# WAVES AND THEIR EFFECTS ON PILE-SUPPORTED STRUCTURES

## **R.L.WIEGEL**

### University of California, Berkeley, U.S.A.

#### SUMMARY

Three methods of presenting wave data are discussed: the significant wave (including the "design wave" concept), the wave spectrum, and the directional spectra. Their use in calculating wave forces on pile supported structures is described, with a discussion of the relative usefulness of the linear versus the non-linear approach. The concept of virtual mass is described, and how this leads to a type of non-linearity which is of great importance in the reversing flow field associated with wave motions. Finally, a plea is made for more wave data in order that adequate wave climates can be obtained for use by the design engineer, by the constructor, and by the operator.

#### INTRODUCTION

We are all aware of the tremendous forces exerted by hurricane and gale generated water waves on structures in the ocean. Man, since ancient times, has been constructing boats, breakwaters, and docks in a manner which he has hoped would be adequate to withstand these forces, often with success, but often failing. In recent years our knowledge of the physics of the phenomenon has been developed rather rapidly, permitting us to make better designs now than previously. Some concepts and details of the present state of our knowledge will be presented in this lecture.

Wind blowing over the ocean's surface drags water along with it, thus forming a current, while at the same time it generates waves. Many of the waves grow so steep that they become unstable and break, and in this breaking process they generate a substantial amount of turbulence. One of the most noticeable features of these waves is their irregularity, both in time and in space. Owing to the nature of the wind, the waves generated by the wind blowing over the water surface move in a continuous spread of directions, as measured from the direction of the mean wind velocity. Once the waves leave the generating area, they become smoother in appearance and are known as swell. Due largely to dispersion and angular spreading, the energy density decreases with distance travelled from the storm.

Three methods have been developed to represent these waves. The simplest method is to use the concept of a "significant wave" designated by a height  $(H_s)$ , period  $(T_s)$  and direction (see Wiegel, 1964). Another method utilizes a "one-dimensional spectrum," that is, the wave energy density as a continuous function of both component wave frequency and direction. Both the one-dimensional and directional spectra are based upon the concept of linear superposition of component waves and assuming the statistical independence of phase

angles amongst the frequency components. Although most of the wave data that are available have been obtained using the significant wave concept, a substantial amount of data is becoming available in the form of one-dimensional spectra.

Almost no directional spectra of ocean waves are available. Obtaining information of this type requires an array of wave gages, the use of an electronic analog to digital converter, and the use of a high-speed digital computer. Furthermore, the mathematical techniques necessary to obtain reliable directional spectra are difficult to use at the present time from a practical standpoint. However, it is expected that in the future many designs will be made which are based upon directional spectra, with the spectra being a few generalized types.

There are two principal reasons, beside the availability of data, which make the significant wave concept useful to the design engineer. One has to do with the problem of the conception of a design in the mind of an engineer, which, because of the large number of variables involved, requires a rather simple visualization of the variables. The second reason is that water waves are not a linear phenomenon, and in relatively shallow water where many structures are built, certain non-linearities are of controlling importance; the significant wave height, period and direction can be used together with the most appropriate non-linear theory for calculations. A variation of this concept is the use of the "design wave," a wave which has been estimated to be the most extreme which will be encountered during the life of a structure. Ultimately, it is expected that the mathematics of non-linear superposition will be developed sufficiently for the directional spectra concept to be used even in shallow water.

It is necessary to have information on the "wave climate" in the area of interest for the planning and design phases, and synoptic wave data for the construction and operation phases. Traditionally, the wave climate has been represented by "wave roses" or tables which have been obtained from visual observations, from wave recorders, or from hindcasts from weather maps. It would be of much greater benefit to the engineer to have wave data in the form of cumulative distribution functions in order to be able to make an economic design based upon the numerical probability of occurrence. In addition, it would also be better to have wave data in another form for use in planning construction and other operations; in the form of continuous observations, measurements, or hindcasts so that the statistical properties could be determined of the number of consecutive days the waves will be less than, or greater than, some safe or economic combination of height, period and direction. Continuous records would also permit the calculation of "wave spectra," and if an appropriate array were used, it would permit the calculation of "directional spectra" for a site.

Finally, a design philosophy is needed. Owing to the lack of statistical information, details of the forcing functions, and our inability to predict in advance our changing needs, it is usually necessary to develop a "plateau" type of design, rather than attempting to design for a sharply tuned optimum design.

#### LINEAR THEORY FOR PROGRESSIVE WAVES

## Linear Wave Theory

The coordinate system usually used is to take x in the plane of the undisturbed water surface and y as the vertical coordinate, measured positive up from the undisturbed water surface. The undisturbed water depth is designated as d. Sometimes the vertical coordinate is taken as measured positive up from the ocean floor, being designated by S.

The wave surface is given by

$$S_{s} = y_{s} + d = \frac{1}{2} H \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) + d \qquad (1)$$

where H is the wave height, L is the wave length, T is the wave period, t is time, and the subscript - refers to the wave surface. The wave length, L, and wave speed, C, are given by

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L}$$
(2)

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L}$$
(3)

where g is the acceleration of gravity. The horizontal component of water particle velocity, u, the local acceleration,  $\partial u/\partial t$ , and the pressure, p, are given by

$$u = \frac{\pi H}{T} \frac{\cosh 2\pi S/L}{\sinh 2\pi d/L} \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$
(4)

$$\frac{\partial u}{\partial t} = \frac{2\pi^2 H}{T^2} \frac{\cosh 2\pi S/L}{\sinh 2 d/L} \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$
(5)

$$\rho + \rho gy = \frac{1}{2} \rho g H \frac{\cosh 2\pi S/L}{\cosh 2\pi d/L} \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$
(6)

where  $\rho$  is the mass density of the water.

Similar expressions are available for the vertical components, and expressions are available of the water particle displacements (see Wiegel, 1964).

## Wave Forces on Piles

In a frictionless, incompressible fluid the force exerted on a fixed rigid submerged body may be expressed as (Lamb, 1945, p. 93)

$$F_{I} = (M_{o} + M_{a}) f_{f} = \rho BC_{M} f_{f}$$
(7)

where  $F_I$  is the inertia force,  $M_o$  is the mass of the displaced fluid,  $M_a$  is the so-called added mass which is dependent upon the shape of the body and the flow characteristics around the body, and  $f_f$  is the acceleration of the fluid at the center of the body were no body present.  $C_M$  has been found theoretically to be equal to 2.0 for a right circular cylinder by several investigators (see, for example, Lamb, 1945). The product of the coefficient of mass,  $C_M$ , the volume of a body, B, and the mass density of the fluid,  $\rho$ , is often called the "virtual mass" of a body (i.e.,  $M_o + M_a$ ) in an unsteady flow (Dryden, Murnagham and Bateman, 1956, p. 97).  $C_M$  is sometimes expressed as

$$C_{M} = 1 + C_{a}$$
(8)

where  $C_a$  is the coefficient of added mass.<sup>\*</sup> The mass of the fluid displaced by the body enters into Eq. 7, with one part of the inertial force being due to the pressure gradient in the fluid which causes the fluid acceleration (or deceleration). This force per unit length of cylinder,  $F_p$ , is given by

$$F_{p} = \oint p dy = \rho \frac{dU}{dt} \int x dy = \rho A_{o} \frac{dU}{dt}$$
(9)

in which A is the cross sectional area of the cylinder and  $\phi$  is a contour integral (McNown, 1957) which follows from the well-known relationship in fluid mechanics for irrotational flow

$$-\frac{1}{\rho}\frac{dp}{dx} = \frac{dU}{dt}$$
(10)

where dp/dx is the pressure gradient in the fluid in the absence of the body. In many papers on aerodynamic studies using wind tunnels F is called the "horizontal buoyancy" (see, for example, Bairstow, 1939). <sup>P</sup> The added mass term, expressed by  $C_a \ \rho \ A_o$  per unit length of cylinder, results from the acceleration of the flow around the body caused by the presence of the body. As the fluid is being accelerated around the body on the fluid), the fluid decelerating around the downstream face of the body will exert a smaller or larger force on the downstream face, depending upon whether the flow is accelerating or decelerating. This concept can be seen more clearly for the case of a body being accelerated or decelerated, through a fluid. The force necessary to do this is proportional to the mass per unit length of the cylinder, M<sub>c</sub>, plus the added mass, M<sub>a</sub>,

$$F_{I} = (M_{c} + C_{a} \rho A_{o}) \frac{dU}{dt} = (M_{c} + M_{a}) \frac{dU}{dt}$$
(11)

The leading face of the cylinder pushes on the fluid causing it to accelerate, and the fluid decelerating on the rear side of the cylinder pushes on the cylinder (with the equivalent reaction of the cylinder). In accelerated motion, the reaction at the front must be greater than the reaction at the rear as the fluid decelerating at the rear was not accelerated as much, when it was at the front, as the fluid in front is being accelerated at that instant.

