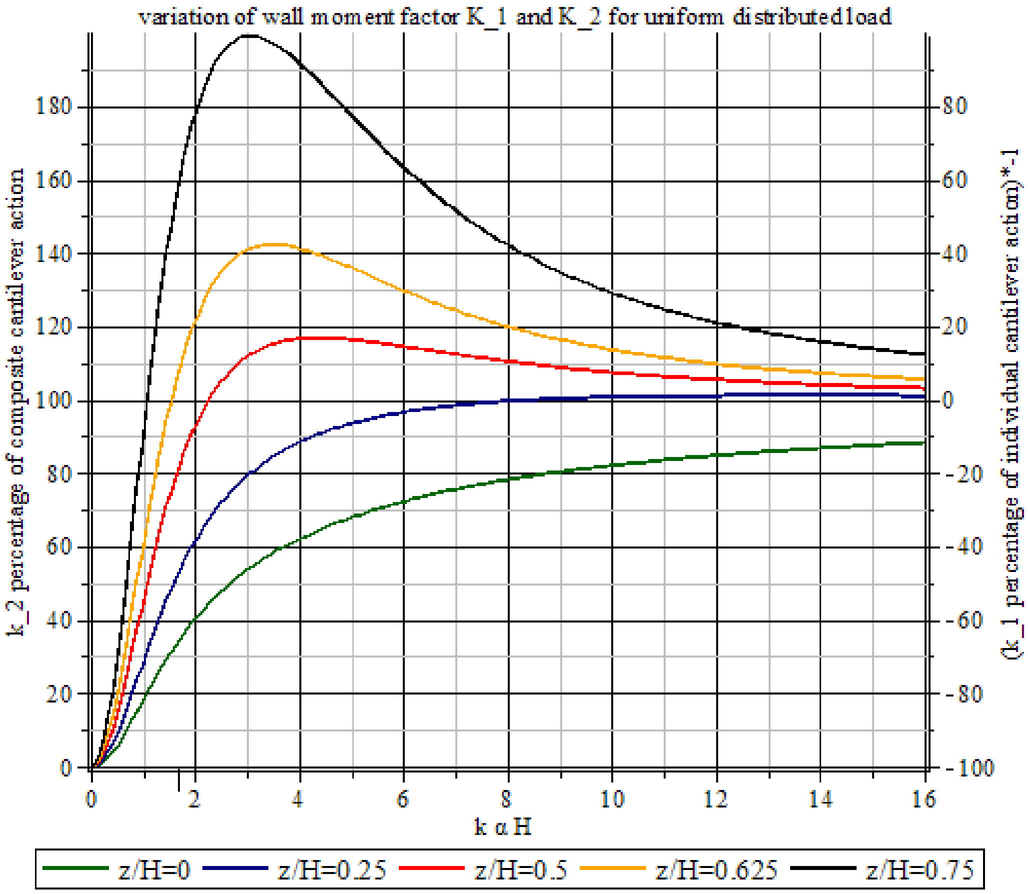


Figure 2.11 : Variation of wall moment composite and individual action factor K_1 and K_2

Design example 1:

Consider a typical system of coupled shear walls with one row of opening shown in Figure 2.12. The dimensions are given in
Table 1. It is assumed that the system consists of 20 stories and it is subjected to a uniform distributed load with a magnitude of 17kN/m.
To determine the stress, bending moment, axial force and the deflection of the walls and connecting beams the curves from the previous sections have been used.

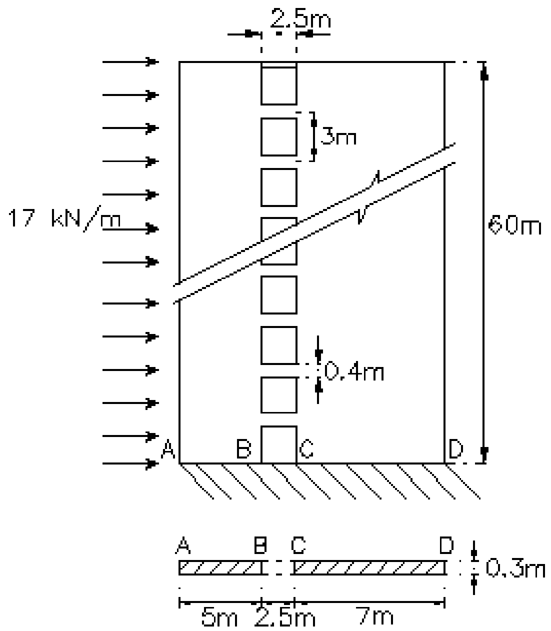


Figure 2.12: Example structure

i. Determine the area and the second moment of area of the connecting beams and the shear walls illustrated in Figure 2.12. It should be noted that in this example a high modulus of elasticity is used for the walls and beams since the walls system is assumed to be uncracked.

Table 1: properties of the coupled walls

$2d_1$	[m]	5
d_1	[m]	2.5
$2d_2$	[m]	7
d_2	[m]	3.5
b	[m]	2.5
l	[m]	8.5
h	[m]	3
H	[m]	60
t_{wall}	[m]	0.3
t_{beam}	[m]	0.4
G	[kN/m^2]	15000000
E	[kN/m^2]	36000000
ν		0.2

Table 2: Cross sectional properties of connected beams and shear walls

I_1	[m^4]	3.125
I_2	[m^4]	8.575
i_t	[m^4]	11.7
A_1	[m^2]	1.5
A_2	[m^2]	2.1
$A = A_1 + A_2$	[m^2]	3.6
A_b	[m^2]	0.12
I_b	[m^4]	0.0016
λ		1.2
$r = \frac{12EI_b}{b^2GA_b}\lambda$		0.073728
$I_e = \frac{I_b}{1+r}$	[m^4]	0.00149

ii. Determine the stiffness parameter $K\alpha H$

$$k^2 = \left(1 + \frac{A.i_t}{A_1.A_2.l^2}\right) = \left(1 + \frac{3.6 * 11.7}{1.5 * 2.1 * 8.5^2}\right) = 1.18$$

$$k = 1.088$$

$$\alpha^2 = \left(\frac{12.I_e.l^2}{b^3.h.i_t}\right) = \left(\frac{12 * 0.00149 * 8^2}{2.5^3 * 3 * 11.7}\right) = 0.00235$$

$$\alpha = 0.0485$$

$$k\alpha H = 1.099 * 0.0456 * 60 = 3.17$$

iii. By using the curves in Figure 2.5 and equation (2.26) the axial force at any level of the walls can be determined.

iv. The bending moment throughout the height in each wall can be determined according to the equation (2.34) and (2.35). The variation of the axial force and the bending moment for this example have been shown in Figure 2.13 to indicate the effect of coupling on the magnitude of bending moment in the walls. As can be seen, the positive bending moment at the upper level of the walls turned into negative which is the consequence of the bending moment in the connecting beams. Furthermore it can be noticed from the bending moment curves that, the effect of the coupling reduces toward the base but it is still efficient.

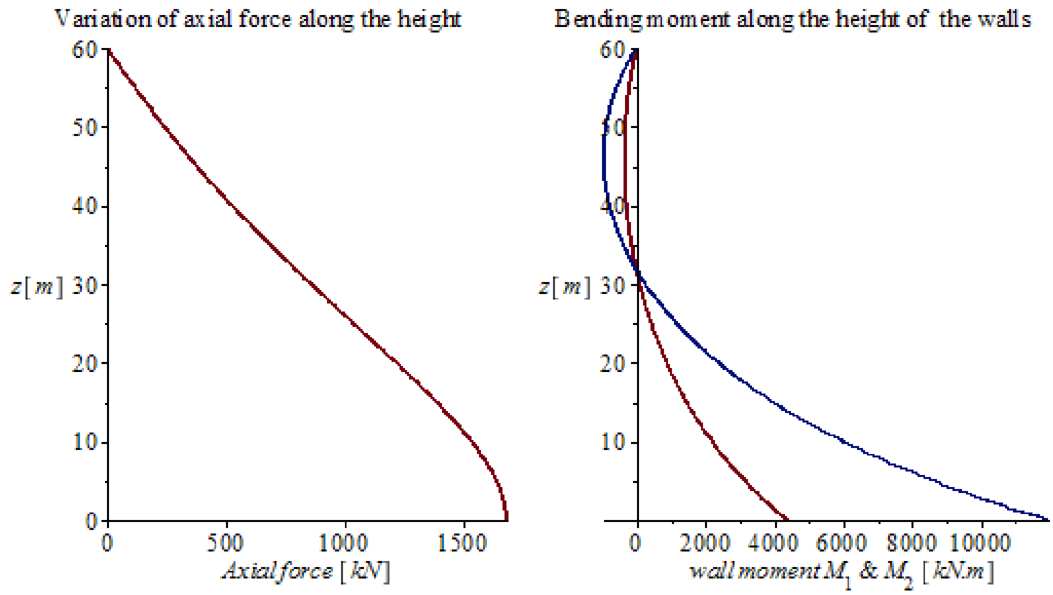


Figure 2.13: Distribution of the axial force and bending moment in the walls

v. To determine the max shear force, the curve from the Figure 2.6 has been used. The maximum shear occurs at the level of $\frac{z}{H} = 0.412$ and the maximum shear factor F_2 at this level is about 0.36. as a result the maximum shear flow will be:

$$q_{max} = \frac{wH}{k^2 l}.F_2 = \frac{17 * 60}{1.185 * 8.5} * 0.36 = 36.5 \frac{\text{kN}}{\text{m}}$$

Consequently the maximum shear force in connecting beams will be:

$$Q_{max} = q_{max}.h = 109.4\ kN$$

The maximum possible moment in any connecting beam is:

$$M_{max} = Q_{max}.\frac{b}{2} = 109.36 * \frac{2.5}{2} = 136.7\ kN.m$$

It should be noted that, with this method the amount of shear force in the connecting beams can be overestimated. The reason is that here, it is assumed that the maximum shear flow is constant throughout the height of a story though, the accurate shear force in a connecting beam is equal to the integration of the shear flow from $\frac{h}{2}$ above to $\frac{h}{2}$ under the considered beam. The precise maximum shear force in this example is equal to 108.17 kN. Note that, because in this example the beams have intermediate stiffness, the difference is negligible but for the beams with higher stiffness the difference will be considerable.

vi. To determine the maximum deflection at the top, Figure 2.7 will be used to figure out the deflection factor and then the top deflection will be given by equation (2.46). For $k = 1.088$ and $k\alpha H = 3.17$, F_3 is equal to 0.35.

$$x(H) = \frac{wH^4}{8Ei_t}.F_3 = \frac{17 * 60^4}{8 * 36 * 10^6 * 11.7} * 0.35 = 0.0228\, m$$

As can be seen due to coupling, the maximum deflection at the top is reduced by to 0.355 of its origin value, this means that if the walls were not coupled the maximum deflection would be:

$$x(H) = \frac{wH^4}{8Ei_t} = 0.0653\, m$$

vii. Now the effect of the coupling will be considered by determining the percentage of moment carrying by individual cantilever action (K_1) and the percentage carrying by composite cantilever action(K_2). Using the curves in Figure 2.11 gives :

$$K_1 = 45\% \quad K_2 = 55\%$$

It is known that the maximum moment is applied at the base of the wall.

$$m = \frac{1}{2}w.H^2 = \frac{1}{2} * 17 * 60^2 = 30600\, kN.m$$

Part of the moment carried by individual action: $K_1.m = 0.45 * 30600 = 13770\, kN.m$

The bending moment in each wall is proportional to second moment of area.

Moment on wall 1: $M_1 = \frac{I_1}{i_t}.m.K_1 = \frac{3.125}{11.7} * 13770 = 3678\ \ kN.m$

Moment on wall 1: $M_1 = \frac{I_2}{i_t}.m.K_1 = \frac{8.575}{11.7} * 13770 = 10092\, kN.m$

Part of the moment carried by composite action: $K_2.m = 0.55 * 30600 = 16830\, kN.m$

viii. Having the percentage of the composite cantilever action and the individual cantilever action, the stress at the extreme fibre of the walls can be calculated by using beam theory.

Effective composite second moment of area of the wall system:

$$I_g = I_1 + I_2 + \frac{A_1.A_2}{A}.l^2 = 3.125 + 8.575 + \frac{1.5 * 2.1}{3.6} * 8^2 = 67.7\, m^4$$

Centre of the gravity for composite wall system is positioned at a distance of 7.45 m from the left corner of the wall which is shown in Figure 2.12.

$$\sigma_A = \frac{3677.88 * 2.5}{3.125} + \frac{16830 * 7.45}{67.7} = 4617.8 \frac{kN}{m^2}$$

$$\sigma_B = -\frac{3677.88 * 2.5}{3.125} + \frac{16830 * 2.45}{67.7} = -2390.1 \frac{kN}{m^2}$$

$$\sigma_C = \frac{10092.12 * 3.5}{8.575} - \frac{16830 * 0.05}{67.6} = 4109.9 \frac{kN}{m^2}$$

$$\sigma_D = -\frac{10092.12 * 3.5}{8.575} - \frac{16830 * 7.05}{67.6} = -5701.1 \frac{kN}{m^2}$$

It is worth to mention that the stress for the extreme fibre of the walls for the uncoupled walls in which the walls behave individually will be:

$$\sigma_A = -\sigma_B = \frac{\left(\frac{I_1}{i_t}.m\right).y}{I_1} = \frac{30600 * 2.5}{11.7} = 6538.5 \frac{kN}{m^2}$$

$$\sigma_C = -\sigma_D = \frac{\left(\frac{I_2}{i_t}.m\right).y}{I_2} = \frac{30600 * 3.5}{11.7} = 9153.9 \frac{kN}{m^2}$$

ix. Further, the axial force in the beams and the shear force in the walls at the most heavily loaded section can be determine. Using equation (2.51) and (2.52) gives the shear force at the base of the walls which is proportional to their flexural rigidity.

$$S_1 = \frac{w.H.I_1}{i_t}\left(1 - \frac{z}{H}\right) - \left(\frac{I_1}{i_t} l - \frac{b}{2} - d_1\right).q(z) = \frac{17 * 60 * 3.125}{11.7} = 272.5\ kN$$

$$S_2 = \frac{w.H.I_2}{i_t}\left(1 - \frac{z}{H}\right) - \left(\frac{I_2}{i_t} l - \frac{b}{2} - d_2\right).q(z) = \frac{17 * 60 * 8.575}{11.7} = 747.6\ kN$$

The curve in Figure 2.9 gives the axial force factor at the base (F_4) equal to 2.17 . By using equation (2.58) the axial load can be determined.

$$n = w\left[\frac{I_2}{i_t} - \frac{1}{k^2.l}\left(\frac{I_1}{i_t}.l - \frac{b}{2} - d_1\right).F_4\right]$$
$$= 17 * \left[\frac{8.575}{11.7} - \frac{1}{1.185 * 8.5}\left(\frac{3.125}{11.7} * 8.5 - \frac{2.5}{2} - 2.5\right) * (2.17)\right] = 17.9 \text{ kN}$$

Besides the maximum axial force, the magnitude of the axial force for the maximum loaded beam in shear is also important:

$$n = (17 * [0.7329 + 0.1469 * 0.0033]) * 3 = 37.4\ kN$$

3 Walls supported on elastic foundation:

In the previous chapter, the behaviour of the coupled shear walls supported on the rigid foundation for uniform distributed load has been considered. However in reality the shear walls are mostly supported on piles or a portal frame to provide the required open areas. As a result, relative rotational and or vertical displacements will occur at the base between the two coupled walls which leads into different behaviour and internal force of the walls. These types of the supports can be modelled as linear or rotational springs. It has to be noted that, the general equation and the approach of the solution will be the same as the walls on rigid foundation. But in this case, the relative displacement at the base will not be zero anymore and it has to be taken into account.

The walls can be supported by a single grade beam on the elastic foundation or by two individually grade beams. All these cases will be considered in this chapter and the next chapter.

In the case of individually supported walls on the elastic foundation, the relative rotational and vertical displacements of the walls at base are identical to the rotational and vertical displacement of the base beams. The relative vertical and rotational displacements are illustrated in Figure 2.3 d,e.

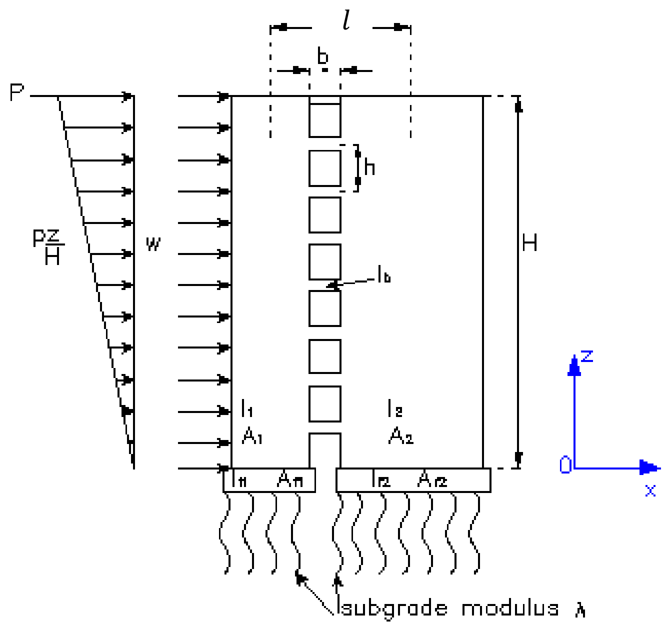


Figure 3.1: Coupled shear walls on elastic foundation

According to the continuous method, the connecting beams will be replaced by continuous mediums. The mediums will be cut along the line of contraflexure in the mediums. The relative displacements at the cut ends will be considered. The total displacement δ, at the cut ends consist of :

$$\delta = \delta_1 + \delta_2 + \delta_3 + \delta_4 \tag{3.1}$$

δ_1, δ_2 and δ_3 has been explained in chapter 1 and given by equation (2.1), (2.2) and (2.5) respectively.

$$\delta_4 = \delta_\theta . l - \delta_v \tag{3.2}$$

In which, δ_θ and δ_v are rotation and vertical displacement at the base respectively. l is the distance between the centroidal axes of the walls.

$$\delta_v = \frac{N_0}{k_{v1}} + \frac{N_0}{k_{v2}} = N_0\left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}}\right) = \frac{N_0}{k_v} \tag{3.3}$$

It is known that the slope and the curvature of both walls are identical at each level because, it is assumed that the horizontal deflection of both walls is identical at any height. Hence, at the base the slope of wall 1 and 2 will be also equal.

$$\delta_\theta = \frac{M_{10}}{k_{\theta1}} = \frac{M_{20}}{k_{\theta2}} = \frac{M_{10} + M_{20}}{k_{\theta1} + k_{\theta2}} \tag{3.4}$$

In the previous equations:

M_{10} is the bending moment in wall 1 at base

M_{20} is the bending moment in wall 2 at bas

N_0 is the axial load in the walls at the base.

k_{v1} is the vertical stiffness under wall 1

k_{v2} is the vertical stiffness under wall 2

$k_{\theta1}$ is the rotational stiffness of the foundation under the wall 1

$k_{\theta2}$ is the rotational stiffness of the foundation under the wall

λ is the modulus of elasticity of the foundation

The rotational and the vertical stiffness of the foundation can be expressed as a function of foundations modulus of elasticity, the area (A_f) and the second moment of area (I_f) of the grade beam.

$$k_{\theta 1} = \lambda I_{f1} \quad \text{and} \quad k_{\theta 2} = \lambda I_{f2} \tag{3.5}$$

$$k_{v1} = \lambda A_{f1} \quad \text{and} \quad k_{v2} = \lambda A_{f2} \tag{3.6}$$

Note that the subscript 1 and 2 are related to foundation beam 1 and 2.

In the case of elastic foundation the equation of relative displacements at the contraflexure line of the connecting beams will be:

$$\delta = l\left(\frac{d}{dz}x(z)\right) + \frac{1}{12}\frac{\left(\frac{d}{dz}N(z)\right)b^3h}{EI_e} - \frac{\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left(\int_0^z N(z)dz\right)}{E} + \frac{(m_{10}+m_{20}).l}{\lambda I_{f1}+\lambda I_{f2}} - N(0)\left(\frac{1}{\lambda A_{f1}}+\frac{1}{\lambda A_{f2}}\right) = 0 \tag{3.7}$$

3.1 Walls subjected to uniform distributed load

To derive the equation of the normal force and the lateral deflection, the same approach as the one for walls on the rigid foundation will be used. Since the first derivative of the equation of compatibility (equation (3.7)) does not consist the third and the fourth terms, it can be concluded that, here the same differential equation will be hold as the one for the rigid foundation. The only difference is the boundary condition at the base.

By substituting the value of the relative displacement δ_4 into equation (3.7) the boundary condition at the base can be expressed in the terms of axial force. The second boundary condition at level H is the same as the case of rigid foundation.

$$\delta = 0 + \frac{1}{12}\frac{\left(\frac{d}{dz}N(z)\right)b^3h}{EI_e} - 0 + \frac{(m(0)-N(0).l).l}{\lambda I_{f1}+\lambda I_{f2}} - N(0)\left(\frac{A_{f1}+A_{f2}}{\lambda A_{f1}A_{f2}}\right) = 0 \tag{3.8}$$

$$\left(\frac{d}{dz}N(z)\right) + \frac{m(0).l}{\lambda I_f}\left(\frac{12EI_e}{b^3h}\right) - \frac{(N(0).l).l}{\lambda I_f}\left(\frac{12EI_e}{b^3h}\right) - N(0)\left(\frac{A_f.I_f}{\lambda A_{f1}A_{f2}.l^2}\right).\left(\frac{12EI_e.l^2}{b^3hI_f}\right) = 0 \tag{3.9}$$

$$\left(\frac{d}{dz}N(z)\right) + \frac{m(0).E}{\lambda}\left(\frac{\alpha_f^2}{l}\right) - \frac{E.N(0)}{\lambda}(\ \alpha_f^2) - N(0).\frac{E}{\lambda}.(k_f^2-1).(\ \alpha_f^2) \tag{3.10}$$

Simplifying the above equation, gives the boundary condition for the normal force in the walls as follows:

1. $At\ z = 0$

$$\frac{d}{dz}N(z) = \frac{E\alpha_f^2 k_f^2}{\lambda}.N(z) - \frac{m(z).E\alpha_f^2}{\lambda.l} \quad (3.11)$$

2. $At\ \ z = H$

$$N(H) = 0 \quad (3.12)$$

Where:

$$\alpha_f^2 = \frac{12.I_e l^2}{I_f b^3 h} \quad (3.13)$$

$$k_f^2 = 1 + \frac{A_f.I_f}{A_{f1}A_{f2}l^2} \quad (3.14)$$

$$I_f = I_{f1} + I_{f2} \quad (3.15)$$

$$A_f = A_{f1} + A_{f2} \quad (3.16)$$

The obtained ODE can be solved by using the boundary conditions. Further, the shear flow in the connecting mediums can be derived by differentiating from the equation of normal force in the walls.

$$\begin{aligned} N_{elastic,dist}(z) = -\frac{1}{2}\Bigg(\bigg(\Big(\big(Hk^2(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - 2\lambda)\alpha^2 - 2E\alpha_f^2k_f^2\big).sinh(k\alpha z) \\ - \lambda\alpha.(2 + k^2(H-z)^2\alpha^2)k\Big).cosh(k\alpha H) \\ + \Big(\big(-Hk^2(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - 2\lambda)\alpha^2 + 2E\alpha_f^2k_f^2\big).cosh(k\alpha z) \\ - E(2 + k^2(H-z)^2\alpha^2)k_f^2\alpha_f^2\Big).sinh(k\alpha H) + 2cosh(k\alpha z)k\alpha\lambda \\ + 2sinh(k\alpha z)E\alpha_f^2k_f^2\bigg)w\Bigg) / \Big(k^4 l\left(E\alpha_f^2k_f^2.sinh(k\alpha H) + k\alpha\lambda.cosh(k\alpha H)\right)\alpha^2\Big) \end{aligned} \quad (3.17)$$

$$q_{elastic,dist} = \frac{1}{2}\Bigg(w\Big(\Big(\big(Hk^2\big(EHk^2\alpha_f^2 - EH\alpha_f^2k_f^2 - 2\lambda\big)\alpha^2 - 2E\alpha_f^2k_f^2\big).cosh(k\alpha z)k\alpha$$
$$+ 2k^3(H-z)\alpha^3\lambda\Big).cosh(k\alpha H)$$
$$+ \Big(\big(-Hk^2\big(EHk^2\alpha_f^2 - EH\alpha_f^2k_f^2 - 2\lambda\big)\alpha^2 + 2E\alpha_f^2k_f^2\big).sinh(k\alpha z)k\alpha$$
$$+ 2k_f^2k^2(H-z)\alpha^2E\alpha_f^2\Big).sinh(k\alpha H) + 2Ecosh(k\alpha z)k\alpha\alpha_f^2k_f^2 \tag{3.18}$$
$$+ 2sinh(k\alpha z).k^2\alpha^2\lambda\Big)\Bigg) / \Big(k^4 l\left(E\alpha_f^2k_f^2 sinh(k\alpha H) + \alpha k\lambda.cosh(k\alpha H)\right)\alpha^2\Big)$$

According to equation (2.13) the total bending moment in the walls is a superposition of the external bending moment due to the applied load and the reverse bending moment due to the shear force in the connecting beams. Hence the equation of bending moment in the walls will be as following:

$$M_{elastic,dist,1,2}(z) = \frac{1}{2}\Bigg(w\Big(\Big(\big(Hk^2\big(EHk^2\alpha_f^2 - EH\alpha_f^2k_f^2 - 2\lambda\big)\alpha^2 - 2E\alpha_f^2k_f^2\big).sinh(k\alpha z) -$$
$$k(2 + k^2(H-z)^2\alpha^2)\alpha\lambda\Big).cosh(k\alpha H) +$$
$$\Big(\big(-Hk^2\big(EHk^2\alpha_f^2 - EH\alpha_f^2k_f^2 - 2\lambda\big)\alpha^2 + 2E\alpha_f^2k_f^2\big).cosh(k\alpha z) - \tag{3.19}$$
$$k_f^2(2 + k^2(H-z)^2\alpha^2)E\alpha_f^2\Big).sinh(k\alpha H) + 2E.sinh(k\alpha z)\alpha_f^2k_f^2 + 2cosh(k\alpha z)\alpha k\lambda\Big)\Bigg) /$$
$$\Big(k^4\left(E\alpha_f^2k_f^2.sinh(k\alpha H) + \alpha k\lambda.cosh(k\alpha H)\right)\alpha^2\Big) + 1/2w(H-z)^2$$

The equation of deflection can be derived by integrating two times from equation (3.20) and using the following boundary conditions.

$$\frac{d^2}{dz^2}x(z) = \frac{1}{Ei_t}(m(z) - N(z).l) \tag{3.20}$$

Boundary conditions

1. At $x = 0$

$$x(z) = 0 \tag{3.21}$$

2. $At\ x = 0$

$$\frac{d}{dz}x(z) = \frac{m_{10} + m_{20}}{k_{\theta 1} + k_{\theta 2}} = \frac{m(0) - N(0)l}{\lambda I_f} \tag{3.22}$$

It is assumed that, only vertical and rotational displacements occur at the base. Therefore, the horizontal deflection at z=0 will be zero. Further, it is known that, in case of elastic foundation the slope at the base is inversely proportional to the rotational stiffness of the sub-grade. The equation of lateral deflection in the case of walls supported on individual elastic support is given in the following.

$$
\begin{aligned}
x_{elastic,dist}(z) = \frac{1}{2}\Bigg(\Bigg(&-I_f\lambda\Big(-2Hk^2\alpha^2\lambda \\
&+\Big(\big(H^2k^4-H^2k^2k_f^2\big)\alpha^2-2k_f^2\Big)E\alpha_f^2\Big).sinh\big(k\alpha(H-z)\big) \\
&-k\alpha z\Bigg(\Bigg(-\frac{1}{2}k^2\alpha^2\Bigg((k-1)k^2z\left(H^2-\frac{2}{3}zH+\frac{1}{6}z^2\right).(k+1)\alpha^2+4H-2z\Bigg)\lambda \\
&+\Big(\big(H^2k^4-H^2k^2k_f^2\big)\alpha^2-2k_f^2\Big)E\alpha_f^2\Bigg)I_f \\
&-k^2Ei_t\alpha^2(-2+(H^2k^4-H^2\,k^2)\alpha^2)\Bigg)\lambda.cosh(k\alpha H) \\
&+\Bigg(\frac{1}{2}\lambda\Bigg(-4Hk^2\alpha^2\lambda \\
&+\Bigg((k-1)k^4z^2\left(H^2-\frac{2}{3}zH+\frac{1}{6}z^2\right).(k+1)k_f^2\alpha^4 \\
&+2\left(H^2k^2-k_f^2(H^2+z^2)\right)k^2\alpha^2-4k_f^2\Bigg)E\alpha_f^2\Bigg)\,I_f \\
&+k^4zEi_t\alpha^4\left(2\lambda+E\alpha_f^2Hk^2(k_f-1).(k_f+1)\right).H\Bigg)\,.sinh(k\alpha H) \\
&+2\lambda\Big(I_fk\alpha\lambda.cosh(k\alpha z)+EI_f\alpha_f^2.sinh(k\alpha z)k_f^2 \\
&-k\left(\left(Ez\alpha_f^2k_f^2+\lambda\right)I_f-Ezi_t\alpha^2k^2\right)\alpha\Big)\Bigg).w\Bigg) \\
&/\left(EI_f\alpha^4k^6\lambda i_t.\left(E\alpha_f^2k_f^2.sinh(k\alpha H)+k\alpha\lambda.cosh(k\alpha H)\right)\right)
\end{aligned}
\tag{3.23}
$$

Design example 2:

To clarify the effect of elastic support on the coupled shear walls, the previous problem of section 0 will be solved for the case walls supported on elastic foundation. Further the results will be compared with the results obtained from the previous example. The walls system and the dimensions are shown in Figure 3.2. The dimensions of the walls system are assumed to be the same as the previous example. The only difference is the modulus of elasticity of the sub-grade and the foundation beams cross sections which are given in Table 3 and Table 4.It is assumed that the length and the thickness of the foundation beams are identical to the dimensions of the walls above, to just demonstrate the effect of the elasticity of the foundation.

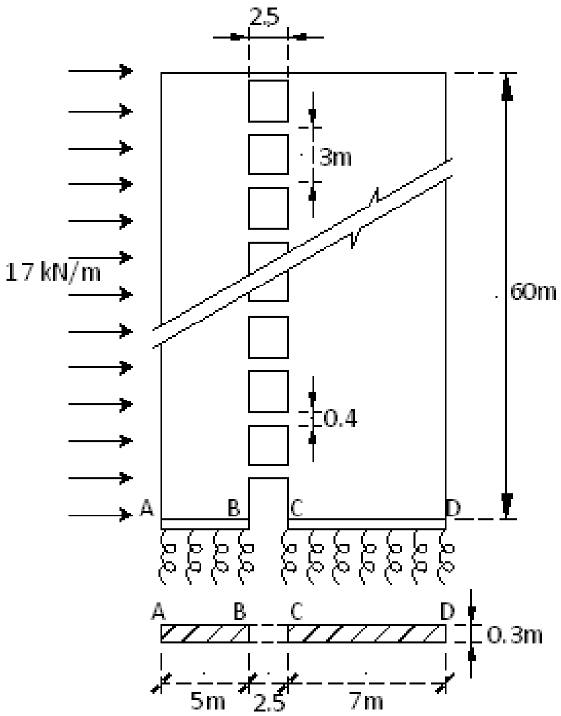


Figure 3.2 : Example structure

I. Since the dimensions of the walls system did not change the value of the stiffness factor $k\alpha H$ will be the same as before, 3.17.

II. Substituting the above parameters in the equation of shear flow, the variation of shear flow along the height of walls system, can be determined. To demonstrate the effect of the elastic foundation the variation of shear flow and bending moment for both walls on the rigid foundation and walls on the elastic foundation, have been plotted by using Maple program (see Figure 3.3)

Table 3 : Properties of foundation beams

b_{f1}	[m]	0.3
l_{f2}	[m]	5
b_{f2}	[m]	0.3
l_{f2}	[m]	7
A_{f1}	[m^2]	1.5
A_{f2}	[m^2]	2.1
I_{f1}	[m^4]	3.125
I_{f2}	[m^4]	8.575
$A_f = A_{f1} + A_{f2}$	[m^2]	3.6
$I_f = I_{f1} + I_{f2}$	[m^4]	11.7

Table 4 : Properties of elastic foundation

$\lambda_{foundation}$	[kN/m^3]	102000
$k_{\theta 1} = \lambda I_{f1}$	[kN.m]	318750
$k_{\theta 2} = \lambda I_{f2}$	[kN.m]	874650
$k_{v1} = \lambda A_{f1}$	[kN/m]	153000
$k_{v2} = \lambda A_{f2}$	[kN/m]	214200
$\alpha_f^2 = \frac{12 I_e l^2}{I_f b^3 h}$	[1/m^2]	0.002356
$k_f^2 = 1 + \frac{A_f I_f}{A_{f1} A_{f2} l^2}$		1.185072

As can be seen in Figure 3.3, the axial force in the walls has been increased in the case of the elastic foundation compared to the walls on the rigid foundation. From the bending curves, it can be observed that the bending moment value at the lower part of the walls has been reduced though; this value in the upper levels has been raised. Besides, the part of walls with negative moment has also been enlarged which occurs due to larger axial force in case of the elastic foundation. The most important thing is the effect of the elastic foundation which diminishes from the base to the top of the walls.

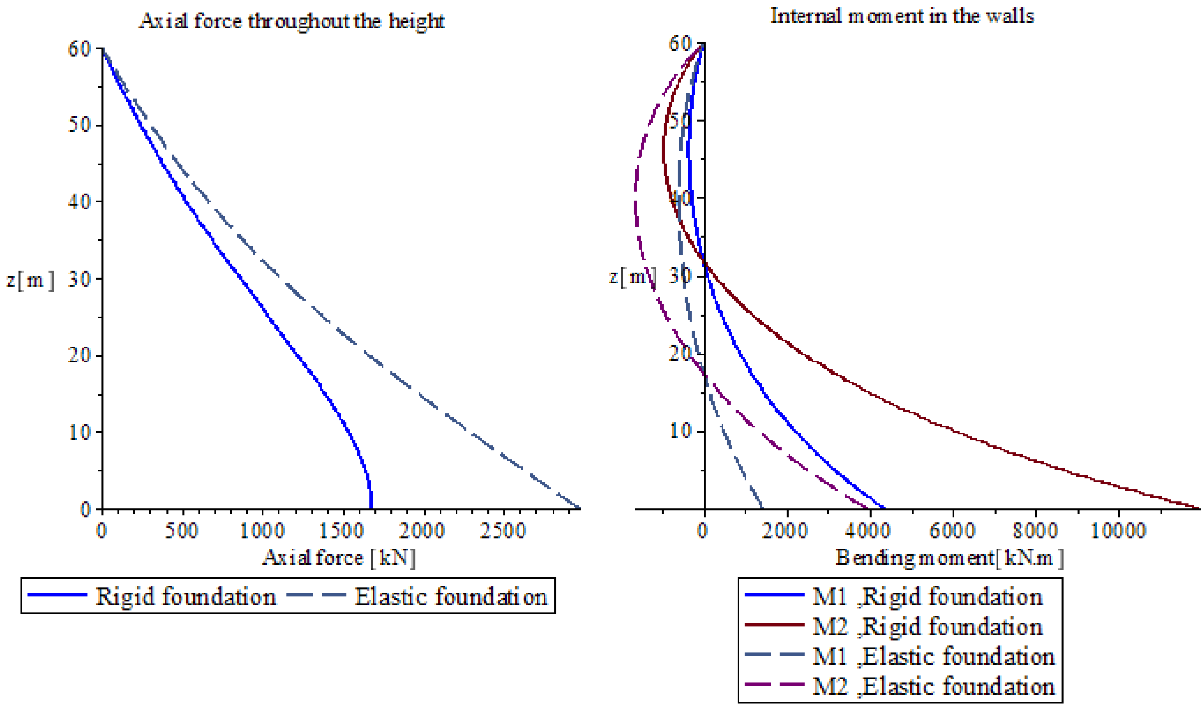


Figure 3.3 : Variation of the axial force and bending moment for walls supported on rigid and elastic foundation

III. The maximum shear force in the connecting beams for the values given in this example occurs at the level of $\frac{z}{H} = 0.03721$ which gives the maximum shear equal to $q_{max} = 68.432 \frac{kN}{m}$. The variation of shear flow in the connecting beams throughout the height is illustrated in Figure 3.4 for both walls on the rigid foundation and the elastic foundation. Noticing that the position of the maximum loaded beam has been lowered and the shear force at base in not zero anymore in the case of the elastic foundation. Furthermore, the shear flow has been considerably increased for the walls supported on the elastic foundation compared to the rigid foundation. Note that the shear flow in the beams is inversely proportional to the rigidity of the sub-grade which means the higher the modulus of the elasticity of the sub-soil the smaller the shear flow in the connecting beams will be.

Consequently the maximum shear force in connecting beams will be:
$Q_{max} = q_{max}.h = 68.43 * 3 = 205.3\ kN$

The maximum possible moment in any connecting beam is:
$$M_{max} = Q_{max}.\frac{b}{2} = 205.29 * \frac{2.5}{2} = 256.61\ \ kN.m$$

Comparing the result with the previous example, it can be seen that the maximum shear force and the maximum moment in the connecting beams are both almost doubled. This means that for the elastic foundation a larger amount of reinforcement will be required.

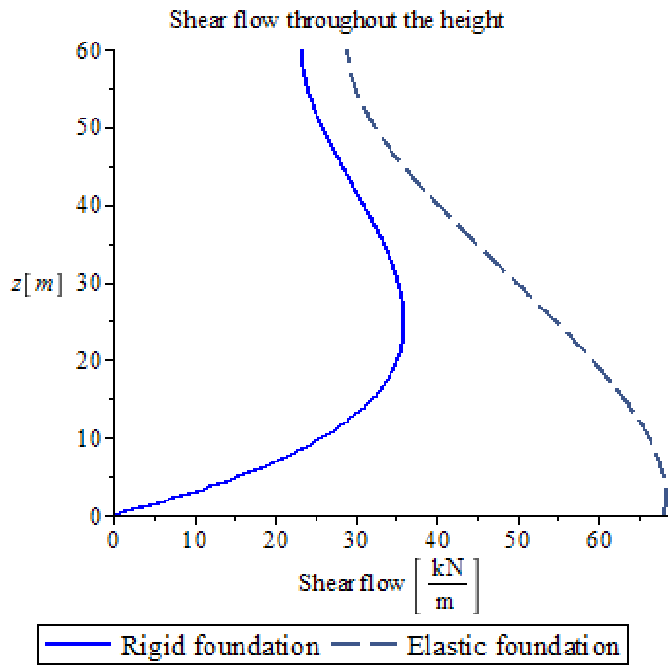


Figure 3.4: Shear flow along the height for rigid and elastic foundation

IV. For the rigid foundation, as well as for the walls supported on the elastic foundation the maximum lateral deflection occurs at the top of the walls. As it was expected the lateral deflection for the walls on the elastic foundation is significantly larger than the deflection of walls on the rigid foundation because the rotational displacement at the base is not restrained. A comparison of the deflection along the height of the walls supported on the rigid and the elastic foundation has been done which is shown in Figure 3.5. The maximum deflection at the top is equal to 0.271 m.

Table 5 : Stress at the extreme fibres at base for both rigid foundation and individually elastic foundation

Stress at the outer fibres [kN/m^2] / Foundations	σ_A	σ_B	σ_C	σ_D
Rigid foundation	4617.8	−2390.1	4109.9	−5701.1
Elastic support	3125.8	832.2	192.1	−3019.2

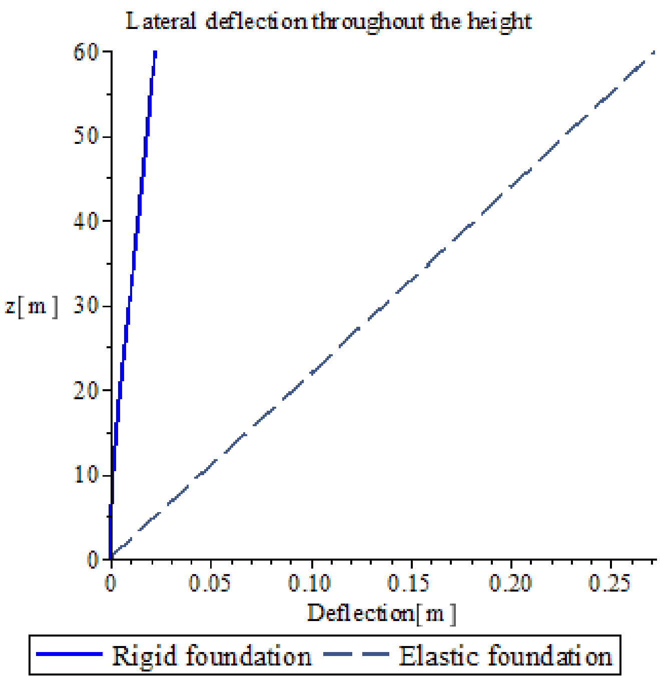


Figure 3.5 : lateral deflection for walls supported on elastic and rigid foundation

V. Stress of extreme fibre of the walls at the base is the result of the stress due to bending and the stress due to axial force on each wall.

$$\sigma_A = \frac{(m(0) - N(0).l).I_1}{i_t}.\frac{c_1}{I_1} + \frac{N(0)}{A_1} = \frac{(30600 - 2968.54 * 8.5) * 2.5}{11.7} + \frac{2968.54}{1.5} = 3125.8\ \frac{kN}{m^2}$$

$$\sigma_B = -\frac{(m(0) - N(0).l).I_1}{i_t}.\frac{c_2}{I_1} + \frac{N(0)}{A_1} = 832.2\ \frac{kN}{m^2}$$

$$\sigma_C = \frac{(m(0) - N(0).l).I_2}{i_t}.\frac{c_3}{I_2} - \frac{N(0)}{A_2} = \frac{(30600 - 2968.54 * 8.5) * 3.5}{11.7} - \frac{2968.54}{2.1} = 192.1\ \frac{kN}{m^2}$$

$$\sigma_D = -\frac{(m(0) - N(0).l).I_2}{i_t}.\frac{c_4}{I_2} + \frac{N(0)}{A_2} = -3019.2\ \frac{kN}{m^2}$$

Comparing the results with previous example (illustrated in Table 5) it can be seen that the absolute value of the stress at extreme fibres of the walls has been significantly decreased. Since the walls are supported on the elastic foundation a part of bending moment and axial force will cause displacement at the base which leads into reduction of the stresses at the base.

3.2 Prove validity of the derived equations

It is very important to show that the derived equations in this chapter are correct since the equation depend on many parameters. To assure that these equations are correct the equations related to rigid foundation have to be obtained when the modulus of elasticity of the sub grade tens to infinite. Therefore the previous example will be reanalyzed for four different foundation stiffness:

$$\lambda = 102000, 1002000, 10002000\ and\ 100002000\ kN/m^3$$

The results of axial force, shear force, bending moment and lateral deflection of the walls versus height have been drawn in Figure 3.6 to Figure 3.9 for these four elastic foundation and rigid foundation.

From the figures it can be noticed that, by increasing the modulus of elasticity of the foundation, the behaviour of the coupled shear walls becomes similar to the behaviour of coupled shear walls on the rigid foundation as expected.

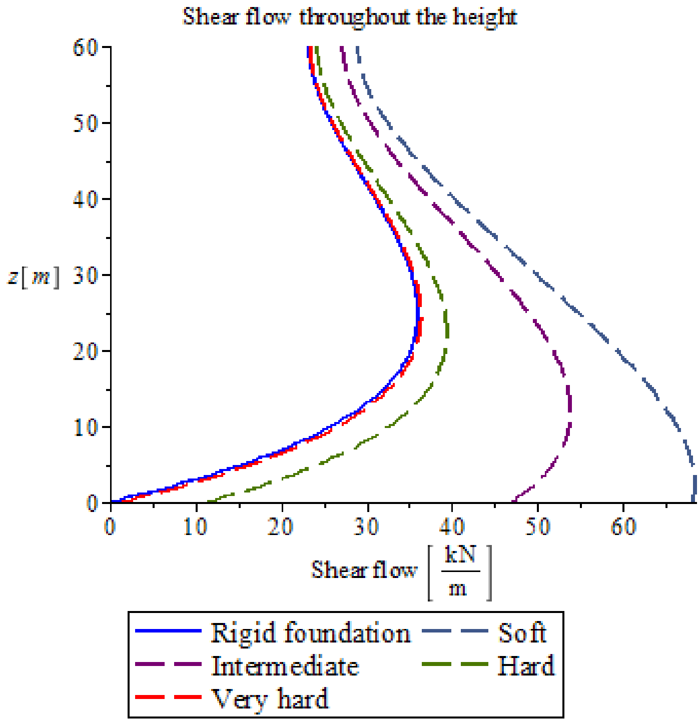


Figure 3.6: Shear flow throughout the height for different foundation stiffness

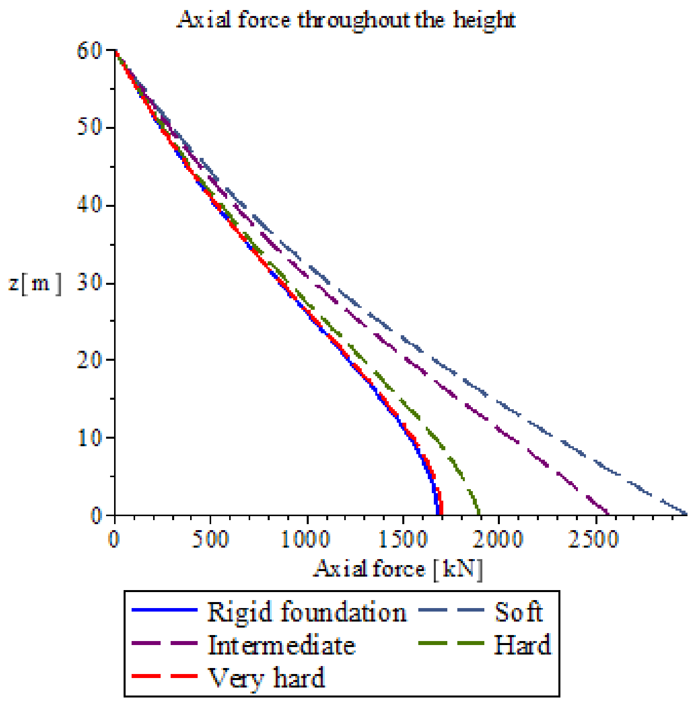


Figure 3.7: Axial force throughout the height for different foundation stiffness

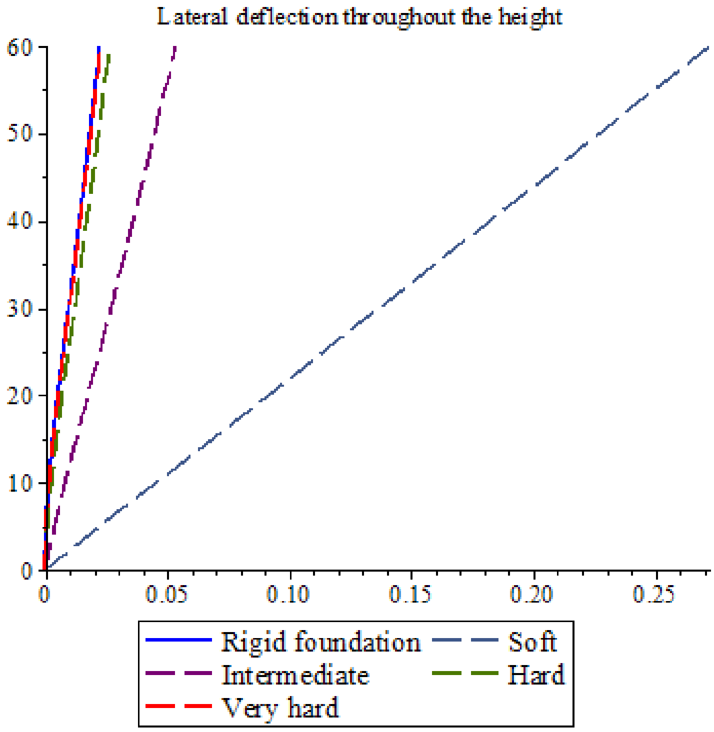


Figure 3.8: Deflection at top of the walls for different foundation stiffness

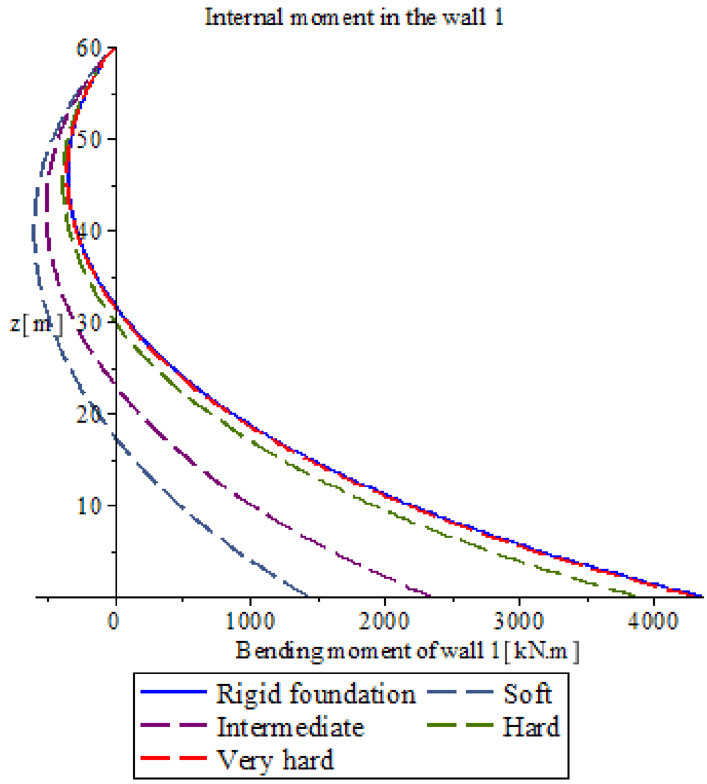


Figure 3.9: Bending comment in wall 1 throughout the height for different foundation stiffness

The equation of the internal forces and the lateral deflection for the walls subjected to a concentrated point load at the top and subjected to triangularly distributed load are given in the follows:

3.3 Walls subjected to point load

The equation of the normal force, shear flow, bending moment and the lateral deflection of the walls subjected to a point load at the top are given below. Note that, in this section the same approach has been used as the one used for the walls subjected to a uniform distributed load. The only difference is the external bending moment applied on the walls.

$$m(z) = p(H - z) \tag{3.24}$$

Axial force in the walls

$$\begin{aligned} N_{elastic,point}(z) = -\Bigg(\bigg(\Big(\big(EH(k - k_f) \,.\, (k + k_f)\alpha_f^2 - \lambda \big) \,.\, sinh(k\alpha z) \\ - k\alpha\lambda(H - z) \Big) \,.\, cosh(k\alpha H) \\ - \Big(\big(EH(k - k_f).\, (k + k_f)\alpha_f^2 - \lambda \big) \,.\, cosh(k\alpha z) \\ + E\alpha_f^2 k_f^2 (H - z) \Big) .\, sinh(k\alpha H) \bigg) \; P \Bigg) \\ / \Big(k^2 l \big(k\alpha\lambda.\, cosh(k\alpha H) + \; E\alpha_f^2 k_f^2.\, sinh(k\alpha H) \big) \Big) \end{aligned} \tag{3.25}$$

Shear flow in the connecting mediums

$$\begin{aligned} q_{elastic,point}\,(z) = \Bigg(\bigg(\Big(\big(EH(k - k_f) \,.\, (k + k_f)\alpha_f^2 - \lambda \big).\; cosh(k\alpha z)k\alpha + k\alpha\lambda \Big) \,.\, cosh(k\alpha H) \\ - \Big(\big(EH(k - k_f).\, (k + k_f)\alpha_f^2 - \lambda \big) \,.\, sinh(k\alpha z)k\alpha - E\alpha_f^2 k_f^2 \Big) \,.\, sinh(k\alpha H) \bigg) P \Bigg) \\ / \Big(k^2 l \big(k\alpha\lambda \,.\, cosh(k\alpha H) + E\alpha_f^2 k_f^2.\, \sinh(\, k\alpha H \,) \big) \Big) \end{aligned} \tag{3.26}$$

Internal bending moment in the walls

$$M_{elastic,point,1,2}(z) = P(H-z) + \Bigg(\Bigg(\Big(\big(EH(k-k_f).(k+k_f)\alpha_f^2-\lambda\big).sinh(k\alpha z) - k\alpha\lambda(H-z)\Big).cosh(k\alpha H) - \Big(\big(EH(k-k_f).(k+k_f)\alpha_f^2-\lambda\big).cosh(k\alpha z) + E\alpha_f^2k_f^2(H-z)\Big).sinh(k\alpha H)\Bigg)P\Bigg) / \Big(k^2\big(k\alpha\lambda.cosh(k\alpha H) + E\alpha_f^2k_f^2.sinh(k\alpha H)\big)\Big) \tag{3.27}$$

Lateral deflection

$$x_{elastic,point}(z) = \Bigg(\Bigg(-I_f\lambda\big(EHk^2\alpha_f^2 - EH\alpha_f^2k_f^2-\lambda\big).\sinh\big(k\alpha(H-z)\big) + \bigg(\frac{1}{2}\Big(-2\lambda + E\Big(\alpha^2\Big(-\frac{1}{3}z+H\Big)k_f^2z^2k^4 + \Big(-\alpha^2\Big(-\frac{1}{3}z+H\Big)k_f^2z^2+2H\Big)k^2 - 2Hk_f^2\Big)\alpha_f^2\Big).\lambda I_f + E\alpha^2 i_t\Big(\lambda + E\alpha_f^2Hk^2(k_f-1).(k_f+1)\Big)zk^2\bigg).sinh(k\alpha H) - \alpha\lambda\Bigg(\bigg(\Big(-1-\frac{1}{2}\alpha^2\Big(-\frac{1}{3}z+H\Big)zk^4 + \frac{1}{2}\alpha^2\Big(-\frac{1}{3}z+H\Big)zk^2\Big).\lambda + E\alpha_f^2H(k-k_f)(k+k_f)\bigg)I_f - EHk^2\alpha^2i_t(k-1)(k+1)\Bigg).cosh(k\alpha H)zk\Bigg)p\Bigg) / \Big(EI_f\alpha^2k^4\lambda i_t\big(E.sinh(k\alpha H)\alpha_f^2k_f^2 + cosh(k\alpha H)\alpha k\lambda\big)\Big) \tag{3.28}$$

3.4 Walls subjected to triangularly distributed load

As well as the walls subjected to point load, the walls subjected to a triangularly distributed load has been studied. The equation related to normal forces and lateral deflections are given below.

$$m(z) = \frac{p}{6H}(H-z)^2.(2H+z) \tag{3.29}$$

Axial force in the walls

$$\begin{aligned} N_{elastic,triangular}(z) &= -\frac{1}{3}\Bigg(p\Bigg(\Bigg(\Bigg(H^2\left(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - \frac{3}{2}\lambda\right)k^2\alpha^2 + 3\lambda\Bigg).sinh(k\alpha z) \\ &- \left(\left(H+\frac{1}{2}z\right)(H-z)^2k^2\alpha^2 + 3z\right)\alpha k\lambda\Bigg).cosh(k\alpha H) \\ &+ \Bigg(\left(-H^2\left(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - \frac{3}{2}\lambda\right)k^2\alpha^2 - 3\lambda\right).cosh(k\alpha z) \\ &- k_f^2\left(\left(H+\frac{1}{2}z\right)(H-z)^2k^2\alpha^2 + 3z\right)\alpha_f^2E\Bigg).sinh(k\alpha H) \\ &+ 3H\left(\alpha k\lambda.cosh(k\alpha z) + sinh(k\alpha z)E\alpha_f^2k_f^2\right)\Bigg)\Bigg) \\ &/\left(\alpha^2k^4Hl\left(\lambda k\alpha.cosh(k\alpha H) + E\alpha_f^2k_f^2.sinh(k\alpha H)\right)\right) \end{aligned} \tag{3.30}$$

Shear flow in the connecting mediums

$$q_{elastic,triangular}(z) = \frac{1}{3}\Bigg(p\Bigg(\Bigg(\Bigg(H^2\left(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - \frac{3}{2}\lambda\right)k^2\alpha^2 + 3\lambda\Bigg).cosh(k\alpha z)\,k\alpha - \left(\frac{1}{2}(H-z)^2k^2\alpha^2 - 2\left(H+\frac{1}{2}z\right)(H-z)k^2\alpha^2 + 3\right)\alpha k\lambda\Bigg).cosh(k\alpha H) + \Bigg(\Bigg(-H^2\left(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - \frac{3}{2}\lambda\right)k^2\alpha^2 - 3\lambda\Bigg).sinh(k\alpha z)k\alpha - k_f^2\left(\frac{1}{2}(H-z)^2k^2\alpha^2 - 2\left(H+\frac{1}{2}z\right)(H-z)\,k^2\alpha^2 + 3\right)\alpha_f^2E\Bigg).sinh(k\alpha H) + 3H\left(\alpha^2k^2\lambda.sinh(k\alpha z) + cosh(k\alpha z)\,k\alpha E\alpha_f^2k_f^2\right)\Bigg)\Bigg) \Big/ \left(\alpha^2k^4Hl\left(\lambda k\alpha.cosh(k\alpha H) + E\alpha_f^2k_f^2.sinh(k\alpha H)\right)\right) \tag{3.31}$$

Internal bending moment in the walls

$$M_{elastic,triangular,1,2}(z) = \frac{1}{6}\frac{p(H-z)^2.(2H+z)}{H} + \frac{1}{3}\Bigg(p\Bigg(\Bigg(\Bigg(H^2\left(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - \frac{3}{2}\lambda\right)k^2\alpha^2 + 3\lambda\Bigg).sinh(k\alpha z) - \left(\left(H+\frac{1}{2}z\right)(H-z)^2k^2\alpha^2 + 3z\right)\alpha k\lambda\Bigg).cosh(k\alpha H) + \Bigg(\Bigg(-H^2\left(HE\alpha_f^2k^2 - HE\alpha_f^2k_f^2 - \frac{3}{2}\lambda\right)k^2\alpha^2 - 3\lambda\Bigg).cosh(k\alpha z) - k_f^2\left(\left(H+\frac{1}{2}z\right)(H-z)^2k^2\alpha^2 + 3z\right)\alpha_f^2E\Bigg).sinh(k\alpha H) + 3H\left(\alpha k\lambda.cosh(k\alpha z) + sinh(k\alpha z)E\alpha_f^2k_f^2\right)\Bigg)\Bigg) \Big/ \left(\alpha^2k^4H\left(\lambda k\alpha.cosh(k\alpha H) + E\alpha_f^2k_f^2.sinh(k\alpha H)\right)\right) \tag{3.32}$$

Lateral deflection

$$
\begin{aligned}
x_{elastic,triangular}(z) \\
= \frac{1}{3}\Bigg(p\Bigg(-\lambda\Bigg(\Big(-\frac{3}{2}k^2H^2\alpha^2+3\Big)\lambda \\
+ E\alpha_f^2H^3k^2\alpha^2(k-k_f)(k+k_f)\Bigg)I_f.sinh(k\alpha(H-z)) \\
+ \Bigg(\frac{1}{2}\lambda\Bigg((-3H^2\alpha^2k^2+6)\lambda \\
+ k^2\alpha_f^2E\Bigg((k-1)k^2k_f^2z^2\ \Big(H^3-\frac{1}{2}H^2z+\frac{1}{20}z^3\Big)(k+1\ )\alpha^2+2H^3k^2 \\
- 2H^3k_f^2-z^3k_f^2\Bigg)\alpha^2\Bigg)I_f \\
+ k^2zE\Bigg(\Big(\frac{3}{2}k^2H^2\alpha^2-3\Big)\lambda \\
+ E\alpha_f^2H^3k^4\alpha^2(k_f-1)(k_f+1\ )\Bigg)\alpha^2i_t\Bigg).sinh(k\alpha H) \\
- \lambda\Bigg(k\alpha z\Bigg(\Bigg(\Bigg(3-\frac{1}{2}(k-1)k^4z\Big(H^3-\frac{1}{2}H^2z+\frac{1}{20}z^3\Big)(k+1)\alpha^4 \\
-\frac{3}{2}\Big(H^2-\frac{1}{3}z^2\Big)k^2\alpha^2\Bigg)\lambda+E\alpha_f^2H^3k^2\alpha^2(k-k_f)(k+k_f)\Bigg)I_f \\
- EH^3k^4\alpha^4i_t(k-1)(k+1)\Bigg).cosh(k\alpha H) \\
- 3\Bigg(I_fk\alpha\lambda.cosh(k\alpha z)+EI_f\alpha_f^2.sinh(k\alpha z)\ k_f^2 \\
- k\alpha\Big(\big(Ez\alpha_f^2k_f^2+\lambda)I_f-zEi_t\alpha^2k^2\big)\Big)H\Bigg)\Bigg)\Bigg) \\
/\Big(EHI-f\ \alpha^4k^6\lambda i_t\big(E.sinh(k\alpha H)\alpha_f^2k_f^2+cosh(k\alpha H)\alpha k\lambda\big)\Big)
\end{aligned}
\qquad (3.33)
$$

4 Walls on elastic foundation with base stiffened beam

In the previous sections the behaviour of the coupled shear walls supported on the rigid and the elastic foundations have been considered. As has been shown, the effect of the coupling will be reduced in the case of coupled shear walls on the elastic foundation. It should be noted that in most cases the coupled shear walls are stiffened by a beam (called grade or stiffened beam) at the base, however sometimes the coupled shear walls are stiffened with more than one beam along the height of the walls. In this part the static behaviour of the coupled shear walls stiffened by a bottom beam supported on the elastic foundation will be studied. As well as the previous parts the continuous method will be used. This study is based on the work of A.Coull [4].

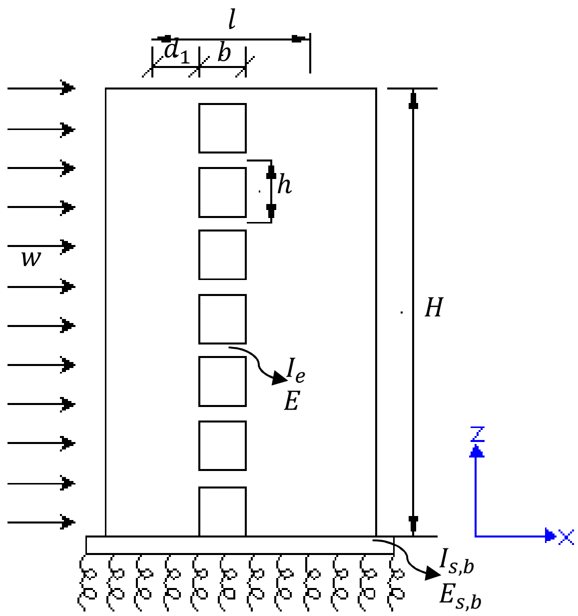


Figure 4.1: Coupled shear walls stiffened by base beam

As have been shown in the previous part, a cut will be made along the point of contraflexure (Figure 2.3). Now the relative vertical deflection of the cut ends of the laminas can be written as equation (4.1) which should be equal to zero. This relative displacement consists of deflection of walls due to axial force and bending moment, deflection of beams due to shear and bending (more detail can be found in chapter 1) and relative vertical and rotational displacement at base due to foundation displacement (δ_v and δ_θ shown in Figure 4.3)

$$\delta = l.\frac{dx}{dz} - \frac{b^3 h}{12EI_e} q(z) - \frac{1}{E}.\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\int_0^z N(z)dz + l.\delta_\theta - \delta_v = 0 \tag{4.1}$$

The same as before the moment curvature relation for the wall is:

$$Ei_t.\frac{d^2x}{dz^2} = m(z) - N(z).l \tag{4.2}$$

Considering the top part of the wall which is shown in the Figure 4.2 the axial force in each wall can be given by:

$$N(z) = \int_z^H qdz \tag{4.3}$$

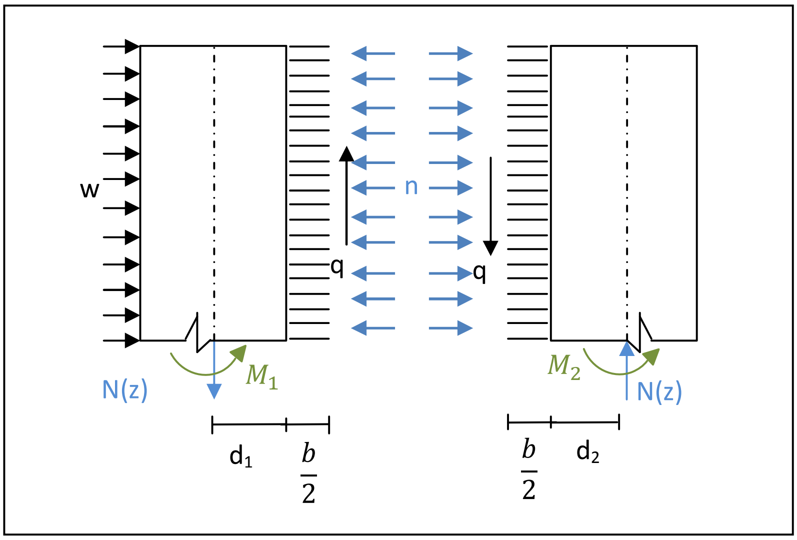


Figure 4.2 : Axial force in the walls

Since the first derivative of the compatibility equation (4.1)does not contains the effect of the rotational and vertical deflection at the base, it can be concluded that second order differential equation which have been used for the rigid foundation will also govern here. Note that the boundary conditions will be different.

$$\frac{d^2}{dz^2}N(z) - k^2\alpha^2.N(z) = -\frac{\alpha^2}{l}.m(z) \tag{4.4}$$

Where $m(z)$ is the applied external moment on the walls and α^2 and k^2 have been illustrated in the equations (2.20)and (2.21).

The compatibility equation for the stiff beam at the base is:

$$l.\frac{dx}{dz} - \frac{b^3}{12E_{s,b}I_{s,b}}Q - \frac{1}{E}.\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^0 N(z)dz + l.\delta_\theta - \delta_v = 0 \tag{4.5}$$

Using the compatibility equation (4.1) at z=0 and assuming it equal to compatibility equation for stiff beam (equation (4.5)), the shear force in the stiff beam can be derived.

$$Q_0 = \psi . q(0) \tag{4.6}$$

$$\psi = \frac{E_{s,b} . I_{s,b}}{EI_e} . h \tag{4.7}$$

In which E, $E_{s,b}$, I_e and $I_{s,b}$ are the modulus of elasticity and second moment inertia of the lamellas and stiff beam at the base respectively. Note, that the rotational displacements (δ_θ) for both walls are assumed to e identical, independent of their dimensions.

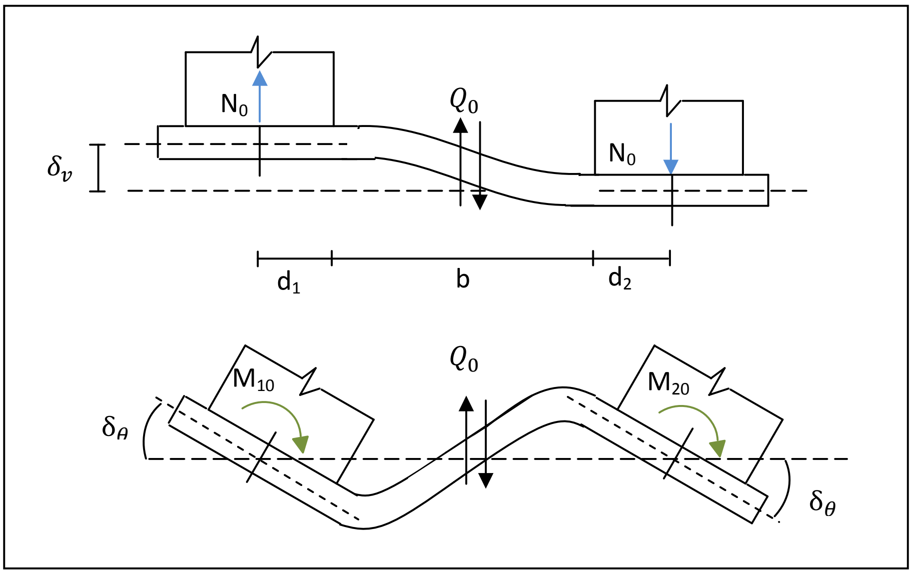


Figure 4.3 : Vertical and rotational deflection at base

$$\delta_v = \frac{(N(0) + Q_0)}{k_v} \tag{4.8}$$

$$\delta_\theta = \frac{M_{10} + M_{20}}{k_\theta} = \frac{m(0) - N(0). l - Q_0 . l}{k_\theta} \tag{4.9}$$

Where k_v and k_θ are the equivalent vertical and rotational stiffness of the soil respectively, given by:

$$\frac{1}{k_v} = \frac{1}{k_{v1}} + \frac{1}{k_{v2}} \tag{4.10}$$

$$k_\theta = k_{\theta 1} + k_{\theta 2} \tag{4.11}$$

Note that, k_{v1}, k_{v1}, $k_{\theta 1}$ and $k_{\theta 1}$ are the vertical and rotational stiffness of soil under wall 1 and 2 which are given by equation (3.5) and (3.6).By considering the moment equilibrium in Figure 4.4 the summation of moment in the walls at base under the beam can be derived.

$$M_{10} = -\int_0^H q(z)dz.\left(\frac{b}{2} + d_1\right) + m(0) - Q_0.\left(\frac{b}{2} + d_1\right) \tag{4.12}$$

$$M_{20} = -\int_0^H q(z)dz.\left(\frac{b}{2} + d_2\right) - Q_0.\left(\frac{b}{2} + d_2\right) \tag{4.13}$$

$$M_{10} + M_{20} = -\int_0^H q(z)dz.l - Q_0.l + m(0) = m(0) - Q_0.l - N(0).l \tag{4.14}$$

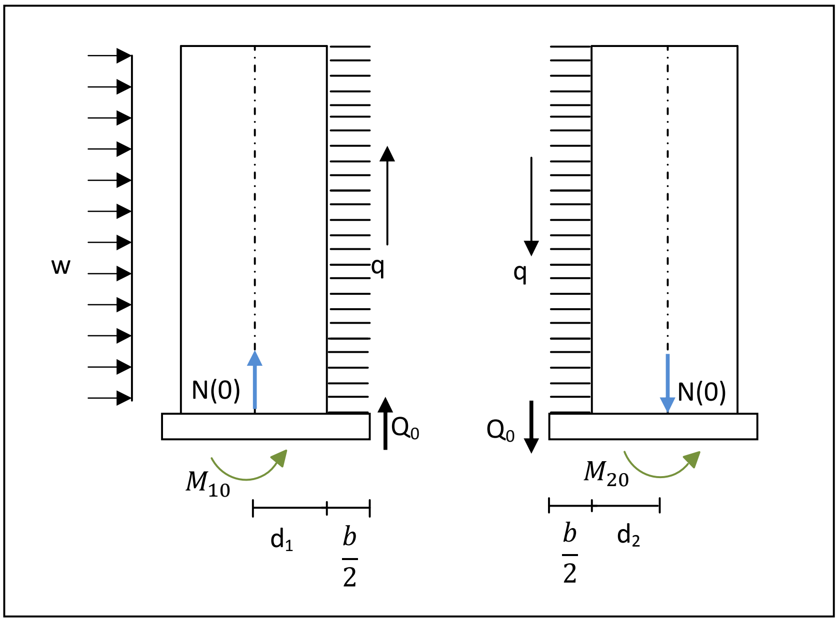


Figure 4.4 : Moment of the walls at the base with stiffened beam

Substituting the equation (4.6)-(4.9) into compatibility equation (4.1) gives the boundary condition at the base. Furthermore, the boundary condition at the top of the walls remains equal to the top boundary condition in the previous case since, there is no stiffened beam at the top.

$$-\frac{b^3 h}{12EI_e}q(0) - \frac{1}{E}.\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^0 N(z)dz + l.\left(\frac{m(0) - N(0).l - Q_0.l}{k_\theta}\right) - \left(\frac{N(0) + Q_0}{k_v}\right) = 0 \tag{4.15}$$

Boundary conditions

1. $x = 0$

$$q(0).\left(1+\psi.\mu_f\right) = \frac{\lambda_r}{l}.m(0) - N(0).\mu_f \tag{4.16}$$

2. $x = H$

$$N(H) = 0 \tag{4.17}$$

In which:

$$\mu_f = \lambda_r + \lambda_v$$
$$\lambda_r = \frac{12EI_e l^2}{k_\theta b^3 h}, \lambda_v = \frac{12EI_e}{k_v b^3 h} \tag{4.18}$$

Note that, E in the above equations refers to the modulus of elasticity of the connecting mediums. Since in this chapter the modulus of elasticity of the shear walls and connecting mediums are assumed to be equal no difference has been made between the notation for the young's modulus of beams and shear walls. Walls subjected to uniform distributed load

Solving the differential equation (4.4) and using the above boundary conditions the axial force in the walls can be obtained.

$$\begin{aligned} N_{stiffened,dist}(z) &= -\frac{1}{2}\Bigg(w\Bigg(\Big(\Big(\big(H\lambda_r k^2 - 2 + (-H-2\psi)\,\mu_f\big)\,Hk^2\alpha^2 - 2\mu_f\Big).sinh(k\alpha z) \\ &- \alpha(2 + k^2(H-z)^2\alpha^2)\,k\big(\psi\mu_f + 1\big)\Big).cosh(k\alpha H) \\ &+ \Big(\big(-(H\lambda_r k^2 - 2 + (-H-2\psi)\mu_f)Hk^2\alpha^2 + 2\mu_f\big).cosh(k\alpha z) \\ &- \mu_f(2 + k^2(H-z)^2\alpha^2)\Big).sinh(k\alpha H) + 2k\alpha\big(\psi\mu_f + 1\big).cosh(k\alpha z) \\ &+ 2.sinh(k\alpha z)\mu_f\Bigg)\Bigg) / \left(\alpha^2\left(k\alpha\big(\psi\mu_f + 1\big).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\right).lk^4\right) \end{aligned} \tag{4.19}$$

By differentiating from the axial force equation, the equation of the shear force in the lamellas can be derived. Further, for the lateral deflection the same differential equation as the one for the rigid foundation will be govern.

$$q_{stiffened,dist}(z) = \frac{1}{2}\Bigg(\bigg(\Big(\big(k^2H(H\lambda_r k^2 - 2 + (-H-2\psi)\mu_f)\alpha^2 - 2\mu_f\big).cosh(k\alpha z) + 2k^2\alpha^2(\mu_f\psi + 1)(H-z)\Big).cosh(k\alpha H) + \Big(\big(-k^2H(H\lambda_r k^2 - 2 + (-H-2\psi)\mu_f)\alpha^2 + 2\mu_f\big).sinh(k\alpha z) + 2k\alpha\mu_f(H-z)\Big).sinh(k\alpha H) + 2k\alpha(\mu_f\psi+1).sinh(k\alpha z) + 2.cosh(k\alpha z)\mu_f\bigg)w\Bigg)/\Big(k^3\alpha l\ \big(k(\mu_f\psi+1)\alpha.cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\big)\Big) \quad (4.20)$$

By considering equation (4.2) and differentiating two time and substituting the relative equation of normal force (equation (4.19)) the equation of lateral deflection as a function of height can be obtained.

To derive the equation of deflection, the following boundary conditions have been used.

