MODIFIED BOND MODEL FOR SHEAR IN SLABS UNDER CONCENTRATED LOADS

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Abstract

Slabs subjected to concentrated loads close to supports, as occurring for truck loads on slab bridges, are less studied than beams in shear or slab-column connections in punching. To predict the shear capacity for this case, the Bond Model for concentric punching shear was studied initially. Modifications to this model resulted in the Modified Bond Model, which takes into account the enhanced capacity from the direct transfer of the load to the support, is able to deal with moment sign changes as occurring near continuous supports, and can take into account the reduction in capacity, resulting from the geometry when the load is placed close to the edge. The model is then compared to the results of experiments on slabs subjected to concentrated loads close to supports. As compared to the Eurocode and the ACI code, the Modified Bond Model leads to a significantly better prediction of the experimental results. The Modified Bond Model is one of the few models available to describe the shear capacity of slabs subjected to concentrated loads close to supports and can be used for design and assessment.

Keywords: Bridge Engineering, Concentrated loads, Design, Punching, Shear, Slabs

1 Introduction

The shear problem is typically addressed by studying two well-defined cases: 1) the one-way shear capacity of beams (beam shear), and 2) the two-way shear capacity of slabs (punching shear). One-way shear in beams is most often studied on small, heavily reinforced, slender beams, tested in four point bending (Collins & al. 2008, Reineck & al. 2013), resulting in the semi-empirical expressions as given in NEN-EN 1992-1-1:2005 (CEN 2005) and ACI 318-11 (ACI Committee 318 2011) for the beam shear capacity. Two-way shear in slabs is studied on slab-column specimens (ASCE-ACI Task Committee 426 1974). These experiments form the basis of the semi-empirical punching shear provisions as given in NEN-EN 1992-1-1:2005 and ACI 318-11.

Besides these two standard cases of the shear problem that have been widely studied over the past decades, other loading cases, often at the intersection of beam shear and punching shear, arise in practice. An example is the shear capacity of existing reinforced concrete solid slab bridges, subjected to concentrated live loads, that result in large shear stresses at the support when the loads are located close to the support (Lantsoght & al. 2013). The available code provisions are not fully suitable to determine the shear capacity of slabs subjected to concentrated loads close to supports, a problem at the intersection between one-way and two-way shear.

To describe the behaviour of reinforced concrete slabs subjected to concentrated loads close to the support, a new model is proposed. This model is a combination of load-bearing quadrants and strips, and is based on the Bond Model (Alexander 1990, Alexander & Simmonds 1992). The resulting Modified Bond Model can be considered a mechanical model, in which the concept of a limiting one-way shear stress is incorporated. Where most beam shear and punching shear models make a strict distinction between these two modes of failure, the Bond Model considers the shear-carrying behaviour as an action of two-way quadrants and one-way strips in arching action. As such, it is the most suitable model for the considered case that is a combination of one-way shear and two-way shear.

2 Bond Model for concentric punching shear

The Bond Model for concentric punching shear (Alexander 1990, Alexander & Simmonds 1992) is a mechanical model for slab-column connections that combines radial arching action and the concept of a critical shear stress, as used for beam shear. Shear, V, (moment gradient) results where the magnitude of the force T or lever arm z varies along the length of the member. As such, shear is carried by a combination of beam action and arching action:

$$V = \frac{d(Tz)}{dx} = \frac{d(T)}{dx} z + \frac{d(z)}{dx} T \tag{1}$$

For slabs, arching action is the dominant mechanism in the radial direction. It is assumed that the load is distributed in the radial directions from the column by strips working in arching. Four strips branching out from the column and parallel to the reinforcement are considered, Fig. 1. The strips separate the column from the slab quadrants. The length of the strips, l_{strip}, is determined from the column to a remote end, a position of zero shear.