It is unfortunate that the terms added mass and virtual mass have entered the literature as they tend to confuse our concept of the phenomenon. MacCamy and Fuchs (1954; see also Wiegel, 1964, p. 273) solved the diffraction problem of waves moving around a vertical right circular cylinder extending from the ocean bottom through the water surface, using linear wave theory. They solved for the potential, obtained the distorted pressure field from this potential, and integrated the x-component of force around the pile which resulted from this pressure field. In our coordinate system, their solution is

$$F_{Ih}(S) = \frac{\rho g H L}{\pi} \frac{\cosh 2\pi S/L}{\cosh 2\pi d/L} f_A(D/L) \sin \left(-\frac{2\pi t}{T} - \beta\right)$$
(12)

where

$$f_{A} (D/L) = \frac{1}{\{[J_{1}' (\pi D/L)]^{2} + [Y_{1}' (\pi D/L)]^{2}\}^{\frac{1}{2}}}$$
(13)

\*In many papers the term virtual mass is used for the term added mass. Owing to this, care must be exercised in reading the literature on the subject.

in which J<sub>1</sub> and Y<sub>1</sub> are Bessel functions of the first and second kinds, respectively, and the prime indicates differentiation.  $\beta$  is the angle of phase lag, and will not be shown here as  $\beta < 5^{\circ}$  for values of D/L < 1/10, although it is very large for large values of D/L. When D/L  $\rightarrow$  0,  $f_{\Delta}$  (D/L)  $\rightarrow \frac{1}{2} \pi$  ( $\pi$ D/L)<sup>2</sup>, and

$$F_{\rm Ih}(S) \rightarrow 2 \frac{2\pi^2 H}{T^2} \frac{\pi D^2}{4} \frac{\cosh 2\pi S/L}{\sinh 2\pi d/L} \sin \left(-\frac{2\pi t}{T} - \beta\right)$$
(14)

Neglecting  $\beta$  for small values of D/L, it can be seen that this is the commonly accepted equation for the inertial force, with  $C_M = 2$ .

In a real fluid, owing to viscosity, there is an additional force, known as the drag force,  $F_D$ . This force consists of two parts, one due to the shear stress of the fluid on the body, and the other due to the pressure differential around the body caused by flow separation. The most common equation used in the design of pile supported structures is due to Morison, O'Brien, Johnson and Schaaf (1953), and is:

$$F = F_{D} + F_{I} = \frac{1}{2} C_{D} \rho_{W} A |V| V + C_{M} \rho_{W} B \frac{dV}{dt}$$
(15)

where A is the projected area and B is the volume of the pile. As V and dV/dt vary with position, it is better to use the following equation where  $F_h(S)$  is the force per unit length of a circular pile. Consider the case of a pile installed vertically in water of depth d, extending from the bottom through the surface. The water particles move in an orbit due to the waves, with both horizontal and vertical components of velocity and acceleration, u, v, du/dt and dv/dt, respectively. The horizontal component of wave induced force, per unit length of pile, is given by

$$F_{h}(S) = \frac{1}{2} C_{D} \rho_{W} D |u| u + C_{M} \rho_{W} \frac{\pi D^{2}}{4} \frac{du}{dt}$$
(16)

Here, du/dt is

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$
(17)

If we consider only linear theory, the convective acceleration (the last three terms on the right-hand side of Eq. 17) can be neglected, leaving only the local acceleration; i.e.,  $du/dt \approx \partial u/\partial t$ . u and  $\partial u/\partial t$  are given by Eqs. 4 and 5. It can be seen that the drag and inertia forces are in quadrature, so that the maximum total force "leads the crest" of the wave. The larger the drag force relative to the inertia force, the closer will be the maximum total force to the passage of the wave crest past the pile. As will be pointed out in a later section, there is a relationship between  $C_D$  and  $C_M$ , so that Eq. 16 is quite complicated, although it is not usually treated as such.

If a circular structure is placed at an angle to the waves, the vertical component of wave induced force can be treated in a similar manner, using v and  $\partial v/\partial t$  as well as u and  $\partial u/\partial t$ .

If strictly linear theory is used the total horizontal component of wave force acting on a vertical circular pile can be obtained by integrating  $F_h(S)$  dS from 0 to d. Very often in practice, one integrates  $F_h(S)$  from 0 to  $S_s$ , obtaining results which are somewhere between the results for linear wave theory and those for second order wave theory. A digital computer program for this operation is available for this purpose, as are graphs and tables of results (Cross, 1964; Cross and Wiegel, 1965). Much time and money have been spent in obtaining prototype and laboratory values of  $C_D$  and  $C_M$ . Most of the work has been done by private companies and is not available.\* Some data which are available for  $C_D$  are given in Fig. 1 (Wiegel, Beebe and Moon, 1957). It is evident that there is a considerable scatter of both  $C_D$ ; this is also true for the values of  $C_M$ . One of the main reasons for this is that the analysis of the data was based upon two simplifications: First, that linear theory could be used to reduce the basic data, and second, that each wave (and force) of a series of irregular waves could be analyzed as one of a series of uniform waves having the height and period of the individual wave in the record.

Agerschou and Edens (1966) reanalyzed the published data of Wiegel, Beebe and Moon (1957) and some unpublished data of Bretschneider, using both linear theory and Stokes Fifth Order theory. They concluded that for the range of variables covered, the fifth-order approach was not superior to the use of linear theory. They recommended for design purposes, if linear theory is used, that  $C_{\mathrm{D}}$  should be between 1.0 and 1.4, and that  $C_{\mathrm{M}}$  should be 2.0, these values being obtained for circular piles 6-5/8, 8-5/8, 12-3/4, 16 and 24 inches in diameter. (It should be noted here that the theoretical value of  ${\rm C}_{\rm M}$  for a circular cylinder in potential flow is 2.0.) Wilson (1965; see alsc,  $\overset{\text{M}}{\text{Wilson}}$  and Reid, 1963) report average values of C<sub>D</sub> = 1.0 and C<sub>M</sub> = 1.45 fo<sup>,</sup> a 30-inch diameter pile. At a recent conference, one design engineer stated he used values of  $C_{\rm D}$  ranging from 0.5 to 1.5 and  $C_{\rm M}$  from 1.3 to 2.0, depending upon his client (Design and Analysis of Offshore Drilling Structures: Continuing Education in Engineering Short Course, University of California, Berkeley, California, 16-21 September 1968). The results reported above were obtained either as values of  $\mathrm{C}_\mathrm{D}$  and  $\mathrm{C}_\mathrm{M}$  at that portion of a wave cycle for which  $\mathrm{F}_\mathrm{D}$  = max and  $F_{I} = 0$ , and vice-versa, or for the best average values of  $C_{D}$  and  $C_{M}$  throughout a wave cycle, assuming  $C_{D}$  and  $C_{M}$  to be constant. Both of these methods of obtaining and reporting the coefficients should be refined, as the coefficients are dependent upon each other, and are also time dependent as well as dependent upon the flow conditions.

In the significant wave approach, the significant wave height,  $\rm H_s$ , and significant wave period,  $\rm T_s$ , are substituted for H and T in the above equations, treating the significant wave as one of a train of waves of uniform height and period. In the design wave approach, the chosen values of H and T are used in a similar manner.

## \*\*One Dimensional Wave Spectra Approach

Recently there have been several papers published on the study of wave forces exerted on circular piles, using probability theory. In these studies it was assumed that the continuous spectrum of component waves could be superimposed linearly, that the process was both stationary and ergodic, and that the phase relationship among the component waves was Gaussian.