1. At $x = 0$

$$x(z) = 0 \quad (4.21)$$

$$\frac{d}{dz}x(z) = \frac{m(0) - N(0)l + \psi.l.\frac{d}{dz}N(0)}{\frac{Ei_t\alpha^2}{\lambda_r}} \quad (4.22)$$

The equation of lateral deflection for each type of loading can be derived by applying these boundary conditions and differentiating two times from the equation of moment curvature of the coupled walls.

$$x_{stiffened,dist}(z) = -\frac{1}{2}\Bigg(w\bigg(\Big(H\big(\lambda_r Hk^2 - 2 + (-H-2\psi)\mu_f\big)k^2\alpha^2 - 2\mu_f\Big).sinh\big(k\alpha(H-z)\big) + z\alpha\Big(-\frac{1}{2}z(k+1)\Big(H^2 - \frac{2}{3}zH + \frac{1}{6}z^2\Big)k^4(k-1).(\psi\mu_f+1).\alpha^4 + \Big(\lambda_r H^2(\psi\lambda_r - \psi\mu_f - 1)k^4 + 2k^2\lambda_r H^2 + (\psi\mu_f+1)z - H\big(2 + (H+2\psi)\mu_f\big)\Big)k^2\alpha^2 + 2\lambda_r k^2 - 2\mu_f\Big).kcosh(k\alpha H) + \Big(-\frac{1}{2}\mu_f z^2(k+1)\Big(H^2 - \frac{2}{3}zH + \frac{1}{6}z^2\Big)k^4(k-1)\alpha^4 + k^2\Big(\lambda_r H^2 z(\lambda_r - \mu_f)k^4 - \lambda_r H(H+2z)k^2 + z^2\mu_f + H\big(2 + (H+2\psi)\mu_f\big)\Big)\alpha^2 + 2\mu_f\Big).sinh(k\alpha H) - 2k\alpha(\psi\mu_f+1).cosh(k\alpha z) - 2sinh(k\alpha z)\mu_f - 2\alpha k(k^2 z\lambda_r - \psi\mu_f - z\mu_f - 1)\bigg)\Bigg) / \Big(E\alpha^4\big(k\alpha(\psi\mu_f+1).cosh(k\alpha H) + \mu_f sinh(k\alpha H)\big)k^6 i_t\Big) \quad (4.23)$$

The internal moment in the walls at each level can be calculated by substituting equation (4.24) into equation (4.2).

$$Ei_t.\frac{d^2x}{dz^2} = M_1 + M_2 \tag{4.24}$$

Which gives the total moment in the walls equal to:

$$M_1 + M_2 = m(z) - N(z).l \tag{4.25}$$

As a result the summation of the internal moments in the coupled walls as a function of height will be equal to:

$$(M_1 + M_2)_{stiffened,dist}(z) = \frac{1}{2}\Bigg(w\bigg(\Big(\big(H.\big(H\lambda_r k^2 - 2 + (-H - 2\psi)\mu_f\big)k^2\alpha^2 - 2\mu_f\big).sinh(k\alpha z) + (-2 + k^2(k-1).(k+1).(H-z)^2\alpha^2)\big(\mu_f\psi + 1\big).k\alpha\Big).cosh(k\alpha H) + \Big(\big(-H\big(H\lambda_r k^2 - 2 + (-H - 2\psi)\mu_f\big).k^2\alpha^2 + 2\mu_f\big).cosh(k\alpha z) + \mu_f.(-2 + k^2(k-1).(k+1).(H-z)^2.\alpha^2)\Big).sinh(k\alpha H) + 2k\alpha\big(\mu_f\psi + 1\big).cosh(k\alpha z) + 2sinh(k\alpha z)\mu_f\bigg)\Bigg)/\Big(\alpha^2 k^4\left(k\alpha\big(\mu_f\psi + 1\big).cosh(k\alpha H) + \mu_f sinh(k\alpha H)\right)\Big) \tag{4.26}$$

Note that, to prove the validity of the equations related to elastic foundation, the modulus of elasticity of the foundation is assumed to be infinite. The results should be equal to the equations related to rigid foundation. This is done by using the Maple programme and it is given in appendix 3.

Design example 3

Consider the walls system which is given in the previous example. But is this example the coupled walls are stiffened at the base with a continuous beam (See Figure 4.5). At the first step it is assumed that the beam has the same dimensions of the connecting beams to consider only the effect of the stiffening of the coupled walls. In the second part of this example the dimensions of the grade beam will be changed to determine the effect of beam's properties on the behaviour of the walls.

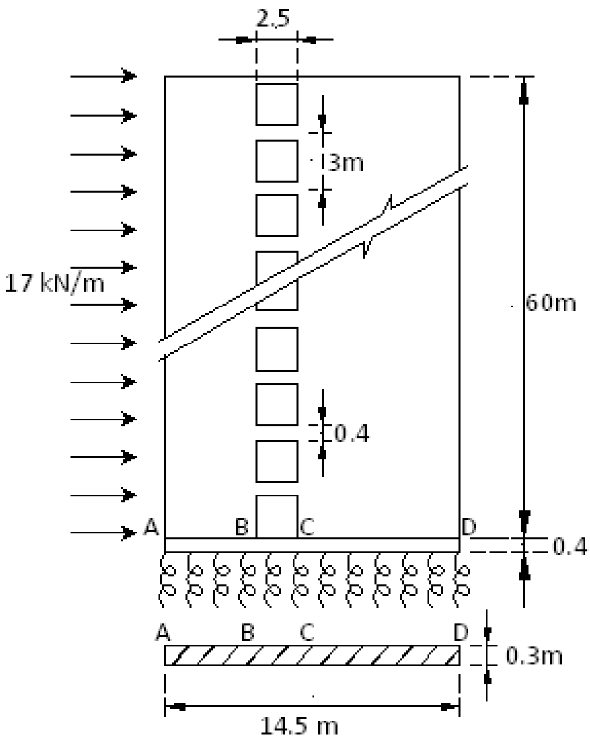


Figure 4.5 : Example structure

The properties of the base beam are given Table 6. Further the dimensions of the walls and the connecting beams remain the same as before.

I. By inserting the above parameter in equations (4.19) and (4.20) the deviation of the shear force and the axial force along the height will be given. A comparison has been made between the walls supported on individual elastic foundation and the walls supported on elastic foundation by a stiffened beam at base. The result is given in Figure 4.6

Table 6 : Properties of the stiffened beam at base

$h_{s,b}$	[m]	0.4
$t_{s,b}$	[m]	0.3
$E_{s,b}$	[kN/m²]	36000000
$I_{s,b}$	[m⁴]	0.0016
$\psi = \frac{E_{s,b}I_{s,b}}{EI_e}h$	[m]	3.221184
$\lambda_r = \frac{12EI_e l^2}{k_\theta b^3 h}$	[1/m]	0.831419
$\lambda_v = \frac{12EI_e}{k_v b^3 h}$	[1/m]	0.153872
k_θ	[kN.m]	1193400
k_v	[kN/m]	89250
$\mu_f = \lambda_r + \lambda_v$	[1/m]	0.985291

According to the curves, the axial force in the walls has been reduced which leads into increasing the bending moment in the walls since the summation of the bending moment and the axial force times the level arm is equal to the external moment applied on the system. As is known, the axial force at each level of the wall is equal to the integration of the shear flow from the top to that specific level which leads to increasing the difference at the lower part of the walls.

As is mentioned before, the bending moment in walls in this case is larger than the bending moment when the walls are supported on individual elastic foundation. The reason is that the rotational displacement at the base is partly restrained by the stiffened beam at base.

II. As well as the axial force, the variation of the shear force in the connecting beams has been drawn throughout the height of system which is shown in Figure 4.7. According to the curve the maximum shear force occurs at $\frac{z}{H} = 0.118$ which is significantly higher than the previous example.

Maximum shear force per unit of length: $q_{max} = 61.018\frac{kN}{m}$
Maximum shear force in connecting beam: $Q_{max} = 61.08 * 3 = 183.24\ kN$
Maximum possible moment in beams: $183.24 * \frac{2.5}{2} = 228.81\ kN.m$

The curves in Figure 4.7 show that the shear force in case of walls supported on a stiffened beam is smaller compared to the walls supported on individual foundations. The reason could be the smaller vertical displacement at the base which is partly restrained by the stiffened beam. It is worth to mention that since the shear force in the beams is smaller, the amount of required reinforcement in the beams will be smaller as well. Therefore, applying beams at the base of shear walls could reduce the cost of the structure.

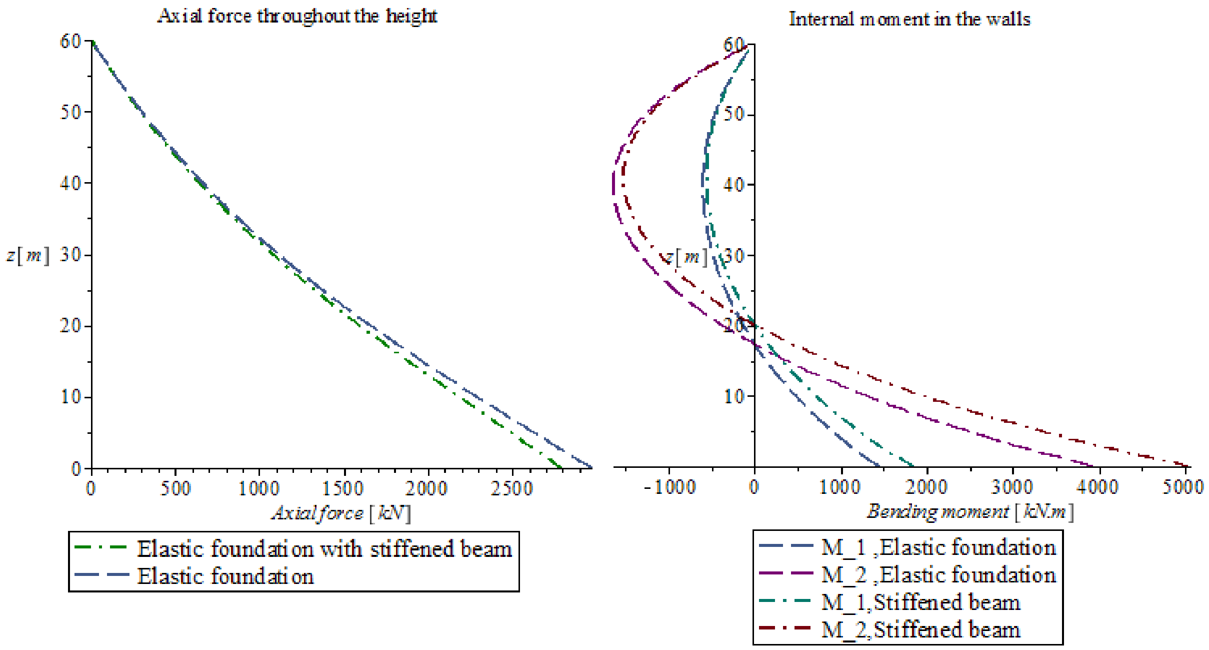


Figure 4.6 : Axial force and bending moment for walls on individual elastic foundation and walls with stiffened beam

III. Lateral deflection is one of the most important aspects about the studying the behaviour of the shear walls. There are some limitations concerning the maximum lateral deflection which have to be satisfied in the design of tall structure. Therefore, it is very important to reduce the deflection of the walls. As can be observed from Figure 4.8 the reduction of the lateral deflection due to applying a stiffened beam at base is not significant. The reason is that in this example the base beam is not stiff enough to resist the deflection at base. However the deflection is reduced from 27.1 mm in case of two individual elastic foundations to 26 mm in case of coupled walls stiffened with a base beam.

IV. Stress at extreme fibre of the walls at the base is the result of the stress due to bending and the stress due to axial force on each wall.

$$\sigma_A = \frac{(m(0) - N(0).l).I_1}{i_t}.\frac{c_1}{I_1} + \frac{N(0)}{A_1} = \frac{(30600 - 2789.44 * 8.5) * 2.5}{11.7} + \frac{2789.44}{1.5} = 3331.8\ \frac{kN}{m^2}$$

$$\sigma_B = -\frac{(m(0) - N(0).l).I_1}{i_t}.\frac{c_2}{I_1} + \frac{N(0)}{A_1} = 387.5\ \frac{kN}{m^2}$$

$$\sigma_C = \frac{(m(0) - N(0).l).I_2}{i_t}.\frac{c_3}{I_2} - \frac{N(0)}{A_2} = \frac{(30600 - 2789.44 * 8.5) * 3.5}{11.7} - \frac{2789.44}{2.1} = 732.8\ \frac{kN}{m^2}$$

$$\sigma_D = -\frac{(m(0) - N(0).l).I_2}{i_t}.\frac{c_4}{I_2} + \frac{N(0)}{A_2} = -3389.4\ \frac{kN}{m^2}$$

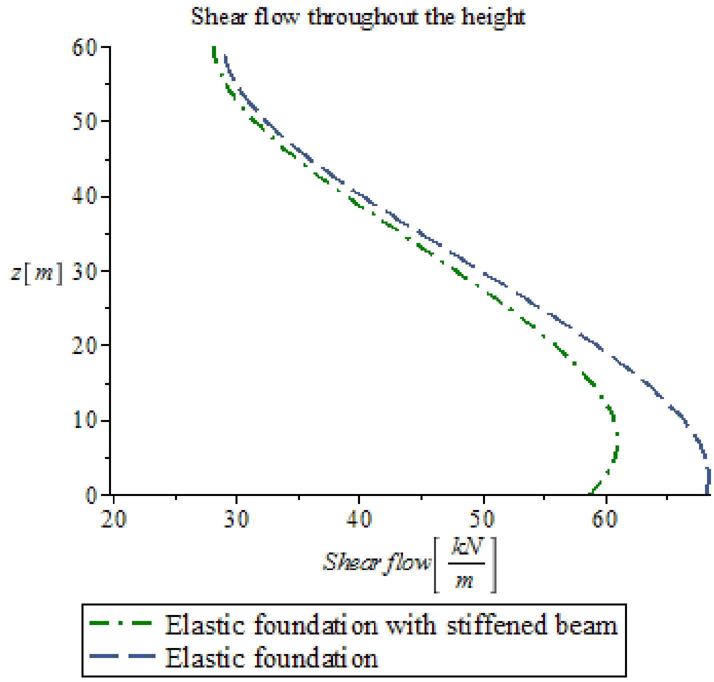


Figure 4.7 : Shear force in the connecting beams along the height of the walls

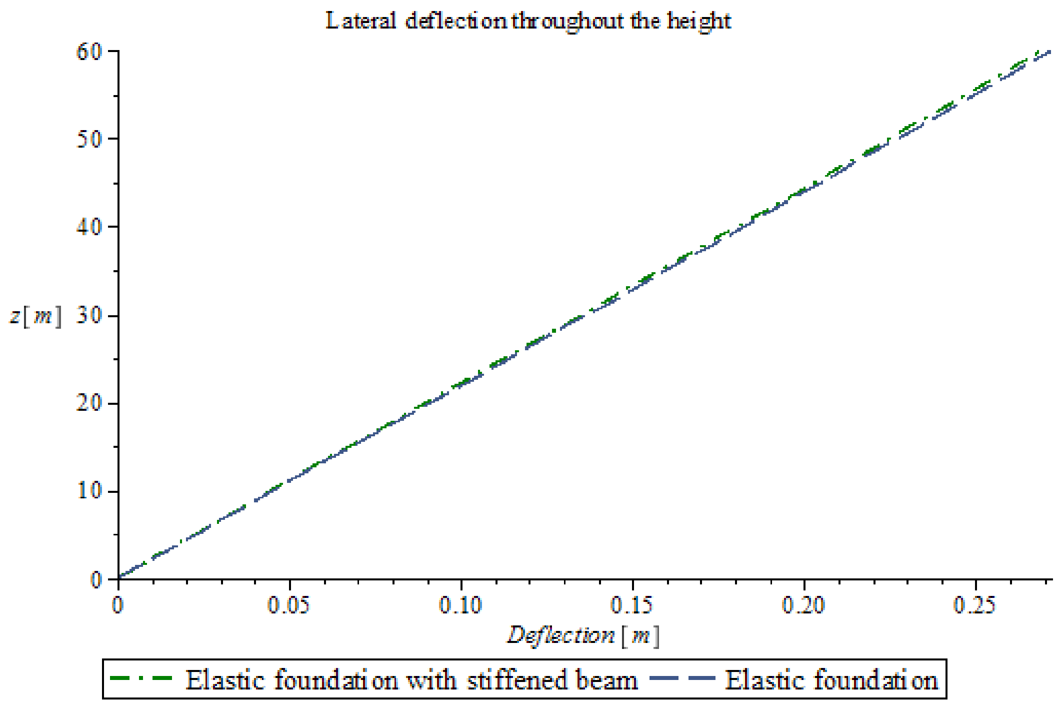


Figure 4.8 : Lateral deflection for the walls supported on individual foundation and walls with stiffened beam

Comparing the results with the previous example it can be seen that the absolute value of the stress at extreme fibres of the walls has been increased. It can be concluded that the behaviour of the shear walls supported on a stiffened beam is more similar to the behaviour of the walls on the rigid foundation.

Design example 4

In this example numerical investigation will be presented in order to consider the influence of the properties of base beam on behaviour of the coupled shear walls. According to equation (4.7) changing the parameters of base beam will change the magnitude of ψ (stiffness factor of base beam to connecting beams). Therefore, the previous walls system given in Figure 4.5 will be considered for three different value of ψ. It is known that, the rotational stiffness of the base k_θ and the vertical stiffness k_v both depend on the dimension of the base beam. This means that by changing the properties of the stiffened base beam the vertical and rotational stiffness will also change. Note that, the modulus of elasticity of the soil is taken equal to $\lambda = 10200\ [\frac{kN}{m^3}]$ in all cases. The base beam's properties are given in Table 7 to Table 12.

Table 7: Properties of base beam 1

Base beam 1		
$h_{s,b}$	[m]	0.4
$t_{s,b}$	[m]	0.3
$E_{s,b}$	[kN/m^2]	36000000
$I_{s,b}$	[m^4]	0.0016
$\psi = E_{s,b} \cdot \frac{I_{s,b}}{EI_e} h$	[m]	3.221184

Table 8: Vertical and rotational stiffness of base beam 1

$\lambda_r = \frac{12EI_e l^2}{k_\theta b^3 h}$	[1/m]	0.831419
$\lambda_v = \frac{12EI_e}{k_v b^3 h}$	[1/m]	0.153872
k_θ	[kN.m]	1193400
k_v	[kN/m]	89250
$\mu_f = \lambda_r + \lambda_v$	[1/m]	0.985291

Table 9 : Properties of base beam 2

Base beam 2		
$h_{s,b}$	[m]	0.8
$t_{s,b}$	[m]	0.6
$E_{s,b}$	[kN/m^2]	36000000
$I_{s,b}$	[m^4]	0.0256
$\psi = E_{s,b} \cdot \frac{I_{s,b}}{EI_e} h$	[m]	51.53894

Table 10 : Vertical and rotational stiffness of base beam 2

$\lambda_r = \frac{12EI_e l^2}{k_\theta b^3 h}$	[1/m]	0.41571
$\lambda_v = \frac{12EI_e}{k_v b^3 h}$	[1/m]	0.076936
k_θ	[kN.m]	2386800
k_v	[kN/m]	178500
$\mu_f = \lambda_r + \lambda_v$	[1/m]	0.492646

Table 11: Properties of base beam 3

Base beam 3		
$h_{s,b}$	[m]	1.2
$t_{s,b}$	[m]	0.8
$E_{s,b}$	[kN/m^2]	36000000
$I_{s,b}$	[m^4]	0.1152
$\psi = E_{s,b} \cdot \frac{I_{s,b}}{EI_e} h$	[m]	231.9252

Table 12 : Vertical and rotational stiffness of base beam 3

$\lambda_r = \frac{12EI_e l^2}{k_\theta b^3 h}$	[1/m]	0.311782
$\lambda_v = \frac{12EI_e}{k_v b^3 h}$	[1/m]	0.057702
k_θ	[kN.m]	3182400
k_v	[kN/m]	238000
$\mu_f = \lambda_r + \lambda_v$	[1/m]	0.369484

The variation of the shear force in the connecting beams, lateral deflection, axial force in the walls and bending moment in the walls throughout the height of the system have been demonstrated in Figure 4.9 to Figure 4.12. Comparing the results it can be noticed that, by increasing the stiffness of the base beam, the axial force in the walls and shear force in the connecting beams are greatly decreased. But the total bending moment in the walls has been increased. As was expected the lateral deflection at the top has been decreased by increasing the stiffness of the base beam. Further, the level of the maximum loaded beam has been raised for the base beam with higher stiffness. The results show that increasing the stiffness of the base beam does not improve the behaviour of the coupled shear walls regarding the internal forces in the walls and lintels, though it was expected. However, with regard to lateral deflection of the walls, increasing the stiffness of base beam will considerably improve the behaviour of the coupled walls. It is worth to mention that increasing the stiffness of the stiffened beam when it is located at the higher level of the walls, will may improve the behaviour of the coupled walls system.

According to equation (4.6), (4.8) and (4.9) and the shear force in the base beam, the rotational and the vertical displacement at the base can be determined.

$$Q_1 = \psi_1 . q_1(0) = 3.22 * 58.7 = 189.04\ kN$$

$$Q_2 = 51.53 * 18.72 = 964.8\ kN$$

$$Q_3 = 231.92 * 5.34 = 1240.5\ kN$$

$$\delta_{v,1} = \frac{N_1(0) + Q_1}{k_v} = \frac{2789.1 + 189.04}{89250} = 3.33 * 10^{-2}\ m$$

$$\delta_{v,2} = \frac{2035 + 964.8}{178500} = 1.68 * 10^{-2}\ m$$

$$\delta_{v,3} = \frac{1782.8 + 1240.5}{238000} = 1.27 * 10^{-2}\ m$$

$$\delta_{\theta,1} = \frac{m(0) - (N_1(0) + Q_1).l}{k_\theta} = \frac{30600 - (2789.1 + 189.04) * 8.5}{1193400} = 4.4 * 10^{-3}\ rad$$

$$\delta_{\theta,1} = \frac{30600 - (2035 + 964.8) * 8.5}{2386800} = 2.1 * 10^{-3}\ rad$$

$$\delta_{\theta,1} = \frac{30600 - (1782.8 + 1240.5) * 8.5}{3182400} = 1.5 * 10^{-3}\ rad$$

As can be seen by increasing the stiffness of the base beam the shear force in the base beam will increase. Furthermore, the vertical and the rotational displacements at base will decrease by increasing the stiffness of the base beam since, the rotational and vertical stiffness of the base are directly proportional to the base beam stiffness.

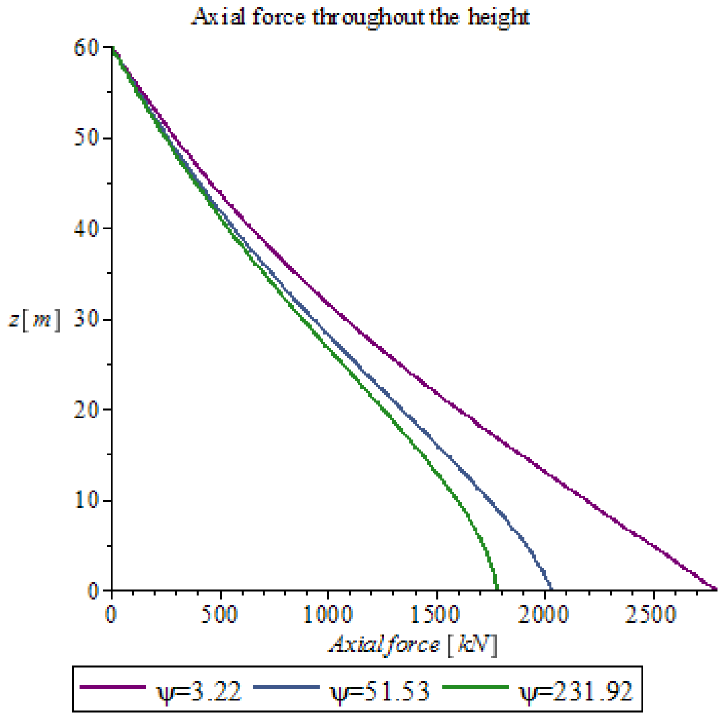


Figure 4.9 : Variation of axial force for the walls supported on different base beams

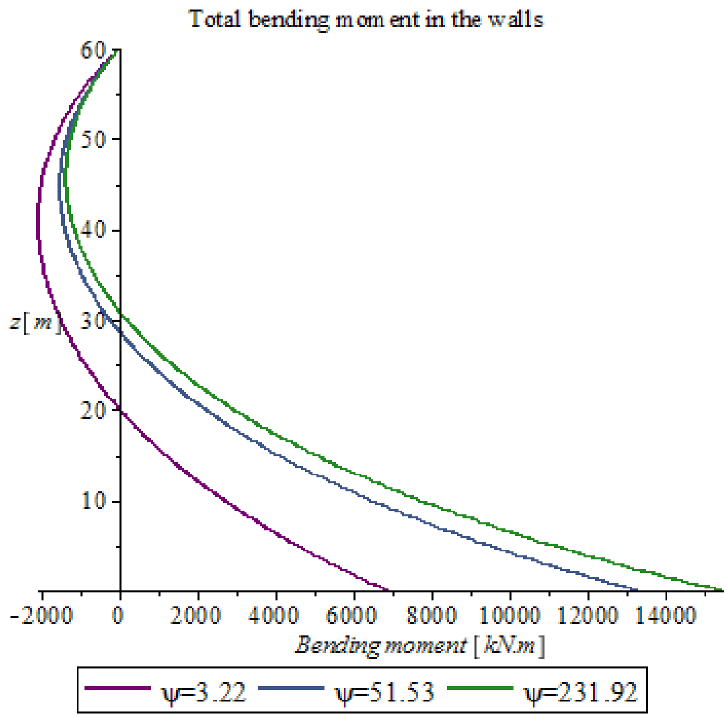


Figure 4.10 : Variation of bending moment for the walls supported on different base beams

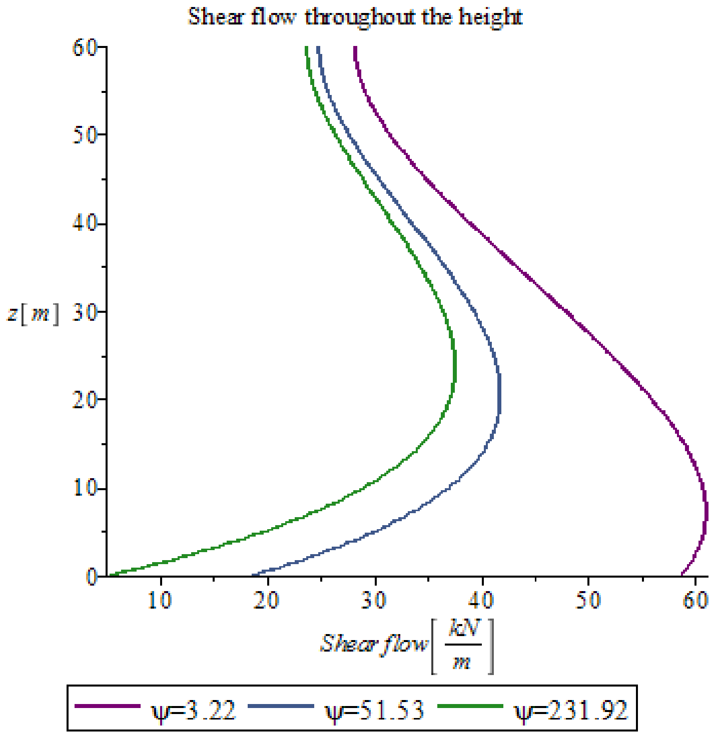


Figure 4.11 : Variation of shear flow for the walls supported on different base beams

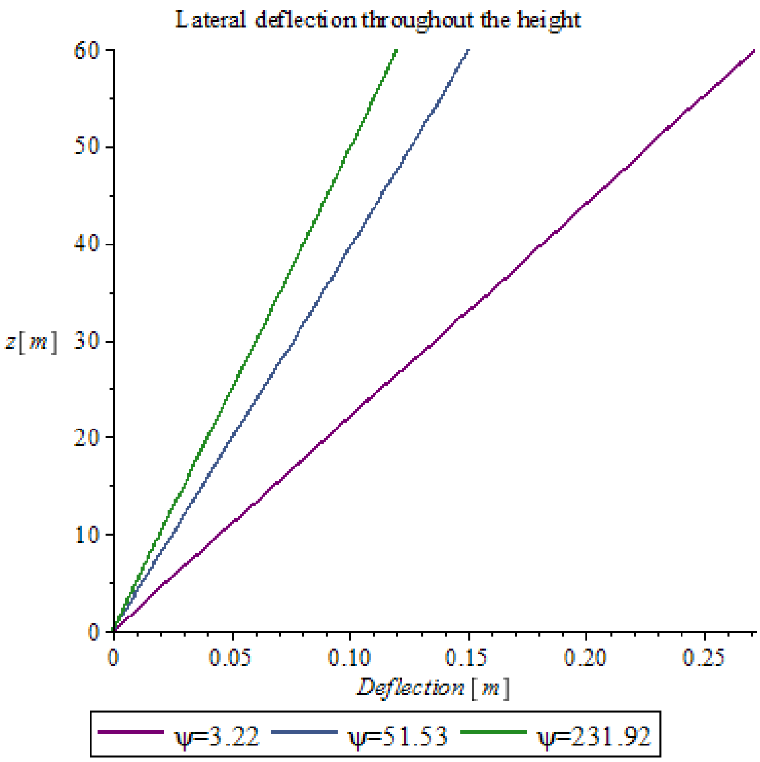


Figure 4.12: Lateral deflection for the walls supported on different base beams

4.1 Walls subjected to a point load at top

For the walls carrying a point load and the walls carrying a triangularly distributed load, the same approach has been taken as the one has been taken for the uniform distributed load. The results are given in the equation (4.27) To (4.34).

Axial force in the shear walls

$$N_{stiffened,point}(z) = \Bigg(\bigg(\Big(\big((-H-\psi)\mu_f + Hk^2\lambda_r - 1\big).sinh(k\alpha z) - k\alpha(\psi\mu_f+1)(H-z)\Big).cosh(k\alpha H) - sinh(k\alpha H)\Big(\big((-H-\psi)\mu_f + Hk^2\lambda_r - 1\big).cosh(k\alpha z) + \mu_f(H-z)\Big)\bigg)P\Bigg) / \Big(lk^2\big(k\alpha(\psi\mu_f+1).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\big)\Big) \tag{4.27}$$

Shear force in the connecting beams

$$q_{stiffened,point}(z) = \Bigg(P\bigg(\Big(\big((-H-\psi)\mu_f + Hk^2\lambda_r - 1\big).cosh(k\alpha z) + \psi\mu_f + 1\Big)\alpha k.cosh(k\alpha H) - \Big(\alpha k\big((-H-\psi)\mu_f + Hk^2\lambda_r - 1\big).sinh(k\alpha z) - \mu_f\Big).sinh(k\alpha H)\bigg)\Bigg) / \Big(lk^2\big(k\alpha(\psi\mu_f+1).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\big)\Big) \tag{4.28}$$

Internal bending moment of the shear walls

$$(M_1+M_2)_{stiffened,point}(z) = \Bigg(\bigg(\Big(\big(k^2\lambda_r H - 1 + (-H-\psi)\mu_f\big).sinh(k\alpha z) + k\alpha(k-1)(k+1)(\psi\mu_f+1)(H-z)\Big).cosh(k\alpha H) + sinh(k\alpha H)\Big(\big(-k^2\lambda_r H + 1 + (H+\psi)\mu_f\big).cosh(k\alpha z) + \mu_f(k-1)(k+1)(H-z)\Big)\bigg)P\Bigg) / \Big(k^2\big(k\alpha(\psi\mu_f+1).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\big)\Big) \tag{4.29}$$

Lateral deflection of the walls system

$$x_{stiffened\,,point}(z) = -\Bigg(\Bigg(\left(k^2\lambda_r H - 1 + (-H-\psi)\mu_f\right).\,sinh\big(k\alpha(H-z)\big)$$
$$+\Bigg(\bigg(-\frac{1}{2}z\left(H-\frac{1}{3}z\right)(\psi\mu_f+1)\alpha^2 + \lambda_r H\left(\psi\lambda_r - \psi\mu_f - 1\right)\bigg)k^4$$
$$+\left(\frac{1}{2}z\left(H-\frac{1}{3}z\right)(\psi\mu_f+1)\alpha^2 + 2\lambda_r H\right)k^2 - 1 + (-H-\psi)\mu_f\Bigg)k\alpha z.\,cosh(k\alpha H)$$
$$+ sinh(k\alpha H)\Bigg(z\left(-\frac{1}{2}\mu_f z\left(H-\frac{1}{3}z\right)\alpha^2 + \lambda_r H\left(\lambda_r - \mu_f\right)\right)k^4 \tag{4.30}$$
$$+\left(\frac{1}{2}\mu_f z^2\left(H-\frac{1}{3}z\right)\alpha^2 - \lambda_r(H+z)\right)k^2 + 1 + (H+\psi)\mu_f\Bigg)\Bigg)P\Bigg)$$
$$/\Big(\big(k\alpha(\psi\mu_f+1).\,cosh(k\alpha H) + \mu_f.\,sinh(k\alpha H)\big)\;k^4 E\alpha^2 i_t\Big)$$

4.2 Walls subjected to triangularly distributed load

Axial force in the shear walls

$$N_{stiffened,triangular}(z)$$

$$= -\frac{1}{3}\Bigg(p\Bigg(\Bigg(\Bigg(k^2\left(H\lambda_r k^2 - \frac{3}{2} + \left(-H - \frac{3}{2}\psi\right)\mu_f\right)H^2\alpha^2 + 3\psi\mu_f + 3\Bigg).sinh(k\alpha z)$$
$$- k\left(k^2(H-z)^2\left(H + \frac{1}{2}z\right)\alpha^2 + 3z\right)\alpha(\psi\mu_f + 1)\Bigg).cosh(k\alpha H)$$
$$+ \left(\left(-k^2\left(H\lambda_r k^2 - \frac{3}{2} + \left(-H - \frac{3}{2}\psi\right)\mu_f\right)H^2\alpha^2 - 3\psi\mu_f - 3\right).cosh(k\alpha z)\right.$$
$$\left.- \left(k^2(H-z)^2\left(H + \frac{1}{2}z\right)\alpha^2 + 3z\right)\mu_f\right).sinh(k\alpha H)$$
$$+ 3H\big(k\alpha(\psi\mu_f + 1).cosh(k\alpha z) + sinh(k\alpha z)\mu_f\big)\Bigg)\Bigg)$$
$$/\left(k^4\alpha^2\left(k\alpha(\psi\mu_f + 1).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\right)\ Hl\right) \tag{4.31}$$

Shear force in the connecting beams

$$q_{stiffened,triangular}(z)$$

$$= \frac{1}{3}\Bigg(p\Bigg(k\Bigg(\left(k^2H^2\left(H\lambda_r k^2 - \frac{3}{2} + \left(-H - \frac{3}{2}\psi\right)\mu_f\right)\alpha^2 + 3\psi\mu_f + 3\right).cosh(k\alpha z)$$
$$+ \frac{3}{2}(\psi\mu_f + 1)(-2 + (H^2 - z^2)k^2\alpha^2)\Bigg)\alpha.cosh(k\alpha H)$$
$$+ \left(-k\alpha\left(k^2H^2\left(H\lambda_r k^2 - \frac{3}{2} + \left(-H - \frac{3}{2}\psi\right)\mu_f\right)\alpha^2 + 3\psi\mu_f + 3\right).sinh(k\alpha z)\right.$$
$$\left.+ \frac{3}{2}\mu_f(-2 + (H^2 - z^2)\ k^2\alpha^2)\right).sinh(k\alpha H)$$
$$+ 3k\alpha H\big(k\alpha(\psi\mu_f + 1).sinh(k\alpha z) + cosh(k\alpha z)\mu_f\big)\Bigg)\Bigg)$$
$$/\left(k^4\ H\left(k\alpha(\psi\mu_f + 1)cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\right)l\alpha^2\right) \tag{4.32}$$

Internal bending moment of the shear walls

$$(M_1+M_2)_{stiffened\,,triangular}$$

$$= \frac{1}{3}\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(\Big(H\lambda_r k^2 - \frac{3}{2} + \Big(-\frac{3}{2}\psi - H\Big)\mu_f\Big)\; H^2k^2\alpha^2 + 3\psi\mu_f + 3\Bigg).sinh(k\alpha H)$$
$$+ \Bigg((k-1)(H-z)^2\Big(H+\frac{1}{2}z\Big)k^2(k+1)\alpha^2 - 3z\Bigg)\; k\big(\psi\mu_f + 1\big)\alpha\Bigg).cosh(k\alpha H)$$
$$+ \Bigg(\Bigg(-\Big(H\lambda_r k^2 - \frac{3}{2} + \Big(-\frac{3}{2}\psi - H\Big)\mu_f\Big)H^2k^2\alpha^2 - 3\psi\mu_f - 3\Bigg).cosh(k\alpha z)$$
$$+ \Bigg((k-1)(H-z)^2\Big(H+\frac{1}{2}z\Big)k^2(k+1)\alpha^2 - 3z\Bigg)\mu_f\Bigg).sinh(k\alpha H) \quad (4.33)$$
$$+ 3\big(k\alpha\big(\psi\mu_f+1\big).cosh(k\alpha z) + sinh(k\alpha z)\mu_f\big)H\Bigg)p\Bigg)$$
$$/\Big(Hk^4\Big(k\alpha\big(\psi\mu_f + 1\big).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\Big)\alpha^2\Big)$$

Lateral defection of the walls system

$$
\begin{aligned}
&x_{stiffened\ ,triangular}(z)\\
&= -\frac{1}{3}\Bigg(p\Bigg(\Bigg(\lambda_r H^3\alpha^2k^4 - H^2\alpha^2\left(H\mu_f + \frac{3}{2}\mu_f\psi + \frac{3}{2}\right)k^2 + 3\mu_f\psi\\
&+ 3\Bigg).sinh\big(k\alpha(H-z)\big)\\
&+ \left(\alpha^2\left(-\frac{1}{2}(\psi\mu_f + 1)z\left(H^3 - \frac{1}{2}zH^2 + \frac{1}{20}z^3\right)\alpha^2 + \lambda_r H^3\big(\psi\lambda_r - \psi\mu_f - 1\big)\right)k^6\right.\\
&+ 2\left(\frac{1}{4}(\psi\mu_f + 1)z\left(H^3 - \frac{1}{2}zH^2 + \frac{1}{20}\ z^3\right)\alpha^2 +\ \lambda_r H^3\right)k^4\alpha^2\\
&- \left(H^3\mu_f +\ \left(\frac{3}{2}\psi\mu_f + \frac{3}{2}\right)H^2 - \frac{1}{2}z^2(\psi\mu_f + 1)\right)\alpha^2k^2 + 3\psi\mu_f\\
&\left.+ 3\right)k\alpha z.cosh(k\alpha H)\\
&+ \Bigg(\Bigg(\left(-\frac{1}{2}z\mu_f\left(H^3 - \frac{1}{2}zH^2 + \frac{1}{20}z^3\right)\alpha^2\right.\\
&\left.+ \lambda_r H^3\big(\lambda_r - \mu_f\big)\right)zk^6\alpha^2 - \left(-\frac{1}{2}z^2\mu_f\left(H^3 - \frac{1}{2}zH^2 + \frac{1}{20}z^3\right)\alpha^2\right.\\
&\left.+ \lambda_r H^2\left(H + \frac{3}{2}z\right)\right)k^4\alpha^2 + \left(\left(H^3\mu_f + \left(\frac{3}{2}\psi\mu_f + \frac{3}{2}\right)H^2 + \frac{1}{2}z^3\mu_f\right)\alpha^2 + 3z\lambda_r\right)k^2\\
&- 3\psi\mu_f - 3\Bigg).sinh(k\alpha H)\\
&-\ 3\Big(k\alpha\big(\psi\mu_f + 1\big).cosh(k\alpha z) + \mu_f.sinh(k\alpha z)\\
&+ k\alpha\big(k^2z\lambda_r - \psi\mu - f - z\mu_f - 1\big)\Big)H\Bigg)\Bigg)\\
&/\left(EH\alpha^4\left(k\alpha\big(\psi\mu_f + 1\big).cosh(k\alpha H) + \mu_f.sinh(k\alpha H)\right)\ i_t k^6\right)
\end{aligned}
\tag{4.34}
$$

5 Analysis of coupled shear walls using discrete method

As has been mentioned before, there are two main methods to analyse the coupled shear walls: the continuous method and the discrete method. Up to now the continuous method for the coupled shear walls with one row of opening has been studied. Furthermore, the related equations and the design curves have been derived. In this chapter the analysis of the coupled shear walls based on the discrete method will be considered. First, the discrete method will be explained and in the second part of this chapter the results obtained from the both methods will be compared.

5.1 Introduction

In analysing the coupled shear walls using the discrete method, the walls will be treated as frame structures. Opposed to the continuous method there is no limitation regarding to symmetry and dimensions variety throughout the height of the walls for the discrete method. This method can be used for any types of coupled shear walls with one or several rows of openings. The only disadvantage of this method is the requirement of a frame analysis computer programme. Though the results from the discrete method could be very accurate, the modelling of the coupled walls system and analysing the results could be very time consuming specially, in case of walls with more than two rows of opening or with larger story numbers.

As has been mentioned, in this method a system of coupled shear walls will simulated by an equivalent frame which will be formed by the centre lines of the walls section and connecting beams. Furthermore, the cross sectional properties of the walls and connecting beams will be assigned to the columns and centre beams in the equivalent frame, respectively. To assure that the behaviour of the frame model is identical to the behaviour of the coupled walls, it is required to assign a very high stiffness to the end beams which are extended from centre line of walls to the end of the connecting beams. Therefore these end beams are assumed to have an infinite large area and moment of inertia.

It is worth to mention that if a frame analysis computer programme does not have the capability to define a member with infinite axial and flexural rigidity, a reasonable large value of about 10^3 -10^4 times the corresponding values for the connecting beams will give a fairly adequate result. Additionally, some advisable values based on the length ratio of the connecting beams $\frac{d_1}{0.5\,b}$ are given by Joseph Schwaighofer to choose the appropriate cross sectional properties for the end beams. These values are given in Table 13. (For more details see Figure 5.1).

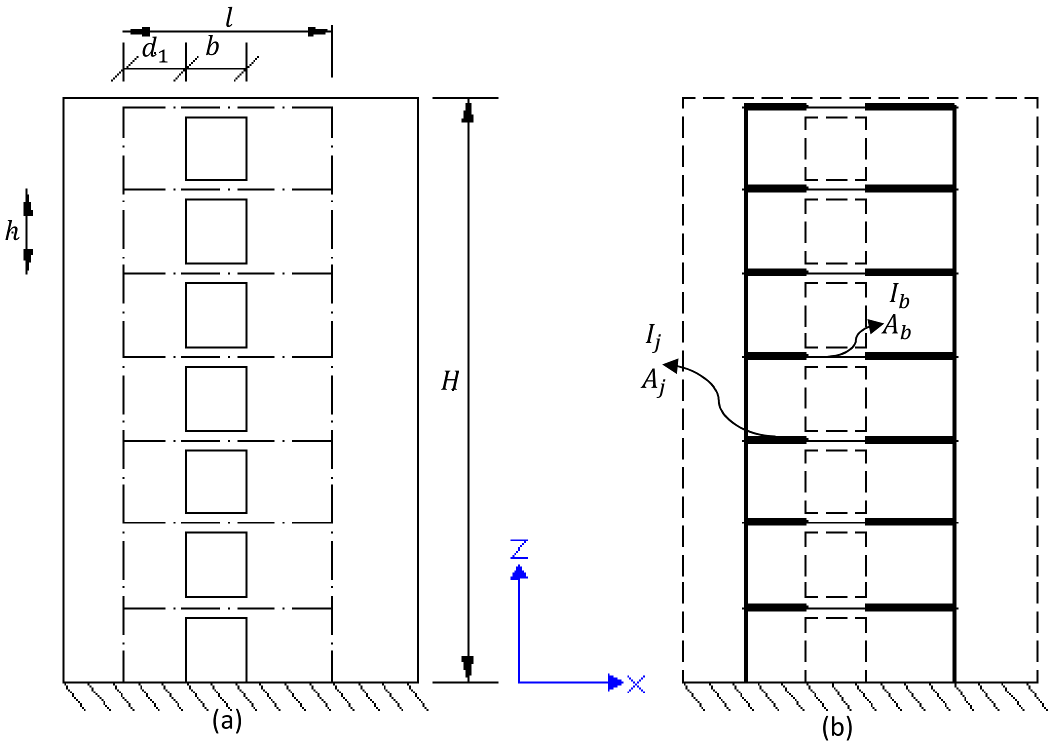


Figure 5.1 : Coupled shear walls and equivalent frame

Table 13 : Coefficient of axial and flexural rigidity for end beams in discrete method

$\frac{d_1}{0.5\,b}$	K_1	K_2
0.5	50	200
1.0	100	700
2.0	200	2600
3.0	300	6300
5.0	500	21500

(Joseph Schwaighofer, 1969)

$A_j = K_1.A_b$
$I_j = K_2.I_b$
b is the clear span of connecting beams
d_1 is the half of the width of wall 1
A_j is the cross sectional area which should be assigned to the end beams
I_j is the second moment of area which should be assigned to the end beams
A_b is the cross sectional area of the connecting beams (centre beams)
I_b is the second moment of area of the connecting beams

5.2 Numerical investigation

For the numerical investigation, the system of coupled shear walls which was given in the previous examples have been analysed by using the discrete method. For this reason the frame analysis computer programme, Matrix Frame, has been be used. The coupled shear walls system has been modelled as an equivalent frame, illustrated in Figure 5.2. Two rigid supports have been defined under the columns. The output of the programme is given in appendix 4.

Design example 5

In the first model it is assumed that, the walls are supported on a rigid foundation. The results will be compared with the continuous method on the rigid foundation. Note that, for all members the modulus of elasticity (E) is assumed to be equal to $36*10^6$ kN/m^2. Further, a horizontal uniform distributed load of 17 kN/m is applied on the frame. The effect of self-weight of the members has been ignored since in the continuous method the effect of self-weight does not considered. For this model four different cross sections have been defined : for the left wall, for the right wall, for the connecting beams and for the end beams. The cross sectional properties are given below in Table 14 and Table 15. More details can be found in design example 1.

Table 14 : Properties of columns and beams

Section of left column		P4
Section area	[m^2]	1.5
Section moment of inertia	[m^4]	3.125
Section of right column		**P3**
Section area	[m^2]	2.1
Section moment of inertia	[m^4]	8.575
Section of middle beam		**P1**
Section area	[m^2]	$1.2*10^{-1}$
Section moment of inertia	[m^4]	$1.6*10^{-3}$
Section elasticity	[kN/m^2]	$3.6*10^{7}$
Section poisson ratio		0.2

Table 15 : Properties of end beams

Section of side beam		P12
Section area	[m^2]	21.4
Section moment of inertia	[m^4]	4.16
Section elasticity	[kN/m^2]	$3.6*10^{15}$
Section poisson ratio		0.2

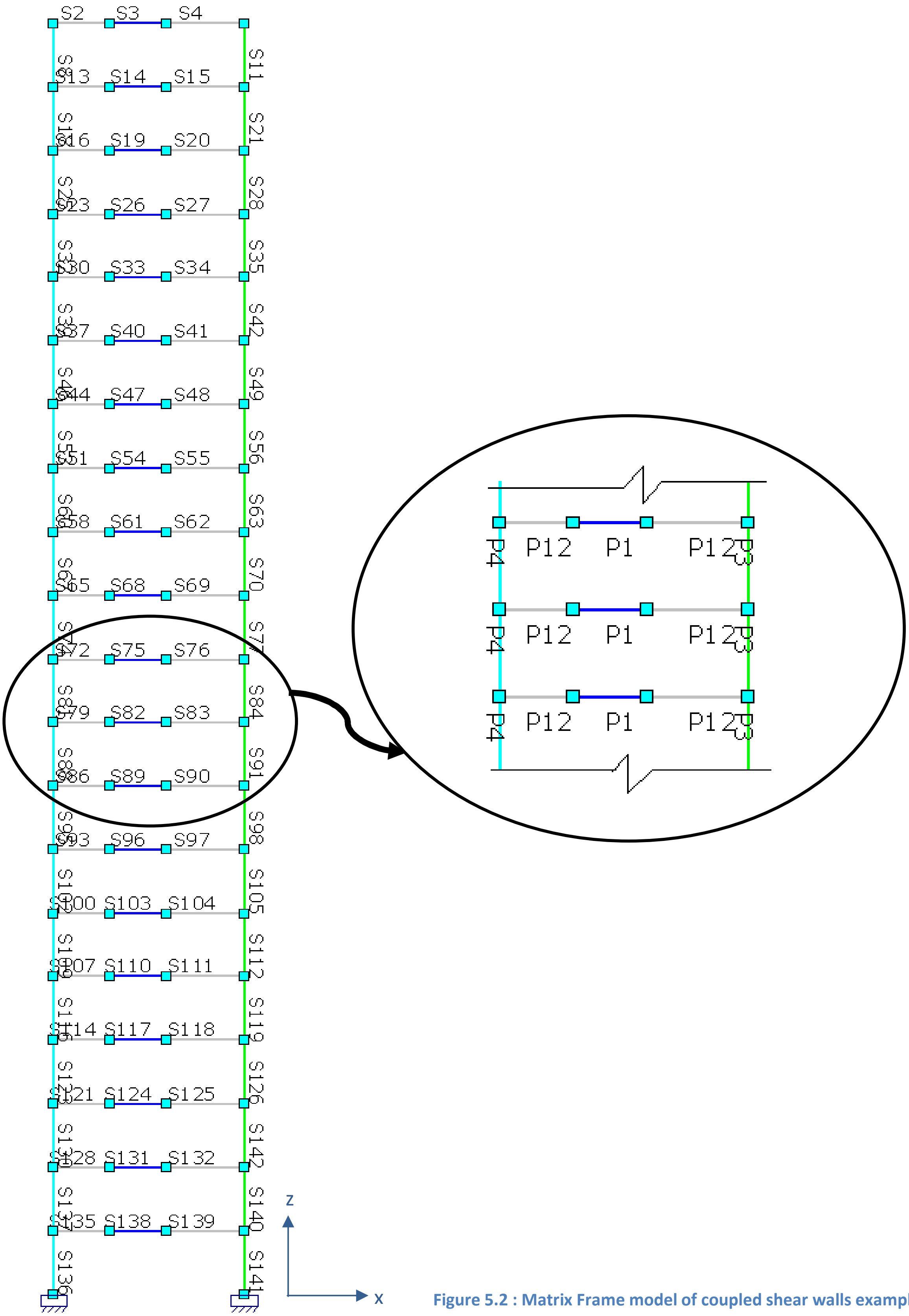


Figure 5.2 : Matrix Frame model of coupled shear walls example 1

Note that, a linear elastic analysis has been used for this example. The Matrix Frame programme gives the result of the analysis as a report in Pdf format. A comparison has been made between the extreme values resulted from the Matrix Frame and from the continuous method, given in Table 16 and Table 17. Additionally, the variation of the axial force and bending moment in wall 1 and 2 along the height has been drawn for both continuous and discrete method. (See Figure 5.3 to Figure 5.5)

Table 16 : Result of internal forces for walls supported on rigid foundation according to continuous and discrete method

Walls on rigid foundation		Continuous method	Matrix frame
Maximum axial force in the walls	[kN]	1681.9	1712.5
Bending moment at base in the wall 1	[kN.m]	4354.67	4482.8
Bending moment at base in the wall 2	[kN.m]	11949.21	11550.9
Maximum shear force in the connecting beams	[kN]	109.36	110.3
Bending moment in the connecting beams	[kN.m]	136.7	137.9
Maximum deflection at the top	[m]	0.022	0.022

Table 17 : Stresses at base of walls supported on rigid foundation according to continuous and discrete method

Walls on rigid foundation		Continuous method	Matrix frame
Stress at A	[kN/m^2]	4617.77	4.73*1000
Stress at B	[kN/m^2]	-2390.06	-2.44*1000
Stress at C	[kN/m^2]	4109.87	3.9*1000
Stress at D	[kN/m^2]	-5701.1	-5.53*1000

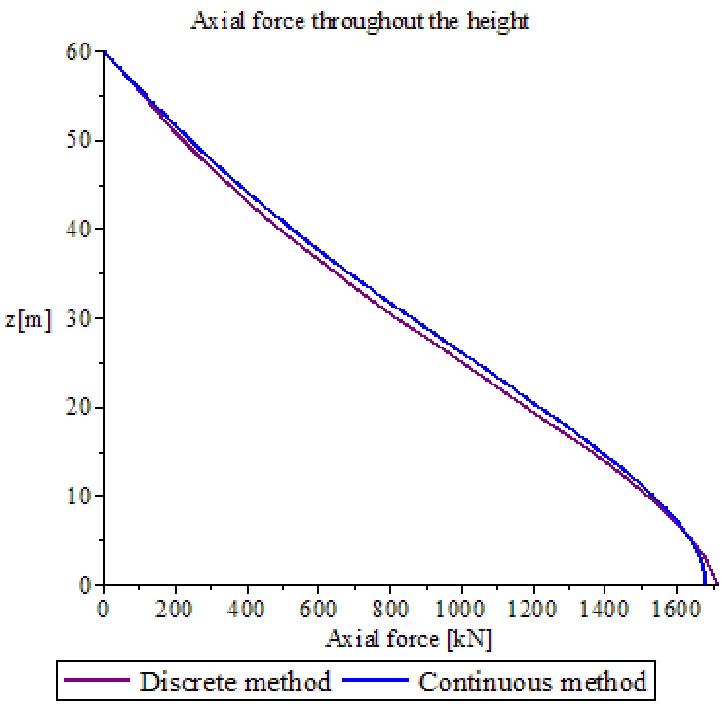


Figure 5.3 : Axial force throughout the height of walls on rigid foundation

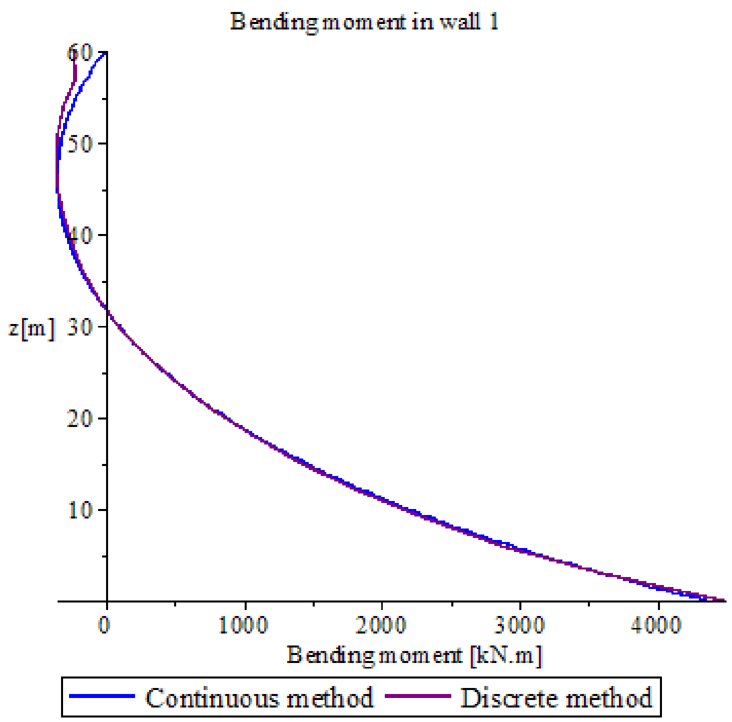


Figure 5.4 : Comparison of bending moment in wall 1 between continuous and discrete method

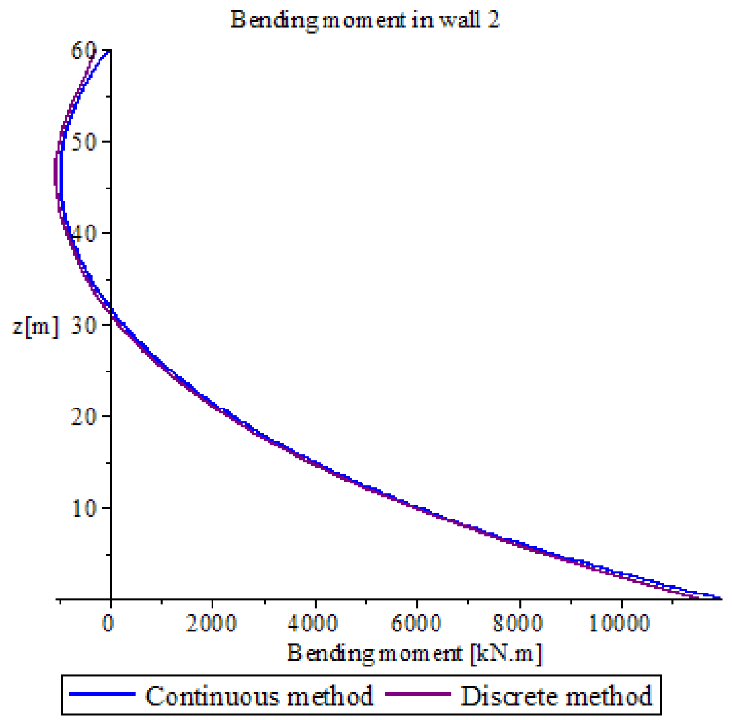


Figure 5.5 : Comparison of bending moment in wall 2 between continuous and discrete method

The investigation has shown that, the results from both methods are fairly similar however the extreme values from the discrete method are within 1% to 3% larger than the values from the continuous method. One of the major differences between both methods is that, the axial force and the bending moment at the top of the walls are equal to zero according to the continuous method, though the discrete method gives a nonzero value for the bending moment and axial force at the top of the walls. The reason is that in the Matrix Frame programme the walls between each two stories are assumed to be a column in which the axial force along the height is constant. As a result the axial force at the top of the walls could not be zero which leads into a nonzero value for the bending moment at the top. Note that, in Figure 5.3 the magnitude of the axial force at the top is taken equal to zero to draw the curve.

It should be mention that, the above curves have been drawn by using the mean values form the Matrix Frame programme for each column. The exact values from the results of Matrix Frame programme will give stepwise curves for both bending and axial force. Since the walls in-between each two stories is considered as a column, there are two different bending moments at each level resisted by the walls, one at the top and one at the bottom. Further the value of the axial force changes at the connection point between two different stories.

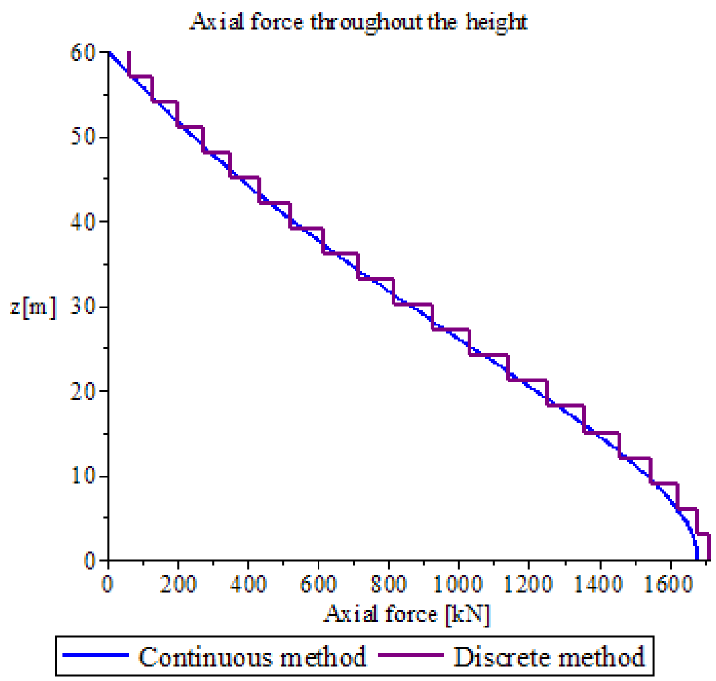


Figure 5.6 : Exact value of axial force according to Matrix Frame programme

According to the results, it can be concluded the accuracy of the continuous method in case of the rigid foundation is fairly acceptable. But to draw a definitive conclusion it is needed to analyzing different models with different dimensions and comparing the results. This will be done in the next chapter.

Design example 6:

In this example the behaviour of the coupled shear walls supported on an individual elastic foundation will be considered by using the discrete method. Consider the same structure of the design example 2 when it is supported on an elastic foundation. The coupled walls system is modelled as an equivalent frame structure identical to the frame structure in previous example. The only difference is that, here the walls are supported on elastic foundations. The rotational and the vertical stiffness of the sub soil are given in Table 18. The frame is subjected to a horizontal uniformly distributed load of 17 kN/m. The effect of self-weight does not considered.

Table 18 : Properties of elastic foundation

left Support		
k_x	[kN/m]	fixed
k_z	[kN/m]	153000.00
k_r	[kN.m]	318750.00

Right support		
k_x	[kN/m]	fixed
k_z	[kN/m]	214200.00
k_r	[kN.m]	874650.00

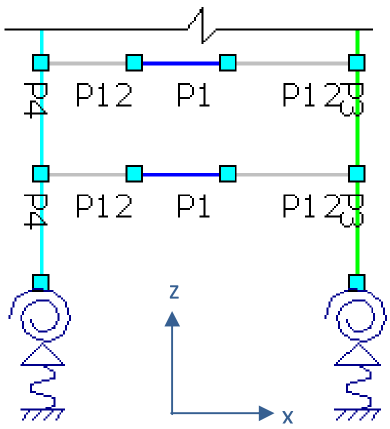


Figure 5.7 : Equivalent frame on elastic support

As well as the previous example, a linear elastic analysis has been carried out. The elastic foundation has been modelled as two linear springs under both columns (see Figure 5.7). The horizontal deflection of the springs is restrained at base. The extreme values of the internal forces and the lateral deflection are given in the following table. The output of the programme can be found in appendix 5.

Table 19 : Result of internal forces for walls supported on elastic foundation according to continuous and discrete method

Walls on elastic foundation		Continuous method	Matrix frame
Maximum axial force in the walls	[kN]	2968.55	2971.5
Bending moment at base in the wall 1	[kN.m]	1433.58	1440.3
Bending moment at base in the wall 2	[kN.m]	3933.74	3945.1
Maximum shear force in the connecting beams	[kN]	205.29	222.0
Bending moment in the connecting beams	[kN.m]	256.61	277.5
Maximum deflection at the top	[m]	0.271	0.271

Table 20 : Stresses at the base of walls supported on elastic foundation according to continuous and discrete method

Walls on elastic foundation		Continuous method	Matrix frame
Stress at A	[kN/m^2]	3125.9	3.13*1000
Stress at B	[kN/m^2]	832.16	0.83*1000
Stress at C	[kN/m^2]	192.01	0.2*1000
Stress at D	[kN/m^2]	3019.2	3.03*1000

It can be observed from above tables that, the extreme value of the internal forces and stresses form both methods are very similar. Figure 5.8 and Figure 5.9 illustrate the comparison of axial force and bending moment in wall 1 along the height between discrete and continuous method. According to the curves the differences between both methods are more negligible at the intermediate levels. Note that, to draw the curves the exact values obtained from the MatrixFrame program have been used and not the mean values which were used in the previous example. As can be seen from the bending moment curve, the connection at each story sustains two different bending moments, one at the top and one at the bottom. Regarding to the Matrix Frame results, the connections have to be designed for the larger value at each level which is almost 30% larger than the obtained value from the continuous method. As a result, the connections designed by the discrete method should be about 30% stiffer than the connection designed according to the continuous method.

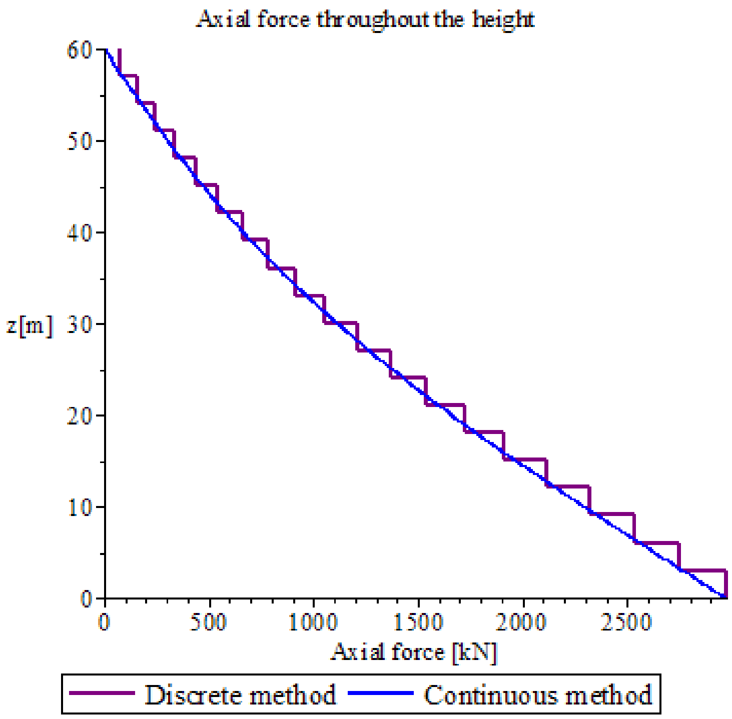


Figure 5.8 : Axial force throughout the height according to discrete and continuous method

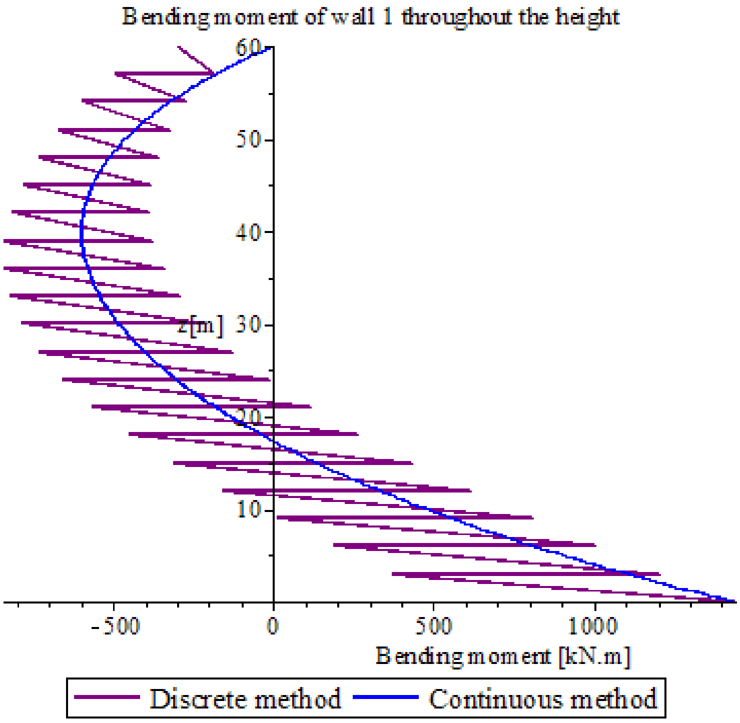


Figure 5.9 : Bending moment in wall 1 according to discrete and continuous method

Design example 7

In this example the behaviour of a system of coupled shear walls with a stiffened beam at the base, supported on an elastic foundation will be studied by using the Matrix Frame programme. Consider the same system given in design example 3. The coupled walls system is modelled as an equivalent frame in frame analysis programme MatrixFrame. The cross sectional properties of the beams and columns are given in Table 14 and Table 15. As well as previous example the elastic foundation has been modelled as two linear springs under the columns. The rotational and vertical stiffness of the springs are given in table Table 18.

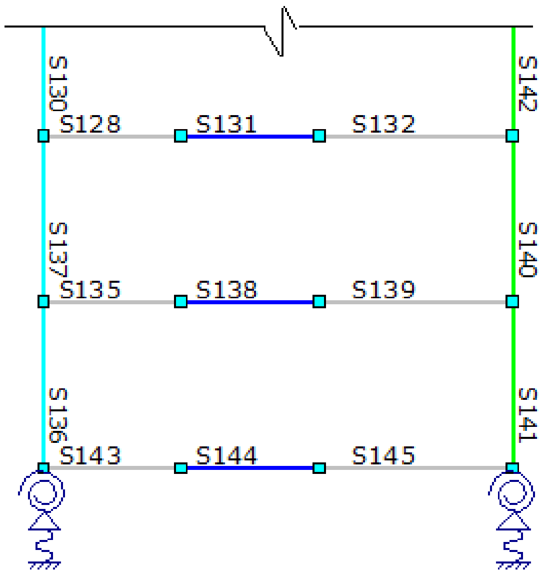


Figure 5.10 : Equivalent frame of the walls on Elastic foundation with base beam

The cross sectional properties of the base beam is assumed to be identical to the connecting beams. As is known, the part of the base beam under the walls cannot rotate freely, due to the high stiffness of the walls. Therefore, the base beam in the equivalent frame is modelled into three beams from which the two sides' beams have higher axial rigidity to prevent the free rotation of the beam. (See Figure 5.10 for more details). The properties of the tow end beams at base are given below.

Table 21 : Cross sectional properties of end beam at base.

Section of side beam		P12
Section area	$[m^2]$	21.4
Section moment of inertia	$[m^4]$	4.16
Section elasticity	$[kN/m^2]$	$3.6*10^{15}$
Section poisson ratio		0.2

A linear elastic analysis has been carried out. The effect of self-weight has been ignored. The only applied load on the structure is a horizontal uniformly distributed load of 17 kN/m. The input and the output of the analysis can be found in appendix 6. Further, a comparison has been made between the results obtained from the continuous method and results from the discrete method. According to the results which are given in Table 22 the differences between the continuous and the discrete method are not considerable as was expected. It is worth to mention that, in this example the differences between the results from the continuous and discrete method are larger than the differences in the previous example. According to Table 22 the differences are within 1 to 5 percent.

Table 22 : Results of internal forces for walls stiffened with base beam according to continuous and discrete method

Walls on elastic foundation		Continuous method	Matrix frame
Maximum axial force in the walls	[kN]	2789.1	2797.4
Bending moment at base in the wall 1	[kN.m]	1840.97	2141.3
Bending moment at base in the wall 2	[kN.m]	5051.64	4809.6
Maximum shear force in the connecting beams	[kN]	183.01	195.7
Bending moment in the connecting beams	[kN.m]	228.77	244.7
Maximum deflection at the top	[m]	0.270	0.271
Shear force in the base beam	[kN]	189.03	192.5

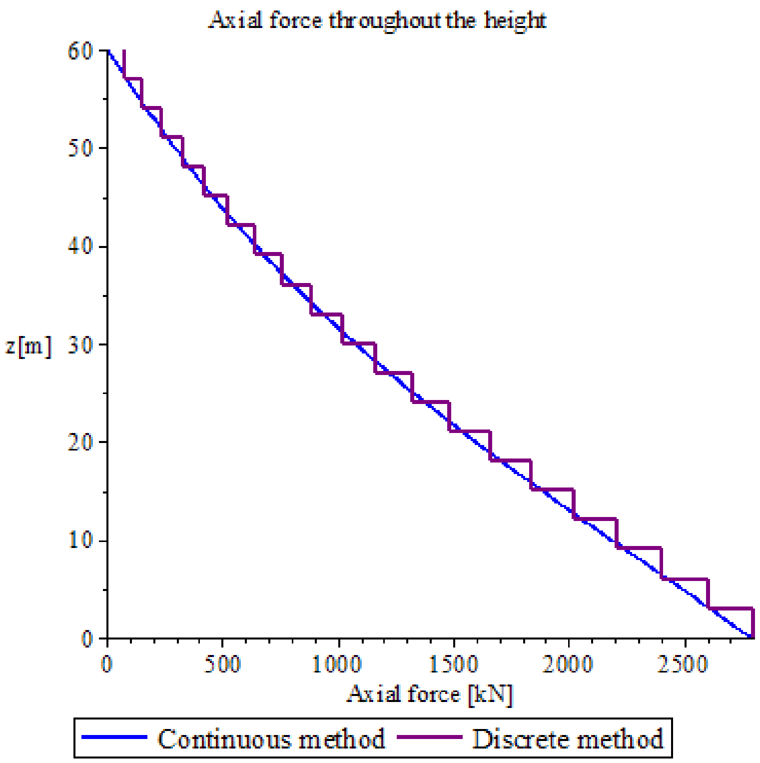


Figure 5.11 : Variation of axial force along the height for continuous and discrete method

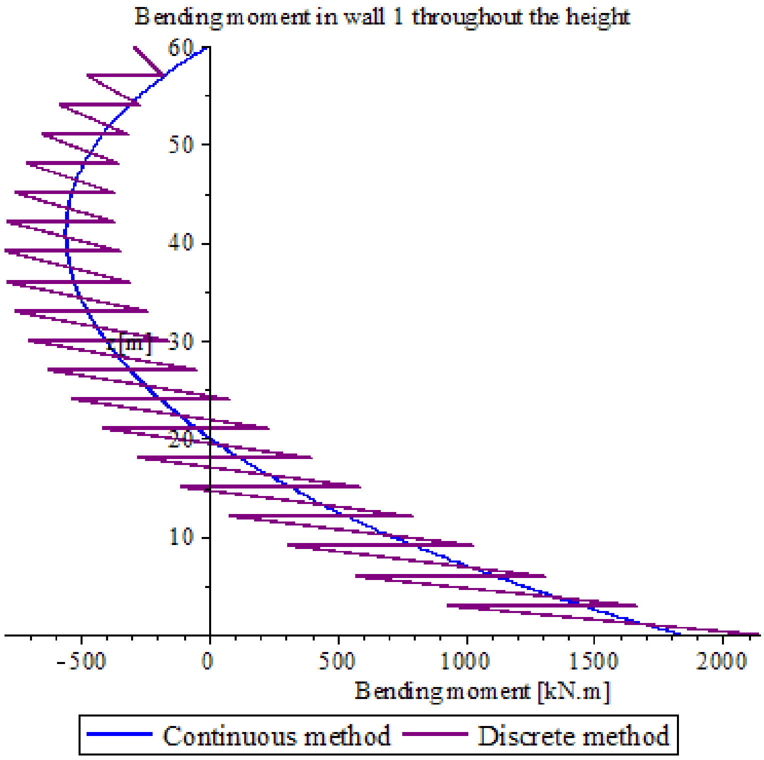


Figure 5.12: Variation of bending moment along the height for continuous and discrete method

6 Investigating the effect of stiffness of walls and connecting beams

Up to now a certain Young's modulus of elasticity has been used for both shear walls and the connecting beams to analyze a system of coupled shear walls. It is known that, in reality reinforced concrete with different strengths may be used for different members of a system. Furthermore, to take the effect of possible cracking into account the young 'modulus should be adjusted. Therefore, it is important to study the effect of Young's modulus variation on behaviour of the coupled shear walls. For this reason, a system of coupled shear walls subjected to uniform distributed load will be analyzed again when different Young's modulus has been assigned to shear walls and connecting beams.