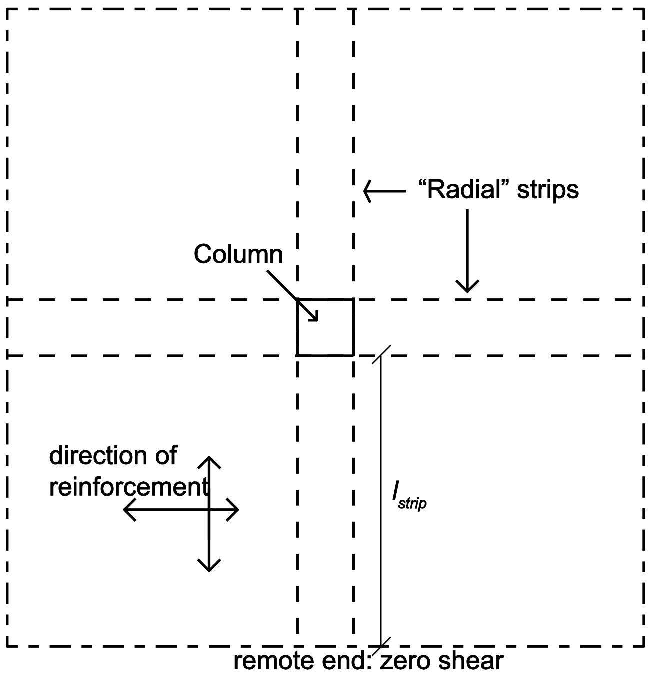

Fig. 1 Slab-column connection: division into radial strips and quadrants (Alexander & Simmonds 1992).

The strips are loaded in shear on their side faces only and are considered as cantilever beams, fixed into the column (Fig. 2). These cantilevers have negative and positive moment capacities of M_{neg} and M_{pos} that are summed into M_s, the total flexural capacity of the strip. The length l_w is the loaded length of the strip, and q the uniformly distributed load. The loading term q is an estimate of the shear that can be delivered by the adjacent quadrant of the slab to one side face of the strip. For a strip with two side faces, the total uniformly distributed load on the strip is $2q$. Using force and moment equilibrium of the cantilever strip (Fig. 2) results in the following expressions for the total flexural capacity M_s and the concentrated load at the column $P_{AS,1}$:

$$M_s = \frac{2ql_w^{\ 2}}{2} \tag{2}$$

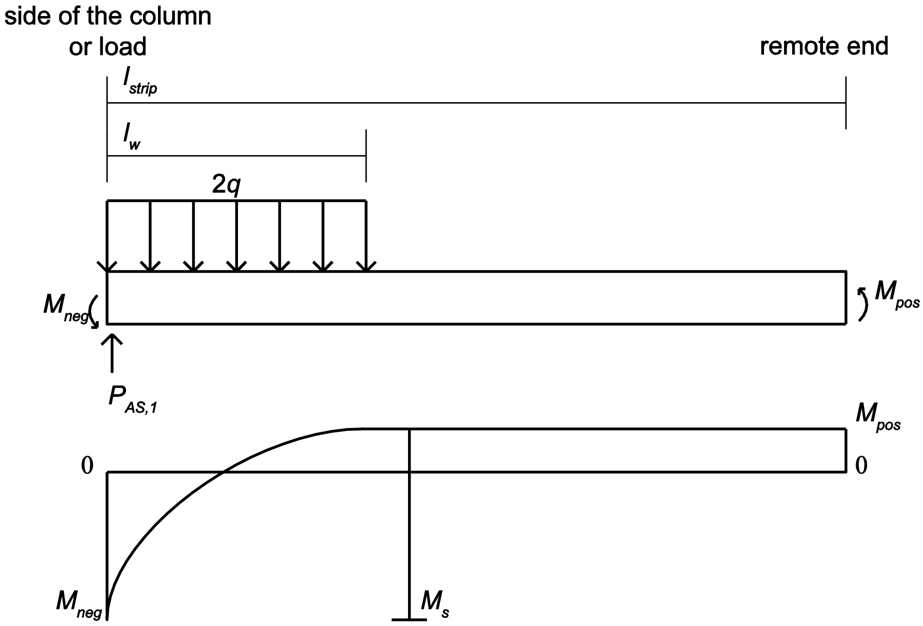

Fig. 2 Loading of strip over loaded length l_w and resulting moment diagram (Alexander & Simmonds 1992).