Some years ago the author obtained both the wave and force spectral densities for a pile installed at the end of the pier at Davenport, California, as shown in Fig. 2. It was not evident why the form of the two spectral densities should be so similar considering the fact that the product |u| u occurs in Eq. 16. Professor Leon E. Borgman (1956) studied this problem in detail and developed the following theory.

\*It appears that the results of a long term prototype study of wave forces on piles, by a consortium of oil companies, will be released at the Offshore Technology Conference, to be held in Houston, Texas, 19-21 May 1969. The basic wave force equation is Eq. 16, which may be expressed as a function of time as  $\$ 

$$F(t) = C_{1} |V(t)| V(t) + C_{2} A(t)$$
(18)

Here F(t) is the time history of the horizontal component of force per unit length of circular pile at an elevation S above the ocean floor, and

$$C_1 = \frac{1}{2} \rho_w C_D D \tag{19a}$$

$$C_2 = \rho_w C_M \pi D^2 / 4$$
 (19b)

The theoretical covariance function for F(t) using ensemble averaging with the Gaussian random wave model is

$$R_{FF}(\tau) = C_1^2 \sigma^4 G\left(R_{VV}(\tau)/\sigma^2\right) + C_2^2 R_{AA}(\tau)$$
(20)

where  $R_{VV}(\tau)$  and  $R_{AA}(\tau)$  are the covariance functions of the horizontal component water particle velocity, V(t), and local acceleration A(t) (i.e., u and  $\partial u/\partial t$ ), where

$$\sigma^{2} = 2 \int_{-\infty}^{\infty} S_{VV}(f) df$$
(21)

and

$$G(r) = \left[ \left( 2 + 4r^2 \right)^2 \operatorname{arc\,sin} r + 6r \sqrt{1 - r^2} \right] / \pi$$
 (22)

in which G(r) = G(R<sub>VV</sub>( $\tau$ )/ $\sigma^2$ ), and f is the frequency of the component wave (f = 1/T).

The covariance function  $R_{\rm VV}(\tau)$  and  $R_{\rm AA}(\tau)$  are calculated from the spectral densities  $S_{\rm VV}(f)$  and  $S_{\rm AA}(f)$  by use of the Fourier transforms

$$R_{VV}(\tau) = \int_{-\infty}^{\infty} S_{VV}(f) e^{i2\pi f\tau} df$$
(23a)

$$R_{AA}(\tau) = \int_{-\infty}^{\infty} S_{AA}(f) e^{i2\pi f\tau} df$$
(23b)

where

$$S_{VV}(f) = \frac{(2\pi f)^2 \cosh^2 2\pi S/L}{\sinh^2 2\pi d/L} S_{\eta\eta}(f) = T_V(f) S_{\eta\eta}(f)$$
 (24a)

$$S_{AA}(f) = \frac{(2\pi f)^4 \cosh^2 2\pi S/L}{\sinh^2 2\pi d/L} S_{\eta\eta}(f) = T_A(f) S_{\eta\eta}(f)$$
 (24b)

and

$$(2\pi f)^2 = \frac{2\pi g}{L} \tanh 2\pi d/L$$
 (25)

The functions  $T_V(f)$  and  $T_A(f)$  are called transfer functions. The fundamental quantity  $S_{\eta\eta}(f)$  is the spectral density of the water waves, and is obtained from the Fourier transform

$$S_{\eta\eta}(f) = \int_{\infty}^{\infty} R_{\eta\eta}(\tau) e^{-i2\pi f\tau} d\tau$$

in which  $R_{\eta\eta}(\tau)$  is the averaged lagged product of  $\eta(t)$  (i.e., average of  $\eta(t) \eta(t + \tau)$ ) where  $\eta(t)$  is the time history of the wave motion at the location of the pile (i.e.,  $\eta(t) = y_s(t)$ ).

Borgman found that Eq. 22 could be expressed in series form as

$$G(\mathbf{r}) = \frac{1}{\pi} \left( 8 \ \mathbf{r} + \frac{4\mathbf{r}^3}{3} + \frac{\mathbf{r}^5}{15} + \frac{\mathbf{r}^7}{70} + \frac{5\mathbf{r}^9}{1008} + \dots \right)$$
(27)

and that the series converges quite rapidly for  $0 \le r \le 1$ . He found that for r = 1, the first term  $G_1(r) = 8r/\pi$  differed from G(r) by only 15%, and that the cubic approximation  $G_3(r) = (8r + 4r^3/3)/\pi$  differed from G(r) by only 1.1%. Substituting the first term of the series into Eq. 20 results in

$$R_{FF}(\tau) = \frac{C_{1}^{2} \sigma^{4}}{\pi} \left( \frac{8 R_{FF}(\tau)}{\sigma^{2}} + \dots \right) + C_{2}^{2} R_{AA}(\tau)$$
(28)

The Fourier transform of this is:

$$S_{FF}(f) = \frac{C^2 \sigma^4}{\pi} \left( \frac{8 S_{VV}(f)}{\sigma^2} + \dots \right) + C_2^2 S_{AA}(f)$$
 (29)

which is the desired force spectral density.

Borgman made a numerical analysis of the situation shown in Fig. 2. The mumerical integration of S (f) gave  $\sigma^2 = 1.203 \text{ ft}^2/\sec^2$  and a least square fitting of the theoretical covariance of F(t) against the measured force covariance gave estimates of C<sub>D</sub> = 1.88 and C<sub>M</sub> = 1.73. The transfer functions T<sub>V</sub>(f) and T<sub>A</sub>(f) were calculated and plotted; it could be seen that T<sub>A</sub>(f) was nearly constant in the range of circular frequencies (2 $\pi$ /T) for which most of the wave energy was associated. The calculated and measured force spectral densities are shown in Fig. 3. The reason for the excellent fit is that for the conditions of the experiment T<sub>V</sub>(f) was nearly constant and the linear approximation to G(r), G<sub>1</sub>(r), was a reliable approximation.

Jen (1968) made a model study of the forces exerted by waves on a 6-inch diameter pile in the 200 ft. long by 8 ft. wide by 6 ft. deep wave tank at the University of California, Berkeley. In addition to using periodic waves, irregular waves were generated by a special wave generator using as an input the magnetic tape recording of waves measured in the ocean. The dimensions of the waves relative to the diameter of the pile were such that the forces were largely inertial. Jen found for the regular waves that  $C_M \approx 2.0$ , and using Borgman's method to analyze the results of the irregular waves tests found  $C_M \approx 2.1$  to 2.2. The reason for this close agreement between theory and measurement of  $C_M$  is probably due to the small value of H/D, which resulted in quasipotential flow (This will be discussed in a subsequent section).

Equation 29 permits the calculation of the force spectral density at a point. This is useful but the design engineer usually needs the total force on a pile, and the total moment about the bottom. In addition, the total force and the total moment on an entire structure is needed. These problems have been considered by Borgman (1966; 1967; 1968) and Foster (1968). In obtaining a solution to this problem, the integration of the force distribution is performed from the ocean bottom to the still water level as this is in

keeping with linear wave theory. There is no difficulty in obtaining the solution for the inertia force, but cross product terms appear in the solution for **\*** the drag force.<sup>\*</sup> Borgman made use of the linearization of G(r) by restricting it to the first term of the series given by Eq. 27 to obtain the approximate solution for the total force spectral density S<sub>00</sub>(f).

$$S_{QQ}(f) \approx S_{\eta\eta}(f) \left\{ \frac{8}{\pi} \left[ \frac{2\pi f C_1}{\sinh 2\pi d/L} \int_0^d \sigma(S) \cosh(2\pi S/L) dS \right]^2 + \left[ \frac{(2\pi f)^2 C_2}{\sinh 2\pi d/L} \int_0^d \cosh(2\pi S/L) dS \right]^2 \right\}$$
(30)

in which

$$\int_{0}^{d} \cosh(2\pi S/L) \, dS = \frac{\sinh 2\pi d/L}{2\pi/L}$$
(31)

The first integral in Eq. 30 cannot be prevaluated, but must be calculated for each sea-surface spectral density used.