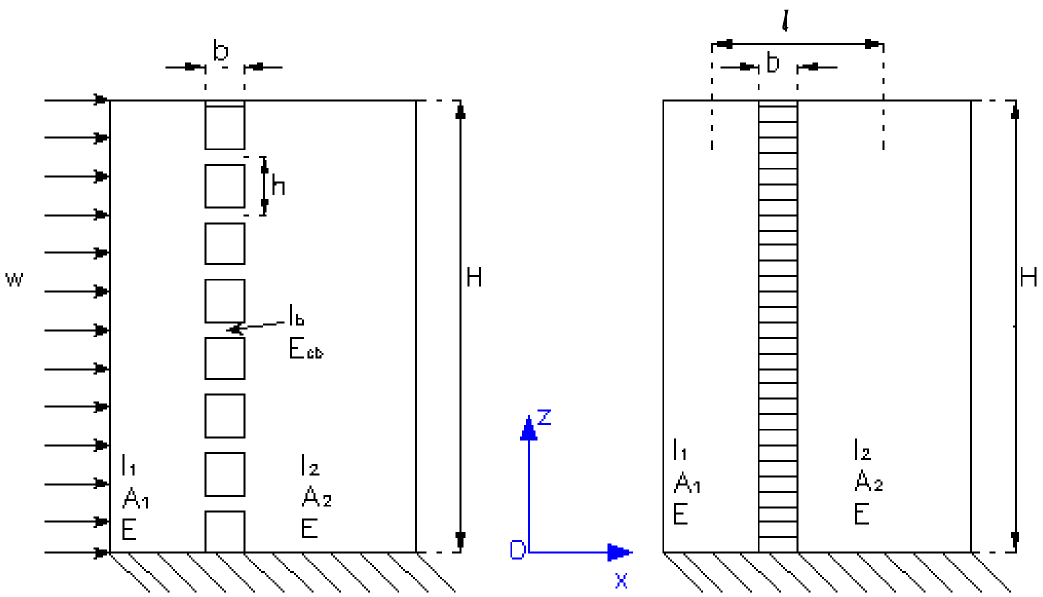


Figure 6.1: Coupled shear walls according to continuous method

6.1 Walls on rigid foundation analyzed by the continuous method

Consider a system of coupled shear walls supported on the rigid foundation with one row of openings, shown in Figure 6.1. The same approach as explained in chapter 2 will be taken here. But in this section the Young's modulus of the connecting beams will be defined by E_{cb}. The connecting beams will be replaced by continuous mediums. The continuous mediums will be cut along their contraflexure points. Considering the relative displacement at the cut ends, gives the following equation.

$$\delta = \delta_1 + \delta_2 + \delta_3 = 0 \quad (6.1)$$

$$\delta = l\left(\frac{d}{dz}x(z)\right) + \frac{1}{12}\frac{\left(\frac{d}{dz}N(z)\right)b^3h}{E_{cb}I_e} - \frac{\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left(\int_0^z N(z)dz\right)}{E} = 0 \quad (6.2)$$

Note that, relative displacements have been illustrated in Figure 2.3 and given by equation (2.1), (2.2) and (2.5). Since in reality there is no relative displacement at the cut ends, the summations of displacements are substituted equal to zero.

Considering the moment equilibrium and vertical force equilibrium in the walls (see Figure 2.4) the equation of moment curvature and the relation of normal force and shear flow in the mediums can be obtained.

$$\left(\int_z^H q(z)dz\right) = N(z) \quad (6.3)$$

$$\left(\frac{d^2}{dz^2}x(z)\right) = \frac{m(z) - lN(z)}{(EI_1 + EI_2)} \quad (6.4)$$

Differentiating from equation (6.2) and substituting the walls' moment - curvature relationship into it, the equation of normal force in the shear walls will be derived.

$$\frac{d^2}{dz^2}N(z) - k^2\alpha^2\zeta^2.N(z) = -\frac{\alpha^2}{l}\zeta^2.m(z) \quad (6.5)$$

Where:

$$\left(\frac{12.I_e.l^2}{b^3.h.i_t}\right) = \alpha^2 \quad (6.6)$$

$$\left(1 + \frac{A.i_t}{A_1.A_2.l^2}\right) = k^2 \quad (6.7)$$

$$\zeta^2 = \frac{E_{cb}}{E} \quad (6.8)$$

To get rid of the dimensions, a stiffness ratio (ζ^2) has been defined which is equal to modulus of elasticity of the connecting beams (E_{cb}) over modulus of elasticity of the walls (E). Considering the result, it can be seen that by substituting $\zeta = 1$ into equation (6.5) , the ordinary differential equation (2.22) will be obtained. It is worth to mention that, the order of ζ is identical to the order of α. The required boundary conditions to solve the obtained ODE are given below.

Boundary conditions

1. $At\ z = 0$

$$\frac{d}{dz}N(z) = 0 \tag{6.9}$$

2. $At\ z = H$

$$N(H) = 0 \tag{6.10}$$

Note that, first boundary condition can be obtained by inserting z=0 in the compatibility equation (6.2) Solving the ODE of normal force by applying the given boundary conditions gives the equation of axial force in the wall as:

$$N_{diff\ E,dist}(z) = w.\frac{H^2}{k^2.l}.\left(\frac{1}{2}.\left(1-\frac{z}{H}\right)^2 + \frac{1}{(k\alpha\zeta H)^2}.\left(1-\frac{\cosh(k\alpha\zeta z) + k\alpha\zeta H.sinh(k\alpha\zeta(H-z))}{\cosh(k\alpha\zeta H)}\right)\right) \tag{6.11}$$

As can be seen, the solution of the ODE is similar to equation (2.25) where, α is replaced by $\zeta\alpha$. As well as the axial force, the shear flow and the lateral deflection of the walls' system can be obtained by substituting $\alpha\zeta$ instead of α into the relevant equations. Note that, the calculations have been done by using Maple program and are given in appendix 7.

$$q_{diff\ E,dist}(z) = \frac{wH}{k^2l}.\left(\frac{1}{k\alpha\zeta H.cosh(k\alpha\zeta H)}\big(sinh(k\alpha\zeta z) - k\alpha\zeta H.cosh(k\alpha\zeta(H-z))\big) + k\alpha\zeta H.cosh(k\alpha\zeta H) - k\alpha\zeta z.cosh(k\alpha\zeta H)\right) \tag{6.12}$$

The required boundary conditions to solve the ODE of deflection are similar to the ones given in chapter 1. The only difference is the forth boundary condition at $z = H$ (equation (2.44)), in which α will be replaced by $\alpha\zeta$.

Boundary conditions

1. $At\ x = 0$

$$x(z) = 0 \tag{6.13}$$

$$\frac{dx}{dz}|_{z=0} = 0 \tag{6.14}$$

2. $At\ X = H$

$$\frac{d^2}{dz^2}|_{z=H} = 0 \tag{6.15}$$

$$\frac{d^3}{dz^3}x(z) - (k\alpha\zeta)^2.\frac{d}{dz}x(z) = \frac{1}{Ei_t}.\left(\frac{d}{dz}m(z) - \alpha^2\zeta^2(k^2-1).\int_0^H m(z)dz\right) \tag{6.16}$$

As has been mentioned before, to derive the ODE related to lateral deflection, the second derivative of the normal force has to be substituted into the first derivative of the compatibility equation (6.2).(2.7)

$$\left(\frac{d^4}{dz^4}x(z)\right) - \left(\frac{d^2}{dz^2}x(z)\right)k^2\alpha^2\zeta^2 = \frac{1}{Ei_t}.\left(\frac{d^2}{dz^2}m(z) - m(z).(k^2-1)\alpha^2\zeta^2\right) \tag{6.17}$$

Solving the above ODE, by applying the boundary conditions gives the equation of lateral deflection of the coupled walls system as following:

$$\begin{aligned} x_{diffE,dist}(z) = \frac{1}{4}\frac{1}{k^6\alpha^4\zeta^4\ Ei_t.cosh(k\alpha\zeta H)}\Bigg(w\Big(4H.sinh\big(k\alpha\zeta(H-z)\big)k\alpha\zeta \\ + k^2\alpha^2\zeta^2 z\left(\zeta^2 z(k+1)k^2\left(H^2 - \frac{2}{3}zH + \frac{1}{6}z^2\right)(k-1)\alpha^2 + 4H \right. \\ \left. - 2z\right).cosh(k\alpha\zeta H) - 4.sinh(k\alpha\zeta H)k\alpha\zeta H + 4.cosh(k\alpha\zeta z) - 4\Big)\Bigg) \end{aligned} \tag{6.18}$$

Moreover, composite cantilever factor in the coupled shear walls can be obtained by considering the stress at the base of the walls. For more details see Figure 2.10. The composite cantilever factor for the case, when two different modulus of elasticity are applied to shear walls and connecting beams will be:

$$k_2 = \frac{200}{(k\alpha\zeta H)^2.\left(1-\frac{z}{H}\right)^2}.\left(\frac{-cosh(k\alpha\zeta z) - k\alpha H\zeta.sinh(k\alpha\zeta H - k\alpha\zeta z)}{cosh(k\alpha\zeta H)} + 1 + \frac{k^2\alpha^2\zeta^2}{2}(H-z)^2\right) \quad (6.19)$$

Individual cantilever action is equal to :

$$k_1 = 100 - k_2 \quad (6.20)$$

6.1 Walls on an elastic foundation analysed by the continuous method

The coupled shear walls supported on elastic foundation for both walls on individual foundation and walls stiffened by a base beam have been studied, for the case of shear walls and connecting beams with different modulus of elasticity. According to the results, to determine equations related to coupled walls system with different modulus of elasticity, the stiffness factor α should be replaced by $\alpha\zeta$ in the relative equations for coupled walls system with a unique modulus of elasticity. The calculations and the derived equations can be found in appendix 8.

6.1 Considering the reinforcement ratio in the connecting beams

In the previous sections a few examples have been shown for walls on different types of foundations, according to the continuous and the discrete method. As is known, there are some limits in Euro code for the maximum and the minimum reinforcement ratio in the beams. Thus, it is important to prove whether the design of the connecting beams in these examples satisfies the Euro code's requirements or not.

For reinforcement design, the results from the analysis of coupled shear walls supported on rigid foundations (design example 1) have been used. Since the dimensions of connecting beams throughout the height of the walls are constant, the heaviest loaded beam has been chosen. The internal forces in this beam are illustrated in Figure 6.2, which are obtained from discrete method by using the frame analysis programme, Matrix Frame.

<u>Load combination:</u>

As is known, the design of the reinforcement will be done in ULS. Therefore a combination of dead loads and live loads should be applied on the structure to determine the internal forces in connecting beams. Note that in previous sections for comparing the results from discrete method and continuous method the effect of self- weight of the coupled walls system had been ignored. But for reinforcement design the weight of the structure will be taken into accounts. (More details are given in example 2).

$$L.C = 1.5\ vriable\ load + 1.2\ permanet\ load$$

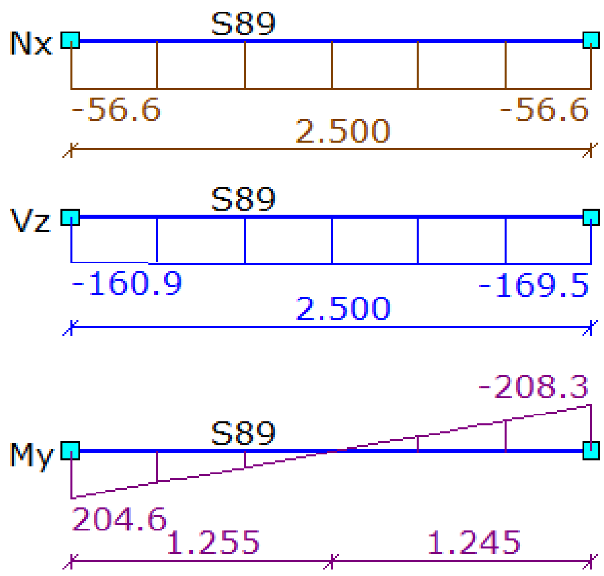


Figure 6.2 : Internal forces of the connecting beam with high stiffness

The beam is 2.5 meter long with a height of 400 mm and thickness of 300 mm. the strength of the concrete is assumed to be C45/55 for the whole structure.

Bending reinforcement

According to Euro code part 9.2.1.2 in monolithic constructions, the section at the support should be designed for a bending moment arising from partial fixity of at least β_1 of the maximum bending moment in the span. The recommended value is 0.15.

$M_{Ed,max} = 1.15 * 208.3 = 239.54 \ kN.m$

Assumption:

- Environmental conditions: XC2
- Concrete strength class : C45/55
- Minimum cover according to EC2, section 4.4.1.1:[2] $c_{nom} = c_{min} + \Delta c_{dev} = 25 + 10 = 35 \ mm$

The amount of the reinforcement area can be determined by using standard table of GTB 2010 (table 11.7) which is based on the following equation:

[2] Based on the recommended values in Euro code 2 NEN-EN 1992-1-1

$$M_{Ed} = A_s.f_{yd}.d\left(1 - 0.52\,\rho\frac{f_{yd}}{f_{cd}}\right) \qquad (6.21)$$

$$d = h - \left(\emptyset_{stirrup} + \frac{\emptyset}{2} + c\right) = 400 - \left(12 + \frac{25}{2} + 35\right) = 340.5\ mm$$

$$\frac{M_{Ed}}{bd^2} = \frac{240}{0.3 * 0.3405^2} = 6900 \xrightarrow{yields} 100\rho = 1.84$$

$$A_s = \rho.b.d = 1.84 * 10^{-2} * 300 * 340.5 = 1879.56\ mm^2$$

If steel bars of Ø25 will be used, 4 bars are needed at the top and at the bottom of the cross section, which gives a reinforcement area of $4 * \pi\frac{25^2}{4} = 1963.49\ mm^2$

$$\rho = \frac{1963.4}{340.5 * 300} = 0.019$$

Maximum and minimum reinforcement ratio

According to EC2, 9.2.1.1 the area of longitudinal reinforcement should not be taken less than $A_{s,min}$

$$A_{s,min} = 0.26\frac{f_{ctm}}{f_{yk}}b_t.d = 0.26 * \frac{3.8}{500} * 300 * 340.5 = 201.8\ mm^2$$

Furthermore, the cross sectional area of tension or compression reinforcement should not exceed $A_{s,max}$ outside the lap locations. The value of $A_{s,max}$ for beams may be found in national annex but the recommended value according to Euro code is 0.04 times the cross section.

$$A_{s,max} = 0.04\,A_c = 0.04 * 300 * 400 = 4800\ mm^2$$

As can be seen the design reinforcement ration is within the minimum and maximum allowed limits.

Shear reinforcement:

According to section 6.2.2 of NEN-EN 1992-1-1, the shear force capacity can be determined with the following equation:

$$V_{Rd,c} = \left[C_{Rd,c}.k(100.\rho_1.f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right].b_w.d \quad (6.22)$$

In which :

$V_{Rd,c}$ is the shear resistance

$C_{Rd,c}$ =0.12

f_{ck} is the characteristic compression strength

k $= 1 + \sqrt{\frac{200}{d}} \leq 2.0$

ρ_1 $\frac{A_{sl}}{b_w d} \leq 0.02$

A_{sl} is the area of the tension reinforcement

b_w is the smallest width

k_1 $= 0.15$

σ_{cp} $\frac{N_{Ed}}{A_c}$

N_{Ed} is the normal force in the cross section as a result of the load or pre-stress force

A_c is the area of the concrete cross section

$$f_{ck} = 45\ MPa$$

$$k = 1 + \sqrt{\frac{200}{340.5}} = 1.76$$

$$\rho_1 = \frac{1963.49}{300 * 340.5} = 0.019$$

$$\sigma_{cp} = \frac{56.7 * 10^3}{300 * 400} = 0.47$$

$$V_{Rd,c} = \left[0.12 * 1.76(100 * 0.019 * 45)^{\frac{1}{3}} - 0.15 * 0.47\right] * 300 * 340.5 * 10^{-3} = 87.83\ kN < 169.5\ kN$$

As can be seen the shear capacity is much lower than the applied shear force on the beam, therefore shear reinforcements are required. Note that the total shear force on the beam has to be carried by shear reinforcement. It is assummed that the diameter of stirups is equal to 12 mm, hence the distance between the stirrups can be detremined by using the following equation

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z.cot\alpha.f_{yd}} \quad (6.23)$$

In which:

A_{sw} is the area of the shear force reinforcment

s is the centre to centre distance of the shear reinforcement

f_{yd} is the calculation value of the yield strenght of the reinforcement

z is the internal lever arm. When thre is no normal force, z=0.9d

α is the angle between the compression diagonals: $1 < cot\alpha < 2.5$

V_{Ed} is the present shear force

$$A_{sw} = 2 * \pi * \frac{12^2}{4} = 226.19\ mm^2$$

$$z = 0.9 * 340.5 = 306.45\ mm$$

$cot\alpha$ is assumed to be equal to 2.5

As a result the centre to centre distance beween the stirrups will be equal to :

$$s = \frac{z.cot\alpha.f_{yd}.A_{sw}}{V_{Ed}} = \frac{306.45 * 2.5 * 435 * 226.19}{169.5 * 10^3} = 444.72\ mm$$

- The maximum spacing between the stirrups should not exceed $S_{l,max}$.

[3] $S_{l,max} = 300\ mm$

[4] $S_{l,max} = 0.75d(1 + cot\alpha) = 0.75 * 340.5(1 + 0) = 255.37 < 444.72\ mm$

[3] According to Dutch national annex

As can be seen the calculated centre to centre distance of the stirrups is larger than the maximum allowable spacing between stirrus. Therefore, the maximum spacing should be applied.

Note, that in the previous equations α is the inclination of the shear reinforcement to the longitudinal axis of the beam.

- The transverse spacing of the legs in a series of shear links should not exceed $S_{t,max}$ (section 9.2.2.(8))

$$S_{t,max} = 0.75d = 255.37 \leq 600\ mm$$

$$S_t = 300 - 2(35+6) = 218\ mm\ \leq S_{t,max} \rightarrow satisfies\ the\ requirements$$

Consequently , Ø12_255 wil be used as shear reinforcement.

Besides the centre to centre disctance of the stirrups, the value of the shear reinforcement should also be checcked. The value of the shear reinforcement should be larger than $\rho_{w,min}$. The recommended value for $\rho_{w,min}$ according to 9.2.2(5) EC2 is given below.

$$\rho_{w,min} = \frac{0.08\sqrt{f_{ck}}}{f_{yk}} = 0.08 * \frac{\sqrt{45}}{500} = 1.07 * 10^{-3}$$

$$\rho_{applied} = \frac{A_{sw}}{b.s} = \frac{226.19}{300 * 255} = 2.95 * 10^{-3} > \rho_{w,min} \rightarrow satisfies\ the\ requirement$$

Maximum shear resistance:

The maximum allowable shear resistance in a member is given by the following equations (NEn-EN 1992 section 6.2.3(3), equation6.9) which should be larger than the force in the compression diagonals.

$$V_{Rd,max} = \alpha_{cw} b_w z \frac{v_1 f_{cd}}{(cot\theta + tan\theta)}$$

Where:

α_{cw} is a coefficient taking account of the state of the stress in the compression chord

v_1 is a strenght reduction factor for concrete cracked in shear

[4] According to EC2 clause 9.2.2(6)

z is the inner lever arm, for a member with constant depth,corresponding to the bending moment in the element under consideration. In the shear anaysis of reinforced concrete without axial force, the approximation value z=0.9d may normally be used.

f_{cd} is the design value of the conrete compression force in the direction of the longitudinal memebr axis

θ is the angle between the concrete compression strut and the beam axis perpendicualr to the shear force

The recommended value for v_1 is equal to 0.6 for $f_{ck} < 60\ MPa$ and the α_{cw} is equal to 1 for non-prestressed structures. By using the above parameter the maximum shear resistance can be obtained.

$$V_{Rd,max} = 1 * 300 * 306.45 * \frac{0.6 * 30}{(2.5 + 0.4)} * 10^{-3} = 570.631\ kN$$

The force in the diagonal is:

$$V_{Ed,diagonal} = \frac{V_{Ed}}{sin\theta} = \frac{161.5}{sin21.8} = 434.87\ kN < V_{Rd,max} \rightarrow satidfies\ the\ requirements$$

According to the analysis the requirements for the longitudinal and the shear reinforcement are satisfied in the connecting beams. One might ask: will the requirements be still satisfied when a smaller Young's modulus will be assigned to the beams? The answer is yes because decreasing the Young's modulus leads into decreasing the stiffness and consequently reducing the internal forces. To prove this statement the Young's modulus of the connecting beams in this example will be reduced to $18*10^6$ and the system will be analysed again. The internal forces in the maximum loaded beam have been illustrated in Figure 6.3 (obtained by applying the discrete method, using the Matrix Frame programme).

Comparing the results, it can be seen that, the internal forces for the beams with lower stiffness are smaller than the internal forces for beams with higher stiffness. Therefore, it can be concluded that the high stiffness of the connecting beams does not cause any problem in the design. Because designing connecting beams regarding to higher stiffness, will be conservative regarding to reinforcement.

It should be mention that all the dimensions and magnitude of the loads in this example are based on some assumptions. However the values are very similar to the values which are used in reality but, we cannot definitively conclude that high stiffness of the beams does not bring up any problems for reinforcement design. In some cases, assigning a very high stiffness to the connecting beams will leads into a very large reinforcement ratio which will not satisfy the requirements according to Euro code.

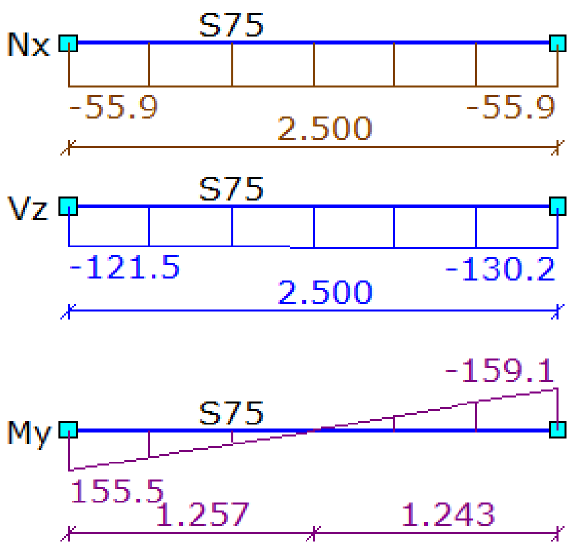


Figure 6.3 : Internal force in the connecting beam with low stiffness

It is worth to mention that if the walls are supported on individual elastic foundations, the internal forces of the connecting beams will be increased. If the bending moment and shear force in the beams are too large, not only the Young's modulus of the beams has to be decreased but the height of the beams should also be increased. Therefore, in the case of walls on elastic foundation preferably lower stiffness have to be assigned to the connecting beams to reduce the internal forces carrying by beams.

6.2 Connecting beams with lower Young's modulus

To find out the influence of the beams' stiffness on the behaviour of the coupled shear walls, the design example 1 of the walls on rigid foundation will be recalculated by assigning a lower modulus of elasticity to connecting beams.

Desing example 8

Consider the coupled shear walls system given in Figure 2.12 .The dimensions and the properties are given in Table 1 and Table 2. The only parameter which has been changed, is the stiffness of the connecting beams. In this example it is assumed that the young's modulus of the beams is equal to $14.25 * 10^6$. The system will be analysed by using the continuous method, considering only uniform distributed load on the system. The calculations have been done in Maple programme. Note that the equations of internal forces have been derived in section 6.1. The results are given in Table 23.

$$\zeta^2 = \frac{14.25 * 10^6}{36 * 10^6} = 0.3958$$

$\zeta = 0.6291$

Table 23: Results of internal forces for walls on rigid foundation for different beams' stiffness

Walls on rigid foundation		E_{cb}=36*10^6	E_{cb}=14.25*10^6
Maximum axial force in the walls	[kN]	1681.9	1221.56
Bending moment at base in the wall 1	[kN.m]	4354.67	5399.76
Bending moment at base in the wall 2	[kN.m]	11949.21	14816.94
Maximum deflection at the top	[m]	0.022	0.032
Composite cantilever action K_2	[%]	55	40.21
Maximum shear force in the connecting beams	[kN]	109.36	74.74
Bending moment in the connecting beams	[kN.m]	136.7	93.43
level of the maximum loaded beam	[m]	24.72	32.28

According to the results, it can be seen that the axial force in the walls and composite cantilever action percentage have been decreased by decreasing the stiffness of the connecting beams. Furthermore, lateral deflection and the bending moments in the walls have been increased for the beams with lower stiffness. The other aspect which has been changed due to the stiffness change, is the level of the maximum loaded beam. By reducing the stiffness of the connecting beams, the level of the maximum shear flow has been grown.

Moreover, the shear force and the bending moment in the connecting beams have been decreased by decreasing the Young's modulus of the beams. As a result, designing the reinforcement of the beams based on the high stiffness will be more conservative. However, it is known that assigning a very high Young's modulus to the connecting beams is not realistic since, the stiffness of the beams will be decreased due to the crack formation.

Considering all the results, it can be concluded that, the higher the stiffness of the connecting beams, the higher the efficiency of the coupling of walls will be. Connecting the shear walls with less stiff beams will cause small internal forces in the beams and large bending moments in the walls.

7 Effect of dimensions on behaviour of the coupled walls

Up to now a system of coupled shear walls has been analyzed for different foundation types and properties. In this section the effect of the dimension of the walls and the connecting beams on the behaviour of the system and coupling efficiency will be considered. The results from the continuous and the discrete methods will be compared to find out the effect of the dimensions on discrepancy between both methods.

7.1 Effect of the beams' length:

The first aspect which can affect the behaviour of coupled walls is the length of the connecting beams. To investigate this effect, a system of the coupled walls will be analysed for five different lengths of the connecting: 1, 2.5,4,5 and 7 meter. It is expected that by enlarging the length of the beams, the efficiency of the coupling will decrease and the individual action of the walls will be increased.

Consider a coupled walls system supported on the rigid foundation shown in Figure 7.1. The dimensions are given in Table 24. The walls will be analysed according to the continuous method by using the Maple programme and according to the discrete method by using the MatrixFrame programme.

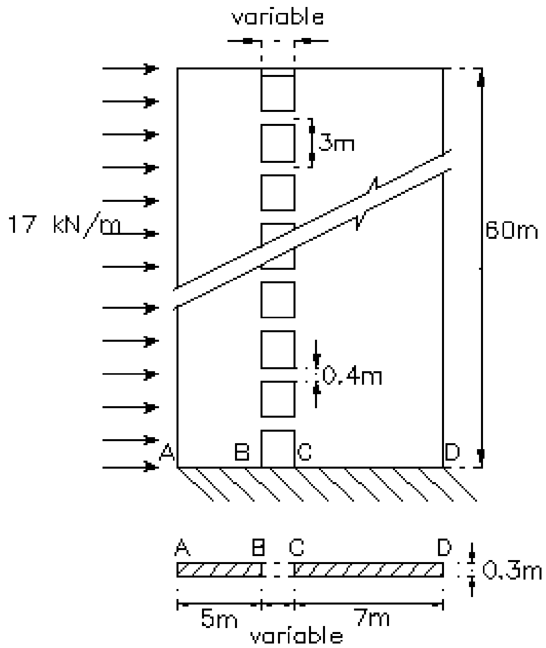


Figure 7.1 : Design example

As can be seen in Table 25, the stiffness factor ($k\alpha H$) will be decreased as the length of the beam increased. Further it can be seen that the effect of the beams length on α is much larger than its effect on k since, α is proportional to the cube of length and k is proportional to the square of length.

Table 24: Properties of the coupled walls system

$2d_1$	[m]	5
d_1	[m]	2.5
$2d_2$	[m]	7
d_2	[m]	3.5
b	[m]	variable
l	[m]	variable
h	[m]	3
H	[m]	60
t_{wall}	[m]	0.3
t_{beam}	[m]	0.4
G	[kN/m^2]	15000000
E	[kN/m^2]	36000000
ν		0.2
w	[kN/m]	17

Table 25 : Stiffness properties of the coupled walls systems

$b\ [m]$	1	2.5	4	5	6
$k^2 = \left(1 + \frac{Ai_t}{A_1 A_2 l^2}\right)$	1.2729	1.1851	1.1337	1.1105	1.0929
$\alpha^2 = \left(\frac{12I_e l^2}{b^2 h i_t}\right)$	0.0183	0.0024	0.0008	0.0004	0.0004
$k\alpha H$	9.1695	3.1702	1.8414	1.1902	1.1902

To compare the results of the continuous and the discrete method, the axial force in the walls, bending moment at base, maximum shear force in the connecting beam and the deflection at the top have been drown versus the length of the beams, shown in Figure 7.2 to Figure 7.5.

As was expected by increasing the length of the connecting beams, axial force in the walls as well as the shear force in the beams will decrease because, the stiffness factor ($k\alpha H$) has been decreased. Furthermore, it can be seen from the curves that the difference of axial force between the discrete and the continuous method have been diminished by enlarging the length of connecting beams. The reason is that increasing the length of the connecting beams will lead into increasing the size of the opening regarding to the walls system. As a result the coupling effect of the system will decrease and the individual action will increase.

Furthermore, it should be mention that the bending moment in both walls will increase when the walls are connected by longer beams. Additionally, as can be seen the similarity of the bending moment between both methods will decrease as the length of the beams increase. The larger the connecting beams the more similar the bending moment in wall 1 will be, though the differences between both methods in wall 2 is almost constant by increasing the length.

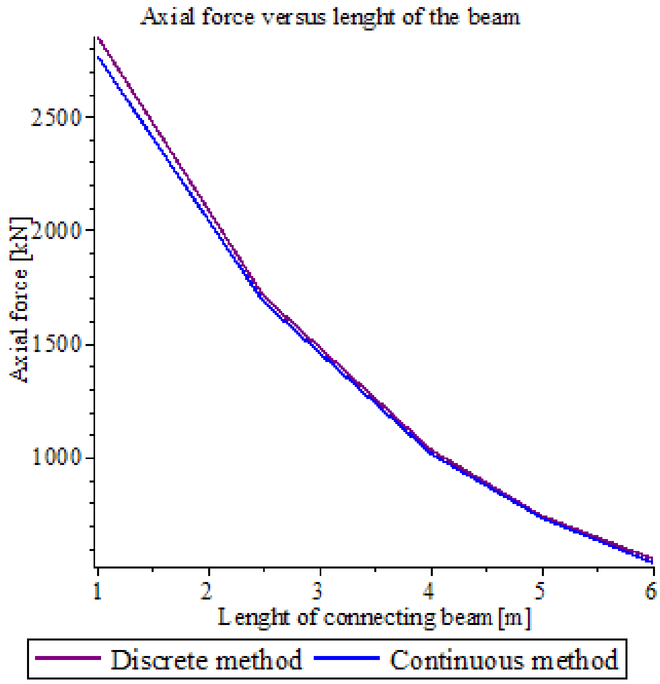


Figure 7.2 : Variation of axial force in the walls versus the length of the beam

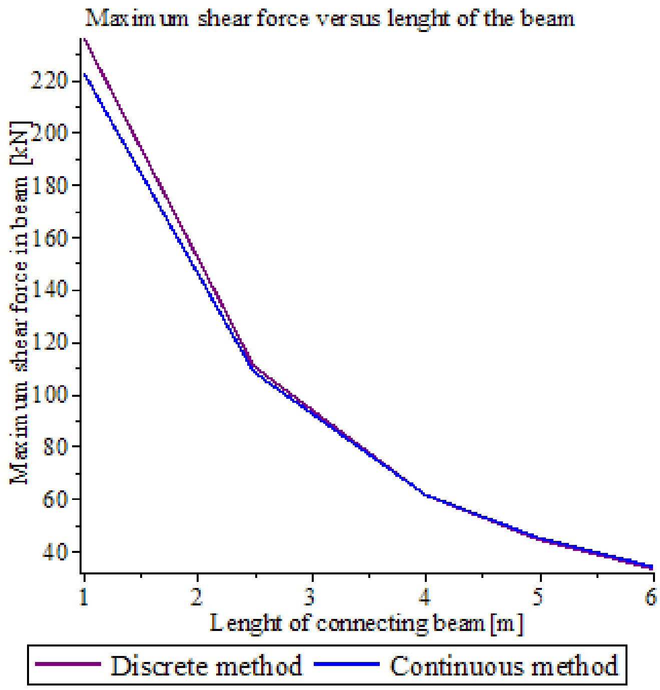


Figure 7.3: Variation of maximum shear force in the beams versus length of the beams

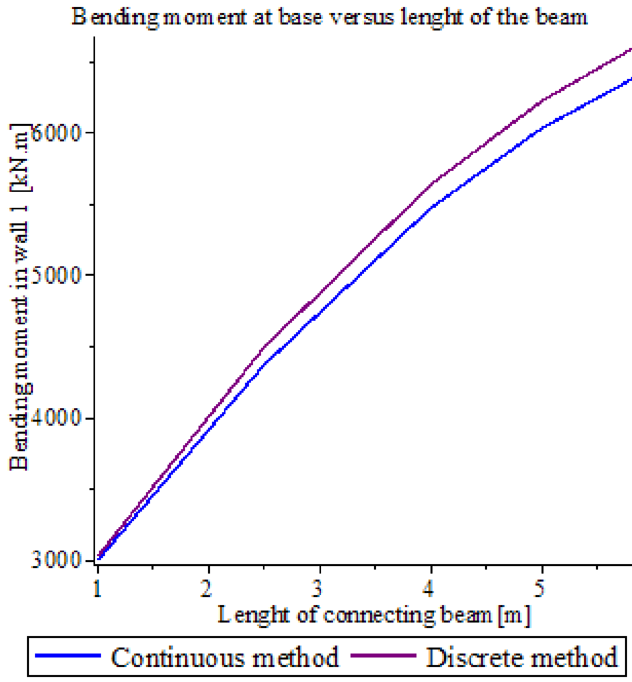


Figure 7.4: Variation of bending moment in walls 1 at the base versus length of the beams

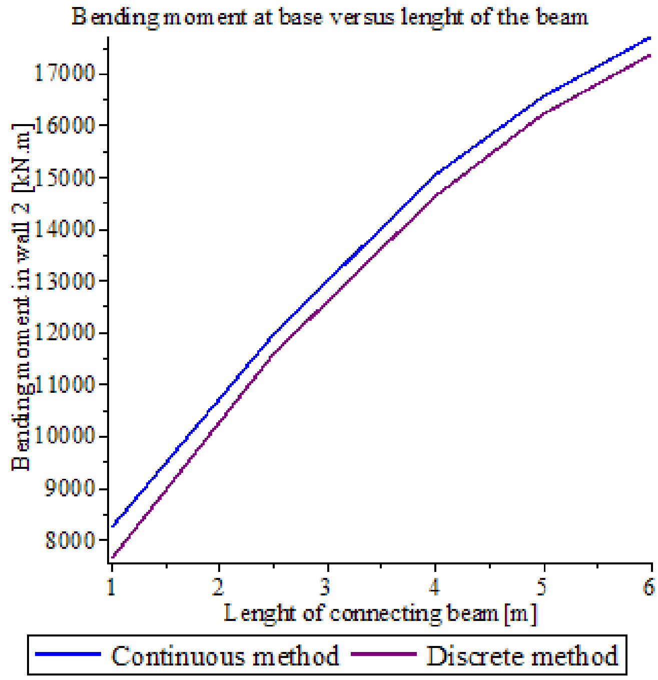


Figure 7.5: Variation of bending moment in wall 2 at base versus length of the beam

7.2 Effect of the height of the connecting beams

For the second consideration the height if the connecting beams will be changed and the behaviour of the coupled walls will be studied. For this reason, the coupled walls system shown in Figure 7.6 will be analysed again by assuming four different heights of the connecting beams, 0.3, 0.4, 0.5 and 0.6 meter. The stiffness factor of each case has been given in Table 26. The results are illustrated in Figure 7.7 to Figure 7.10 as axial force, shear force and the bending moment versus height of the connecting beams.

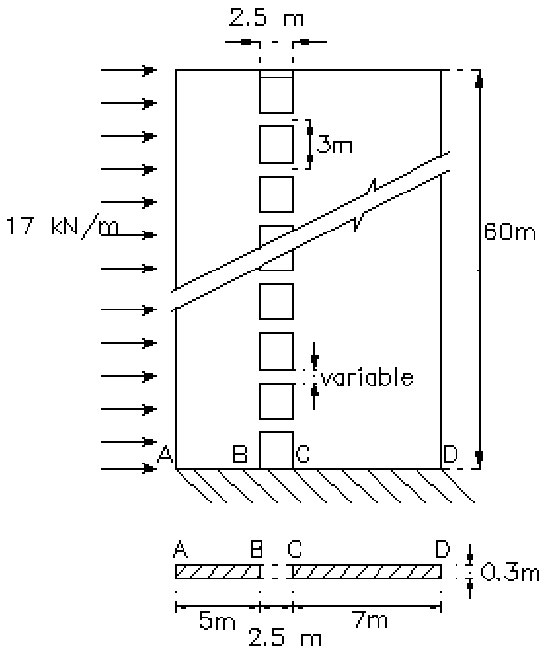


Figure 7.6 :Design example

Table 26 : Stiffness properties of the coupled walls systems

h_{beam}	0.3	0.4	0.5	0.6
$k^2 = \left(1 + \frac{Ai_t}{A_1A_2l^2}\right)$	1.1851	1.1851	1.1851	1.1851
$\alpha^2 = \left(\frac{12I_el^2}{b^2hi_t}\right)$	0.0010	0.0024	1.1851	0.0073
$k\alpha H$	2.0907	3.1702	4.3473	5.5891

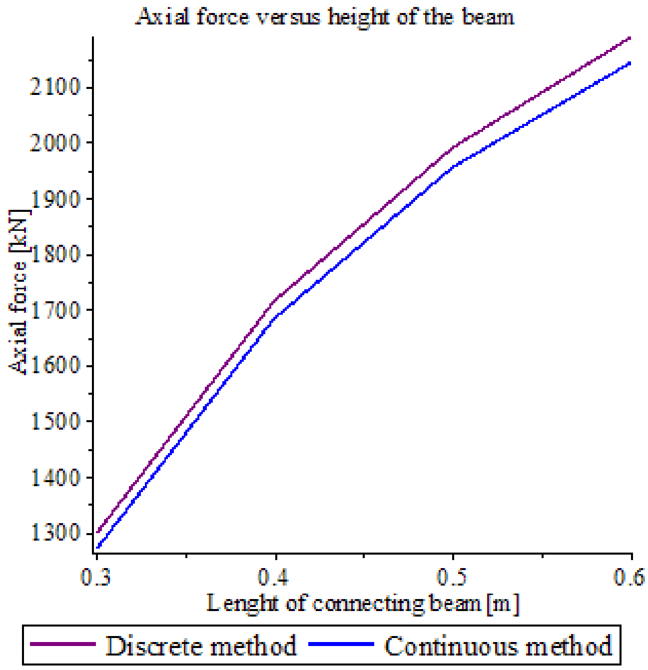


Figure 7.7: Variation of axial force in the walls versus height of the beam

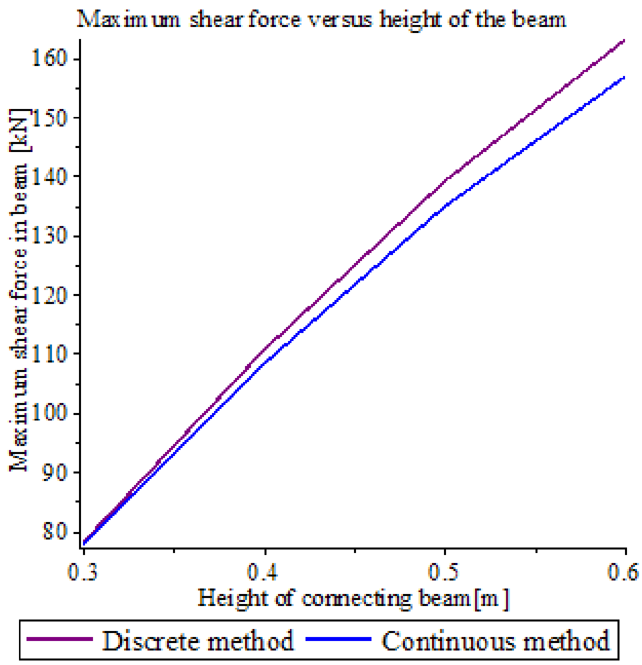


Figure 7.8 : Variation of maximum shear force in the beams versus height of the beam

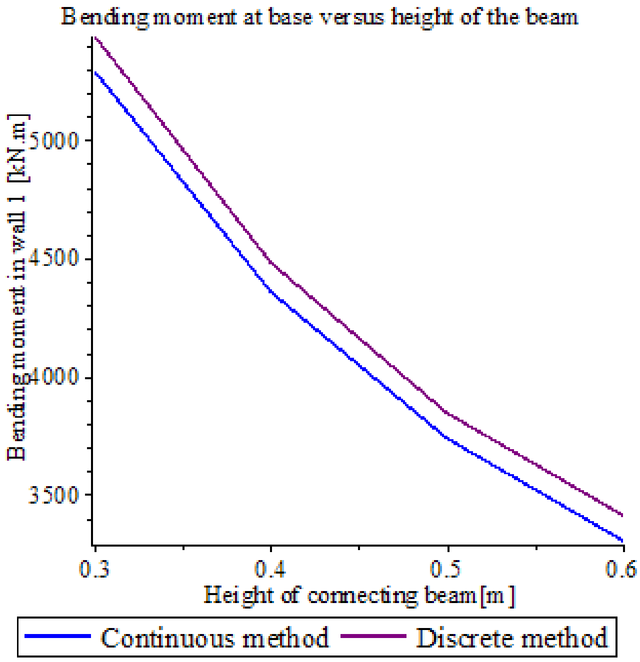


Figure 7.9 : Variation of bending moment at the base of wall 1 versus height of the beams

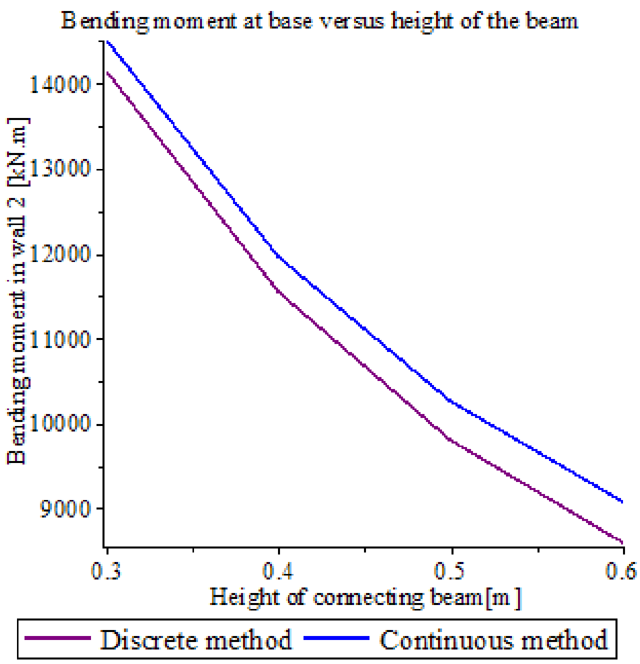


Figure 7.10: Variation of bending moment at the base of wall 2 versus height of the beams

Considering the previous curves it can be seen that, changing the height of the lintels only affects the accuracy of the shear force in connecting beams. By increasing the height of the beams the differences between the continuous method and the discrete method will also increase. Further, it can be seen that changing the height of the lintels does not have any effect on accuracy of axial force and bending moment in the walls.

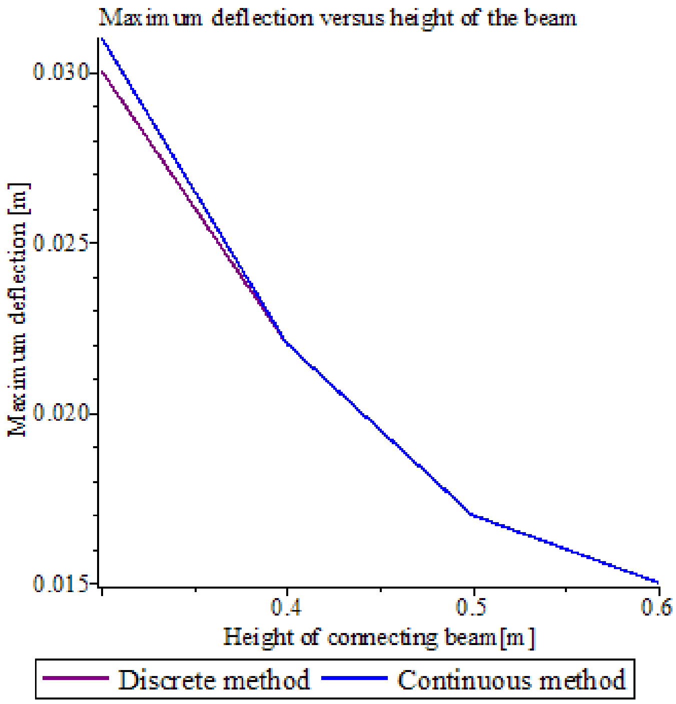


Figure 7.11: Variation of lateral deflection at top versus height of the beams

As can be seen in Figure 7.11 lateral deflections according to both methods are very similar. The differences are very small and can be neglected. Therefore it can be concluded that changing the height of the connecting beams does not affect the accuracy of the lateral deflection.

Altogether it can be concluded that changing the height of the lintels does not influence the accuracy of the results considering the continuous method. The precision of the shear force in the lintels was the only aspect which was affected by increasing the height of the beams. However the differences are negligible.

7.3 Effect of the height of story

The next aspect which can affect the behaviour of the coupled shear walls is the story height. Therefore in this consideration the story height will be changed. The coupled walls system shown in Figure 7.12 will be analysed for four different story height, 2.5, 3, 3.5 and 5 meter. The results are given in Figure 7.13 to Figure 7.16.

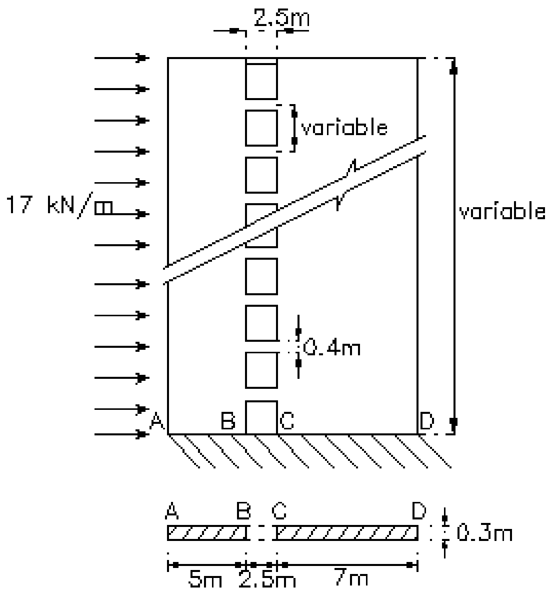


Figure 7.12 : Design example

Table 27 : Stiffness properties of the coupled walls systems

h	2.5	3	3.5	5
$k^2 = \left(1 + \frac{Ai_t}{A_1 A_2 l^2}\right)$	1.1851	1.1851	1.1851	1.1851
$\alpha^2 = \left(\frac{12 I_e l^2}{b^2 h i_t}\right)$	0.0028	0.0024	0.0020	0.0014
$k\alpha H$	2.8940	3.1702	3.4242	4.0927

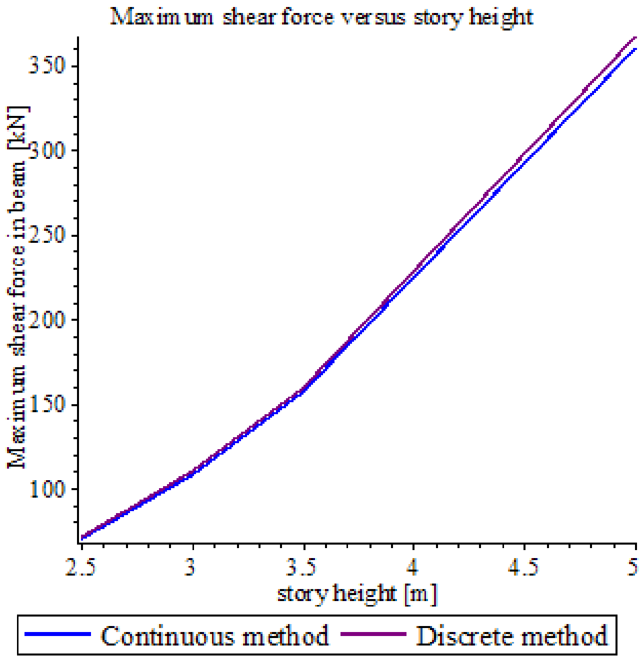


Figure 7.13: Variation of maximum shear force in the beams versus story height

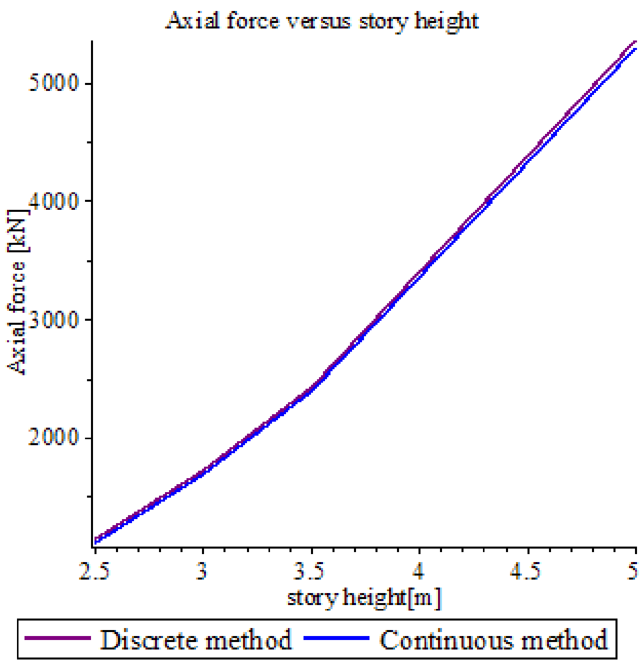


Figure 7.14 : Variation of axial force in the walls versus story height

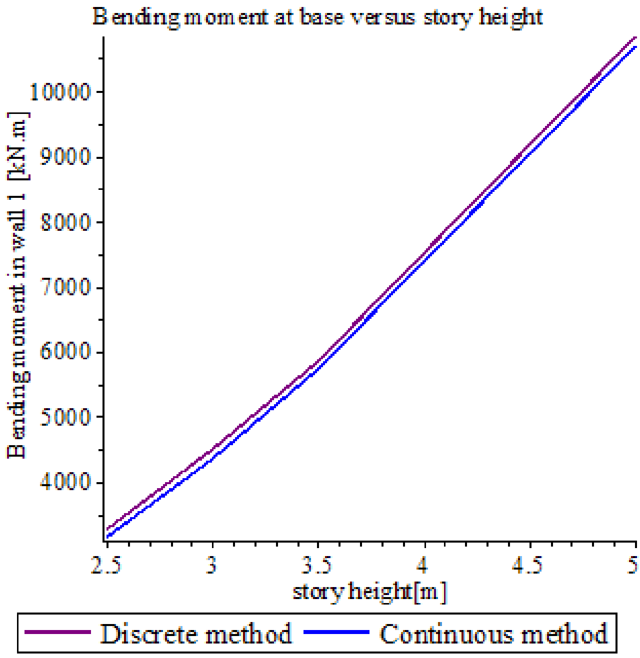


Figure 7.15 : Variation of bending moment at the base under wall 1 versus story height

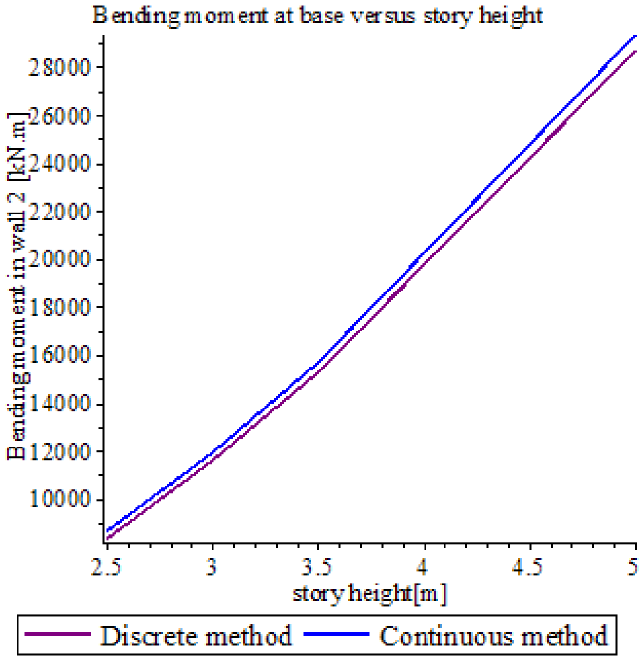


Figure 7.16 : Variation of bending moment at the base under wall 2 versus story height

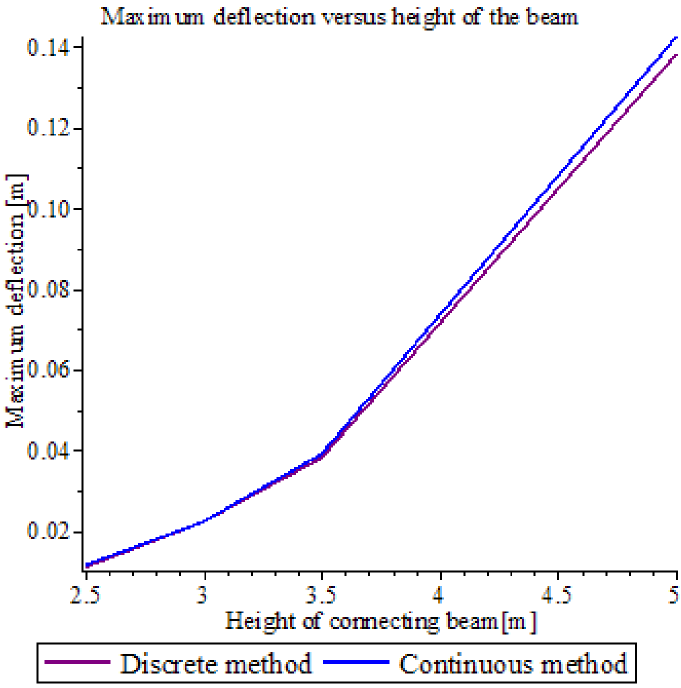


Figure 7.17 : Variation of lateral deflection at top versus story height

As can be seen from the previous curves the larger the story height of a system of the coupled shear walls, the larger the differences between the continuous method and the discrete method will be. Further, it can be seen that the differences in bending moment is smaller than the differences of the axial force in the walls. It is known that, the summation of the bending moment in the walls and the axial force times the distance between the centroidal axes of the walls are always equal to external bending moment applied on the structure. The reason that the differences concerning the bending moment are smaller is that, bending moment here has been considered per wall therefore the differences will be shared between both walls. Consequently the differences will be not as considerable as the differences of the axial force in the walls.

7.4 Effect of the width ratio of the walls

The last aspect which can be considered is the effect of the width ratio of wall 1 to wall 2 on accuracy of the continuous method. Assume the structure showed in Figure 7.18. In this example the width of the wall 1 and the span of opening remain constant. The width of wall two will be varied from 1 times width wall 1 to 10 times width wall 1.

The systems have been analyzed according to the continuous and the discrete method. A comparison has been made between both results which are shown in Table 29 and Table 30.

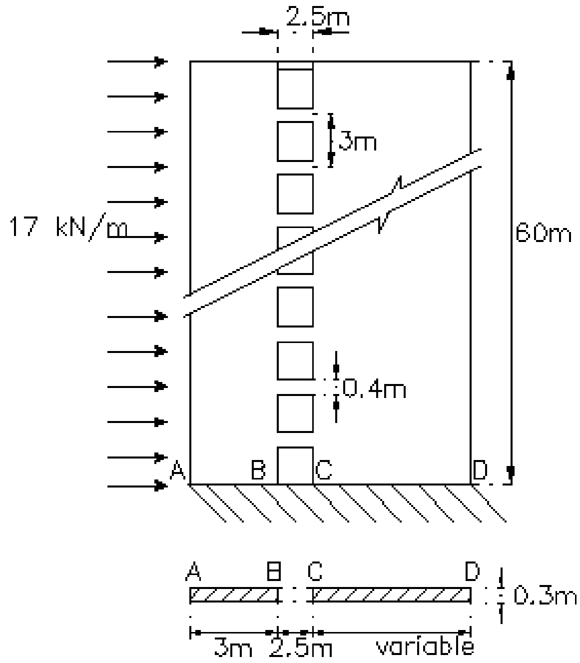


Figure 7.18 : Design example

Table 28 : various width ration and the stiffness properties

width ratio	1	2	3	4	5	6	7	8	9	10
$k^2 = \left(1 + \frac{Ai_t}{A_1A_2l^2}\right)$	1.09917	1.20663	1.38754	1.60937	1.85746	2.1235	2.4024	2.6907	2.9863	3.2876
$\alpha^2 = \left(\frac{12I_e l^2}{b^2 h i_t}\right)$	0.00854	0.00307	0.00145	0.00086	0.00059	0.0004	0.0003	0.0002	0.0002	0.0002
$k\alpha H$	5.81584	3.65591	2.69895	2.24442	1.99161	1.83431	1.72840	1.65285	1.5965	1.55313

Table 29 : Axial force according to continuous and discrete method for various width ratios

Width ratio	1	2	3	4	5	6	7	8	9	10
Continuous method [kN]	3618.5	2157.6	1306.1	841.3	575.3	413.6	310.1	239.6	190.48	154.7
Discrete method [kN]	3643.3	2190.5	1334.6	866	595.3	429.7	322.8	250.5	199.5	162.5
Difference in %	0.679	1.497	2.131	2.844	3.354	3.738	3.924	4.323	4.520	4.761

Comparing the axial force in the walls it can be seen that, by increasing the width ratio of the walls the accuracy of the results will be decreased. Analysing the coupled walls systems with a width ratio equal or smaller than 6 according to the continuous method gives difference of about 3% which is acceptable. For the walls with a width ratio of 10 the difference of axial force will become about 5%. Further it can be seen that a system of symmetric coupled walls gives the most accurate results.

Table 30 : Bending moment in wall 1 according to continuous and discrete method for various width ratio

Width ratio	1	2	3	4	5	6	7	8	9	10
Continuous method [kN]	5314.5	1848.2	828.6	468.7	313.1	235.8	193.3	168	152.1	141.6
Discrete method [kN]	5348.9	1721.7	696.3	341.32	190.34	116.2	75.8	52.17	37.4	27.63
Difference in %	0.648	6.839	15.96	27.17	39.20	50.70	60.74	68.94	75.40	80.48

As can be seen, for the walls with a width ratio larger than 6 the difference of bending moment in wall 1 between the continuous and the discrete method are enormous. Therefore it is recommended to not using the continuous method for the walls with a width ratio larger than 6.

Finlay it can be concluded that changing the dimensions of the coupled walls system does not influence the accuracy of the results except the higher width ratio which considerably decrease the accuracy. By changing other aspects, some distinction will occur between the results from the continuous method and the discrete method, but the variations are within the range of 1 to 3 percent which simply can be neglected.

It is worth to mention that the curves in this chapter have been drawn based on a few examples and just by connecting the resulting points by straight line to each other. Doing this gives a few lines with different slopes for each comparison however , the resulting curves should be smooth.

8 Practical design

In the previous chapters the behavior of the coupled shear walls has been studied according to the continuous method. As can be seen for the walls on the elastic foundation the related equations of internal forces and deflections are very lengthy which makes them difficult to be used in practice. Further, these equations are dependent on many stiffness parameters and dimensions which increases the probability of errors in the calculations. Therefore, to make these equations more applicable three excel sheets have been made by applying the related equations. These equations are written in Visual Basic for Applications by using the excel programs. There are three different sheets for three different types of foundations which can be used, for walls on a rigid foundation, walls on two individual elastic foundation and walls supported by a grade beam on elastic foundation. By inserting the dimensions and properties of coupled shear walls and connecting beams in these excel sheets the following results will be given

- Axial force in the shear walls
- Maximum positive and negative bending moment in the shear walls
- The height at which maximum bending moment in the walls occur
- Maximum shear force in the connecting beams
- The height at which maximum shear force in beams occur
- Maximum deflection at top

The most important advantage of these excel sheets is that there is no requirements for time consuming modeling of the coupled walls structure. It is worth to mention that obtained results have good accuracy.

8.1 Practical example

To show the application of these excel sheets, two different examples will be shown. The first example is a system of coupled shear walls on rigid foundation. The results from this example will be compared with a hand calculation which is commonly used in practice. The second example is a coupled shear walls supported by a foundation beam on piles. This example will be analysed by using the excel sheet and the results will be used for reinforcement design.

Design example 1

Consider a coupled shear walls on a rigid foundation with one row of opening illustrated in Figure 8.1. The building contains 30 floors with a total height of 90 meters. The coupled walls are symmetrical with a width of 6.3 meter and the opening span is 2.4 meter. The walls are subjected to lateral wind load with a magnitude of 3 kN/m^2. Calculate the stresses under the walls and deflection at the top.

Table 31: dimensions and properties of coupled walls system

Height of story	[m]	3
Total height of building	[m]	90
Width of shear wall	[m]	6.3
Thickness of wall	[m]	0.250
Height of connecting beams	[m]	0.6
Width of connecting beams	[m]	0.250
Modulus of elasticity of walls	[kN/m^2]	25*10^6
Modulus of elasticity of lintels	[kN/m^2]	15*10^6

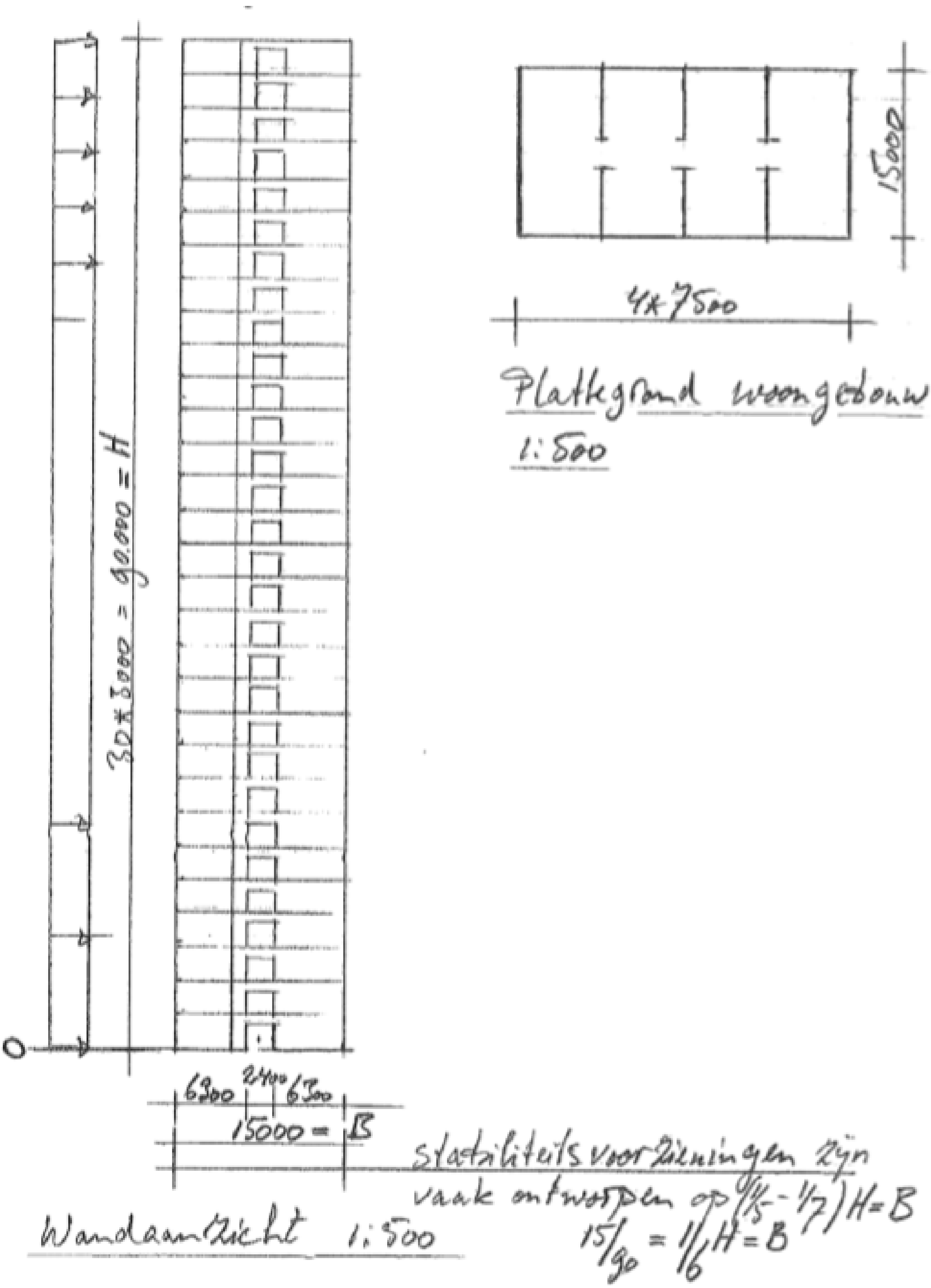


Figure 8.1 :Side view and plan of the coupled walls system, example 1

Hand calculation

Stress under walls

Horizontal load, wind = 3 $\frac{kN}{m^2}$

$$q_{wind,d} = 3 * 7.5 = 22.5\ \frac{kN}{m}$$

$$w = \frac{1}{6}250 * 15000^2 = 9 * 10^9\ mm^3$$

$$M_{wind}(0) = \frac{1}{2}22.5 * 90^2 = 91 * 10^3\ kN.m$$

$$\sigma_{wind} = \frac{M_{wind}}{w} = \frac{91 * 10^3}{9 * 10^9} = \pm 10\ N/mm^2$$

Shear force in lintels

$$z = 0.67 * 15 = 10.05\ m$$

$$V_{lintels} = \frac{M_{wind}(0) - M_{wind}(3)}{z} = \frac{91.1 * 10^3 - 85.2 * 10^3}{10} = 590\ kN$$

$$M_{wind}(0) = \frac{1}{2} * 22.5 * 90^2 = 91.1 * 10^3\ kN.m$$

$$M_{wind}(3) = \frac{1}{2} * 22.5 * 87^2 = 85.2 * 10^3\ kN.m$$

If the load is uniform distributed $V_{lintels} = \frac{590}{2} = 295\ kN$

To check the strength

$$V_{lintel,d} = \frac{1}{2}\left(590 + \frac{590}{2}\right) = 442\ kN$$

$$M_{lintel,d} = V_{lintel,d} * \frac{l}{2} = 442 * \frac{2.4}{2} = 531\ kN.m$$

Lateral deformation

$$u_{bending} = \frac{ql^4}{8EI} = \frac{15 * 90^2}{8 * 25000 * 10^3 * 70.31} = 0.07\ m = 70\ mm$$

$$I = \frac{1}{12} * 0.25 * 15^3 = 70.31\ m^4$$

$$q_{wind,rep} = 2 * 7.5 = 15\frac{kN}{m}$$

Deformation of lintels

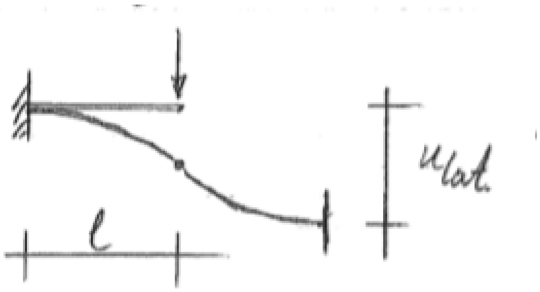


Figure 8.2 : Deformation of lintels

$$u_{lintel} = \frac{Vl^3}{3EI} * 2 = \frac{290 * 1.2^3}{3 * 15000 * 10^3 * 4.5 * 10^{-3}} * 2 = 0.005\ m = 5mm$$

$$V_{rep} = \frac{442}{1.5} = 290\ kN$$

$$I = \frac{1}{12} * 0.25 * 0.6^3 = 4.5 * 10^{-3}\ m^4$$

Shear deformation

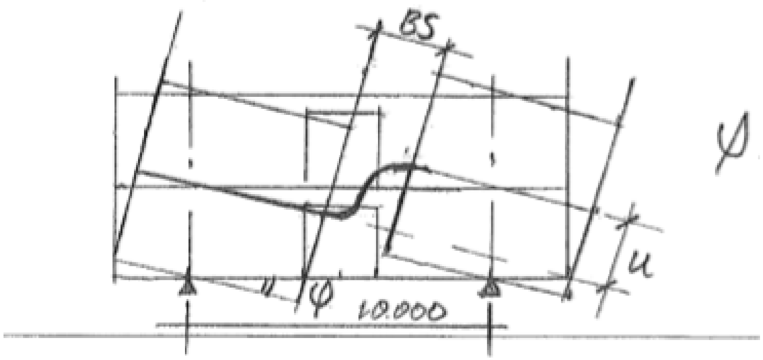


Figure 8.3: Deformation of walls due to shear

$$\varphi = \frac{0.5u_{lintel}}{5000 - 0.5BS} = \frac{0.5 * 5}{5000 - 0.5 * 2400} = 6.57 * 10^{-4}$$

$$u_{top} = \varphi * H = 6.75 * 20^{-4} * 90 * 10^3 = 59\ mm$$

If the load is uniform distributed $V_{lintel,rep} = \frac{295}{1.5} = 196.7\ \ kN$

$$u_{lintel} = \frac{Vl^3}{3EI} * 2 = \frac{196.7*1.2^3}{3*15000*10^3*4.5*10^{-3}} = 3.33\ mm$$

$$u_{top} = 39.4\ mm$$

Total deformation

$$u_{total} = u_{shear} + u_{bending} + u_{rotational} = 70 + 40 + \frac{(70 + 40)}{3} = 147.7\ mm$$

Using excel sheets

The result from analysing according to the continuous method is given in Table 32. More details can be found in appendix 9.

Stress under wall

$$\sigma = \frac{(m(z) - (N).l).I_1 c_1}{i_t} + \frac{N}{A_1} = \frac{(18785.26) * 3.15}{10.41} + \frac{6155.69}{1.575} = \pm 9.6\ N/mm^2$$

Note that, here A_1 is the cross section of wall 1 which is equal to cross section of wall 2 due to symmetry. c_1 is the distance of the centre of the gravity of wall 1 or 2 to the most extreme fibres of wall 1 or 2.

Axial force in the walls:

It is assumed that the axial force in the walls are equal to summation of shear walls in the lintels.
$N = 29 * 295 = 8555\ kN$

Table 32: internal forces and deflection according to hand calculation and excel sheet

Internal forces and deflection	Excel sheet	Hand calculation	
The height at which maximum shear occurs	28.95		[m]
Maximum shear force in lintels is equal to	296.08	295	[kN]
Maximum axial force in walls is equal to	6155.69	8555	[kN]
Maximum deflection (rotation at base not concluded)	0.111	0.11	[m]
Maximum deflection (rotation at base concluded)	0.148	0.147	[m]
The height of maximum positive moment	0.00		[m]
Maximum bending moment in wall 1	18785.26	45500	[kN.m]
Maximum bending moment in wall 2	18785.26	45500	[kN.m]
The height of maximum negative moment	69.67		[m]
Maximum negative bending moment in wall 1	-1381.56		[kN.m]
Maximum negative bending moment in wall 2	-1381.56		[kN.m]

As can be seen the maximum bending moment in the walls according to excel sheet is considerably smaller than the bending moment according to hand calculation. The reason is that when the walls are coupled a part of bending moment applied on the walls will be resisted by axial force. But in hand calculation the effect of the axial force has been ignored. Therefore, the obtained bending moment is significantly larger. Further as can be seen the stress under the walls are smaller according to excel sheet compared to hand calculation. Here also the effect of the coupling in the hand calculations has been neglected.

One of the important aspects in analysing the coupled shear walls, is the lateral deflection at top. As can be seen, the lateral deflection of the coupled walls system according to excel sheet and hand calculation are almost the same. It is worth to mention that, in hand calculations it was assumed that rotation at base will cause a lateral deflection of about one third of the total defection due to bending and shear deformation. Therefore, in the above table two different lateral deflections have been presented which gives 1) total deformation of the walls by considering the rotation at base and 2) total deformation of the walls when rotation at base is not included.

Further, it can be noticed that both calculations give the same results for the maximum shear force in lintels. Note that it is assumed that the forces will be uniformly distributed over the height of lintel.

Considering the results, it can be concluded that hand calculation is more conservative, though the excel sheet based on continuous method gives more accurate results.

Design example 2

Consider the coupled shear walls shown in Figure 8.4. The walls have a T shape cross section. The walls are stiffened with a foundation beam at base which is supported by 4 piles. The dimensions and stiffness parameter can be taken form Table 33. The structure consists of 13 floors with a total height of 38.48 meter. The magnitude of wind load is assumed to be 25 kN/m. The span of lintels will be changed throughout the height from 1.1 in the first five floors to 2.15 in the upper levels. Therefore, two different variations will be considered with two different opening spans.

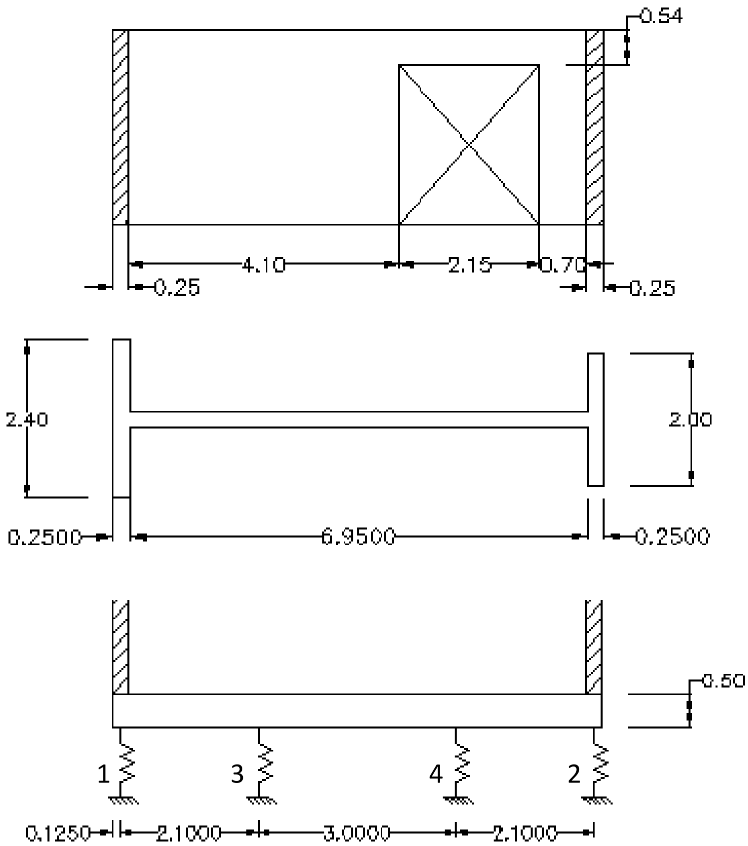


Figure 8.4 : Side view and plan of the shear walls, example 2

Table 33: Dimensions and properties of coupled walls system

Height of story	[m]	2.96
Total height of building	[m]	38.48
Thickness of wall	[m]	0.25
Height of connecting beams	[m]	0.535
Width of connecting beams	[m]	0.25
Modulus of elasticity of walls	[kN/m^2]	11*10^6
Modulus of elasticity of lintels	[kN/m^2]	11*10^6
Height of stiffened beam	[m]	1.2
Width of stiffened beam	[m]	0.5

The stiffness of the piles are as follows:

Outer piles $k_1 = k_2 = 9.8 * 10^5 \frac{kN}{m}$

Inner piles $k_3 = k_4 = 7 * 10^4 \frac{kN}{m}$

Equivalent rotational and vertical stiffness of piles

Note that, in this example, it is assumed that the piles 1 and 3 work together under wall 1 and pile 2 works under walls 2. The equivalent rotational and vertical stiffness of the piles are as following.

$$\frac{1}{k_{v,total}} = \frac{1}{k_{v,1}} + \frac{1}{k_{v,2}} = \frac{1}{9.8 * 10^5 + 7 * 10^4} + \frac{1}{9.8 * 10^5} \rightarrow k_{v,total} = 506896.6 \frac{kN}{m}$$

$$k_{\theta,total} = \sum k_i.(x_i - x_m)^2 = 9.8 * 10^5 * (0.125 - 3.725)^2 + 7 * 10^4 * (2.225 - 3.725)^2 + 7 * 10^4 * (5.225 - 3.725)^2 + 9.8 * 10^5 * (7.325 - 3.725)^2 = 25716600\ kN.m/rad$$

$$x_m = \frac{\sum k_i x_i}{\sum k_i} = \frac{125 * 9.8 * 10^5 + 2225 * 7 * 10^4 + 5225 * 7 * 10^4 + 7325 * 9.8 * 10^5}{2 * 9.8 * 10^5 + 2 * 7 * 10^4} = 3725\ mm$$

By using the continuous method the internal forces in the walls and lintels can be obtained. The results for both variations are given in Table 34. Considering the results, the reinforcement design can be done for the maximum forces. More details about calculations can be found in appendix 10.