$$P_{AS,1} = 2ql_w \tag{3}$$

Solving Eq. (2) for the unknown loaded length l_w and substituting this into Eq. (3) results in:

$$P_{AS,1} = 2\sqrt{M_s q} \tag{4}$$

To find the maximum column axial load P_{AS}, the capacity of all four strips can be summed:

$$P_{AS} = 8\sqrt{M_s q} \tag{5}$$

The maximum value of the loading term q is based on the equivalence between the maximum value of beam action shear and a limiting nominal one-way shear stress as prescribed by the codes. Using the one-way shear capacity from ACI 318-11, empirically defined as the inclined cracking load (Morrow & Viest 1957), was found to lead to the best results (Alexander & Simmonds 1992). When the maximum value of the loading term is limited by beam action shear, it is expressed as follows:

$$q_{ACI} = 0.1667d\sqrt{f_c^{'}} \tag{6}$$

In Eq. (6), q_{ACI} is given in [kN/m] with the concrete compressive strength f_c' in [MPa] and the effective depth d in [mm].

3 Development of the Modified Bond Model

3.1 Loads close to the support

To apply the Bond Model to the case of slabs subjected to concentrated loads close to supports, it is necessary to take into account direct load transfer. For one-way slabs, the different properties in the span direction and in the transverse direction need to be taken into account. The four cantilevering strips branching out from the load can be studied together to sketch the assumed moment distribution, Fig. 3, showing the geometry and layout from the slab shear experiments carried out at Delft University of Technology (Lantsoght & al. 2013).

For the x-direction strip between the concentrated load and the support, direct transfer of the load from its point of application to the support needs to be taken into account. Regan described the punching capacity of slabs under concentrated loads close to supports by considering the 4 sides of the punching perimeter separately (Regan 1982). To take into account the beneficial influence of direct load transfer, the capacity of the side of the punching perimeter parallel and closest to the support was enhanced with a factor $2d_x / a_v$, in which d_x is the effective depth to the reinforcement in the x-direction and a_v is the face-to-face distance between the load and the support. Similarly, it is proposed to increase the capacity of the strip between the load and the support by enhancing the capacity with $2d_x / a_v$ for $0.5d_x < a_v < 2d_x$ and 4 for $a_v \leq 0.5d_x$.

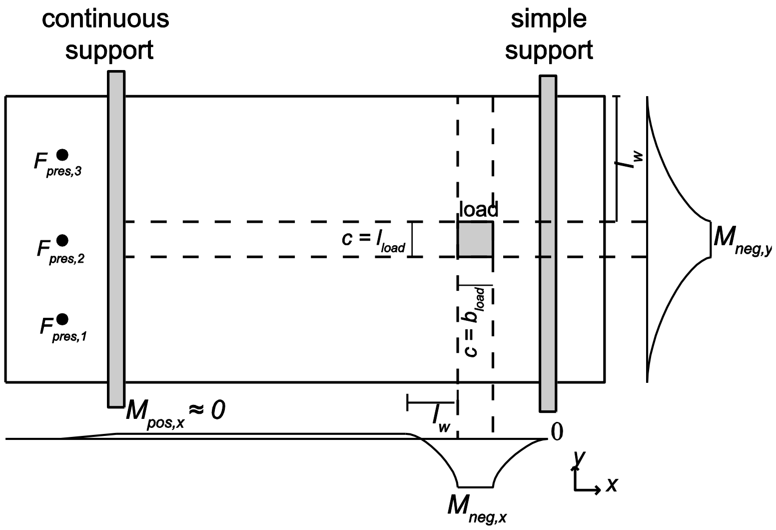

Fig. 3 Application of Bond Model to case of a slab under concentrated load.