The total moment about the bottom is

$$S_{MM}(f) \approx S_{\eta\eta}(f) \left\{ \frac{8}{\pi} \left[ \frac{2\pi f C_1}{\sinh 2\pi d/L} \int_0^d S \sigma(S) \cosh(2\pi S/L) dS \right]^2 + \left[ \frac{(2\pi f)^2 C_2}{\sinh 2\pi d/L} \int_0^d S \cosh(2\pi S/L) dS \right]^2 \right\}$$
(32)

in which

$$\int_{0}^{d} S \cosh(2\pi S/L) dS = \frac{1}{(2\pi/L)^{2}} \left[1 - \cosh 2\pi d/L + (2\pi d/L) \cosh 2\pi d/L\right]$$
(33)

As in the case of Eq. 30, the first integral cannot be prevaluated.

Borgman (1967; 1968) has found this linearization of the drag term to be the equivalent of using  $(V_{\rm rms} \sqrt{8/\pi}) V(t)$  in place of |V(t)| V(t) in Eq. 18; the physical reason for this is not clear, however. It should be pointed out here, that another linearization has been used by nearly every investigator in the past, with essentially no discussion; that is, the use of  $\partial u/\partial t$  rather than du/dt (see Eq. 17). Work is needed to determine the size of error introduced by this linearization compared with the size of the error introduced by the linearization of the drag term.

A relatively simple transfer function has been obtained by Borgman (1966; 1967) to calculate the total force and overturning moment the pile array of an offshore platform, and the reader is referred to the original work for information thereof.

\* A solution to this problem has been obtained by A. Malhotra and J. Penzien, University of California, Berkeley, California, and is to be published soon.

#### One Dimensional Wave Spectra

There have been a number of papers published on one dimensional wave spectra (see, for example, National Academy of Sciences, 1963), and a large number of measured wave spectra have been published (see, for example, Moskowitz, Pierson and Mehr, 1963). There are several possible ways of using actual spectra, one being a simulation technique (Borgman, 1968) for a large number of spectra, or a large number of wave time histories reconstituted from spectra. Another way to use spectra is to develop a "standard" set of spectra. There have been a number of such standards suggested. One of these has been given by Scott (1965), who re-examined the data of Darbyshire (1959) and Moskowitz, Pierson and Mehr (1963), and then recommended the following equation as being a better fit of the ocean data

$$S(\omega)/H_{s}^{2} = 0.214 \exp \left[\frac{(\omega - \omega_{o})^{2}}{0.065 \{(\omega - \omega_{o}) + 0.26\}}\right]^{2}$$
 (33a)

for 
$$-0.26 < (\omega - \omega_0) < 1.65$$
 (33b)

(33c)

where  $\omega = 2\pi f$  (in radians per second),  $\omega$  is the spectrum peak frequency, H<sub>s</sub> is the significant wave height (in feet), and the energy spectral density  $S(\omega)$  is defined by

**\*\*** 
$$S(\omega) = \frac{1}{\pi} S_{\eta\eta}(f)$$

It is also defined by

\*\* 
$$S(\omega) = \frac{1}{2} \sum_{i=1}^{\delta \omega} a_{i}^{2}/\delta \omega$$
 (34)

in which the summation is over the frequency interval  $\omega,~\omega$  +  $\delta\omega,$  and  $a_{i}$  is the amplitude of the i<sup>th</sup> component, with

$$y_{s} = \sum_{i=1}^{n} y_{i} = \sum_{i} a_{i} \cos(\omega_{i} t + \phi_{i})$$
(35)

in which  $\phi_i$  is the phase angle of the  $i\frac{th}{t}$  component. The factor  $\frac{1}{2}$  enters as  $\sum_{i=1}^{n} a_i^2/2$  is the mean value of  $y_s^2$  during the motion. The term  $a_i^2/\delta \omega$  is used, as the concept of  $a_i$  tends to lose physical significance (i.e.,  $a_i^2 \neq 0$ ) as  $n \neq \infty$ , whereas  $a_i^2/\delta \omega$  does not; hence the value of using the energy density as a function of frequency.

Scott also found, using linear regression, that

$$1/f_{0} = 0.19 H_{s} + 8.5$$
 (36a)

$$1/\omega_{0} = 0.03 \text{ H}_{2} + 1.35$$
 (36b)

$$T = 0.085 H + 7.1$$
 (36c)

where T is the average period (in seconds) of all waves in the record, and can be shown to be

$$T = 2\pi \left( \frac{m_0}{m_2} \right)^{\frac{1}{2}}$$
(37)

where

$$m_{k} = \int_{0}^{\infty} \omega^{k} S_{\eta\eta}(\omega) d\omega$$
 (38)

For k = 0, we have the "variance," m, and for a narrow (i.e., "Rayleigh" spectrum) we have

$$H_{s} = 4 m^{\frac{1}{2}}$$
(39)

Using quadratic regression, Scott found

$$f_{o} = (0.501/T) + (1.43/T^{2})$$
(40a)

$$\omega_{0} = (3.15/T) + (8.98/T^{2})$$
(40b)

It is of considerable importance to the engineering profession to develop means by which the spectral approach can be studied in the laboratory. In studying some of the problems, it is necessary to know the relationship between the one-dimensional spectra in the ocean and the spectra generated in a windwave tank (Plate and Nath, 1968). Comparison of a number of wave spectra measured in the ocean, in lakes and in wave tanks have been made by Hess, Hidy and Plate (1968). Their results, shown in Figure 4, are fully developed seas' wind-wave energy density spectra. The high frequency portion of the spectra all tend to lie close to a single curve, with energy density being approximately proportional to  $\omega^{-5}$  as predicted by the Phillips' equilibrium theory (see Wiegel and Cross, 1966, for a physical explanation of this). A close inspection of these data by Plate and Nath (1968) led them to conclude that the high frequency portion of the energy spectral density curve varies from the  $\omega^{-5}$  "law," being proportional to  $\omega^{-7}$  near the spectral peak, and being proportional to about  $\omega^{-4}$  in the highest frequency range of the spectra. It would appear from the one example of Wiegel and Cross (1966), Figure 5, in which they compared a normalized measured laboratory wind-wave energy density spectrum with one calculated by use of Miles' theory, together with other physical reasoning, that a theoretically sound basis exists for the development of a "standard" set of spectra.

The argument for the high frequency portion of the energy density spectra being proportional to  $\omega^{-5}$  is as follows (Wiegel and Cross, 1966). For a train of uniform periodic progressive waves, the maximum wave steepness is generally considered to be

$$\frac{H}{L} \approx \frac{1}{7} \tanh 2\pi d/L$$
 (41a)

which, for deep water, reduces to

$$\frac{H}{L} = \frac{H}{(g/2\pi) T^2} \approx \frac{1}{7}$$
(42b)

$$H^2 \approx \frac{4\pi^2 g^2}{49(2\pi f)^2}$$
 (43)

from which

$$H^2/\omega = H^2/2\pi f \approx \frac{4\pi^2 g^2}{49 (2\pi f)^5} = \frac{4\pi^2 g^2}{49 \omega^5}$$
 (44)

If the energy spectral density is proportional to  $(H/2)^2/\omega$ , then it must also be proportional to  $\omega^{-5}$ .

In order for the design engineer to use with confidence the work of the type proposed by Borgman, it would be desirable to measure  $S_{VV}(f)$  and  $S_{AA}(f)$  as a function of  $S_{\eta\eta}(f)$  in both the ocean and in the laboratory to see how reliable the linear transfer functions are for different sea states.

#### Directional Wave Spectra

Before directional spectra can be used in the design of structures in relatively deep water it is necessary to have measurements of such spectra, and to understand them sufficiently to be able to choose a "design" directional spectra. Two sets of measurements have been made in the ocean (Chase, et al., 1957; Longuet-Higgins, Cartwright and Smith, 1963), a few in a bay (Stevens, 1965) and a few in the laboratory (Mobarek, 1965; Mobarek and Wiegel, 1967; Fan, 1968).