Table 34 : Internal forces and deflection of the shear walls for short and long span

Internal forces and deflection	Span=2.15	Span=1.1	
The height at which maximum shear occurs	13.67	9.83	[m]
Maximum shear force in lintels is equal to	183.03	282.25	[kN]
Maximum axial force in walls is equal to	1703	2263.9	[kN]
Maximum deflection	0.071	0.053	[m]
The height of maximum positive moment	0	0	[m]
Maximum bending moment in wall 1	8686.72	5761.9	[kN.m]
Maximum bending moment in wall 2	105	619.6	[kN.m]
Shear force in the foundation beam	330	468.9	[kN]

Stress in walls due to vertical forces

Self weight of concrete: $q_e = 24\frac{kN}{m3}$ $\rightarrow$ $q_{e,total} = 1.2 * 24 * 38.48 = 1108.22\ \ kN/m\text{^}2$

Permanent load per floor: $q_p = 36\frac{kN}{m}$ $\rightarrow$ $q_{p,total} = 36 * 13 = 468\ \ kN/m$

Variable load per floor : $q_v = \frac{1.4kN}{m}$ $\rightarrow$ $q_{v,total} = 1.4 * 13 = 18.2\ kN/m$

Load combination:

$1.2q_p + 1.5q_v = 1.2 * 468 + 1.5 * 18.2 = 588.9\ \ \ kN/m$

Maximum stress in the walls due to vertical forces : $\sigma = \frac{588.9}{0.25} + 1108.22 = 3463.8\frac{kN}{m^2} = 3.46\ \ N/mm^2$

Minimum stress in the walls due to vertical forces : $\sigma = 0.9 * \frac{468}{0.25} + 24 * 38.48 = 2608.3\ \frac{kN}{m^2} = 2.6\ \ N/mm^2$

Stress in the walls due to wind

$$\sigma_{max} = \frac{(m(z) - (N).l).I_1.c_1}{i_t} + \frac{N}{A_1} = \frac{(8686.7) * 1.49}{3.266} + \frac{1703}{1.625} = 5.01\ N/mm^2$$

$$\sigma_{min} = -\frac{(m(z) - (N).l).I_1.c_1}{i_t} + \frac{N}{A_1} = -\frac{(8686.7) * 2.853}{3.266} + \frac{1703}{1.625} = -6.5\ N/mm^2$$

Total stress in the walls

Maximum stress: $\sigma_{compression} = 3.46 + 1.5 * 6.5 = 13.21\ \frac{N}{mm2}$
Using concrete C28/35, then design strength of concrete will be $f_{cd} = \frac{28}{1.5} = 18.67\ N/mm^2$

$f_{cd} > \sigma_{compression}$ satisfy the requirement

Minimum stress: $\sigma_{tension} = -2.6 + 1.5 * 5.01 = 4.9\ \frac{N}{mm2}$
The walls could be in tension which does not satisfy the requirements. The design of shear walls subjected to tension is beyond the scope of this thesis.

Reinforcement design for foundation beam

$M_{Ed,max} = 1.5 * 330 * 2.15/2 = 532.12\ \ kN.m$

Assumption:

- Environmental conditions: XC2
- Concrete strength class : C28/35
- Minimum cover according to EC2, section 4.4.1.1:[5] $c_{nom} = c_{min} + \Delta c_{dev} = 25 + 10 = 35\ mm$

The required reinforcement area can be determined by using standard table of GTB 2010 (table 11.2).

$$d = h - \left(\emptyset_{stirrup} + \frac{\emptyset}{2} + c\right) = 1200 - \left(12 + \frac{25}{2} + 35\right) = 1140.5\ mm$$

$$\frac{M_{Ed}}{bd^2 f_{cd}} = \frac{533}{0.5 * 1.1405^2 * 18.7} = 43.82 \xrightarrow{yields} minimum\ reinforcment$$

$$A_s = \rho.b.d = 1.456 * 10^{-3} * 1140.5 * 500 = 830.28\ \ mm^2$$

4 bars are required at the top and at the bottom of the cross section if steel bars of Ø25 will be used. As a result the applied reinforcement area will be $6 * \pi \frac{25^2}{4} = 981.74\ \ mm^2$

$$\rho = \frac{981.74}{1140.5 * 500} = 0.0017$$

Maximum and minimum reinforcement ratio

According to EC2, 9.2.1.1 the area of longitudinal reinforcement should not be taken less than $A_{s,min}$

$$A_{s,min} = 0.26 \frac{f_{ctm}}{f_{yk}} b_t.d = 0.26 * \frac{2.8}{500} * 500 * 1140.5 = 830.28\ mm^2$$

$$A_{smax} = 0.04\ A_c = 0.04 * 500 * 1140.5 = 22810\ mm^2$$

As can be seen the design reinforcement ratio is within the minimum and maximum allowed limits.

Shear reinforcement:

According to section 6.2.2 of NEN-EN 1992-1-1, the shear force capacity can be determined with the following equation:

$$V_{Rd,c} = \left[C_{Rd,c}.k(100.\rho_1.f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right].b_w.d \qquad (8.1)$$

In which:

[5] Based on the recommended values in Euro code 2 NEN-EN 1992-1-1

$C_{Rd,c}$ =0.12

k $= 1 + \sqrt{\frac{200}{d}} \leq 2.0 = 1 + \sqrt{\frac{200}{1140.5}} = 1.41$

ρ_1 $\frac{A_{sl}}{b_w d} \leq 0.02$

k_1 $= 0.15$

σ_{cp} $\frac{N_{Ed}}{A_c}$

N_{Ed} is the normal force in the cross section as a result of the load or pre-stress force

$$V_{Rd,c} = \left[0.12 * 1.41(100 * 0.0051 * 35)^{\frac{1}{3}}\right] * 500 * 1140.5 = 252.16 \text{ kN} < \mathrm{V_{Ed}} = 468.9 * 1.5 = 703.4 \text{ kN}$$

As can be seen the shear capacity is much lower than the applied shear force on the beam, therefore shear reinforcements are required. It is assummed that the diameter of stirups is equal to 12 mm, hence the distance between the stirrups can be detremined by using following equation

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z.cot\alpha.f_{yd}}$$

$$A_{sw} = 2 * \pi * \frac{10^2}{4} = 157.1\ mm^2$$

It has to mention that,

s is the centre to centre distance of the shear reinforcement

z is the internal lever arm. When thre is no normal force, z=0.9d

α is the angle between the compression diagonals: $1 < cot\alpha < 2.5$ which is assummed to be equal to 2.5

$$z = 0.9 * 1140.5 = 1026.45\ mm$$

Therefore, the centre to centre distance beween the stirrups (s) will be equal to :

$$s = \frac{z.cot\alpha.f_{yd}.A_{sw}}{V_{Ed}} = \frac{1026.45 * 2.5 * 435 * 157.1}{703.4 * 10^3} = 249.3\ mm$$

- Note that, the maximum spacing between the stirrups should not exceed $S_{l,max}$.

  [6] $S_{l,max} = 300\ mm$

  [7] $S_{l,max} = 0.75d(1 + cot\alpha) = 0.75 * 1140.5(1 + 0) = 855.37mm$

  Where α is the inclination of the shear reinforcement to the longitudinal axis of the beam.

As can be seen the derived value for centre to centre distance of stirrups is smaller than the maximum allowable distance.

- The transverse apacing of the legs in a series of shear links should not exceed $S_{t,max}$:

$$S_{t,max} = 0.75d = 855.37 < 600\ mm$$

$$S_t = h - 2\left(c + \frac{d_{stirrups}}{2}\right) = 1200 - 2\left(35 + \frac{10}{2}\right) = 1120\ mm > S_{t,max}$$

Since the transverse distance is larger than the maximum allwoable value, the stirrups should be applied in three rows. Therefore the transverse distance will be:

$$S_t = h - 2\left(c + \frac{d_{stirrups}}{2}\right) = 0.5\left(1200 - 2\left(35 + \frac{8}{2}\right)\right) = 561\ mm < S_{t,max}$$
$$\rightarrow staisfy\ the\ requirements$$

$$A_{sw} = 3 * \pi * \frac{8^2}{4} = 150.7\ mm^2$$

$$s = \frac{z.cot\alpha.f_{yd}.A_{sw}}{V_{Ed}} = \frac{1026.45 * 2.5 * 435 * 150.7}{703.4 * 10^3} = 239.3\ mm$$

Consequently ,2 Ø12_235 wil be used as shear reinforcement.

[6] According to Dutch national annex
[7] According to EC2 clause 9.2.2(6)

Besides the centre to centre disctance of the stirrups, the value of the shear reinforcement should also be controled. The value of the shear reinforcement ratio should be larger than $\rho_{w,min}$. The recommended value for $\rho_{w,min}$ according to 9.2.2(5) EC2 is given below.

$$\rho_{w,min} = \frac{0.08\sqrt{f_{ck}}}{f_{yk}} = 0.08 * \frac{\sqrt{35}}{500} = 9.46 * 10^{-4}$$

$$\rho_{applied} = \frac{A_{sw}}{b.s} = \frac{150.7}{500 * 235} = 1.28 * 10^{-3} > \rho_{w,min} \rightarrow satisfies\ the\ requirement$$

Maximum shear resistance:

The maximum allowable shear resistance in a member is given by the following equations (NEn-EN 1992 section 6.2.3(3), equation 6.9) which should be larger than the force in the compression diagonals.

$$V_{Rd,max} = \alpha_{cw} b_w z \frac{v_1 f_{cd}}{(cot\theta + tan\theta)}$$

Where:

α_{cw} is a coefficient taking account of the state of the stress in the compression chord

v_1 is a strenght reduction factor for concrete cracked in shear

θ is the angle between the concrete compression strut and the beam axis perpendicualr to the shear force

The recommended value for v_1 is equal to 0.6 for $f_{ck} < 60\ MPa$ and the α_{cw} is equal to 1 for non-prestressed structures. By using the above parameter the maximum shear resistance can be obtained.

$$V_{Rd,max} = 1 * 500 * 1026.45 * \frac{0.6 * 18.7}{(2.5 + 0.4)} * 10^{-3} = 1982.1\ kN$$

The force in the diagonal is:

$$V_{Ed,diagonal} = \frac{V_{Ed}}{sin\theta} = \frac{703.4}{sin21.8} = 1894.1\ kN < V_{Rd,max} \rightarrow\ satidfies\ the\ requirements$$

9 Conclusions and discussions

This chapter presents the answers to the main research questions and thereby the conclusions to this research. The findings of this research will be discussed by flagging limitations and by making Implications for further research.

9.1 Conclusions

The main objective of this paper was to study the behaviour of coupled shear walls according to the continuous and the discrete method. The following conclusions can be drawn from this study.

- In this thesis the previous study about analysis of coupled shear walls according to continuous method has been revised. The obtained results are exactly identical to the results given in other researches.

- The behaviour of the coupled shear walls subjected to lateral loading has been considered according to the continuous method for walls on rigid foundation, two individual elastic foundations and walls on an elastic foundation stiffened by a foundation beam. The results have been given as some equations and curves from which the axial force in the walls, bending moment in the walls, shear force in the connecting beams and lateral deflection of the coupled walls system can be obtained. Note that, in this thesis the continuous method has been extended for analysis of coupled shear walls system with different stiffness for the connecting beams and shear walls which is more applicable in practice.

- Further, the coupled shear walls systems have been studied according to the discrete method. Comparisons have been made between the results obtained from the continuous and discrete method. According to the results it can be concluded that the continuous method gives very good results with high accuracy to analyse the coupled shear walls.

- Since the obtained equations for walls supported on elastic foundation are very large, it will be difficult to use them in practical examples. Therefore by applying these equations a programme has been written in VBA for analysing the coupled shear walls which is given as two excel sheets for Dutch and English users. These excel sheets can be simply used in practical examples to get a rapid evaluation of internal forces of the coupled walls and lintels and lateral deflection, without any requirements of modelling the coupled walls system. Note that comparing to the frame analysis programme this excels sheet gives a very good result with an accuracy of 1 to 3 percent.

9.2 Limitation and future research

This research aimed to extend the application of the continuous method for analysing coupled shear walls supported by elastic foundation and with different stiffness for walls and connecting beams. However there are still some limitations in the continuous method. One of the important limitations is that the cross section of the shear walls and connecting beams are asummed constant along the height. Therefore, an extensive study for walls with variable cross section along the height is recommended.

Further, as can be noticed from the results, the continuous method and discrete method have been compared in this paper. One other important method to analysing coupled shear walls is the finite element method. An interesting fact would be to compare the results of FEM with the continuous method.

10 Appendix 1:

10.1 Verification of the equivalent flexural rigidity of the connecting beams

Consider a beam with a length of L, cross sectional area of A_b and second moment of area of I_b. If a point load, F, will be applied at the end beam, the beam will deflect. The total deflection of a beam subjected to a point load consists of shear and bending deflection.

a. It is known that the bending deflection of a beam due to a point load is [8] ;

$$\delta_{b,bending} = \frac{F.L^3}{3E_b.I_b}$$

b. The shearing deflection of a beam due to a point load is [9] :

$$\delta_{b,shear} = \frac{F.L}{G_b.A_b}.\lambda$$

E_b and G_b are the modulus of elasticity and shear modulus of the beam, respectively.

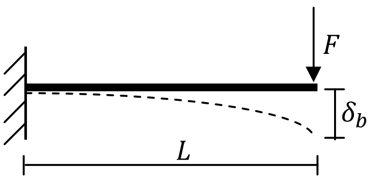


Figure 10.1: cantilever beam with appoint load

In this thesis it is assumed that the shear flow $q(z)$ is applied at the point of contraflexure (middle point) of the continuous mediums. As a result, each connecting beam is subjected to a shear force of $q(z).h$, which is assumed to be applied at the midpoint of the span. Note that h is the height of one story. Consequently, F can be substituted by $q(z).h$ in the above equations. Further, the length of the beam is assumed to be equal to $\frac{b}{2}$ since, the relative displacement at the midpoint of the beams has to be considered. By substituting the following parameter in the above equations, the total deflection of the beam can be obtained.

- $F = q(z).h$
- $L = \frac{b}{2}$ is half of the span of the connecting beams
- λ is the form factor of the cross section which is equal to 1.2 for a rectangular cross section.
- A_b is the cross sectional area of the connecting beams
- I_b is the second moment area of the connecting beams

[8] According to GTB 2010 . table 7.1
[9] According to study of J.A.Newlin, deflection of beams with special reference to shear deformation

As a result the total deformation of the connecting beams could be written as:

$$\delta_2 = -2.\left(\delta_{b,bending} + \delta_{b,shear}\right) = -2.\left(h.q(z).\frac{\left(\frac{b}{2}\right)^3}{3E_b.I_b} + h.q(z).\frac{\frac{b}{2}}{G_b.A_b}.\lambda \right)$$

Note that, the factor 2 has been used because the relative displacement is considered at the midpoint of the beam. Therefore the relative displacement is equal to two times the displacement of half span.

$$\delta_2 = -\left[\frac{b^3}{12E_bI_b} + \frac{b}{G_b.A_b}.\lambda\right].hq(z)$$

$$\delta_2 = -\left[\frac{1}{I_b} + \frac{12E_b}{G_b.A_b.b^2}.\lambda\right].\frac{b^3h}{12E_b}.q(z) = -\left[\frac{G_b.A_b.b^2 + 12.E_b.I_b.\lambda}{I_b.G_b.A_b.b^2}\right].\frac{b^3.h}{12E_b}.q(z)$$

$$= -\left[\frac{1 + \frac{(12.E_b.I_b)}{G_b.A_b.b^2}.\lambda}{I_b}\right].\frac{b^3.h}{12E_b}.q(z)$$

Assuming the following parameters:

$$r = \frac{(12.E_b.I_b)}{G_b.A_b.b^2}.\lambda \quad \text{and} \quad \frac{1}{I_e} = \left[\frac{1+r}{I_b}\right]$$

$$\delta_2 = -\left[\frac{1+r}{I_b}\right].\frac{b^3.h}{12E_b}.q(z) = -\frac{1}{I_e}.\frac{b^3.h}{12E_b}.q(z)$$

10.2 Maple output for the walls subjected to uniform distributed load on rigid foundation

Maple file of walls on rigid foundation subjected to uniform distributed load:

$$\delta_1 := l \cdot \frac{dx}{dz};$$

$$l\left(\frac{d}{dz}x(z)\right) \tag{1}$$

$$\delta_2 := \frac{dN}{dz} \cdot \frac{b^3 \cdot h}{12 \cdot E \cdot I_e};$$

$$\frac{1}{12}\,\frac{\left(\frac{d}{dz}N(z)\right)b^3 h}{E\,\mathrm{I}_e} \tag{2}$$

$$\delta_3 := -\frac{1}{E} \cdot \left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^z N(z)\,dz;$$

$$-\frac{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\left(\int_0^z N(z)\,dz\right)}{E} \tag{3}$$

$$\delta := \delta_1 + \delta_2 + \delta_3;$$

$$l\left(\frac{d}{dz}x(z)\right) + \frac{1}{12}\,\frac{\left(\frac{d}{dz}N(z)\right)b^3 h}{E\,\mathrm{I}_e} - \frac{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\left(\int_0^z N(z)\,dz\right)}{E} \tag{4}$$

$$\mathrm{M}_1 := m(z) - \left(\frac{b}{2} + d_1\right) \cdot \int_z^H \mathrm{q}(z)\,dz - M_b = E \cdot I_1 \frac{d}{dz}\left(\frac{dx}{dz}\right);$$

$$m(z) - \left(\frac{1}{2}b + d_1\right)\left(\int_z^H q(z)\,dz\right) - M_b = E\,\mathrm{I}_1\left(\frac{d^2}{dz^2}x(z)\right) \tag{5}$$

$$\mathrm{M}_2 := -\left(\frac{b}{2} + d_2\right) \cdot \int_z^H \mathrm{q}(z)\,dz + M_b = E \cdot I_2 \frac{d}{dz}\left(\frac{dx}{dz}\right);$$

$$-\left(\frac{1}{2}b + d_2\right)\left(\int_z^H q(z)\,dz\right) + M_b = E\,\mathrm{I}_2\left(\frac{d^2}{dz^2}x(z)\right) \tag{6}$$

>

$$(M_1 + M_2);$$

$$m(z) - \left(\frac{1}{2}b + d_1\right)\left(\int_z^H q(z)\,dz\right) - \left(\frac{1}{2}b + d_2\right)\left(\int_z^H q(z)\,dz\right) = E\,\mathrm{I}_1\left(\frac{d^2}{dz^2}x(z)\right) + E\,\mathrm{I}_2\left(\frac{d^2}{dz^2}x(z)\right) \tag{7}$$

>

> $sol_1 := collect(M_1 + M_2, diff);$

(8)

$$sol_1 := m(z) - \left(\frac{1}{2}\,b + d_1\right)\left(\int_z^H q(z)\,\mathrm{d}z\right) - \left(\frac{1}{2}\,b + d_2\right)\left(\int_z^H q(z)\,\mathrm{d}z\right) = \left(E\,\mathrm{I}_1 + E\,\mathrm{I}_2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\,x(z)\right) \quad (8)$$

> $sol_2 := collect\left(sol_1, \int_z^H q(z)\,\mathrm{d}z\right);$

$$sol_2 := \left(-b - d_1 - d_2\right)\left(\int_z^H q(z)\,\mathrm{d}z\right) + m(z) = \left(E\,\mathrm{I}_1 + E\,\mathrm{I}_2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\,x(z)\right) \quad (9)$$

> $sol_3 := algsubs(\left(-b - d_1 - d_2\right) = -l, sol_2);$

$$sol_3 := -\left(\int_z^H q(z)\,\mathrm{d}z\right) l + m(z) = \left(E\,\mathrm{I}_1 + E\,\mathrm{I}_2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\,x(z)\right) \quad (10)$$

> $sol_4 := algsubs\left(\left(\int_z^H q(z)\,\mathrm{d}z\right) = N(z), sol_3\right);$

$$sol_4 := -l\,N(z) + m(z) = \left(E\,\mathrm{I}_1 + E\,\mathrm{I}_2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\,x(z)\right) \quad (11)$$

> $sol_5 := collect\left(collect\left(algsubs\left(\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\,x(z)\right) = \frac{-l\,N(z) + m(z)}{\left(E\,\mathrm{I}_1 + E\,\mathrm{I}_2\right)}, \frac{\mathrm{d}}{\mathrm{d}\,z}\,\delta = 0\right), \frac{\mathrm{d}^2}{\mathrm{d}z^2}\,N(z)\right), N(z)\right);$

$$sol_5 := \frac{1}{12}\,\frac{\left(-12\,\mathrm{I}_e A_2\,\mathrm{I}_1 - 12\,\mathrm{I}_e A_2\,\mathrm{I}_2 - 12\,\mathrm{I}_e A_1\,\mathrm{I}_1 - 12\,\mathrm{I}_e A_1\,\mathrm{I}_2 - 12\,l^2 A_1 A_2\,\mathrm{I}_e\right) N(z)}{E\,A_1 A_2\left(\mathrm{I}_1 + \mathrm{I}_2\right)\mathrm{I}_e} + \frac{1}{12}\,\frac{\left(b^3\,h\,A_1 A_2\,\mathrm{I}_1 + b^3\,h\,A_1 A_2\,\mathrm{I}_2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\,N(z)\right)}{E\,A_1 A_2\left(\mathrm{I}_1 + \mathrm{I}_2\right)\mathrm{I}_e} + \frac{l\,m(z)}{E\left(\mathrm{I}_1 + \mathrm{I}_2\right)} = 0 \quad (12)$$

> $eq_1 := \frac{\mathrm{d}^2}{\mathrm{d}z^2}\,N(z) - \mathrm{N}(z)\cdot\left(\frac{12\cdot I_e\cdot l^2}{b^3\cdot h\cdot i_t}\right)\cdot\left(1 + \frac{A\cdot i_t}{A_1\cdot A_2\cdot l^2}\right) = -\frac{m(z)\cdot l}{i_t}\cdot\left(\frac{12\cdot I_e}{b^3\cdot h}\right);$

$$eq_1 := \frac{\mathrm{d}^2}{\mathrm{d}z^2}\,N(z) - \frac{12\,N(z)\,\mathrm{I}_e\,l^2\left(1 + \frac{A\,i_t}{A_1 A_2\,l^2}\right)}{b^3\,h\,i_t} = -\frac{12\,m(z)\,l\,\mathrm{I}_e}{i_t\,b^3\,h} \quad (13)$$

> $eq_{normal\,force} := algsubs\left(\left(\frac{12\cdot I_e\cdot l}{b^3\cdot h\cdot i_t}\right) = \frac{\alpha^2}{l}, algsubs\left(\left(\frac{12\cdot I_e\cdot l^2}{b^3\cdot h\cdot i_t}\right) = \alpha^2, \left(algsubs\left(\left(1 + \frac{A\cdot i_t}{A_1\cdot A_2\cdot l^2}\right) = k^2, eq_1\right)\right)\right)\right);$

$$eq_{normal\,force} := -N(z)\,k^2\,\alpha^2 + \frac{d^2}{dz^2}N(z) = -\frac{m(z)\,\alpha^2}{l} \tag{14}$$

> $eq_{deflec,\,1} := solve\left(algsubs\left(E\,\mathrm{I}_1 + E\,\mathrm{I}_2 = E\cdot i_t, sol_4\right), N(z)\right);$

$$eq_{deflec,\,1} := -\frac{\left(\frac{d^2}{dz^2}x(z)\right)E\,i_t - m(z)}{l} \tag{15}$$

> $eq_{deflec,\,2} := diff\left(eq_{deflec,\,1}, z, z\right);$

$$eq_{deflec,\,2} := -\frac{\left(\frac{d^4}{dz^4}x(z)\right)E\,i_t - \left(\frac{d^2}{dz^2}m(z)\right)}{l} \tag{16}$$

> $eq_{deflec,\,3} := diff(\delta, z);$

$$eq_{deflec,\,3} := l\left(\frac{d^2}{dz^2}x(z)\right) + \frac{1}{12}\,\frac{\left(\frac{d^2}{dz^2}N(z)\right)b^3\,h}{E\,\mathrm{I}_e} - \frac{\left(\frac{1}{A_1}+\frac{1}{A_2}\right)N(z)}{E} \tag{17}$$

> $eq_{deflec,\,4} := algsubs\left(N(z) = eq_{deflec,\,1}, algsubs\left(\frac{d^2}{dz^2}N(z) = eq_{deflec,\,2}, eq_{deflec,\,3}\right)\right);$

$$eq_{deflec,\,4} := \frac{1}{12}\,\frac{1}{E\,\mathrm{I}_e\,l\,A_1\,A_2}\Bigg(12\,l^2\left(\frac{d^2}{dz^2}x(z)\right)E\,\mathrm{I}_e\,A_1\,A_2 - b^3\,h\,A_1\,A_2\left(\frac{d^4}{dz^4}x(z)\right)E\,i_t + b^3\,h\,A_1\,A_2\left(\frac{d^2}{dz^2}m(z)\right) + 12\,\mathrm{I}_e\,A_1\left(\frac{d^2}{dz^2}x(z)\right)E\,i_t - 12\,\mathrm{I}_e\,A_1\,m(z) + 12\,\mathrm{I}_e\,A_2\left(\frac{d^2}{dz^2}x(z)\right)E\,i_t - 12\,\mathrm{I}_e\,A_2\,m(z)\Bigg) \tag{18}$$

> $ode_1 := eq_{deflec,\,4} = 0;$

$$ode_1 := \frac{1}{12}\,\frac{1}{E\,\mathrm{I}_e\,l\,A_1\,A_2}\Bigg(12\,l^2\left(\frac{d^2}{dz^2}x(z)\right)E\,\mathrm{I}_e\,A_1\,A_2 - b^3\,h\,A_1\,A_2\left(\frac{d^4}{dz^4}x(z)\right)E\,i_t + b^3\,h\,A_1\,A_2\left(\frac{d^2}{dz^2}m(z)\right) + 12\,\mathrm{I}_e\,A_1\left(\frac{d^2}{dz^2}x(z)\right)E\,i_t - 12\,\mathrm{I}_e\,A_1\,m(z) + 12\,\mathrm{I}_e\,A_2\left(\frac{d^2}{dz^2}x(z)\right)E\,i_t - 12\,\mathrm{I}_e\,A_2\,m(z)\Bigg) = 0 \tag{19}$$

>

> $ode_2 := algsubs\left(-\frac{12\,\mathrm{I}_e}{b^3\,h\,\mathrm{i}_t\,E\,A_2} - \frac{12\,\mathrm{I}_e}{b^3\,h\,\mathrm{i}_t\,E\,A_1} = -\frac{(k^2-1)\cdot\alpha^2}{\mathrm{i}_t\,E}, algsubs\left(\frac{12\,\mathrm{I}_e\,l^2}{b^3\,h\,\mathrm{i}_t} + \frac{12\,\mathrm{I}_e}{b^3\,h\,A_2} + \frac{12\,\mathrm{I}_e}{b^3\,h\,A_1} = k^2\cdot\alpha^2, collect\left(collect\left(expand\left((ode_1)\cdot\left(\frac{l\cdot I_e\cdot 12}{h\cdot b^3\cdot i_t}\right)\right), \frac{d^2}{dz^2}x(z)\right), m(z)\right)\right)\right);$

$$ode_2 := -\frac{m(z)\,(k^2-1)\,\alpha^2}{i_t E} + \frac{\frac{d^2}{dz^2} m(z)}{i_t E} + \left(\frac{d^2}{dz^2} x(z)\right) k^2 \alpha^2 - \left(\frac{d^4}{dz^4} x(z)\right) = 0 \qquad (20)$$

> $ode_3 := \left(\frac{d^4}{dz^4} x(z)\right) - \left(\frac{d^2}{dz^2} x(z)\right) k^2 \alpha^2 = \frac{1}{i_t \cdot E} \cdot \left(\frac{d^2}{dz^2} m(z) - m(z)\,(k^2-1)\,\alpha^2\right);$

$$ode_3 := \frac{d^4}{dz^4} x(z) - \left(\frac{d^2}{dz^2} x(z)\right) k^2 \alpha^2 = \frac{\frac{d^2}{dz^2} m(z) - m(z)\,(k^2-1)\,\alpha^2}{i_t E} \qquad (21)$$

>

> $m(z) := \frac{w\cdot(H-z)^2}{2};\ \frac{d^2}{dz^2} m(z);$

$$m := z \rightarrow \frac{1}{2} w\,(H-z)^2$$

$$w \qquad (22)$$

>

> $dsolve(ode_3, x(z));$

$$x(z) = \frac{1}{2} \frac{1}{k^4 E\, i_t\, \alpha^2} \left(2 k^2 \_C2\, E\, i_t\, e^{k\alpha z} + 2 \_C1\, k^2 E\, i_t\, e^{-k\alpha z} - z^2 w + \frac{1}{2} \alpha^2 k^4 w H^2 z^2 - \frac{1}{2} w \alpha^2 H^2 k^2 z^2 - \frac{1}{3} z^3 H w k^4 \alpha^2 + \frac{1}{3} w \alpha^2 H z^3 k^2 + \frac{1}{12} z^4 w k^4 \alpha^2 - \frac{1}{12} w \alpha^2 z^4 k^2\right) + \_C3\, z + \_C4 \qquad (23)$$

> $ics := x(0) = 0,\ D(x)(0) = 0,\ D^{(2)}(x)(H) = 0,\ \left(D^{(3)}(x)(H) - (k\cdot\alpha)^2 \cdot D(x)(H)\right) = \frac{1}{E\cdot i_t} \cdot \left(\frac{d}{dz} m(H) - \alpha^2 \cdot (k^2-1) \cdot \int_0^H m(z)\, dz\right);$

$$ics := x(0) = 0,\ D(x)(0) = 0,\ D^{(2)}(x)(H) = 0,\ D^{(3)}(x)(H) - k^2 \alpha^2 D(x)(H) = -\frac{1}{6} \frac{\alpha^2 (k^2-1)\, w H^3}{E\, i_t} \qquad (24)$$

>

> $sol_{deflection} := combine(convert(dsolve(\{ode_3, ics\}, x(z)), trig));$

$$sol_{deflection} := x(z) = \frac{1}{24} \frac{1}{k^6 E\, i_t\, \alpha^4 \cosh(k\alpha H)} \big(24 H w k \alpha \sinh(k\alpha H - k\alpha z) + 24 w \cosh(k\alpha z) - 12 z^2 w k^2 \alpha^2 \cosh(k\alpha H) + 6 \alpha^4 k^6 w H^2 z^2 \cosh(k\alpha H) - 6 w \alpha^4 H^2 k^4 z^2 \cosh(k\alpha H) - 4 z^3 H w k^6 \alpha^4 \cosh(k\alpha H) + 4 w \alpha^4 H z^3 k^4 \cosh(k\alpha H) + z^4 w k^6 \alpha^4 \cosh(k\alpha H) - w \alpha^4 z^4 k^4 \cosh(k\alpha H) + 24 z H w k^2 \alpha^2 \cosh(k\alpha H) - 24 w k \alpha H \sinh(k\alpha H) - 24 w\big) \qquad (25)$$

> simplify(**(25)**, 'size')

$$x(z) = -\frac{1}{k^6 E\, i_t\, \alpha^4 \cosh(k\,\alpha\, H)} \left(w\left(-H\sinh(k\,\alpha\,(H-z))\,\alpha\, k - \frac{1}{4}\,\alpha^2\, z\, k^2 \left(z\,(k+1)\,(k-1)\, k^2 \left(H^2 - \frac{2}{3}\, z\, H + \frac{1}{6}\, z^2\right)\alpha^2 + 4\,H - 2\,z\right)\cosh(k\,\alpha\,H) + k\,\alpha\,H\sinh(k\,\alpha\,H) - \cosh(k\,\alpha\,z) + 1\right)\right) \quad \textbf{(26)}$$

> *simplify*(**(25)**)

$$x(z) = \frac{1}{24}\,\frac{1}{k^6 E\, i_t\, \alpha^4 \cosh(k\,\alpha\,H)}\left(w\left(6\,\alpha^4 k^6 H^2 z^2 \cosh(k\,\alpha\,H) - 4\,z^3 H\, k^6 \alpha^4 \cosh(k\,\alpha\,H) + z^4 k^6 \alpha^4 \cosh(k\,\alpha\,H) - 6\,\alpha^4 H^2 k^4 z^2 \cosh(k\,\alpha\,H) + 4\,\alpha^4 H z^3 k^4 \cosh(k\,\alpha\,H) - \alpha^4 z^4 k^4 \cosh(k\,\alpha\,H) + 24\,z\,H\,k^2 \alpha^2 \cosh(k\,\alpha\,H) - 12\,z^2 k^2 \alpha^2 \cosh(k\,\alpha\,H) + 24\,H\,k\,\alpha \sinh(H\,\alpha\,k - \alpha\,k\,z) - 24\,k\,\alpha\,H\sinh(k\,\alpha\,H) + 24\cosh(k\,\alpha\,z) - 24\right)\right) \quad \textbf{(27)}$$

> $deflection_{max} := expand(algsubs(z = H, sol_{deflection}));$

$$deflection_{max} := x(H) = -\frac{1}{8}\,\frac{w\,H^4}{k^2 E\, i_t} + \frac{1}{8}\,\frac{w\,H^4}{E\, i_t} - \frac{w\,H\sinh(k\,\alpha\,H)}{k^5 E\, i_t\, \alpha^3 \cosh(k\,\alpha\,H)} - \frac{w}{k^6 E\, i_t\, \alpha^4 \cosh(k\,\alpha\,H)} + \frac{1}{2}\,\frac{H^2 w}{k^4 E\, i_t\, \alpha^2} + \frac{w}{k^6 E\, i_t\, \alpha^4} \quad \textbf{(28)}$$

> $F_3 := expand\left(\frac{8\cdot E\cdot i_t}{w\cdot H^4}\cdot deflection_{max}\right);$

$$F_3 := \frac{8\,E\, i_t\, x(H)}{w\,H^4} = -\frac{1}{k^2} + 1 - \frac{8\sinh(k\,\alpha\,H)}{H^3 k^5 \alpha^3 \cosh(k\,\alpha\,H)} - \frac{8}{H^4 k^6 \alpha^4 \cosh(k\,\alpha\,H)} + \frac{4}{H^2 k^4 \alpha^2} + \frac{8}{H^4 k^6 \alpha^4} \quad \textbf{(29)}$$

> $x(H) = \frac{w\,H^4}{8\cdot E\cdot i_t}\cdot F_3$

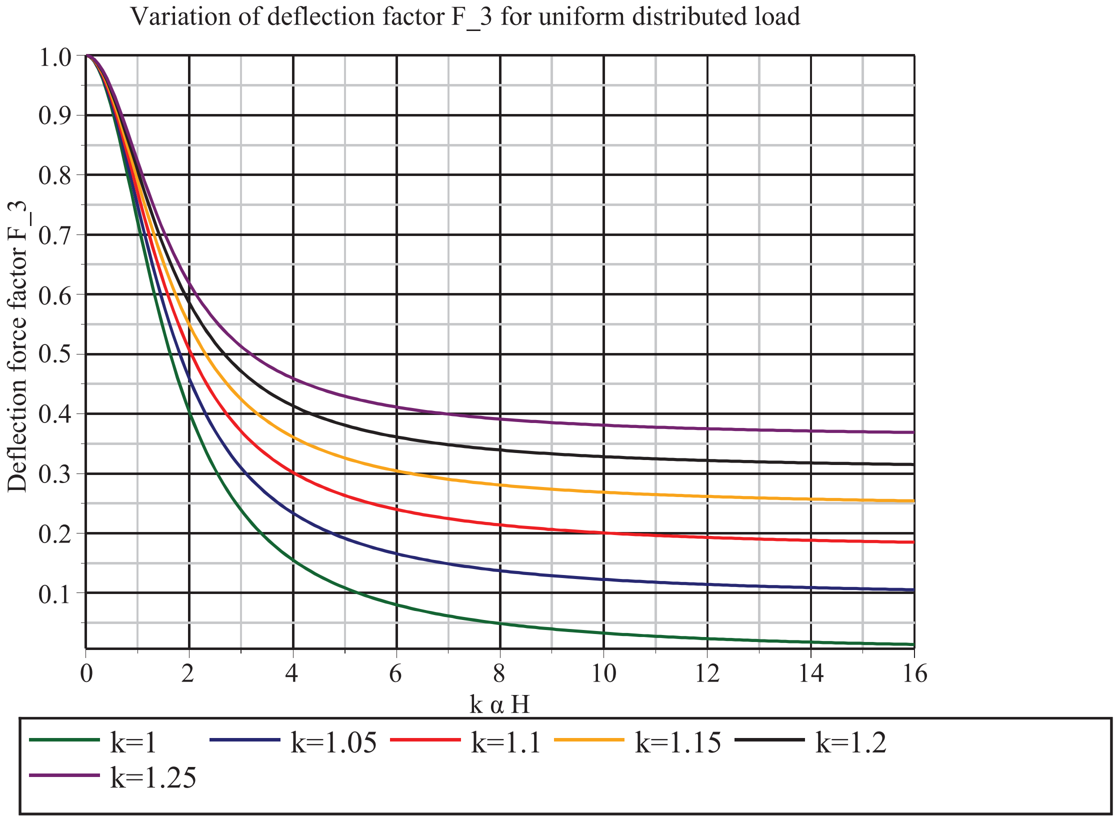
$$x(H) = \left(x(H) = \frac{1}{8}\,\frac{1}{E\, i_t}\left(w\,H^4\left(-\frac{1}{k^2} + 1 - \frac{8\sinh(k\,\alpha\,H)}{H^3 k^5 \alpha^3 \cosh(k\,\alpha\,H)} - \frac{8}{H^4 k^6 \alpha^4 \cosh(k\,\alpha\,H)} + \frac{4}{H^2 k^4 \alpha^2} + \frac{8}{H^4 k^6 \alpha^4}\right)\right)\right) \quad \textbf{(30)}$$

> $F_{3,\, distributed} := 1 - \frac{1}{y^2}\left(1 - \frac{4}{x^2} + \frac{8}{x^4\cdot\cosh(x)}\cdot(1 + x\cdot\sinh(x) - \cosh(x))\right);$

$$F_{3,\, distributed} := 1 - \frac{1 - \frac{4}{x^2} + \frac{8\,(1 + x\sinh(x) - \cosh(x))}{x^4 \cosh(x)}}{y^2} \quad \textbf{(31)}$$

> $plot([subs(y = 1, F_{3,\, distributed}), subs(y = 1.05, F_{3,\, distributed}), subs(y = 1.1, F_{3,\, distributed}), subs(y$

$= 1.15, F_{3,\ distributed}), subs(y = 1.2, F_{3,\ distributed}), subs(y = 1.25, F_{3,\ distributed})], x = 0..16,$
color = ["DarkGreen", "NavyBlue", "Red", "Orange", "Black", "Niagara Purple"], *legend*
= ["k=1", "k=1.05", "k=1.1", "k=1.15", "k=1.2", "k=1.25"], *title*
= "Variation of deflection factor F_3 for uniform distributed load", *labels* = ["k α H",
"Deflection force factor F_3"], *labeldirections* = ["horizontal", "vertical"]);



> restart;

> $ode := diff(N(z), z, z) - (k\cdot\alpha)^2 \cdot N(z) = -\frac{\alpha^2}{l}\cdot\left(\frac{w\cdot(H-z)^2}{2}\right);$

$$ode := \frac{d^2}{dz^2} N(z) - k^2 \alpha^2 N(z) = -\frac{1}{2}\frac{\alpha^2 w (H-z)^2}{l} \quad (1)$$

$dsolve(ode);$

$$N(z) = e^{-k\alpha z}\_C2 + e^{k\alpha z}\_C1 + \frac{1}{2}\frac{\left(2+k^2 (H-z)^2 \alpha^2\right) w}{k^4 l \alpha^2} \quad (2)$$

> $ics := N(H) = 0, D(N)(0) = 0;$

$$ics := N(H) = 0, D(N)(0) = 0 \quad (3)$$

> $convert(dsolve(\{ode, ics\}), trig);$

$$N(z) = -\frac{1}{2}\frac{(\cosh(k\alpha z) - \sinh(k\alpha z))\, w\, (k\alpha H(\cosh(k\alpha H) + \sinh(k\alpha H)) + 1)}{k^4 l \alpha^2 \cosh(k\alpha H)} + \frac{1}{2}\frac{(\cosh(k\alpha z) + \sinh(k\alpha z))\, w\, (H(\cosh(k\alpha H) - \sinh(k\alpha H))\alpha k - 1)}{k^4 l \alpha^2 \cosh(k\alpha H)} + \frac{1}{2}\frac{\left(2+k^2 (H-z)^2\alpha^2\right) w}{k^4 l \alpha^2} \quad (4)$$

> $N_{dist,\,load} := \frac{1}{2}\frac{(\cosh(k\alpha z) + \sinh(k\alpha z))\, w\, (H(\cosh(k\alpha H) - \sinh(k\alpha H))\alpha k - 1)}{k^4 l \alpha^2 \cosh(k\alpha H)} - \frac{1}{2}\frac{(\cosh(k\alpha z) - \sinh(k\alpha z))\, w\, (k\alpha H(\cosh(k\alpha H) + \sinh(k\alpha H)) + 1)}{k^4 l \alpha^2 \cosh(k\alpha H)} + \frac{1}{2}\frac{\left(2+k^2 (H-z)^2\alpha^2\right) w}{k^4 l \alpha^2};$

$$N_{dist,\,load} := -\frac{1}{2}\frac{(\cosh(k\alpha z) - \sinh(k\alpha z))\, w\, (k\alpha H(\cosh(k\alpha H) + \sinh(k\alpha H)) + 1)}{k^4 l \alpha^2 \cosh(k\alpha H)} + \frac{1}{2}\frac{(\cosh(k\alpha z) + \sinh(k\alpha z))\, w\, (H(\cosh(k\alpha H) - \sinh(k\alpha H))\alpha k - 1)}{k^4 l \alpha^2 \cosh(k\alpha H)} + \frac{1}{2}\frac{\left(2+k^2 (H-z)^2\alpha^2\right) w}{k^4 l \alpha^2} \quad (5)$$

> $sa_1 := simplify(combine(N_{dist,\,load}), size);$

$$sa_1 := \frac{1}{2}\frac{1}{k^4 l \alpha^2 \cosh(k\alpha H)}\left(\left(-2 H \sinh(k\alpha(H-z))\alpha k + \left(2 + k^2 (H-z)^2 \alpha^2\right)\cosh(k\alpha H) - 2\cosh(k\alpha z)\right) w\right) \quad (6)$$

> $simplify(N_{dist,\ load})$;

$$\frac{1}{2}\,\frac{1}{k^4\, l\, \alpha^2 \cosh(k\,\alpha\,H)}\big(w\,\big(H^2\cosh(k\,\alpha\,H)\,\alpha^2\,k^2-2\,H\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\,z+\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\,z^2+2\,H\cosh(k\,\alpha\,H)\,\sinh(k\,\alpha\,z)\,\alpha\,k-2\,H\sinh(k\,\alpha\,H)\,\cosh(k\,\alpha\,z)\,\alpha\,k+2\cosh(k\,\alpha\,H)-2\cosh(k\,\alpha\,z)\big)\big) \tag{7}$$

> $\mathrm{algsubs}\Big(k^2\cdot\alpha^2\cdot H^2=y^2,\ algsubs\Big(z\,H\,k^2\,\alpha^2=x\cdot y^2,\ algsubs\Big(k\,\alpha\,H=\mathrm{y},\ \mathrm{algsubs}\Big(k\,\alpha\,z=\mathrm{y}\cdot\mathrm{x},\ \frac{\mathrm{k}^2\cdot l}{H^2\cdot w}\cdot\big(simplify(N_{dist,\ load})\big)\Big)\Big)\Big)\Big)$;

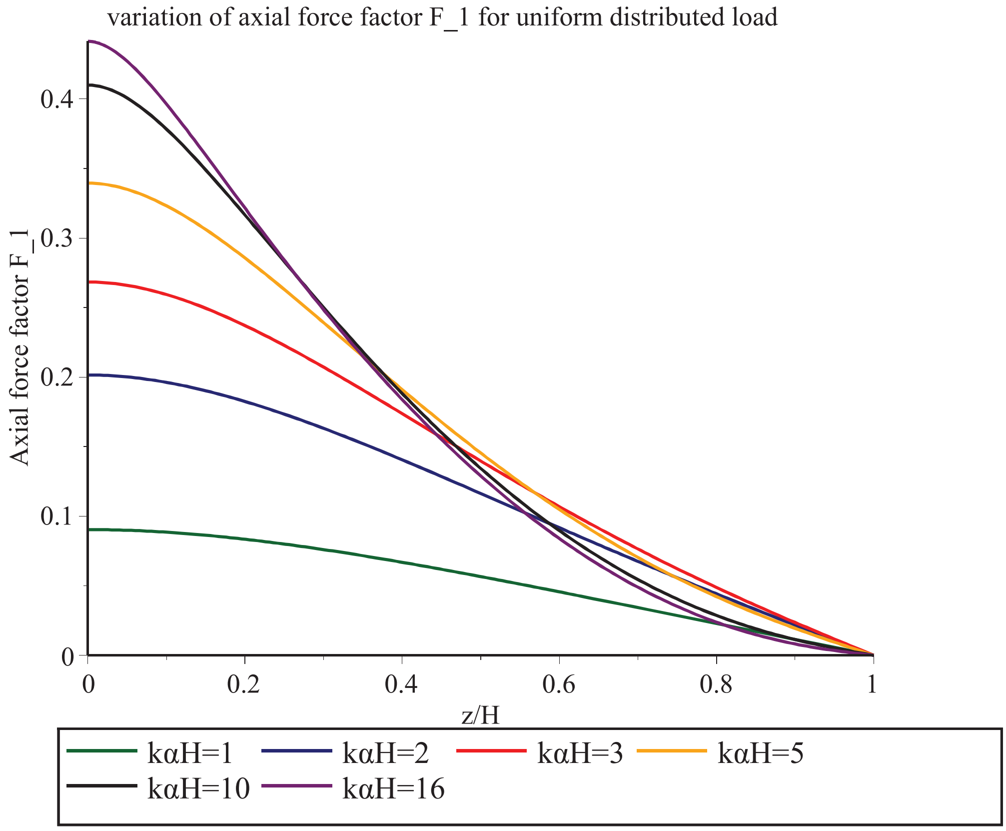
$$\frac{1}{2}\,\frac{1}{\cosh(y)\,H^2\,k^2\,\alpha^2}\big(k\,\alpha\,(-2\cosh(y)\,x\,y+2\cosh(y)\,\sinh(y\,x)-2\sinh(y)\,\cosh(y\,x))\,H+\cosh(y)\,y^2+\cosh(y)\,x^2\,y^2+2\cosh(y)-2\cosh(y\,x)\big) \tag{8}$$

substitting $\mathrm{k}\alpha H=y$, $\frac{z}{H}=x$

> $F_1 := \frac{1}{2}\,\frac{1}{\cosh(y)\,\mathrm{y}^2}\big(\mathrm{y}\cdot(-2\cosh(y)\,x\,y-2\sinh(y)\cosh(y\,x)+2\cosh(y)\sinh(y\,x))+\cosh(y)\,y^2+\cosh(y)\,x^2\,y^2+2\cosh(y)-2\cosh(y\,x)\big)$;

$$F_1 := \frac{1}{2}\,\frac{1}{\cosh(y)\,y^2}\big(y\,(-2\cosh(y)\,x\,y+2\cosh(y)\,\sinh(y\,x)-2\sinh(y)\,\cosh(y\,x))+\cosh(y)\,y^2+\cosh(y)\,x^2\,y^2+2\cosh(y)-2\cosh(y\,x)\big) \tag{9}$$

> $plot\big([subs(y=1, F_1), subs(y=2, F_1), subs(y=3, F_1), subs(y=5, F_1), subs(y=10, F_1), subs(y=16, F_1)], x=0..1,$ $color$ = ["DarkGreen", "NavyBlue", "Red", "Orange", "Black", "Niagara Purple"], $legend$ = ["kαH=1", "kαH=2", "kαH=3", "kαH=5", "kαH=10", "kαH=16"], $title$ = "variation of axial force factor F_1 for uniform distributed load", $labels$ = ["z/H", "Axial force factor F_1"], $labeldirections$ = ["horizontal", "vertical"]);



> $N_{dist} := F_1 \cdot \left(\frac{w \cdot H^2}{k^2 \cdot l}\right)$:

> $\mathrm{q}_{dist} := -diff\left(N_{dist,\ load}, z\right);$

$$q_{dist} := \frac{1}{2} \frac{\left(\sinh(k\,\alpha\,z)\,k\,\alpha - \cosh(k\,\alpha\,z)\,k\,\alpha\right)\,w\,\left(k\,\alpha\,H\,\left(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)\right) + 1\right)}{k^4\,l\,\alpha^2\,\cosh(k\,\alpha\,H)} - \frac{1}{2} \frac{1}{k^4\,l\,\alpha^2\,\cosh(k\,\alpha\,H)} \left(\left(\sinh(k\,\alpha\,z)\,k\,\alpha + \cosh(k\,\alpha\,z)\,k\,\alpha\right)\,w\,\left(H\,\left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\,\alpha\,k - 1\right)\right) + \frac{(H-z)\,w}{k^2\,l} \quad \mathbf{(10)}$$

> $sim_1 := simplify\left(convert\left(simplify\left(q_{dist} \cdot \frac{k^2 \cdot l}{w \cdot H}\right), trig\right)\right);$

$$sim_1 := \frac{1}{k\,H\,\alpha\,\cosh(k\,\alpha\,H)} \left(H\sinh(k\,\alpha\,z)\,\sinh(k\,\alpha\,H)\,\alpha\,k - H\cosh(k\,\alpha\,z)\,\cosh(k\,\alpha\,H)\,\alpha\,k + k\,\alpha\,H\cosh(k\,\alpha\,H) - k\,\alpha\cosh(k\,\alpha\,H)\,z + \sinh(k\,\alpha\,z)\right) \quad \mathbf{(11)}$$

>

> $algsubs\left(\left(k\,\alpha\,(H-z)\right)=(y\cdot(1-x)),\,algsubs\left(k\,\alpha\,z=y\cdot x,\,algsubs\left(k\,\alpha\,H=y,\,sim_1\right)\right)\right);$

$$\frac{\sinh(y\,x)\,\sinh(y)\,y-\cosh(y\,x)\,\cosh(y)\,y-\cosh(y)\,x\,y+\cosh(y)\,y+\sinh(y\,x)}{\cosh(y)\,H\,k\,\alpha} \quad \textbf{(12)}$$

> $F_2 := \frac{\sinh(y\,x)+\cosh(y)\,y-\cosh(y\cdot(1-x))\,y-\cosh(y)\,y\,x}{\cosh(y)\cdot y};$

$$F_2 := \frac{\sinh(y\,x)+\cosh(y)\,y-\cosh(y\,(1-x))\,y-\cosh(y)\,x\,y}{\cosh(y)\,y} \quad \textbf{(13)}$$

>

> $q_{z,\,distibuted} := \frac{w\cdot H}{k^2\cdot l}\cdot sim_1;$

$$q_{z,\,distibuted} := \frac{1}{k^3\,\alpha\,l\cosh(k\,\alpha\,H)}\left(w\,\left(H\sinh(k\,\alpha\,z)\,\sinh(k\,\alpha\,H)\,\alpha\,k - H\cosh(k\,\alpha\,z)\,\cosh(k\,\alpha\,H)\,\alpha\,k + k\,\alpha\,H\cosh(k\,\alpha\,H) - k\,\alpha\cosh(k\,\alpha\,H)\,z + \sinh(k\,\alpha\,z)\right)\right) \quad \textbf{(14)}$$

> calculate-position-of-the-max-shear:

$q_{max} := combine\left(diff\left(sim_1, z\right)\right);$

$$\frac{H\sinh(H\,\alpha\,k-\alpha\,k\,z)\,\alpha\,k+\cosh(k\,\alpha\,z)-\cosh(k\,\alpha\,H)}{H\cosh(k\,\alpha\,H)} \quad \textbf{(15)}$$

> substituting-y,x:

$f_1 := -\frac{-y\cdot\sinh(y\cdot(1-x))-\cosh(y\cdot x)+\cosh(y)}{\cosh(y)};$

$$-\frac{-y\sinh(y\,(1-x))-\cosh(y\,x)+\cosh(y)}{\cosh(y)} \quad \textbf{(16)}$$

$solve\left(subs\left(y=2, f_1\right)=0, x\right);$

$$1,\ \frac{1}{2}\ln\left(\frac{2\,e^2+1}{e^2-2}\right) \quad \textbf{(17)}$$

> $\left(\frac{1}{y}\ln\left(\frac{y\cdot e^y+1}{e^y-y}\right)\right)$

$$\frac{\ln\left(\frac{y\,e^y+1}{e^y-y}\right)}{y} \quad \textbf{(18)}$$

> $f_{star}(y) := \frac{1}{\cosh(y)\,y}\left(\sinh\left(y\,\frac{\ln\left(\frac{y\,e^y+1}{e^y-y}\right)}{y}\right)+\cosh(y)\,y-\cosh\left(y\left(-\frac{\ln\left(\frac{y\,e^y+1}{e^y-y}\right)}{y}+1\right)\right)y-\cosh(y)\,\frac{\ln\left(\frac{y\,e^y+1}{e^y-y}\right)}{y}\,y\right);$

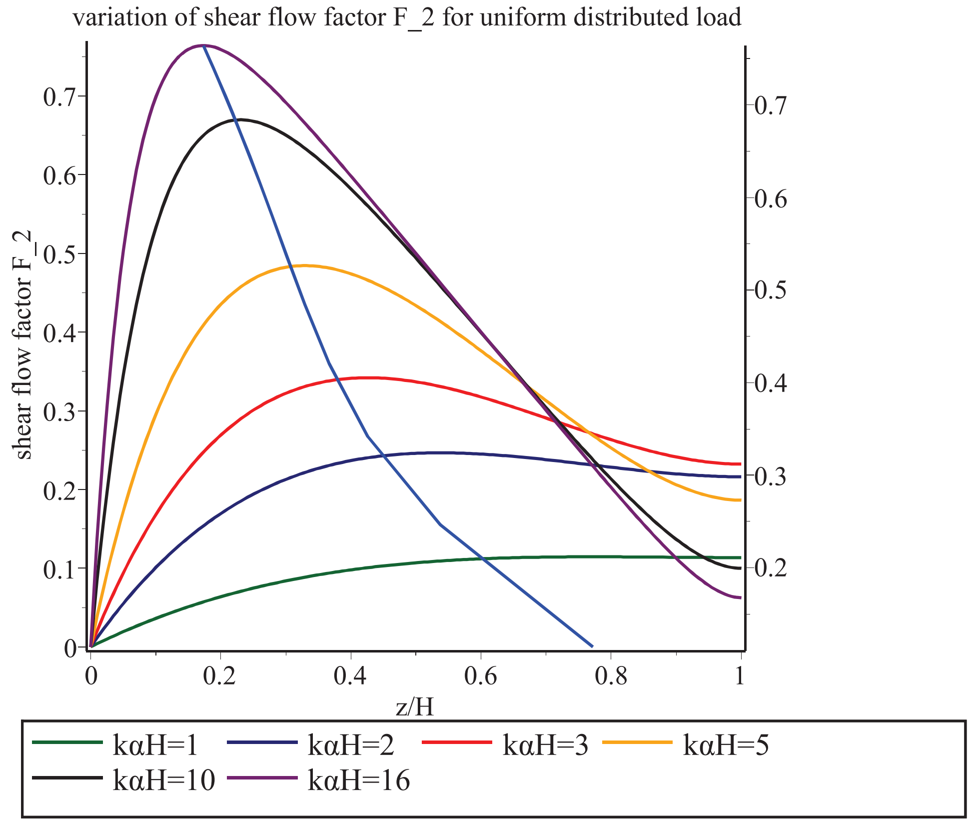
$$f_{star} := y \rightarrow \frac{1}{\cosh(y)\, y} \left(\sinh\left(\frac{y \ln\left(\frac{y\, e^{y} + 1}{e^{y} - y} \right)}{y} \right) + \cosh(y)\, y - \cosh\left(y \left(-\frac{\ln\left(\frac{y\, e^{y} + 1}{e^{y} - y} \right)}{y} + 1 \right) \right) y + \frac{(-1) \cosh(y) \ln\left(\frac{y\, e^{y} + 1}{e^{y} - y} \right) y}{y} \right) \quad \textbf{(19)}$$

> *with*(*plots*) :

> *dualaxisplot*$\Big($*plot*$\big($[*subs*($y=1, F_2$), *subs*($y=2, F_2$), *subs*($y=3, F_2$), *subs*($y=5, F_2$), *subs*($y=10, F_2$), *subs*($y=16, F_2$)], $x = 0..1$, *color* = ["DarkGreen", "NavyBlue", "Red", "Orange", "Black", "Niagara Purple"], *legend* = ["kαH=1", "kαH=2", "kαH=3", "kαH=5", "kαH=10", "kαH=16"], *title* = "variation of shear flow factor F_2 for uniform distributed load", *labels* = ["z/H", "shear flow factor F_2"], *labeldirections* = ["horizontal", "vertical"]$\big)$, *plot*$\Big($*Vector*$\Big($

$$\left[\frac{1}{1} \cdot \ln\left(\frac{1 \cdot e^{1} + 1}{e^{1} - 1} \right), \frac{1}{2} \ln\left(\frac{2\, e^{2} + 1}{e^{2} - 2} \right), \frac{1}{3} \ln\left(\frac{3\, e^{3} + 1}{e^{3} - 3} \right), \frac{1}{4} \ln\left(\frac{4\, e^{4} + 1}{e^{4} - 4} \right), \frac{1}{5} \ln\left(\frac{5\, e^{5} + 1}{e^{5} - 5} \right), \frac{1}{6} \ln\left(\frac{6\, e^{6} + 1}{e^{6} - 6} \right), \frac{1}{7} \ln\left(\frac{7\, e^{7} + 1}{e^{7} - 7} \right), \frac{1}{8} \ln\left(\frac{8\, e^{8} + 1}{e^{8} - 8} \right), \frac{1}{9} \ln\left(\frac{9\, e^{9} + 1}{e^{9} - 9} \right), \frac{1}{10} \ln\left(\frac{10\, e^{10} + 1}{e^{10} - 10} \right), \frac{1}{11} \ln\left(\frac{11\, e^{11} + 1}{e^{11} - 11} \right), \frac{1}{12} \ln\left(\frac{12\, e^{12} + 1}{e^{12} - 12} \right), \frac{1}{13} \ln\left(\frac{13\, e^{13} + 1}{e^{13} - 13} \right), \frac{1}{14} \ln\left(\frac{14\, e^{14} + 1}{e^{14} - 14} \right), \frac{1}{15} \ln\left(\frac{15\, e^{15} + 1}{e^{15} - 15} \right), \frac{1}{16} \ln\left(\frac{16\, e^{16} + 1}{e^{16} - 16} \right) \right]$$

$\Big)$, *Vector*([$f_{star}(1), f_{star}(2), f_{star}(3), f_{star}(4), f_{star}(5), f_{star}(6), f_{star}(7), f_{star}(8), f_{star}(9), f_{star}(10), f_{star}(11), f_{star}(12), f_{star}(13), f_{star}(14), f_{star}(15), f_{star}(16)$]), *style* = polygon, color = blue$\Big)$, *title* = "variation of shear flow factor F_2 for uniform distributed load"$\Big)$;



\>

\> **shear-of-each-connecting-beam:**

\> $Q_{z,\ distributed} := expand\left(simplify\left(\int_{z_i - \frac{h}{2}}^{z_i + \frac{h}{2}} q_{z,\ distibuted}\, dz\right)\right);$

$$Q_{z,\ distributed} := \frac{w\,H\,h}{k^2\,l} - \frac{w\,h\,z_i}{k^2\,l} - \frac{2\,w\,H \sinh\left(\frac{1}{2}\,k\,\alpha\,h\right) \cosh\left(k\,\alpha\,z_i\right)}{k^3\,\alpha\,l} + \frac{2\,w\,H \sinh\left(k\,\alpha\,H\right) \sinh\left(\frac{1}{2}\,k\,\alpha\,h\right) \sinh\left(k\,\alpha\,z_i\right)}{k^3\,\alpha\,l \cosh\left(k\,\alpha\,H\right)} + \frac{2\,w \sinh\left(\frac{1}{2}\,k\,\alpha\,h\right) \sinh\left(k\,\alpha\,z_i\right)}{k^4\,\alpha^2\,l \cosh\left(k\,\alpha\,H\right)} \tag{20}$$

\>

\> $solve\left(\frac{d}{d\,z}\,q_{dist} = 0, z\right);$

(21)

$$H, \frac{\ln\left(-\frac{k\alpha H e^{k\alpha H}+1}{k\alpha H - e^{k\alpha H}}\right)}{k\alpha} \quad \textbf{(21)}$$

```
> normal-force-beam;
```

$$normal - force - beam \quad \textbf{(22)}$$

$$> S_1 := \frac{w\cdot H\cdot I_1}{i_t}\left(1-\frac{z}{H}\right) - \left(\frac{I_1}{i_t}\cdot l - \frac{b}{2} - d_1\right)\cdot q_{z,\, distibuted};$$

$$S_1 := \frac{w\,H\,\mathrm{I}_1\left(1-\frac{z}{H}\right)}{i_t} - \frac{1}{k^3\,\alpha\,l\cosh(k\,\alpha\,H)}\left(\left(\frac{\mathrm{I}_1\,l}{i_t} - \frac{1}{2}\,b - d_1\right) w\,\big(H\sinh(k\,\alpha\,z)\,\sinh(k\,\alpha\,H)\,\alpha\,k - H\cosh(k\,\alpha\,z)\,\cosh(k\,\alpha\,H)\,\alpha\,k + k\,\alpha\,H\cosh(k\,\alpha\,H) - k\,\alpha\cosh(k\,\alpha\,H)\,z + \sinh(k\,\alpha\,z)\big)\right) \quad \textbf{(23)}$$

$$> S_2 := \frac{w\cdot H\cdot I_2}{(i_t)}\left(1-\frac{z}{H}\right) + \left(\frac{I_1}{i_t}\cdot l - \frac{b}{2} - d_1\right)\cdot q_{z,\, distibuted};$$

$$S_2 := \frac{w\,H\,\mathrm{I}_2\left(1-\frac{z}{H}\right)}{i_t} + \frac{1}{k^3\,\alpha\,l\cosh(k\,\alpha\,H)}\left(\left(\frac{\mathrm{I}_1\,l}{i_t} - \frac{1}{2}\,b - d_1\right) w\,\big(H\sinh(k\,\alpha\,z)\,\sinh(k\,\alpha\,H)\,\alpha\,k - H\cosh(k\,\alpha\,z)\,\cosh(k\,\alpha\,H)\,\alpha\,k + k\,\alpha\,H\cosh(k\,\alpha\,H) - k\,\alpha\cosh(k\,\alpha\,H)\,z + \sinh(k\,\alpha\,z)\big)\right) \quad \textbf{(24)}$$

\>

$$> n := \left(\frac{1}{2}\cdot\frac{d}{dz}\,S_1 - \frac{1}{2}\cdot\frac{d}{dz}\,S_2 + \frac{1}{2}\cdot w\right);$$

$$n := -\frac{1}{2}\,\frac{w\,\mathrm{I}_1}{i_t} - \frac{1}{k^3\,\alpha\,l\cosh(k\,\alpha\,H)}\left(\left(\frac{\mathrm{I}_1\,l}{i_t} - \frac{1}{2}\,b - d_1\right) w\,\big(H\cosh(k\,\alpha\,z)\,k^2\,\alpha^2\sinh(k\,\alpha\,H) - H\sinh(k\,\alpha\,z)\,k^2\,\alpha^2\cosh(k\,\alpha\,H) - k\cosh(k\,\alpha\,H)\,\alpha + \cosh(k\,\alpha\,z)\,k\,\alpha\big)\right) + \frac{1}{2}\,\frac{w\,\mathrm{I}_2}{i_t} + \frac{1}{2}\,w \quad \textbf{(25)}$$

$$> n := \frac{1}{2}\cdot\left(\frac{d}{dz}\,S_1 - \frac{d}{dz}\,S_2 + w\right);$$

$$n := -\frac{1}{2}\,\frac{w\,\mathrm{I}_1}{i_t} - \frac{1}{k^3\,\alpha\,l\cosh(k\,\alpha\,H)}\left(\left(\frac{\mathrm{I}_1\,l}{i_t} - \frac{1}{2}\,b \right.\right. \quad \textbf{(26)}$$

$$- d_1 \Big) w \left(H \cosh(k\,\alpha\,z)\, k^2 \alpha^2 \sinh(k\,\alpha\,H) - H \sinh(k\,\alpha\,z)\, k^2 \alpha^2 \cosh(k\,\alpha\,H) \right.$$

$$\left. - k \cosh(k\,\alpha\,H)\,\alpha + \cosh(k\,\alpha\,z)\, k\,\alpha \right) \Bigg) + \frac{1}{2}\, \frac{w\, \mathrm{I}_2}{i_t} + \frac{1}{2}\, w$$

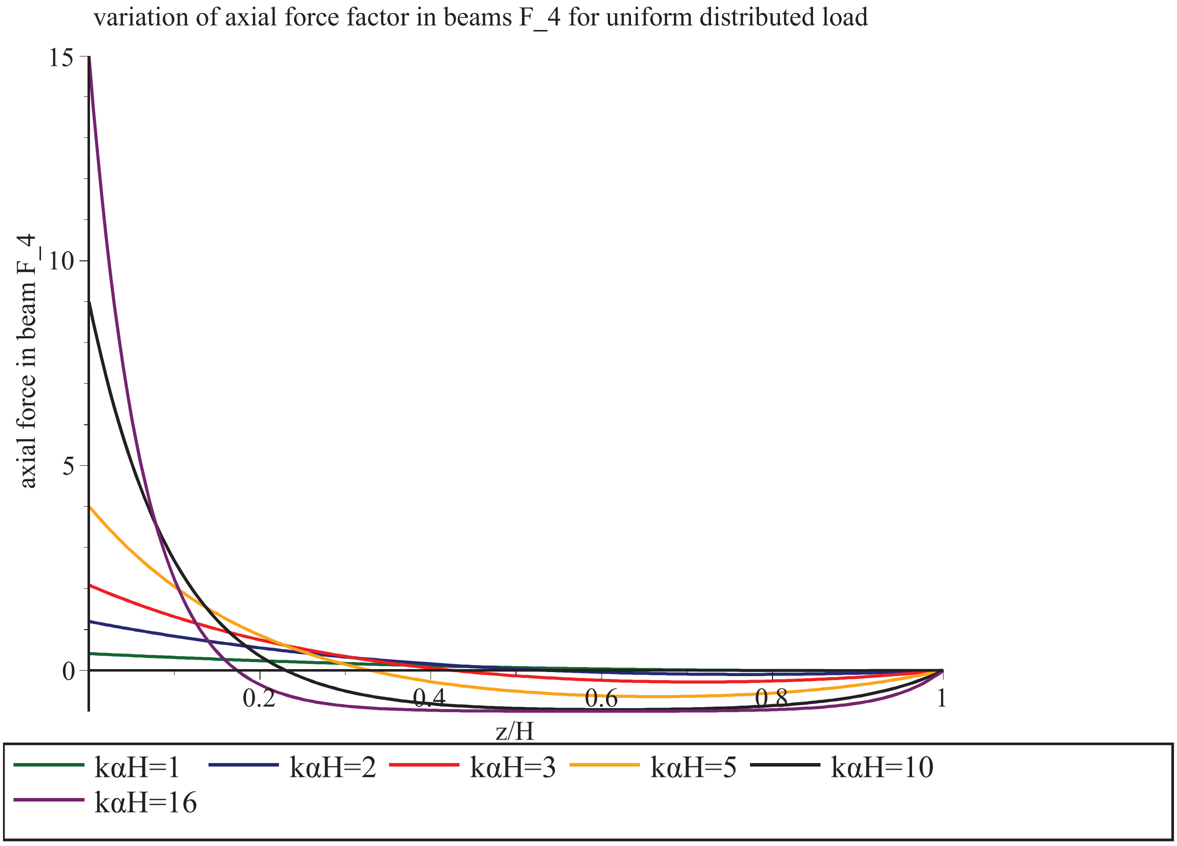
> $F_{4,\, distributed} := -\left(\mathrm{n} - \left(-\frac{1}{2}\, \frac{\mathrm{I}_1\, w}{i_t} + \frac{1}{2}\, \frac{\mathrm{I}_2\, w}{i_t} + \frac{1}{2}\, w \right) \right) \cdot \left(\frac{k^2 \cdot l}{w} \right) \cdot \left(\frac{1}{\left(\frac{\mathrm{I}_1\, l}{i_t} - \frac{1}{2}\, b - d_1 \right)} \right);$

$$F_{4,\, distributed} := \frac{1}{k\,\alpha \cosh(k\,\alpha\,H)} \left(H \cosh(k\,\alpha\,z)\, k^2 \alpha^2 \sinh(k\,\alpha\,H) - H \sinh(k\,\alpha\,z)\, k^2 \alpha^2 \cosh(k\,\alpha\,H) - k \cosh(k\,\alpha\,H)\,\alpha + \cosh(k\,\alpha\,z)\, k\,\alpha \right) \quad \textbf{(27)}$$

> $F_{4,\, plot} := \left(\frac{1}{\cosh(y)} \cdot (\cosh(y \cdot x) + y \cdot \sinh(y \cdot (1 - x))) - 1 \right);$

$$F_{4,\, plot} := \frac{\cosh(y\,x) + y \sinh(y\,(1 - x))}{\cosh(y)} - 1 \quad \textbf{(28)}$$

> $plot([subs(y=1, F_{4,\, plot}), subs(y=2, F_{4,\, plot}), subs(y=3, F_{4,\, plot}), subs(y=5, F_{4,\, plot}), subs(y=10, F_{4,\, plot}), subs(y=16, F_{4,\, plot})], x=0..1,$ *color* = ["DarkGreen", "NavyBlue", "Red", "Orange", "Black", "Niagara Purple"], *legend* = ["kαH=1", "kαH=2", "kαH=3", "kαH=5", "kαH=10", "kαH=16"], *title* = "variation of axial force factor in beams F_4 for uniform distributed load", *labels* = ["z/H", "axial force in beam F_4"], *labeldirections* = ["horizontal", "vertical"]);




```
> alternative-method;
```

$$alternative - method \tag{29}$$

```
> es[1] := ((1/2*w*(H-z)^2 - (N[dist, load])*l)*c[1])/i[t] + (N[dist, load])/A[1];
```

$$es_1 := \frac{\left(\left(\frac{1}{2} \cdot w \cdot (H-z)^2 - \left(N_{dist,\ load}\right) \cdot l\right) \cdot c_1}{i_t} + \frac{\left(N_{dist,\ load}\right)}{A_1};$$

$$es_1 := \frac{1}{i_t} \Bigg(\bigg(\frac{1}{2}\, w\, (H-z)^2 - \Big(-\frac{1}{2}\, \frac{(\cosh(k\,\alpha\, z) - \sinh(k\,\alpha\, z))\, w\, (k\,\alpha\, H\,(\cosh(k\,\alpha\, H) + \sinh(k\,\alpha\, H)) + 1)}{k^4\, l\, \alpha^2 \cosh(k\,\alpha\, H)} + \frac{1}{2}\, \frac{(\cosh(k\,\alpha\, z) + \sinh(k\,\alpha\, z))\, w\, (H\,(\cosh(k\,\alpha\, H) - \sinh(k\,\alpha\, H))\,\alpha\, k - 1)}{k^4\, l\, \alpha^2 \cosh(k\,\alpha\, H)} + \frac{1}{2}\, \frac{(2 + k^2\, (H-z)^2\, \alpha^2)\, w}{k^4\, l\, \alpha^2} \Big)\, l \bigg)\, c_1 \Bigg) + \frac{1}{A_1} \Big(\tag{30}$$

$$-\frac{1}{2}\,\frac{(\cosh(k\,\alpha\,z)-\sinh(k\,\alpha\,z))\,w\,(k\,\alpha\,H\,(\cosh(k\,\alpha\,H)+\sinh(k\,\alpha\,H))+1)}{k^4\,l\,\alpha^2\cosh(k\,\alpha\,H)}$$
$$+\frac{1}{2}\,\frac{(\cosh(k\,\alpha\,z)+\sinh(k\,\alpha\,z))\,w\,(H\,(\cosh(k\,\alpha\,H)-\sinh(k\,\alpha\,H))\,\alpha\,k-1)}{k^4\,l\,\alpha^2\cosh(k\,\alpha\,H)}$$
$$+\frac{1}{2}\,\frac{\left(2+k^2\,(H-z)^2\,\alpha^2\right)w}{k^4\,l\,\alpha^2}\Bigg)$$

$> es_2 := \dfrac{w\cdot(H-z)^2}{2\cdot\left(i_t+\dfrac{A_1\cdot A_2}{A}\cdot l^2\right)}\cdot\left(\dfrac{A_2\cdot l}{A}+c_1\right)\cdot\left(\dfrac{k_2}{100}\right);$

$$es_2 := \frac{1}{100}\,\frac{w\,(H-z)^2\left(\frac{A_2\,l}{A}+c_1\right)k_2}{2\,i_t+\frac{2\,A_1\,A_2\,l^2}{A}} \qquad \textbf{(31)}$$

$> es_3 := \dfrac{1}{2}\cdot\dfrac{w\cdot(H-z)^2\cdot c_1}{i_t}\cdot\dfrac{(100-k_2)}{100};$

$$es_3 := \frac{1}{2}\,\frac{w\,(H-z)^2\,c_1\left(1-\frac{1}{100}\,k_2\right)}{i_t} \qquad \textbf{(32)}$$

$>$

$> es_4 := \left(simplify\left(convert\left(solve\left(es_1=es_2+es_3,k_2\right),trig\right)\right)\right)\cdot\dfrac{(H^2-2\,H\,z+z^2)\cdot k^2\,\alpha^2}{200};$

$$es_4 := \frac{1}{2}\,\frac{1}{A_2\,(H-z)^2\,A_1\cosh(k\,\alpha\,H)\,l^2\,k^2}\Big(\left(\left(l^2\,A_1\,A_2+A\,i_t\right)\left(H^2\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\right.\right.$$
$$-2\,H\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\,z+\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\,z^2-2\,H\cosh(k\,\alpha\,z)\sinh(k\,\alpha\,H)\,\alpha\,k$$
$$\left.+2\,H\sinh(k\,\alpha\,z)\cosh(k\,\alpha\,H)\,\alpha\,k-2\cosh(k\,\alpha\,z)+2\cosh(k\,\alpha\,H)\right)\left(H^2-2\,H\,z\right.$$
$$\left.+z^2\right)\Big) \qquad \textbf{(33)}$$

$>$

$> expand\left(algsubs\left(\dfrac{i_t\;A}{l^2\;A_1\;A_2}=k^2-1,\ expand(\ \textbf{(33)}\ )\right)\right);$

$$\frac{1}{2}\,H^2\,\alpha^2\,k^2-H\,\alpha^2\,k^2\,z+\frac{1}{2}\,\alpha^2\,k^2\,z^2-\frac{H\cosh(k\,\alpha\,z)\sinh(k\,\alpha\,H)\,\alpha\,k}{\cosh(k\,\alpha\,H)}+H\sinh(k\,\alpha\,z)\,\alpha\,k$$
$$-\frac{\cosh(k\,\alpha\,z)}{\cosh(k\,\alpha\,H)}+1 \qquad \textbf{(34)}$$

$> F_5 := combine\left(-\dfrac{\cosh(k\,\alpha\,z)}{\cosh(k\,\alpha\,H)}-\dfrac{k\,\alpha\,H\cosh(k\,\alpha\,z)\sinh(k\,\alpha\,H)}{\cosh(k\,\alpha\,H)}+k\,\alpha\,H\sinh(k\,\alpha\,z),trig\right)$

$$+1+factor\left(\frac{1}{2}k^2\alpha^2H^2-k^2\alpha^2Hz+\frac{1}{2}k^2\alpha^2z^2\right);$$

$$F_5 := \frac{-H\sinh(H\alpha k-\alpha k z)\,\alpha k-\cosh(k\alpha z)}{\cosh(k\alpha H)}+1+\frac{1}{2}k^2(H-z)^2\alpha^2 \quad \textbf{(35)}$$

> $algsubs(k\alpha z=y\cdot x, algsubs(k\alpha H=x, F_5));$

$$\frac{\sinh(xy-x)\,x-\cosh(yx)}{\cosh(x)}+\frac{1}{2}k^2(H-z)^2\alpha^2+1 \quad \textbf{(36)}$$

> $K_{2,\,plot} := \frac{200}{x^2\cdot(1-y)^2}\cdot\left(1+\frac{1}{2}\cdot x^2\cdot(1-y)^2+\frac{-\cosh(yx)+\sinh(yx-x)\,x}{\cosh(x)}\right);$

$$K_{2,\,plot} := \frac{200\left(1+\frac{1}{2}x^2(1-y)^2+\frac{\sinh(xy-x)\,x-\cosh(yx)}{\cosh(x)}\right)}{x^2(1-y)^2} \quad \textbf{(37)}$$

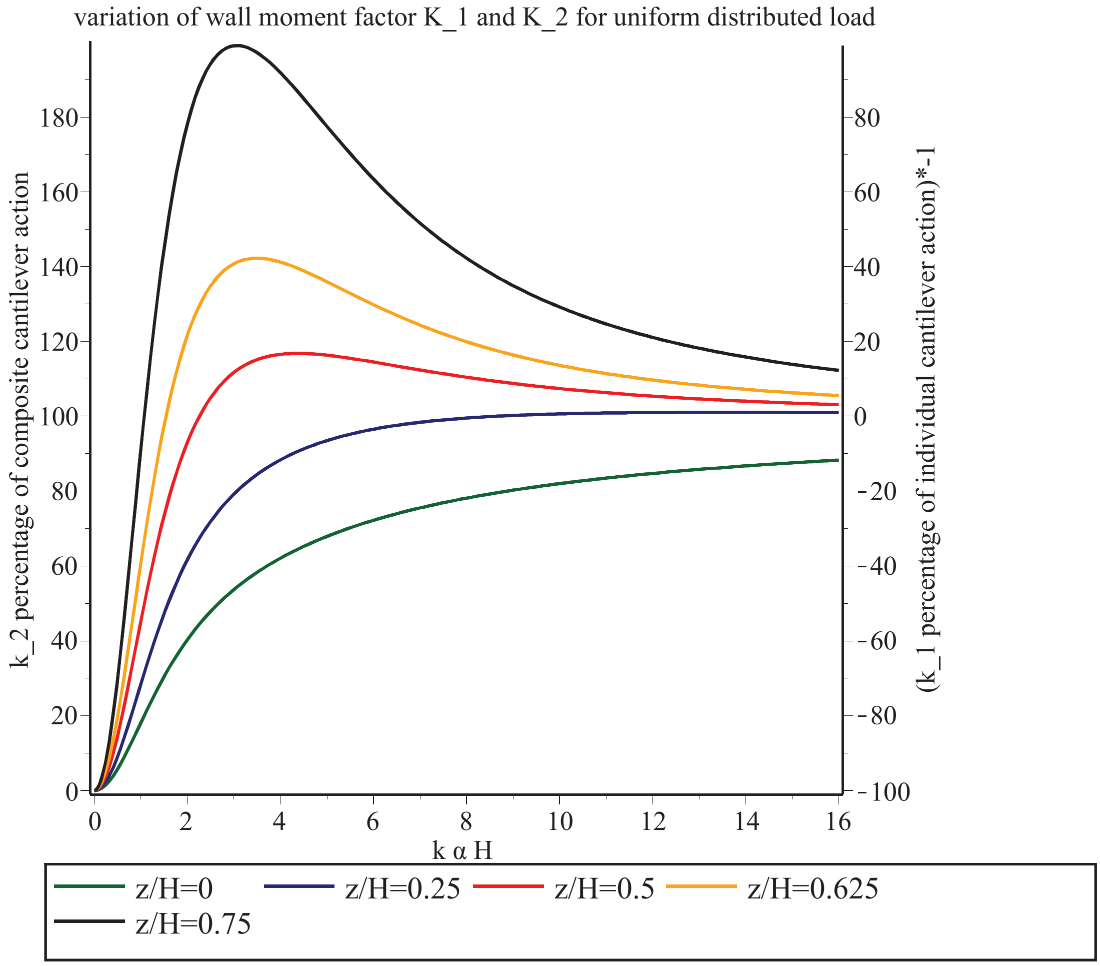
> $K_{1,\,plot} := 100-K_{2,\,plot};$

$$K_{1,\,plot} := 100-\frac{200\left(1+\frac{1}{2}x^2(1-y)^2+\frac{\sinh(xy-x)\,x-\cosh(yx)}{\cosh(x)}\right)}{x^2(1-y)^2} \quad \textbf{(38)}$$

>

> *with*(*plots*) :

> *dualaxisplot*(*plot*([*subs*($y=0, K_{2,\,plot}$), *subs*($y=0.25, K_{2,\,plot}$), *subs*($y=0.5, K_{2,\,plot}$), *subs*($y=0.625, K_{2,\,plot}$), *subs*($y=0.75, K_{2,\,plot}$)], $x=0..16$, *color* = ["DarkGreen", "NavyBlue", "Red", "Orange", "Black"], *legend* = ["z/H=0", "z/H=0.25", "z/H=0.5", "z/H=0.625", "z/H=0.75"], *labels* = ["k α H ", "k_2 percentage of composite cantilever action "], *labeldirections* = ["horizontal", "vertical"]), *plot*([*subs*($y=0, -K_{1,\,plot}$), *subs*($y=0.25, -K_{1,\,plot}$), *subs*($y=0.5, -K_{1,\,plot}$), *subs*($y=0.625, -K_{1,\,plot}$), *subs*($y=0.75, -K_{1,\,plot}$)], $x=0..16$, *color* = ["DarkGreen", "NavyBlue", "Red", "Orange", "Black"], *legend* = ["z/H=0", "z/H=0.25", "z/H=0.5", "z/H=0.625", "z/H=0.75"], *labels* = ["k α H ", "(k_1 percentage of individual cantilever action)·-1 "], *labeldirections* = ["horizontal", "vertical"]), *title* = "variation of wall moment factor K_1 and K_2 for uniform distributed load");


variation of wall moment factor K_1 and K_2 for uniform distributed load
k_2 percentage of composite cantilever action
(k_1 percentage of individual cantilever action)*-1
k α H
0
20
40
60
80
100
120
140
160
180
-100
-80
-60
-40
-20
0
20
40
60
80
0
2
4
6
8
10
12
14
16
z/H=0
z/H=0.25
z/H=0.5
z/H=0.625
z/H=0.75

11 Appendix 2:

In this part, the equations and the design curves for the walls subjected to point load and triangularly distributed load on rigid foundation will be given. For this case also the continuous approach has been used. It is worth to mention that, for both point load and triangularly load the ordinary differential equation for the normal force and the deflection will be the same as ones for the uniform distributed load. The only difference between the load cases is the applied moment on the shear walls. The load configurations have been illustrated in Figure 11.1.