The original Bond Model considered a slab of infinite dimensions. For the application to bridge deck slabs with finite sizes and given support conditions, it is necessary to take into account the difference in amount of shear that flows to the two supports bordering the studied span. A sketch of this situation with a load placed at a distance $a < L/2$ from the support on a slab of span length L is given in **Fig. 4**. If the load is increased from 0 to P_{max}, failure is induced following the cruciform shape outlined in **Fig. 4**. At the moment of failure, more load will be transferred to the closest support than to the farthest support. As such, it can be estimated that the interface shear between the strips and the quadrants $v_{2,x}$ towards the closest support will be larger than the shear $v_{1,x}$ facing the farthest support.

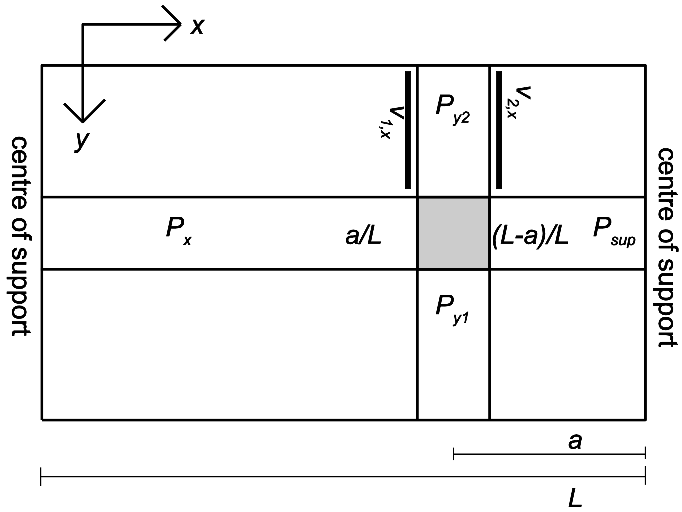

Fig. 4 Idea behind unequal loading of the strips.

If $v_{2,x}$ reaches the inclined cracking load q_{ACI} failure of the section is reached. At that moment, the shear at $v_{1,x}$ will be less. The ratio between the shears $v_{1,x}$ and $v_{2,x}$ at the strip is given as:

$$\frac{v_{1,x}}{v_{2,x}} = \frac{\frac{a}{L}}{\frac{L-a}{L}} = \frac{a}{L-a} \tag{7}$$

When $v_{2,x}$ reaches the inclined cracking load q_{ACI}, $v_{1,x}$ will have reached only a fraction of this load:

$$v_{1,x} = \frac{a}{L-a} 0.1667\sqrt{f_c^{'}} d \tag{8}$$

As a result, the total load on the *y*-direction strip at failure is:

$$v_{1,x} + v_{2,x} = \left(1 + \frac{a}{L-a}\right) 0.1667\sqrt{f_c^{'}} d = \frac{L}{L-a} 0.1667\sqrt{f_c^{'}} d \tag{9}$$

The capacity of the strips (as sketched in Fig. 4) can then be expressed as:

$$P_{y1} = P_{y2} = (v_{1,x} + v_{2,x}) l_w = \sqrt{2M_{neg,y}(v_{1,x} + v_{2,x})} = \sqrt{2M_{neg,y} \frac{L}{L-a} 0.1667\sqrt{f_c^{'}} d_x} \tag{10}$$