Mobarek (1965) checked several methods that had been suggested for obtaining the directional spectra from an array of wave gages, and found none of them too reliable. However, making use of simulated inputs, he was able to choose the most reliable method and to devise correction factors. Some of his measurements are shown in Figure 6. Values in the ordinate are in terms of the wave energy, E, rather than the energy density, S<sub>nn</sub>(f). When normalized, his laboratory results were found to be similar to normalized values of the measurements made in the ocean by Longuet-Higgins, et al. (1963), as can be seen in Figure 7. At the suggestion of Professor Leon E. Borgman, Dr. Mobarek compared the circular normal probability function (the solid curve in Figure 7) with the normalized data and found the comparison to be excellent.

The probability density of the circular normal distribution function is \*\* given by (Gumbel, 1952 and Court, 1952):

$$P(\alpha, K) = \frac{1}{I_{o}(K)} \exp(K \cos \alpha)$$
(45)

where  $\alpha$  is the angle measured from the mean  $(\theta_m - \theta)$ , K is a measure of the concentration about the mean, and I<sub>0</sub>(K) involves an incomplete Bessel function of the first kind of zero order for an imaginary argument. The larger K, the greater the concentration of energy; it is analogous to the reciprocal of the standard deviation of the linear normal distribution.

It has been found that much useful information on directional spectra can be obtained from the outputs of two wave recorders, through use of the co-spectra and quadrature spectra to calculate the linear coherence and the mean wave direction (Munk, Miller, Snodgrass and Barber, 1963; Snodgrass, Groves, Hasselman,

)

Miller, Munk and Powers, 1966). It appeared to the author that if the directional spectra were represented by the circular normal distribution function it should be possible to obtain the necessary statistical parameters in a similar manner. It was believed that such a simplified approach could provide data of sufficient accuracy for many practical purposes. As a result of discussions with Professor Leon Borgman, a theory was developed by Borgman (1967) to do this, and tables were calculated to provide a practical means to obtain the required information.

Borgman (1967) used a slightly different representation of the directional spectra

$$S_{\eta\eta 2}(f, \alpha) = S_{\eta\eta 1}(f) \exp[-K \cos(\theta - \theta_m)]/2\pi I_o(K)$$
(46)

where the  $2\pi$  in the denominator indicates an area under the curve of  $2\pi$  rather than unity, f is the component wave frequency in cycles sec, and  $S_{\eta\eta1}(f)$  is the one-dimensional spectral density. The estimation of the parameters  $S_{\eta\eta1}(f)$ , K(f) and  $\theta_m(f)$  is achieved by cross-spectral analysis based on a sea surface record at two locations.  $S_{\eta\eta1}(f)$  and the co- and quadrature spectral densities for the two recordings are computed by the usual time series procedures. The theoretical relations between measured and unknown quantities is

$$\frac{C(f)}{S_{\eta\eta_1}(f)} = \int_{0}^{2\pi} \frac{\exp[K \cos(\theta - \theta_m)]}{2\pi I_0(K)} \cos[k\overline{D} \cos(\theta - \overline{\beta})] d\theta$$
(47)

$$\frac{Q(f)}{S_{\eta\eta1}(f)} + \int_{0}^{2\pi} \frac{\exp[K\cos(\theta - \theta_{m})]}{2\pi I_{0}(K)} \sin[k\overline{D}\cos(\theta - \overline{\beta})] d\theta$$
(48)

where  $\overline{D}$  is the distance between the pair of recorders, k is the wave number  $(2\pi/L)$  and  $\overline{\beta}$  is the direction from wave recorder #1 to wave recorder #2. For a given frequency, all quantities are known except  $\theta_m$  and K. Hence these two equations represent two nonlinear equations with two unknowns. Borgman has prepared tables which enable one to solve for  $\theta_m$  and K, given  $C(f)/S_{\eta\eta 1}(f)$  and  $Q(f)/S_{\eta\eta 1}(f)$ . Two solutions, symmetric about the direction between the pair of recorders result. This ambiguity may be eliminated by using three wave gages instead of two, or in many applications using other information regarding the main direction of the directional spectra. The relationship between the parameter K and the directional width of the spectrum can be seen in Figure 8.

Using simulation techniques devised by Professor Leon Borgman, Dr. Fan (1968), continuing the work of Mobarek, made an extensive study of the effects of different lengths of data, lag numbers, wave recorder spacings, filters, and different samples on the calculation of directional spectra, using several methods, using a known circular normal distribution input. An example of the effect of gage spacings, relative to the component wave length, on the estimates can be seen in Figure 9. He then used the "best" combination to obtain the directional spectra of waves generated in a model basin by wind blowing over the water surface. As a result of this study it appears that, for the case of waves being generated in a nearly stationary single storm, the directional spectra can be two parameters and should be tested for use in the design of an offshore structure.

The results were sufficiently good to encourage Borgman and Suzuki to develop a new method for obtaining useful information on directional spectra by measuring the time histories of the x and y components of wave induced force on a sphere mounted a few feet above the ocean bottom, together with the wave pressure time history at the sphere. The results of this work (Suzuki, 1968) indicated that a practical method is available to the engineer for measuring the approximate directional spectra of ocean waves.

## NON-LINEAR PROBLEMS

There are several types of non-linearities involved in the problem of wave induced forces on offshore structures. One, which is due to the term |u| u of Eq. 16, is important in the wave spectra approach; a method of overcoming the handicap has been described in a previous section. A second enters through the term du/dt in Eq. 16, which has been linearized through the use of  $\partial u/\partial t$  in place of du/dt. A third non-linearity enters through the generation of eddies, and will be discussed subsequently.

The most commonly considered non-linearity is associated with non-linear \*\* wave theories. Two of these are the Stokes and the Cnoidal wave theories (see, for example, Wiegel, 1964). The first is best used for relatively deep water, and the second is best used for relatively shallow water. No attempt will be made to describe these theories in detail herein; rather a few equations will be given to indicate the general nature of the difference between these theories and the linear theory.

To the third order, the Stokes (Stokes, 1880; Skjelbreia, 1959) wave profile is given by

$$\frac{y_s}{L} = A_1 \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) + A_2 \cos 4\pi \left(\frac{x}{L} - \frac{t}{T}\right) + A_3 \cos 6\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$
(49)

where the coefficients  ${\rm A}^{}_1,~{\rm A}^{}_2$  and  ${\rm A}^{}_3$  are related to the wave height by

$$H/d = (L/d) [2A_1 + 2\pi^2 A_1^3 F_3 (d/L)]$$
(50)

where

$$A_2 = A_1^2 \cdot f_2 (d/L), \quad A_3 = \pi^2 A_1^3 \cdot f_3 (d/L)$$
 (51)

with  $f_2$  (d/L) and  $f_3$  (d/L) being functions of d/L.

The waves have steeper crests and flatter troughs than linear waves, and there is a mass transport of water in the direction of wave advance. The equations for water particle velocities and accelerations will not be presented herein as extensive tables of functions are needed for their use (or the availability of a high speed digital computer).

When the wave length becomes quite long comapred with the water depth, about L/d > 10 (the value depending upon H/d as well), the Cnoidal wave theory is perhaps a better approximation than is the theory of Stokes waves. The theory was originally derived by Korteweg and de Vries (1895). To the first approximation the wave profile is given by S<sub>c</sub>, measured from the ocean bottom

$$S_{s} = S_{t} + H cn^{2} [2 K(k) (x/L - t/T), k]$$
(52)

where cn is the "cnoidal" Jacobian elliptical function and K(k) is the complete elliptic integral of the first kind of modulus k,  $S_t$  is the elevation of the wave trough above the bottom, and is given by

$$\frac{S_{t}}{H} - \frac{d}{H} + 1 = \frac{16 d^{3}}{3L^{2} H} \{K(k) [K(k) - E(k)]\}$$
(53)

where E(k) is the complete elliptic integral of the second kind of modulus k. The wave length is

$$L = \sqrt{\frac{16 d^3}{3H}} \cdot kK(k)$$
(54)

and the period is related to the modulus k through

1

$$T\sqrt{\frac{g}{d}} = \sqrt{\frac{16d}{3H}} \left\{ \frac{kK(k)}{\sqrt{1 + \frac{H}{d} \left[ -1 + \frac{1}{k^2} \left[ 2 - 3 \frac{E(k)}{K(k)} \right] \right]}} \right\}$$
(55)

The equation for water particle velocities and acceleration and graphs which permit the use of the Cnoidal wave theory have been prepared by Wiegel (1964; see Masch and Wiegel, 1961 for tables of functions).