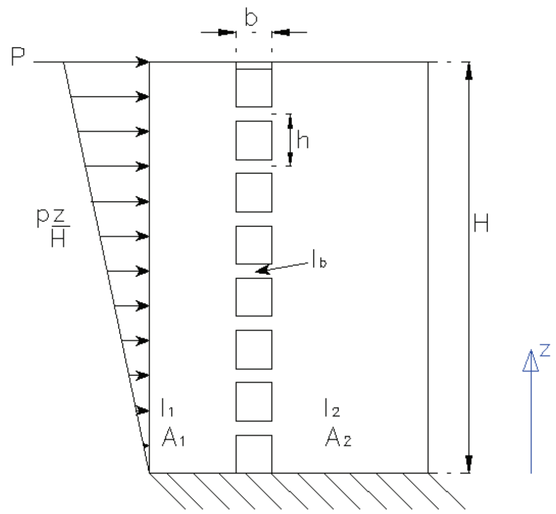


Figure 11.1 : load configuration on the walls

A.1: Point load:

The design curves for the case of the point load have been illustrated in the Figure 11.2 to Figure 11.4.

$$m(z) = p(H - z) \quad (11.1)$$

$$N_{point,load} = \frac{pH}{k^2 l}.\left(\left(1 - \frac{z}{H}\right) - \frac{1}{cosh(k\alpha H).k\alpha H}.sinh\left(k\alpha H\left(1 - \frac{z}{H}\right)\right)\right) \quad (11.2)$$

It should be mentioned that the variation of axial force in the wall trough the height of the wall for an applied point load at the top of the wall is linear.

Shear force:

$$q_{point,load} = -\frac{pH\left(-\frac{1}{H} + \frac{cosh\left(k\alpha(H-z)\right)}{cosh(k\alpha H)H}\right)}{k^2 l} \tag{11.3}$$

Axial force in the connecting beams

$$n = -\frac{\left(\frac{I_1 l}{i_t} - \frac{1}{2}b - d_1\right).psinh\left(k\alpha(H-z)\right).\alpha}{kl.cosh(k\alpha H)} \tag{11.4}$$

Deflection

$$x_{point,load} = \frac{1}{3}.\frac{1}{Ei_t}\left(pH^3\left(\frac{1}{2}.\frac{(-1+k^2).\left(\frac{3z^2}{H^2} - \frac{z^3}{H^3}\right)}{k^2} + \frac{3\left(\frac{z}{k^2\alpha^2H^3} - \frac{sinh(k\alpha H) - sinh(k\alpha H - k\alpha z)}{k^3\alpha^3H^3.cosh(k\alpha H)}\right)}{k^2}\right)\right) \tag{11.5}$$

Maximum deflection at the top

$$x_{max,point} = \frac{1}{3}.\frac{pH^3\left(1 - \frac{3\left(\frac{1}{3} + \frac{sinh(k\alpha H)}{k^3\alpha^3H^3.cosh(k\alpha H)} - \frac{1}{k^2\alpha^2H^2}\right)}{k^2}\right)}{Ei_t} \tag{11.6}$$

By differentiating from equation (11.3) and substituting it equal to zero the position of the maximum shear flow can be determined. As can be seen in Figure 11.2 by dashed line the maximum shear flow in the case of walls subjected to a point load occurs at the top of the walls.

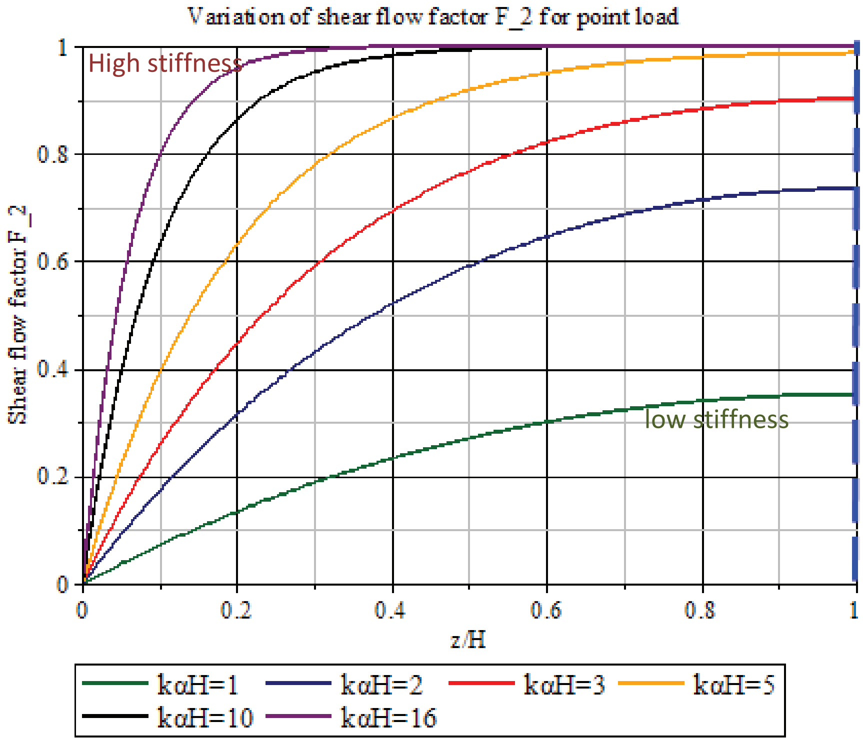


Figure 11.2 : Variation of the shear flow for the walls subjected to point load

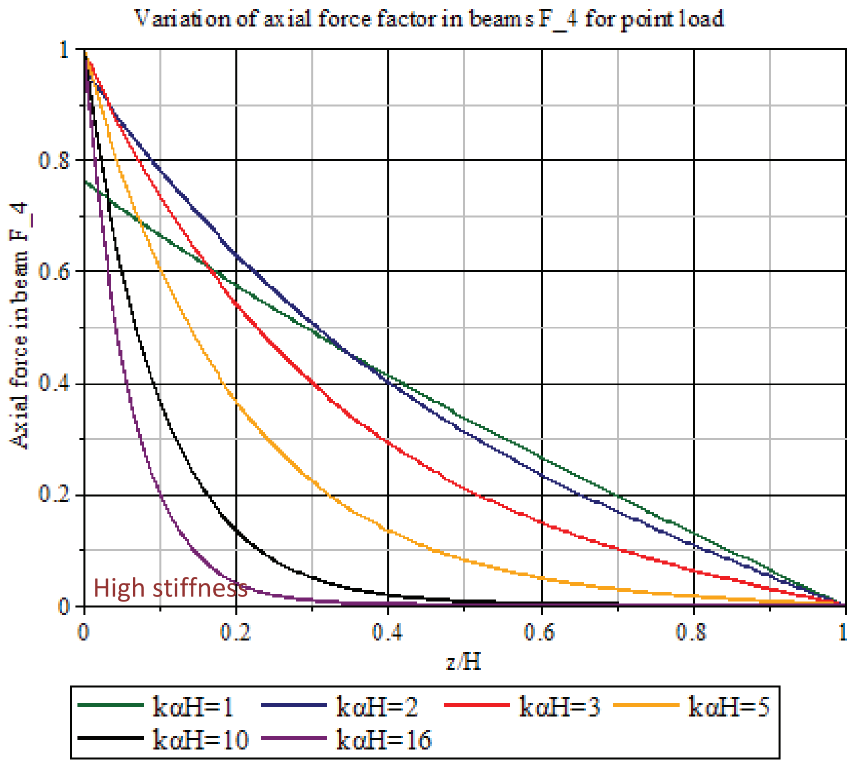


Figure 11.3 : variation of the axial force factor in the connecting beams for walls subjected to point load

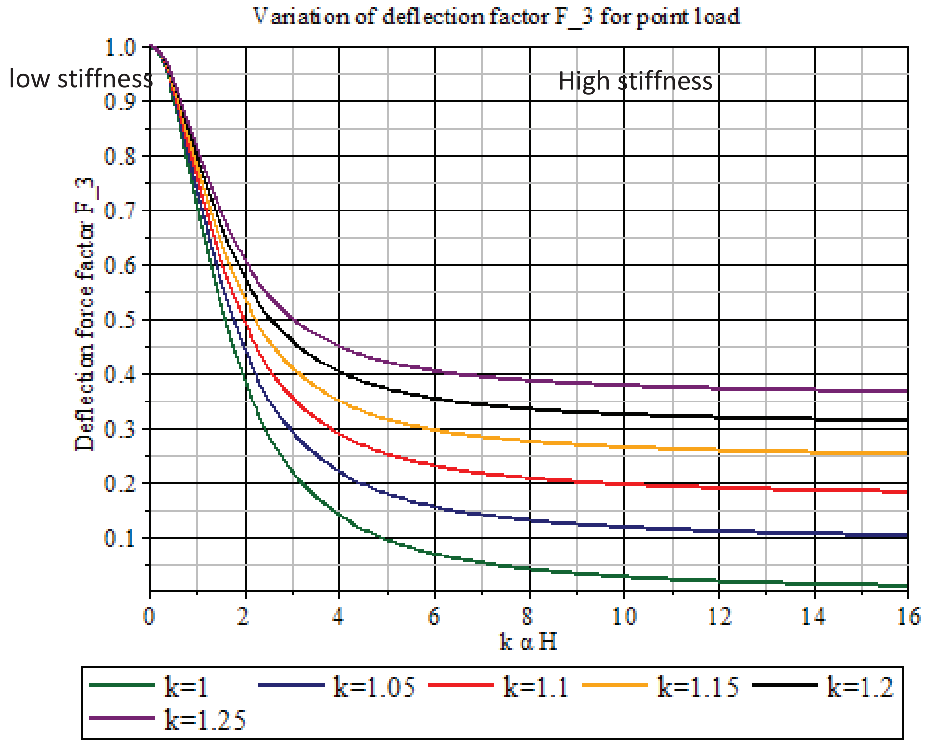


Figure 11.4 : variation of the maximum deflection factor for walls subjected to a point load at the top

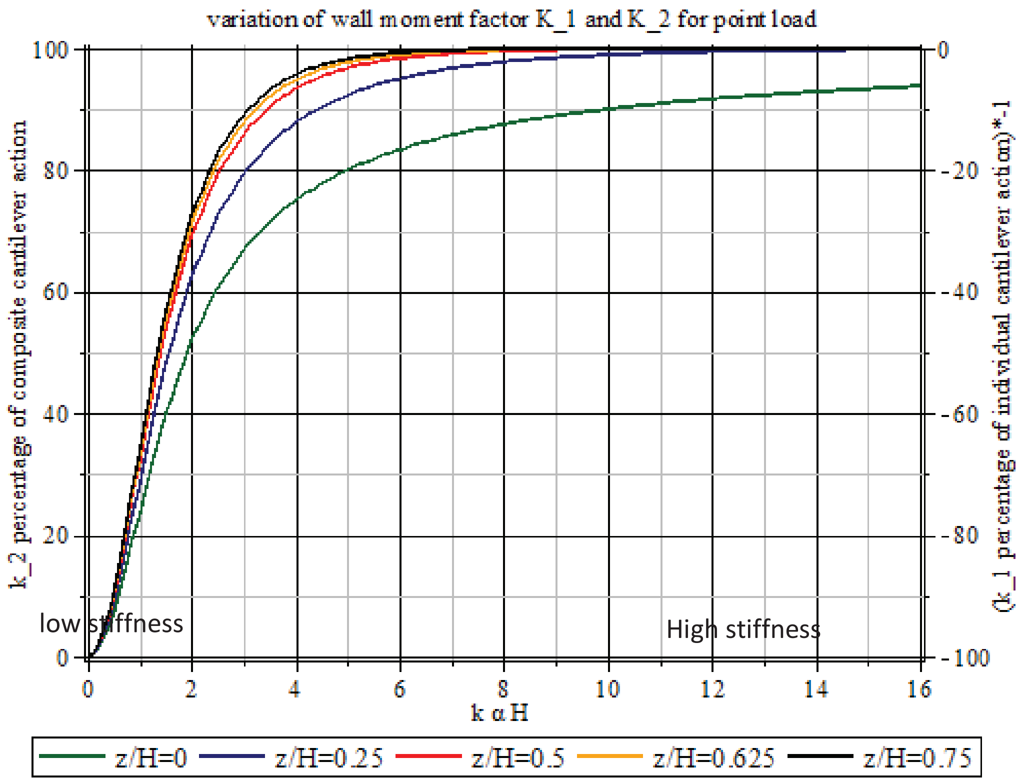


Figure 11.5 : variation of wall moment composite and individual action factor K_1 and K_2 for walls subjected to point load

A.2: Triangularly distributed load:

$$m(z) = \frac{p}{6H}.(H-z)^2.(2H+z) \tag{11.7}$$

The design curves for the case of walls subjected to triangularly distributed load have been illustrated in the Figure 11.6 to Figure 11.10.

Axial force in the walls

$$N_{triang,load} = \frac{pH^2}{k^2 l}\left(\frac{sinh(k\alpha H) - \frac{(k\alpha H)}{2} + \frac{1}{(k\alpha H)}}{(k\alpha H)^2.cosh(k\alpha H)}.sinh(k\alpha H - k\alpha z) - \frac{cosh(k\alpha H - k\alpha z)}{(k\alpha H)^2} + \frac{1}{2}\left(1-\frac{z}{H}\right)^2 - \frac{1}{6}\left(1-\frac{z}{H}\right)^3 + \frac{1}{(k\alpha H)^2}.\left(\frac{z}{H}\right)\right) \tag{11.8}$$

Shear force

$$q_{triang,load} = \frac{pH}{lk^2}.\left(\frac{\left(sinh(k\alpha H) - \frac{1}{2}k\alpha H + \frac{1}{k\alpha H}\right).cosh(k\alpha H - k\alpha z)}{k\alpha H.cosh(k\alpha H)} - \frac{sinh(k\alpha H - k\alpha z)}{k\alpha H} + \left(1-\frac{z}{H}\right) - \frac{1}{2}\left(1-\frac{z}{H}\right)^2 - \frac{1}{(k\alpha H)^2}\right) \tag{11.9}$$

Axial force in the connecting beams

$$n_{beam} = \frac{1}{lk^2}\left(\left(\frac{I_1 l}{i_t} - \frac{1}{2}b - d_1\right).pH^2.\left(\frac{1}{H^2.cosh(k\alpha H)}\left(\left(sinh(k\alpha H) - \frac{1}{2}k\alpha H + \frac{1}{k\alpha H}\right).sinh(k\alpha H - k\alpha z)\right) - \frac{cosh(k\alpha H - k\alpha z)}{H^2} + \frac{1}{H^2} - \frac{1-\frac{z}{H}}{H^2}\right)\right) + \frac{1}{2}.\frac{pzI_1}{Hi_t} \tag{11.10}$$

Lateral deflection

$$x_{triang,load} = \frac{1}{2}.\frac{pH^4}{Ei_t}.\left(\frac{1}{60}.\left(\frac{(k^2-1)}{k^2}\right).\left(20\left(\frac{z}{H}\right)^2 - 10.\left(\frac{z}{H}\right)^3 + \left(\frac{z}{H}\right)^5\right)\right.$$
$$+ \frac{1}{k^2.(k\alpha H)^2}.\left(\left(\frac{z}{H}\right).\left(1-\frac{2}{(k\alpha H)^2}\right) - \frac{1}{3}\left(\frac{z}{H}\right)^3\right.$$
$$+ \frac{2}{(k\alpha H)^2.cosh(k\alpha H)}.\left(cosh(k\alpha z) - 1\right.$$
$$\left.\left.\left.+ \left(\frac{1}{k\alpha H} - \frac{k\alpha H}{2}\right).\left(sinh(k\alpha H) - sinh(k\alpha H - k\alpha z)\right)\right)\right)\right) \quad (11.11)$$

Maximum deflection at the top

$$x_{max,triang} = \frac{11}{120}.\frac{1}{Ei_t}\left(pH^4\left(1-\frac{1}{k^2}+\frac{120}{11}.\frac{\frac{1}{3}-\frac{1+sinh(k\alpha H)\left(\frac{1}{2}k\alpha H-\frac{1}{k\alpha H}\right)}{cosh(k\alpha H).k^2\alpha^2H^2}}{k^4\alpha^2H^2}\right)\right) \quad (11.12)$$

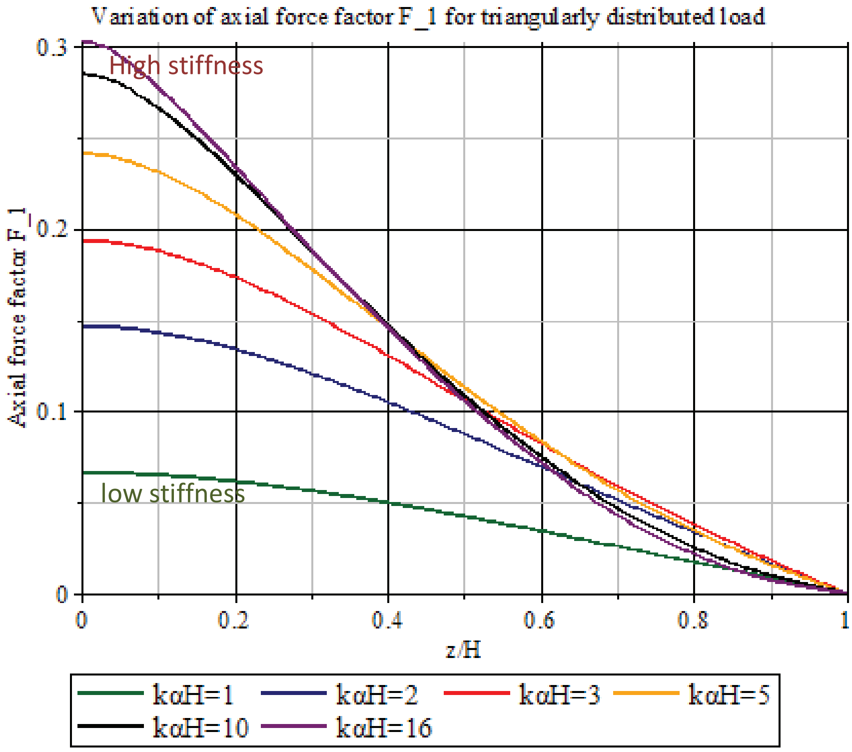


Figure 11.6 : variation of the axial force factor in the walls subjected to the triangularly distributed load

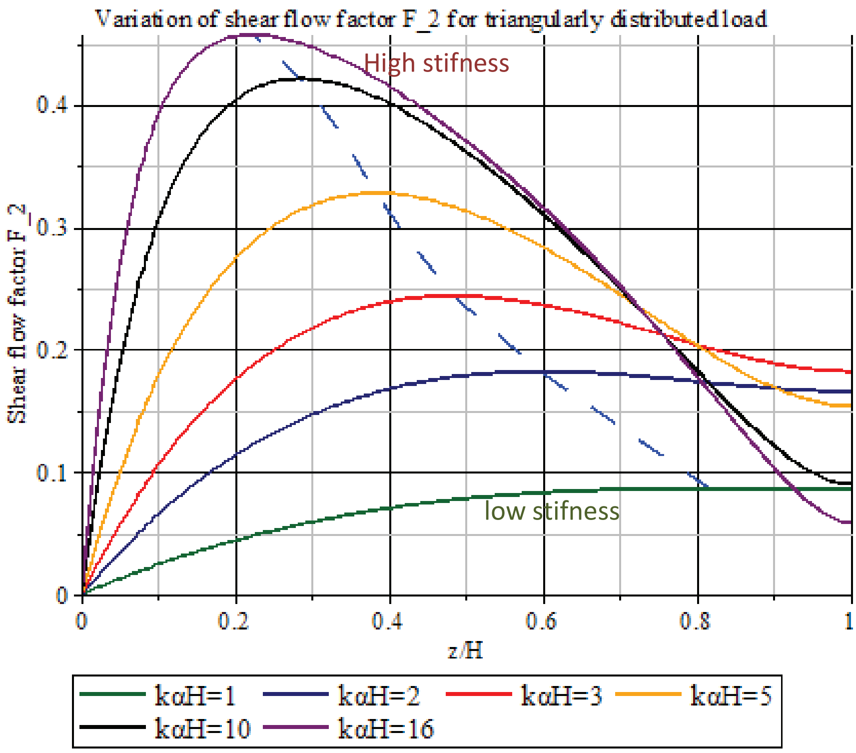


Figure 11.7 : variation of the shear flow in factor for the walls subjected to the triangularly distributed load

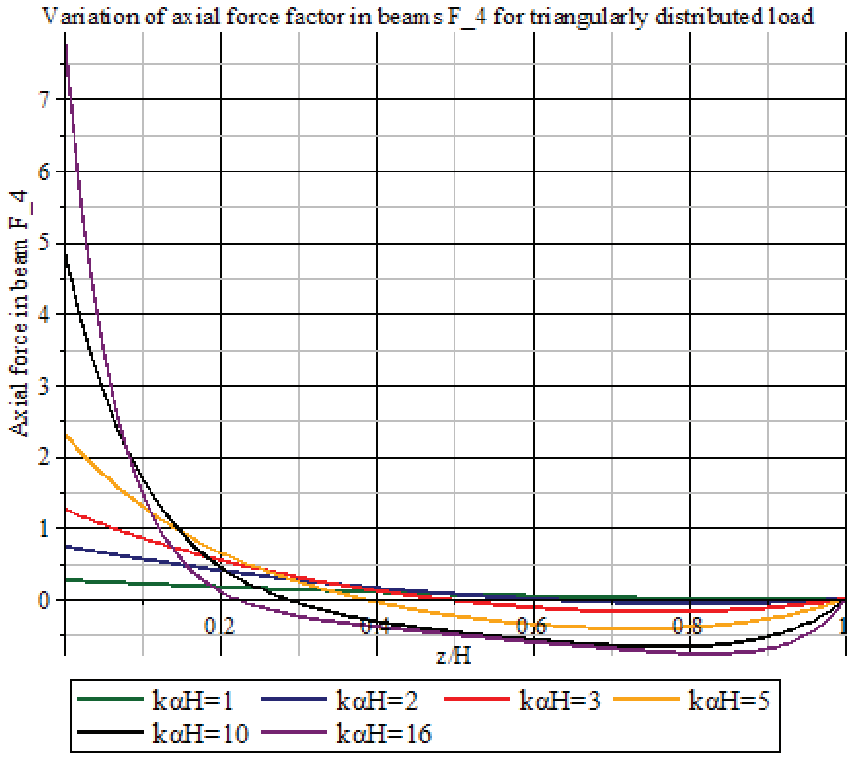


Figure 11.8 : variation of the normal force in the connecting beams for the walls subjected to triangularly distributed load

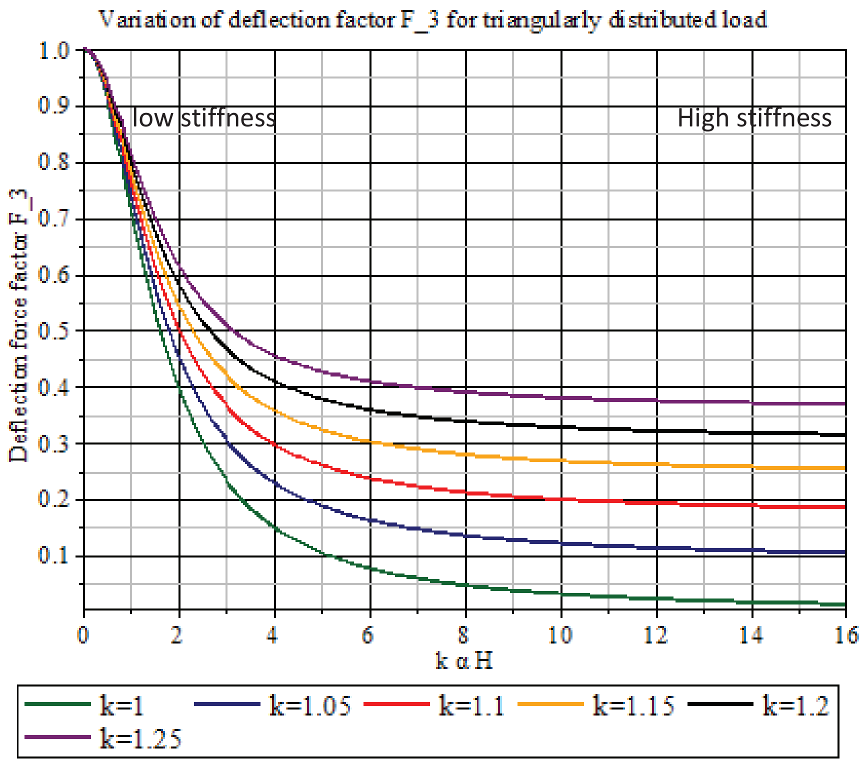


Figure 11.9 : variation of the maximum deflection for the walls subjected to triangularly distributed load

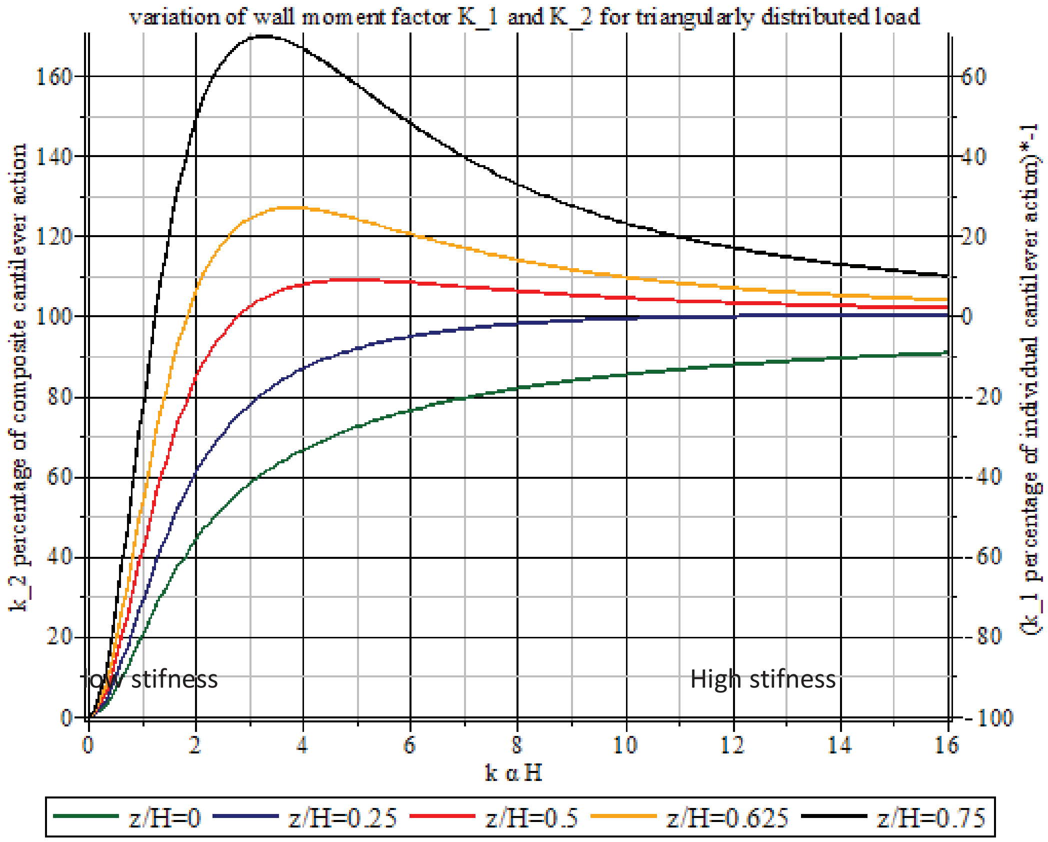


Figure 11.10 : variation of wall moment composite and individual action factor K_1 and K_2 for walls subjected to triangularly distributed load

12 Appendix 3:

As has been mentioned before, to prove the validity of the equations related to the walls on flexible foundation, the elasticity of the soil in derived equations should be assumed equal to infinity. The results have to be identical to the results from the walls on rigid foundations. This work has been done in Maple programme for all three types of loading. In this part only the calculations related to walls subjected to uniform distributed loads will be given.

12.1 Walls supported on individual elastic foundation

restart;

$$N_{elastic,\,uniform}(z) := -\frac{1}{2}\,\frac{1}{k^4\, l\,\left(E\,\alpha_f^2\,k_f^2\sinh(k\,\alpha\,H) + k\,\alpha\,\lambda\cosh(k\,\alpha\,H)\right)\alpha^2}\left(\left(\left(\left(H\,k^2\left(H\,E\,\alpha_f^2\,k^2 - H\,E\,\alpha_f^2\,k_f^2 - 2\,\lambda\right)\alpha^2 - 2\,E\,\alpha_f^2\,k_f^2\right)\sinh(k\,\alpha\,z) - \lambda\,k\,\alpha\left(2 + k^2\,(H-z)^2\,\alpha^2\right)\right)\cosh(k\,\alpha\,H) + \left(\left(-H\,k^2\left(H\,E\,\alpha_f^2\,k^2 - H\,E\,\alpha_f^2\,k_f^2 - 2\,\lambda\right)\alpha^2 + 2\,E\,\alpha_f^2\,k_f^2\right)\cosh(k\,\alpha\,z) - \alpha_f^2\,k_f^2\,E\left(2 + k^2\,(H-z)^2\,\alpha^2\right)\right)\sinh(k\,\alpha\,H) + 2\cosh(k\,\alpha\,z)\,k\,\alpha\,\lambda + 2\sinh(k\,\alpha\,z)\,E\,\alpha_f^2\,k_f^2\right)w\right);$$

$$z \rightarrow -\frac{1}{2}\left(\left(\left(\left(H\,k^2\left(H\,E\,\alpha_f^2\,k^2 - H\,E\,\alpha_f^2\,k_f^2 - 2\,\lambda\right)\alpha^2 - 2\,E\,\alpha_f^2\,k_f^2\right)\sinh(k\,\alpha\,z) - \lambda\,k\,\alpha\left(2 + k^2\,(H-z)^2\,\alpha^2\right)\right)\cosh(k\,\alpha\,H) + \left(\left(-H\,k^2\left(H\,E\,\alpha_f^2\,k^2 - H\,E\,\alpha_f^2\,k_f^2 - 2\,\lambda\right)\alpha^2 + 2\,E\,\alpha_f^2\,k_f^2\right)\cosh(k\,\alpha\,z) - E\,\alpha_f^2\,k_f^2\left(2 + k^2\,(H-z)^2\,\alpha^2\right)\right)\sinh(k\,\alpha\,H) + 2\cosh(k\,\alpha\,z)\,k\,\alpha\,\lambda + 2\sinh(k\,\alpha\,z)\,E\,\alpha_f^2\,k_f^2\right)w\right) \Big/ \left(k^4\,l\left(E\,\alpha_f^2\,k_f^2\sinh(k\,\alpha\,H) + \lambda\,k\,\alpha\cosh(k\,\alpha\,H)\right)\alpha^2\right) \quad \textbf{(1)}$$

$$\mathrm{N}_{elastic,\,uni,\,infinite} := combine\left(limit\left(N_{elastic,\,uniform}(z), \lambda = +\mathrm{infinity}\right)\right);$$

$$\frac{1}{2}\,\frac{1}{\cosh(k\,\alpha\,H)\,k^4\,l\,\alpha^2}\left(-2\,w\,H\,k\,\alpha\sinh(k\,\alpha\,H - k\,\alpha\,z) + 2\,w\cosh(k\,\alpha\,H) + w\cosh(k\,\alpha\,H)\,k^2\,\alpha^2\,H^2 - 2\,w\cosh(k\,\alpha\,H)\,k^2\,\alpha^2\,H\,z + w\cosh(k\,\alpha\,H)\,k^2\,\alpha^2\,z^2 - 2\,w\cosh(k\,\alpha\,z)\right) \quad \textbf{(2)}$$

$$N_{rigid,\,uniform} := combine\left(\frac{1}{2}\,\frac{(\cosh(k\,\alpha\,z) + \sinh(k\,\alpha\,z))\,w\,((\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H))\,k\,\alpha\,H - 1)}{k^4\,l\,\alpha^2\cosh(k\,\alpha\,H)} - \frac{1}{2}\,\frac{(\cosh(k\,\alpha\,z) - \sinh(k\,\alpha\,z))\,w\,(1 + k\,\alpha\,H\,(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)))}{k^4\,l\,\alpha^2\cosh(k\,\alpha\,H)} + \frac{1}{2}\,\frac{w\left(2 + k^2\,(H-z)^2\,\alpha^2\right)}{k^4\,l\,\alpha^2}\right);$$

$$\frac{1}{2}\,\frac{1}{\cosh(k\,\alpha\,H)\,k^4\,l\,\alpha^2}\left(-2\,w\,H\,k\,\alpha\sinh(k\,\alpha\,H - k\,\alpha\,z) + 2\,w\cosh(k\,\alpha\,H) + w\cosh(k\,\alpha\,H)\,k^2\,\alpha^2\,H^2 - 2\,w\cosh(k\,\alpha\,H)\,k^2\,\alpha^2\,H\,z + w\cosh(k\,\alpha\,H)\,k^2\,\alpha^2\,z^2 - 2\,w\cosh(k\,\alpha\,z)\right) \quad \textbf{(3)}$$

$$evalb\left(\mathrm{N}_{elastic,\,uni,\,infinite} = N_{rigid,\,uniform}\right);$$

$$true \quad \textbf{(4)}$$

deflection;

$$> x_{rigid,\,uniform} := \frac{1}{24}\,\frac{1}{k^6\,E\,i_t\,\alpha^4\cosh(k\,\alpha\,H)}\left(24\,H\,w\,k\,\alpha\sinh(k\,\alpha\,H - k\,\alpha\,z) + 24\,w\cosh(k\,\alpha\,z)\right.$$

$$- 12 z^2 w k^2 \alpha^2 \cosh(k \alpha H) + 6 \alpha^4 k^6 w H^2 z^2 \cosh(k \alpha H) - 6 w \alpha^4 H^2 k^4 z^2 \cosh(k \alpha H) - 4 z^3 H w k^6 \alpha^4 \cosh(k \alpha H) + 4 w \alpha^4 H z^3 k^4 \cosh(k \alpha H) + z^4 w k^6 \alpha^4 \cosh(k \alpha H) - w \alpha^4 z^4 k^4 \cosh(k \alpha H) + 24 z H w k^2 \alpha^2 \cosh(k \alpha H) - 24 w k \alpha H \sinh(k \alpha H) - 24 w\Big)$$

;

$$x_{rigid,\ uniform} := \frac{1}{24} \frac{1}{k^6 E i_t \alpha^4 \cosh(k \alpha H)} \Big(24 w H k \alpha \sinh(k \alpha H - k \alpha z) + 24 w \cosh(k \alpha z) - 12 w \cosh(k \alpha H) k^2 \alpha^2 z^2 + 6 \alpha^4 k^6 w H^2 z^2 \cosh(k \alpha H) - 6 w \alpha^4 H^2 k^4 z^2 \cosh(k \alpha H) - 4 z^3 H w k^6 \alpha^4 \cosh(k \alpha H) + 4 w \alpha^4 H z^3 k^4 \cosh(k \alpha H) + z^4 w k^6 \alpha^4 \cosh(k \alpha H) - w \alpha^4 z^4 k^4 \cosh(k \alpha H) + 24 w \cosh(k \alpha H) k^2 \alpha^2 H z - 24 w H k \alpha \sinh(k \alpha H) - 24 w\Big) \qquad \textbf{(5)}$$

> $x_{elastic,\ unidist}(z) := \frac{1}{2} \Big(w \Big(\Big(\Big(-2 k^2 l^2 I_f H \lambda + 2 i_t E \Big(H^2 \Big(\frac{1}{2} - \frac{1}{2} k_f^2 - l^2 + k^2 l^2\Big) k^2 \alpha^2 + 1 - k_f^2 - 2 l^2\Big)\Big) \sinh(\alpha k z) + z \alpha \Big(\frac{1}{2} \Big((k-1) z \Big(H^2 - \frac{2}{3} H z + \frac{1}{6} z^2\Big) k^2 (k+1) \alpha^2 + 4 H - 2 z\Big) \lambda l^2 k^2 I_f + i_t \big(H^2 k^2 (k^4 l^2 + 2 l^2 + k_f^2 - 3 k^2 l^2 - 1) \alpha^2 - 2 k^2 l^2 - 2 + 4 l^2 + 2 k_f^2\big) E\Big) k\Big) \lambda \alpha^2 \cosh(\alpha k H)^2 + \Big(\Big(-2 \Big(-k^2 l^2 I_f H \lambda + i_t E \Big(H^2 \Big(\frac{1}{2} - \frac{1}{2} k_f^2 - l^2 + k^2 l^2\Big) k^2 \alpha^2 + 1 - k_f^2 - 2 l^2\Big)\Big) \lambda \alpha \cosh(\alpha k z) + 2 H \Big(-l^2 \lambda I_f \alpha_f^2 k_f^2 + \Big(H \alpha_f^2 \Big(-\frac{1}{2} + l^2\Big) k^2 (k_f - 1)(k_f + 1) E + \lambda (-1 + 2 l^2 + k_f^2)\Big) i_t \alpha^2\Big) E k \sinh(\alpha k z) + \alpha \Big(\frac{1}{2} \Big(\alpha_f^2 k_f^2 E (k-1) z^2 \Big(H^2 - \frac{2}{3} H z + \frac{1}{6} z^2\Big) k^2 (k+1) \alpha^2 + 4 \alpha_f^2 \Big(H - \frac{1}{2} z\Big) k_f^2 z E - 4 H \lambda\Big) \lambda l^2 k^2 I_f + i_t E \Big(H \Big(z H k^2 \alpha_f^2 (k_f - 1)(k_f + 1) (k^2 l^2 + 1 - 2 l^2) E + 2 \lambda \Big(l^2 (H + z) k^2 - \Big(-\frac{1}{2} + \frac{1}{2} k_f^2 + l^2\Big) (2 z + H)\Big)\Big) k^2 \alpha^2 + (2 - 2 k_f^2 - 4 l^2) \lambda\Big)\Big)\Big) \sinh(\alpha k H) + 2 \lambda \Big(\lambda k l^2 I_f \cosh(\alpha k z) + 2 i_t E \alpha \Big(-\frac{1}{2} + \frac{1}{2} k_f^2 + l^2\Big) \sinh(\alpha k z) + \big(-l^2 \lambda I_f + \alpha^2 z E i_t (k^2 l^2 + 1 - 2 l^2 - k_f^2)\big) k\Big) \alpha \cosh(\alpha k H) - 2 E \Big(H \Big(-l^2 \lambda I_f \alpha_f^2 k_f^2 + \Big(H \alpha_f^2 \Big(-\frac{1}{2} + l^2\Big) k^2 (k_f - 1)(k_f + 1) E + \lambda (-1 + 2 l^2 + k_f^2)\Big) i_t \alpha^2\Big) \alpha k \sinh(\alpha k H) + 2 \lambda \Big(-\frac{1}{2} l^2 I_f \alpha_f^2 k_f^2 + i_t \alpha^2 \Big(-\frac{1}{2} + \frac{1}{2} k_f^2 + l^2\Big)\Big)\Big) (-1 + \cosh(\alpha k z)) \sinh(\alpha k H)\Big)\Big) \Big/ \Big(k^6 \alpha^4 E i_t l^2 \lambda I_f \cosh(\alpha k H) \big(E \alpha_f^2 k_f^2 \sinh(\alpha k H) + \alpha k \lambda \cosh(\alpha k H)\big)\Big);$

$$x_{elastic,\ unidist} := z \rightarrow \frac{1}{2} \Big(w \Big(\Big(\Big(-2 k^2 l^2 I_f H \lambda + 2 i_t E \Big(H^2 \Big(\frac{1}{2} - \frac{1}{2} k_f^2 - l^2 + k^2 l^2\Big) k^2 \alpha^2 + 1 - k_f^2 - 2 l^2\Big)\Big) \sinh(k \alpha z) + z \alpha \Big(\Big(\frac{1}{2} (k-1) z \Big(H^2 - \frac{2}{3} H z + \frac{1}{6} z^2\Big) k^2 (k+1) \alpha^2 + 2 H - z\Big) \lambda l^2 k^2 I_f + i_t \big(H^2 k^2 (k^4 l^2 + 2 l^2 + k_f^2 - 3 k^2 l^2 - 1) \alpha^2 - 2 k^2 l^2 - 2 + 4 l^2 + 2 \qquad \textbf{(6)}$$

$$k_f^2\big)E\big)k\big)\lambda\alpha^2\cosh(k\alpha H)^2+\Big(\Big(-\Big(-2k^2l^2 I_f H\lambda+2i_tE\Big(H^2\Big(\frac{1}{2}-\frac{1}{2}k_f^2-l^2+k^2l^2\Big)k^2\alpha^2+1-k_f^2-2l^2\Big)\Big)\lambda\alpha\cosh(k\alpha z)+2H\Big(-l^2\lambda I_f\alpha_f^2k_f^2+\Big(H\alpha_f^2\Big(-\frac{1}{2}+l^2\Big)k^2(k_f-1)(k_f+1)E+\lambda(-1+2l^2+k_f^2)\Big)i_t\alpha^2\Big)Ek\sinh(k\alpha z)+\alpha\Big(\Big(\frac{1}{2}E\alpha_f^2 k_f^2(k-1)z^2\Big(H^2-\frac{2}{3}Hz+\frac{1}{6}z^2\Big)k^2(k+1)\alpha^2+2\alpha_f^2\Big(H-\frac{1}{2}z\Big)k_f^2zE-2H\lambda\Big)\lambda l^2k^2I_f+i_tE\Big(H\Big(Hzk^2\alpha_f^2(k_f-1)(k_f+1)(k^2l^2+1-2l^2)E+2\lambda\Big(l^2(H+z)k^2-\Big(-\frac{1}{2}+\frac{1}{2}k_f^2+l^2\Big)(2z+H)\Big)\Big)k^2\alpha^2+(2-2k_f^2-4l^2)\lambda\Big)\Big)\sinh(k\alpha H)+2\lambda\Big(\lambda k l^2 I_f\cosh(k\alpha z)+2i_tE\alpha\Big(-\frac{1}{2}+\frac{1}{2}k_f^2+l^2\Big)\sinh(k\alpha z)+\Big(-l^2\lambda I_f+\alpha^2zEi_t(k^2l^2+1-2l^2-k_f^2)\Big)k\Big)\Big)\alpha\cosh(k\alpha H)-2E\Big(H\Big(-l^2\lambda I_f\alpha_f^2k_f^2+\Big(H\alpha_f^2\Big(-\frac{1}{2}+l^2\Big)k^2(k_f-1)(k_f+1)E+\lambda(-1+2l^2+k_f^2)\Big)i_t\alpha^2\Big)\alpha k\sinh(k\alpha H)+2\lambda\Big(-\frac{1}{2}l^2I_f\alpha_f^2k_f^2+i_t\alpha^2\Big(-\frac{1}{2}+\frac{1}{2}k_f^2+l^2\Big)\Big)\Big)(-1+\cosh(k\alpha z))\sinh(k\alpha H)\Big)\Big)\Big/\Big(k^6\alpha^4Ei_tl^2\lambda I_f\cosh(k\alpha H)\big(E\alpha_f^2k_f^2\sinh(k\alpha H)+\lambda k\alpha\cosh(k\alpha H)\big)\Big)$$

>

> $x_{elastic,\,uni,\,infinite} := combine(limit(x_{elastic,\,unidist}(z), \lambda = +\text{infinity}));$

$$x_{elastic,\,uni,\,infinite} := \frac{1}{24}\frac{1}{k^6Ei_t\alpha^4\cosh(k\alpha H)}\big(24wHk\alpha\sinh(k\alpha H-k\alpha z)+24w\cosh(k\alpha z)-12w\cosh(k\alpha H)k^2\alpha^2z^2+6\alpha^4k^6wH^2z^2\cosh(k\alpha H)-6w\alpha^4H^2k^4z^2\cosh(k\alpha H)-4z^3Hwk^6\alpha^4\cosh(k\alpha H)+4w\alpha^4Hz^3k^4\cosh(k\alpha H)+z^4wk^6\alpha^4\cosh(k\alpha H)-w\alpha^4z^4k^4\cosh(k\alpha H)+24w\cosh(k\alpha H)k^2\alpha^2Hz-24wHk\alpha\sinh(k\alpha H)-24w\big) \qquad \textbf{(7)}$$

> $evalb(x_{rigid,\,uniform} = x_{elastic,\,uni,\,infinite});$

true **(8)**

>

12.2 Walls supported on elastic foundation with stiffened base beam

restart;

> $\delta_1 := l \cdot \frac{dx}{dz}$;

$$\delta_1 := l \left(\frac{d}{dz} x(z) \right) \quad \textbf{(1)}$$

(2)

$\delta_2 := \frac{dN}{dz} \cdot \frac{b^3 \cdot h}{12 \cdot E \cdot I_e}$;

$$\frac{1}{12} \frac{\left(\frac{d}{dz} N(z) \right) b^3 h}{E \, I_e} \quad \textbf{(3)}$$

$\delta_3 := -\frac{1}{E} \cdot \left(\frac{1}{A_1} + \frac{1}{A_2} \right) \int_0^z N(z) \, dz$;

$$-\frac{\left(\frac{1}{A_1} + \frac{1}{A_2} \right) \left(\int_0^z N(z) \, dz \right)}{E} \quad \textbf{(4)}$$

$\delta_v := (N(0) + Q_0) \cdot \left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}} \right)$;

$$(N(0) + Q_0) \left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}} \right) \quad \textbf{(5)}$$

$\delta_\theta := \left(\frac{M_{10} + M_{20}}{k_{\theta 1} + k_{\theta 2}} \right) \cdot l$;

$$\frac{(M_{10} + M_{20}) \, l}{k_{\theta 1} + k_{\theta 2}} \quad \textbf{(6)}$$

$\delta_4 := \delta_\theta - \delta_v$;

$$\frac{(M_{10} + M_{20}) \, l}{k_{\theta 1} + k_{\theta 2}} - (N(0) + Q_0) \left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}} \right) \quad \textbf{(7)}$$

$\delta := \delta_1 + \delta_2 + \delta_3 + \delta_4$;

$$l \left(\frac{d}{dz} x(z) \right) + \frac{1}{12} \frac{\left(\frac{d}{dz} N(z) \right) b^3 h}{E \, I_e} - \frac{\left(\frac{1}{A_1} + \frac{1}{A_2} \right) \left(\int_0^z N(z) \, dz \right)}{E} + \frac{(M_{10} + M_{20}) \, l}{k_{\theta 1} + k_{\theta 2}} - (N(0) + Q_0) \left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}} \right) \quad \textbf{(8)}$$

>

$eq_{normal,\, force} := -N(z) \, k^2 \alpha^2 + \frac{d^2}{dz^2} N(z) = -\frac{m(z) \, \alpha^2}{l}$;

$$-N(z)\,k^2\,\alpha^2 + \frac{d^2}{dz^2}N(z) = -\frac{m(z)\,\alpha^2}{l} \tag{9}$$

$$\left(1 + \frac{A \cdot i_t}{A_1 \cdot A_2 \cdot l^2}\right) = k^2; \left(\frac{12 \cdot I_e \cdot l^2}{b^3 \cdot h \cdot i_t}\right) = \alpha^2;$$

$$\frac{12\,\mathrm{I}_e\,l^2}{b^3\,h\,i_t} = \alpha^2 \tag{10}$$

$$ics := N(H) = 0, \mathrm{D}(N)(0)\cdot\left(1 + \psi\cdot\mu_f\right) = \mu_f \cdot N(0) - \frac{\lambda_r}{l}\cdot m(0);$$

$$N(H) = 0, \mathrm{D}(N)(0)\left(\psi\,\mu_f + 1\right) = \mu_f N(0) - \frac{\lambda_r\,m(0)}{l} \tag{11}$$

$$m(z) := \frac{w\cdot(H-z)^2}{2};$$

$$z \to \frac{1}{2}\,w\,(H-z)^2 \tag{12}$$

> $convert\left(dsolve\left(eq_{normal,\,force}\right), trig\right);$

$$N(z) = \left(\cosh(k\,\alpha\,z) + \sinh(k\,\alpha\,z)\right)\_C2 + \left(\cosh(k\,\alpha\,z) - \sinh(k\,\alpha\,z)\right)\_C1 + \frac{1}{2}\,\frac{w\left(2 + k^2\,(H-z)^2\,\alpha^2\right)}{k^4\,l\,\alpha^2} \tag{13}$$

$$n_{\text{uniform, load, }elastic} := convert\left(dsolve\left(\left\{eq_{normal,\,force}, ics\right\}, N(z)\right), trig\right);$$

$$N(z) = -\frac{1}{2}\Big(\left(\cosh(k\,\alpha\,z) + \sinh(k\,\alpha\,z)\right) w \Big(H^2\left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\alpha^2\,k^4\,\lambda_r$$
$$- H^2\left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\alpha^2\,k^2\,\mu_f - 2\,H\,(\cosh(k\,\alpha\,H)$$
$$- \sinh(k\,\alpha\,H))\,\alpha^2\,k^2\,\psi\,\mu_f - 2\,H\left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\alpha^2\,k^2 + 2\,\alpha\,k\,\psi\,\mu_f$$
$$- 2\left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\mu_f + 2\,k\,\alpha + 2\,\mu_f\Big)\Big) \Big/ \Big(k^4\,l\,\big(\alpha\,k\,\psi\,\mu_f\,(\cosh(k\,\alpha\,H)$$
$$+ \sinh(k\,\alpha\,H)) + \left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\alpha\,k\,\psi\,\mu_f + \alpha\,k\,(\cosh(k\,\alpha\,H)$$
$$+ \sinh(k\,\alpha\,H)) + \left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\alpha\,k + \mu_f\left(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)\right)$$
$$- \left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\mu_f\big)\,\alpha^2\Big) + \frac{1}{2}\Big(w\,\Big(H^2\,(\cosh(k\,\alpha\,H)$$
$$+ \sinh(k\,\alpha\,H))\,\alpha^2\,k^4\,\lambda_r - H^2\left(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)\right)\alpha^2\,k^2\,\mu_f - 2\,H\,(\cosh(k\,\alpha\,H)$$
$$+ \sinh(k\,\alpha\,H))\,\alpha^2\,k^2\,\psi\,\mu_f - 2\,H\left(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)\right)\alpha^2\,k^2 - 2\,\alpha\,k\,\psi\,\mu_f$$
$$- 2\,\mu_f\left(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)\right) - 2\,k\,\alpha + 2\,\mu_f\Big)\left(\cosh(k\,\alpha\,z) - \sinh(k\,\alpha\,z)\right)\Big) \Big/$$
$$\Big(k^4\,l\,\big(\alpha\,k\,\psi\,\mu_f\left(\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,H)\right) + \left(\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,H)\right)\alpha\,k\,\psi\,\mu_f$$

$$+\alpha k\left(\cosh(k\alpha H)+\sinh(k\alpha H)\right)+\left(\cosh(k\alpha H)-\sinh(k\alpha H)\right)\alpha k$$
$$+\mu_f\left(\cosh(k\alpha H)+\sinh(k\alpha H)\right)-\left(\cosh(k\alpha H)-\sinh(k\alpha H)\right)\mu_f\Big)\alpha^2\Big)$$
$$+\frac{1}{2}\,\frac{w\left(2+k^2(H-z)^2\alpha^2\right)}{k^4\,l\,\alpha^2}$$

$$\overset{\text{simplify}}{=}$$

$$N(z)=-\frac{1}{2}\Big(w\Big(-2\cosh(k\alpha H)\sinh(k\alpha z)\mu_f-2k\alpha\cosh(k\alpha H)$$
$$+2\sinh(k\alpha H)\cosh(k\alpha z)\mu_f+2\cosh(k\alpha z)\alpha k-2\mu_f\sinh(k\alpha H)+2\sinh(k\alpha z)\mu_f$$
$$+H^2\cosh(k\alpha H)\sinh(k\alpha z)\alpha^2k^4\lambda_r-H^2\cosh(k\alpha H)\alpha^3k^3\psi\mu_f$$
$$-H^2\sinh(k\alpha H)\cosh(k\alpha z)\alpha^2k^4\lambda_r-\cosh(k\alpha H)\alpha^3k^3\psi z^2\mu_f$$
$$-H^2\cosh(k\alpha H)\sinh(k\alpha z)\alpha^2k^2\mu_f+H^2\sinh(k\alpha H)\cosh(k\alpha z)\alpha^2k^2\mu_f$$
$$+2H\sinh(k\alpha H)\alpha^2k^2z\mu_f+2H\cosh(k\alpha H)\alpha^3k^3z-H^2\alpha^2k^2\mu_f\sinh(k\alpha H)$$
$$-\sinh(k\alpha H)\alpha^2k^2z^2\mu_f-2H\cosh(k\alpha H)\sinh(k\alpha z)\alpha^2k^2$$
$$+2H\sinh(k\alpha H)\cosh(k\alpha z)\alpha^2k^2-2\alpha k\psi\mu_f\cosh(k\alpha H)+2\cosh(k\alpha z)\alpha k\psi\mu_f$$
$$-H^2\cosh(k\alpha H)\alpha^3k^3-\cosh(k\alpha H)\alpha^3k^3z^2+2H\cosh(k\alpha H)\alpha^3k^3\psi z\mu_f$$
$$-2H\cosh(k\alpha H)\sinh(k\alpha z)\alpha^2k^2\psi\mu_f+2H\sinh(k\alpha H)\cosh(k\alpha z)\alpha^2k^2\psi\mu_f\Big)\Big)\Big/$$
$$\left(k^4\,l\left(\alpha k\psi\mu_f\cosh(k\alpha H)+k\alpha\cosh(k\alpha H)+\mu_f\sinh(k\alpha H)\right)\alpha^2\right)$$

$$\overset{\text{simplify}}{=}$$

$$N(z)=-\frac{1}{2}\Big(w\Big(\Big(\Big(\left(k^2\left(H\lambda_r k^2-2+(-H-2\psi)\mu_f\right)H\alpha^2-2\mu_f\right)\sinh(k\alpha z)-k\Big(2 \tag{16}$$
$$+k^2(H-z)^2\alpha^2\Big)\left(\psi\mu_f+1\right)\alpha\Big)\cosh(k\alpha H)+\Big(\Big(-k^2\Big(H\lambda_r k^2-2+\big(-H$$
$$-2\psi\big)\mu_f\Big)H\alpha^2+2\mu_f\Big)\cosh(k\alpha z)-\left(2+k^2(H-z)^2\alpha^2\right)\mu_f\Big)\sinh(k\alpha H)$$
$$+2k\alpha\left(\psi\mu_f+1\right)\cosh(k\alpha z)+2\sinh(k\alpha z)\mu_f\Big)\Big)\Big/\Big(k^4\Big(k\alpha\left(\psi\mu_f+1\right)\cosh(k\alpha H)$$
$$+\mu_f\sinh(k\alpha H)\Big)l\,\alpha^2\Big)$$

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>
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> shear-of-lamellas;
```

$$shear-of-lamellas \tag{17}$$

> $q_{\text{uniform, }load\text{, }elastic} := simplify\left(-\frac{d}{dz}\, n_{\text{uniform, load, }elastic}\right);$

$$q_{uniform,\ load,\ elastic} := -\left(\frac{d}{dz}N(z)\right) = -\frac{1}{2}\Big(w\big(H^2\sinh(k\alpha z)\sinh(k\alpha H)\alpha^2k^4\lambda_r - H^2\cosh(k\alpha z)\cosh(k\alpha H)\alpha^2k^4\lambda_r - H^2\sinh(k\alpha z)\sinh(k\alpha H)\alpha^2k^2\mu_f + H^2\cosh(k\alpha z)\cosh(k\alpha H)\alpha^2k^2\mu_f - 2H\sinh(k\alpha z)\sinh(k\alpha H)\alpha^2k^2\psi\mu_f + 2H\cosh(k\alpha z)\cosh(k\alpha H)\alpha^2k^2\psi\mu_f - 2H\alpha^2k^2\psi\mu_f\cosh(k\alpha H) + 2\cosh(k\alpha H)\alpha^2k^2\psi z\mu_f - 2H\sinh(k\alpha z)\sinh(k\alpha H)\alpha^2k^2 + 2H\cosh(k\alpha z)\cosh(k\alpha H)\alpha^2k^2 - 2H\alpha^2k^2\cosh(k\alpha H) + 2\cosh(k\alpha H)\alpha^2k^2z - 2H\sinh(k\alpha H)\alpha k\mu_f - 2\sinh(k\alpha z)\alpha k\psi\mu_f + 2\sinh(k\alpha H)\alpha kz\mu_f - 2\sinh(k\alpha z)\sinh(k\alpha H)\mu_f - 2\sinh(k\alpha z)k\alpha + 2\cosh(k\alpha z)\cosh(k\alpha H)\mu_f - 2\cosh(k\alpha z)\mu_f\big)\Big)\Big/\Big(k^3\alpha l\big(\alpha k\psi\mu_f\cosh(k\alpha H) + k\alpha\cosh(k\alpha H) + \mu_f\sinh(k\alpha H)\big)\Big) \quad \textbf{(18)}$$

> $simplify(\,\textbf{(18)}, 'size'\,)$

$$-\left(\frac{d}{dz}N(z)\right) = \frac{1}{2}\Big(w\Big(\Big(\big(H(H\lambda_rk^2 - 2 + (-H-2\psi)\mu_f)k^2\alpha^2 - 2\mu_f\big)\cosh(k\alpha z) + 2k^2\alpha^2(\psi\mu_f+1)(H-z)\Big)\cosh(k\alpha H) + \Big(\big(-H(H\lambda_rk^2 - 2 + (-H-2\psi)\mu_f)k^2\alpha^2 + 2\mu_f\big)\sinh(k\alpha z) + 2k\alpha\mu_f(H-z)\Big)\sinh(k\alpha H) + 2k\alpha\big((\psi\mu_f+1)\sinh(k\alpha z) + 2\cosh(k\alpha z)\mu_f\big)\Big)\Big)\Big/\Big(lk^3\big(k\alpha(\psi\mu_f+1)\cosh(k\alpha H) + \mu_f\sinh(k\alpha H)\big)\alpha\Big) \quad \textbf{(19)}$$

```
> internal-moment;
```

$$internal - moment \quad \textbf{(20)}$$

> $m_{internal\text{, uni, 1, 2}} := m(z) - n_{\text{uniform, load, }elastic}\cdot l;$

$$m_{internal,\ uni,\ 1,\ 2} := -l\,N(z) + \frac{1}{2}w(H-z)^2 = -l\Bigg(-\frac{1}{2}\Big(\big(\cosh(k\alpha z) + \sinh(k\alpha z)\big)w\Big(H^2\big(\cosh(k\alpha H) - \sinh(k\alpha H)\big)\alpha^2k^4\lambda_r - H^2\big(\cosh(k\alpha H) - \sinh(k\alpha H)\big)\alpha^2k^2\mu_f - 2H\big(\cosh(k\alpha H) - \sinh(k\alpha H)\big)\alpha^2k^2\psi\mu_f \quad \textbf{(21)}$$

$$-2H(\cosh(k\alpha H)-\sinh(k\alpha H))\alpha^2k^2+2\alpha k\psi\mu_f-2(\cosh(k\alpha H)$$

$$-\sinh(k\alpha H))\mu_f+2k\alpha+2\mu_f\Big)\Big)\Big/\Big(k^4 l\Big(\alpha k\psi\mu_f(\cosh(k\alpha H)+\sinh(k\alpha H))$$

$$+(\cosh(k\alpha H)-\sinh(k\alpha H))\alpha k\psi\mu_f+\alpha k(\cosh(k\alpha H)+\sinh(k\alpha H))$$

$$+(\cosh(k\alpha H)-\sinh(k\alpha H))\alpha k+\mu_f(\cosh(k\alpha H)+\sinh(k\alpha H))$$

$$-(\cosh(k\alpha H)-\sinh(k\alpha H))\mu_f\Big)\alpha^2\Big)+\frac{1}{2}\Big(w\Big(H^2(\cosh(k\alpha H)$$

$$+\sinh(k\alpha H))\alpha^2k^4\lambda_r-H^2(\cosh(k\alpha H)+\sinh(k\alpha H))\alpha^2k^2\mu_f-2H(\cosh(k\alpha H)$$

$$+\sinh(k\alpha H))\alpha^2k^2\psi\mu_f-2H(\cosh(k\alpha H)+\sinh(k\alpha H))\alpha^2k^2-2\alpha k\psi\mu_f$$

$$-2\mu_f(\cosh(k\alpha H)+\sinh(k\alpha H))-2k\alpha+2\mu_f\Big)(\cosh(k\alpha z)-\sinh(k\alpha z))\Big)\Big/$$

$$\Big(k^4 l\Big(\alpha k\psi\mu_f(\cosh(k\alpha H)+\sinh(k\alpha H))+(\cosh(k\alpha H)-\sinh(k\alpha H))\alpha k\psi\mu_f$$

$$+\alpha k(\cosh(k\alpha H)+\sinh(k\alpha H))+(\cosh(k\alpha H)-\sinh(k\alpha H))\alpha k$$

$$+\mu_f(\cosh(k\alpha H)+\sinh(k\alpha H))-(\cosh(k\alpha H)-\sinh(k\alpha H))\mu_f\Big)\alpha^2\Big)$$

$$+\frac{1}{2}\frac{w\left(2+k^2(H-z)^2\alpha^2\right)}{k^4 l\alpha^2}\Bigg)+\frac{1}{2}w(H-z)^2$$

> *simplify*(**(21)**, '*size*')

$$-lN(z)+\frac{1}{2}w(H-z)^2=\frac{1}{2}\Big(w\Big(\Big(\Big(H\big(H\lambda_r k^2-2+(-H-2\psi)\mu_f\big)k^2\alpha^2 \qquad \textbf{(22)}$$

$$-2\mu_f\Big)\sinh(k\alpha z)+\left(-2+k^2(k-1)(k+1)(H-z)^2\alpha^2\right)\left(\psi\mu_f+1\right)k\alpha\Big)$$

$$\cosh(k\alpha H)+\Big(\Big(-H\big(H\lambda_r k^2-2+(-H-2\psi)\mu_f\big)k^2\alpha^2+2\mu_f\Big)\cosh(k\alpha z)+\Big(-2$$

$$+k^2(k-1)(k+1)(H-z)^2\alpha^2\Big)\mu_f\Big)\sinh(k\alpha H)+2k\alpha\left(\psi\mu_f+1\right)\cosh(k\alpha z)$$

$$+ 2\sinh(k\alpha z)\,\mu_f\Big)\Big)\Big/\Big(k^4\left(k\alpha\left(\psi\mu_f+1\right)\cosh(k\alpha H)+\mu_f\sinh(k\alpha H)\right)\alpha^2\Big)$$

> $M_1 + M_2 = \frac{1}{2}\Big(\Big(\Big(\Big(H\left(H\lambda_r k^2 - 2 + (-H - 2\psi)\mu_f\right)k^2\alpha^2 - 2\mu_f\Big)\sinh(k\alpha z) + \alpha\Big(-2 + k^2(k-1)(k+1)(H-z)^2\alpha^2\Big)k\left(\mu_f\psi+1\right)\Big)\cosh(k\alpha H) + \Big(\Big(-H\left(H\lambda_r k^2 - 2 + (-H-2\psi)\mu_f\right)k^2\alpha^2 + 2\mu_f\Big)\cosh(k\alpha z) + \mu_f\Big(-2 + k^2(k-1)(k+1)(H-z)^2\alpha^2\Big)\Big)\sinh(k\alpha H) + 2k\alpha\left(\mu_f\psi+1\right)\cosh(k\alpha z) + 2\sinh(k\alpha z)\,\mu_f\Big)w\Big)\Big/\Big(\alpha^2 k^4\left(k\alpha\left(\mu_f\psi + 1\right)\cosh(k\alpha H) + \mu_f\sinh(k\alpha H)\right)\Big)$

$$M_1 + M_2 = \frac{1}{2}\Big(w\Big(\Big(\Big(H\left(H\lambda_r k^2 - 2 + (-H-2\psi)\mu_f\right)k^2\alpha^2 - 2\mu_f\Big)\sinh(k\alpha z) + \Big(-2 + k^2(k-1)(k+1)(H-z)^2\alpha^2\Big)\left(\psi\mu_f+1\right)k\alpha\Big)\cosh(k\alpha H) + \Big(\Big(-H\left(H\lambda_r k^2 - 2 + (-H-2\psi)\mu_f\right)k^2\alpha^2 + 2\mu_f\Big)\cosh(k\alpha z) + \Big(-2 + k^2(k-1)(k+1)(H-z)^2\alpha^2\Big)\mu_f\Big)\sinh(k\alpha H) + 2k\alpha\left(\psi\mu_f+1\right)\cosh(k\alpha z) + 2\sinh(k\alpha z)\,\mu_f\Big)\Big)\Big/\Big(k^4\left(k\alpha\left(\psi\mu_f+1\right)\cosh(k\alpha H) + \mu_f\sinh(k\alpha H)\right)\alpha^2\Big) \quad \textbf{(23)}$$