$$P_x = 2\sqrt{M_{neg,x} q_{ACI,x}} \tag{11}$$

$$P_{sup} = \frac{2d_x}{a_v} \times 2\sqrt{M_{neg,x} q_{ACI,x}} \tag{12}$$

$$w_{ACI,x} = 0.1667 d_y \sqrt{f_c^{'}} \tag{13}$$

$$w_{ACI,y} = 0.1667 d_x \sqrt{f_c^{'}} \tag{14}$$

3.2 Loads close to a continuous support

A first extension of the model deals with loads applied close to the continuous support, in which the hogging moment reinforcement can increase the total flexural capacity. As the sagging and hogging moment capacities are not activated in the same cross-sections of the strips and because yielding of the hogging reinforcement is not assumed in the model, the following expression is proposed to take into account the effect of the hogging moment reinforcement:

$$M_s = M_{neg} + \lambda_{moment} M_{pos} \tag{15}$$

The factor λ_{moment} ranges from 0 (for simply supported edges) to 1 (for fully restrained cases) and equals (Fig. 5):

$$\lambda_{moment} = \frac{M_{sup}}{M_{span}} \leq 1 \tag{16}$$

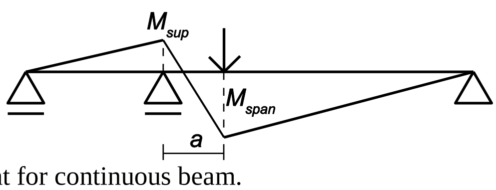

Fig. 5 Support and span moment for continuous beam.

When the concentrated load is placed close to a continuous support, two quadrants experience the change in moment from hogging moment M_{sup} to sagging moment M_{span}. As a result, the combined effect of the top and bottom reinforcement should be taken into account on the three strips that border these two quadrants: the two *y*-direction strips as well as the *x*-direction strip between the load and the support. The resulting strip capacities are then:

$$P_{y1} = P_{y2} = \sqrt{2M_{s,y}\frac{L}{L-a} \times 0.1667\sqrt{f_c^{'}}d_x} \tag{17}$$

$$P_x = 2\sqrt{M_{neg,x}q_{ACI,x}} \tag{18}$$

$$P_{sup} = \frac{2d_x}{a_v} \times 2\sqrt{M_{s,x}q_{ACI,x}} \tag{19}$$

3.3 Loads close to a free edge

For loads applied right at the edge, only 2 quadrants and 3 strips can develop. For this case, the strips in the *x*-direction are loaded from one side only and the strip in the *y*-direction is loaded from both sides. As the load is placed farther away from the free edge, a small *y*-direction strip develops, influenced by the presence of the edge.

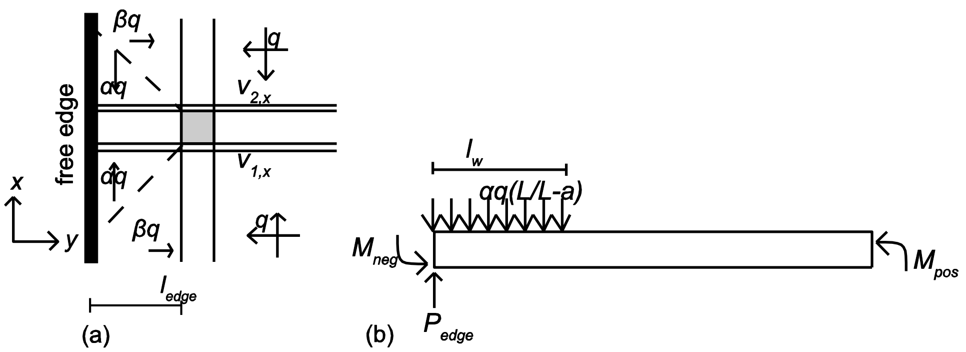

Fig. 6 Modified Bond Model for loading close to a free edge: (a) resulting strips and quadrants, and loading in the quadrants; (b) loading on *y*-direction strip between load and free edge.