Professor Robert Dean (1968) has made analytical studies of the wave profiles predicted by these and other theories, including his "stream function wave theory," in order to determine the probable useful ranges of the theories. His results are shown in Figures 10 and 11.

It is necessary to be able to calculate the height of the wave crest above the water surface in order to determine the deck height on an offshore platform, and the work of Dean cited above is useful for this purpose. It is also important to be able to estimate the regions of reliability of the several theories in the prediction of water particle velocities and accelerations. Dr. Bernard Le Méhauté and his co-workers (Le Méhauté, Divoky and Lin, 1968) have made careful laboratory studies of the water particle velocities of "shallow water waves" for several values of H, T, and d and compared their measurements with predictions made using a number of linear and non-linear wave theories. An example of their results is shown in Fig. 12. They concluded, that while no theory was found to be exceptionally accurate, the Cnoidal wave theory of Keulegan and Patterson appeared to be most adequate for the range of wave parameters and water depths studied. It appears that much more work of this type is needed.

The water particle velocities and accelerations given by the most valid non-linear theory are used in Eq. 16 to calculate the force on a pile. These velocities and accelerations are usually calculated for the so-called "design wave," which is usually the wave considered by design engineers to be the largest wave the structure might encounter during its useful life.

Another reason for the variability of the data is associated with the wake. The formation of eddies in the lee of a circular cylinder in uniform steady flow has been studied by a number of persons. It has been found that the relation-ship among the frequency (cycles per second) of the eddies,  $f_e$ , the diameter of the cylinder, D, and the flow velocity, V, is given by the Strouhal number,  $N_e$ ,

$$N_{s} \left( 1 - \frac{19.7}{N_{R}} \right) = \frac{f_{e}}{V} \approx N_{s}$$
(56)

where  $N_R$  is the Reynolds number. Except in the range of laminar flow, the Reynolds number effect can be neglected. For flow in the sub-critical range

 $(N_R < about 2.0 \times 10^5)$ ,  $N_S \approx 0.2$ . For  $N_R > 2.0 \times 10^5$ , there appears to be a considerable variation of  $N_S$ ; in fact, it is most likely that a spectrum of eddy frequencies exists (see Wiegel, 1964, p. 268 for a discussion of this). The most extensive data on  $N_S$  at very high Reynolds numbers, as well as data on  $C_D$  and the pressure distribution around a circular cylinder with its axis oriented normal to a steady flow, has been given by Rosko (1961), some of which are shown in Figure 13.

What is the significance of N<sub>s</sub> for the type of oscillating flow that exists in wave motion? Consider the horizontal component of water particle velocity as given by Eq. 4. For deep water, the equation is approximately

$$u = (\pi H/T) \cos 2\pi t/T$$
(57)

at x - 0. Then, using an average of u to represent V; i.e.,

$$V_{w} \approx u_{avg} \approx \pi H/2T$$
 (58)

where  $V_w$  is the "average" horizontal component of water particle velocity due to a train of waves of height H and period T. For at least one eddy to have time to form it is necessary for

$$T > 1/f_e \approx 2DT/\pi H N_s$$
 (59a)

And, if  $N_{c} \approx 0.2$ 

 $H > 10 D/\pi$ 

Keulegan and Carpenter (1958) studied both experimentally and theoretically the problem of the forces exerted on bodies in an oscillating flow. The oscillations were of the standing wave type in which the wave length was long compared with the water depth so that the horizontal component of water particle velocity was nearly uniform from top to bottom. Furthermore, the body was placed with its center in the node of the standing wave. They found that  $C_M$  and  $C_D$  depended upon the number  $u_{max}$  T/D where  $u = u_{max} \cos 2\pi t/T$ . They observed that when  $u_{max}$  T/D was relatively small, no eddy formed, that a single eddy formed when  $u_{max}$  T/D was about 15, and that numerous eddies formed for large values of the parameter. It is useful to note that this leads to a conclusion similar to Eq. 59. For example, if one used the deep water wave equation for  $u_{max} = \pi H/T$ , then

$$u_{max} T/D > \pi H/D > 15$$

and

 $H > 15D/\pi$ 

It appears from the work described above that a high Reynolds number oscillating flow can exist which is quite different from high Reynolds number rectilinear flow unless the wave heights are much larger than the diameter of the circular cylinder. It would appear that the Keulegan-Carpenter number is of greater significance in correlating  $C_D$  and  $C_M$  with flow conditions than is Reynolds number (Wiegel, 1964, p. 259), and that the ratio H/D should be held constant to correlate model and prototype results, or at least should be the appropriate value to indicate the prototype and model flows are in the same "eddy regime" (see Paape and Breusers, 1967, for similar results for a cylinder oscillating in water).

(61)

(60)

(59Ъ)

When the Keulegan-Patterson number is large enough that eddies form, an oscillating "lift" force will occur with a frequency twice that of the wave frequency. For a vertical pile the "lift" force will be in the horizontal plane normal to the direction of the drag force. Essentially no information has been published on the coefficient  $C_L$  for water wave type of flow. In uniform rectilinear flows it has about the same numerical value as  $C_D$ .

Photographs taken of flow starting from rest, in the vicinity of a circular cylinder for the simpler case of a non-reversing flow, show that it takes time for separation to occur and eddies to form. The effect of time on the flow, and hence on  $C_D$  and  $C_M$  has been studied by Sarpkaya and Garrison (1963; see also Sarpkaya, 1963). A theory was developed which was used as a guide in analyzing laboratory data taken of the uniform acceleration of a circular cylinder in one direction. Figure 14 shows the relationship they found between  $C_D$  and  $C_M$  was found which was dependent upon  $\ell/d$ , where  $\ell$  is the distance traveled by the cylinder from its rest position and D is the cylinder diameter. They indicated the "steady state" (i.e., for large value of  $\ell/D$ ) values of  $C_D = 1.2$  and  $C_M = 1.3$ .

The results shown in Figure 14 are different than those found by McNown and Keulegan (1959) for the relationship between  $C_D$  and  $C_M$  in oscillatory flow, Figure 15. They measured the horizontal force exerted on a horizontal circular cylinder placed in a standing water wave, with the cylinder being parallel to the bottom, far from both the free surface and the bottom, and with the axis of the cylinder normal to the direction of motion of the water particles. The axis of the cylinder was placed at the node of the standing wave so that the water particle motion was only horizontal (in the absence of the cylinder). Their results are shown in Figure 15. Here, T is the wave period and  $T_e$  is the period of a pair of eddies shedding in steady flow at a velocity characteristic of the unsteady flow. In their figure, the characteristic velocity was taken as the maximum velocity. They found that if  $T/T_e$  was 0.1 or less, separation and eddy formation were relatively unimportant, with the inertial effects being approximately those for the classical unseparated flow, and if  $T/T_e$  was greater than 10, the motion was quasi-steady.

#### WAVE CLIMATES

In preparing feasibility studies, in designing, in constructing and in operating coastal and offshore structures and facilities it is necessary to have reliable information on surface water waves. These structures and facilities include harbors, pipelines on the bottom, offshore oil structures and drilling vessels, buoys for use in mooring tankers, dredging for offshore mineral recovery, lightering craft and equipment, and waste disposal systems.

It is necessary to have information on the "wave climate" in the area of interest for the planning and design phases, and synoptic wave data for the construction and operation phases. Traditionally, the wave climate has been represented by "wave roses" or tables which have been obtained from visual observations, from wave recorders, or from hindcasts from weather maps. It would be of much greater benefit to the engineer to have wave data in the form of cumulative distribution functions in order to be able to make an economic design based upon the numerical probability of occurrence. In addition, it would also be better to have wave data in another form for use in planning construction and other operations; in the form of continuous observations, measurements, or hindcasts so that the statistical properties could be determined of the number of consecutive days the waves will be less than, or greater than, some safe or economic combination of height, period and direction.