```
> laterla-deflection;
```

$$laterla - deflection \quad \textbf{(24)}$$

> $restart;$

> $ode_3 := \left(\frac{d^2}{dz^2}x(z)\right) = \frac{1}{E\cdot i_t}\cdot(\,\mathrm{m}(z) - N(z)\cdot l\,);$

$$ode_3 := \frac{d^2}{dz^2}x(z) = \frac{m(z) - N(z)\,l}{E\,i_t} \quad \textbf{(25)}$$

> $ics_3 := x(0) = 0,\ \mathrm{D}(x)(0) = \frac{(m(0) - N(0)\cdot l + \psi\cdot\mathrm{D}(N)(0)\cdot l)}{\left(\frac{\mathrm{E}\cdot\mathrm{i}_t\cdot\alpha^2}{\lambda_r}\right)};$

$$ics_3 := x(0) = 0,\ \mathrm{D}(x)(0) = \frac{(m(0) - N(0)\,l + \psi\,\mathrm{D}(N)(0)\,l)\,\lambda_r}{E\,i_t\,\alpha^2} \quad \textbf{(26)}$$

>

> $N(z) := \frac{1}{2}\Big(w\Big(\cosh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\sinh(k\alpha H) - \cosh(k\alpha z)\,\mu_f\alpha^2 k^2 H^2\sinh(k\alpha H) - \sinh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\cosh(k\alpha H) + \sinh(k\alpha z)\,\mu_f\alpha^2 k^2 H^2\cosh(k\alpha H) + \alpha^3 k^3 H^2\psi\mu_f\cosh(k\alpha H) - 2zHk^2\alpha^2\mu_f\sinh(k\alpha H) + z^2k^3\alpha^3\psi\mu_f\cosh(k\alpha H) + 2k\alpha\cosh(k\alpha H) - 2\cosh(k\alpha z)\,\mu_f\sinh(k\alpha H) - 2\cosh(k\alpha z)\,k\alpha + 2\sinh(k\alpha z)\,\mu_f\cosh(k\alpha H) + 2\sinh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\cosh(k\alpha H)$

$$-2\cosh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\sinh(k\alpha H) - 2zHk^3\alpha^3\psi\mu_f\cosh(k\alpha H) + 2\mu_f\sinh(k\alpha H) - 2\sinh(k\alpha z)\,\mu_f + \alpha^3 k^3 H^2\cosh(k\alpha H) + z^2k^3\alpha^3\cosh(k\alpha H) - 2zHk^3\alpha^3\cosh(k\alpha H) + z^2k^2\alpha^2\mu_f\sinh(k\alpha H) + \mu_f\alpha^2k^2H^2\sinh(k\alpha H) + 2k\alpha\psi\mu_f\cosh(k\alpha H) - 2\cosh(k\alpha z)\,Hk^2\alpha^2\sinh(k\alpha H) - 2\cosh(k\alpha z)\,k\alpha\psi\mu_f + 2\sinh(k\alpha z)\,Hk^2\alpha^2\cosh(k\alpha H)\Big)\Big)\Big/\Big(k^4 l\,\big(\mu_f\sinh(k\alpha H) + k\alpha\cosh(k\alpha H) + k\alpha\psi\mu_f\cosh(k\alpha H)\big)\,\alpha^2\Big);$$

$$N := z \to \frac{1}{2}\Big(w\Big(z^2k^3\alpha^3\cosh(k\alpha H) + \alpha^3k^3H^2\cosh(k\alpha H) - 2zHk^2\alpha^2\mu_f\sinh(k\alpha H) + z^2k^3\alpha^3\psi\mu_f\cosh(k\alpha H) + \cosh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\sinh(k\alpha H) - \cosh(k\alpha z)\,\mu_f\alpha^2k^2H^2\sinh(k\alpha H) - \sinh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\cosh(k\alpha H) + \sinh(k\alpha z)\,\mu_f\alpha^2k^2H^2\cosh(k\alpha H) + \alpha^3k^3H^2\psi\mu_f\cosh(k\alpha H) - 2\cosh(k\alpha z)\,\mu_f\sinh(k\alpha H) - 2\cosh(k\alpha z)\,k\alpha + 2\sinh(k\alpha z)\,\mu_f\cosh(k\alpha H) - 2\sinh(k\alpha z)\,\mu_f + 2k\alpha\cosh(k\alpha H) + 2\mu_f\sinh(k\alpha H) + 2\sinh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\cosh(k\alpha H) - 2\cosh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\sinh(k\alpha H) - 2zHk^3\alpha^3\psi\mu_f\cosh(k\alpha H) + \mu_f\alpha^2k^2H^2\sinh(k\alpha H) + 2k\alpha\psi\mu_f\cosh(k\alpha H) - 2zHk^3\alpha^3\cosh(k\alpha H) + z^2k^2\alpha^2\mu_f\sinh(k\alpha H) - 2\cosh(k\alpha z)\,Hk^2\alpha^2\sinh(k\alpha H) - 2\cosh(k\alpha z)\,k\alpha\psi\mu_f + 2\sinh(k\alpha z)\,Hk^2\alpha^2\cosh(k\alpha H)\Big)\Big)\Big/\Big(k^4 l\,\big(\mu_f\sinh(k\alpha H) + k\alpha\cosh(k\alpha H) + k\alpha\psi\mu_f\cosh(k\alpha H)\big)\,\alpha^2\Big) \qquad \textbf{(27)}$$

> $m(z) := \frac{w\cdot(H-z)^2}{2};$

$$m := z \to \frac{1}{2}\,w\,(H-z)^2 \qquad \textbf{(28)}$$

>

> $convert(dsolve(ode_3, x(z)), trig);$

$$x(z) = \iint \frac{1}{2}\Big(w\Big(-\alpha^3k^3H^2\cosh(k\alpha H) - z^2k^3\alpha^3\cosh(k\alpha H) + 2H\psi\mu_f\alpha^2k^2\sinh(H\alpha k - \alpha k z) - \alpha^3k^3H^2\psi\mu_f\cosh(k\alpha H) + 2zHk^2\alpha^2\mu_f\sinh(k\alpha H) - z^2k^3\alpha^3\psi\mu_f\cosh(k\alpha H) - \lambda_r H^2\alpha^2k^4\sinh(H\alpha k - \alpha k z) + H^2\mu_f\alpha^2k^2\sinh(H\alpha k - \alpha k z) - 2H\cosh(k\alpha H)\,\alpha^3k^5z + H^2\sinh(k\alpha H)\,\alpha^2k^4\mu_f + \sinh(k\alpha H)\,\alpha^2k^4z^2\mu_f - 2H\cosh(k\alpha H)\,\alpha^3k^5\psi z\mu_f + 2H\alpha^2k^2\sinh(H\alpha k - \alpha k z) \qquad \textbf{(29)}$$

$$+H^2\cosh(k\alpha H)\,\alpha^3 k^5\psi\mu_f+\cosh(k\alpha H)\,\alpha^3 k^5\psi z^2\mu_f-2H\sinh(k\alpha H)\,\alpha^2 k^4 z\mu_f$$
$$+2\cosh(k\alpha z)\,k\alpha-2k\alpha\cosh(k\alpha H)+2\sinh(k\alpha z)\,\mu_f-2\mu_f\sinh(k\alpha H)$$
$$+2zHk^3\alpha^3\psi\mu_f\cosh(k\alpha H)+2\mu_f\sinh(H\alpha k-\alpha kz)+2zHk^3\alpha^3\cosh(k\alpha H)$$
$$-z^2k^2\alpha^2\mu_f\sinh(k\alpha H)-\mu_f\alpha^2k^2H^2\sinh(k\alpha H)-2k\alpha\psi\mu_f\cosh(k\alpha H)$$
$$+2\cosh(k\alpha z)\,k\alpha\psi\mu_f+H^2\cosh(k\alpha H)\,\alpha^3k^5+\cosh(k\alpha H)\,\alpha^3k^5z^2\big)\big)\Big/$$
$$\left(E\alpha^2k^4 i_t\left(\mu_f\sinh(k\alpha H)+k\alpha\cosh(k\alpha H)+k\alpha\psi\mu_f\cosh(k\alpha H)\right)\right)\,\mathrm{d}z\,\mathrm{d}z+\_C1\,z$$
$$+\_C2$$

> *simplify*(*combine*(*convert*(*value*(*dsolve*({ode_3, ics_3}, $x(z)$)), *trig*), *trig*), *size*);

$$x(z)=-\frac{1}{2}\Bigg(w\Bigg(\Big(H\big(\lambda_r Hk^2-2+(-H-2\psi)\,\mu_f\big)k^2\alpha^2-2\mu_f\Big)\sinh(\alpha k(H-z))+z\alpha\Bigg(\tag{30}$$
$$-\frac{1}{2}z(k+1)\left(H^2-\frac{2}{3}zH+\frac{1}{6}z^2\right)k^4(k-1)(\psi\mu_f+1)\,\alpha^4+\Big(\lambda_r H^2\big(\psi\lambda_r-\psi\mu_f$$
$$-1\big)k^4+2k^2\lambda_r H^2+(\psi\mu_f+1)z-H\big(2+(H+2\psi)\,\mu_f\big)\Big)k^2\alpha^2+2\lambda_r k^2-2\mu_f\Bigg)$$
$$k\cosh(k\alpha H)+\Bigg(-\frac{1}{2}\mu_f z^2(k+1)\left(H^2-\frac{2}{3}zH+\frac{1}{6}z^2\right)k^4(k-1)\,\alpha^4$$
$$+k^2\Big(\lambda_r H^2 z(\lambda_r-\mu_f)k^4-\lambda_r H(H+2z)k^2+z^2\mu_f+H\big(2+(H+2\psi)\,\mu_f\big)\Big)\alpha^2$$
$$+2\mu_f\Bigg)\sinh(k\alpha H)-2k\alpha(\psi\mu_f+1)\cosh(k\alpha z)-2\sinh(k\alpha z)\,\mu_f-2\alpha k\big(k^2 z\lambda_r$$
$$-\psi\mu_f-z\mu_f-1\big)\Bigg)\Bigg)\Bigg/\Big(E\alpha^4\big(k\alpha(\psi\mu_f+1)\cosh(k\alpha H)+\mu_f\sinh(k\alpha H)\big)k^6 i_t\Big)$$

>

> **prove-validity-of-the-equations;**

$$prove-validity-of-the-equations \tag{31}$$

> $N_{elastic,\,uniform}(z) := \frac{1}{2}\Big(w\Big(\cosh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\sinh(k\alpha H)$

$$-\cosh(k\alpha z)\,\mu_f\alpha^2k^2H^2\sinh(k\alpha H)-\sinh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\cosh(k\alpha H)$$
$$+\sinh(k\alpha z)\,\mu_f\alpha^2k^2H^2\cosh(k\alpha H)+\alpha^3k^3H^2\psi\mu_f\cosh(k\alpha H)$$
$$-2zHk^2\alpha^2\mu_f\sinh(k\alpha H)+z^2k^3\alpha^3\psi\mu_f\cosh(k\alpha H)+2k\alpha\cosh(k\alpha H)$$
$$-2\cosh(k\alpha z)\,\mu_f\sinh(k\alpha H)-2\cosh(k\alpha z)\,k\alpha+2\sinh(k\alpha z)\,\mu_f\cosh(k\alpha H)$$
$$+2\sinh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\cosh(k\alpha H)-2\cosh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\sinh(k\alpha H)$$
$$-2zHk^3\alpha^3\psi\mu_f\cosh(k\alpha H)+2\mu_f\sinh(k\alpha H)-2\sinh(k\alpha z)\,\mu_f$$
$$+\alpha^3k^3H^2\cosh(k\alpha H)+z^2k^3\alpha^3\cosh(k\alpha H)-2zHk^3\alpha^3\cosh(k\alpha H)$$
$$+z^2k^2\alpha^2\mu_f\sinh(k\alpha H)+\mu_f\alpha^2k^2H^2\sinh(k\alpha H)+2k\alpha\psi\mu_f\cosh(k\alpha H)$$

$$-2\cosh(k\alpha z)\,H k^2\alpha^2\sinh(k\alpha H)-2\cosh(k\alpha z)\,k\alpha\psi\mu_f$$
$$+2\sinh(k\alpha z)\,H k^2\alpha^2\cosh(k\alpha H)\Big)\Big)\Big/\Big(k^4 l\,\big(\mu_f\sinh(k\alpha H)+k\alpha\cosh(k\alpha H)$$
$$+k\alpha\psi\mu_f\cosh(k\alpha H)\big)\,\alpha^2\Big);$$

$$N_{elastic,\,uniform} := z\rightarrow \frac{1}{2}\,\Big(w\,\Big(\cosh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\sinh(k\alpha H)$$
$$-\cosh(k\alpha z)\,\mu_f\alpha^2 k^2 H^2\sinh(k\alpha H)-\sinh(k\alpha z)\,k^4\lambda_r H^2\alpha^2\cosh(k\alpha H)$$
$$+\sinh(k\alpha z)\,\mu_f\alpha^2 k^2 H^2\cosh(k\alpha H)-2\,z H k^2\alpha^2\mu_f\sinh(k\alpha H)$$
$$+z^2 k^3\alpha^3\psi\mu_f\cosh(k\alpha H)+\alpha^3 k^3 H^2\psi\mu_f\cosh(k\alpha H)-2\sinh(k\alpha z)\,\mu_f$$
$$-2\cosh(k\alpha z)\,k\alpha-2\cosh(k\alpha z)\,\mu_f\sinh(k\alpha H)+2\sinh(k\alpha z)\,\mu_f\cosh(k\alpha H)$$
$$+2\,k\alpha\cosh(k\alpha H)-2\,z H k^3\alpha^3\cosh(k\alpha H)+z^2 k^2\alpha^2\mu_f\sinh(k\alpha H)$$
$$-2\cosh(k\alpha z)\,H k^2\alpha^2\sinh(k\alpha H)-2\cosh(k\alpha z)\,k\alpha\psi\mu_f$$
$$+2\sinh(k\alpha z)\,H k^2\alpha^2\cosh(k\alpha H)+2\,\mu_f\sinh(k\alpha H)+z^2 k^3\alpha^3\cosh(k\alpha H)$$
$$+\alpha^3 k^3 H^2\cosh(k\alpha H)+\mu_f\alpha^2 k^2 H^2\sinh(k\alpha H)+2\,k\alpha\psi\mu_f\cosh(k\alpha H)$$
$$+2\sinh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\cosh(k\alpha H)-2\cosh(k\alpha z)\,H\psi\mu_f k^2\alpha^2\sinh(k\alpha H)$$
$$-2\,z H k^3\alpha^3\psi\mu_f\cosh(k\alpha H)\Big)\Big)\Big/\Big(k^4 l\,\big(\mu_f\sinh(k\alpha H)+k\alpha\cosh(k\alpha H)$$
$$+k\alpha\psi\mu_f\cosh(k\alpha H)\big)\,\alpha^2\Big) \qquad \textbf{(32)}$$

> $N_{elastic,\ \mathrm{uniform,\ infinite}} := combine\big(limit\big(N_{elastic,\,uniform}(z),\ \{\lambda_r=0,\ \mu_f=0\}\big)\big);$

$$N_{elastic,\,uniform,\,infinite} := \frac{1}{2}\,\frac{1}{\cosh(k\alpha H)\,k^4 l\,\alpha^2}\,\big(w H^2\cosh(k\alpha H)\,\alpha^2 k^2$$
$$-2\,w H\cosh(k\alpha H)\,\alpha^2 k^2 z+w\cosh(k\alpha H)\,\alpha^2 k^2 z^2-2\,H\sinh(H\alpha k-\alpha k z)\,\alpha k w$$
$$+2\,w\cosh(k\alpha H)-2\,w\cosh(k\alpha z)\big) \qquad \textbf{(33)}$$

> $N_{rigid} := \frac{1}{2}\,\frac{1}{\cosh(k\alpha H)\,k^4 l\,\alpha^2}\big(-2\,w H k\alpha\sinh(k\alpha H-k\alpha z)+2\,w\cosh(k\alpha H)$
> $+w\cosh(k\alpha H)\,k^2\alpha^2 H^2-2\,w\cosh(k\alpha H)\,k^2\alpha^2 H z+w\cosh(k\alpha H)\,k^2\alpha^2 z^2$
> $-2\,w\cosh(k\alpha z)\big);$

$$N_{rigid} := \frac{1}{2}\,\frac{1}{\cosh(k\alpha H)\,k^4 l\,\alpha^2}\,\big(w H^2\cosh(k\alpha H)\,\alpha^2 k^2-2\,w H\cosh(k\alpha H)\,\alpha^2 k^2 z$$
$$+w\cosh(k\alpha H)\,\alpha^2 k^2 z^2-2\,H\sinh(H\alpha k-\alpha k z)\,\alpha k w+2\,w\cosh(k\alpha H)$$
$$-2\,w\cosh(k\alpha z)\big) \qquad \textbf{(34)}$$

> $evalb(N_{elastic,\ \mathrm{uniform,\ infinite}}=N_{rigid});$

true **(35)**

> $x_{elastic,\,uni,\,l}(z) := -\frac{1}{2}\Bigg(\Bigg(\Big(k^2\big(\lambda_r H k^2 - 2 + (-H - 2\psi)\mu_f\big) H\alpha^2 - 2\mu_f\Big)\sinh(\alpha k (H - z)) + k z \alpha\Big(-\frac{1}{2} k^4 z (k+1)\big(H^2 - \frac{2}{3} z H + \frac{1}{6} z^2\big)(\psi\mu_f + 1)(k-1)\alpha^4 + k^2\big(\lambda_r H^2(\psi\lambda_r - \psi\mu_f - 1) k^4 + 2 k^2 \lambda_r H^2 + (\psi\mu_f + 1) z - (2 + (H + 2\psi)\mu_f) H\big)\alpha^2 + 2\lambda_r k^2 - 2\mu_f\Big)\cosh(k\alpha H) + \Big(-\frac{1}{2} k^4 z^2 (k+1)\big(H^2 - \frac{2}{3} z H + \frac{1}{6} z^2\big)(k-1)\mu_f \alpha^4 + k^2\big(\lambda_r H^2 z(\lambda_r - \mu_f) k^4 - \lambda_r H (H + 2z) k^2 + z^2\mu_f + (2 + (H + 2\psi)\mu_f) H\big)\alpha^2 + 2\mu_f\Big)\sinh(k\alpha H) - 2k\alpha(\psi\mu_f + 1)\cosh(k\alpha z) - 2\sinh(k\alpha z)\mu_f - 2k\alpha\big(k^2 z\lambda_r - \psi\mu_f - z\mu_f - 1\big)\Bigg) w\Bigg) \Big/ \Big(\big(k\alpha(\psi\mu_f + 1)\cosh(k\alpha H) + \mu_f \sinh(k\alpha H)\big) k^6 i_t \alpha^4 E\Big);$

$$x_{elastic,\,uni,\,l} := z \rightarrow -\frac{1}{2}\Bigg(\Bigg(\Big(k^2\big(\lambda_r H k^2 - 2 + (-H - 2\psi)\mu_f\big) H\alpha^2 - 2\mu_f\Big)\sinh(k\alpha (H - z)) + k\alpha z\Big(-\frac{1}{2} k^4 z (k+1)\big(H^2 - \frac{2}{3} z H + \frac{1}{6} z^2\big)(\psi\mu_f + 1)(k-1)\alpha^4 + k^2\big(\lambda_r H^2(\psi\lambda_r - \psi\mu_f - 1) k^4 + 2 k^2 \lambda_r H^2 + (\psi\mu_f + 1) z - (2 + (H + 2\psi)\mu_f) H\big)\alpha^2 + 2 k^2\lambda_r - 2\mu_f\Big)\cosh(k\alpha H) + \Big(-\frac{1}{2} k^4 z^2 (k+1)\big(H^2 - \frac{2}{3} z H + \frac{1}{6} z^2\big)(k-1)\mu_f \alpha^4 + k^2\big(\lambda_r H^2 z(\lambda_r - \mu_f) k^4 - \lambda_r H (H + 2z) k^2 + z^2\mu_f + (2 + (H + 2\psi)\mu_f) H\big)\alpha^2 + 2\mu_f\Big)\sinh(k\alpha H) - 2k\alpha(\psi\mu_f + 1)\cosh(k\alpha z) - 2\sinh(k\alpha z)\mu_f - 2k\alpha\big(k^2 z\lambda_r - \psi\mu_f - z\mu_f - 1\big)\Bigg) w\Bigg) \Big/ \Big(\big(k\alpha(\psi\mu_f + 1)\cosh(k\alpha H) + \mu_f \sinh(k\alpha H)\big) k^6 i_t \alpha^4 E\Big) \quad \textbf{(36)}$$

> $x_{elastic,\,uniform,\,infinite} := combine\big(limit\big(x_{elastic,\,uni,\,l}(z), \{\lambda_r = 0, \mu_f = 0, \lambda_v = 0\}\big)\big);$

$$x_{elastic,\,uniform,\,infinite} := \frac{1}{24}\,\frac{1}{k^6\alpha^4 i_t E\cosh(k\alpha H)}\big(6H^2\cosh(k\alpha H)\alpha^4 k^6 w z^2 - 4H\cosh(k\alpha H)\alpha^4 k^6 w z^3 + \cosh(k\alpha H)\alpha^4 k^6 w z^4 - 6H^2\cosh(k\alpha H)\alpha^4 k^4 w z^2 + 4H\cosh(k\alpha H)\alpha^4 k^4 w z^3 - \cosh(k\alpha H)\alpha^4 k^4 w z^4 + 24 w H\cosh(k\alpha H)\alpha^2 k^2 z - 12 w\cosh(k\alpha H)\alpha^2 k^2 z^2 + 24H\sinh(H\alpha k - \alpha k z)\alpha k w - 24H\sinh(k\alpha H)\alpha k w + 24 w\cosh(k\alpha z) - 24 w\big) \quad \textbf{(37)}$$

> $x_{rigid,\,uniform} := \frac{1}{24}\,\frac{1}{k^6 E i_t \alpha^4\cosh(k\alpha H)}\big(24 H w k\alpha\sinh(k\alpha H - k\alpha z) + 24 w\cosh(k\alpha z) - 12 z^2 w k^2\alpha^2\cosh(k\alpha H) + 6\alpha^4 k^6 w H^2 z^2\cosh(k\alpha H) - 6 w\alpha^4 H^2 k^4 z^2\cosh(k\alpha H) - 4 z^3 H w k^6\alpha^4\cosh(k\alpha H) + 4 w\alpha^4 H z^3 k^4\cosh(k\alpha H) + z^4 w k^6\alpha^4\cosh(k\alpha H)$

$$- w\,\alpha^4 z^4 k^4 \cosh(k\,\alpha\,H) + 24\,z\,H\,w\,k^2\,\alpha^2 \cosh(k\,\alpha\,H) - 24\,w\,k\,\alpha\,H \sinh(k\,\alpha\,H) - 24\,w\Big)$$

;

$$x_{rigid,\ uniform} := \frac{1}{24}\,\frac{1}{k^6\,\alpha^4\,i_t\,E\cosh(k\,\alpha\,H)}\Big(6\,H^2\cosh(k\,\alpha\,H)\,\alpha^4\,k^6\,w\,z^2 - 4\,H\cosh(k\,\alpha\,H)\,\alpha^4\,k^6\,w\,z^3 + \cosh(k\,\alpha\,H)\,\alpha^4\,k^6\,w\,z^4 - 6\,H^2\cosh(k\,\alpha\,H)\,\alpha^4\,k^4\,w\,z^2 + 4\,H\cosh(k\,\alpha\,H)\,\alpha^4\,k^4\,w\,z^3 - \cosh(k\,\alpha\,H)\,\alpha^4\,k^4\,w\,z^4 + 24\,w\,H\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\,z - 12\,w\cosh(k\,\alpha\,H)\,\alpha^2\,k^2\,z^2 + 24\,H\sinh(H\,\alpha\,k - \alpha\,k\,z)\,\alpha\,k\,w - 24\,H\sinh(k\,\alpha\,H)\,\alpha\,k\,w + 24\,w\cosh(k\,\alpha\,z) - 24\,w\Big) \tag{38}$$

> $evalb\left(\mathrm{x}_{elastic,\ \mathrm{uniform},\ infinite} = x_{rigid,\ uniform}\right);$

$$true \tag{39}$$

13 Appendix 4:

Output of MatrixFrame program for analysis of shear walls on rigid foundation

Job Name	Rigid foundation	Job Number	1
Part Description	standard dimension	Structural Engineer	M.SH
Client		Units	m, kN, kNm
File	C:\Users\Acer\Dropbox\courses\gaduation work\working maple\worked_example_rigid_f.mxe		

Structure Info

Project Type	Nodes	Members	Supports	Sections	Loads Cases	Loads Comb.
2D-Frame	82	100	2	4	2	6

Members

Member	Node B	Release B	Release E	Node E	Section	X-B	Z-B	X-E	Z-E	Length
S2	K2	NVM	NVM	K3	P12	0.000	-60.000	2.500	-60.000	2.500
S3	K3	NVM	NVM	K4	P1	2.500	-60.000	5.000	-60.000	2.500
S4	K4	NVM	NVM	K5	P12	5.000	-60.000	8.500	-60.000	3.500
S8	K7	NVM	NVM	K2	P4	0.000	-57.000	0.000	-60.000	3.000
S11	K10	NVM	NVM	K5	P3	8.500	-57.000	8.500	-60.000	3.000
S13	K7	NVM	NVM	K11	P12	0.000	-57.000	2.500	-57.000	2.500
S14	K11	NVM	NVM	K12	P1	2.500	-57.000	5.000	-57.000	2.500
S15	K12	NVM	NVM	K10	P12	5.000	-57.000	8.500	-57.000	3.500
S16	K13	NVM	NVM	K14	P12	0.000	-54.000	2.500	-54.000	2.500
S18	K13	NVM	NVM	K7	P4	0.000	-54.000	0.000	-57.000	3.000
S19	K14	NVM	NVM	K15	P1	2.500	-54.000	5.000	-54.000	2.500
S20	K15	NVM	NVM	K16	P12	5.000	-54.000	8.500	-54.000	3.500
S21	K16	NVM	NVM	K10	P3	8.500	-54.000	8.500	-57.000	3.000
S23	K17	NVM	NVM	K18	P12	0.000	-51.000	2.500	-51.000	2.500
S25	K17	NVM	NVM	K13	P4	0.000	-51.000	0.000	-54.000	3.000
S26	K18	NVM	NVM	K19	P1	2.500	-51.000	5.000	-51.000	2.500
S27	K19	NVM	NVM	K20	P12	5.000	-51.000	8.500	-51.000	3.500
S28	K20	NVM	NVM	K16	P3	8.500	-51.000	8.500	-54.000	3.000
S30	K21	NVM	NVM	K22	P12	0.000	-48.000	2.500	-48.000	2.500
S32	K21	NVM	NVM	K17	P4	0.000	-48.000	0.000	-51.000	3.000
S33	K22	NVM	NVM	K23	P1	2.500	-48.000	5.000	-48.000	2.500
S34	K23	NVM	NVM	K24	P12	5.000	-48.000	8.500	-48.000	3.500
S35	K24	NVM	NVM	K20	P3	8.500	-48.000	8.500	-51.000	3.000
S37	K25	NVM	NVM	K26	P12	0.000	-45.000	2.500	-45.000	2.500
S39	K25	NVM	NVM	K21	P4	0.000	-45.000	0.000	-48.000	3.000
S40	K26	NVM	NVM	K27	P1	2.500	-45.000	5.000	-45.000	2.500
S41	K27	NVM	NVM	K28	P12	5.000	-45.000	8.500	-45.000	3.500
S42	K28	NVM	NVM	K24	P3	8.500	-45.000	8.500	-48.000	3.000
S44	K29	NVM	NVM	K30	P12	0.000	-42.000	2.500	-42.000	2.500
S46	K29	NVM	NVM	K25	P4	0.000	-42.000	0.000	-45.000	3.000
S47	K30	NVM	NVM	K31	P1	2.500	-42.000	5.000	-42.000	2.500
S48	K31	NVM	NVM	K32	P12	5.000	-42.000	8.500	-42.000	3.500
S49	K32	NVM	NVM	K28	P3	8.500	-42.000	8.500	-45.000	3.000
S51	K33	NVM	NVM	K34	P12	0.000	-39.000	2.500	-39.000	2.500
S53	K33	NVM	NVM	K29	P4	0.000	-39.000	0.000	-42.000	3.000
S54	K34	NVM	NVM	K35	P1	2.500	-39.000	5.000	-39.000	2.500
S55	K35	NVM	NVM	K36	P12	5.000	-39.000	8.500	-39.000	3.500
S56	K36	NVM	NVM	K32	P3	8.500	-39.000	8.500	-42.000	3.000
S58	K37	NVM	NVM	K38	P12	0.000	-36.000	2.500	-36.000	2.500
S60	K37	NVM	NVM	K33	P4	0.000	-36.000	0.000	-39.000	3.000
S61	K38	NVM	NVM	K39	P1	2.500	-36.000	5.000	-36.000	2.500
S62	K39	NVM	NVM	K40	P12	5.000	-36.000	8.500	-36.000	3.500
S63	K40	NVM	NVM	K36	P3	8.500	-36.000	8.500	-39.000	3.000
S65	K41	NVM	NVM	K42	P12	0.000	-33.000	2.500	-33.000	2.500
S67	K41	NVM	NVM	K37	P4	0.000	-33.000	0.000	-36.000	3.000
S68	K42	NVM	NVM	K43	P1	2.500	-33.000	5.000	-33.000	2.500
S69	K43	NVM	NVM	K44	P12	5.000	-33.000	8.500	-33.000	3.500
S70	K44	NVM	NVM	K40	P3	8.500	-33.000	8.500	-36.000	3.000
S72	K45	NVM	NVM	K46	P12	0.000	-30.000	2.500	-30.000	2.500
S74	K45	NVM	NVM	K41	P4	0.000	-30.000	0.000	-33.000	3.000
S75	K46	NVM	NVM	K47	P1	2.500	-30.000	5.000	-30.000	2.500
S76	K47	NVM	NVM	K48	P12	5.000	-30.000	8.500	-30.000	3.500
S77	K48	NVM	NVM	K44	P3	8.500	-30.000	8.500	-33.000	3.000
S79	K49	NVM	NVM	K50	P12	0.000	-27.000	2.500	-27.000	2.500
S81	K49	NVM	NVM	K45	P4	0.000	-27.000	0.000	-30.000	3.000
S82	K50	NVM	NVM	K51	P1	2.500	-27.000	5.000	-27.000	2.500
S83	K51	NVM	NVM	K52	P12	5.000	-27.000	8.500	-27.000	3.500
S84	K52	NVM	NVM	K48	P3	8.500	-27.000	8.500	-30.000	3.000
S86	K53	NVM	NVM	K54	P12	0.000	-24.000	2.500	-24.000	2.500
S88	K53	NVM	NVM	K49	P4	0.000	-24.000	0.000	-27.000	3.000
S89	K54	NVM	NVM	K55	P1	2.500	-24.000	5.000	-24.000	2.500
S90	K55	NVM	NVM	K56	P12	5.000	-24.000	8.500	-24.000	3.500
S91	K56	NVM	NVM	K52	P3	8.500	-24.000	8.500	-27.000	3.000
S93	K57	NVM	NVM	K58	P12	0.000	-21.000	2.500	-21.000	2.500
S95	K57	NVM	NVM	K53	P4	0.000	-21.000	0.000	-24.000	3.000
S96	K58	NVM	NVM	K59	P1	2.500	-21.000	5.000	-21.000	2.500

Student version

Member	Node B	B	Release E	Node E	Section	X-B	Z-B	X-E	Z-E	Length
S97	K59	NVM	NVM	K60	P12	5.000	-21.000	8.500	-21.000	3.500
S98	K60	NVM	NVM	K56	P3	8.500	-21.000	8.500	-24.000	3.000
S100	K61	NVM	NVM	K62	P12	0.000	-18.000	2.500	-18.000	2.500
S102	K61	NVM	NVM	K57	P4	0.000	-18.000	0.000	-21.000	3.000
S103	K62	NVM	NVM	K63	P1	2.500	-18.000	5.000	-18.000	2.500
S104	K63	NVM	NVM	K64	P12	5.000	-18.000	8.500	-18.000	3.500
S105	K64	NVM	NVM	K60	P3	8.500	-18.000	8.500	-21.000	3.000
S107	K65	NVM	NVM	K66	P12	0.000	-15.000	2.500	-15.000	2.500
S109	K65	NVM	NVM	K61	P4	0.000	-15.000	0.000	-18.000	3.000
S110	K66	NVM	NVM	K67	P1	2.500	-15.000	5.000	-15.000	2.500
S111	K67	NVM	NVM	K68	P12	5.000	-15.000	8.500	-15.000	3.500
S112	K68	NVM	NVM	K64	P3	8.500	-15.000	8.500	-18.000	3.000
S114	K69	NVM	NVM	K70	P12	0.000	-12.000	2.500	-12.000	2.500
S116	K69	NVM	NVM	K65	P4	0.000	-12.000	0.000	-15.000	3.000
S117	K70	NVM	NVM	K71	P1	2.500	-12.000	5.000	-12.000	2.500
S118	K71	NVM	NVM	K72	P12	5.000	-12.000	8.500	-12.000	3.500
S119	K72	NVM	NVM	K68	P3	8.500	-12.000	8.500	-15.000	3.000
S121	K73	NVM	NVM	K74	P12	0.000	-9.000	2.500	-9.000	2.500
S123	K73	NVM	NVM	K69	P4	0.000	-9.000	0.000	-12.000	3.000
S124	K74	NVM	NVM	K75	P1	2.500	-9.000	5.000	-9.000	2.500
S125	K75	NVM	NVM	K76	P12	5.000	-9.000	8.500	-9.000	3.500
S126	K76	NVM	NVM	K72	P3	8.500	-9.000	8.500	-12.000	3.000
S128	K77	NVM	NVM	K78	P12	0.000	-6.000	2.500	-6.000	2.500
S130	K77	NVM	NVM	K73	P4	0.000	-6.000	0.000	-9.000	3.000
S131	K78	NVM	NVM	K79	P1	2.500	-6.000	5.000	-6.000	2.500
S132	K79	NVM	NVM	K80	P12	5.000	-6.000	8.500	-6.000	3.500
S135	K81	NVM	NVM	K82	P12	0.000	-3.000	2.500	-3.000	2.500
S136	K1	NVM	NVM	K81	P4	0.000	0.000	0.000	-3.000	3.000
S137	K81	NVM	NVM	K77	P4	0.000	-3.000	0.000	-6.000	3.000
S138	K82	NVM	NVM	K83	P1	2.500	-3.000	5.000	-3.000	2.500
S139	K83	NVM	NVM	K84	P12	5.000	-3.000	8.500	-3.000	3.500
S140	K84	NVM	NVM	K80	P3	8.500	-3.000	8.500	-6.000	3.000
S141	K6	NVM	NVM	K84	P3	8.500	0.000	8.500	-3.000	3.000
S142	K80	NVM	NVM	K76	P3	8.500	-6.000	8.500	-9.000	3.000
-	-	-	-	-	-	m	m	m	m	m

Sections

Section	Section Name	Area	Iy	Material	Angle
P1	R300x400	1.2000e-01	1.6000e-03	C45/55	0
P3	R300x7000	2.1000e+00	8.5750e+00	C45/55	0
P4	R300x5000	1.5000e+00	3.1250e+00	C45/55	0
P12		2.4000e+01	4.1600e+00	Mat. 3	0
-	-	m2	m4	-	°

Section Shapes

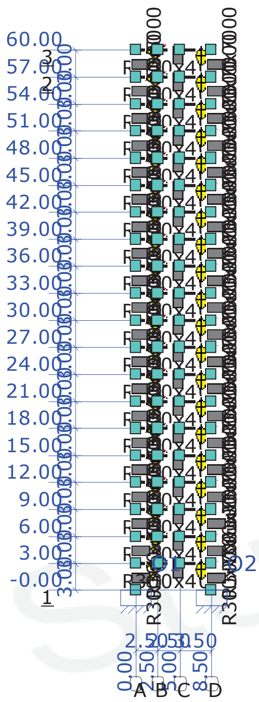
Section	Tapered	hB	hE	tf	tw	tf2	B	b1	b2	Castellate	Height
P1	No	0.400	0.400	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
P3	No	7.000	7.000	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
P4	No	5.000	5.000	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
-	-	m	m	m	m	m	m	m	m	-	m

Materials

Material Name	Density	Youngs mod.	Lin. Exp.
C45/55	24.00	3.6000e+07	10.0000e-06
Mat. 3	0.00	3.6000e+15	10.0000e-06
-	kN/m3	kN/m2	C°m

Supports

Support	Node	X	Z	Yr	AngleYr
O1	K1	fixed	fixed	fixed	0
O2	K6	fixed	fixed	fixed	0
-	-	kN/m	kN/m	kNmrad	°



Pic. Geometrie: Doorgaande Ligger

Loads Cases

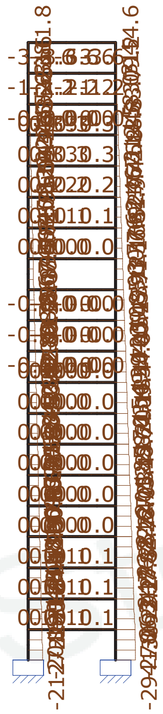
Type	Value Begin	Value End	Dist. Begin	Dist. End	Direction	Member/Node
B.G.2: Wind load						
q	17.00	17.00	0.000	3.000(L)	Z'	S8,S18,S25,S32, S39,S46,S53,S60, S67,S74,S81,S88, S95,S102,S109, S116,S123,S130, S136-S137
Sum of loads	**X:**	**1020,00**	**kN Z: 0.00**	**kN**		
-	**-**	**-**	**m**	**m**	**- -**	

Persistent Loads Combinations

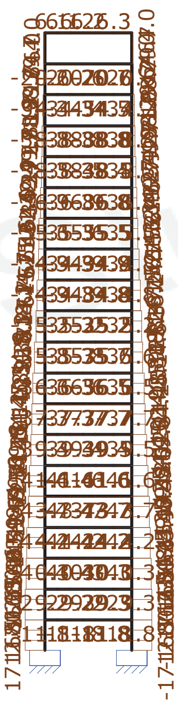
L.C.	Description	Pe.C.1	Pe.C.2
B.G.1	Permanent actions	1.20	1.35
B.G.2	Wind load	1.50	-

Analysis Assumptions

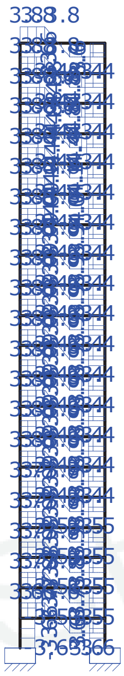
Linear Elastic Analysis performed



Pic. B.G.1: Permanent actions Normaalkracht (Nx)

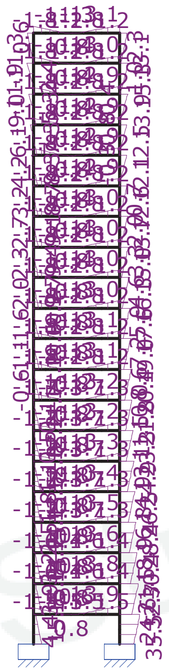


Pic. B.G.2: Wind load Normaalkracht (Nx)

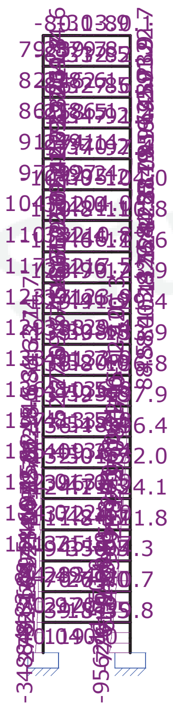


Pic. B.G.1: Permanent actions Dwarskracht (Vz)

Pic. B.G.2: Wind load Dwarskracht (Vz)



Pic. B.G.1: Permanent actions Momenten (My)



Pic. B.G.2: Wind load Momenten (My)

L.C. Extreme Member Forces

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S2	B.G.2	**239.68**	0.00	0.000	**79.79**	0.000	0.000	T	**6.14**	**-63.96**	**-63.96**	**-63.96**
S3	B.G.2	**79.79**	0.00	0.000	**-80.10**	1.248	0.000	T	**6.25**	**-63.96**	**-63.96**	**-63.96**
S4	B.G.2	**-80.10**	0.00	0.000	**-303.94**	0.000	0.000	T	**6.27**	**-63.96**	**-63.96**	**-63.96**
S8	B.G.2	**144.59**	0.00	0.000	**239.68**	0.000	0.000	T	**63.96**	**57.20**	**57.20**	**6.20**
S11	B.G.2	**322.66**	0.00	0.000	**303.94**	0.000	0.000	C	**-63.96**	**-6.24**	**-6.24**	**-6.24**

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S13	B.G.2	**246.55**	0.00	0.000	**82.12**	0.000	0.000	C	**-20.61**	**-65.77**	**-65.77**	**-65.77**
S14	B.G.2	**82.12**	0.00	0.000	**-82.31**	1.249	0.000	C	**-20.69**	**-65.77**	**-65.77**	**-65.77**
S15	B.G.2	**-82.31**	0.00	0.000	**-312.51**	0.000	0.000	C	**-20.61**	**-65.77**	**-65.77**	**-65.77**
S16	B.G.2	**258.50**	0.00	0.000	**86.14**	0.000	0.000	C	**-34.43**	**-68.95**	**-68.95**	**-68.95**
S18	B.G.2	**204.79**	0.00	0.000	**391.13**	0.000	0.000	T	**129.73**	**87.62**	**87.62**	**36.62**
S19	B.G.2	**86.14**	0.00	0.000	**-86.23**	1.249	0.000	C	**-34.54**	**-68.95**	**-68.95**	**-68.95**
S20	B.G.2	**-86.23**	0.00	0.000	**-327.54**	0.000	0.000	C	**-34.69**	**-68.95**	**-68.95**	**-68.95**
S21	B.G.2	**591.90**	0.00	0.000	**635.18**	0.000	0.000	C	**-129.73**	**14.42**	**14.42**	**14.42**
S23	B.G.2	**274.10**	0.00	0.000	**91.36**	0.000	0.000	C	**-38.78**	**-73.10**	**-73.10**	**-73.10**
S25	B.G.2	**227.59**	0.00	0.000	**463.29**	0.000	0.000	T	**198.67**	**104.06**	**104.06**	**53.06**
S26	B.G.2	**91.36**	0.00	0.000	**-91.38**	1.250	0.000	C	**-38.80**	**-73.09**	**-73.09**	**-73.09**
S27	B.G.2	**-91.38**	0.00	0.000	**-347.21**	0.000	0.000	C	**-38.85**	**-73.09**	**-73.09**	**-73.09**
S28	B.G.2	**772.66**	0.00	0.000	**919.44**	0.000	0.000	C	**-198.67**	**48.93**	**48.93**	**48.93**
S30	B.G.2	**292.24**	0.00	0.000	**97.41**	0.000	0.000	C	**-38.53**	**-77.93**	**-77.93**	**-77.93**
S32	B.G.2	**229.15**	0.00	0.000	**501.69**	0.000	0.000	T	**271.77**	**116.35**	**116.35**	**65.35**
S33	B.G.2	**97.41**	0.00	0.000	**-97.42**	1.250	0.000	C	**-38.42**	**-77.93**	**-77.93**	**-77.93**
S34	B.G.2	**-97.42**	0.00	0.000	**-370.17**	0.000	0.000	C	**-38.46**	**-77.93**	**-77.93**	**-77.93**
S35	B.G.2	**856.85**	0.00	0.000	**1119.88**	0.000	0.000	C	**-271.77**	**87.67**	**87.67**	**87.67**
S37	B.G.2	**312.00**	0.00	0.000	**103.99**	0.000	0.000	C	**-36.93**	**-83.20**	**-83.20**	**-83.20**
S39	B.G.2	**210.96**	0.00	0.000	**521.39**	0.000	0.000	T	**349.70**	**128.98**	**128.98**	**77.98**
S40	B.G.2	**104.00**	0.00	0.000	**-104.00**	1.250	0.000	C	**-36.81**	**-83.20**	**-83.20**	**-83.20**
S41	B.G.2	**-104.00**	0.00	0.000	**-395.20**	0.000	0.000	C	**-36.80**	**-83.20**	**-83.20**	**-83.20**
S42	B.G.2	**849.06**	0.00	0.000	**1227.03**	0.000	0.000	C	**-349.70**	**125.99**	**125.99**	**125.99**
S44	B.G.2	**332.45**	0.00	0.000	**110.81**	0.000	0.000	C	**-35.58**	**-88.66**	**-88.66**	**-88.66**
S46	B.G.2	**169.41**	0.00	0.000	**522.95**	0.000	0.000	T	**432.90**	**143.35**	**143.35**	**92.35**
S47	B.G.2	**110.81**	0.00	0.000	**-110.82**	1.250	0.000	C	**-35.50**	**-88.65**	**-88.65**	**-88.65**
S48	B.G.2	**-110.82**	0.00	0.000	**-421.11**	0.000	0.000	C	**-35.52**	**-88.65**	**-88.65**	**-88.65**
S49	B.G.2	**755.70**	0.00	0.000	**1244.26**	0.000	0.000	C	**-432.90**	**162.85**	**162.85**	**162.85**
S51	B.G.2	**352.63**	0.00	0.000	**117.54**	0.000	0.000	C	**-34.88**	**-94.04**	**-94.04**	**-94.04**
S53	B.G.2	**101.96**	0.00	0.000	**501.86**	0.000	0.000	T	**521.55**	**158.80**	**158.80**	**107.80**
S54	B.G.2	**117.54**	0.00	0.000	**-117.56**	1.250	0.000	C	**-34.88**	**-94.04**	**-94.04**	**-94.04**
S55	B.G.2	**-117.56**	0.00	0.000	**-446.69**	0.000	0.000	C	**-34.82**	**-94.04**	**-94.04**	**-94.04**
S56	B.G.2	**581.83**	0.00	0.000	**1176.81**	0.000	0.000	C	**-521.55**	**198.33**	**198.33**	**198.33**
S58	B.G.2	**371.59**	0.00	0.000	**123.86**	0.000	0.000	C	**-34.94**	**-99.09**	**-99.09**	**-99.09**
S60	B.G.2	**6.29**	0.00	0.000	**454.60**	0.000	0.000	T	**615.59**	**174.94**	**174.94**	**123.94**
S61	B.G.2	**123.86**	0.00	0.000	**-123.88**	1.250	0.000	C	**-34.84**	**-99.09**	**-99.09**	**-99.09**
S62	B.G.2	**-123.88**	0.00	0.000	**-470.71**	0.000	0.000	C	**-34.75**	**-99.09**	**-99.09**	**-99.09**
S63	B.G.2	**329.35**	0.00	0.000	**1028.52**	0.000	0.000	C	**-615.59**	**233.06**	**233.06**	**233.06**
S65	B.G.2	**388.29**	0.00	0.000	**129.42**	0.000	0.000	C	**-35.20**	**-103.55**	**-103.55**	**-103.55**
S67	B.G.2	**-118.71**	0.00	0.000	**377.88**	0.640	0.000	T	**714.68**	**191.03**	**191.03**	**140.03**
S68	B.G.2	**129.42**	0.00	0.000	**-129.45**	1.250	0.000	C	**-35.17**	**-103.55**	**-103.55**	**-103.55**
S69	B.G.2	**-129.45**	0.00	0.000	**-491.87**	0.000	0.000	C	**-35.17**	**-103.55**	**-103.55**	**-103.55**
S70	B.G.2	**-3.43**	0.00	0.000	**800.06**	0.013	0.000	C	**-714.68**	**267.83**	**267.83**	**267.83**
S72	B.G.2	**401.65**	0.00	0.000	**133.88**	0.000	0.000	C	**-35.78**	**-107.11**	**-107.11**	**-107.11**
S74	B.G.2	**-274.69**	0.00	0.000	**269.58**	1.409	0.000	T	**818.23**	**206.92**	**206.92**	**155.92**
S75	B.G.2	**133.87**	0.00	0.000	**-133.90**	1.250	0.000	C	**-35.73**	**-107.11**	**-107.11**	**-107.11**
S76	B.G.2	**-133.90**	0.00	0.000	**-508.79**	0.000	0.000	C	**-35.65**	**-107.11**	**-107.11**	**-107.11**
S77	B.G.2	**-420.31**	0.00	0.000	**488.43**	1.388	0.000	C	**-818.23**	**302.91**	**302.91**	**302.91**
S79	B.G.2	**410.48**	0.00	0.000	**136.81**	0.000	0.000	C	**-36.58**	**-109.47**	**-109.47**	**-109.47**
S81	B.G.2	**-463.13**	0.00	0.000	**126.96**	2.284	0.000	T	**925.34**	**222.20**	**222.20**	**171.20**
S82	B.G.2	**136.82**	0.00	0.000	**-136.85**	1.250	0.000	C	**-36.55**	**-109.47**	**-109.47**	**-109.47**
S83	B.G.2	**-136.85**	0.00	0.000	**-519.98**	0.000	0.000	C	**-36.51**	**-109.47**	**-109.47**	**-109.47**
S84	B.G.2	**-927.00**	0.00	0.000	**88.48**	2.739	0.000	C	**-925.34**	**338.49**	**338.49**	**338.49**
S86	B.G.2	**413.49**	0.00	0.000	**137.82**	0.000	0.000	C	**-37.70**	**-110.27**	**-110.27**	**-110.27**
S88	B.G.2	**-686.26**	0.00	0.000	**-52.66**	0.000	0.000	T	**1034.81**	**236.70**	**236.70**	**185.70**
S89	B.G.2	**137.82**	0.00	0.000	**-137.85**	1.250	0.000	C	**-37.74**	**-110.27**	**-110.27**	**-110.27**
S90	B.G.2	**-137.85**	0.00	0.000	**-523.79**	0.000	0.000	C	**-37.70**	**-110.27**	**-110.27**	**-110.27**
S91	B.G.2	**-1531.92**	0.00	0.000	**-407.02**	0.000	0.000	C	**-1034.81**	**374.97**	**374.97**	**374.97**
S93	B.G.2	**409.21**	0.00	0.000	**136.39**	0.000	0.000	C	**-39.39**	**-109.13**	**-109.13**	**-109.13**
S95	B.G.2	**-946.34**	0.00	0.000	**-272.77**	0.000	0.000	T	**1145.08**	**250.02**	**250.02**	**199.02**
S96	B.G.2	**136.39**	0.00	0.000	**-136.43**	1.250	0.000	C	**-39.44**	**-109.13**	**-109.13**	**-109.13**
S97	B.G.2	**-136.43**	0.00	0.000	**-518.38**	0.000	0.000	C	**-39.46**	**-109.13**	**-109.13**	**-109.13**
S98	B.G.2	**-2246.12**	0.00	0.000	**-1008.13**	0.000	0.000	C	**-1145.07**	**412.66**	**412.66**	**412.66**
S100	B.G.2	**396.03**	0.00	0.000	**132.00**	0.000	0.000	C	**-41.60**	**-105.61**	**-105.61**	**-105.61**
S102	B.G.2	**-1245.51**	0.00	0.000	**-537.12**	0.000	0.000	T	**1254.21**	**261.63**	**261.63**	**210.63**
S103	B.G.2	**132.00**	0.00	0.000	**-132.04**	1.250	0.000	C	**-41.62**	**-105.61**	**-105.61**	**-105.61**

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S104	B.G.2	**-132.04**	0.00	0.000	**-501.68**	0.000	0.000	C	**-41.57**	**-105.61**	**-105.61**	**-105.61**
S105	B.G.2	**-3084.00**	0.00	0.000	**-1727.73**	0.000	0.000	C	**-1254.20**	**452.09**	**452.09**	**452.09**
S107	B.G.2	**372.08**	0.00	0.000	**124.02**	0.000	0.000	C	**-43.71**	**-99.23**	**-99.23**	**-99.23**
S109	B.G.2	**-1586.12**	0.00	0.000	**-849.48**	0.000	0.000	T	**1359.82**	**271.05**	**271.05**	**220.05**
S110	B.G.2	**124.02**	0.00	0.000	**-124.05**	1.250	0.000	C	**-43.72**	**-99.23**	**-99.23**	**-99.23**
S111	B.G.2	**-124.05**	0.00	0.000	**-471.34**	0.000	0.000	C	**-43.71**	**-99.23**	**-99.23**	**-99.23**
S112	B.G.2	**-4063.19**	0.00	0.000	**-2582.31**	0.000	0.000	C	**-1359.82**	**493.63**	**493.63**	**493.63**
S114	B.G.2	**335.25**	0.00	0.000	**111.75**	0.000	0.000	C	**-44.18**	**-89.40**	**-89.40**	**-89.40**
S116	B.G.2	**-1972.59**	0.00	0.000	**-1214.03**	0.000	0.000	T	**1459.05**	**278.35**	**278.35**	**227.35**
S117	B.G.2	**111.75**	0.00	0.000	**-111.76**	1.250	0.000	C	**-44.18**	**-89.40**	**-89.40**	**-89.40**
S118	B.G.2	**-111.75**	0.00	0.000	**-424.66**	0.000	0.000	C	**-44.19**	**-89.40**	**-89.40**	**-89.40**
S119	B.G.2	**-5203.71**	0.00	0.000	**-3591.85**	0.000	0.000	C	**-1459.04**	**537.29**	**537.29**	**537.29**
S121	B.G.2	**283.06**	0.00	0.000	**94.37**	0.000	0.000	C	**-40.30**	**-75.47**	**-75.47**	**-75.47**
S123	B.G.2	**-2416.42**	0.00	0.000	**-1637.35**	0.000	0.000	T	**1548.45**	**285.19**	**285.19**	**234.19**
S124	B.G.2	**94.37**	0.00	0.000	**-94.32**	1.250	0.000	C	**-40.30**	**-75.48**	**-75.48**	**-75.48**
S125	B.G.2	**-94.32**	0.00	0.000	**-358.48**	0.000	0.000	C	**-40.30**	**-75.48**	**-75.48**	**-75.48**
S126	B.G.2	**-6523.40**	0.00	0.000	**-4779.05**	0.000	0.000	C	**-1548.44**	**581.45**	**581.45**	**581.45**
S128	B.G.2	**212.49**	0.00	0.000	**70.87**	0.000	0.000	C	**-29.25**	**-56.65**	**-56.65**	**-56.65**
S130	B.G.2	**-2944.57**	0.00	0.000	**-2133.36**	0.000	0.000	T	**1623.93**	**295.90**	**295.90**	**244.90**
S131	B.G.2	**70.87**	0.00	0.000	**-70.75**	1.251	0.000	C	**-29.25**	**-56.65**	**-56.65**	**-56.65**
S132	B.G.2	**-70.75**	0.00	0.000	**-269.02**	0.000	0.000	C	**-29.25**	**-56.65**	**-56.65**	**-56.65**
S135	B.G.2	**119.77**	0.00	0.000	**39.98**	0.000	0.000	C	**-11.80**	**-31.92**	**-31.92**	**-31.92**
S136	B.G.2	**-4482.84**	0.00	0.000	**-3488.78**	0.000	0.000	T	**1712.49**	**356.86**	**356.86**	**305.86**
S137	B.G.2	**-3608.55**	0.00	0.000	**-2732.08**	0.000	0.000	T	**1680.57**	**317.66**	**317.66**	**266.66**
S138	B.G.2	**39.97**	0.00	0.000	**-39.82**	1.252	0.000	C	**-11.80**	**-31.92**	**-31.92**	**-31.92**
S139	B.G.2	**-39.82**	0.00	0.000	**-151.53**	0.000	0.000	C	**-11.80**	**-31.92**	**-31.92**	**-31.92**
S140	B.G.2	**-9714.10**	0.00	0.000	**-7761.12**	0.000	0.000	C	**-1680.57**	**650.99**	**650.99**	**650.99**
S141	B.G.2	**-11550.95**	0.00	0.000	**-9562.57**	0.000	0.000	C	**-1712.48**	**662.79**	**662.79**	**662.79**
S142	B.G.2	**-8030.14**	0.00	0.000	**-6164.92**	0.000	0.000	C	**-1623.92**	**621.74**	**621.74**	**621.74**
-	**-**	**kNm**	**kNm**	**m**	**kNm**	**m**	**m**	**-**	**kN**	**kN**	**kN**	**kN**

L.C. Extreme Nodal Displacements

Node	L.C.	X	Z	Ry
K2	B.G.2	**0.0216**	**-0.0010**	**-0.367e-03**
K3		**0.0216**	**-0.0001**	**-0.367e-03**
K4		**0.0216**	**-0.0006**	**-0.373e-03**
K5		**0.0216**	**0.0007**	**-0.373e-03**
K7		**0.0205**	**-0.0010**	**-0.372e-03**
K10		**0.0204**	**0.0007**	**-0.376e-03**
K11		**0.0205**	**-0.0001**	**-0.372e-03**
K12		**0.0204**	**-0.0006**	**-0.376e-03**
K13		**0.0193**	**-0.0010**	**-0.380e-03**
K14		**0.0193**	**0.0000**	**-0.380e-03**
K15		**0.0193**	**-0.0006**	**-0.382e-03**
K16		**0.0193**	**0.0007**	**-0.382e-03**
K17		**0.0182**	**-0.0010**	**-0.390e-03**
K18		**0.0182**	0.0000	**-0.390e-03**
K19		**0.0182**	**-0.0007**	**-0.390e-03**
K20		**0.0182**	**0.0007**	**-0.390e-03**
K21		**0.0170**	**-0.0010**	**-0.400e-03**
K22		**0.0170**	**0.0000**	**-0.400e-03**
K23		**0.0170**	**-0.0007**	**-0.400e-03**
K24		**0.0170**	**0.0007**	**-0.400e-03**
K25		**0.0158**	**-0.0009**	**-0.410e-03**
K26		**0.0158**	**0.0001**	**-0.410e-03**
K27		**0.0158**	**-0.0008**	**-0.410e-03**
K28		**0.0158**	**0.0007**	**-0.410e-03**
K29		**0.0145**	**-0.0009**	**-0.420e-03**
K30		**0.0145**	**0.0001**	**-0.420e-03**
K31		**0.0145**	**-0.0008**	**-0.420e-03**
K32		**0.0145**	**0.0007**	**-0.420e-03**
K33		**0.0133**	**-0.0009**	**-0.428e-03**
K34		**0.0133**	**0.0002**	**-0.428e-03**
K35		**0.0132**	**-0.0009**	**-0.428e-03**
K36		**0.0132**	**0.0006**	**-0.428e-03**
K37		**0.0120**	**-0.0008**	**-0.434e-03**
K38		**0.0120**	**0.0002**	**-0.434e-03**
K39		**0.0119**	**-0.0009**	**-0.435e-03**

Node	L.C.	X	Z	Ry
K40	B.G.2	0.0119	0.0006	-0.435e-03
K41		0.0106	-0.0008	-0.438e-03
K42		0.0106	0.0003	-0.438e-03
K43		0.0106	-0.0010	-0.439e-03
K44		0.0106	0.0006	-0.439e-03
K45		0.0093	-0.0008	-0.439e-03
K46		0.0093	0.0003	-0.439e-03
K47		0.0093	-0.0010	-0.439e-03
K48		0.0093	0.0005	-0.439e-03
K49		0.0080	-0.0007	-0.434e-03
K50		0.0080	0.0004	-0.434e-03
K51		0.0080	-0.0010	-0.435e-03
K52		0.0080	0.0005	-0.435e-03
K53		0.0067	-0.0007	-0.425e-03
K54		0.0067	0.0004	-0.425e-03
K55		0.0067	-0.0010	-0.426e-03
K56		0.0067	0.0005	-0.426e-03
K57		0.0055	-0.0006	-0.409e-03
K58		0.0055	0.0004	-0.409e-03
K59		0.0054	-0.0010	-0.410e-03
K60		0.0054	0.0004	-0.410e-03
K61		0.0043	-0.0005	-0.386e-03
K62		0.0043	0.0004	-0.386e-03
K63		0.0042	-0.0010	-0.386e-03
K64		0.0042	0.0004	-0.386e-03
K65		0.0032	-0.0004	-0.353e-03
K66		0.0032	0.0004	-0.353e-03
K67		0.0031	-0.0009	-0.354e-03
K68		0.0031	0.0003	-0.354e-03
K69		0.0022	-0.0004	-0.311e-03
K70		0.0022	0.0004	-0.311e-03
K71		0.0021	-0.0008	-0.311e-03
K72		0.0021	0.0003	-0.311e-03
K73		0.0013	-0.0003	-0.258e-03
K74		0.0013	0.0004	-0.258e-03
K75		0.0013	-0.0007	-0.256e-03
K76		0.0013	0.0002	-0.256e-03
K77		0.0006	-0.0002	-0.190e-03
K78		0.0006	0.0003	-0.190e-03
K79		0.0006	-0.0005	-0.188e-03
K80		0.0006	0.0001	-0.188e-03
K81		0.0002	-0.0001	-0.106e-03
K82		0.0002	0.0002	-0.106e-03
K83		0.0002	-0.0003	-0.103e-03
K84		0.0002	0.0001	-0.103e-03
-	-	m	m	rad

L.C. Support Reactions

L.C.	Support	Node	X	Z	My
B.G.2	O1	K1	-356.86	1712.49	4482.84
	O2	K6	-662.79	-1712.48	11550.95
	Sum Reactions		**-1019.65**	**0.01**	
	Sum Loads		**1020.00**	**0.00**	
-	-	-	kN	kN	kNm

14 Appendix 5:

Output of MatrixFrame program for analysis of shear walls supported on individual elastic foundation

Job Name	Elastic individual supports	Job Number	1
Part Description	standard dimensions	Structural Engineer	M.SH
Client		Units	m, kN, kNm
File	C:\Users\Acer\Dropbox\courses\gaduation work\working maple\worked_example_elastic_individual.mxe		

Structure Info

Project Type	Nodes	Members	Supports	Sections	Loads Cases	Loads Comb.
2D-Frame	82	100	2	4	2	6

Members

Member	Node B	B	Release E	Node E	Section	X-B	Z-B	X-E	Z-E	Length
S2	K2	NVM	NVM	K3	P12	0.000	-60.000	2.500	-60.000	2.500
S3	K3	NVM	NVM	K4	P1	2.500	-60.000	5.000	-60.000	2.500
S4	K4	NVM	NVM	K5	P12	5.000	-60.000	8.500	-60.000	3.500
S8	K7	NVM	NVM	K2	P4	0.000	-57.000	0.000	-60.000	3.000
S11	K10	NVM	NVM	K5	P3	8.500	-57.000	8.500	-60.000	3.000
S13	K7	NVM	NVM	K11	P12	0.000	-57.000	2.500	-57.000	2.500
S14	K11	NVM	NVM	K12	P1	2.500	-57.000	5.000	-57.000	2.500
S15	K12	NVM	NVM	K10	P12	5.000	-57.000	8.500	-57.000	3.500
S16	K13	NVM	NVM	K14	P12	0.000	-54.000	2.500	-54.000	2.500
S18	K13	NVM	NVM	K7	P4	0.000	-54.000	0.000	-57.000	3.000
S19	K14	NVM	NVM	K15	P1	2.500	-54.000	5.000	-54.000	2.500
S20	K15	NVM	NVM	K16	P12	5.000	-54.000	8.500	-54.000	3.500
S21	K16	NVM	NVM	K10	P3	8.500	-54.000	8.500	-57.000	3.000
S23	K17	NVM	NVM	K18	P12	0.000	-51.000	2.500	-51.000	2.500
S25	K17	NVM	NVM	K13	P4	0.000	-51.000	0.000	-54.000	3.000
S26	K18	NVM	NVM	K19	P1	2.500	-51.000	5.000	-51.000	2.500
S27	K19	NVM	NVM	K20	P12	5.000	-51.000	8.500	-51.000	3.500
S28	K20	NVM	NVM	K16	P3	8.500	-51.000	8.500	-54.000	3.000
S30	K21	NVM	NVM	K22	P12	0.000	-48.000	2.500	-48.000	2.500
S32	K21	NVM	NVM	K17	P4	0.000	-48.000	0.000	-51.000	3.000
S33	K22	NVM	NVM	K23	P1	2.500	-48.000	5.000	-48.000	2.500
S34	K23	NVM	NVM	K24	P12	5.000	-48.000	8.500	-48.000	3.500
S35	K24	NVM	NVM	K20	P3	8.500	-48.000	8.500	-51.000	3.000
S37	K25	NVM	NVM	K26	P12	0.000	-45.000	2.500	-45.000	2.500
S39	K25	NVM	NVM	K21	P4	0.000	-45.000	0.000	-48.000	3.000
S40	K26	NVM	NVM	K27	P1	2.500	-45.000	5.000	-45.000	2.500
S41	K27	NVM	NVM	K28	P12	5.000	-45.000	8.500	-45.000	3.500
S42	K28	NVM	NVM	K24	P3	8.500	-45.000	8.500	-48.000	3.000
S44	K29	NVM	NVM	K30	P12	0.000	-42.000	2.500	-42.000	2.500
S46	K29	NVM	NVM	K25	P4	0.000	-42.000	0.000	-45.000	3.000
S47	K30	NVM	NVM	K31	P1	2.500	-42.000	5.000	-42.000	2.500
S48	K31	NVM	NVM	K32	P12	5.000	-42.000	8.500	-42.000	3.500
S49	K32	NVM	NVM	K28	P3	8.500	-42.000	8.500	-45.000	3.000
S51	K33	NVM	NVM	K34	P12	0.000	-39.000	2.500	-39.000	2.500
S53	K33	NVM	NVM	K29	P4	0.000	-39.000	0.000	-42.000	3.000
S54	K34	NVM	NVM	K35	P1	2.500	-39.000	5.000	-39.000	2.500
S55	K35	NVM	NVM	K36	P12	5.000	-39.000	8.500	-39.000	3.500
S56	K36	NVM	NVM	K32	P3	8.500	-39.000	8.500	-42.000	3.000
S58	K37	NVM	NVM	K38	P12	0.000	-36.000	2.500	-36.000	2.500
S60	K37	NVM	NVM	K33	P4	0.000	-36.000	0.000	-39.000	3.000
S61	K38	NVM	NVM	K39	P1	2.500	-36.000	5.000	-36.000	2.500
S62	K39	NVM	NVM	K40	P12	5.000	-36.000	8.500	-36.000	3.500
S63	K40	NVM	NVM	K36	P3	8.500	-36.000	8.500	-39.000	3.000
S65	K41	NVM	NVM	K42	P12	0.000	-33.000	2.500	-33.000	2.500
S67	K41	NVM	NVM	K37	P4	0.000	-33.000	0.000	-36.000	3.000
S68	K42	NVM	NVM	K43	P1	2.500	-33.000	5.000	-33.000	2.500
S69	K43	NVM	NVM	K44	P12	5.000	-33.000	8.500	-33.000	3.500
S70	K44	NVM	NVM	K40	P3	8.500	-33.000	8.500	-36.000	3.000
S72	K45	NVM	NVM	K46	P12	0.000	-30.000	2.500	-30.000	2.500
S74	K45	NVM	NVM	K41	P4	0.000	-30.000	0.000	-33.000	3.000
S75	K46	NVM	NVM	K47	P1	2.500	-30.000	5.000	-30.000	2.500
S76	K47	NVM	NVM	K48	P12	5.000	-30.000	8.500	-30.000	3.500
S77	K48	NVM	NVM	K44	P3	8.500	-30.000	8.500	-33.000	3.000
S79	K49	NVM	NVM	K50	P12	0.000	-27.000	2.500	-27.000	2.500
S81	K49	NVM	NVM	K45	P4	0.000	-27.000	0.000	-30.000	3.000
S82	K50	NVM	NVM	K51	P1	2.500	-27.000	5.000	-27.000	2.500
S83	K51	NVM	NVM	K52	P12	5.000	-27.000	8.500	-27.000	3.500
S84	K52	NVM	NVM	K48	P3	8.500	-27.000	8.500	-30.000	3.000
S86	K53	NVM	NVM	K54	P12	0.000	-24.000	2.500	-24.000	2.500
S88	K53	NVM	NVM	K49	P4	0.000	-24.000	0.000	-27.000	3.000
S89	K54	NVM	NVM	K55	P1	2.500	-24.000	5.000	-24.000	2.500
S90	K55	NVM	NVM	K56	P12	5.000	-24.000	8.500	-24.000	3.500
S91	K56	NVM	NVM	K52	P3	8.500	-24.000	8.500	-27.000	3.000
S93	K57	NVM	NVM	K58	P12	0.000	-21.000	2.500	-21.000	2.500
S95	K57	NVM	NVM	K53	P4	0.000	-21.000	0.000	-24.000	3.000

Member	Node B	Release B	Release E	Node E	Section	X-B	Z-B	X-E	Z-E	Length
S96	K58	NVM	NVM	K59	P1	2.500	-21.000	5.000	-21.000	2.500
S97	K59	NVM	NVM	K60	P12	5.000	-21.000	8.500	-21.000	3.500
S98	K60	NVM	NVM	K56	P3	8.500	-21.000	8.500	-24.000	3.000
S100	K61	NVM	NVM	K62	P12	0.000	-18.000	2.500	-18.000	2.500
S102	K61	NVM	NVM	K57	P4	0.000	-18.000	0.000	-21.000	3.000
S103	K62	NVM	NVM	K63	P1	2.500	-18.000	5.000	-18.000	2.500
S104	K63	NVM	NVM	K64	P12	5.000	-18.000	8.500	-18.000	3.500
S105	K64	NVM	NVM	K60	P3	8.500	-18.000	8.500	-21.000	3.000
S107	K65	NVM	NVM	K66	P12	0.000	-15.000	2.500	-15.000	2.500
S109	K65	NVM	NVM	K61	P4	0.000	-15.000	0.000	-18.000	3.000
S110	K66	NVM	NVM	K67	P1	2.500	-15.000	5.000	-15.000	2.500
S111	K67	NVM	NVM	K68	P12	5.000	-15.000	8.500	-15.000	3.500
S112	K68	NVM	NVM	K64	P3	8.500	-15.000	8.500	-18.000	3.000
S114	K69	NVM	NVM	K70	P12	0.000	-12.000	2.500	-12.000	2.500
S116	K69	NVM	NVM	K65	P4	0.000	-12.000	0.000	-15.000	3.000
S117	K70	NVM	NVM	K71	P1	2.500	-12.000	5.000	-12.000	2.500
S118	K71	NVM	NVM	K72	P12	5.000	-12.000	8.500	-12.000	3.500
S119	K72	NVM	NVM	K68	P3	8.500	-12.000	8.500	-15.000	3.000
S121	K73	NVM	NVM	K74	P12	0.000	-9.000	2.500	-9.000	2.500
S123	K73	NVM	NVM	K69	P4	0.000	-9.000	0.000	-12.000	3.000
S124	K74	NVM	NVM	K75	P1	2.500	-9.000	5.000	-9.000	2.500
S125	K75	NVM	NVM	K76	P12	5.000	-9.000	8.500	-9.000	3.500
S126	K76	NVM	NVM	K72	P3	8.500	-9.000	8.500	-12.000	3.000
S128	K77	NVM	NVM	K78	P12	0.000	-6.000	2.500	-6.000	2.500
S130	K77	NVM	NVM	K73	P4	0.000	-6.000	0.000	-9.000	3.000
S131	K78	NVM	NVM	K79	P1	2.500	-6.000	5.000	-6.000	2.500
S132	K79	NVM	NVM	K80	P12	5.000	-6.000	8.500	-6.000	3.500
S135	K81	NVM	NVM	K82	P12	0.000	-3.000	2.500	-3.000	2.500
S136	K1	NVM	NVM	K81	P4	0.000	0.000	0.000	-3.000	3.000
S137	K81	NVM	NVM	K77	P4	0.000	-3.000	0.000	-6.000	3.000
S138	K82	NVM	NVM	K83	P1	2.500	-3.000	5.000	-3.000	2.500
S139	K83	NVM	NVM	K84	P12	5.000	-3.000	8.500	-3.000	3.500
S140	K84	NVM	NVM	K80	P3	8.500	-3.000	8.500	-6.000	3.000
S141	K6	NVM	NVM	K84	P3	8.500	0.000	8.500	-3.000	3.000
S142	K80	NVM	NVM	K76	P3	8.500	-6.000	8.500	-9.000	3.000
-	**-**	**-**	**-**	**-**	**-**	**m**	**m**	**m**	**m**	**m**

Sections

Section	Section Name	Area	Iy	Material	Angle
P1	R300x400	1.2000e-01	1.6000e-03	C45/55	0
P3	R300x7000	2.1000e+00	8.5750e+00	C45/55	0
P4	R300x5000	1.5000e+00	3.1250e+00	C45/55	0
P12		2.1400e+01	4.1600e+00	Mat. 3	0
-	**-**	**m2**	**m4**	**-**	**°**

Section Shapes

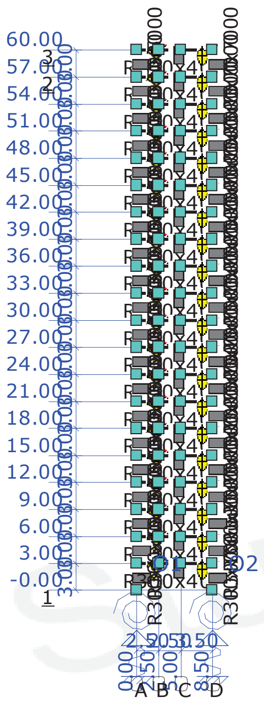
Section	Tapered	hB	hE	tf	tw	tf2	B	b1	b2	Castellate	Height
P1	No	0.400	0.400	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
P3	No	7.000	7.000	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
P4	No	5.000	5.000	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
-	**-**	**m**	**m**	**m**	**m**	**m**	**m**	**m**	**m**	**-**	**m**

Materials

Material Name	Density	Youngs mod.	Lin. Exp.
C45/55	24.00	3.6000e+07	10.0000e-06
Mat. 3	0.00	3.6000e+15	10.0000e-06
-	**kN/m3**	**kN/m2**	**C°m**

Supports

Support	Node	X	Z	Yr	AngleYr
O1	K1	fixed	153000	318750	0
O2	K6	fixed	214200	874650	0
-	**-**	**kN/m**	**kN/m**	**kNmrad**	**°**



Pic. Geometrie: Doorgaande Ligger

Loads Cases

Type	Value Begin	Value End	Dist. Begin	Dist. End	Direction	Member/Node
B.G.2: Wind load						
q	17.00	17.00	0.000	3.000(L)	Z'	S8,S18,S25,S32, S39,S46,S53,S60, S67,S74,S81,S88, S95,S102,S109, S116,S123,S130, S136-S137

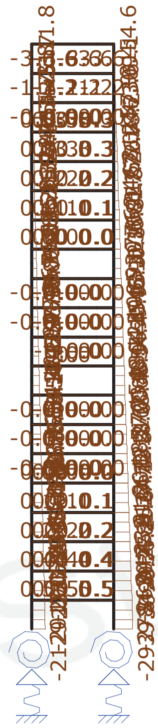
Sum of loads	**X:**	**1020,00**	**kN Z: 0.00**	**kN**		
-	**-**	**-**	**m**	**m**	**- -**	

Persistent Loads Combinations

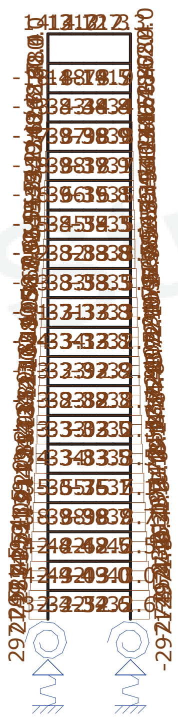
L.C.	Description	Pe.C.1	Pe.C.2
B.G.1	Permanent actions	1.20	1.35
B.G.2	Wind load	1.50	-

Analysis Assumptions

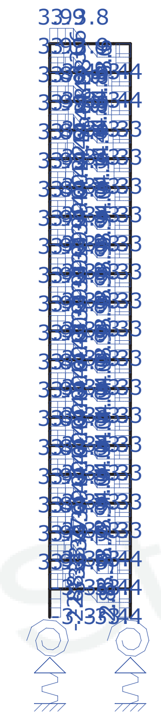
Linear Elastic Analysis performed



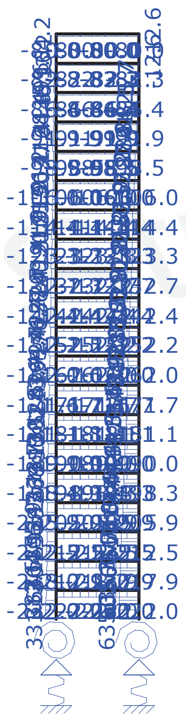
Pic. B.G.1: Permanent actions Normaalkracht (Nx)



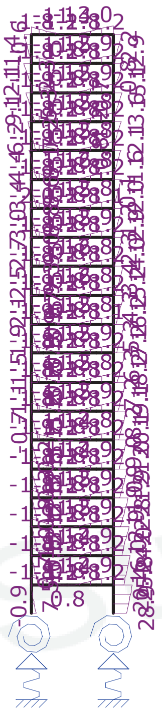
Pic. B.G.2: Wind load Normaalkracht (Nx)



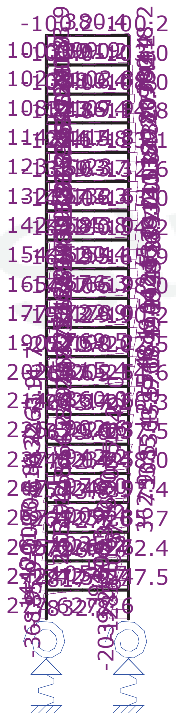
Pic. B.G.1: Permanent actions Dwarskracht (Vz)



Pic. B.G.2: Wind load Dwarskracht (Vz)



Pic. B.G.1: Permanent actions Momenten (My)



Pic. B.G.2: Wind load Momenten (My)

L.C. Extreme Member Forces

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S2	B.G.2	**300.02**	0.00	0.000	**99.86**	0.000	0.000	T	**14.34**	**-80.02**	**-80.02**	**-80.02**
S3	B.G.2	**99.88**	0.00	0.000	**-100.23**	1.248	0.000	T	**12.68**	**-80.04**	**-80.04**	**-80.04**
S4	B.G.2	**-100.23**	0.00	0.000	**-380.38**	0.000	0.000	T	**12.29**	**-80.04**	**-80.04**	**-80.04**
S8	B.G.2	**183.93**	0.00	0.000	**300.00**	0.000	0.000	T	**80.04**	**64.19**	**64.19**	**13.19**
S11	B.G.2	**418.20**	0.00	0.000	**380.38**	0.000	0.000	C	**-80.04**	**-12.61**	**-12.61**	**-12.61**

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S13	B.G.2	**308.61**	0.00	0.000	**102.85**	0.000	0.000	C	**-18.43**	**-82.31**	**-82.31**	**-82.31**
S14	B.G.2	**102.79**	0.00	0.000	**-103.02**	1.249	0.000	C	**-18.69**	**-82.32**	**-82.32**	**-82.32**
S15	B.G.2	**-103.02**	0.00	0.000	**-391.15**	0.000	0.000	C	**-19.46**	**-82.32**	**-82.32**	**-82.32**
S16	B.G.2	**323.94**	0.00	0.000	**107.90**	0.000	0.000	C	**-33.79**	**-86.36**	**-86.36**	**-86.36**
S18	B.G.2	**271.65**	0.00	0.000	**492.56**	0.000	0.000	T	**162.38**	**99.13**	**99.13**	**48.13**
S19	B.G.2	**107.94**	0.00	0.000	**-108.04**	1.249	0.000	C	**-34.41**	**-86.39**	**-86.39**	**-86.39**
S20	B.G.2	**-108.05**	0.00	0.000	**-410.45**	0.000	0.000	C	**-34.82**	**-86.40**	**-86.40**	**-86.40**
S21	B.G.2	**792.21**	0.00	0.000	**809.37**	0.000	0.000	C	**-162.36**	**5.72**	**5.72**	**5.72**
S23	B.G.2	**344.48**	0.00	0.000	**114.82**	0.000	0.000	C	**-37.89**	**-91.87**	**-91.87**	**-91.87**
S25	B.G.2	**325.15**	0.00	0.000	**595.56**	0.000	0.000	T	**248.74**	**115.64**	**115.64**	**64.64**
S26	B.G.2	**114.81**	0.00	0.000	**-114.84**	1.250	0.000	C	**-38.92**	**-91.86**	**-91.86**	**-91.86**
S27	B.G.2	**-114.84**	0.00	0.000	**-436.37**	0.000	0.000	C	**-38.91**	**-91.86**	**-91.86**	**-91.86**
S28	B.G.2	**1078.92**	0.00	0.000	**1202.66**	0.000	0.000	C	**-248.74**	**41.25**	**41.25**	**41.25**
S30	B.G.2	**369.31**	0.00	0.000	**123.09**	0.000	0.000	C	**-38.91**	**-98.48**	**-98.48**	**-98.48**
S32	B.G.2	**360.98**	0.00	0.000	**669.62**	0.000	0.000	T	**340.64**	**128.38**	**128.38**	**77.38**
S33	B.G.2	**123.09**	0.00	0.000	**-123.10**	1.250	0.000	C	**-38.15**	**-98.48**	**-98.48**	**-98.48**
S34	B.G.2	**-123.09**	0.00	0.000	**-467.79**	0.000	0.000	C	**-37.89**	**-98.48**	**-98.48**	**-98.48**
S35	B.G.2	**1273.00**	0.00	0.000	**1515.30**	0.000	0.000	C	**-340.59**	**80.76**	**80.76**	**80.76**
S37	B.G.2	**397.63**	0.00	0.000	**132.61**	0.000	0.000	C	**-36.86**	**-106.04**	**-106.04**	**-106.04**
S39	B.G.2	**383.64**	0.00	0.000	**730.29**	0.000	0.000	T	**439.15**	**141.05**	**141.05**	**90.05**
S40	B.G.2	**132.55**	0.00	0.000	**-132.56**	1.250	0.000	C	**-36.05**	**-106.04**	**-106.04**	**-106.04**
S41	B.G.2	**-132.55**	0.00	0.000	**-503.71**	0.000	0.000	C	**-35.84**	**-106.04**	**-106.04**	**-106.04**
S42	B.G.2	**1389.23**	0.00	0.000	**1740.80**	0.000	0.000	C	**-439.07**	**117.19**	**117.19**	**117.19**
S44	B.G.2	**428.97**	0.00	0.000	**142.96**	0.000	0.000	C	**-35.84**	**-114.41**	**-114.41**	**-114.41**
S46	B.G.2	**389.16**	0.00	0.000	**781.35**	0.000	0.000	T	**545.16**	**156.23**	**156.23**	**105.23**
S47	B.G.2	**142.97**	0.00	0.000	**-142.98**	1.250	0.000	C	**-34.49**	**-114.38**	**-114.38**	**-114.38**
S48	B.G.2	**-143.01**	0.00	0.000	**-543.33**	0.000	0.000	C	**-35.33**	**-114.38**	**-114.38**	**-114.38**
S49	B.G.2	**1433.35**	0.00	0.000	**1892.95**	0.000	0.000	C	**-545.11**	**153.20**	**153.20**	**153.20**
S51	B.G.2	**462.50**	0.00	0.000	**154.14**	0.000	0.000	C	**-32.77**	**-123.33**	**-123.33**	**-123.33**
S53	B.G.2	**378.89**	0.00	0.000	**818.08**	0.000	0.000	T	**659.54**	**171.90**	**171.90**	**120.90**
S54	B.G.2	**154.15**	0.00	0.000	**-154.17**	1.250	0.000	C	**-33.83**	**-123.32**	**-123.32**	**-123.32**
S55	B.G.2	**-154.16**	0.00	0.000	**-585.81**	0.000	0.000	C	**-33.79**	**-123.32**	**-123.32**	**-123.32**
S56	B.G.2	**1412.96**	0.00	0.000	**1976.67**	0.000	0.000	C	**-659.49**	**187.90**	**187.90**	**187.90**
S58	B.G.2	**497.59**	0.00	0.000	**165.94**	0.000	0.000	C	**-33.79**	**-132.66**	**-132.66**	**-132.66**
S60	B.G.2	**343.86**	0.00	0.000	**841.39**	0.000	0.000	T	**782.86**	**191.34**	**191.34**	**140.34**
S61	B.G.2	**165.87**	0.00	0.000	**-165.90**	1.250	0.000	C	**-33.53**	**-132.71**	**-132.71**	**-132.71**
S62	B.G.2	**-165.90**	0.00	0.000	**-630.35**	0.000	0.000	C	**-33.28**	**-132.70**	**-132.70**	**-132.70**
S63	B.G.2	**1338.02**	0.00	0.000	**1998.75**	0.000	0.000	C	**-782.82**	**220.24**	**220.24**	**220.24**
S65	B.G.2	**533.86**	0.00	0.000	**178.02**	0.000	0.000	C	**-31.74**	**-142.37**	**-142.37**	**-142.37**
S67	B.G.2	**289.73**	0.00	0.000	**841.50**	0.000	0.000	T	**915.57**	**209.42**	**209.42**	**158.42**
S68	B.G.2	**177.95**	0.00	0.000	**-177.98**	1.250	0.000	C	**-33.30**	**-142.37**	**-142.37**	**-142.37**
S69	B.G.2	**-177.98**	0.00	0.000	**-676.29**	0.000	0.000	C	**-33.79**	**-142.37**	**-142.37**	**-142.37**
S70	B.G.2	**1215.97**	0.00	0.000	**1968.35**	0.000	0.000	C	**-915.52**	**250.80**	**250.80**	**250.80**
S72	B.G.2	**570.76**	0.00	0.000	**190.14**	0.000	0.000	C	**-34.30**	**-152.24**	**-152.24**	**-152.24**
S74	B.G.2	**216.94**	0.00	0.000	**823.56**	0.000	0.000	T	**1057.97**	**227.71**	**227.71**	**176.71**
S75	B.G.2	**190.22**	0.00	0.000	**-190.25**	1.250	0.000	C	**-33.10**	**-152.19**	**-152.19**	**-152.19**
S76	B.G.2	**-190.23**	0.00	0.000	**-722.90**	0.000	0.000	C	**-32.77**	**-152.18**	**-152.18**	**-152.18**
S77	B.G.2	**1042.53**	0.00	0.000	**1892.23**	0.000	0.000	C	**-1057.89**	**283.23**	**283.23**	**283.23**
S79	B.G.2	**607.51**	0.00	0.000	**202.50**	0.000	0.000	C	**-33.28**	**-162.02**	**-162.02**	**-162.02**
S81	B.G.2	**125.09**	0.00	0.000	**787.66**	0.000	0.000	T	**1210.12**	**246.35**	**246.35**	**195.35**
S82	B.G.2	**202.48**	0.00	0.000	**-202.51**	1.250	0.000	C	**-32.91**	**-162.00**	**-162.00**	**-162.00**
S83	B.G.2	**-202.54**	0.00	0.000	**-769.55**	0.000	0.000	C	**-32.77**	**-162.00**	**-162.00**	**-162.00**
S84	B.G.2	**819.45**	0.00	0.000	**1765.43**	0.000	0.000	C	**-1210.06**	**315.33**	**315.33**	**315.33**
S86	B.G.2	**643.68**	0.00	0.000	**214.62**	0.000	0.000	C	**-33.79**	**-171.63**	**-171.63**	**-171.63**
S88	B.G.2	**15.15**	0.00	0.000	**732.61**	0.000	0.000	T	**1372.10**	**264.65**	**264.65**	**213.65**
S89	B.G.2	**214.57**	0.00	0.000	**-214.61**	1.250	0.000	C	**-32.81**	**-171.67**	**-171.67**	**-171.67**
S90	B.G.2	**-214.61**	0.00	0.000	**-815.42**	0.000	0.000	C	**-32.26**	**-171.66**	**-171.66**	**-171.66**
S91	B.G.2	**548.11**	0.00	0.000	**1589.01**	0.000	0.000	C	**-1372.05**	**346.97**	**346.97**	**346.97**
S93	B.G.2	**678.98**	0.00	0.000	**226.27**	0.000	0.000	C	**-33.28**	**-181.08**	**-181.08**	**-181.08**
S95	B.G.2	**-114.73**	0.00	0.000	**658.94**	0.410	0.000	T	**1543.79**	**283.39**	**283.39**	**232.39**
S96	B.G.2	**226.30**	0.00	0.000	**-226.34**	1.250	0.000	C	**-32.98**	**-181.06**	**-181.06**	**-181.06**
S97	B.G.2	**-226.34**	0.00	0.000	**-860.06**	0.000	0.000	C	**-32.51**	**-181.06**	**-181.06**	**-181.06**
S98	B.G.2	**225.35**	0.00	0.000	**1363.51**	0.000	0.000	C	**-1543.73**	**379.39**	**379.39**	**379.39**
S100	B.G.2	**712.42**	0.00	0.000	**237.52**	0.000	0.000	C	**-34.30**	**-189.93**	**-189.93**	**-189.93**
S102	B.G.2	**-263.89**	0.00	0.000	**564.18**	0.898	0.000	T	**1724.87**	**301.52**	**301.52**	**250.52**
S103	B.G.2	**237.48**	0.00	0.000	**-237.53**	1.250	0.000	C	**-33.79**	**-190.00**	**-190.00**	**-190.00**

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S104	B.G.2	-237.54	0.00	0.000	-902.53	0.000	0.000	C	-33.54	-190.01	-190.01	-190.01
S105	B.G.2	-148.97	0.00	0.000	1085.39	0.362	0.000	C	-1724.78	411.45	411.45	411.45
S107	B.G.2	743.85	0.00	0.000	247.81	0.000	0.000	C	-35.58	-198.40	-198.40	-198.40
S109	B.G.2	-432.27	0.00	0.000	448.59	1.407	0.000	T	1914.85	319.12	319.12	268.12
S110	B.G.2	247.89	0.00	0.000	-247.96	1.250	0.000	C	-35.69	-198.34	-198.34	-198.34
S111	B.G.2	-247.95	0.00	0.000	-942.14	0.000	0.000	C	-35.07	-198.34	-198.34	-198.34
S112	B.G.2	-579.22	0.00	0.000	753.54	1.304	0.000	C	-1914.79	444.25	444.25	444.25
S114	B.G.2	772.10	0.00	0.000	257.34	0.000	0.000	C	-38.91	-205.90	-205.90	-205.90
S116	B.G.2	-616.22	0.00	0.000	311.48	1.936	0.000	T	2113.15	334.73	334.73	283.73
S117	B.G.2	257.33	0.00	0.000	-257.41	1.250	0.000	C	-38.92	-205.90	-205.90	-205.90
S118	B.G.2	-257.39	0.00	0.000	-978.03	0.000	0.000	C	-38.66	-205.90	-205.90	-205.90
S119	B.G.2	-1075.28	0.00	0.000	362.92	2.243	0.000	C	-2113.12	479.40	479.40	479.40
S121	B.G.2	796.84	0.00	0.000	265.58	0.000	0.000	C	-42.75	-212.50	-212.50	-212.50
S123	B.G.2	-808.83	0.00	0.000	155.91	2.481	0.000	T	2319.03	347.08	347.08	296.08
S124	B.G.2	265.58	0.00	0.000	-265.65	1.250	0.000	C	-42.56	-212.50	-212.50	-212.50
S125	B.G.2	-265.66	0.00	0.000	-1009.34	0.000	0.000	C	-42.50	-212.48	-212.48	-212.48
S126	B.G.2	-1650.14	0.00	0.000	-97.25	0.000	0.000	C	-2319.01	517.63	517.63	517.63
S128	B.G.2	817.34	0.00	0.000	272.40	0.000	0.000	C	-42.88	-217.97	-217.97	-217.97
S130	B.G.2	-1003.20	0.00	0.000	-11.99	0.000	0.000	T	2531.45	355.90	355.90	304.90
S131	B.G.2	272.43	0.00	0.000	-272.44	1.250	0.000	C	-43.01	-217.95	-217.95	-217.95
S132	B.G.2	-272.44	0.00	0.000	-1035.23	0.000	0.000	C	-43.01	-217.94	-217.94	-217.94
S135	B.G.2	832.75	0.00	0.000	277.65	0.000	0.000	C	-32.45	-222.05	-222.05	-222.05
S136	B.G.2	-1440.28	0.00	0.000	-368.94	0.000	0.000	T	2971.51	382.62	382.62	331.62
S137	B.G.2	-1201.66	0.00	0.000	-185.91	0.000	0.000	T	2749.41	364.08	364.08	313.08
S138	B.G.2	277.62	0.00	0.000	-277.49	1.250	0.000	C	-32.52	-222.05	-222.05	-222.05
S139	B.G.2	-277.51	0.00	0.000	-1054.66	0.000	0.000	C	-32.58	-222.05	-222.05	-222.05
S140	B.G.2	-3093.87	0.00	0.000	-1285.65	0.000	0.000	C	-2749.42	602.74	602.74	602.74
S141	B.G.2	-3945.07	0.00	0.000	-2039.21	0.000	0.000	C	-2971.47	635.29	635.29	635.29
S142	B.G.2	-2320.92	0.00	0.000	-640.80	0.000	0.000	C	-2531.49	560.04	560.04	560.04
-	-	kNm	kNm	m	kNm	m	m	-	kN	kN	kN	kN

L.C. Extreme Nodal Displacements

Node	L.C.	X	Z	Ry
K1	B.G.2	0.0000	-0.0194	-4.519e-03
K2		0.2714	-0.0208	-4.413e-03
K3		0.2714	-0.0098	-4.413e-03
K4		0.2714	-0.0006	-4.421e-03
K5		0.2714	0.0149	-4.421e-03
K6		0.0000	0.0139	-4.510e-03
K7		0.2581	-0.0208	-4.420e-03
K10		0.2581	0.0149	-4.425e-03
K11		0.2581	-0.0098	-4.420e-03
K12		0.2581	-0.0006	-4.425e-03
K13		0.2449	-0.0208	-4.431e-03
K14		0.2449	-0.0098	-4.431e-03
K15		0.2448	-0.0006	-4.433e-03
K16		0.2448	0.0149	-4.433e-03
K17		0.2316	-0.0208	-4.443e-03
K18		0.2316	-0.0097	-4.443e-03
K19		0.2315	-0.0007	-4.444e-03
K20		0.2315	0.0149	-4.444e-03
K21		0.2182	-0.0208	-4.457e-03
K22		0.2182	-0.0097	-4.457e-03
K23		0.2182	-0.0007	-4.457e-03
K24		0.2182	0.0149	-4.457e-03
K25		0.2048	-0.0208	-4.472e-03
K26		0.2048	-0.0096	-4.472e-03
K27		0.2048	-0.0008	-4.473e-03
K28		0.2048	0.0148	-4.473e-03
K29		0.1914	-0.0207	-4.488e-03
K30		0.1914	-0.0095	-4.488e-03
K31		0.1913	-0.0009	-4.489e-03
K32		0.1913	0.0148	-4.489e-03
K33		0.1779	-0.0207	-4.505e-03
K34		0.1779	-0.0094	-4.505e-03
K35		0.1778	-0.0010	-4.505e-03
K36		0.1778	0.0148	-4.505e-03
K37		0.1643	-0.0207	-4.521e-03

Node	L.C.	X	Z	Ry
K38	B.G.2	0.1643	-0.0094	-4.521e-03
K39		0.1643	-0.0011	-4.521e-03
K40		0.1643	0.0148	-4.521e-03
K41		0.1507	-0.0206	-4.536e-03
K42		0.1507	-0.0093	-4.536e-03
K43		0.1507	-0.0012	-4.537e-03
K44		0.1507	0.0147	-4.537e-03
K45		0.1371	-0.0206	-4.550e-03
K46		0.1371	-0.0092	-4.550e-03
K47		0.1371	-0.0012	-4.551e-03
K48		0.1371	0.0147	-4.551e-03
K49		0.1234	-0.0205	-4.563e-03
K50		0.1234	-0.0091	-4.563e-03
K51		0.1234	-0.0013	-4.564e-03
K52		0.1234	0.0146	-4.564e-03
K53		0.1097	-0.0204	-4.573e-03
K54		0.1097	-0.0090	-4.573e-03
K55		0.1097	-0.0014	-4.574e-03
K56		0.1097	0.0146	-4.574e-03
K57		0.0960	-0.0203	-4.581e-03
K58		0.0960	-0.0089	-4.581e-03
K59		0.0960	-0.0015	-4.582e-03
K60		0.0960	0.0145	-4.582e-03
K61		0.0822	-0.0202	-4.585e-03
K62		0.0822	-0.0088	-4.585e-03
K63		0.0822	-0.0016	-4.586e-03
K64		0.0822	0.0145	-4.586e-03
K65		0.0685	-0.0201	-4.586e-03
K66		0.0685	-0.0087	-4.586e-03
K67		0.0684	-0.0017	-4.587e-03
K68		0.0684	0.0144	-4.587e-03
K69		0.0547	-0.0200	-4.582e-03
K70		0.0547	-0.0086	-4.582e-03
K71		0.0547	-0.0018	-4.584e-03
K72		0.0547	0.0143	-4.584e-03
K73		0.0410	-0.0199	-4.574e-03
K74		0.0410	-0.0084	-4.574e-03
K75		0.0409	-0.0018	-4.575e-03
K76		0.0409	0.0142	-4.575e-03
K77		0.0273	-0.0197	-4.560e-03
K78		0.0273	-0.0083	-4.560e-03
K79		0.0272	-0.0019	-4.561e-03
K80		0.0272	0.0141	-4.561e-03
K81		0.0136	-0.0196	-4.542e-03
K82		0.0136	-0.0082	-4.542e-03
K83		0.0136	-0.0019	-4.540e-03
K84		0.0136	0.0140	-4.540e-03
-	-	m	m	rad

L.C. Support Reactions

L.C.	Support	Node	X	Z	My
B.G.2	O1	K1	-382.62	2971.50	1440.28
	O2	K6	-635.29	-2971.47	3945.07
	Sum Reactions		**-1017.90**	**0.04**	
	Sum Loads		**1020.00**	**0.00**	
-	-	-	kN	kN	kNm

15 Appendix 6:

Output of the MatrixFrame program for analysis of shear walls supported on elastic foundation with a stiffened beam at base

Job Name	example stiffened beam	Job Number	
Part Description	Elastic foundation	Structural Engineer	M.Sh
Client		Units	m, kN, kNm
File	h:\Desktop\worked_example_elastic_stiffned_beam.mxe		

Structure Info

Project Type	Nodes	Members	Supports	Sections	Loads Cases	Loads Comb.
2D-Frame	84	103	2	4	2	0

Members

Member	Node B	Release B	Release E	Node E	Section	X-B	Z-B	X-E	Z-E	Length
S2	K2	NVM	NVM	K3	P12	0,000	-60,000	2,500	-60,000	2,500
S3	K3	NVM	NVM	K4	P1	2,500	-60,000	5,000	-60,000	2,500
S4	K4	NVM	NVM	K5	P12	5,000	-60,000	8,500	-60,000	3,500
S8	K7	NVM	NVM	K2	P4	0,000	-57,000	0,000	-60,000	3,000
S11	K10	NVM	NVM	K5	P3	8,500	-57,000	8,500	-60,000	3,000
S13	K7	NVM	NVM	K11	P12	0,000	-57,000	2,500	-57,000	2,500
S14	K11	NVM	NVM	K12	P1	2,500	-57,000	5,000	-57,000	2,500
S15	K12	NVM	NVM	K10	P12	5,000	-57,000	8,500	-57,000	3,500
S16	K13	NVM	NVM	K14	P12	0,000	-54,000	2,500	-54,000	2,500
S18	K13	NVM	NVM	K7	P4	0,000	-54,000	0,000	-57,000	3,000
S19	K14	NVM	NVM	K15	P1	2,500	-54,000	5,000	-54,000	2,500
S20	K15	NVM	NVM	K16	P12	5,000	-54,000	8,500	-54,000	3,500
S21	K16	NVM	NVM	K10	P3	8,500	-54,000	8,500	-57,000	3,000
S23	K17	NVM	NVM	K18	P12	0,000	-51,000	2,500	-51,000	2,500
S25	K17	NVM	NVM	K13	P4	0,000	-51,000	0,000	-54,000	3,000
S26	K18	NVM	NVM	K19	P1	2,500	-51,000	5,000	-51,000	2,500
S27	K19	NVM	NVM	K20	P12	5,000	-51,000	8,500	-51,000	3,500
S28	K20	NVM	NVM	K16	P3	8,500	-51,000	8,500	-54,000	3,000
S30	K21	NVM	NVM	K22	P12	0,000	-48,000	2,500	-48,000	2,500
S32	K21	NVM	NVM	K17	P4	0,000	-48,000	0,000	-51,000	3,000
S33	K22	NVM	NVM	K23	P1	2,500	-48,000	5,000	-48,000	2,500
S34	K23	NVM	NVM	K24	P12	5,000	-48,000	8,500	-48,000	3,500
S35	K24	NVM	NVM	K20	P3	8,500	-48,000	8,500	-51,000	3,000
S37	K25	NVM	NVM	K26	P12	0,000	-45,000	2,500	-45,000	2,500
S39	K25	NVM	NVM	K21	P4	0,000	-45,000	0,000	-48,000	3,000
S40	K26	NVM	NVM	K27	P1	2,500	-45,000	5,000	-45,000	2,500
S41	K27	NVM	NVM	K28	P12	5,000	-45,000	8,500	-45,000	3,500
S42	K28	NVM	NVM	K24	P3	8,500	-45,000	8,500	-48,000	3,000
S44	K29	NVM	NVM	K30	P12	0,000	-42,000	2,500	-42,000	2,500
S46	K29	NVM	NVM	K25	P4	0,000	-42,000	0,000	-45,000	3,000
S47	K30	NVM	NVM	K31	P1	2,500	-42,000	5,000	-42,000	2,500
S48	K31	NVM	NVM	K32	P12	5,000	-42,000	8,500	-42,000	3,500
S49	K32	NVM	NVM	K28	P3	8,500	-42,000	8,500	-45,000	3,000
S51	K33	NVM	NVM	K34	P12	0,000	-39,000	2,500	-39,000	2,500
S53	K33	NVM	NVM	K29	P4	0,000	-39,000	0,000	-42,000	3,000
S54	K34	NVM	NVM	K35	P1	2,500	-39,000	5,000	-39,000	2,500
S55	K35	NVM	NVM	K36	P12	5,000	-39,000	8,500	-39,000	3,500
S56	K36	NVM	NVM	K32	P3	8,500	-39,000	8,500	-42,000	3,000
S58	K37	NVM	NVM	K38	P12	0,000	-36,000	2,500	-36,000	2,500
S60	K37	NVM	NVM	K33	P4	0,000	-36,000	0,000	-39,000	3,000
S61	K38	NVM	NVM	K39	P1	2,500	-36,000	5,000	-36,000	2,500
S62	K39	NVM	NVM	K40	P12	5,000	-36,000	8,500	-36,000	3,500
S63	K40	NVM	NVM	K36	P3	8,500	-36,000	8,500	-39,000	3,000
S65	K41	NVM	NVM	K42	P12	0,000	-33,000	2,500	-33,000	2,500
S67	K41	NVM	NVM	K37	P4	0,000	-33,000	0,000	-36,000	3,000
S68	K42	NVM	NVM	K43	P1	2,500	-33,000	5,000	-33,000	2,500
S69	K43	NVM	NVM	K44	P12	5,000	-33,000	8,500	-33,000	3,500
S70	K44	NVM	NVM	K40	P3	8,500	-33,000	8,500	-36,000	3,000
S72	K45	NVM	NVM	K46	P12	0,000	-30,000	2,500	-30,000	2,500
S74	K45	NVM	NVM	K41	P4	0,000	-30,000	0,000	-33,000	3,000
S75	K46	NVM	NVM	K47	P1	2,500	-30,000	5,000	-30,000	2,500
S76	K47	NVM	NVM	K48	P12	5,000	-30,000	8,500	-30,000	3,500
S77	K48	NVM	NVM	K44	P3	8,500	-30,000	8,500	-33,000	3,000
S79	K49	NVM	NVM	K50	P12	0,000	-27,000	2,500	-27,000	2,500
S81	K49	NVM	NVM	K45	P4	0,000	-27,000	0,000	-30,000	3,000
S82	K50	NVM	NVM	K51	P1	2,500	-27,000	5,000	-27,000	2,500
S83	K51	NVM	NVM	K52	P12	5,000	-27,000	8,500	-27,000	3,500
S84	K52	NVM	NVM	K48	P3	8,500	-27,000	8,500	-30,000	3,000
S86	K53	NVM	NVM	K54	P12	0,000	-24,000	2,500	-24,000	2,500
S88	K53	NVM	NVM	K49	P4	0,000	-24,000	0,000	-27,000	3,000
S89	K54	NVM	NVM	K55	P1	2,500	-24,000	5,000	-24,000	2,500
S90	K55	NVM	NVM	K56	P12	5,000	-24,000	8,500	-24,000	3,500
S91	K56	NVM	NVM	K52	P3	8,500	-24,000	8,500	-27,000	3,000
S93	K57	NVM	NVM	K58	P12	0,000	-21,000	2,500	-21,000	2,500
S95	K57	NVM	NVM	K53	P4	0,000	-21,000	0,000	-24,000	3,000
S96	K58	NVM	NVM	K59	P1	2,500	-21,000	5,000	-21,000	2,500

Member	Node B	Release B	Release E	Node E	Section	X-B	Z-B	X-E	Z-E	Length
S97	K59	NVM	NVM	K60	P12	5,000	-21,000	8,500	-21,000	3,500
S98	K60	NVM	NVM	K56	P3	8,500	-21,000	8,500	-24,000	3,000
S100	K61	NVM	NVM	K62	P12	0,000	-18,000	2,500	-18,000	2,500
S102	K61	NVM	NVM	K57	P4	0,000	-18,000	0,000	-21,000	3,000
S103	K62	NVM	NVM	K63	P1	2,500	-18,000	5,000	-18,000	2,500
S104	K63	NVM	NVM	K64	P12	5,000	-18,000	8,500	-18,000	3,500
S105	K64	NVM	NVM	K60	P3	8,500	-18,000	8,500	-21,000	3,000
S107	K65	NVM	NVM	K66	P12	0,000	-15,000	2,500	-15,000	2,500
S109	K65	NVM	NVM	K61	P4	0,000	-15,000	0,000	-18,000	3,000
S110	K66	NVM	NVM	K67	P1	2,500	-15,000	5,000	-15,000	2,500
S111	K67	NVM	NVM	K68	P12	5,000	-15,000	8,500	-15,000	3,500
S112	K68	NVM	NVM	K64	P3	8,500	-15,000	8,500	-18,000	3,000
S114	K69	NVM	NVM	K70	P12	0,000	-12,000	2,500	-12,000	2,500
S116	K69	NVM	NVM	K65	P4	0,000	-12,000	0,000	-15,000	3,000
S117	K70	NVM	NVM	K71	P1	2,500	-12,000	5,000	-12,000	2,500
S118	K71	NVM	NVM	K72	P12	5,000	-12,000	8,500	-12,000	3,500
S119	K72	NVM	NVM	K68	P3	8,500	-12,000	8,500	-15,000	3,000
S121	K73	NVM	NVM	K74	P12	0,000	-9,000	2,500	-9,000	2,500
S123	K73	NVM	NVM	K69	P4	0,000	-9,000	0,000	-12,000	3,000
S124	K74	NVM	NVM	K75	P1	2,500	-9,000	5,000	-9,000	2,500
S125	K75	NVM	NVM	K76	P12	5,000	-9,000	8,500	-9,000	3,500
S126	K76	NVM	NVM	K72	P3	8,500	-9,000	8,500	-12,000	3,000
S128	K77	NVM	NVM	K78	P12	0,000	-6,000	2,500	-6,000	2,500
S130	K77	NVM	NVM	K73	P4	0,000	-6,000	0,000	-9,000	3,000
S131	K78	NVM	NVM	K79	P1	2,500	-6,000	5,000	-6,000	2,500
S132	K79	NVM	NVM	K80	P12	5,000	-6,000	8,500	-6,000	3,500
S135	K81	NVM	NVM	K82	P12	0,000	-3,000	2,500	-3,000	2,500
S136	K1	NVM	NVM	K81	P4	0,000	0,000	0,000	-3,000	3,000
S137	K81	NVM	NVM	K77	P4	0,000	-3,000	0,000	-6,000	3,000
S138	K82	NVM	NVM	K83	P1	2,500	-3,000	5,000	-3,000	2,500
S139	K83	NVM	NVM	K84	P12	5,000	-3,000	8,500	-3,000	3,500
S140	K84	NVM	NVM	K80	P3	8,500	-3,000	8,500	-6,000	3,000
S141	K6	NVM	NVM	K84	P3	8,500	0,000	8,500	-3,000	3,000
S142	K80	NVM	NVM	K76	P3	8,500	-6,000	8,500	-9,000	3,000
S143	K1	NVM	NVM	K85	P12	0,000	0,000	2,500	0,000	2,500
S144	K85	NVM	NVM	K86	P1	2,500	0,000	5,000	0,000	2,500
S145	K86	NVM	NVM	K6	P12	5,000	0,000	8,500	0,000	3,500
-	-	-	-	-	-	m	m	m	m	m

Sections

Section	Section Name	Area	Iy	Material	Angle
P1	R300x400	1.2000e-01	1.6000e-03	C45/55	0
P3	R300x7000	2.1000e+00	8.5750e+00	C45/55	0
P4	R300x5000	1.5000e+00	3.1250e+00	C45/55	0
P12		2.1400e+01	4.1600e+00	Mat. 3	0
-	-	m2	m4	-	°

Section Shapes

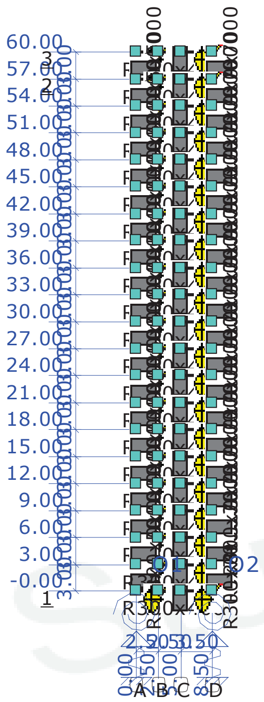
Section	Tapered	hB	hE	tf	tw	tf2	B	b1	b2	Castellate	Height
P1	No	0.400	0.400	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
P3	No	7.000	7.000	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
P4	No	5.000	5.000	0.000	0.000	0.000	0.300	0.000	0.000	No	0.000
-	-	m	m	m	m	m	m	m	m	-	m

Materials

Material Name	Density	Youngs mod.	Lin. Exp.
C45/55	24.00	3.6000e+07	10.0000e-06
Mat. 3	0.00	3.6000e+15	10.0000e-06
-	kN/m3	kN/m2	C°m

Supports

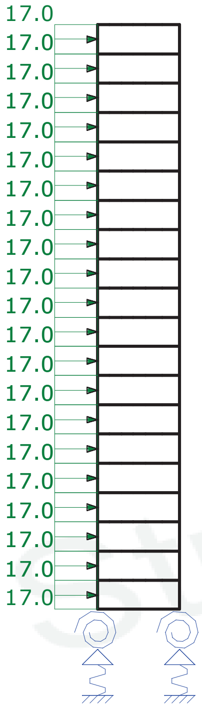
Support	Node	X	Z	Yr	AngleYr
O1	K1	fixed	153000	318750	0
O2	K6	fixed	214200	874650	0
-	-	kN/m	kN/m	kNmrad	°



Pic. Geometrie: Doorgaande Ligger

B.G.2: Wind load

Type	Value Begin	Value End	Dist. Begin	Dist. End	Direction	Member/Node
B.G.2: Wind load						
q	17,00	17,00	0,000	3,000(L)	Z'	S8,S18,S25,S32, S39,S46,S53,S60, S67,S74,S81,S88, S95,S102,S109, S116,S123,S130, S136-S137
Sum of loads		**X: 1.020,00**	**kN Z: 0,00**	**kN**		
-	**-**	**-**	**m**	**m**	**- -**	



B.G.2: Wind load

L.C. Extreme Member Forces

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S2	B.G.2	**290.32**	0.00	0.000	**96.66**	0.000	0.000	T	**12.29**	**-77.49**	**-77.49**	**-77.49**
S3	B.G.2	**96.66**	0.00	0.000	**-97.01**	1.248	0.000	T	**11.53**	**-77.47**	**-77.47**	**-77.47**
S4	B.G.2	**-97.02**	0.00	0.000	**-368.18**	0.000	0.000	T	**12.29**	**-77.47**	**-77.47**	**-77.47**
S8	B.G.2	**178.54**	0.00	0.000	**290.33**	0.000	0.000	T	**77.46**	**62.77**	**62.77**	**11.77**
S11	B.G.2	**405.77**	0.00	0.000	**368.16**	0.000	0.000	C	**-77.46**	**-12.54**	**-12.54**	**-12.54**
S13	B.G.2	**298.67**	0.00	0.000	**99.47**	0.000	0.000	C	**-20.48**	**-79.63**	**-79.63**	**-79.63**
S14	B.G.2	**99.48**	0.00	0.000	**-99.71**	1.249	0.000	C	**-19.62**	**-79.68**	**-79.68**	**-79.68**
S15	B.G.2	**-99.70**	0.00	0.000	**-378.59**	0.000	0.000	C	**-19.46**	**-79.67**	**-79.67**	**-79.67**
S16	B.G.2	**313.50**	0.00	0.000	**104.50**	0.000	0.000	C	**-34.82**	**-83.58**	**-83.58**	**-83.58**
S18	B.G.2	**266.04**	0.00	0.000	**477.17**	0.000	0.000	T	**157.15**	**95.88**	**95.88**	**44.88**
S19	B.G.2	**104.48**	0.00	0.000	**-104.58**	1.249	0.000	C	**-35.49**	**-83.62**	**-83.62**	**-83.62**
S20	B.G.2	**-104.59**	0.00	0.000	**-397.28**	0.000	0.000	C	**-33.79**	**-83.62**	**-83.62**	**-83.62**
S21	B.G.2	**766.83**	0.00	0.000	**784.34**	0.000	0.000	C	**-157.12**	**5.84**	**5.84**	**5.84**
S23	B.G.2	**333.48**	0.00	0.000	**111.15**	0.000	0.000	C	**-39.94**	**-88.91**	**-88.91**	**-88.91**
S25	B.G.2	**316.95**	0.00	0.000	**579.53**	0.000	0.000	T	**240.77**	**113.03**	**113.03**	**62.03**
S26	B.G.2	**111.15**	0.00	0.000	**-111.18**	1.250	0.000	C	**-40.13**	**-88.93**	**-88.93**	**-88.93**
S27	B.G.2	**-111.18**	0.00	0.000	**-422.46**	0.000	0.000	C	**-38.91**	**-88.94**	**-88.94**	**-88.94**
S28	B.G.2	**1045.88**	0.00	0.000	**1164.11**	0.000	0.000	C	**-240.74**	**39.41**	**39.41**	**39.41**
S30	B.G.2	**357.58**	0.00	0.000	**119.17**	0.000	0.000	C	**-38.91**	**-95.38**	**-95.38**	**-95.38**
S32	B.G.2	**350.54**	0.00	0.000	**650.39**	0.000	0.000	T	**329.71**	**125.45**	**125.45**	**74.45**
S33	B.G.2	**119.19**	0.00	0.000	**-119.19**	1.250	0.000	C	**-39.36**	**-95.35**	**-95.35**	**-95.35**
S34	B.G.2	**-119.20**	0.00	0.000	**-452.94**	0.000	0.000	C	**-38.91**	**-95.35**	**-95.35**	**-95.35**
S35	B.G.2	**1233.66**	0.00	0.000	**1468.35**	0.000	0.000	C	**-329.67**	**78.23**	**78.23**	**78.23**
S37	B.G.2	**384.92**	0.00	0.000	**128.40**	0.000	0.000	C	**-36.86**	**-102.64**	**-102.64**	**-102.64**
S39	B.G.2	**368.91**	0.00	0.000	**708.06**	0.000	0.000	T	**425.07**	**138.55**	**138.55**	**87.55**
S40	B.G.2	**128.33**	0.00	0.000	**-128.33**	1.250	0.000	C	**-37.15**	**-102.67**	**-102.67**	**-102.67**
S41	B.G.2	**-128.31**	0.00	0.000	**-487.71**	0.000	0.000	C	**-36.86**	**-102.68**	**-102.68**	**-102.68**
S42	B.G.2	**1336.22**	0.00	0.000	**1686.61**	0.000	0.000	C	**-425.01**	**116.79**	**116.79**	**116.79**
S44	B.G.2	**415.03**	0.00	0.000	**138.37**	0.000	0.000	C	**-34.82**	**-110.63**	**-110.63**	**-110.63**
S46	B.G.2	**369.04**	0.00	0.000	**753.86**	0.000	0.000	T	**527.74**	**153.77**	**153.77**	**102.77**
S47	B.G.2	**138.35**	0.00	0.000	**-138.36**	1.250	0.000	C	**-35.30**	**-110.68**	**-110.68**	**-110.68**
S48	B.G.2	**-138.37**	0.00	0.000	**-525.79**	0.000	0.000	C	**-34.82**	**-110.69**	**-110.69**	**-110.69**
S49	B.G.2	**1363.31**	0.00	0.000	**1823.91**	0.000	0.000	C	**-527.66**	**153.53**	**153.53**	**153.53**
S51	B.G.2	**446.99**	0.00	0.000	**149.06**	0.000	0.000	C	**-34.82**	**-119.20**	**-119.20**	**-119.20**

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S53	B.G.2	**348.28**	0.00	0.000	**784.06**	0.000	0.000	T	**638.43**	**170.76**	**170.76**	**119.76**
S54	B.G.2	**149.01**	0.00	0.000	**-149.03**	1.250	0.000	C	**-34.22**	**-119.22**	**-119.22**	**-119.22**
S55	B.G.2	**-149.02**	0.00	0.000	**-566.34**	0.000	0.000	C	**-34.82**	**-119.23**	**-119.23**	**-119.23**
S56	B.G.2	**1325.49**	0.00	0.000	**1889.09**	0.000	0.000	C	**-638.33**	**187.87**	**187.87**	**187.87**
S58	B.G.2	**480.26**	0.00	0.000	**160.13**	0.000	0.000	C	**-33.79**	**-128.04**	**-128.04**	**-128.04**
S60	B.G.2	**305.60**	0.00	0.000	**795.29**	0.000	0.000	T	**757.65**	**188.73**	**188.73**	**137.73**
S61	B.G.2	**160.09**	0.00	0.000	**-160.12**	1.250	0.000	C	**-33.76**	**-128.08**	**-128.08**	**-128.08**
S62	B.G.2	**-160.14**	0.00	0.000	**-608.43**	0.000	0.000	C	**-32.77**	**-128.09**	**-128.09**	**-128.09**
S63	B.G.2	**1226.04**	0.00	0.000	**1891.80**	0.000	0.000	C	**-757.55**	**221.92**	**221.92**	**221.92**
S65	B.G.2	**514.10**	0.00	0.000	**171.39**	0.000	0.000	C	**-33.79**	**-137.06**	**-137.06**	**-137.06**
S67	B.G.2	**241.14**	0.00	0.000	**785.86**	0.000	0.000	T	**885.74**	**207.07**	**207.07**	**156.07**
S68	B.G.2	**171.37**	0.00	0.000	**-171.40**	1.250	0.000	C	**-33.63**	**-137.11**	**-137.11**	**-137.11**
S69	B.G.2	**-171.38**	0.00	0.000	**-651.31**	0.000	0.000	C	**-33.28**	**-137.12**	**-137.12**	**-137.12**
S70	B.G.2	**1070.60**	0.00	0.000	**1834.49**	0.000	0.000	C	**-885.62**	**254.63**	**254.63**	**254.63**
S72	B.G.2	**547.93**	0.00	0.000	**182.66**	0.000	0.000	C	**-33.79**	**-146.10**	**-146.10**	**-146.10**
S74	B.G.2	**155.40**	0.00	0.000	**755.24**	0.000	0.000	T	**1022.85**	**225.45**	**225.45**	**174.45**
S75	B.G.2	**182.64**	0.00	0.000	**-182.67**	1.250	0.000	C	**-33.66**	**-146.12**	**-146.12**	**-146.12**
S76	B.G.2	**-182.66**	0.00	0.000	**-694.13**	0.000	0.000	C	**-32.77**	**-146.12**	**-146.12**	**-146.12**
S77	B.G.2	**859.86**	0.00	0.000	**1721.91**	0.000	0.000	C	**-1022.72**	**287.35**	**287.35**	**287.35**
S79	B.G.2	**581.11**	0.00	0.000	**193.65**	0.000	0.000	C	**-33.79**	**-154.98**	**-154.98**	**-154.98**
S81	B.G.2	**48.70**	0.00	0.000	**703.31**	0.000	0.000	T	**1168.98**	**243.70**	**243.70**	**192.70**
S82	B.G.2	**193.68**	0.00	0.000	**-193.72**	1.250	0.000	C	**-33.81**	**-154.96**	**-154.96**	**-154.96**
S83	B.G.2	**-193.74**	0.00	0.000	**-736.13**	0.000	0.000	C	**-33.79**	**-154.96**	**-154.96**	**-154.96**
S84	B.G.2	**593.97**	0.00	0.000	**1553.98**	0.000	0.000	C	**-1168.84**	**320.00**	**320.00**	**320.00**
S86	B.G.2	**612.90**	0.00	0.000	**204.29**	0.000	0.000	C	**-33.79**	**-163.45**	**-163.45**	**-163.45**
S88	B.G.2	**-78.90**	0.00	0.000	**629.74**	0.304	0.000	T	**1323.95**	**261.71**	**261.71**	**210.71**
S89	B.G.2	**204.29**	0.00	0.000	**-204.33**	1.250	0.000	C	**-34.19**	**-163.45**	**-163.45**	**-163.45**
S90	B.G.2	**-204.34**	0.00	0.000	**-776.43**	0.000	0.000	C	**-34.82**	**-163.46**	**-163.46**	**-163.46**
S91	B.G.2	**270.77**	0.00	0.000	**1330.10**	0.000	0.000	C	**-1323.80**	**353.11**	**353.11**	**353.11**
S93	B.G.2	**642.73**	0.00	0.000	**214.22**	0.000	0.000	C	**-34.82**	**-171.37**	**-171.37**	**-171.37**
S95	B.G.2	**-227.30**	0.00	0.000	**533.97**	0.835	0.000	T	**1487.40**	**279.26**	**279.26**	**228.26**
S96	B.G.2	**214.23**	0.00	0.000	**-214.27**	1.250	0.000	C	**-34.90**	**-171.40**	**-171.40**	**-171.40**
S97	B.G.2	**-214.27**	0.00	0.000	**-814.21**	0.000	0.000	C	**-34.56**	**-171.41**	**-171.41**	**-171.41**
S98	B.G.2	**-114.71**	0.00	0.000	**1047.19**	0.296	0.000	C	**-1487.25**	**387.30**	**387.30**	**387.30**
S100	B.G.2	**669.86**	0.00	0.000	**223.23**	0.000	0.000	C	**-35.84**	**-178.63**	**-178.63**	**-178.63**
S102	B.G.2	**-396.10**	0.00	0.000	**415.39**	1.394	0.000	T	**1658.81**	**296.00**	**296.00**	**245.00**
S103	B.G.2	**223.26**	0.00	0.000	**-223.31**	1.250	0.000	C	**-35.87**	**-178.63**	**-178.63**	**-178.63**
S104	B.G.2	**-223.33**	0.00	0.000	**-848.53**	0.000	0.000	C	**-35.33**	**-178.63**	**-178.63**	**-178.63**
S105	B.G.2	**-564.34**	0.00	0.000	**699.49**	1.340	0.000	C	**-1658.65**	**421.28**	**421.28**	**421.28**
S107	B.G.2	**693.38**	0.00	0.000	**231.17**	0.000	0.000	C	**-36.61**	**-184.89**	**-184.89**	**-184.89**
S109	B.G.2	**-584.84**	0.00	0.000	**273.68**	1.984	0.000	T	**1837.44**	**311.67**	**311.67**	**260.67**
S110	B.G.2	**231.13**	0.00	0.000	**-231.17**	1.250	0.000	C	**-36.69**	**-184.92**	**-184.92**	**-184.92**
S111	B.G.2	**-231.18**	0.00	0.000	**-878.38**	0.000	0.000	C	**-36.61**	**-184.91**	**-184.91**	**-184.91**
S112	B.G.2	**-1085.40**	0.00	0.000	**284.19**	2.377	0.000	C	**-1837.27**	**456.53**	**456.53**	**456.53**
S114	B.G.2	**712.62**	0.00	0.000	**237.58**	0.000	0.000	C	**-36.10**	**-190.02**	**-190.02**	**-190.02**
S116	B.G.2	**-794.29**	0.00	0.000	**108.54**	2.611	0.000	T	**2022.37**	**326.44**	**326.44**	**275.44**
S117	B.G.2	**237.54**	0.00	0.000	**-237.57**	1.250	0.000	C	**-36.21**	**-190.04**	**-190.04**	**-190.04**
S118	B.G.2	**-237.58**	0.00	0.000	**-902.77**	0.000	0.000	C	**-36.10**	**-190.05**	**-190.05**	**-190.05**
S119	B.G.2	**-1685.76**	0.00	0.000	**-207.03**	0.000	0.000	C	**-2022.17**	**492.91**	**492.91**	**492.91**
S121	B.G.2	**726.55**	0.00	0.000	**242.16**	0.000	0.000	C	**-32.26**	**-193.74**	**-193.74**	**-193.74**
S123	B.G.2	**-1029.97**	0.00	0.000	**-81.68**	0.000	0.000	T	**2212.42**	**341.60**	**341.60**	**290.60**
S124	B.G.2	**242.19**	0.00	0.000	**-242.17**	1.250	0.000	C	**-32.47**	**-193.74**	**-193.74**	**-193.74**
S125	B.G.2	**-242.15**	0.00	0.000	**-920.34**	0.000	0.000	C	**-32.51**	**-193.76**	**-193.76**	**-193.76**
S126	B.G.2	**-2369.35**	0.00	0.000	**-783.00**	0.000	0.000	C	**-2212.21**	**528.78**	**528.78**	**528.78**
S128	B.G.2	**733.86**	0.00	0.000	**244.66**	0.000	0.000	C	**-23.42**	**-195.68**	**-195.68**	**-195.68**
S130	B.G.2	**-1308.15**	0.00	0.000	**-303.46**	0.000	0.000	T	**2406.17**	**360.40**	**360.40**	**309.40**
S131	B.G.2	**244.65**	0.00	0.000	**-244.59**	1.250	0.000	C	**-23.44**	**-195.70**	**-195.70**	**-195.70**
S132	B.G.2	**-244.57**	0.00	0.000	**-929.54**	0.000	0.000	C	**-23.42**	**-195.71**	**-195.71**	**-195.71**
S135	B.G.2	**733.12**	0.00	0.000	**244.34**	0.000	0.000	C	**-9.66**	**-195.51**	**-195.51**	**-195.51**
S136	B.G.2	**-2141.25**	0.00	0.000	**-929.16**	0.000	0.000	T	**2797.35**	**429.53**	**429.53**	**378.53**
S137	B.G.2	**-1662.20**	0.00	0.000	**-574.30**	0.000	0.000	T	**2601.87**	**388.13**	**388.13**	**337.13**
S138	B.G.2	**244.39**	0.00	0.000	**-244.30**	1.250	0.000	C	**-9.69**	**-195.48**	**-195.48**	**-195.48**
S139	B.G.2	**-244.32**	0.00	0.000	**-928.46**	0.000	0.000	C	**-9.66**	**-195.47**	**-195.47**	**-195.47**
S140	B.G.2	**-3956.13**	0.00	0.000	**-2203.01**	0.000	0.000	C	**-2601.63**	**584.37**	**584.37**	**584.37**
S141	B.G.2	**-4809.63**	0.00	0.000	**-3027.65**	0.000	0.000	C	**-2797.10**	**593.99**	**593.99**	**593.99**
S142	B.G.2	**-3132.54**	0.00	0.000	**-1449.05**	0.000	0.000	C	**-2405.94**	**561.16**	**561.16**	**561.16**

Member	L.C.	Mb	Mmax	xMmax	Me	x-M0	x-M0	TC	Nmax	Vb	Vmax	Ve
S143	B.G.2	**722.00**	0.00	0.000	**240.61**	0.000	0.000	-	0.00	**-192.54**	**-192.54**	**-192.54**
S144	B.G.2	**240.64**	0.00	0.000	**-240.67**	1.250	0.000	-	0.00	**-192.53**	**-192.53**	**-192.53**
S145	B.G.2	**-240.66**	0.00	0.000	**-914.51**	0.000	0.000	-	0.00	**-192.52**	**-192.52**	**-192.52**
-	**-**	**kNm**	**kNm**	**m**	**kNm**	**m**	**m**	**-**	**kN**	**kN**	**kN**	**kN**

L.C. Extreme Support Reactions

Support	Node	L.C.	Xmax	Z	My	L.C.	X	Zmax	My	L.C.	X	Z	Mymax
O1	K1					B.G.2	-429.53	**2989.90**	1419.25	B.G.2	-429,53	2.989,90	**1.419,25**
O1	K1	B.G.2	**-429.53**	2989.90	1419.25								
O2	K6									B.G.2	-593,99	-2.989,63	**3.895,11**
O2	K6	B.G.2	**-593.99**	-2989.63	3895.11	B.G.2	-593.99	**-2989.63**	3895.11				
Global extreme values													
O2	K6	B.G.2	**-593.99**	-2989.63	3895.11								
O1	K1					B.G.2	-429.53	**2989.90**	1419.25				
O2	K6					B.G.2	-593.99	**-2989.63**	3895.11				
O2	K6									-	0,00	0,00	**0,00**
-	**-**	**-**	**kN**	**kN**	**kNm**	**-**	**kN**	**kN**	**kNm**		**kN**	**kN**	**kNm**

L.C. Extreme Nodal Displacements

Node	L.C.	X	Z	Ry
K1	B.G.2	0,0000	**-0,0195**	**-4.453e-03**
K2		**0,2708**	**-0,0209**	**-4.418e-03**
K3		**0,2708**	**-0,0099**	**-4.418e-03**
K4		**0,2708**	**-0,0006**	**-4.426e-03**
K5		**0,2708**	**0,0149**	**-4.426e-03**
K6		0,0000	**0,0140**	**-4.453e-03**
K7		**0,2575**	**-0,0209**	**-4.425e-03**
K10		**0,2575**	**0,0149**	**-4.430e-03**
K11		**0,2575**	**-0,0098**	**-4.425e-03**
K12		**0,2575**	**-0,0006**	**-4.430e-03**
K13		**0,2442**	**-0,0209**	**-4.435e-03**
K14		**0,2442**	**-0,0098**	**-4.435e-03**
K15		**0,2442**	**-0,0006**	**-4.437e-03**
K16		**0,2442**	**0,0149**	**-4.437e-03**
K17		**0,2309**	**-0,0209**	**-4.447e-03**
K18		**0,2309**	**-0,0098**	**-4.447e-03**
K19		**0,2309**	**-0,0007**	**-4.448e-03**
K20		**0,2309**	**0,0149**	**-4.448e-03**
K21		**0,2175**	**-0,0209**	**-4.461e-03**
K22		**0,2175**	**-0,0097**	**-4.461e-03**
K23		**0,2175**	**-0,0007**	**-4.461e-03**
K24		**0,2175**	**0,0149**	**-4.461e-03**
K25		**0,2041**	**-0,0208**	**-4.476e-03**
K26		**0,2041**	**-0,0096**	**-4.476e-03**
K27		**0,2041**	**-0,0008**	**-4.476e-03**
K28		**0,2041**	**0,0149**	**-4.476e-03**
K29		**0,1907**	**-0,0208**	**-4.491e-03**
K30		**0,1907**	**-0,0096**	**-4.491e-03**
K31		**0,1907**	**-0,0009**	**-4.491e-03**
K32		**0,1907**	**0,0149**	**-4.491e-03**
K33		**0,1772**	**-0,0208**	**-4.506e-03**
K34		**0,1772**	**-0,0095**	**-4.506e-03**
K35		**0,1772**	**-0,0009**	**-4.507e-03**
K36		**0,1772**	**0,0148**	**-4.507e-03**
K37		**0,1636**	**-0,0207**	**-4.521e-03**
K38		**0,1636**	**-0,0094**	**-4.521e-03**
K39		**0,1636**	**-0,0010**	**-4.522e-03**
K40		**0,1636**	**0,0148**	**-4.522e-03**
K41		**0,1500**	**-0,0207**	**-4.535e-03**
K42		**0,1500**	**-0,0093**	**-4.535e-03**
K43		**0,1500**	**-0,0011**	**-4.536e-03**
K44		**0,1500**	**0,0148**	**-4.536e-03**
K45		**0,1364**	**-0,0206**	**-4.548e-03**
K46		**0,1364**	**-0,0093**	**-4.548e-03**
K47		**0,1364**	**-0,0012**	**-4.549e-03**
K48		**0,1364**	**0,0147**	**-4.549e-03**
K49		**0,1228**	**-0,0206**	**-4.558e-03**
K50		**0,1228**	**-0,0092**	**-4.558e-03**
K51		**0,1227**	**-0,0013**	**-4.559e-03**

Node	L.C.	X	Z	Ry
K52	B.G.2	**0,1227**	**0,0147**	**-4.559e-03**
K53		**0,1091**	**-0,0205**	**-4.566e-03**
K54		**0,1091**	**-0,0091**	**-4.566e-03**
K55		**0,1090**	**-0,0014**	**-4.567e-03**
K56		**0,1090**	**0,0146**	**-4.567e-03**
K57		**0,0954**	**-0,0204**	**-4.570e-03**
K58		**0,0954**	**-0,0090**	**-4.570e-03**
K59		**0,0953**	**-0,0014**	**-4.571e-03**
K60		**0,0953**	**0,0146**	**-4.571e-03**
K61		**0,0816**	**-0,0203**	**-4.571e-03**
K62		**0,0816**	**-0,0089**	**-4.571e-03**
K63		**0,0816**	**-0,0015**	**-4.572e-03**
K64		**0,0816**	**0,0145**	**-4.572e-03**
K65		**0,0679**	**-0,0202**	**-4.567e-03**
K66		**0,0679**	**-0,0088**	**-4.567e-03**
K67		**0,0679**	**-0,0016**	**-4.568e-03**
K68		**0,0679**	**0,0144**	**-4.568e-03**
K69		**0,0542**	**-0,0201**	**-4.558e-03**
K70		**0,0542**	**-0,0087**	**-4.558e-03**
K71		**0,0542**	**-0,0016**	**-4.559e-03**
K72		**0,0542**	**0,0144**	**-4.559e-03**
K73		**0,0406**	**-0,0200**	**-4.544e-03**
K74		**0,0406**	**-0,0086**	**-4.544e-03**
K75		**0,0405**	**-0,0016**	**-4.544e-03**
K76		**0,0405**	**0,0143**	**-4.544e-03**
K77		**0,0270**	**-0,0198**	**-4.523e-03**
K78		**0,0270**	**-0,0085**	**-4.523e-03**
K79		**0,0269**	**-0,0017**	**-4.521e-03**
K80		**0,0269**	**0,0142**	**-4.521e-03**
K81		**0,0134**	**-0,0197**	**-4.493e-03**
K82		**0,0134**	**-0,0085**	**-4.493e-03**
K83		**0,0134**	**-0,0017**	**-4.491e-03**
K84		**0,0134**	**0,0141**	**-4.491e-03**
K85		0,0000	**-0,0084**	**-4.453e-03**
K86		0,0000	**-0,0016**	**-4.453e-03**
-	-	m	m	rad

L.C. Extreme Deflections

Member	L.C.	Node Begin		Member		Node End	
		X	Z	Z' dist	Z'	X	Z
S3	B.G.2	0,271	-0,010	1.969	**-0.0002**	0,271	-0,001
S8	B.G.2	0,258	-0,021	1.555	**0.0000**	0,271	-0,021
S11	B.G.2	0,258	0,015	1.488	**0.0000**	0,271	0,015
S14	B.G.2	0,258	-0,010	1.970	**-0.0002**	0,258	-0,001
S18	B.G.2	0,244	-0,021	1.567	**0.0000**	0,258	-0,021
S19	B.G.2	0,244	-0,010	1.971	**-0.0002**	0,244	-0,001
S21	B.G.2	0,244	0,015	1.503	**0.0000**	0,258	0,015
S25	B.G.2	0,231	-0,021	1.570	**0.0000**	0,244	-0,021
S26	B.G.2	0,231	-0,010	1.972	**-0.0002**	0,231	-0,001
S28	B.G.2	0,231	0,015	1.513	**0.0000**	0,244	0,015
S32	B.G.2	0,218	-0,021	1.572	**0.0000**	0,231	-0,021
S33	B.G.2	0,218	-0,010	1.972	**-0.0002**	0,218	-0,001
S35	B.G.2	0,218	0,015	1.522	**0.0000**	0,231	0,015
S39	B.G.2	0,204	-0,021	1.575	**0.0000**	0,218	-0,021
S40	B.G.2	0,204	-0,010	1.972	**-0.0002**	0,204	-0,001
S42	B.G.2	0,204	0,015	1.529	**0.0000**	0,218	0,015
S46	B.G.2	0,191	-0,021	1.582	**0.0000**	0,204	-0,021
S47	B.G.2	0,191	-0,010	1.972	**-0.0002**	0,191	-0,001
S49	B.G.2	0,191	0,015	1.536	**0.0000**	0,204	0,015
S53	B.G.2	0,177	-0,021	1.592	**0.0000**	0,191	-0,021
S54	B.G.2	0,177	-0,010	1.972	**-0.0003**	0,177	-0,001
S56	B.G.2	0,177	0,015	1.544	**0.0000**	0,191	0,015
S60	B.G.2	0,164	-0,021	1.606	**0.0000**	0,177	-0,021
S61	B.G.2	0,164	-0,009	1.972	**-0.0003**	0,164	-0,001
S63	B.G.2	0,164	0,015	1.553	**0.0000**	0,177	0,015
S67	B.G.2	0,150	-0,021	1.625	**0.0000**	0,164	-0,021
S68	B.G.2	0,150	-0,009	1.972	**-0.0003**	0,150	-0,001

Member	L.C.	Node Begin		Member		Node End	
		X	Z	Z' dist	Z'	X	Z
S70	B.G.2	0,150	0,015	1.565	**0.0000**	0,164	0,015
S74	B.G.2	0,136	-0,021	1.653	**0.0000**	0,150	-0,021
S75	B.G.2	0,136	-0,009	1.972	**-0.0003**	0,136	-0,001
S77	B.G.2	0,136	0,015	1.583	**0.0000**	0,150	0,015
S81	B.G.2	0,123	-0,021	1.696	**0.0000**	0,136	-0,021
S82	B.G.2	0,123	-0,009	1.972	**-0.0003**	0,123	-0,001
S84	B.G.2	0,123	0,015	1.610	**0.0000**	0,136	0,015
S88	B.G.2	0,109	-0,020	1.771	**0.0000**	0,123	-0,021
S89	B.G.2	0,109	-0,009	1.972	**-0.0004**	0,109	-0,001
S91	B.G.2	0,109	0,015	1.660	**0.0000**	0,123	0,015
S95	B.G.2	0,095	-0,020	1.922	**0.0000**	0,109	-0,020
S96	B.G.2	0,095	-0,009	1.972	**-0.0004**	0,095	-0,001
S98	B.G.2	0,095	0,015	1.779	**0.0000**	0,109	0,015
S103	B.G.2	0,082	-0,009	1.972	**-0.0004**	0,082	-0,001
S109	B.G.2	0,068	-0,020	0.990	**0.0000**	0,082	-0,020
S110	B.G.2	0,068	-0,009	1.972	**-0.0004**	0,068	-0,002
S112	B.G.2	0,068	0,014	1.145	**0.0000**	0,082	0,015
S116	B.G.2	0,054	-0,020	1.195	**0.0000**	0,068	-0,020
S117	B.G.2	0,054	-0,009	1.972	**-0.0004**	0,054	-0,002
S119	B.G.2	0,054	0,014	1.314	**0.0000**	0,068	0,014
S123	B.G.2	0,041	-0,020	1.292	**0.0000**	0,054	-0,020
S124	B.G.2	0,041	-0,009	0.528	**0.0004**	0,041	-0,002
S126	B.G.2	0,041	0,014	1.377	**0.0000**	0,054	0,014
S130	B.G.2	0,027	-0,020	1.345	**0.0000**	0,041	-0,020
S131	B.G.2	0,027	-0,009	0.528	**0.0004**	0,027	-0,002
S136	B.G.2	0,000	-0,020	1.401	**0.0000**	0,013	-0,020
S137	B.G.2	0,013	-0,020	1.379	**0.0000**	0,027	-0,020
S138	B.G.2	0,013	-0,008	0.529	**0.0004**	0,013	-0,002
S140	B.G.2	0,013	0,014	1.429	**0.0000**	0,027	0,014
S141	B.G.2	0,000	0,014	1.443	**0.0000**	0,013	0,014
S142	B.G.2	0,027	0,014	1.409	**0.0000**	0,041	0,014
S144	B.G.2	0,000	-0,008	1.972	**-0.0004**	0,000	-0,002
-	**-**	**m**	**m**	**m**	**m**	**m**	**m**

16 Appendix 7:

Coupled walls system with different modulus of elasticity supported on the rigid foundation according to the continuous method.

$restart;$

$$\delta_1 := l \cdot \frac{dx}{dz}:$$

$$\delta_2 := \frac{dN}{dz} \cdot \frac{b^3 \cdot h}{12 \cdot E_{cb} \cdot I_e}:$$

$$\delta_3 := -\frac{1}{E} \cdot \left(\frac{1}{A_1} + \frac{1}{A_2} \right) \int_0^z N(z)\, dz:$$

$$\delta := \delta_1 + \delta_2 + \delta_3;$$

$$l \left(\frac{d}{dz} x(z) \right) + \frac{1}{12} \frac{\left(\frac{d}{dz} N(z) \right) b^3 h}{E_{cb} I_e} - \frac{\left(\frac{1}{A_1} + \frac{1}{A_2} \right) \left(\int_0^z N(z)\, dz \right)}{E} \quad \textbf{(1)}$$

$$M_1 := m(z) - \left(\frac{b}{2} + d_1 \right) \cdot \int_z^H q(z)\, dz - M_0 = E \cdot I_1 \frac{d}{dz} \left(\frac{dx}{dz} \right):$$

$$M_2 := -\left(\frac{b}{2} + d_2 \right) \cdot \int_z^H q(z)\, dz + M_0 = E \cdot I_2 \frac{d}{dz} \left(\frac{dx}{dz} \right):$$

$$(M_1 + M_2);$$

$$m(z) - \left(\frac{1}{2} b + d_1 \right) \left(\int_z^H q(z)\, dz \right) - \left(\frac{1}{2} b + d_2 \right) \left(\int_z^H q(z)\, dz \right) = E\, I_1 \left(\frac{d^2}{dz^2} x(z) \right) + E\, I_2 \left(\frac{d^2}{dz^2} x(z) \right) \quad \textbf{(2)}$$

$$sol_1 := algsubs\left(\left(\int_z^H q(z)\, dz \right) = N(z), algsubs\left((-b - d_1 - d_2) = -l, collect\left(collect(M_1 + M_2, diff), \int_z^H q(z)\, dz \right) \right) \right);$$

$$-l\, N(z) + m(z) = (E\, I_1 + E\, I_2) \left(\frac{d^2}{dz^2} x(z) \right) \quad \textbf{(3)}$$

$$s_1 := collect\left(collect\left(algsubs\left(\left(\frac{d^2}{dz^2} x(z) \right) = \frac{-l\, N(z) + m(z)}{(E\, I_1 + E\, I_2)}, \frac{d}{dz} \delta = 0 \right), \frac{d^2}{dz^2} N(z) \right), N(z) \right);$$

$$\frac{1}{12} \frac{\left(-12\, l^2 I_e A_1 A_2 E_{cb} - 12\, I_1 I_e A_1 E_{cb} - 12\, I_1 I_e A_2 E_{cb} - 12\, I_2 I_e A_1 E_{cb} - 12\, I_2 I_e A_2 E_{cb} \right) N(z)}{E\, A_1 A_2 (I_1 + I_2) E_{cb} I_e} + \frac{1}{12} \frac{\left(E\, b^3 h\, I_1 A_1 A_2 + E\, b^3 h\, I_2 A_1 A_2 \right) \left(\frac{d^2}{dz^2} N(z) \right)}{E\, A_1 A_2 (I_1 + I_2) E_{cb} I_e} + \frac{m(z)\, l}{E\, (I_1 + I_2)} = 0 \quad \textbf{(4)}$$

$$s_2 := collect\left(collect\left(simplify\left(\frac{s_1\cdot(12\cdot E\cdot A_1\,A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,E_{cb}\,\mathrm{I}_e)}{(b^3\,h\,E\,A_1\,A_2\,\mathrm{I}_1+b^3\,h\,E\,A_1\,A_2\,\mathrm{I}_2)}\right),\left(\frac{d^2}{dz^2}N(z)\right)\right),N(z)\right);$$

$$\frac{(-12\,l^2\,\mathrm{I}_e\,A_1\,A_2\,E_{cb}-12\,\mathrm{I}_1\,\mathrm{I}_e\,A_1\,E_{cb}-12\,\mathrm{I}_1\,\mathrm{I}_e\,A_2\,E_{cb}-12\,\mathrm{I}_2\,\mathrm{I}_e\,A_1\,E_{cb}-12\,\mathrm{I}_2\,\mathrm{I}_e\,A_2\,E_{cb})\,N(z)}{E\,A_1\,A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}+\frac{(E\,b^3\,h\,\mathrm{I}_1\,A_1\,A_2+E\,b^3\,h\,\mathrm{I}_2\,A_1\,A_2)\left(\frac{d^2}{dz^2}N(z)\right)}{E\,A_1\,A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}+\frac{12\,\mathrm{I}_e\,m(z)\,l\,E_{cb}}{E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}=0 \qquad (5)$$

$$simplify\left(\frac{(-12\,E_{cb}\,\mathrm{I}_e\,A_2\,\mathrm{I}_1-12\,E_{cb}\,\mathrm{I}_e\,A_2\,\mathrm{I}_2-12\,E_{cb}\,\mathrm{I}_e\,A_1\,\mathrm{I}_1-12\,E_{cb}\,\mathrm{I}_e\,A_1\,\mathrm{I}_2-12\,l^2\,A_1\,A_2\,E_{cb}\,\mathrm{I}_e)\,N(z)}{E\,A_1\,A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}\right)+simplify\left(\frac{(b^3\,h\,E\,A_1\,A_2\,\mathrm{I}_1+b^3\,h\,E\,A_1\,A_2\,\mathrm{I}_2)\left(\frac{d^2}{dz^2}N(z)\right)}{E\,A_1\,A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}\right)+\frac{12\,l\,m(z)\,E_{cb}\,\mathrm{I}_e}{E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}=0;$$

$$-\frac{12\,E_{cb}\,\mathrm{I}_e\,(l^2\,A_1\,A_2+\mathrm{I}_1\,A_1+\mathrm{I}_1\,A_2+\mathrm{I}_2\,A_1+\mathrm{I}_2\,A_2)\,N(z)}{b^3\,h\,E\,A_1\,A_2\,(\mathrm{I}_1+\mathrm{I}_2)}+\frac{d^2}{dz^2}N(z)+\frac{12\,\mathrm{I}_e\,m(z)\,l\,E_{cb}}{E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}=0 \qquad (6)$$

assumme E_cb/E is equal to ζ^2 and we rewrite the ordinary differential equation

$$\left(\frac{12\cdot I_e\cdot l^2}{b^3\cdot h\cdot i_t}\right)=\alpha^2:\left(1+\frac{A\cdot i_t}{A_1\cdot A_2\cdot l^2}\right)=k^2:(I_1+I_2)=i_t:\frac{E_{cb}}{E}=\zeta^2:$$

$$\frac{d^2}{dz^2}N(z)-\frac{E_{cb}}{E}\cdot\frac{12\,\mathrm{I}_e\cdot l^2}{b^3\,h\cdot i_t}\cdot\left(1+\frac{A\cdot i_t}{A_1\cdot A_2\cdot l^2}\right)N(z)+\frac{12\,\mathrm{I}_e\cdot l}{b^3\,h\cdot i_t}\cdot\frac{E_{cb}}{E}\cdot m(z)=0;$$

$$\frac{d^2}{dz^2}N(z)-\frac{12\,E_{cb}\,\mathrm{I}_e\,l^2\left(1+\frac{A\,i_t}{A_1\,A_2\,l^2}\right)N(z)}{E\,b^3\,h\,i_t}+\frac{12\,\mathrm{I}_e\,l\,E_{cb}\,m(z)}{b^3\,h\,i_t\,E}=0 \qquad (7)$$

$$eq_{normal,\,force} := \frac{d^2}{dz^2}N(z)-k^2\cdot\alpha^2\cdot\zeta^2\cdot N(z)+\frac{\alpha^2}{l}\cdot\zeta^2\cdot m(z)=0;$$

$$\frac{d^2}{dz^2}N(z)-k^2\,\alpha^2\,\zeta^2\,N(z)+\frac{\alpha^2\,\zeta^2\,m(z)}{l}=0 \qquad (8)$$

Deflection:

$$eq_{moment,\,cur} := -l\,N(z)+m(z)=(E\cdot i_t)\left(\frac{d^2}{dz^2}x(z)\right);$$

$$-l\,N(z)+m(z)=E\,i_t\left(\frac{d^2}{dz^2}x(z)\right) \qquad (9)$$

$$so_1 := solve(eq_{moment,\,cur},N(z));$$

$$-\frac{E\,i_t\left(\frac{d^2}{dz^2}x(z)\right)-m(z)}{l} \tag{10}$$

$so_2 := diff\left(so_1, z, z\right);$

$$-\frac{E\,i_t\left(\frac{d^4}{dz^4}x(z)\right)-\left(\frac{d^2}{dz^2}m(z)\right)}{l} \tag{11}$$

$eq_{deflec,\,4} := algsubs\left(N(z)=so_1, algsubs\left(\frac{d^2}{dz^2}N(z)=so_2, \frac{d}{dz}\delta=0\right)\right);$

$$-\frac{1}{12}\frac{1}{E_{cb}\,I_e\,l\,E\,A_1\,A_2}\Bigg(E^2\left(\frac{d^4}{dz^4}x(z)\right)b^3\,h\,A_1\,A_2\,i_t-12\,l^2\left(\frac{d^2}{dz^2}x(z)\right)E_{cb}\,I_e\,E\,A_1\,A_2 -E\left(\frac{d^2}{dz^2}m(z)\right)b^3\,h\,A_1\,A_2-12\,E\left(\frac{d^2}{dz^2}x(z)\right)I_e\,A_1\,E_{cb}\,i_t-12\,E\left(\frac{d^2}{dz^2}x(z)\right)I_e\,A_2\,E_{cb}\,i_t +12\,m(z)\,I_e\,A_1\,E_{cb}+12\,m(z)\,I_e\,A_2\,E_{cb}\Bigg)=0 \tag{12}$$

$eq_{deflection,\,4} := algsubs\left(\left(-\frac{12\,I_e\,E_{cb}}{h\,b^3\,i_t\,E^2\,A_2}-\frac{12\,I_e\,E_{cb}}{h\,b^3\,i_t\,E^2\,A_1}\right)=-\frac{(k^2-1)\cdot\alpha^2\cdot\zeta^2}{i_t\,E}, algsubs\left(\left(\frac{12\,I_e\,E_{cb}}{h\,b^3\,E\,A_2}+\frac{12\,l^2\,I_e\,E_{cb}}{h\,b^3\,i_t\,E}+\frac{12\,I_e\,E_{cb}}{h\,b^3\,E\,A_1}\right)=k^2\cdot\alpha^2\cdot\zeta^2, collect\left(collect\left(expand\left(\left(eq_{deflec,\,4}\right)\cdot\left(\frac{12\cdot I_e\cdot l}{b^3\cdot h\cdot i_t}\cdot\frac{E_{cb}}{E}\right)\right), \frac{d^2}{dz^2}x(z)\right), m(z)\right)\right)\right);$

$$-\frac{m(z)\,(k^2-1)\,\alpha^2\,\zeta^2}{i_t\,E}+\frac{\frac{d^2}{dz^2}m(z)}{i_t\,E}+\left(\frac{d^2}{dz^2}x(z)\right)k^2\,\alpha^2\,\zeta^2-\left(\frac{d^4}{dz^4}x(z)\right)=0 \tag{13}$$

$m(z) := \frac{w\cdot(H-z)^2}{2};$

$$z\rightarrow\frac{1}{2}\,w\,(H-z)^2 \tag{14}$$

$convert\left(dsolve\left(eq_{normal,\,force}\right), trig\right);$

$$N(z)=\left(\cosh(k\,\alpha\,\zeta\,z)+\sinh(k\,\alpha\,\zeta\,z)\right)\_C2+\left(\cosh(k\,\alpha\,\zeta\,z)-\sinh(k\,\alpha\,\zeta\,z)\right)\_C1 +\frac{1}{2}\frac{w\left(2+k^2\,\zeta^2\,(H-z)^2\,\alpha^2\right)}{k^4\,l\,\alpha^2\,\zeta^2} \tag{15}$$

$ics := N(H)=0, D(N)(0)=0;$

$$N(H)=0,\ D(N)(0)=0 \tag{16}$$

$$simplify(combine(convert(dsolve(\{eq_{normal,\,force}, ics\}), trig)), size);$$

$$N(z) = \frac{1}{2} \frac{1}{k^4 l \alpha^2 \zeta^2 \cosh(k \alpha \zeta H)} \left(w\left(-2 H \sinh(k \alpha \zeta (H-z)) \zeta \alpha k + \left(2 + k^2 \zeta^2 (H-z)^2 \alpha^2\right) \cosh(k \alpha \zeta H) - 2\cosh(k \alpha \zeta z)\right)\right) \quad (17)$$

$$N_{uni,\,l,\,rigid} := \frac{1}{2} \frac{1}{k^4 l \alpha^2 \zeta^2 \cosh(k \alpha \zeta H)} \left(w\left(-2 k \alpha \zeta H \sinh(k \alpha \zeta (H-z)) + \left(2 + k^2 \zeta^2 (H-z)^2 \alpha^2\right) \cosh(k \alpha \zeta H) - 2\cosh(k \alpha \zeta z)\right)\right);$$

$$\frac{1}{2} \frac{1}{k^4 l \alpha^2 \zeta^2 \cosh(k \alpha \zeta H)} \left(w\left(-2 H \sinh(k \alpha \zeta (H-z)) \zeta \alpha k + \left(2 + k^2 \zeta^2 (H-z)^2 \alpha^2\right) \cosh(k \alpha \zeta H) - 2\cosh(k \alpha \zeta z)\right)\right) \quad (18)$$

$$\frac{1}{2} \frac{w\left(-2 k \alpha H \sinh(k \alpha (H-z)) + \left(2 + k^2 (H-z)^2 \alpha^2\right) \cosh(k \alpha H) - 2 \cosh(k \alpha z)\right)}{k^4 l \alpha^2 \cosh(k \alpha H)}$$

shear force:

$$q_{uni,\,load} := -diff(N_{uni,\,l,\,rigid}, z);$$

$$-\frac{1}{2} \frac{1}{k^4 l \alpha^2 \zeta^2 \cosh(k \alpha \zeta H)} \left(w\left(2 H \cosh(k \alpha \zeta (H-z)) k^2 \alpha^2 \zeta^2 - 2 k^2 \zeta^2 (H-z) \alpha^2 \cosh(k \alpha \zeta H) - 2 \sinh(k \alpha \zeta z) k \alpha \zeta\right)\right) \quad (19)$$

position of maximum shear

$$h_{\max,\,shear} := simplify\left(diff(q_{uni,\,load}, z) \cdot \frac{k^2 \cdot l}{w \cdot H}\right);$$

$$\frac{H \sinh(k \alpha \zeta (H-z)) \zeta \alpha k - \cosh(k \alpha \zeta H) + \cosh(k \alpha \zeta z)}{H \cosh(k \alpha \zeta H)} \quad (20)$$

$$-\frac{-y \sinh(y(1-x)) - \cosh(y x) + \cosh(y)}{\cosh(y)}$$

axial force in beams:

$$S_1 := \frac{w \cdot H \cdot I_1}{i_t}\left(1 - \frac{z}{H}\right) - \left(\frac{I_1}{i_t} \cdot l - \frac{b}{2} - d_1\right) \cdot q_{uni,\,load}:$$

$$S_2 := \frac{w \cdot H \cdot I_2}{(i_t)}\left(1 - \frac{z}{H}\right) + \left(\frac{I_1}{i_t} \cdot l - \frac{b}{2} - d_1\right) \cdot q_{uni,\,load}:$$

$$n := \frac{1}{2} \cdot \left(\frac{d}{dz} S_1 - \frac{d}{dz} S_2 + w\right);$$

$$-\frac{1}{2} \frac{w I_1}{i_t} + \frac{1}{2} \frac{1}{k^4 l \alpha^2 \zeta^2 \cosh(k \alpha \zeta H)} \left(\left(\left(\frac{I_1 l}{i_t} - \frac{1}{2} b - d_1\right) w \left(-2 H \sinh(k \alpha \zeta (H \right.\right.\right. \quad (21)$$

$$-z)) k^3 \alpha^3 \zeta^3 + 2 k^2 \zeta^2 \alpha^2 \cosh(k \alpha \zeta H) - 2 \cosh(k \alpha \zeta z) k^2 \alpha^2 \zeta^2 \Big) \Big) + \frac{1}{2} \frac{w \, \mathrm{l}_2}{i_t} + \frac{1}{2} w$$

individual and composite factor:

$$es_1 := \frac{\left(\frac{1}{2} \cdot w \cdot (H-z)^2 - (N_{uni,\, l,\, rigid}) \cdot l\right) \cdot c_1}{i_t} + \frac{(N_{uni,\, l,\, rigid})}{A_1};$$

$$\frac{1}{i_t} \Bigg(\Bigg(\frac{1}{2} w (H-z)^2 - \frac{1}{2} \frac{1}{k^4 \alpha^2 \zeta^2 \cosh(k \alpha \zeta H)} \Big(w \big(-2 H \sinh(k \alpha \zeta (H-z)) \zeta \alpha k + \big(2 + k^2 \zeta^2 (H-z)^2 \alpha^2\big) \cosh(k \alpha \zeta H) - 2 \cosh(k \alpha \zeta z) \big) \Big) \Bigg) c_1 \Bigg) + \frac{1}{2} \frac{1}{k^4 l \alpha^2 \zeta^2 \cosh(k \alpha \zeta H) A_1} \Big(w \big(-2 H \sinh(k \alpha \zeta (H-z)) \zeta \alpha k + \big(2 + k^2 \zeta^2 (H-z)^2 \alpha^2\big) \cosh(k \alpha \zeta H) - 2 \cosh(k \alpha \zeta z) \big) \Big) \tag{22}$$

$$es_2 := \frac{w \cdot (H-z)^2}{2 \cdot \left(i_t + \frac{A_1 \cdot A_2}{A} \cdot l^2\right)} \cdot \left(\frac{A_2 \cdot l}{A} + c_1\right) \cdot \left(\frac{k_2}{100}\right);$$

$$\frac{1}{100} \frac{w (H-z)^2 \left(\frac{A_2 l}{A} + c_1\right) k_2}{2 i_t + \frac{2 A_1 A_2 l^2}{A}} \tag{23}$$

$$es_3 := \frac{1}{2} \cdot \frac{w \cdot (H-z)^2 \cdot c_1}{i_t} \cdot \frac{(100 - k_2)}{100};$$

$$\frac{1}{2} \frac{w (H-z)^2 c_1 \left(1 - \frac{1}{100} k_2\right)}{i_t} \tag{24}$$

$$es_4 := solve(es_1 = es_2 + es_3, k_2) \cdot \frac{(H - z)^2 \cdot k^2 \alpha^2 \cdot \zeta^2}{200};$$

$$\frac{1}{2} \frac{1}{A_2 A_1 \cosh(k \alpha \zeta H) l^2 k^2} \Big(\big(l^2 A_1 A_2 + A i_t\big) \big(H^2 \cosh(k \alpha \zeta H) \zeta^2 \alpha^2 k^2 - 2 H \cosh(k \alpha \zeta H) \zeta^2 \alpha^2 k^2 z + \cosh(k \alpha \zeta H) \zeta^2 \alpha^2 k^2 z^2 - 2 H \sinh(k \alpha \zeta (H-z)) \zeta \alpha k + 2 \cosh(k \alpha \zeta H) - 2 \cosh(k \alpha \zeta z) \big) \Big) \tag{25}$$

$$expand\left(algsubs\left(\frac{i_t A}{l^2 A_1 A_2} = k^2 - 1, \text{expand}(\text{es}_4)\right)\right);$$

$$\frac{1}{2} H^2 \zeta^2 \alpha^2 k^2 - H \zeta^2 \alpha^2 k^2 z + \frac{1}{2} \zeta^2 \alpha^2 k^2 z^2 - \frac{k \alpha \zeta H \sinh(k \alpha \zeta H) \cosh(k \alpha \zeta z)}{\cosh(k \alpha \zeta H)} \tag{26}$$

$$+k\,\alpha\,\zeta\,H\sinh(k\,\alpha\,\zeta\,z)+1-\frac{\cosh(k\,\alpha\,\zeta\,z)}{\cosh(k\,\alpha\,\zeta\,H)}$$

$$combine\left(k\,\alpha\,\zeta\,H\sinh(k\,\alpha\,\zeta\,z)-\frac{k\,\alpha\,\zeta\,H\cosh(k\,\alpha\,\zeta\,z)\sinh(k\,\alpha\,\zeta\,H)}{\cosh(k\,\alpha\,\zeta\,H)}-\frac{\cosh(k\,\alpha\,\zeta\,z)}{\cosh(k\,\alpha\,\zeta\,H)}\right)+1$$
$$+factor\left(\frac{1}{2}\,k^2\,\alpha^2\,\zeta^2\,H^2-k^2\,\alpha^2\,\zeta^2\,H\,z+\frac{1}{2}\,k^2\,\alpha^2\,\zeta^2\,z^2\right);$$

$$\frac{-k\,\alpha\,\zeta\,H\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)-\cosh(k\,\alpha\,\zeta\,z)}{\cosh(k\,\alpha\,\zeta\,H)}+1+\frac{1}{2}\,k^2\,\zeta^2\,(H-z)^2\,\alpha^2 \tag{27}$$

$$K_2:=\left(\frac{-k\,\alpha\,\zeta\,H\sinh(-k\,\alpha\,\zeta\,z+k\,\alpha\,\zeta\,H)-\cosh(k\,\alpha\,\zeta\,z)}{\cosh(k\,\alpha\,\zeta\,H)}+1+\frac{1}{2}\,k^2\,\zeta^2\,(H-z)^2\,\alpha^2\right)$$
$$\cdot\frac{200}{(H-z)^2\cdot k^2\,\alpha^2\cdot\zeta^2};$$

$$\frac{200\left(\frac{-k\,\alpha\,\zeta\,H\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)-\cosh(k\,\alpha\,\zeta\,z)}{\cosh(k\,\alpha\,\zeta\,H)}+1+\frac{1}{2}\,k^2\,\zeta^2\,(H-z)^2\,\alpha^2\right)}{(H-z)^2\,k^2\,\alpha^2\,\zeta^2} \tag{28}$$

$$K_1:=100-K_2;$$

$$100-\frac{200\left(\frac{-k\,\alpha\,\zeta\,H\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)-\cosh(k\,\alpha\,\zeta\,z)}{\cosh(k\,\alpha\,\zeta\,H)}+1+\frac{1}{2}\,k^2\,\zeta^2\,(H-z)^2\,\alpha^2\right)}{(H-z)^2\,k^2\,\alpha^2\,\zeta^2} \tag{29}$$

deflection:

$$ode_2:=\left(\frac{d^4}{dz^4}\,x(z)\right)-\left(\frac{d^2}{dz^2}\,x(z)\right)k^2\,\alpha^2\,\zeta^2=\frac{1}{i_t\cdot E}\cdot\left(\frac{d^2}{dz^2}\,m(z)-m(z)\,(k^2-1)\,\alpha^2\,\zeta^2\right);$$

$$\frac{d^4}{dz^4}\,x(z)-\left(\frac{d^2}{dz^2}\,x(z)\right)k^2\,\alpha^2\,\zeta^2=\frac{w-\frac{1}{2}\,w\,(H-z)^2\,(k^2-1)\,\alpha^2\,\zeta^2}{i_t\,E} \tag{30}$$

$$ics:=x(0)=0,\ \mathrm{D}(x)(0)=0,\ \mathrm{D}^{(2)}(x)(H)=0,\ \left(\mathrm{D}^{(3)}(x)(H)-(k\cdot\alpha\cdot\zeta)^2\cdot\mathrm{D}(x)(\mathrm{H})\right)=\frac{1}{E\cdot i_t}$$
$$\cdot\left(\frac{d}{dz}\,m(H)-\alpha^2\cdot\zeta^2\cdot(k^2-1)\cdot\int_0^H m(z)\,dz\right);$$

$$x(0)=0,\ \mathrm{D}(x)(0)=0,\ \mathrm{D}^{(2)}(x)(H)=0,\ \mathrm{D}^{(3)}(x)(H)-k^2\,\alpha^2\,\zeta^2\,\mathrm{D}(x)(H)=$$
$$-\frac{1}{6}\,\frac{\alpha^2\,\zeta^2\,(k^2-1)\,w\,H^3}{E\,i_t} \tag{31}$$

$$combine(\mathrm{convert}(\mathrm{dsolve}(\{ode_2,ics\},x(z)),trig));$$

$$x(z)=\frac{1}{24}\,\frac{1}{k^6\,\alpha^4\,\zeta^4\,E\,i_t\cosh(k\,\alpha\,\zeta\,H)}\bigl(6\,w\,H^2\,z^2\,k^6\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)$$
$$-4\,w\,H\,z^3\,k^6\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)+w\,z^4\,k^6\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)$$
$$-6\,w\,H^2\,z^2\,k^4\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)+4\,w\,H\,z^3\,k^4\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)$$
$$-w\,z^4\,k^4\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)+24\,w\,H\,z\,k^2\,\alpha^2\,\zeta^2\cosh(k\,\alpha\,\zeta\,H)$$
$$-12\,w\,z^2\,k^2\,\alpha^2\,\zeta^2\cosh(k\,\alpha\,\zeta\,H)+24\,H\,w\,\zeta\,\alpha\,k\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)$$
$$-24\,H\sinh(k\,\alpha\,\zeta\,H)\,\zeta\,\alpha\,k\,w+24\cosh(k\,\alpha\,\zeta\,z)\,w-24\,w\bigr)\tag{32}$$

$\overset{\text{simplify}}{=}$

$$x(z)=\frac{1}{4}\,\frac{1}{k^6\,\alpha^4\,\zeta^4\,E\,i_t\cosh(k\,\alpha\,\zeta\,H)}\Bigl(w\Bigl(4\,H\sinh(k\,\zeta\,\alpha\,(H-z))\,\zeta\,\alpha\,k+\zeta^2\,z\,k^2\Bigl(\zeta^2\,z\,(k$$
$$+1)\,k^2\Bigl(H^2-\frac{2}{3}\,z\,H+\frac{1}{6}\,z^2\Bigr)\,(k-1)\,\alpha^2+4\,H-2\,z\Bigr)\,\alpha^2\cosh(k\,\alpha\,\zeta\,H)$$
$$-4\,H\sinh(k\,\alpha\,\zeta\,H)\,\zeta\,\alpha\,k+4\cosh(\zeta\,\alpha\,k\,z)-4\Bigr)\Bigr)\tag{33}$$

$$x_{uni,\,rigid}:=\frac{1}{24}\,\frac{1}{k^6\,\alpha^4\,\zeta^4\,i_t\,E\cosh(k\,\alpha\,\zeta\,H)}\bigl(24\,w\,H\,k\,\alpha\,\zeta\sinh(k\,\alpha\,\zeta\,H-k\,\alpha\,\zeta\,z)+24\,w\cosh(k\,\alpha\,\zeta\,z)$$
$$-12\,z^2\,w\,k^2\,\alpha^2\,\zeta^2\cosh(k\,\alpha\,\zeta\,H)+6\,w\,H^2\,z^2\,k^6\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)$$
$$-6\,w\,H^2\,z^2\,k^4\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)-4\,z^3\,H\,w\,k^6\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)$$
$$+4\,z^3\,H\,w\,k^4\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)+z^4\,w\,k^6\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)-z^4\,w\,k^4\,\alpha^4\,\zeta^4\cosh(k\,\alpha\,\zeta\,H)$$
$$+24\,w\,H\,z\,k^2\,\alpha^2\,\zeta^2\cosh(k\,\alpha\,\zeta\,H)-24\,w\,k\,\alpha\,\zeta\,H\sinh(k\,\alpha\,\zeta\,H)-24\,w\bigr);$$

$$M_{1,\,0}:=\frac{I_1}{i_t}\cdot\bigl(m(z)-N_{uni,\,l,\,rigid}\cdot l\bigr);$$

$$\frac{1}{i_t}\Bigl(\mathrm{I}_1\Bigl(\frac{1}{2}\,w\,(H-z)^2-\frac{1}{2}\,\frac{1}{k^4\,\alpha^2\,\zeta^2\cosh(k\,\alpha\,\zeta\,H)}\bigl(w\,\bigl(-2\,H\sinh(k\,\alpha\,\zeta\,(H-z))\,\zeta\,\alpha\,k+\bigl(2$$
$$+k^2\,\zeta^2\,(H-z)^2\,\alpha^2\bigr)\cosh(k\,\alpha\,\zeta\,H)-2\cosh(k\,\alpha\,\zeta\,z)\bigr)\bigr)\Bigr)\Bigr)\tag{34}$$

$$M_{2,\,0}:=\frac{I_2}{i_t}\cdot\bigl(m(z)-N_{uni,\,l,\,rigid}\cdot l\bigr);$$

$$\frac{1}{i_t}\Bigl(\mathrm{I}_2\Bigl(\frac{1}{2}\,w\,(H-z)^2-\frac{1}{2}\,\frac{1}{k^4\,\alpha^2\,\zeta^2\cosh(k\,\alpha\,\zeta\,H)}\bigl(w\,\bigl(-2\,H\sinh(k\,\alpha\,\zeta\,(H-z))\,\zeta\,\alpha\,k+\bigl(2$$
$$+k^2\,\zeta^2\,(H-z)^2\,\alpha^2\bigr)\cosh(k\,\alpha\,\zeta\,H)-2\cosh(k\,\alpha\,\zeta\,z)\bigr)\bigr)\Bigr)\Bigr)\tag{35}$$

Example:

$$N_{example}:=subs\bigl(\alpha=0.048535,\,H=60,\,l=8.5,\,k=1.08861,\,w=17,\,\zeta=0.629153,\,N_{uni,\,l,\,rigid}\bigr):$$

$eval\left(N_{example}, z=0\right);$

$$1221.563781 \quad \mathbf{(36)}$$

$h_{example,\max} := solve\left(subs\left(\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, h_{\max, shear}\right)=0, z\right);$

$$59.99999999, 32.28011024 \quad \mathbf{(37)}$$

$q_{exam,\max} := eval\left(q_{uni,\, load}, \{\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, z=32.44952673\}\right);$

$$24.91620771 \quad \mathbf{(38)}$$

$Q_{\max, exam} := q_{exam,\max}\cdot 3;$

$$74.74862313 \quad \mathbf{(39)}$$

$M_{exam,\max, b} := \dfrac{Q_{\max, exam}\cdot 2.5}{2};$

$$93.43577891 \quad \mathbf{(40)}$$

$x_{example} := subs\left(\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, E=36\cdot 10^{6}, i_t=11.7, x_{uni,\, rigid}\right):$

$x_{exam,\max} := eval\left(x_{example}, z=60\right);$

$$0.03252003159 \quad \mathbf{(41)}$$

$M_{1,0} := \dfrac{I_1}{i_t}\cdot\left(m(z)-N_{uni,\, l,\, rigid}\cdot l\right):$

$eval\left(M_{1,0}, \{\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, z=0, I_1=3.125, i_t=11.7\}\right);$

$$5399.761720 \quad \mathbf{(42)}$$

$M_{2,0} := \dfrac{I_2}{i_t}\cdot\left(m(z)-N_{uni,\, l,\, rigid}\cdot l\right):$

$eval\left(M_{2,0}, \{\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, z=0, I_2=8.575, i_t=11.7\}\right);$

$$14816.94616 \quad \mathbf{(43)}$$

$eval\left(K_2, \{\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, z=0\}\right);$

$$40.21224176 \quad \mathbf{(44)}$$

$eval\left(S_1, \{z=0, H=60, w=17, I_1=3.125, i_t=11.7\}\right);$

$$272.4358974 \quad \mathbf{(45)}$$

$eval\left(S_2, \{z=0, H=60, w=17, I_2=8.575, i_t=11.7\}\right);$

$$747.5641026 \quad \mathbf{(46)}$$

$N_b := 3\cdot eval\left(n, \{\alpha=0.048535, H=60, l=8.5, k=1.08861, w=17, \zeta=0.629153, z=32.44952673, I_1=3.125, i_t=11.7, I_2=8.575, b=2.5, d_1=2.5\}\right);$

$$37.36015311 \quad \mathbf{(47)}$$

$0.629153^{2}\cdot 36$

17 Appendix 8

Coupled walls system with different modulus of elasticity supported on elastic foundation stiffened by a foundation beam.

restart;

$$> \delta_1 := l \cdot \frac{dx}{dz} :$$

$$> \delta_2 := \frac{dN}{dz} \cdot \frac{b^3 \cdot h}{12 \cdot E_{cb} \cdot I_e} :$$

$$> \delta_3 := -\frac{1}{E} \cdot \left(\frac{1}{A_1} + \frac{1}{A_2} \right) \int_0^z N(z)\, dz :$$

$$> \delta_v := (N(0) + Q_0) \cdot \left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}} \right) :$$

$$> \delta_\theta := \left(\frac{M_{10} + M_{20}}{k_{\theta 1} + k_{\theta 2}} \right) \cdot l :$$

$$> \delta_4 := \delta_\theta - \delta_v :$$

$$> \delta := \delta_1 + \delta_2 + \delta_3 + \delta_4;$$

$$\delta := l \left(\frac{d}{dz} x(z) \right) + \frac{1}{12} \frac{\left(\frac{d}{dz} N(z) \right) b^3 h}{E_{cb} I_e} - \frac{\left(\frac{1}{A_1} + \frac{1}{A_2} \right) \left(\int_0^z N(z)\, dz \right)}{E} + \frac{(M_{10} + M_{20})\, l}{k_{\theta 1} + k_{\theta 2}} - (N(0) + Q_0) \left(\frac{1}{k_{v1}} + \frac{1}{k_{v2}} \right) \quad \textbf{(1)}$$

$$> M_1 := m(z) - \left(\frac{b}{2} + d_1 \right) \cdot \int_z^H q(z)\, dz - M_0 = E \cdot I_1 \frac{d}{dz} \left(\frac{dx}{dz} \right);$$

$$M_1 := m(z) - \left(\frac{1}{2} b + d_1 \right) \left(\int_z^H q(z)\, dz \right) - M_0 = E\, I_1 \left(\frac{d^2}{dz^2} x(z) \right) \quad \textbf{(2)}$$

$$> M_2 := -\left(\frac{b}{2} + d_2 \right) \cdot \int_z^H q(z)\, dz + M_0 = E \cdot I_2 \frac{d}{dz} \left(\frac{dx}{dz} \right);$$

$$M_2 := -\left(\frac{1}{2} b + d_2 \right) \left(\int_z^H q(z)\, dz \right) + M_0 = E\, I_2 \left(\frac{d^2}{dz^2} x(z) \right) \quad \textbf{(3)}$$

$$> (M_1 + M_2);$$

$$m(z) - \left(\frac{1}{2} b + d_1 \right) \left(\int_z^H q(z)\, dz \right) - \left(\frac{1}{2} b + d_2 \right) \left(\int_z^H q(z)\, dz \right) = E\, I_1 \left(\frac{d^2}{dz^2} x(z) \right) + E\, I_2 \left(\frac{d^2}{dz^2} x(z) \right) \quad \textbf{(4)}$$

$$> sol_1 := algsubs\left(\left(\left(\int_z^H q(z)\, dz \right) = N(z), algsubs\left((-b - d_1 - d_2) = -l, collect\left(collect(M_1 + M_2, \right.\right.\right.\right.$$

$$diff), \int_z^H q(z)\, dz \Big)\Big)\Big);$$

$$sol_1 := -l\,N(z) + m(z) = (E\,\mathrm{I}_1 + E\,\mathrm{I}_2)\left(\frac{d^2}{dz^2}x(z)\right) \quad \textbf{(5)}$$

> $$s_1 := collect\left(collect\left(algsubs\left(\left(\frac{d^2}{dz^2}x(z)\right) = \frac{-l\,N(z)+m(z)}{(E\,\mathrm{I}_1 + E\,\mathrm{I}_2)}, \frac{d}{dz}\delta = 0\right), \frac{d^2}{dz^2}N(z)\right), N(z)\right);$$

$$s_1 := \frac{1}{12}\frac{1}{A_1 A_2 E\,(\mathrm{I}_1+\mathrm{I}_2)\,E_{cb}\,\mathrm{I}_e}\Big(\big(-12\,l^2\,\mathrm{I}_e A_1 A_2 E_{cb} - 12\,\mathrm{I}_1\,\mathrm{I}_e A_1 E_{cb} - 12\,\mathrm{I}_1\,\mathrm{I}_e A_2 E_{cb} - 12\,\mathrm{I}_2\,\mathrm{I}_e A_1 E_{cb} - 12\,\mathrm{I}_2\,\mathrm{I}_e A_2 E_{cb}\big)\,N(z)\Big) + \frac{1}{12}\frac{\left(E\,b^3\,h\,\mathrm{I}_1 A_1 A_2 + E\,b^3\,h\,\mathrm{I}_2 A_1 A_2\right)\left(\frac{d^2}{dz^2}N(z)\right)}{A_1 A_2 E\,(\mathrm{I}_1+\mathrm{I}_2)\,E_{cb}\,\mathrm{I}_e} + \frac{m(z)\,l}{E\,(\mathrm{I}_1+\mathrm{I}_2)} = 0 \quad \textbf{(6)}$$

> $$s_2 := collect\left(collect\left(simplify\left(\frac{s_1\cdot\left(12\cdot E\cdot A_1 A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,E_{cb}\,\mathrm{I}_e\right)}{\left(b^3\,h\,E\,A_1 A_2\,\mathrm{I}_1 + b^3\,h\,E\,A_1 A_2\,\mathrm{I}_2\right)}\right), \left(\frac{d^2}{dz^2}N(z)\right)\right), N(z)\right);$$

$$s_2 := \frac{1}{A_1 A_2 E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}\Big(\big(-12\,l^2\,\mathrm{I}_e A_1 A_2 E_{cb} - 12\,\mathrm{I}_1\,\mathrm{I}_e A_1 E_{cb} - 12\,\mathrm{I}_1\,\mathrm{I}_e A_2 E_{cb} - 12\,\mathrm{I}_2\,\mathrm{I}_e A_1 E_{cb} - 12\,\mathrm{I}_2\,\mathrm{I}_e A_2 E_{cb}\big)\,N(z)\Big) + \frac{\left(E\,b^3\,h\,\mathrm{I}_1 A_1 A_2 + E\,b^3\,h\,\mathrm{I}_2 A_1 A_2\right)\left(\frac{d^2}{dz^2}N(z)\right)}{E\,A_1 A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h} + \frac{12\,m(z)\,\mathrm{I}_e\,l\,E_{cb}}{E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h} = 0 \quad \textbf{(7)}$$

> $$simplify\left(\frac{1}{E\,A_1 A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}\Big(\big(-12\,E_{cb}\,\mathrm{I}_e A_2\,\mathrm{I}_1 - 12\,E_{cb}\,\mathrm{I}_e A_2\,\mathrm{I}_2 - 12\,E_{cb}\,\mathrm{I}_e A_1\,\mathrm{I}_1 - 12\,E_{cb}\,\mathrm{I}_e A_1\,\mathrm{I}_2 - 12\,l^2 A_1 A_2 E_{cb}\,\mathrm{I}_e\big)\,N(z)\Big)\right) + simplify\left(\frac{\left(b^3\,h\,E\,A_1 A_2\,\mathrm{I}_1 + b^3\,h\,E\,A_1 A_2\,\mathrm{I}_2\right)\left(\frac{d^2}{dz^2}N(z)\right)}{E\,A_1 A_2\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h}\right) + \frac{12\,l\,m(z)\,E_{cb}\,\mathrm{I}_e}{E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h} = 0;$$

$$-\frac{12\,\mathrm{I}_e E_{cb}\left(l^2 A_1 A_2 + \mathrm{I}_1 A_1 + \mathrm{I}_1 A_2 + \mathrm{I}_2 A_1 + \mathrm{I}_2 A_2\right)N(z)}{E\,b^3\,h\,A_1 A_2\,(\mathrm{I}_1+\mathrm{I}_2)} + \frac{d^2}{dz^2}N(z) + \frac{12\,m(z)\,\mathrm{I}_e\,l\,E_{cb}}{E\,(\mathrm{I}_1+\mathrm{I}_2)\,b^3\,h} = 0 \quad \textbf{(8)}$$

> $$\left(\frac{12\cdot I_e\cdot l^2}{b^3\cdot h\cdot i_t}\right) = \alpha^2 : \left(1 + \frac{A\cdot i_t}{A_1\cdot A_2\cdot l^2}\right) = k^2 : (I_1 + I_2) = i_t : \frac{E_{cb}}{E} = \zeta^2 :$$

> $eq_{normal,\, force} := \frac{d^2}{dz^2} N(z) - k^2 \cdot \alpha^2 \cdot \zeta^2 \cdot N(z) + \frac{\alpha^2}{l} \cdot \zeta^2 \cdot m(z) = 0;$

$$eq_{normal,\, force} := \frac{d^2}{dz^2} N(z) - k^2 \alpha^2 \zeta^2 N(z) + \frac{\alpha^2 \zeta^2 m(z)}{l} = 0 \quad \textbf{(9)}$$

> $ics := N(H) = 0, D(N)(0) \cdot (1 + \psi \cdot \mu_f) = \mu_f \cdot N(0) - \frac{\lambda_r}{l} \cdot m(0);$

$$ics := N(H) = 0, D(N)(0)\,(\psi \mu_f + 1) = \mu_f N(0) - \frac{\lambda_r m(0)}{l} \quad \textbf{(10)}$$

> $m(z) := \frac{w \cdot (H - z)^2}{2};$

$$m := z \rightarrow \frac{1}{2} w\,(H - z)^2 \quad \textbf{(11)}$$

> $convert(dsolve(eq_{normal,\, force}), trig);$

$$N(z) = (\cosh(k \alpha \zeta z) + \sinh(k \alpha \zeta z))\,\_C2 + (\cosh(k \alpha \zeta z) - \sinh(k \alpha \zeta z))\,\_C1 + \frac{1}{2} \frac{w\,(2 + k^2 \zeta^2 (H - z)^2 \alpha^2)}{k^4 l \alpha^2 \zeta^2} \quad \textbf{(12)}$$

> $n_{uniform,\, load,\, elastic} := simplify(convert(dsolve(\{eq_{normal,\, force}, ics\}, N(z)), trig), size);$

$$n_{uniform,\, load,\, elastic} := N(z) = -\frac{1}{2} \Big(\Big(\Big(\Big(\big(k^2 \zeta^2 \big(H \lambda_r k^2 - 2 + (-H - 2\psi) \mu_f \big) H \alpha^2 - 2 \mu_f \big) \sinh(k \alpha \zeta z) - k \zeta (\psi \mu_f + 1) \alpha \big(2 + k^2 \zeta^2 (H - z)^2 \alpha^2\big) \Big) \cosh(k \alpha \zeta H) + \Big(\big(-k^2 \zeta^2 \big(H \lambda_r k^2 - 2 + (-H - 2\psi) \mu_f \big) H \alpha^2 + 2 \mu_f \big) \cosh(k \alpha \zeta z) - \mu_f \big(2 + k^2 \zeta^2 (H - z)^2 \alpha^2\big) \Big) \sinh(k \alpha \zeta H) + 2 k \alpha \zeta (\psi \mu_f + 1) \cosh(k \alpha \zeta z) + 2 \sinh(k \alpha \zeta z) \mu_f \Big) w \Big) \Big/ \Big(k^4 \big(k \alpha \zeta (\psi \mu_f + 1) \cosh(k \alpha \zeta H) + \mu_f \sinh(k \alpha \zeta H) \big) l \zeta^2 \alpha^2 \Big) \quad \textbf{(13)}$$

> **shear-of-lamellas;**

$$shear - of - lamellas \quad \textbf{(14)}$$

> $q_{uniform,\, load,\, elastic} := simplify\left(-\frac{d}{dz} n_{uniform,\, load,\, elastic}\right);$

$$q_{uniform,\, load,\, elastic} := -\left(\frac{d}{dz} N(z)\right) = \frac{1}{2} \Big(w \Big(H^2 \cosh(k \alpha \zeta H) \cosh(k \alpha \zeta z) \zeta^2 \alpha^2 k^4 \lambda_r - H^2 \sinh(k \alpha \zeta H) \sinh(k \alpha \zeta z) \zeta^2 \alpha^2 k^4 \lambda_r - H^2 \cosh(k \alpha \zeta H) \cosh(k \alpha \zeta z) \zeta^2 \alpha^2 k^2 \mu_f + H^2 \sinh(k \alpha \zeta H) \sinh(k \alpha \zeta z) \zeta^2 \alpha^2 k^2 \mu_f - 2 H \cosh(k \alpha \zeta H) \cosh(k \alpha \zeta z) \zeta^2 \alpha^2 k^2 \psi \mu_f + 2 H \sinh(k \alpha \zeta H) \sinh(k \alpha \zeta z) \zeta^2 \alpha^2 k^2 \psi \mu_f + 2 H \zeta^2 \alpha^2 k^2 \psi \mu_f \cosh(k \alpha \zeta H) \quad \textbf{(15)}$$

$$-2\cosh(k\alpha\zeta H)\,\zeta^2\alpha^2k^2\psi z\mu_f-2H\cosh(k\alpha\zeta H)\cosh(k\alpha\zeta z)\,\zeta^2\alpha^2k^2$$
$$+2H\sinh(k\alpha\zeta H)\sinh(k\alpha\zeta z)\,\zeta^2\alpha^2k^2+2Hk^2\alpha^2\zeta^2\cosh(k\alpha\zeta H)$$
$$-2\cosh(k\alpha\zeta H)\,\zeta^2\alpha^2k^2z+2H\sinh(k\alpha\zeta H)\,\zeta\alpha k\mu_f-2\sinh(k\alpha\zeta H)\,\zeta\alpha kz\mu_f$$
$$+2\sinh(k\alpha\zeta z)\,\zeta\alpha k\psi\mu_f+2\sinh(k\alpha\zeta z)\,k\alpha\zeta-2\cosh(k\alpha\zeta H)\cosh(k\alpha\zeta z)\,\mu_f$$
$$+2\sinh(k\alpha\zeta H)\sinh(k\alpha\zeta z)\,\mu_f+2\cosh(k\alpha\zeta z)\,\mu_f\Big)\Big)\Big/$$
$$\Big(k^3 l\alpha\zeta\big(\zeta\alpha k\psi\mu_f\cosh(k\alpha\zeta H)+k\alpha\zeta\cosh(k\alpha\zeta H)+\mu_f\sinh(k\alpha\zeta H)\big)\Big)$$

> *simplify*(**(15)**, '*size*');

$$-\left(\frac{d}{dz}N(z)\right)=\frac{1}{2}\Bigg(\Bigg(\Big(\Big(k^2\zeta^2\big(H\lambda_r k^2-2+(-H-2\psi)\mu_f\big)H\alpha^2-2\mu_f\Big)\cosh(k\alpha\zeta z)$$
$$+2k^2\zeta^2\alpha^2(\psi\mu_f+1)(H-z)\Big)\cosh(k\alpha\zeta H)+\Big(\Big(-k^2\zeta^2\big(H\lambda_r k^2-2+(-H$$
$$-2\psi)\mu_f\big)H\alpha^2+2\mu_f\Big)\sinh(k\alpha\zeta z)+2k\zeta\alpha\mu_f(H-z)\Big)\sinh(k\alpha\zeta H)+2k\alpha\zeta\big(\psi\mu_f$$
$$+1\big)\sinh(k\alpha\zeta z)+2\cosh(k\alpha\zeta z)\,\mu_f\Bigg)w\Bigg)\Bigg/\Big(k^3\big(k\alpha\zeta(\psi\mu_f+1)\cosh(k\alpha\zeta H)$$
$$+\mu_f\sinh(k\alpha\zeta H)\big)l\zeta\alpha\Big)\quad\textbf{(16)}$$