To describe the edge effect, the loaded length of the *y*-direction strip between the load and the edge is studied. When the *y*-direction are loaded with *αq(L/L-a)* (Fig. 6) the loaded length will be:

$$l_w = \frac{M_{s,y}}{l_{edge}q_{ACI,y}\left(\frac{L}{L-a}\right)} \text{ if } l_w < l_{edge} \tag{20}$$

$$l_w = \sqrt{\frac{M_{s,y}}{q_{ACI,y}\left(\frac{L}{L-a}\right)}} \text{ if } l_w \geq l_{edge} \tag{21}$$

The distance between the edge and the face of the load, l_{edge} (Fig. 6) is expressed as (with b_r the distance between the centre of the load and the edge and l_{load} the length of the load):

$$l_{edge} = b_r - \frac{l_{load}}{2} \tag{22}$$

If l_w from Eq. (21) is larger than l_{edge} from Eq. (22), the model would be assuming a loaded length of the strip that is longer than what is physically possible. Therefore, for those cases the edge effect needs to be taken into account: the loaded length l_w needs to be limited to the edge length l_{edge}. Not the full load q will be transferred to the strip between the load and the edge when the edge effect is present. Let's now assume that only a fraction $\alpha \times q$ with $\alpha < 1$ can be carried off in the quadrants instead of q. The value of α can be expressed as:

$$\alpha = \frac{l_{edge}}{l_w} \text{ for } l_{edge} \leq l_w \tag{23}$$

Torsion was neglected in the original Bond Model, as the influence of the torsional moments is small when loads are placed on infinitely large slabs. However, when the load is placed close to the edge of a slab, torsional distress influences the capacity. As a result of the influence of torsion, a smaller capacity than predicted by the (Modified) Bond Model is found. The influence of torsion was studied based on experimental results (Lantsoght & al. 2014), and a simplified method suitable for hand calculations was proposed (Fig. 6a). The influence of torsion is taken into account by using the factor $\beta \leq 1$. Because the influence of torsion is related to the edge effect, this influence is assumed to act only in the quadrants between the load and the edge. Moreover, it is assumed that this influence acts only in the y-direction, the weaker direction in which a lower amount of reinforcement is provided. As such, it is assumed that the capacity of the x-direction strips, which carry the larger part of the load in one-way slabs, is reduced and are loaded with $(1 + \beta)q$. The value of the factor β, which takes torsion into account, can be expressed based on the resulting bending moment m_1 and the resulting torsional moment $m_{t,max}$ at the position of the load. Since the goal of the Modified Bond Model is to provide a design method which can be used without the need of finite element programs, a simplification is to use $\beta = 0$ for loading cases where the maximum bending moment and maximum torsional moment in a slab coincide, such as the case of loads close to the edge, and to use $\beta = 1$ when the maximum bending moment coincides with a small torsional moment, such as for loading in the middle of the slab width.

Moreover, the effect of the unequal loading on the y-direction strips due to the finite span length of the studied situation (Fig. 4) needs to be taken into account. The capacity of the strips is then:

$$P_{sup} = \frac{2d_x}{a_v}\sqrt{2(1+\beta)M_{s,x}q_{ACI,x}} \tag{24}$$

$$P_x = \sqrt{2(1+\beta)M_{neg,x}q_{ACI,x}} \tag{25}$$

$$P_y = \sqrt{2M_{s,y}\frac{L}{L-a}0.1667\sqrt{f_c^{'}}d_x} \tag{26}$$

$$P_{edge} = \alpha l_{gov}\frac{L}{L-a}0.1667\sqrt{f_c^{'}}d_x \tag{27}$$

The following symbols are used:

P_{sup}	capacity of the strip between the load and the support
P_x	capacity of the x-direction strip from the load towards the span of the slab
P_y	capacity of the y-direction strip not affected by the edge
P_{edge}	capacity of the y-direction strip between the load and the free edge
d_l	effective depth to the longitudinal reinforcement
a_v	face-to-face distance between the load and the support
$M_{s,x}$	$M_{neg,x} + \lambda_{moment}M_{pos,x}$: the moment capacity in the x-direction
$M_{s,y}$	$M_{neg,y} + \lambda_{moment}M_{pos,y}$: the moment capacity in the y-direction
$M_{neg,x}$	hogging moment capacity of the x-direction reinforcement
$M_{pos,x}$	sagging moment capacity of the x-direction reinforcement
$M_{neg,y}$	hogging moment capacity of the y-direction reinforcement
$M_{pos,y}$	sagging moment capacity of the y-direction reinforcement
λ_{moment}	as given in Eq. (16)
L	span length
a	centre-to-centre distance between the load and the support

f_c' specified concrete compressive strength
d_x effective depth to the x-direction reinforcement
$q_{ACI,x}$ as given in Eq. (13)
α the edge effect factor as given in Eq. (23)
l_{gov} the governing length: l_w or l_{edge} if $l_{edge} \leq l_w$

4 Comparison to experiments

To study the performance of the Modified Bond Model, all results of the experiments on slabs S1 to S10, tested at Delft University of Technology (Lantsoght, van der Veen & al. 2013) are compared to the results obtained with the Modified Bond Model. The maximum concentrated load in the experiment, P_{exp}, is compared to the maximum capacity obtained with the Modified Bond Model, P_{MBM}. Mean values are used for the material properties. The evaluation showed an average ratio P_{exp}/P_{MBM} = 1.33 with a standard deviation of 0.14 and a coefficient of variation of 10.8%. Considering that the problem under study is a shear problem with a large number of parameters that have been varied in the experiments, the statistical results are in agreement with the experiments. The results are shown graphically in Fig. 7a.