As an example, the cumulative significant wave height distribution functions for swell and sea were constructed for one location in the Pacific Ocean, using information obtained from a wave hindcasting study which was made using a three year series of weather maps (Figure 16). The distribution functions are not too useful as both swell and seas must have occurred simultaneously on a number of days; the data were not reported in a manner that permitted the recovery of this information. The few data that are available on the ability of several types of floating structures to perform their functions in waves are given in Tables 1 and 2. These data are not too useful as the capability of a floating structure to work in waves depends upon the wave period, winds, currents and the crew as well as upon the wave height. However, in the absence of other data, these data must be used. Consider either a seaworthy suction hopper dredge, with a flexible suction tube, or a seaworthy tin dredge; an average workable wave height might be taken to be about 5 feet. This limitation on wave height, together with the significant wave height distribution functions given in Figure 16 indicate that these two types of dredges would not be usable for 24% of the time owing to swell and 18% of the time owing to seas that were too high. If one assumes that half of the time the seas were too high occurred simultaneously with swell that were too high, one would estimate that the site was "unworkable" about one-third of the time. Considering the time necessary to get a dredge from the work site to a harbor of refuge and back again the site would be "unworkable" for considerably more than one-third of the time

Similar data are necessary for the safer and more economic use of the oceans for transportation. These data are needed for improved ship design and routings, as well as for improved terminal facilities. In this regard, it should be emphasized that the design of unique ships or shipping techniques interacts with the harbors and offshore facilities of many countries. In a UNESCO report ("Marine Science and Technology: Surveys and Proposals," Report of the Secretary General, E/4487, 24 April 1968) it was pointed out that in 1966, alone, 112 ships larger than 1,000 gross tons were lost.

At the present time there are very few places in the world for which we have sufficient, or even barely adequate wave data. This is especially true of the little-traveled portions of the open oceans.

It is recognized that considerable advances have been made, and are continuing to be made, in our understanding of the basic phenomenon of the generation of waves by winds, and in the development and use of computer programs to calculate wave fields from meteorological inputs. It would be desirable if the programs could be developed in such a way, if this is not already the case, that the required wave data could be recalled at a reasonable cost for any geographical location for which the data became necessary.

Associated with the problems of transforming meteorological data to wave data are the phenomena of wave scattering, dispersion, energy dissipation, refraction, reflection, and diffraction. It is necessary to make reliable measurements of wave characteristics on an ocean-wide basis to obtain the data needed by the engineer to perform his job properly, and by the geophysicist to test and improve his theories. To be useful to the engineers the measurements should be made for a long period of time. Measurements should be made in the open ocean and along the coasts. It would be desirable, for use by both the engineer and the geophysicist, if directional spectra could be measured. At the other extreme it would still be useful to obtain consistent visual observations, especially in areas for which few measurements have been made (i.e., most areas). Much valuable data could be obtained from a study of newspapers, technical publications, harbor logs, etc., by investigators in each country. These data would be of greater value if damage resulting from wave and wind action could be summarized with the wave data.

Two international standards should be used for data reduction, one simplified and one rather sophisticated. It would appear that the standards proposed by L. Draper ("The Analysis and Presentation of Wave Data - A Plea for Uniformity," Proceedings of the Tenth Conference on Coastal Engineering, ASCE, pp. 1-11, 1967) should be used.

## Table 1. WAVE HEIGHTS LIMITING OPERATIONS OF DREDGES AND BARGES (after Santema, 1955)

\_

Equipment and kind of work		Limiting wave height (feet)	Santema, P., About the estimation of the number of days with favorable meteorological and oceanographical
1.	Dredging with a. Seaworthy suction hopper dredge, with rigid suction tube and cutters Seaworthy suction hopper dredge, with flexible	2-3	conditions for engineering operations on the sea coast and in estuaries,
	suction tube c. Suction dredges of the nonpropelled, low pon-	4 <u>7</u> -3	Proc. Fifth Conf. Coastal Eng., Berkeley, Calif.: The Engineering
G	toon type, rigid suction tube I. Bucket dredge nonpropelled, low pontoon type, hard bottom	11	1955, pp. 405-410.
2. I	e. Seaworthy tin dredges Mooring barges alongside a dredge with barge lischarge, or alongside a barge-unloading dredge	41-6 11-21	Glenn, A. H., Progress report on
3. I	Dumping stones, sand, or clay with dump barge with bottom doors (up to 400 tons)	11-3	and hurricane problems in coastal operations $Oil Gas J$ , 49
4. 1	Dumping stones and clay with self-tipping barges up to 600 tons)	11-21	7 (June 22, 1950), 174-77.
5. ] 6. I f	Fransport and sinking fascine mattresses Pumping stones in layers on fascine mattresses rom barges	$1-1\frac{1}{2}$ $1\frac{1}{2}-2\frac{1}{2}$	

# Table 2. Generalized Performance Data for Marine Operations (after Glenn, 1950)

1

	Wave heights <sup>a</sup> (feet) for		
Type of operation	Safe, efficient operation	Marginal operation	Dangerous and/or inefficient operation
Deep sea tug			
Handling oil and water barge	0-2	2-4	>4
Towing oil and water barge	0-4	4-6	>6
Handling derrick barge	0-2	2-3	>3
Handling and towing LST-type vessel	0-3	3-5	>5
Crew boats, 60-90 ft in length			
Underway	0-8	8-15	>15
Loading or unloading crews at platform	0-3	3-5	>5
Supervisor's boats, fast craft, 30-50 ft in length			
Underway at cruising speed	-2	2-4	>4
Loading or unloading personnel at platform or floating equipment	0-2	2-4	>4
LCT-type vessel and cargo luggers			
Underway	0-4	4-5	>5
Loading or unloading at platform	0-3	3-4	>4
Loading or unloading at floating equipment	0-4	4-5	>5
Buoy laying (using small derrick barge)	0-2	2-3	>3
Platform building			
Using ship-mounted derrick	0-4	4-6	>6
Using large derrick barge	0-3	3-5	>5
Pipeline construction	0-3	3-4	>4
Gravity-meter exploration using surface vessel (limiting conditions caused by instrument becoming noisy)	0-4	4-6	>6
Seismograph exploration using craft under 100 ft in length	06	68	>8
Large amphibious aircraft (PBY)			
Sea landings and take-offs	0-1.5	1.5-3	>3
Boat-to-plane transfer operations in water	0-1	1-2	>2
Small amphibious aircraft	0-1	1-2	>2

<sup>a</sup> Wave heights used are those of the average maximum waves. Height limits given are not rigid and will vary to some extent with locality, local wind conditions, experience of personnel, etc.



Coefficient of Drag for Circular Cylindrical Piles of Various Diameters (from Wiegel, Beebe and Moon, 1957)



Figure 2. A comparison of force and sea-surface spectral densities for roll 10, Davenport data. (From Borgmon, 1966)



Figure 3. A comparison of the measured and computed force spectral density for roll 10, Davenport data.

(From Borgman, 1966)





Frequency, Cycles per Second

COMPARISON OF SMOOTHED AND UNSMOOTHED WAVE POWER SPECTRA WITH SHAPE OF THEORETICAL SPECTRUM (from Wiegel and Cross, 1966)

FIGURE 5

H-9426-A





![](_page_26_Figure_0.jpeg)

 $\theta_u$  = the angle in the first quadrant at which the energy level is  $I_2$  the peak energy

FIG. 7 NORMALIZED PLOT OF DIRECTIONAL SPECTRA (FROM MOBAREK, 1965)

![](_page_27_Figure_0.jpeg)

![](_page_27_Figure_1.jpeg)

![](_page_28_Figure_0.jpeg)

FIG. 9 COMPARISON OF DIRECTIONAL SPECTRAL ESTIMATES FOR VARIOUS GAGE SPACINGS (from Fan, 1968)

![](_page_29_Figure_0.jpeg)

![](_page_30_Figure_0.jpeg)

![](_page_31_Figure_0.jpeg)

Figure 12 Horizontal Particle Velocity under the Crest - NEAR-BREAKING WAVE (FROM LÉ MEHAUTÉ, DIVOKY AND LIN, 1968)

![](_page_32_Figure_0.jpeg)

![](_page_32_Figure_1.jpeg)

FIG. 13 STROUHAL NUMBER, DRAG COEFFICIENT AND PRESSURE COEFFICIENT AT HIGH REYNOLDS NUMBER FOR A CIRCULAR CYLINDER.