```
> internal-moment;
```

$$internal-moment\quad\textbf{(17)}$$

> $m_{internal,\,\mathrm{uni},\,1,\,2} := m(z) - n_{\mathrm{uniform,\,load},\,elastic}\cdot l;$

$$m_{internal,\,uni,\,1,\,2}:=-lN(z)+\frac{1}{2}w(H-z)^2=\frac{1}{2}\Bigg(\Bigg(\Big(\Big(\Big(k^2\zeta^2\big(H\lambda_r k^2-2+(-H$$
$$-2\psi)\mu_f\big)H\alpha^2-2\mu_f\Big)\sinh(k\alpha\zeta z)-k\zeta(\psi\mu_f+1)\alpha\big(2+k^2\zeta^2(H-z)^2\alpha^2\big)\Big)$$
$$\cosh(k\alpha\zeta H)+\Big(\Big(-k^2\zeta^2\big(H\lambda_r k^2-2+(-H-2\psi)\mu_f\big)H\alpha^2+2\mu_f\Big)\cosh(k\alpha\zeta z)$$
$$-\mu_f\big(2+k^2\zeta^2(H-z)^2\alpha^2\big)\Big)\sinh(k\alpha\zeta H)+2k\alpha\zeta(\psi\mu_f+1)\cosh(k\alpha\zeta z)$$
$$+2\sinh(k\alpha\zeta z)\,\mu_f\Bigg)w\Bigg)\Bigg/\Big(k^4\big(k\alpha\zeta(\psi\mu_f+1)\cosh(k\alpha\zeta H)$$
$$+\mu_f\sinh(k\alpha\zeta H)\big)\zeta^2\alpha^2\Big)+\frac{1}{2}w(H-z)^2\quad\textbf{(18)}$$

> *simplify*(**(18)**, '*size*');

$$-lN(z)+\frac{1}{2}w(H-z)^2=\frac{1}{2}\Bigg(\Bigg(\Big(\Big(\Big(k^2\zeta^2\big(H\lambda_r k^2-2+(-H-2\psi)\mu_f\big)H\alpha^2$$
$$-2\mu_f\Big)\sinh(k\alpha\zeta z)+k\big(-2+k^2\zeta^2(k-1)(k+1)(H-z)^2\alpha^2\big)\zeta\alpha(\psi\mu_f+1)\Big)$$
$$\cosh(k\alpha\zeta H)+\Big(\Big(-k^2\zeta^2\big(H\lambda_r k^2-2+(-H-2\psi)\mu_f\big)H\alpha^2+2\mu_f\Big)\cosh(k\alpha\zeta z)$$
$$+\big(-2+k^2\zeta^2(k-1)(k+1)(H-z)^2\alpha^2\big)\mu_f\Big)\sinh(k\alpha\zeta H)+2k\alpha\zeta\big(\psi\mu_f$$
$$+1\big)\cosh(k\alpha\zeta z)+2\sinh(k\alpha\zeta z)\,\mu_f\Bigg)w\Bigg)\Bigg/\Big(k^4\zeta^2\alpha^2\big(k\alpha\zeta(\psi\mu_f+1)\cosh(k\alpha\zeta H)\quad\textbf{(19)}$$

$+\mu_f \sinh(k\,\alpha\,\zeta\,H)\,)\,)$