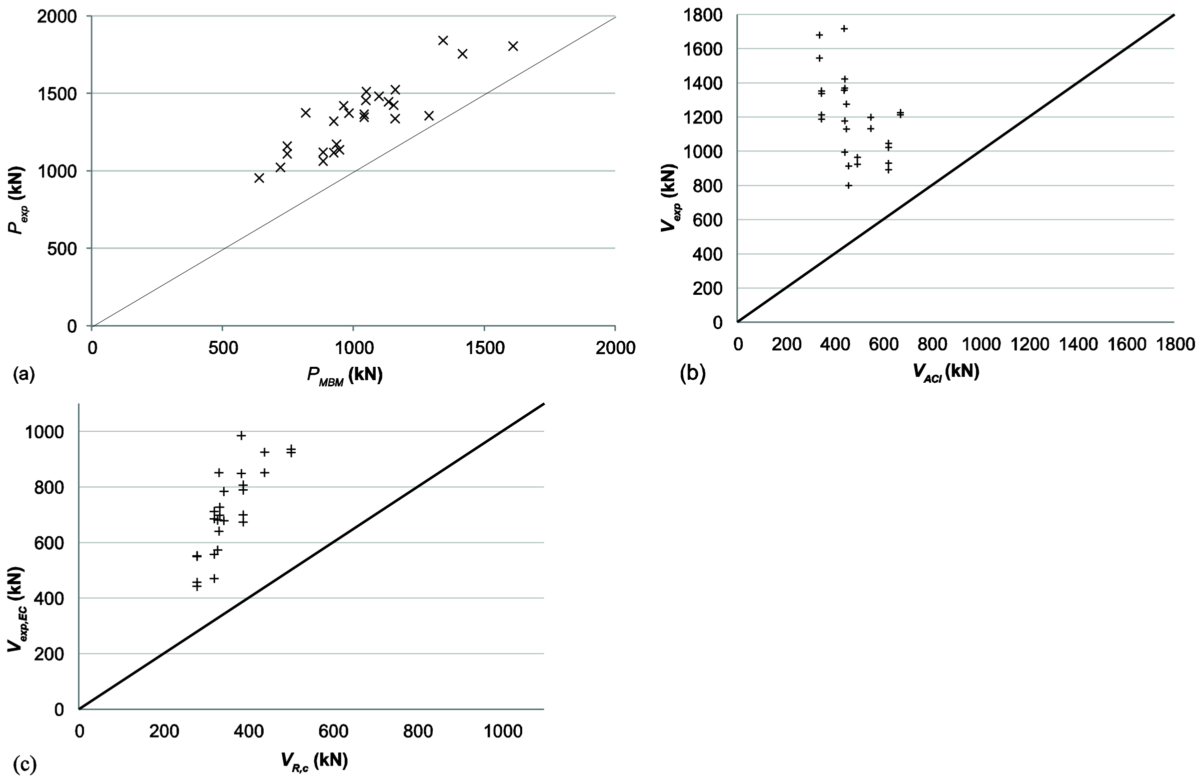

Fig. 7 Comparison between experimental results and (a) Modified Bond Model, (b) ACI 318-11 code, (c) NEN-EN 1992-1-1:2005.

Next, these experimental results are compared to the shear capacities according to ACI 318-11 (ACI Committee 318 2011) and NEN-EN 1992-1-1:2005 (CEN 2005), to evaluate the performance of the Modified Bond Model as compared to existing code provisions. Note that the code provisions predict a maximum sectional shear force, while the Modified Bond Model predicts a maximum concentrated load. The values of the experimental sectional shear force V_{exp} compared to the shear capacity according to ACI 318-11, V_{ACI}, are shown in Fig. 7b. The reduced sectional shear force $V_{exp,EC}$ is compared to the shear capacity according to NEN-EN 1992-1-1:2005, $V_{R,c}$, in Fig. 7c. The reduced sectional shear force $V_{exp,EC}$ takes into account a reduction of the contribution of the loads close to the support ($0.5d_x \leq a_v \leq 2d_x$) to the shear force by $\beta = a_v / 2d_x$ as prescribed by NEN-EN 1992-1-1:2005 §6.2.2 (6). The ratio of V_{exp}/V_{ACI} has an average value of 2.67, with a standard deviation of 1.00 and a coefficient of variation of 37%. For $V_{exp,EC}/V_{R,c}$ the average value is 1.99, with a standard deviation of 0.27 and a coefficient of variation of 13%. The statistical results show that the Modified Bond Model gives a better prediction of the experiments. The 45^o line in Fig. 7a, b and c indicates experimental shear capacities that are as predicted by the method under consideration. When using ACI 318-11 and NEN-EN 1992-1-1:2005 to compare to the experimental results, an increasing conservatism with increasing shear capacities is seen. The Modified Bond Model, on the other hand, shows more consistent results, as the cloud of test results lies above and parallel to the 45^o line.

5 Summary and conclusions

To study the transition zone between one-way and two-way shear, a new mechanical model is presented: the Modified Bond Model. This model is based on the Bond Model for concentric punching shear.

The following changes have been made to the original Bond Model so that it can be applied to one-way slabs subjected to concentrated loads:

- The geometry of the slab is taken into account. The difference in reinforcement ratios in the span and transverse directions is taken into account. The enhancement in the capacity for loads close to the support is built into the model. The unequal stresses that occur in a slab of finite dimensions are taken into account.
- For loads close to the continuous support, the effect of the change in sign of the moment is taken into account for the strips bordering the quadrants over which the sign of the moment changes.
- For loads close to the edge, it needs to be understood that sometimes not the full loaded length can develop, and then the physical length of the strip needs to be taken into account. For loads close to the edge, torsion starts to play a role.

When comparing the proposed model to experimental results from shear tests on one-way slabs subjected to a single concentrated load, it is found that the Modified Bond Model gives a significantly better estimate of the capacity than the shear capacity as prescribed by ACI 318-11 or NEN-EN 1992-1-1:2005.

Acknowledgements

The authors wish to express their gratitude and sincere appreciation to the Dutch Ministry of Infrastructure and the Environment (Rijkswaterstaat) for financing this research work and to InfraQuest for coordinating the cooperation between Delft University of Technology, Rijkswaterstaat and the research institute TNO.

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