(From Roshko 1961)

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![](_page_33_Figure_0.jpeg)

FIG. 14 CORRELATION OF DRAG AND INERTIA COEFFICIENTS (From Sarpkaya and Garrison, 1963)

![](_page_33_Figure_2.jpeg)

![](_page_33_Figure_3.jpeg)

IG.15 INTER-RELATIONSHIP BETWEEN COEFFICIENTS OF COEFFICIENTS OF DRAG AND OF VIRTUAL MASS FOR (a) FLAT PLATES AND (b) CIRCULAR CYLINDERS (From Mc Nown and Keulegan, 1959)

![](_page_34_Figure_0.jpeg)

![](_page_34_Figure_1.jpeg)

a <sub>i</sub>	=	Amplitude of wave component, feet
А	=	Projected area, feet <sup>2</sup>
Ao		Cross sectional area of cylinder, feet <sup>2</sup>
A <sub>1</sub> , A <sub>2</sub> , A	·3 =	Coefficients in Stokes' third order wave theory, dimensionless
В	=	Volume of submerged body, feet <sup>3</sup>
cn	=	Jacobian cnoidal elliptical function, dimensionless
c <sub>1</sub>	=	$\frac{1}{2} \rho_{W} C_{D}$ D; pound-second <sup>2</sup> /feet <sup>3</sup>
c <sub>2</sub>	=	$\rho_{\rm w} C_{\rm m} \pi D^2/4$ , pound-second <sup>2</sup> /foot <sup>2</sup>
C(f)	=	Co-spectra, feet <sup>2</sup> -second
C <sub>a</sub>	union insta	Coefficient of added mass, dimensionless
C <sub>D</sub>		Coefficient of drag, dimensionless
C <sub>m</sub>	-	Coefficient of mass, dimensionless
d	=	Water depth, with no waves present, feet
D	=	Diameter of pile, feet
D	=	Distance between a pair of wave recorders, feet
dp/dx	=	Pressure gradient, pounds/foot <sup>3</sup>
du/dt	-225	Horizontal component of water particle total acceleration, feet/second <sup>2</sup>
dv/dt	<u></u>	Total acceleration, feet/second <sup>2</sup>
E(k)	-	Complete elliptic integral of the second kind of modulus k, dimensionless
f	=	Wave frequency, 1/T, cycles/second
f <sub>2</sub> , f <sub>3</sub>	=	Functions in Stokes' third order wave theory, dimensionless
fe	=	Eddy frequency, cycles/second
ff	=	Acceleration of fluid in general case of unsteady flow, feet/second <sup>2</sup>
fo	=	Wave component frequence for which spectral density peak value occurs, l/second
F		$F_{D} + F_{I}$ , total force, pounds

F(t) = Horizontal component of force, per unit length of pile, statistical theory, pounds/foot

F <sub>D</sub>	=	Drag force, pounds
F <sub>h</sub> (s)		Horizontal component of force on a vertical pile, pounds
FI	=	Inertia force, pounds
F <sub>Ih</sub>		Horizontal component of inertia force, pounds
Fp		Integrated pressure force, pounds
g	=	Acceleration of gravity, feet/second <sup>2</sup>
G(r)	=	Defined by Equation 22, dimensionless
G <sub>1</sub> (r)	=	First term of series representing G(r), dimensionless
Н	=	Wave height, feet
H <sub>d</sub>		Design wave period, seconds
Hs		Significant wave period, seconds
I <sub>o</sub> (k)	-	Incomplete Bessel function of the first kind of zero order for an imaginary argument, dimensionless
J <sub>1</sub>	=	Bessel function of the first kind, dimensionless
k	=	integer, dimensionless
K	=	Measure of concentration about the mean, circular normal distribution function, dimensionless
K(k)		Complete elliptic integral of the first kind of modules k, dimensionless
L	=	Wave length, feet
1	=	Distance traveled by cylinder from position of rest, feet.
m k	=	Statistical moment, defined by Equation 38, feet $^2-\omega^k$
m O	10000	$m_k$ for k = 0, variance of wave surface time history, feet <sup>2</sup>
Ma		Added mass, slugs
Mc		Mass per unit length of cylinder, slugs/foot
Mo		Mass of a submerged body, slugs
N s	==	Strouhal number, dimensionless
р	-	Pressure due to wave, at some point x, y, z in the water, pounds/feet <sup>2</sup>

Ρ( α,Κ)	=	= Circular normal distribution function, dimensionless
Q(f)	=	= Quadrature spectra, feet <sup>2</sup> -second
r		= In G(r), $r = R_{vv}(\tau)/\sigma^2$ , dimensionless
$R_{AA}(\tau)$	=	Covariance function of the horizontal component of water particle acceleration, feet <sup>2</sup> /second <sup>4</sup>
R <sub>FF</sub> (τ)		Covariance function of the horizontal component of force, pounds <sup>2</sup> /foot <sup>2</sup>
R <sub>VV</sub> (τ)	=	Covariance function of the horizontal component of water particle velocity, feet <sup>2</sup> /second <sup>2</sup>
R <sub>ηη</sub> (τ)	=	Covariance function of wave surface, feet <sup>2</sup>
S		Vertical coordinate, measured from the ocean bottom (i.e., $S = 0$ at ocean bottom), feet
S <sub>AA</sub> (f)	=	Wave water particle horizontal component of water particle local acceleration spectral density, feet <sup>2</sup> /second <sup>3</sup>
S <sub>FF</sub> (f)	=	Wave force per unit length of pile (horizontal component) spectral density, pounds <sup>2</sup> -second/feet <sup>2</sup>
S <sub>MM</sub> (f)	-	Total wave induced moment about pile bottom (horizontal component) spectral density, foot <sup>2</sup> -pounds <sup>2</sup> -second
S <sub>QQ</sub> (f)	=	Total wave force (horizontal component) spectral density, pounds <sup>2</sup> -second
S s		Vertical distance from the ocean bottom to the water surface, feet
St	=	Elevation of wave trough above ocean bottom, feet
S <sub>VV</sub> (f)		Wave water particle horizontal component of velocity, feet <sup>2</sup> / second
s <sub>nn</sub> (f)	-	Wave surface spectral density, feet <sup>2</sup> -second
s <sub>nn 1</sub>	=	One dimensional wave energy spectral density in directional spectra theory, feet <sup>2</sup> -second
s <sub>nn2</sub>	=	Directional wave energy spectral density, feet <sup>2</sup> -second/radian
s(ω)	=	Wave spectral density, $S_{\eta\eta}(f)/\pi$ , feet <sup>2</sup> -second
t	=	time, seconds
Т	=	Wave period, seconds

$T_A$	= Transfer function, Equation 24b, 1/second <sup>4</sup>
Td	= Design wave period, seconds
Т <sub>е</sub>	= Eddy period, second
T <sub>s</sub>	= Significant wave period, seconds
т <sub>v</sub>	= Transfer function, Equation 24a, 1/second <sup>2</sup>
U	= Horizontal component of water particle velocity, feet/second
V	= Velocity, feet/second
Vrms	= Root mean square value of V, feet/second
V <sub>w</sub>	= An "average" horizontal component of water particle velocity, feet/second
x	= Horizontal coordinate, $x = 0$ at wave crest, feet
y <sub>s</sub>	= Vertical distance from the undisturbed water surface to the water surface when waves are present, feet
Y <sub>1</sub>	= Bessel function of the second kind, dimensionless
α	= Angle measured from the mean, degrees
β	= A phase angle, diffraction theory for wave force, radians
β	= Direction between a pair of wave recorders, degrees
δω	<pre>= Circular frequency interval, cycles/second</pre>
θ	= Horizontal angle in directional spectra, degrees
$\theta_{m}$	= Direction angle of peak value of directional wave energy spectral density, degrees
ρ	= Mass density of a fluid slugs/foot <sup>3</sup>
ρ <sub>w</sub>	= Mass density of water, slugs/foot <sup>3</sup>
$\sigma^2$	= $2 \int_{-\infty}^{\infty} S_{VV}(f) df$ , feet <sup>2</sup> /second <sup>2</sup>
τ	= Lag in covariance function, seconds
ω	= Circular frequency, $2 \pi / T$ , radians/second
ω	= Circular frequency at which peak value occurs in wave spectrum, radians/second
∂u/∂t	Horizontal component of water particle local acceleration, 2 feet/second

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