```
> laterla-deflection;
```

$$laterla - deflection \tag{20}$$

> $restart;$

> $ode_3 := \left(\frac{d^2}{dz^2}\,x(z)\right) = \frac{1}{E \cdot i_t}\cdot(\,\mathrm{m}(z) - N(z)\cdot l\,);$

$$ode_3 := \frac{d^2}{dz^2}\,x(z) = \frac{m(z) - N(z)\,l}{E\,i_t} \tag{21}$$

> $ics_3 := x(0) = 0, \mathrm{D}(x)(0) = \frac{(m(0) - N(0)\cdot l + \psi\cdot \mathrm{D}(N)(0)\cdot l)}{\left(\frac{\mathrm{E}\cdot \mathrm{i}_t\cdot\alpha^2\cdot\zeta^2}{\lambda_r}\right)};$

$$ics_3 := x(0) = 0, \mathrm{D}(x)(0) = \frac{(m(0) - N(0)\,l + \psi\,\mathrm{D}(N)(0)\,l)\,\lambda_r}{E\,i_t\,\alpha^2\,\zeta^2} \tag{22}$$

> $N(z) := -\frac{1}{2}\,\Big(\Big(\Big(\Big(k^2\,\zeta^2\,\big(H\lambda_r k^2 - 2 + (-H - 2\,\psi)\,\mu_f\big)\,H\alpha^2 - 2\,\mu_f\Big)\sinh(k\,\alpha\,\zeta\,z) - k\,\zeta\,(\psi\,\mu_f + 1)\,\alpha\,\big(2 + k^2\,\zeta^2\,(H - z)^2\,\alpha^2\big)\Big)\cosh(k\,\alpha\,\zeta\,H) + \Big(\Big(-k^2\,\zeta^2\,\big(H\lambda_r k^2 - 2 + (-H - 2\,\psi)\,\mu_f\big)\,H\alpha^2 + 2\,\mu_f\Big)\cosh(k\,\alpha\,\zeta\,z) - \mu_f\big(2 + k^2\,\zeta^2\,(H - z)^2\,\alpha^2\big)\Big)\sinh(k\,\alpha\,\zeta\,H) + 2\,k\,\alpha\,\zeta\,(\psi\,\mu_f + 1)\cosh(k\,\alpha\,\zeta\,z) + 2\sinh(k\,\alpha\,\zeta\,z)\,\mu_f\Big)\,w\Big)\Big/\Big(k^4\,\big(k\,\alpha\,\zeta\,(\psi\,\mu_f + 1)\cosh(k\,\alpha\,\zeta\,H) + \mu_f \sinh(k\,\alpha\,\zeta\,H)\big)\,l\,\zeta^2\,\alpha^2\Big);$

$$N := z \rightarrow -\frac{1}{2}\,\Big(\Big(\Big(\Big(k^2\,\zeta^2\,\big(H\lambda_r k^2 - 2 + (-H - 2\,\psi)\,\mu_f\big)\,H\alpha^2 - 2\,\mu_f\Big)\sinh(k\,\alpha\,\zeta\,z) - k\,\zeta\,(\psi\,\mu_f + 1)\,\alpha\,\big(2 + k^2\,\zeta^2\,(H - z)^2\,\alpha^2\big)\Big)\cosh(k\,\alpha\,\zeta\,H) + \Big(\Big(-k^2\,\zeta^2\,\big(H\lambda_r k^2 - 2 + (-H - 2\,\psi)\,\mu_f\big)\,H\alpha^2 + 2\,\mu_f\Big)\cosh(k\,\alpha\,\zeta\,z) - \mu_f\big(2 + k^2\,\zeta^2\,(H - z)^2\,\alpha^2\big)\Big)\sinh(k\,\alpha\,\zeta\,H) + 2\,k\,\zeta\,(\psi\,\mu_f + 1)\,\alpha\cosh(k\,\alpha\,\zeta\,z) + 2\sinh(k\,\alpha\,\zeta\,z)\,\mu_f\Big)\,w\Big)\Big/\Big(k^4\,\big(k\,\zeta\,(\psi\,\mu_f + 1)\,\alpha\cosh(k\,\alpha\,\zeta\,H) + \mu_f \sinh(k\,\alpha\,\zeta\,H)\big)\,l\,\zeta^2\,\alpha^2\Big) \tag{23}$$

> $m(z) := \frac{w\cdot(H - z)^2}{2};$

$$m := z \rightarrow \frac{1}{2}\,w\,(H - z)^2 \tag{24}$$

> $convert(dsolve(ode_3, x(z)), trig);$

$$x(z) = \iint\Big(-\frac{1}{2}\,\Big(w\,\Big(-2\,H\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^3\,\psi\,z\,\mu_f + 2\,H\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^5\,\psi\,z\,\mu_f \tag{25}$$

$$+2\,k\,\zeta\,\alpha\cosh(k\,\alpha\,\zeta\,H)-2\,k\,\zeta\,\alpha\cosh(k\,\alpha\,\zeta\,z)+2\,\mu_f\sinh(k\,\alpha\,\zeta\,H)$$

$$-2\,H\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^3\,z+H^2\sinh(k\,\alpha\,\zeta\,H)\,\zeta^2\,\alpha^2\,k^2\,\mu_f+\sinh(k\,\alpha\,\zeta\,H)\,\zeta^2\,\alpha^2\,k^2\,z^2\,\mu_f$$

$$-2\cosh(k\,\alpha\,\zeta\,z)\,\zeta\,\alpha\,k\,\psi\,\mu_f+2\,H\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^5\,z-H^2\sinh(k\,\alpha\,\zeta\,H)\,\zeta^2\,\alpha^2\,k^4\,\mu_f$$

$$-\sinh(k\,\alpha\,\zeta\,H)\,\zeta^2\,\alpha^2\,k^4\,z^2\,\mu_f+2\cosh(k\,\alpha\,\zeta\,H)\,\zeta\,\alpha\,k\,\psi\,\mu_f-2\,\mu_f\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)$$

$$-2\,H\,\zeta^2\,\alpha^2\,k^2\,\psi\,\mu_f\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)+H^2\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^3\,\psi\,\mu_f$$

$$+\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^3\,\psi\,z^2\,\mu_f-2\,H\sinh(k\,\alpha\,\zeta\,H)\,\zeta^2\,\alpha^2\,k^2\,z\,\mu_f$$

$$-H^2\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^5\,\psi\,\mu_f-\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^5\,\psi\,z^2\,\mu_f$$

$$+2\,H\sinh(k\,\alpha\,\zeta\,H)\,\zeta^2\,\alpha^2\,k^4\,z\,\mu_f-2\sinh(k\,\alpha\,\zeta\,z)\,\mu_f-2\,k^2\,\zeta^2\,H\,\alpha^2\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)$$

$$+H^2\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^3+\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^3\,z^2-H^2\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^5$$

$$-\cosh(k\,\alpha\,\zeta\,H)\,\zeta^3\,\alpha^3\,k^5\,z^2-H^2\,\zeta^2\,\alpha^2\,k^2\,\mu_f\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)$$

$$+H^2\,\zeta^2\,\alpha^2\,k^4\,\lambda_r\sinh(H\,\zeta\,\alpha\,k-\zeta\,\alpha\,k\,z)\Big)\Big)\Big/\Big(k^4\,\big(\cosh(k\,\alpha\,\zeta\,H)\,\zeta\,\alpha\,k\,\psi\,\mu_f$$

$$+k\,\zeta\,\alpha\cosh(k\,\alpha\,\zeta\,H)+\mu_f\sinh(k\,\alpha\,\zeta\,H)\big)\,\zeta^2\,\alpha^2\,E\,i_t\Big)\Big)\,\mathrm{d}z\,\mathrm{d}z+\_C1\,z+\_C2$$

> *simplify*(*combine*(*convert*(*value*(*dsolve*($\{ode_3, ics_3\}$, $x(z)$)), *trig*), *trig*), *size*);

$$x(z)=\frac{1}{2}\Bigg(w\bigg(\Big(H\big(-H\,\lambda_r\,k^2+2+(H+2\,\psi)\,\mu_f\big)\,k^2\,\zeta^2\,\alpha^2+2\,\mu_f\Big)\sinh(k\,\zeta\,\alpha\,(H-z)) \tag{26}$$

$$+z\,\alpha\bigg(\frac{1}{2}\,z\,(k+1)\,(\psi\,\mu_f+1)\left(H^2-\frac{2}{3}\,z\,H+\frac{1}{6}\,z^2\right)(k-1)\,k^4\,\zeta^4\,\alpha^4+\Big(\lambda_r\,H^2\,\big(-\psi\,\lambda_r$$

$$+\psi\,\mu_f+1\big)\,k^4-2\,\lambda_r\,H^2\,k^2+\big(-\psi\,\mu_f-1\big)\,z+H\big(2+(H+2\,\psi)\,\mu_f\big)\Big)\,k^2\,\zeta^2\,\alpha^2-2\,\lambda_r\,k^2$$

$$+2\,\mu_f\bigg)\,k\,\zeta\cosh(k\,\alpha\,\zeta\,H)+\bigg(\frac{1}{2}\,z^2\,(k+1)\left(H^2-\frac{2}{3}\,z\,H+\frac{1}{6}\,z^2\right)\mu_f\,(k-1)\,k^4\,\zeta^4\,\alpha^4$$

$$+\Big(\lambda_r\,H^2\,z\,\big(-\lambda_r+\mu_f\big)\,k^4+\lambda_r\,H\,(H+2\,z)\,k^2-\mu_f\,z^2-H\big(2+(H+2\,\psi)\,\mu_f\big)\Big)\,k^2\,\zeta^2\,\alpha^2$$

$$-2\,\mu_f\bigg)\sinh(k\,\alpha\,\zeta\,H)+2\,k\,\zeta\,\big(\psi\,\mu_f+1\big)\,\alpha\cosh(k\,\alpha\,\zeta\,z)+2\sinh(k\,\alpha\,\zeta\,z)\,\mu_f-2\,k\,\zeta\,\alpha\,\big($$

$$-k^2\,z\,\lambda_r+\psi\,\mu_f+z\,\mu_f+1\big)\bigg)\Bigg)\Bigg/\Big(\Big(k\,\zeta\,\big(\psi\,\mu_f+1\big)\,\alpha\cosh(k\,\alpha\,\zeta\,H)$$

$$+\mu_f\sinh(k\,\alpha\,\zeta\,H)\Big)\,i_t\,E\,\alpha^4\,k^6\,\zeta^4\Big)$$

18 Appendix 9

Excel sheet for analysis the practical example 1

Dimensions of walls and lintels

d_1	[m]	2.175
d_2	[m]	0.475
b	[m]	2.15
h	[m]	2.96
H	[m]	38.48
l	[m]	4.8
t_{wall}	[m]	0.25
h_{beam}	[m]	0.535
E_{wall}	[kN/m^2]	11000000
E_{beam}	[kN/m^2]	11000000
ν		0.2
G	[kN/m^2]	4583333.33

Stiffness parametri

A_1	[m^2]
A_2	[m^2]
$A = A_1 + A_2$	[m^2]
I_1	[m^4]
I_2	[m^4]
$i_t = I_1 + I_2$	[m^4]
A_b	[m^2]
I_b	[m^4]
I_s	[m^4]
$\zeta = \frac{E_{beam}}{E_{wall}}$	
k	
α	[1/m]

External load

Uniform distributed load	w	[kN/m]	25
Point load	P	[kN]	0
Triangular distributed load	p	[kN/m]	0

Desired height leve

z	[m]

Maximum shear force, bending moment and deformation

The height at which maximum shear occurs	12.82	[m]
Maximum shear force in lintels is equal to	211.34	[kN]
Maximum axial force in walls is equal to	1844.58	[kN]
Maximum deflection	0.130	[m]
The height of maximum positive moment	0.00	[m]
Maximum positive bending moment in wall 1	9555.39	[kN.m]
Maximum positive bending moment in wall 2	99.53	[kN.m]
The height of maximum negative moment	32.28	[m]
Maximum negative bending moment in wall 1	-338.43	[kN.m]
Maximum negative bending moment in wall 2	-3.53	[kN.m]
Shear force in foundation beam	-1205.55	[kN]

Maximum shear force and its location

Max positive and negative bending moment and location es

1.0875
0.2375
1.325
1.714851563
0.017861979
1.732713542
0.13375
0.003
0.003
1
1.1772
0.1212

el

0

Properties of the foundation

k_θ	[kN.m]	1E+14
k_v	[kN/m]	506896.55

Dimension of the foundation beam

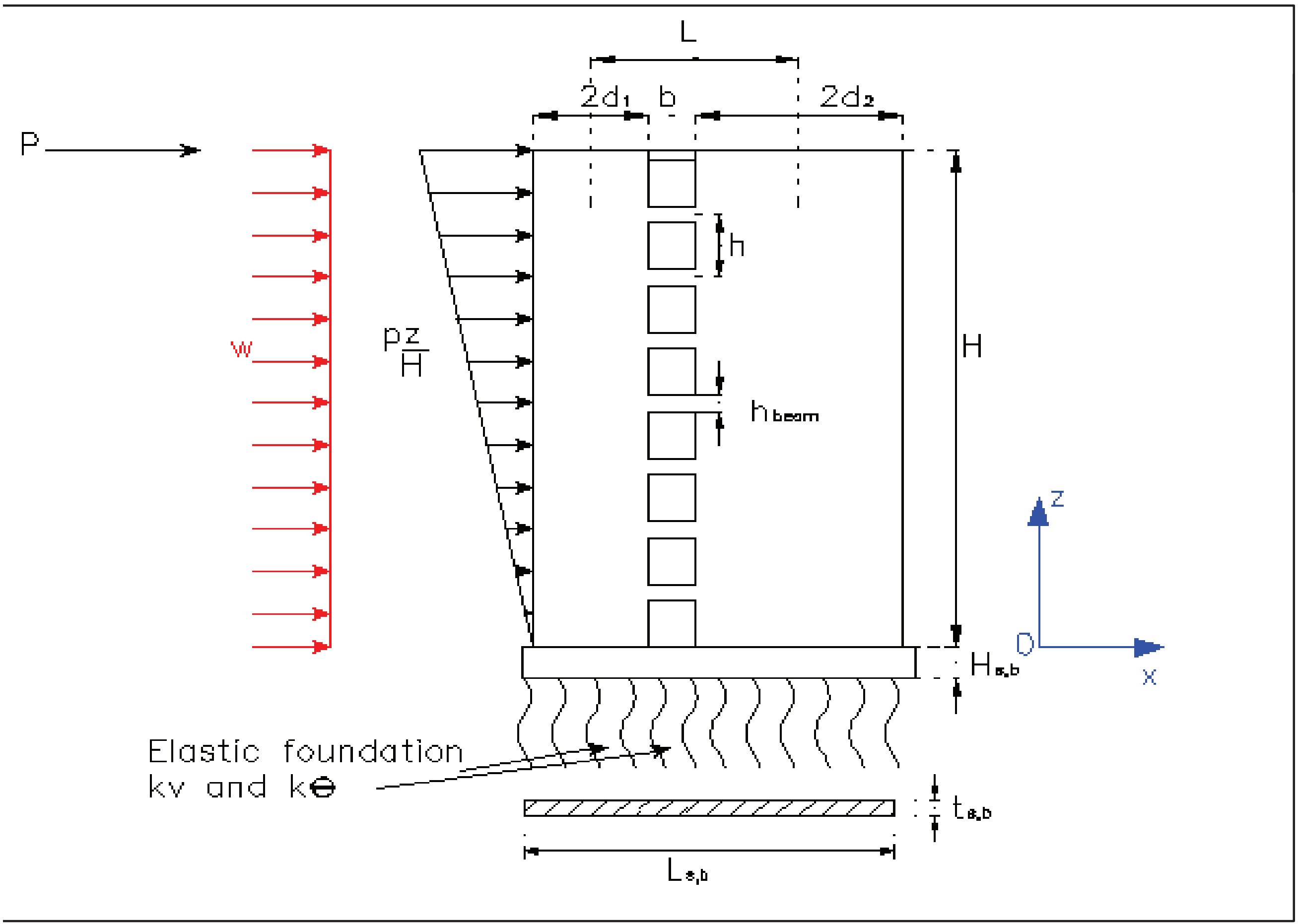
$b_{s,b}$	[m]	0.5
$h_{s,b}$	[m]	1.2
$E_{s,b}$	[kN/m^2]	11000000
$I_{s,b}$	[m^4]	0.072
$\psi = \frac{E_{s,b}.I_{s,b}}{E_b.I_e}.h$	[m]	78.717418
λ_r	[1/m]	2.799E-09
λ_v	[1/m]	0.0239663
μ_f	[1/m]	0.0239663

Axial Force

Shear Force

Lateral Deflection

Bending Moment



19 Appendix 10

Calculation of practical example 2 according to the continuous method

restart;

$$x_{elastic,\ stiffenned} := -\frac{1}{2}\Big(w\Big(\Big(H\big(\lambda_r H k^2 - 2 + (-H-2\psi)\,\mu_f\big)\,k^2\alpha^2 - 2\,\mu_f\Big)\sinh(\alpha\,k\,(H-z)) + z\,\alpha\Big(-\frac{1}{2}\,z\,(k+1)\Big(H^2-\frac{2}{3}\,z\,H+\frac{1}{6}\,z^2\Big)k^4\,(k-1)\,(\psi\,\mu_f+1)\,\alpha^4 + \Big(\lambda_r H^2\big(\psi\,\lambda_r-\psi\,\mu_f-1\big)k^4 + 2\,k^2\lambda_r H^2 + (\psi\,\mu_f+1)\,z - H\big(2+(H+2\,\psi)\,\mu_f\big)\Big)k^2\alpha^2 + 2\,\lambda_r k^2 - 2\,\mu_f\Big)\,k\cosh(k\,\alpha\,H) + \Big(-\frac{1}{2}\,\mu_f z^2\,(k+1)\Big(H^2-\frac{2}{3}\,z\,H+\frac{1}{6}\,z^2\Big)k^4\,(k-1)\,\alpha^4 + k^2\Big(\lambda_r H^2 z\,(\lambda_r-\mu_f)\,k^4 - \lambda_r H\,(H+2\,z)\,k^2 + z^2\mu_f + H\big(2+(H+2\,\psi)\,\mu_f\big)\Big)\alpha^2 + 2\,\mu_f\Big)\sinh(k\,\alpha\,H) - 2\,k\,\alpha\,(\psi\,\mu_f+1)\cosh(k\,\alpha\,z) - 2\sinh(k\,\alpha\,z)\,\mu_f - 2\,\alpha\,k\big(k^2 z\,\lambda_r - \psi\,\mu_f - z\,\mu_f - 1\big)\Big)\Big)\Big/\Big(E\,\alpha^4\big(k\,\alpha\,(\psi\,\mu_f+1)\cosh(k\,\alpha\,H) + \mu_f\sinh(k\,\alpha\,H)\big)\,k^6\,i_t\Big):$$

$x_{plot,\ stiffened,\ 1} := simplify\Big(subs\Big(k=1.1,\ \alpha=0.104908,\ H=38.48,\ w=25,\ E=11\cdot 10^6,\ 1=5.70494,\ \psi=78.71,\ \lambda_r=0.01537,\ \lambda_v=0.023966,\ \mu_f=0.039336,\ i_t=3.266,\ x_{elastic,\ stiffenned}\Big)\Big);$

$$-0.0003498310834\sinh(-4.440545824+0.1153988\,z)+0.00002311395624\,z^2-7.745489691\,10^{-7}\,z^3+5.032152867\,10^{-9}\,z^4+0.002126458833\,z-0.01490507149+0.00007058286952\cosh(0.1153988\,z)+0.000005873727383\sinh(0.1153988\,z) \quad \textbf{(1)}$$

$eval\big(x_{plot,\ stiffened,\ 1},\ z=38.48\big);$

$$0.07129005728 \quad \textbf{(2)}$$

$$N_{elastic,\ stiffened} := \frac{1}{2}\Big(w\Big(\cosh(k\,\alpha\,z)\,k^4\lambda_r H^2\alpha^2\sinh(k\,\alpha\,H) - \cosh(k\,\alpha\,z)\,\mu_f\alpha^2 k^2 H^2\sinh(k\,\alpha\,H) - \sinh(k\,\alpha\,z)\,k^4\lambda_r H^2\alpha^2\cosh(k\,\alpha\,H) + \sinh(k\,\alpha\,z)\,\mu_f\alpha^2 k^2 H^2\cosh(k\,\alpha\,H) + \alpha^3 k^3 H^2\psi\,\mu_f\cosh(k\,\alpha\,H) - 2\,z\,H\,k^2\alpha^2\mu_f\sinh(k\,\alpha\,H) + z^2 k^3\alpha^3\psi\,\mu_f\cosh(k\,\alpha\,H) + 2\,k\,\alpha\cosh(k\,\alpha\,H) - 2\cosh(k\,\alpha\,z)\,\mu_f\sinh(k\,\alpha\,H) - 2\cosh(k\,\alpha\,z)\,k\,\alpha + 2\sinh(k\,\alpha\,z)\,\mu_f\cosh(k\,\alpha\,H) + 2\sinh(k\,\alpha\,z)\,H\,\psi\,\mu_f k^2\alpha^2\cosh(k\,\alpha\,H) - 2\cosh(k\,\alpha\,z)\,H\,\psi\,\mu_f k^2\alpha^2\sinh(k\,\alpha\,H) - 2\,z\,H\,k^3\alpha^3\psi\,\mu_f\cosh(k\,\alpha\,H) + 2\,\mu_f\sinh(k\,\alpha\,H) - 2\sinh(k\,\alpha\,z)\,\mu_f + \alpha^3 k^3 H^2\cosh(k\,\alpha\,H) + z^2 k^3\alpha^3\cosh(k\,\alpha\,H) - 2\,z\,H\,k^3\alpha^3\cosh(k\,\alpha\,H) + z^2 k^2\alpha^2\mu_f\sinh(k\,\alpha\,H) + \mu_f\alpha^2 k^2 H^2\sinh(k\,\alpha\,H) + 2\,k\,\alpha\,\psi\,\mu_f\cosh(k\,\alpha\,H) - 2\cosh(k\,\alpha\,z)\,H\,k^2\alpha^2\sinh(k\,\alpha\,H) - 2\cosh(k\,\alpha\,z)\,k\,\alpha\,\psi\,\mu_f + 2\sinh(k\,\alpha\,z)\,H\,k^2\alpha^2\cosh(k\,\alpha\,H)\Big)\Big)\Big/\Big(k^4\,l\,\big(\mu_f\sinh(k\,\alpha\,H) + k\,\alpha\cosh(k\,\alpha\,H) + k\,\alpha\,\psi\,\mu_f\cosh(k\,\alpha\,H)\big)\,\alpha^2\Big):$$

$N_{plot,\ stiffened,\ 1} := simplify\Big(subs\Big(k=1.1,\ \alpha=0.104908,\ H=38.48,\ w=25,\ E=11\cdot 10^6,\ 1=5.70494,\ \psi=78.71,\ \lambda_r=0.01537,\ \lambda_v=0.023966,\ \mu_f=0.039336,\ i_t=3.266,\ N_{elastic,\ stiffened}\Big)\Big);$

$$1243.889297\sinh(0.1153988\,z)+2953.246836-1249.955157\cosh(0.1153988\,z)-139.3601549\,z+1.810812824\,z^2 \quad \textbf{(3)}$$

$N_{1,\,0} := eval\left(N_{plot,\,stiffened,\,1}, z=0\right);$

$$1703.291679 \tag{4}$$

$q_{elastic,\,stiffened} := \frac{1}{2}\Big(w\big(2\,H\,\psi\,\mu_f k^2\,\alpha^2\cosh(k\,\alpha\,H) + \cosh(k\,\alpha\,z)\,k^4\,\lambda_r H^2\,\alpha^2\cosh(k\,\alpha\,H)$
$- \cosh(k\,\alpha\,z)\,\mu_f\alpha^2\,k^2\,H^2\cosh(k\,\alpha\,H) - \sinh(k\,\alpha\,z)\,k^4\,\lambda_r H^2\,\alpha^2\sinh(k\,\alpha\,H)$
$+ \sinh(k\,\alpha\,z)\,\mu_f\alpha^2\,k^2\,H^2\sinh(k\,\alpha\,H) - 2\cosh(k\,\alpha\,z)\,\mu_f\cosh(k\,\alpha\,H)$
$+ 2\sinh(k\,\alpha\,z)\,\mu_f\sinh(k\,\alpha\,H) + 2\sinh(k\,\alpha\,z)\,k\,\alpha - 2\cosh(k\,\alpha\,z)\,H\,\psi\,\mu_f k^2\,\alpha^2\cosh(k\,\alpha\,H)$
$+ 2\sinh(k\,\alpha\,z)\,H\,\psi\,\mu_f k^2\,\alpha^2\sinh(k\,\alpha\,H) + 2\,k\,\alpha\,H\,\mu_f\sinh(k\,\alpha\,H) - 2\,k\,\alpha\,z\,\mu_f\sinh(k\,\alpha\,H)$
$- 2\,k^2\,\alpha^2\,z\,\psi\,\mu_f\cosh(k\,\alpha\,H) + 2\cosh(k\,\alpha\,z)\,\mu_f - 2\,k^2\,\alpha^2\,z\cosh(k\,\alpha\,H) + 2\,H\,k^2\,\alpha^2\cosh(k\,\alpha\,H)$
$- 2\cosh(k\,\alpha\,z)\,H\,k^2\,\alpha^2\cosh(k\,\alpha\,H) + 2\sinh(k\,\alpha\,z)\,H\,k^2\,\alpha^2\sinh(k\,\alpha\,H)$
$+ 2\sinh(k\,\alpha\,z)\,k\,\alpha\,\psi\,\mu_f\big)\Big) \Big/ \Big(\alpha\,k^3\,l\,\big(\mu_f\sinh(k\,\alpha\,H) + k\,\alpha\cosh(k\,\alpha\,H)$
$+ k\,\alpha\,\psi\,\mu_f\cosh(k\,\alpha\,H)\big)\Big):$

$q_{plot,\,stiffened,\,1} := simplify\Big(subs\Big(k = 1.1\ , \alpha = 0.104908, H = 38.48, w = 25, E = 11\cdot 10^6, l = 5.70494, \psi = 78.71, \lambda_r = 0.01537, \lambda_v = 0.023966, \mu_f = 0.039336, i_t = 3.266, q_{elastic,\,stiffened}\Big)\Big);$

$$139.3601548 - 143.5433322\cosh(0.1153988\,z) + 144.2433252\sinh(0.1153988\,z) - 3.621625644\,z \tag{5}$$

$solve\left(diff\left(q_{plot,\,stiffened,\,1}, z\right) = 0, z\right);$

$$38.47999946,\ 13.67742575 \tag{6}$$

$q_{1,\,\max} := 2.96\cdot eval\left(q_{plot,\,stiffened,\,1}, z = 13.67742575\right);$

$$183.0314110 \tag{7}$$

$Q_1 := 78.71\cdot eval\left(q_{plot,\,stiffened,\,1}, z=0\right);$

$$-329.2578932 \tag{8}$$

$\delta_{v,\,1} := \frac{\left(N_{1,\,0} + Q_1\right)}{506896.55};$

$$0.002710678906 \tag{9}$$

$m_0 := \frac{25\cdot(38.48 - 0)^2}{2};$

$$18508.88000 \tag{10}$$

$M_{stiffened} := \Big(\frac{1}{2}\Big(\Big(\Big(\big(H\big(H\,\lambda_r k^2 - 2 + (-H - 2\,\psi)\,\mu_f\big)\,k^2\,\alpha^2 - 2\,\mu_f\big)\sinh(k\,\alpha\,z) + \alpha\big(-2 + k^2\,(k - 1)\,(k + 1)\,(H - z)^2\,\alpha^2\big)\,k\,\big(\mu_f\psi + 1\big)\Big)\cosh(k\,\alpha\,H) + \Big(\big(-H\big(H\,\lambda_r k^2 - 2 + (-H - 2\,\psi)\,\mu_f\big)\,k^2\,\alpha^2 + 2\,\mu_f\big)\cosh(k\,\alpha\,z) + \mu_f\big(-2 + k^2\,(k - 1)\,(k + 1)\,(H - z)^2\,\alpha^2\big)\Big)\sinh(k\,\alpha\,H) + 2\,k\,\alpha\,\big(\mu_f\psi + 1\big)\cosh(k\,\alpha\,z) + 2\sinh(k\,\alpha\,z)\,\mu_f\Big)\,w\Big) \Big/ \Big(\alpha^2\,k^4\,\big(k\,\alpha\,\big(\mu_f\psi + 1\big)\cosh(k\,\alpha\,H) + \mu_f\sinh(k\,\alpha\,H)\big)\Big)\Big):$

$M_{plot,\,stiffened,\,1} := simplify\big(subs\big(k=1.1\ ,\alpha=0.104908, H=38.48, w=25, E=11\cdot 10^{6}, \mathrm{l}=5.70494, \psi=78.71, \lambda_r=0.01537, \lambda_v=0.023966, \mu_f=0.039336, i_t=3.266, M_{stiffened}\big)\big);$

$M_1 := \frac{eval(M_{plot,\,stiffened,\,1}, z=0)\cdot 3.227}{3.266};$

$$8686.719572 \tag{11}$$

$M_2 := \frac{eval(M_{plot,\,stiffened,\,1}, z=0)\cdot 0.039}{3.266};$

$$104.9835957 \tag{12}$$

$S_1 := \frac{w\cdot H\cdot I_1}{i_t}\left(1-\frac{z}{H}\right) - \left(\frac{I_1}{i_t}\cdot l - \frac{b}{2} - d_1\right)\cdot q_{elastic,\,stiffened} : S_2 := \frac{w\cdot H\cdot I_2}{(i_t)}\left(1-\frac{z}{H}\right) + \left(\frac{I_1}{i_t}\cdot l - \frac{b}{2} - d_1\right)\cdot q_{elastic,\,stiffened} :$

$fn_{beam} := \left(\frac{1}{2}\cdot\frac{\mathrm{d}}{\mathrm{d}z}S_1 - \frac{1}{2}\cdot\frac{\mathrm{d}}{\mathrm{d}z}S_2 + \frac{1}{2}\cdot w\right);$

$$-\frac{1}{2}\frac{w\,\mathrm{I}_1}{i_t} - \frac{1}{2}\Bigg(\left(\frac{\mathrm{I}_1\,l}{i_t} - \frac{1}{2}b - d_1\right) w\,\big(\sinh(\alpha k z)\,k^5\alpha^3\lambda_r H^2\cosh(\alpha k H) - \sinh(\alpha k z)\,k^3\alpha^3\mu_f H^2\cosh(\alpha k H) - \cosh(\alpha k z)\,k^5\alpha^3\lambda_r H^2\sinh(\alpha k H) + \cosh(\alpha k z)\,k^3\alpha^3\mu_f H^2\sinh(\alpha k H) - 2\sinh(\alpha k z)\,k\,\alpha\,\mu_f\cosh(\alpha k H) + 2\cosh(\alpha k z)\,k\,\alpha\,\mu_f\sinh(\alpha k H) + 2\cosh(\alpha k z)\,k^2\alpha^2 - 2\sinh(\alpha k z)\,k^3\alpha^3 H\psi\mu_f\cosh(\alpha k H) + 2\cosh(\alpha k z)\,k^3\alpha^3 H\psi\mu_f\sinh(\alpha k H) - 2\alpha k\mu_f\sinh(\alpha k H) - 2k^2\alpha^2\psi\mu_f\cosh(\alpha k H) + 2\sinh(\alpha k z)\,k\,\alpha\,\mu_f - 2k^2\alpha^2\cosh(\alpha k H) - 2\sinh(\alpha k z)\,k^3\alpha^3 H\cosh(\alpha k H) + 2\cosh(\alpha k z)\,k^3\alpha^3 H\sinh(\alpha k H) + 2\cosh(\alpha k z)\,k^2\alpha^2\psi\mu_f\big)\Bigg) \Big/ \big(\alpha k^3 l\,(\mu_f\sinh(\alpha k H) + k\alpha\cosh(\alpha k H) + k\alpha\psi\mu_f\cosh(\alpha k H))\big) + \frac{1}{2}\frac{w\,\mathrm{I}_2}{i_t} + \frac{1}{2}w \tag{13}$$

$fn_{plot,\,stiffened,\,1} := simplify\big(subs\big(k=1.1\ ,\alpha=0.104908, H=38.48, w=25, E=11\cdot 10^{6}, \mathrm{l}=5.70494, \psi=78.71, \lambda_r=0.01537, \lambda_v=0.023966, \mu_f=0.039336, i_t=3.266, \mathrm{I}_2=0.039, \mathrm{I}_1=3.227, b=2.15, d_1=2.853, fn_{beam}\big)\big);$

$$6.487222524 + 28.30607437\sinh(0.1153988\,z) - 28.44410972\cosh(0.1153988\,z) \tag{14}$$

$eval(fn_{plot,\,stiffened,\,1}, z=14.25159827)\cdot 2.96;$

$$1.926760792 \tag{15}$$

Second part;

Second part **(16)**

$x_{plot,\ stiffened,\ 2} := simplify\Big(subs\Big(k = 1.099\ , \alpha = 0.2155, H = 38.48, w = 25, E = 11 \cdot 10^6, 1 = 5.36, \psi = 112.31, \lambda_r = 0.071024, \lambda_v = 0.125421, \mu_f = 0.196445, i_t = 3.574, \mathrm{x}_{elastic,\ stiffenned}\Big)\Big);$

$$-3.556333850\ 10^{-7} \sinh(-9.113391560 + 0.2368345\ z) + 0.00003580672186\ z^2 - 7.016634206\ 10^{-7}\ z^3 + 4.558624094\ 10^{-9}\ z^4 + 0.0008327370381\ z - 0.001613904258 + 3.559644177\ 10^{-8} \cosh(0.2368345\ z) + 1.280241014\ 10^{-9} \sinh(0.2368345\ z) \quad \textbf{(17)}$$

$eval\big(x_{plot,\ stiffened,\ 2}, z = 38.48\big);$

$$0.05363215431 \quad \textbf{(18)}$$

$N_{plot,\ stiffened,\ 2} := simplify\Big(subs\Big(k = 1.099\ , \alpha = 0.2155, H = 38.48, w = 25, E = 11 \cdot 10^6, 1 = 5.36, \psi = 112.31, \lambda_r = 0.071024, \lambda_v = 0.125421, \mu_f = 0.196445, i_t = 3.574, \mathrm{N}_{elastic,\ stiffened}\Big)\Big);$

$$663.9574251 \sinh(0.2368345\ z) - 663.9725803 \cosh(0.2368345\ z) - 148.5986616\ z + 1.930855789\ z^2 + 2927.886021 \quad \textbf{(19)}$$

$N_{2,\ 0} := eval\big(N_{plot,\ stiffened,\ 2}, z = 0\big);$

$$2263.913441 \quad \textbf{(20)}$$

$q_{plot,\ stiffened,\ 2} := simplify\Big(subs\Big(k = 1.099\ , \alpha = 0.2155, H = 38.48, w = 25, E = 11 \cdot 10^6, 1 = 5.36, \psi = 112.31, \lambda_r = 0.071024, \lambda_v = 0.125421, \mu_f = 0.196445, i_t = 3.574, q_{elastic,\ stiffened}\Big)\Big);$

$$148.5986615 - 157.2480247 \cosh(0.2368345\ z) + 157.2516140 \sinh(0.2368345\ z) - 3.861711576\ z \quad \textbf{(21)}$$

$solve\big(diff\ (q_{plot,\ stiffened,\ 2}, z) = 0, z\big);$

$$38.47997818,\ 9.573751711 \quad \textbf{(22)}$$

$q_{2,\ \max} := 2.96 \cdot eval\big(q_{plot,\ stiffened,\ 2}, z = 9.573751711\big);$

$$282.2558841 \quad \textbf{(23)}$$

$\mathrm{Q}_2 := 112.31 \cdot eval\big(q_{plot,\ stiffened,\ 1}, z = 0\big);$

$$-469.8126538 \quad \textbf{(24)}$$

$\delta_{v,\ 2} := \dfrac{(N_{2,\ 0} + Q_2)}{506896.55};$

$$0.003539382517 \quad \textbf{(25)}$$

$m_0 := \dfrac{25 \cdot (38.48 - 0)^2}{2};$

$$18508.88000 \quad \textbf{(26)}$$

$M_{plot,\ stiffened,\ 2} := simplify\Big(subs\Big(k = 1.099\ , \alpha = 0.2155, H = 38.48, w = 25, E = 11 \cdot 10^6, 1 = 5.36, \psi = 112.31, \lambda_r = 0.071024, \lambda_v = 0.125421, \mu_f = 0.196445, i_t = 3.574, M_{stiffened}\Big)\Big);$

$$-3558.811800 \sinh(0.2368345\ z) + 2815.410931 - 165.5111745\ z + 2.150612974\ z^2 + 3558.893031 \cosh(0.2368345\ z) \quad \textbf{(27)}$$

$$M_{1,s} := \frac{eval\left(M_{plot,\ stiffened,\ 2}, z=0\right)\cdot 3.227}{3.57};$$

$$5761.870836 \tag{28}$$

$$M_{2,s} := \frac{eval\left(M_{plot,\ stiffened,\ 2}, z=0\right)\cdot 0.347}{3.57};$$

$$619.5752030 \tag{29}$$

$fn_{plot,\ stiffened,\ 2} := simplify\Big(subs\Big(k=1.099, \alpha=0.2155, H=38.48, w=25, E=11\cdot 10^6, l=5.36, \psi=112.31, \lambda_r=0.071024, \lambda_v=0.125421, \mu_f=0.196445, i_t=3.574, I_2=0.347, I_1=3.227, b=1.1, d_1=2.853, fn_{beam}\Big)\Big);$

$$7.974975992 + 53.50140026\sinh(0.2368345\,z) - 53.50262149\cosh(0.2368345\,z) \tag{30}$$

$eval\left(fn_{plot,\ stiffened,\ 2}, z=9.829\right)\cdot 2.96;$

$$8.145838304 \tag{31}$$

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